# Computer Algebra
# Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

## 7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/187-7.1.4-f-x-^m-d+e-x^2-^p-a+b-arcsinh-c-x-^n

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# Contents

CHAPTER 1

# Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 663 ]. This is test number [ 187 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 99.40 ( 659 ) | 0.60 ( 4 ) |
| Mathematica | 98.94 ( 656 ) | 1.06 ( 7 ) |
| Maple | 76.32 ( 506 ) | 23.68 ( 157 ) |
| Maxima | 38.46 ( 255 ) | 61.54 ( 408 ) |
| Fricas | 36.35 ( 241 ) | 63.65 ( 422 ) |
| Sympy | 30.02 ( 199 ) | 69.98 ( 464 ) |
| Mupad | 20.06 ( 133 ) | 79.94 ( 530 ) |
| Giac | 14.33 ( 95 ) | 85.67 ( 568 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

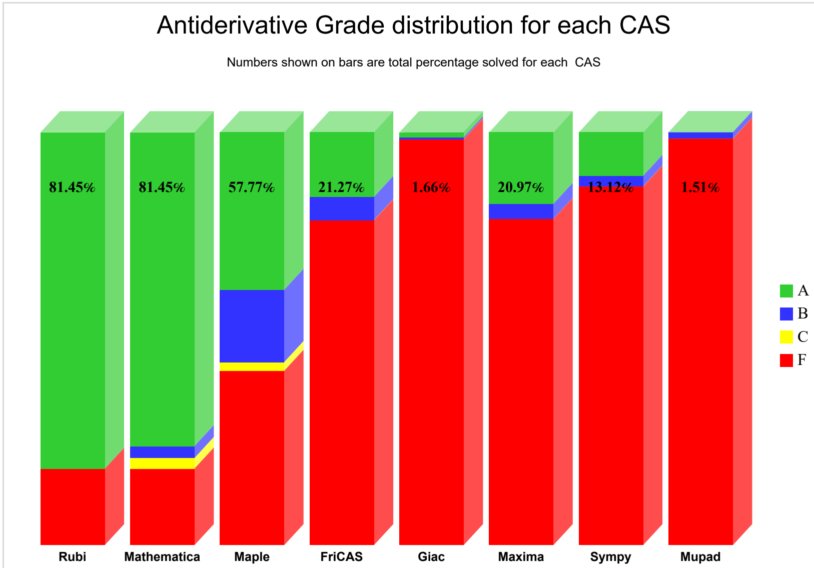
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

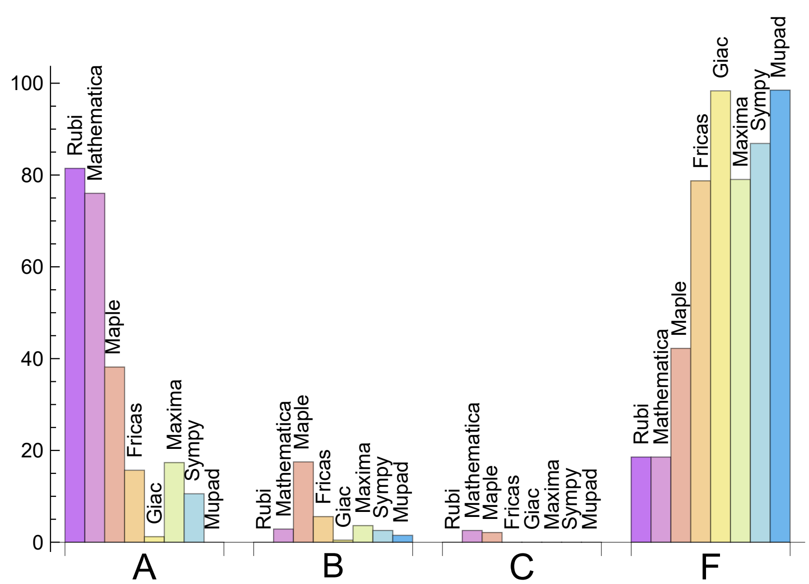
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Mathematica | 76.018 | 2.866 | 2.564 | 18.552 |
| Rubi | 62.745 | 0.151 | 17.949 | 19.155 |
| Maple | 38.160 | 17.496 | 2.112 | 42.232 |
| Maxima | 17.345 | 3.620 | 0.000 | 79.035 |
| Fricas | 15.686 | 5.581 | 0.000 | 78.733 |
| Sympy | 10.558 | 2.564 | 0.000 | 86.878 |
| Giac | 1.207 | 0.452 | 0.000 | 98.341 |
| Mupad | 0.000 | 1.508 | 0.000 | 98.492 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Mathematica | 7 | 0.00 | 100.00 | 0.00 |
| Rubi | 4 | 100.00 | 0.00 | 0.00 |
| Maple | 157 | 100.00 | 0.00 | 0.00 |
| Fricas | 422 | 84.83 | 0.00 | 15.17 |
| Maxima | 408 | 78.19 | 1.96 | 19.85 |
| Sympy | 464 | 86.85 | 12.93 | 0.22 |
| Giac | 568 | 48.06 | 0.35 | 51.58 |
| Mupad | 530 | 0.00 | 100.00 | 0.00 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Fricas | 0.27 |
| Maple | 0.30 |
| Maxima | 0.32 |
| Giac | 0.40 |
| Rubi | 0.87 |
| Mathematica | 2.15 |
| Mupad | 2.69 |
| Sympy | 9.80 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Mupad | 24.11 | 1.00 | 26.00 | 1.00 |
| Giac | 29.62 | 1.07 | 23.00 | 1.00 |
| Sympy | 115.20 | 1.24 | 26.00 | 0.96 |
| Fricas | 144.33 | 1.86 | 99.00 | 1.46 |
| Rubi | 176.63 | 0.96 | 142.00 | 1.00 |
| Maxima | 199.68 | 4.27 | 101.00 | 1.00 |
| Mathematica | 231.62 | 1.11 | 145.50 | 1.07 |
| Maple | 301.52 | 1.67 | 163.00 | 1.22 |

Table 1.6: Leaf size performance for each CAS

# 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

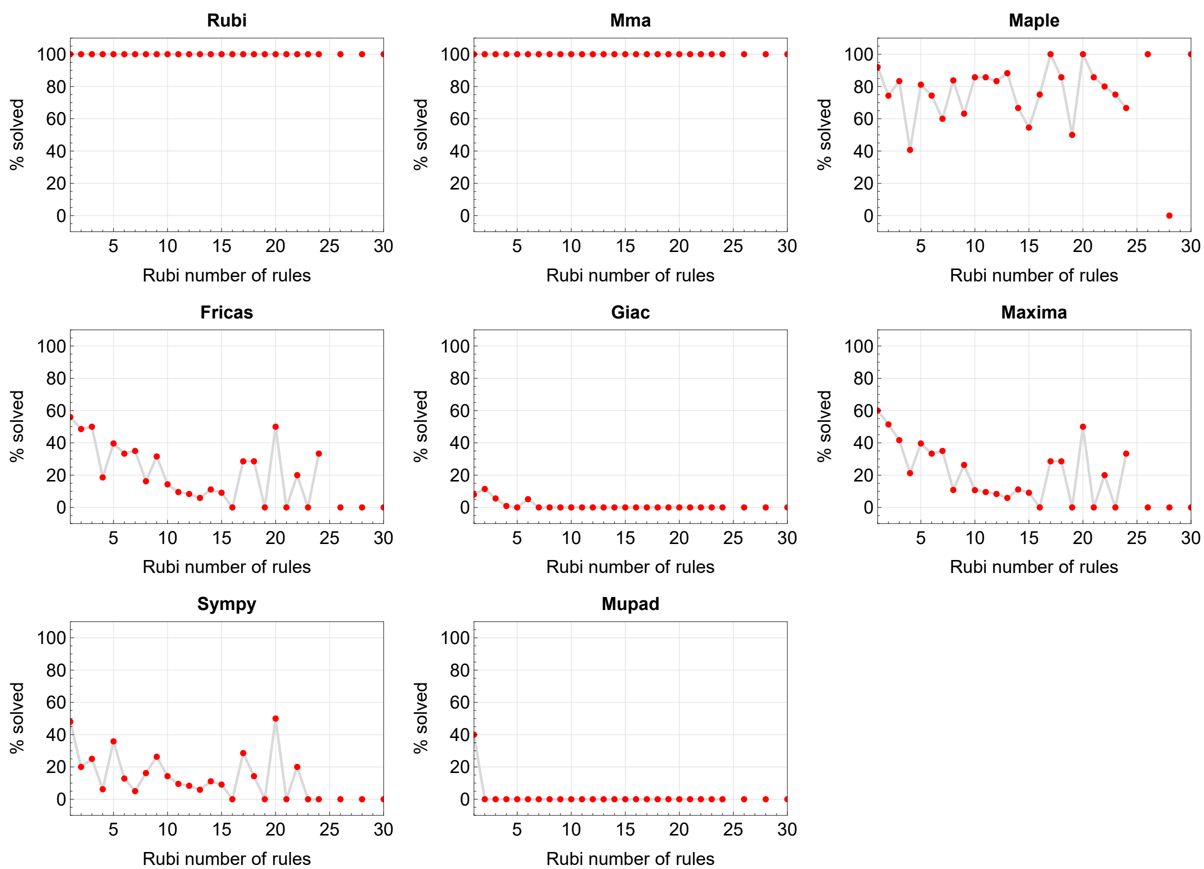



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

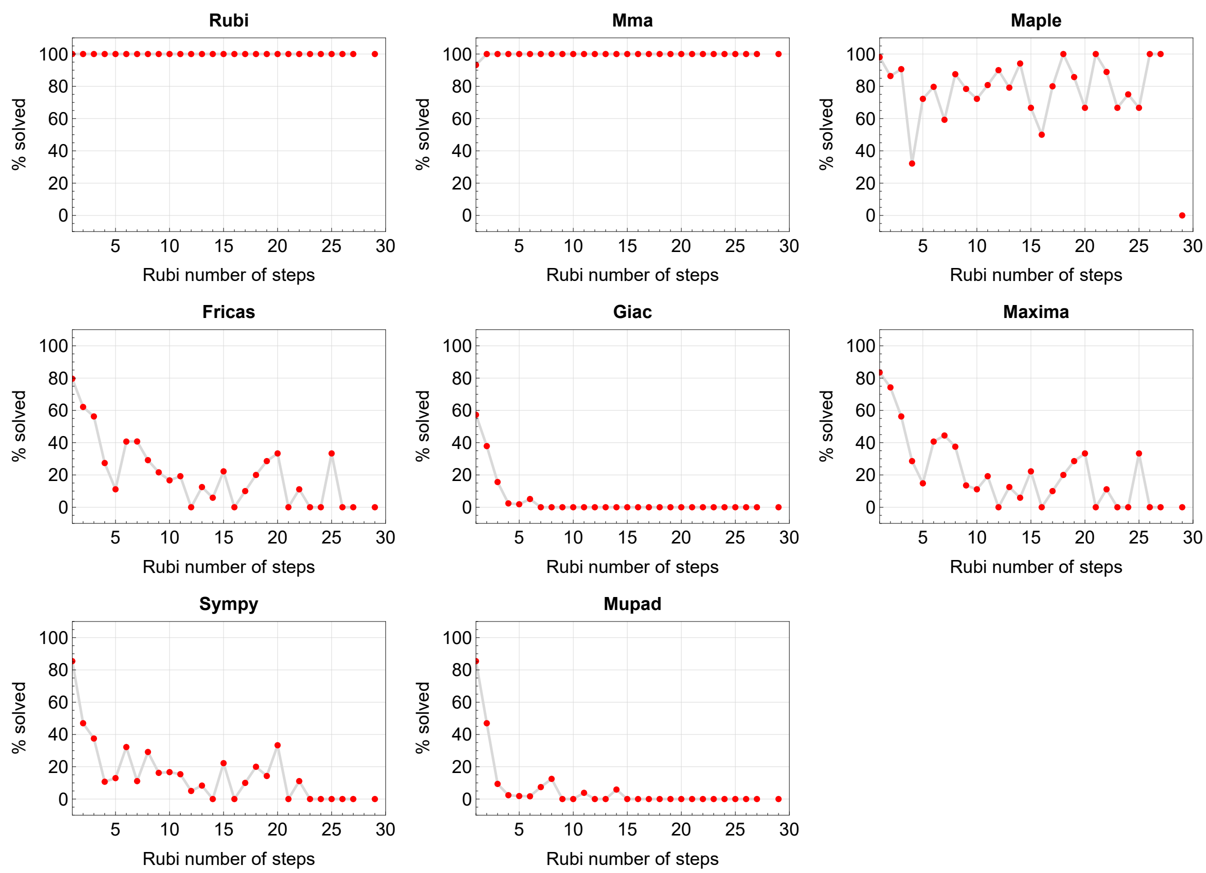



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

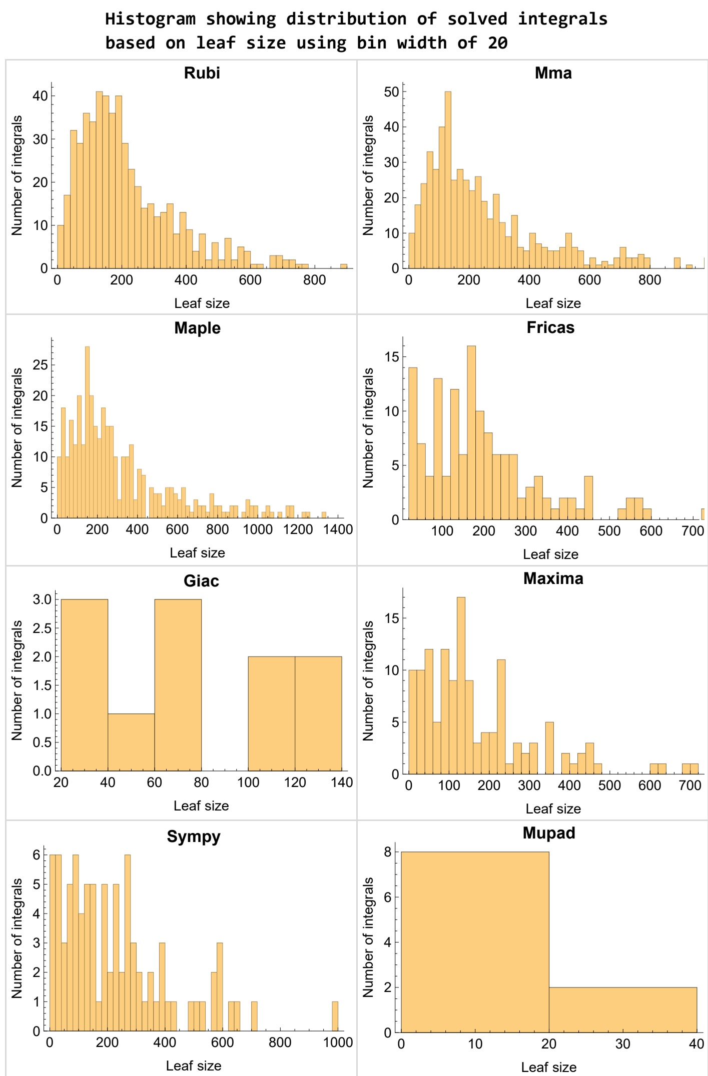



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

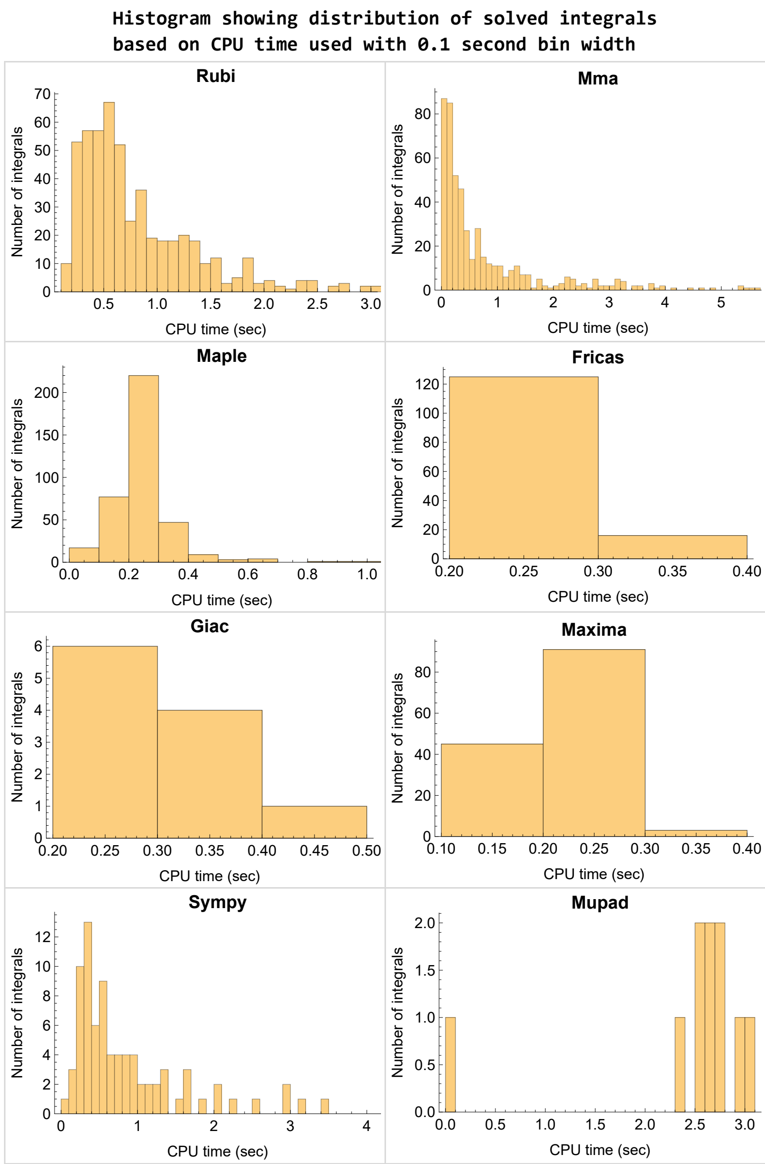



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

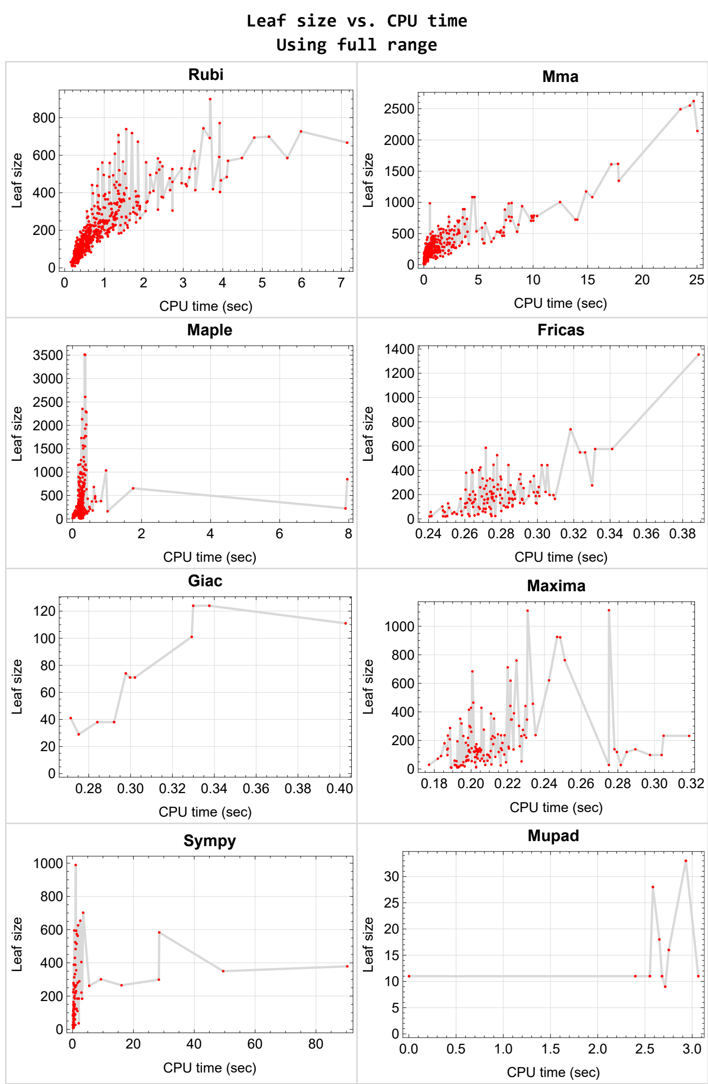



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{188, 189, 190, 321, 322, 323, 324, 325, 326, 327, 341, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {6, 8, 15, 16, 17, 18, 24, 26, 27, 203, 205, 212, 214, 221, 263, 265, 271, 273, 279, 280, 297, 299}

**Mathematica** {564, 575, 581, 587, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604}

**Maple** {612}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.14 Important notes about some of the results

## 1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

### 1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

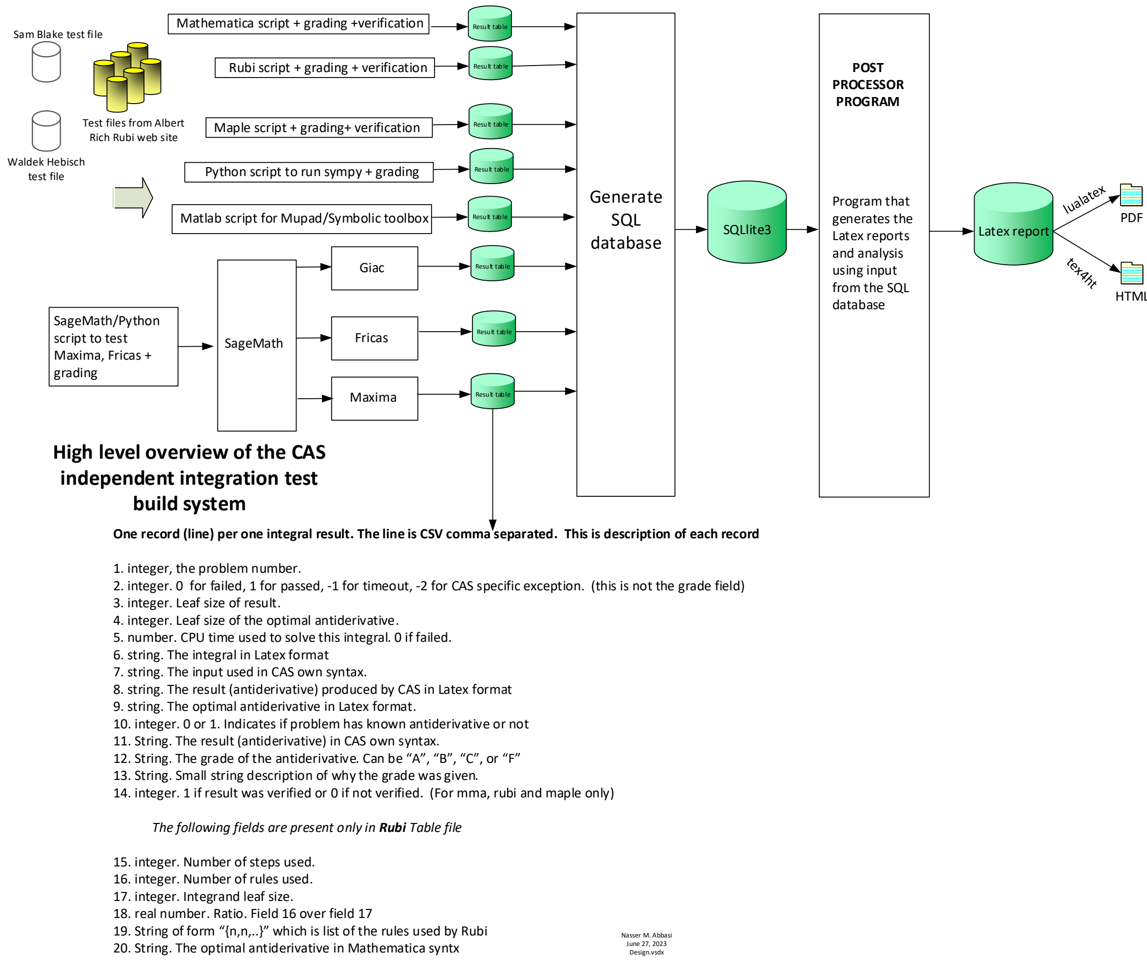
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

## 2.1 List of integrals sorted by grade for each CAS



### 2.1.1 Rubi

**A grade** { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 28, 30, 32, 34, 36, 37, 39, 40, 41, 43, 45, 46, 47, 48, 49, 50, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 70, 71, 72, 73, 74, 76, 78, 79, 80, 81, 82, 83, 84, 85, 87, 89, 90, 91, 92, 93, 94, 95, 97, 99, 100, 101, 102, 103, 104, 105, 106, 108, 110, 111, 112, 113, 114, 115, 116, 118, 120, 121, 122, 123, 125, 127, 128, 129, 130, 131, 133, 135, 136, 137, 138, 139, 141, 143, 144, 145, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 169, 170, 171, 173, 175, 176, 177, 178, 179, 180, 181, 183, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 218, 219, 220, 225, 227, 229, 231, 233, 234, 236, 237, 238, 240, 242, 243, 244, 245, 246, 247, 249, 251, 252, 253, 254, 255, 258, 259, 260, 261, 266, 267, 268, 269, 274, 275, 276, 277, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 300, 302, 304, 306, 308, 310, 312, 314, 316, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 382, 383, 384, 385, 389, 391, 393, 406, 407, 408, 411, 413, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 439, 440, 461, 462, 463, 464, 465, 467, 468, 469, 470, 474, 477, 478, 479, 483, 487, 490, 491, 492, 495, 496, 497, 498, 501, 502, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 529, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 629, 630, 634, 635, 638, 639, 640, 643, 649, 650, 651 }

**B grade** { 217 }

**C grade** { 6, 8, 15, 17, 24, 26, 29, 31, 33, 35, 38, 42, 44, 51, 53, 59, 61, 67, 69, 75, 77, 86, 88, 96, 98, 107, 109, 117, 119, 124, 126, 132, 134, 140, 142, 151, 153, 161, 163, 172, 174, 182, 184, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 224, 226, 228, 230, 232, 235, 239, 241, 248, 250, 256, 257, 262, 263, 264, 265, 270, 271, 272, 273, 278, 279, 280, 287, 288, 289, 296, 297, 298, 299, 301, 303, 305, 307, 309, 311, 313, 315, 317, 319, 320, 338, 339, 340, 347, 348, 349, 381, 388, 390, 392, 412, 414, 438, 472, 473, 482, 485, 486, 494, 503, 528, 530, 626, 631, 644 }

**F normal fail** { 223, 281, 318, 481 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.2 Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 37, 39, 40, 41, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 231, 234, 236, 237, 238, 240, 242, 243, 244, 245, 246, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 440, 461, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 588, 589, 590, 592, 593, 595, 596, 597, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644 }

**B grade** { 31, 33, 35, 42, 44, 228, 230, 233, 295, 551, 564, 581, 586, 587, 591, 594, 598, 600, 601 }

**C grade** { 38, 43, 45, 52, 54, 226, 232, 235, 239, 241, 248, 250, 348, 612, 649, 650, 651 }

**F normal fail** { }

**F(-1) timedout fail** { 449, 451, 453, 632, 633, 636, 637 }

**F(-2) exception fail** { }

### 2.1.3 Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 61, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 86, 88, 96, 98, 107, 109, 112, 113, 114, 115, 116, 117, 119, 124, 126, 127, 132, 133, 134, 135, 140, 141, 142, 143, 144, 150, 151, 153, 154, 156, 158, 161, 162, 163, 165, 167, 172, 174, 175, 177, 178, 179, 180, 181, 182, 184, 198, 199, 200, 201, 202, 204, 206, 207, 208, 209, 210, 211, 213, 215, 216, 217, 218, 219, 220, 222, 224, 226, 228, 235, 237, 244, 246, 252, 253, 254, 271, 272, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 300, 301, 302, 304, 305, 320, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 406, 407, 408, 412, 414, 420, 422, 428, 430, 436, 438, 440, 461, 474, 479, 483, 487, 492, 498, 504, 509, 531, 606, 607, 608, 609, 610, 613, 614, 615, 616, 618, 619, 620, 621, 625, 626 }

**B grade** { 57, 60, 62, 65, 68, 70, 73, 78, 85, 87, 89, 91, 93, 95, 97, 99, 100, 102, 104, 106, 108, 110, 111, 118, 120, 121, 122, 123, 125, 128, 129, 130, 131, 136, 137, 138, 139, 145, 146, 147, 148, 149, 152, 160, 164, 169, 171, 173, 176, 183, 203, 205, 212, 214, 221, 223, 230, 232, 239, 241, 248, 250, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 273, 274, 275, 276, 277, 278, 281, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 303, 307, 309, 310, 311, 312, 313, 314, 315, 317, 319, 411, 413, 419, 421, 427, 429, 435, 437, 439, 624 }

**C grade** { 90, 92, 94, 101, 103, 105, 155, 157, 159, 166, 168, 170, 611, 612 }

**F normal fail** { 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 225, 227, 229, 231, 233, 234, 236, 238, 240, 242, 243, 245, 247, 249, 251, 306, 308, 316, 318, 331, 332, 333, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 477, 478, 481, 482, 485, 486, 490, 491, 494, 495, 496, 497, 501, 502, 503, 507, 508, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 617, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.4 Fricas

**A grade** { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 55, 63, 79, 80, 82, 90, 101, 112, 113, 114, 115, 118, 120, 122, 128, 130, 136, 138, 144, 145, 147, 149, 154, 155, 157, 159, 166, 168, 170, 177, 178, 179, 180, 183, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 244, 258, 260, 266, 268, 274, 276, 282, 283, 284, 285, 290, 292, 294, 328, 329, 330, 342, 343, 344, 345, 393, 440, 487, 492, 504, 509, 606, 607, 608, 609, 610, 613, 614, 615 }

**B grade** { 7, 9, 16, 18, 57, 62, 65, 71, 73, 84, 87, 89, 92, 94, 103, 105, 116, 127, 152, 181, 237, 246, 286, 346, 385, 461, 531, 539, 556, 557, 561, 566, 567, 616, 649, 650, 651 }

**C grade** { }

**F normal fail** { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 85, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 102, 104, 106, 107, 108, 109, 110, 111, 117, 119, 121, 123, 124, 125, 126, 129, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 146, 148, 150, 151, 153, 156, 158, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 287, 288, 289, 291, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 558, 559, 560, 562, 563, 564, 565, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 510, 511, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646 }

### 2.1.5 Maxima

**A grade** { 1, 2, 3, 4, 5, 7, 9, 11, 12, 14, 16, 18, 25, 27, 55, 57, 63, 65, 71, 73, 79, 80, 82, 84, 85, 89, 92, 95, 97, 103, 106, 110, 111, 112, 113, 114, 115, 116, 118, 120, 122, 127, 128, 130, 136, 138, 144, 145, 147, 149, 150, 152, 154, 157, 160, 162, 168, 169, 171, 175, 176, 177, 178, 179, 180, 181, 183, 198, 200, 258, 260, 266, 268, 274, 276, 283, 285, 286, 290, 292, 294, 295, 328, 329, 330, 337, 343, 345, 346, 385, 393, 440, 461, 531, 539, 555, 556, 557, 561, 562, 563, 566, 567, 568, 569, 591, 606, 607, 608, 609, 610, 613, 614, 615, 616 }

**B grade** { 10, 13, 19, 20, 21, 22, 23, 62, 87, 104, 199, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 255 }

**C grade** { }

**F normal fail** { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 59, 60, 61, 67, 69, 75, 77, 86, 88, 90, 91, 93, 94, 96, 98, 99, 100, 101, 102, 105, 107, 108, 109, 117, 119, 124, 125, 126, 132, 134, 140, 142, 151, 153, 155, 156, 158, 159, 161, 163, 164, 165, 166, 167, 170, 172, 173, 174, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 256, 257, 262, 263, 264, 265, 270, 272, 278, 280, 282, 284, 287, 288, 289, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 338, 339, 340, 342, 344, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 537, 538, 544, 545, 550, 551, 554, 558, 559, 560, 564, 565, 573, 574, 580, 586, 590, 592, 594, 595, 596, 597, 598, 605, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 650, 651 }

**F(-1) timedout fail** { 575, 581, 587, 593, 600, 601, 602, 603 }

**F(-2) exception fail** { 56, 58, 64, 66, 68, 70, 72, 74, 76, 78, 81, 83, 121, 123, 129, 131, 133, 135, 137, 139, 141, 143, 146, 148, 252, 253, 254, 259, 261, 267, 269, 271, 273, 275, 277, 279, 281, 291, 293, 334, 335, 336, 534, 535, 536, 540, 541, 542, 543, 546, 547, 548, 549, 552, 553, 570, 571, 572, 576, 577, 578, 579, 582, 583, 584, 585, 588, 589, 599, 604, 611, 612, 617, 632, 636, 647, 648, 649, 652, 653, 654 }

### 2.1.6 Giac

**A grade** { 111, 115, 176, 180, 285, 345, 531, 610 }

**B grade** { 118, 183, 616 }

**C grade** { }

**F normal fail** { 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 56, 64, 72, 79, 81, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 102, 104, 105, 106, 107, 108, 109, 110, 112, 114, 116, 117, 119, 121, 129, 137, 144, 146, 148, 149, 150, 151, 152, 153, 154, 158, 159, 160, 161, 162, 163, 164, 167, 169, 170, 171, 172, 173, 174, 175, 177, 179, 181, 182, 184, 194, 195, 196, 197, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 251, 255, 256, 257, 259, 267, 275, 282, 284, 286, 287, 289, 291, 293, 294, 295, 296, 297, 298, 299, 303, 304, 305, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 331, 332, 333, 337, 338, 342, 344, 346, 347, 349, 350, 351, 352, 355, 357, 358, 359, 365, 367, 373, 375, 380, 382, 383, 384, 385, 389, 391, 392, 393, 406, 407, 408, 412, 413, 414, 420, 422, 428, 430, 436, 438, 439, 440, 461, 463, 465, 468, 470, 474, 479, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 517, 522, 529, 530, 537, 538, 539, 543, 545, 553, 554, 555, 556, 557, 560, 561, 562, 565, 566, 567, 573, 574, 575, 579, 589, 590, 591, 592, 593, 596, 597, 598, 602, 603, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

**F(-1) timedout fail** { 544, 580 }

**F(-2) exception fail** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 37, 38, 47, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 90, 91, 92, 100, 101, 103, 113, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145, 147, 155, 156, 157, 165, 166, 168, 178, 185, 186, 187, 191, 192, 193, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 234, 235, 244, 252, 253, 254, 258, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 288, 290, 292, 300, 301, 302, 310, 312, 320, 321, 322, 323, 328, 329, 330, 334, 335, 336, 339, 340, 343, 348, 356, 360, 362, 364, 366, 368, 370, 372, 374, 376, 378, 381, 388, 390, 394, 397, 399, 401, 402, 403, 411, 415, 417, 419, 421, 423, 425, 427, 429, 431, 433, 435, 437, 441, 443, 445, 447, 449, 451, 453, 455, 456, 457, 462, 464, 466, 467, 469, 471, 472, 473, 477, 478, 481, 482, 513, 514, 515, 516, 518, 519, 520, 521, 523, 524, 525, 526, 528, 534, 535, 536, 540, 541, 542, 546, 547, 548, 549, 550, 551, 552, 558, 559, 563, 564, 568, 569, 570, 571, 572, 576, 577, 578, 581, 582, 583, 584, 585, 586, 587, 588, 594, 595, 599, 600, 601, 604, 605, 606, 607, 608, 609, 613, 614, 615, 634 }

### 2.1.7 Mupad

**A grade** { }

**B grade** { 116, 181, 286, 346, 385, 393, 440, 461, 531, 610 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

**F(-2) exception fail** { }

### 2.1.8 Sympy

**A grade** { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 64, 66, 72, 74, 80, 81, 82, 83, 84, 112, 113, 114, 115, 116, 177, 178, 179, 180, 181, 198, 199, 200, 202, 207, 208, 209, 211, 216, 217, 218, 220, 282, 283, 284, 285, 286, 328, 329, 330, 342, 343, 344, 345, 346, 385, 461, 606, 607, 608, 609, 610, 613, 614, 615, 616 }

**B grade** { 55, 57, 60, 63, 65, 71, 73, 85, 201, 210, 219, 252, 253, 255, 393, 440, 531 }

**C grade** { }

**F normal fail** { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 61, 62, 67, 68, 69, 70, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 187, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 478, 479, 485, 486, 487, 491, 492, 494, 496, 497, 498, 502, 503, 504, 507, 508, 509, 512, 513, 514, 528, 529, 530, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 553, 554, 555, 556, 557, 559, 560, 561, 562, 565, 566, 567, 571, 572, 573, 574, 575, 578, 579, 580, 581, 589, 590, 591, 592, 593, 595, 596, 597, 598, 601, 602, 603, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650 }

**F(-1) timedout fail** { 136, 137, 191, 192, 274, 275, 321, 322, 334, 455, 477, 481, 482, 483, 484, 490, 495, 501, 506, 510, 511, 518, 519, 521, 522, 523, 524, 525, 526, 534, 540, 546, 547, 548, 549, 550, 551, 552, 558, 563, 564, 568, 569, 570, 576, 577, 582, 583, 584, 585, 586, 587, 588, 594, 599, 600, 604, 605, 628, 651 }

**F(-2) exception fail** { 517 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 122 | 87 | 120 | 184 | 113 | 151 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.70 | 0.97 | 1.48 | 0.91 | 1.22 | 0.00 | 0.00 |
| time (sec) | N/A | 0.309 | 0.073 | 0.064 | 0.197 | 0.286 | 0.691 | 0.000 | 0.000 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 132 | 88 | 109 | 166 | 109 | 138 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.73 | 0.91 | 1.38 | 0.91 | 1.15 | 0.00 | 0.00 |
| time (sec) | N/A | 0.307 | 0.041 | 0.033 | 0.198 | 0.281 | 0.599 | 0.000 | 0.000 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 78 | 101 | 145 | 103 | 126 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.99 | 1.42 | 1.01 | 1.24 | 0.00 | 0.00 |
| time (sec) | N/A | 0.304 | 0.063 | 0.045 | 0.200 | 0.286 | 0.369 | 0.000 | 0.000 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 90 | 77 | 85 | 127 | 98 | 117 | 0 | 0 |
| N.S. | 1 | 1.03 | 0.89 | 0.98 | 1.46 | 1.13 | 1.34 | 0.00 | 0.00 |
| time (sec) | N/A | 0.244 | 0.040 | 0.077 | 0.205 | 0.268 | 0.297 | 0.000 | 0.000 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 77 | 86 | 73 | 97 | 83 | 90 | 0 | 0 |
| N.S. | 1 | 1.03 | 1.15 | 0.97 | 1.29 | 1.11 | 1.20 | 0.00 | 0.00 |
| time (sec) | N/A | 0.260 | 0.032 | 0.015 | 0.187 | 0.274 | 0.166 | 0.000 | 0.000 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 153 | 113 | 159 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.38 | 1.02 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.664 | 0.041 | 0.154 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 68 | 74 | 67 | 64 | 156 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 1.12 | 1.02 | 0.97 | 2.36 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.292 | 0.023 | 0.018 | 0.199 | 0.292 | 0.000 | 0.000 | 0.000 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 154 | 111 | 151 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.20 | 0.87 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.686 | 0.038 | 0.119 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 79 | 93 | 83 | 91 | 169 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 1.16 | 1.04 | 1.14 | 2.11 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.285 | 0.026 | 0.023 | 0.198 | 0.285 | 0.000 | 0.000 | 0.000 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 169 | 119 | 163 | 319 | 165 | 230 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.66 | 0.90 | 1.76 | 0.91 | 1.27 | 0.00 | 0.00 |
| time (sec) | N/A | 0.434 | 0.074 | 0.234 | 0.195 | 0.262 | 1.151 | 0.000 | 0.000 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 187 | 115 | 152 | 292 | 161 | 218 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.64 | 0.84 | 1.62 | 0.89 | 1.21 | 0.00 | 0.00 |
| time (sec) | N/A | 0.414 | 0.063 | 0.236 | 0.200 | 0.266 | 0.996 | 0.000 | 0.000 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 149 | 111 | 144 | 261 | 153 | 202 | 0 | 0 |
| N.S. | 1 | 0.95 | 0.71 | 0.92 | 1.66 | 0.97 | 1.29 | 0.00 | 0.00 |
| time (sec) | N/A | 0.414 | 0.062 | 0.184 | 0.200 | 0.263 | 0.649 | 0.000 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 117 | 104 | 116 | 234 | 149 | 190 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.87 | 0.97 | 1.95 | 1.24 | 1.58 | 0.00 | 0.00 |
| time (sec) | N/A | 0.280 | 0.094 | 0.217 | 0.187 | 0.276 | 0.530 | 0.000 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 124 | 95 | 116 | 194 | 133 | 165 | 0 | 0 |
| N.S. | 1 | 0.97 | 0.74 | 0.91 | 1.52 | 1.04 | 1.29 | 0.00 | 0.00 |
| time (sec) | N/A | 0.357 | 0.069 | 0.214 | 0.193 | 0.261 | 0.321 | 0.000 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 243 | 173 | 222 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.41 | 1.01 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.000 | 0.132 | 0.216 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 114 | 124 | 112 | 143 | 228 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.03 | 0.93 | 1.19 | 1.90 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.424 | 0.087 | 0.197 | 0.187 | 0.287 | 0.000 | 0.000 | 0.000 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 251 | 151 | 240 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.34 | 0.81 | 1.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.033 | 0.164 | 0.242 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 134 | 133 | 112 | 137 | 243 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 1.06 | 0.89 | 1.09 | 1.93 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.418 | 0.086 | 0.228 | 0.187 | 0.278 | 0.000 | 0.000 | 0.000 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 226 | 210 | 143 | 202 | 465 | 201 | 289 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.63 | 0.89 | 2.06 | 0.89 | 1.28 | 0.00 | 0.00 |
| time (sec) | N/A | 0.612 | 0.099 | 0.207 | 0.201 | 0.287 | 2.235 | 0.000 | 0.000 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 192 | 139 | 191 | 429 | 197 | 280 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.70 | 0.96 | 2.16 | 0.99 | 1.41 | 0.00 | 0.00 |
| time (sec) | N/A | 0.395 | 0.086 | 0.219 | 0.200 | 0.279 | 1.657 | 0.000 | 0.000 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 190 | 135 | 183 | 388 | 189 | 265 | 0 | 0 |
| N.S. | 1 | 0.94 | 0.67 | 0.91 | 1.92 | 0.94 | 1.31 | 0.00 | 0.00 |
| time (sec) | N/A | 0.576 | 0.098 | 0.228 | 0.211 | 0.295 | 1.234 | 0.000 | 0.000 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 140 | 128 | 143 | 352 | 185 | 253 | 0 | 0 |
| N.S. | 1 | 0.97 | 0.88 | 0.99 | 2.43 | 1.28 | 1.74 | 0.00 | 0.00 |
| time (sec) | N/A | 0.302 | 0.118 | 0.168 | 0.194 | 0.277 | 0.877 | 0.000 | 0.000 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 162 | 119 | 153 | 301 | 169 | 221 | 0 | 0 |
| N.S. | 1 | 0.95 | 0.70 | 0.90 | 1.77 | 0.99 | 1.30 | 0.00 | 0.00 |
| time (sec) | N/A | 0.486 | 0.082 | 0.209 | 0.199 | 0.280 | 0.580 | 0.000 | 0.000 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 348 | 216 | 271 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.57 | 0.98 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.384 | 0.166 | 0.283 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 146 | 163 | 149 | 231 | 276 | 0 | 0 | 0 |
| N.S. | 1 | 0.91 | 1.02 | 0.93 | 1.44 | 1.72 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.551 | 0.112 | 0.162 | 0.196 | 0.330 | 0.000 | 0.000 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 249 | 356 | 244 | 289 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.43 | 0.98 | 1.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.461 | 0.154 | 0.293 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 156 | 171 | 153 | 208 | 289 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.98 | 0.88 | 1.20 | 1.66 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.630 | 0.106 | 0.159 | 0.188 | 0.291 | 0.000 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 170 | 170 | 208 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 1.09 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.856 | 0.175 | 0.351 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 141 | 181 | 146 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.34 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.659 | 0.155 | 0.280 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 99 | 121 | 170 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 1.12 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.534 | 0.168 | 0.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 75 | 167 | 84 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 2.29 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.419 | 0.051 | 0.230 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 59 | 135 | 143 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 1.93 | 2.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.346 | 0.073 | 0.219 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 65 | 207 | 161 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 3.39 | 2.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.437 | 0.068 | 0.212 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 96 | 182 | 176 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.80 | 1.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.565 | 0.113 | 0.195 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 112 | 240 | 217 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 2.12 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.655 | 0.182 | 0.241 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 160 | 247 | 215 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 1.58 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.856 | 0.143 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 181 | 268 | 224 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 1.57 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.870 | 0.220 | 0.205 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 147 | 241 | 140 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.01 | 1.66 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.692 | 0.158 | 0.261 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 117 | 221 | 195 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 1.74 | 1.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.557 | 0.170 | 0.161 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 74 | 61 | 0 | 65 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.35 | 1.11 | 0.00 | 1.18 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.246 | 0.099 | 0.213 | 0.000 | 0.280 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 114 | 216 | 195 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 1.74 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.529 | 0.114 | 0.219 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 115 | 234 | 217 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 2.13 | 1.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.654 | 0.309 | 0.210 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 182 | 253 | 227 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 1.51 | 1.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.874 | 0.383 | 0.230 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 191 | 326 | 242 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.31 | 2.23 | 1.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.968 | 0.315 | 0.229 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 280 | 311 | 268 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.17 | 1.30 | 1.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.228 | 0.425 | 0.234 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 201 | 341 | 247 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 1.83 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.893 | 0.489 | 0.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 95 | 79 | 108 | 0 | 99 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.81 | 1.11 | 0.00 | 1.02 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.289 | 0.139 | 0.229 | 0.000 | 0.283 | 0.000 | 0.000 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 170 | 340 | 247 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 1.85 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.749 | 0.193 | 0.176 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 78 | 56 | 76 | 0 | 98 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.70 | 0.95 | 0.00 | 1.22 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.247 | 0.116 | 0.237 | 0.000 | 0.274 | 0.000 | 0.000 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 164 | 341 | 248 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 1.92 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.699 | 0.149 | 0.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 183 | 289 | 280 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 1.82 | 1.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.915 | 0.448 | 0.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 249 | 298 | 291 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 1.34 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.100 | 0.775 | 0.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 280 | 353 | 344 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.21 | 1.52 | 1.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.309 | 0.605 | 0.312 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 364 | 374 | 333 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 1.27 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.595 | 1.056 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 102 | 106 | 164 | 134 | 158 | 221 | 0 | 0 |
| N.S. | 1 | 0.94 | 0.97 | 1.50 | 1.23 | 1.45 | 2.03 | 0.00 | 0.00 |
| time (sec) | N/A | 0.373 | 0.189 | 0.224 | 0.204 | 0.286 | 0.900 | 0.000 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 114 | 79 | 156 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.66 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.513 | 0.189 | 0.211 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 58 | 63 | 108 | 73 | 127 | 141 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.03 | 1.77 | 1.20 | 2.08 | 2.31 | 0.00 | 0.00 |
| time (sec) | N/A | 0.269 | 0.125 | 0.214 | 0.206 | 0.260 | 0.344 | 0.000 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 69 | 100 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.310 | 0.157 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 91 | 131 | 171 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.47 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.567 | 0.192 | 0.227 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 75 | 155 | 0 | 0 | 110 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.23 | 2.54 | 0.00 | 0.00 | 1.80 | 0.00 | 0.00 |
| time (sec) | N/A | 0.362 | 0.217 | 0.169 | 0.000 | 0.000 | 1.610 | 0.000 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 106 | 185 | 231 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 1.64 | 2.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.593 | 2.484 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 58 | 96 | 501 | 133 | 217 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 1.55 | 8.08 | 2.15 | 3.50 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.299 | 0.168 | 0.164 | 0.217 | 0.300 | 0.000 | 0.000 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 112 | 100 | 195 | 145 | 199 | 301 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.80 | 1.56 | 1.16 | 1.59 | 2.41 | 0.00 | 0.00 |
| time (sec) | N/A | 0.407 | 0.198 | 0.277 | 0.216 | 0.263 | 9.329 | 0.000 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 178 | 154 | 214 | 0 | 0 | 262 | 0 | 0 |
| N.S. | 1 | 1.08 | 0.93 | 1.30 | 0.00 | 0.00 | 1.59 | 0.00 | 0.00 |
| time (sec) | N/A | 0.832 | 0.378 | 0.211 | 0.000 | 0.000 | 5.488 | 0.000 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 68 | 72 | 139 | 85 | 167 | 221 | 0 | 0 |
| N.S. | 1 | 0.88 | 0.94 | 1.81 | 1.10 | 2.17 | 2.87 | 0.00 | 0.00 |
| time (sec) | N/A | 0.286 | 0.146 | 0.171 | 0.213 | 0.273 | 2.929 | 0.000 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 129 | 111 | 152 | 0 | 0 | 185 | 0 | 0 |
| N.S. | 1 | 1.16 | 1.00 | 1.37 | 0.00 | 0.00 | 1.67 | 0.00 | 0.00 |
| time (sec) | N/A | 0.498 | 0.246 | 0.194 | 0.000 | 0.000 | 1.578 | 0.000 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 143 | 180 | 228 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 1.34 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.871 | 0.366 | 0.222 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 122 | 122 | 208 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 1.13 | 1.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.542 | 0.370 | 0.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 151 | 292 | 291 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.97 | 1.88 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.861 | 1.275 | 0.221 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 121 | 125 | 622 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 1.09 | 5.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.603 | 0.300 | 0.164 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 120 | 108 | 226 | 156 | 263 | 379 | 0 | 0 |
| N.S. | 1 | 0.85 | 0.77 | 1.60 | 1.11 | 1.87 | 2.69 | 0.00 | 0.00 |
| time (sec) | N/A | 0.407 | 0.260 | 0.249 | 0.202 | 0.301 | 90.306 | 0.000 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 252 | 196 | 267 | 0 | 0 | 350 | 0 | 0 |
| N.S. | 1 | 1.18 | 0.92 | 1.25 | 0.00 | 0.00 | 1.64 | 0.00 | 0.00 |
| time (sec) | N/A | 1.165 | 0.648 | 0.234 | 0.000 | 0.000 | 49.520 | 0.000 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 75 | 80 | 170 | 96 | 225 | 299 | 0 | 0 |
| N.S. | 1 | 0.81 | 0.86 | 1.83 | 1.03 | 2.42 | 3.22 | 0.00 | 0.00 |
| time (sec) | N/A | 0.291 | 0.154 | 0.195 | 0.210 | 0.265 | 28.365 | 0.000 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 186 | 153 | 202 | 0 | 0 | 265 | 0 | 0 |
| N.S. | 1 | 1.13 | 0.93 | 1.22 | 0.00 | 0.00 | 1.61 | 0.00 | 0.00 |
| time (sec) | N/A | 0.668 | 0.408 | 0.216 | 0.000 | 0.000 | 16.114 | 0.000 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 205 | 257 | 284 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 1.44 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.150 | 0.447 | 0.154 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 197 | 168 | 261 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.25 | 1.07 | 1.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.794 | 0.515 | 0.201 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 214 | 349 | 348 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.70 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.204 | 1.489 | 0.167 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 193 | 179 | 692 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.16 | 1.08 | 4.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.857 | 0.480 | 0.218 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 28 | 26 | 28 | 40 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.81 | 0.88 | 1.25 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.249 | 0.011 | 0.181 | 0.275 | 0.262 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 160 | 108 | 193 | 174 | 161 | 184 | 0 | 0 |
| N.S. | 1 | 1.07 | 0.72 | 1.30 | 1.17 | 1.08 | 1.23 | 0.00 | 0.00 |
| time (sec) | N/A | 0.649 | 0.241 | 0.204 | 0.203 | 0.285 | 3.148 | 0.000 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 134 | 111 | 165 | 0 | 0 | 185 | 0 | 0 |
| N.S. | 1 | 1.06 | 0.88 | 1.31 | 0.00 | 0.00 | 1.47 | 0.00 | 0.00 |
| time (sec) | N/A | 0.584 | 0.337 | 0.161 | 0.000 | 0.000 | 2.078 | 0.000 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 101 | 82 | 133 | 117 | 132 | 124 | 0 | 0 |
| N.S. | 1 | 1.03 | 0.84 | 1.36 | 1.19 | 1.35 | 1.27 | 0.00 | 0.00 |
| time (sec) | N/A | 0.438 | 0.196 | 0.205 | 0.199 | 0.299 | 1.323 | 0.000 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 69 | 107 | 0 | 0 | 122 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.43 | 0.00 | 0.00 | 1.63 | 0.00 | 0.00 |
| time (sec) | N/A | 0.376 | 0.244 | 0.165 | 0.000 | 0.000 | 1.027 | 0.000 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 49 | 72 | 55 | 96 | 60 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 1.71 | 1.31 | 2.29 | 1.43 | 0.00 | 0.00 |
| time (sec) | N/A | 0.244 | 0.149 | 0.201 | 0.197 | 0.284 | 0.797 | 0.000 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 53 | 28 | 0 | 87 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.12 | 1.12 | 0.00 | 3.48 | 0.00 | 0.00 |
| time (sec) | N/A | 0.204 | 0.014 | 0.209 | 0.191 | 0.000 | 0.617 | 0.000 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 57 | 96 | 118 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.71 | 2.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.415 | 0.201 | 0.214 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 52 | 84 | 101 | 132 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 2.05 | 2.46 | 3.22 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.267 | 0.139 | 0.158 | 0.203 | 0.290 | 0.000 | 0.000 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 109 | 185 | 216 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.61 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.618 | 1.949 | 0.196 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 95 | 106 | 373 | 121 | 222 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 1.09 | 3.85 | 1.25 | 2.29 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.432 | 0.175 | 0.194 | 0.202 | 0.304 | 0.000 | 0.000 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 129 | 131 | 279 | 0 | 196 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 0.96 | 2.04 | 0.00 | 1.43 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.463 | 0.287 | 0.196 | 0.000 | 0.309 | 0.000 | 0.000 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 153 | 147 | 261 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.17 | 1.12 | 1.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.697 | 0.453 | 0.293 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | B | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 80 | 87 | 163 | 119 | 165 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 1.01 | 1.90 | 1.38 | 1.92 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.361 | 0.242 | 0.171 | 0.285 | 0.292 | 0.000 | 0.000 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 78 | 168 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 2.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.405 | 0.390 | 0.210 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 52 | 103 | 0 | 127 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.16 | 2.29 | 0.00 | 2.82 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.257 | 0.181 | 0.157 | 0.000 | 0.292 | 0.000 | 0.000 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 66 | 110 | 58 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.29 | 2.16 | 1.14 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.229 | 0.120 | 0.234 | 0.205 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 96 | 143 | 155 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.52 | 1.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.620 | 0.393 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 88 | 102 | 244 | 119 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.10 | 2.62 | 1.28 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.370 | 0.250 | 0.226 | 0.215 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 160 | 269 | 224 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 1.66 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.912 | 2.927 | 0.170 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 142 | 168 | 604 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 1.10 | 3.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.475 | 0.307 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 251 | 202 | 970 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.31 | 1.05 | 5.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.109 | 0.596 | 0.331 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 139 | 132 | 237 | 0 | 218 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 0.90 | 1.62 | 0.00 | 1.49 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.437 | 0.318 | 0.178 | 0.000 | 0.287 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 166 | 166 | 897 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.19 | 1.19 | 6.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.750 | 0.491 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | B | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 104 | 93 | 157 | 138 | 187 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 0.89 | 1.50 | 1.31 | 1.78 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.374 | 0.281 | 0.170 | 0.290 | 0.301 | 0.000 | 0.000 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 74 | 88 | 729 | 137 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 1.10 | 9.11 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.345 | 0.256 | 0.193 | 0.223 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 74 | 72 | 121 | 0 | 165 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 0.96 | 1.61 | 0.00 | 2.20 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.278 | 0.215 | 0.168 | 0.000 | 0.310 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 113 | 100 | 619 | 126 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.93 | 5.73 | 1.17 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.386 | 0.150 | 0.196 | 0.211 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 169 | 209 | 208 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 1.41 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.926 | 0.703 | 0.229 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 140 | 125 | 778 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.83 | 5.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.463 | 0.409 | 0.182 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 256 | 331 | 314 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.34 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.349 | 4.107 | 0.178 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 188 | 239 | 1155 | 236 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 1.15 | 5.55 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.642 | 0.398 | 0.169 | 0.218 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 213 | 121 | 363 | 143 | 0 | 0 | 124 | 0 |
| N.S. | 1 | 1.06 | 0.60 | 1.82 | 0.72 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.573 | 0.145 | 0.277 | 0.221 | 0.000 | 0.000 | 0.338 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 94 | 63 | 74 | 83 | 83 | 82 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.73 | 0.86 | 0.97 | 0.97 | 0.95 | 0.00 | 0.00 |
| time (sec) | N/A | 0.479 | 0.038 | 0.158 | 0.202 | 0.270 | 0.500 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 73 | 48 | 82 | 59 | 55 | 65 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.69 | 1.17 | 0.84 | 0.79 | 0.93 | 0.00 | 0.00 |
| time (sec) | N/A | 0.356 | 0.032 | 0.209 | 0.199 | 0.254 | 0.357 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 42 | 40 | 55 | 62 | 42 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.82 | 1.12 | 1.27 | 0.86 | 0.00 | 0.00 |
| time (sec) | N/A | 0.334 | 0.048 | 0.204 | 0.200 | 0.270 | 0.337 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 47 | 26 | 38 | 24 | 38 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.68 | 0.93 | 1.36 | 0.86 | 1.36 | 0.00 |
| time (sec) | N/A | 0.221 | 0.038 | 0.222 | 0.194 | 0.268 | 0.240 | 0.292 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 11 | 23 | 10 | 0 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.77 | 0.77 | 0.00 | 0.85 |
| time (sec) | N/A | 0.193 | 0.008 | 0.234 | 0.193 | 0.263 | 0.198 | 0.000 | 2.398 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 46 | 57 | 89 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.35 | 1.68 | 2.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.357 | 0.109 | 0.231 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 56 | 25 | 39 | 0 | 71 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 2.07 | 0.93 | 1.44 | 0.00 | 2.63 | 0.00 |
| time (sec) | N/A | 0.230 | 0.049 | 0.244 | 0.192 | 0.256 | 0.000 | 0.302 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 84 | 126 | 150 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 1.58 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.517 | 0.555 | 0.200 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 124 | 120 | 578 | 134 | 158 | 0 | 0 | 0 |
| N.S. | 1 | 0.71 | 0.69 | 3.30 | 0.77 | 0.90 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.382 | 0.128 | 0.271 | 0.206 | 0.271 | 0.000 | 0.000 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 158 | 129 | 338 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.71 | 1.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.612 | 0.617 | 0.175 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 80 | 92 | 321 | 73 | 127 | 0 | 0 | 0 |
| N.S. | 1 | 0.76 | 0.88 | 3.06 | 0.70 | 1.21 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.292 | 0.109 | 0.232 | 0.204 | 0.271 | 0.000 | 0.000 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 120 | 256 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 2.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.349 | 0.400 | 0.166 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 135 | 168 | 331 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.76 | 0.95 | 1.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.644 | 0.401 | 0.203 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 129 | 251 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.23 | 2.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.412 | 0.342 | 0.209 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 150 | 223 | 334 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.75 | 1.11 | 1.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.680 | 2.252 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 80 | 117 | 137 | 133 | 217 | 0 | 0 | 0 |
| N.S. | 1 | 0.75 | 1.10 | 1.29 | 1.25 | 2.05 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.314 | 0.330 | 0.199 | 0.199 | 0.284 | 0.000 | 0.000 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 135 | 130 | 872 | 145 | 199 | 0 | 0 | 0 |
| N.S. | 1 | 0.62 | 0.60 | 4.02 | 0.67 | 0.92 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.412 | 0.194 | 0.216 | 0.205 | 0.266 | 0.000 | 0.000 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 245 | 251 | 799 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.99 | 3.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.906 | 0.640 | 0.219 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 91 | 102 | 559 | 85 | 167 | 0 | 0 | 0 |
| N.S. | 1 | 0.62 | 0.70 | 3.83 | 0.58 | 1.14 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.296 | 0.134 | 0.204 | 0.197 | 0.275 | 0.000 | 0.000 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 196 | 200 | 496 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 1.11 | 2.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.556 | 0.696 | 0.182 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 249 | 210 | 248 | 428 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 1.00 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.973 | 0.631 | 0.270 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 189 | 200 | 230 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 1.13 | 1.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.624 | 0.834 | 0.249 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 218 | 352 | 289 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.81 | 1.30 | 1.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.997 | 3.471 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 188 | 217 | 257 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.18 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.691 | 0.634 | 0.249 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 145 | 140 | 996 | 156 | 263 | 0 | 0 | 0 |
| N.S. | 1 | 0.55 | 0.53 | 3.74 | 0.59 | 0.99 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.421 | 0.229 | 0.230 | 0.220 | 0.276 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 344 | 388 | 1165 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.15 | 3.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.331 | 0.896 | 0.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 100 | 112 | 863 | 96 | 225 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 0.58 | 4.47 | 0.50 | 1.17 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.302 | 0.152 | 0.216 | 0.207 | 0.274 | 0.000 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 267 | 317 | 801 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 1.25 | 3.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.752 | 0.664 | 0.185 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 297 | 339 | 540 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 1.03 | 1.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.368 | 0.712 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 289 | 270 | 285 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 1.05 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.891 | 1.332 | 0.201 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 306 | 424 | 339 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 1.19 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.436 | 6.407 | 0.243 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 285 | 286 | 322 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 1.08 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.989 | 0.992 | 0.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 28 | 26 | 28 | 40 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.81 | 0.88 | 1.25 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.241 | 0.007 | 0.232 | 0.282 | 0.255 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 226 | 119 | 625 | 174 | 161 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.55 | 2.91 | 0.81 | 0.75 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.668 | 0.173 | 0.217 | 0.212 | 0.273 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 200 | 151 | 519 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.79 | 2.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.645 | 0.691 | 0.205 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 145 | 93 | 358 | 117 | 132 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 0.65 | 2.52 | 0.82 | 0.93 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.449 | 0.140 | 0.196 | 0.204 | 0.272 | 0.000 | 0.000 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 121 | 273 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 2.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.405 | 0.656 | 0.211 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 74 | 148 | 55 | 96 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.16 | 2.31 | 0.86 | 1.50 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.246 | 0.152 | 0.236 | 0.207 | 0.283 | 0.000 | 0.000 | 0.000 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 76 | 77 | 28 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 1.64 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.210 | 0.078 | 0.206 | 0.208 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 79 | 129 | 233 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.65 | 1.06 | 1.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.421 | 0.292 | 0.249 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 77 | 169 | 101 | 132 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 2.68 | 1.60 | 2.10 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.277 | 0.192 | 0.235 | 0.209 | 0.284 | 0.000 | 0.000 | 0.000 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 153 | 229 | 333 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.75 | 1.13 | 1.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.667 | 2.285 | 0.203 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 139 | 131 | 166 | 121 | 222 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 0.93 | 1.18 | 0.86 | 1.57 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.471 | 0.235 | 0.221 | 0.205 | 0.300 | 0.000 | 0.000 | 0.000 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 154 | 174 | 238 | 0 | 197 | 0 | 0 | 0 |
| N.S. | 1 | 0.73 | 0.82 | 1.12 | 0.00 | 0.93 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.461 | 0.398 | 0.230 | 0.000 | 0.307 | 0.000 | 0.000 | 0.000 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 222 | 161 | 274 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 0.78 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.762 | 0.550 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 105 | 130 | 176 | 119 | 166 | 0 | 0 | 0 |
| N.S. | 1 | 0.77 | 0.96 | 1.29 | 0.88 | 1.22 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.369 | 0.355 | 0.231 | 0.280 | 0.288 | 0.000 | 0.000 | 0.000 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 146 | 232 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 1.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.436 | 0.331 | 0.195 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 86 | 125 | 0 | 128 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.23 | 1.79 | 0.00 | 1.83 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.272 | 0.264 | 0.240 | 0.000 | 0.300 | 0.000 | 0.000 | 0.000 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 100 | 143 | 58 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 1.88 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.241 | 0.174 | 0.177 | 0.209 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 146 | 231 | 271 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.75 | 1.19 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.684 | 0.683 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 113 | 163 | 239 | 119 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.79 | 1.14 | 1.67 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.381 | 0.605 | 0.194 | 0.202 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 235 | 369 | 342 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.82 | 1.29 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.059 | 6.182 | 0.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 167 | 216 | 968 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.73 | 0.95 | 4.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.481 | 0.324 | 0.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 345 | 222 | 410 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 0.79 | 1.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.203 | 0.936 | 0.253 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 164 | 177 | 400 | 0 | 219 | 0 | 0 | 0 |
| N.S. | 1 | 0.78 | 0.84 | 1.90 | 0.00 | 1.04 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.452 | 0.417 | 0.262 | 0.000 | 0.298 | 0.000 | 0.000 | 0.000 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 241 | 191 | 345 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.19 | 0.94 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.812 | 0.576 | 0.217 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 129 | 136 | 263 | 138 | 188 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.94 | 1.83 | 0.96 | 1.31 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.390 | 0.363 | 0.190 | 0.278 | 0.299 | 0.000 | 0.000 | 0.000 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 99 | 118 | 413 | 137 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.99 | 3.47 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.356 | 0.281 | 0.255 | 0.204 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 99 | 103 | 198 | 0 | 166 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.90 | 1.74 | 0.00 | 1.46 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.293 | 0.286 | 0.192 | 0.000 | 0.286 | 0.000 | 0.000 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 152 | 143 | 425 | 126 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 0.97 | 2.89 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.402 | 0.188 | 0.276 | 0.203 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 244 | 247 | 364 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.94 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.045 | 1.001 | 0.245 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 214 | 165 | 227 | 1257 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.77 | 1.06 | 5.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.490 | 0.329 | 0.248 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 400 | 356 | 409 | 406 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 1.02 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.583 | 5.360 | 0.231 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 297 | 213 | 267 | 372 | 236 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.72 | 0.90 | 1.25 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.642 | 0.360 | 0.216 | 0.216 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 213 | 121 | 363 | 143 | 0 | 0 | 124 | 0 |
| N.S. | 1 | 1.06 | 0.60 | 1.82 | 0.72 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.533 | 0.083 | 0.184 | 0.203 | 0.000 | 0.000 | 0.330 | 0.000 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 94 | 63 | 74 | 83 | 83 | 82 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.73 | 0.86 | 0.97 | 0.97 | 0.95 | 0.00 | 0.00 |
| time (sec) | N/A | 0.468 | 0.023 | 0.209 | 0.203 | 0.272 | 0.417 | 0.000 | 0.000 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 73 | 48 | 82 | 59 | 55 | 65 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.69 | 1.17 | 0.84 | 0.79 | 0.93 | 0.00 | 0.00 |
| time (sec) | N/A | 0.355 | 0.020 | 0.232 | 0.191 | 0.271 | 0.351 | 0.000 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 42 | 40 | 55 | 62 | 42 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.82 | 1.12 | 1.27 | 0.86 | 0.00 | 0.00 |
| time (sec) | N/A | 0.321 | 0.013 | 0.199 | 0.192 | 0.258 | 0.299 | 0.000 | 0.000 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 47 | 26 | 38 | 24 | 38 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.68 | 0.93 | 1.36 | 0.86 | 1.36 | 0.00 |
| time (sec) | N/A | 0.212 | 0.010 | 0.243 | 0.196 | 0.267 | 0.232 | 0.284 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 11 | 23 | 10 | 0 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.77 | 0.77 | 0.00 | 0.85 |
| time (sec) | N/A | 0.184 | 0.001 | 0.228 | 0.193 | 0.277 | 0.204 | 0.000 | 0.002 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 46 | 57 | 89 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.35 | 1.68 | 2.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.350 | 0.092 | 0.201 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 56 | 25 | 39 | 0 | 71 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 2.07 | 0.93 | 1.44 | 0.00 | 2.63 | 0.00 |
| time (sec) | N/A | 0.228 | 0.011 | 0.211 | 0.216 | 0.274 | 0.000 | 0.300 | 0.000 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 84 | 126 | 150 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 1.58 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.510 | 0.114 | 0.250 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 313 | 320 | 257 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.809 | 0.352 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 215 | 188 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.515 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 126 | 118 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.336 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 26 | 36 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 1.50 | 1.08 | 1.08 |
| time (sec) | N/A | 0.248 | 3.334 | 0.260 | 0.298 | 0.280 | 2.258 | 0.287 | 2.577 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 41 | 54 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.71 | 2.25 | 1.08 | 1.08 |
| time (sec) | N/A | 0.448 | 4.911 | 0.283 | 0.283 | 0.269 | 16.142 | 0.278 | 2.735 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 52 | 71 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 2.17 | 2.96 | 1.08 | 1.08 |
| time (sec) | N/A | 0.660 | 5.239 | 0.286 | 0.283 | 0.284 | 117.915 | 0.291 | 2.706 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 618 | 442 | 332 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.72 | 0.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.312 | 0.904 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 390 | 320 | 233 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.82 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.873 | 0.370 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 215 | 179 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.530 | 0.049 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 129 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.337 | 0.044 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 267 | 206 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.600 | 0.161 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 402 | 384 | 286 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.900 | 0.273 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 97 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.273 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 324 | 201 | 329 | 441 | 260 | 388 | 0 | 0 |
| N.S. | 1 | 1.14 | 0.71 | 1.16 | 1.56 | 0.92 | 1.37 | 0.00 | 0.00 |
| time (sec) | N/A | 1.488 | 0.197 | 0.166 | 0.230 | 0.268 | 0.921 | 0.000 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 328 | 186 | 266 | 442 | 240 | 332 | 0 | 0 |
| N.S. | 1 | 1.66 | 0.94 | 1.34 | 2.23 | 1.21 | 1.68 | 0.00 | 0.00 |
| time (sec) | N/A | 1.475 | 0.192 | 0.109 | 0.221 | 0.282 | 0.786 | 0.000 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 236 | 177 | 247 | 346 | 225 | 313 | 0 | 0 |
| N.S. | 1 | 1.15 | 0.86 | 1.20 | 1.68 | 1.09 | 1.52 | 0.00 | 0.00 |
| time (sec) | N/A | 1.135 | 0.179 | 0.114 | 0.222 | 0.275 | 0.548 | 0.000 | 0.000 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 150 | 155 | 182 | 347 | 204 | 269 | 0 | 0 |
| N.S. | 1 | 1.11 | 1.15 | 1.35 | 2.57 | 1.51 | 1.99 | 0.00 | 0.00 |
| time (sec) | N/A | 0.639 | 0.206 | 0.085 | 0.222 | 0.275 | 0.432 | 0.000 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 137 | 135 | 166 | 230 | 178 | 224 | 0 | 0 |
| N.S. | 1 | 1.10 | 1.08 | 1.33 | 1.84 | 1.42 | 1.79 | 0.00 | 0.00 |
| time (sec) | N/A | 0.571 | 0.108 | 0.067 | 0.212 | 0.275 | 0.235 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 228 | 226 | 415 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.37 | 1.36 | 2.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.390 | 0.292 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 167 | 192 | 239 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.27 | 1.47 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.166 | 0.316 | 0.120 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 227 | 241 | 404 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.26 | 1.34 | 2.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.651 | 0.226 | 0.195 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 208 | 245 | 240 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.32 | 1.55 | 1.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.163 | 0.609 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 386 | 506 | 251 | 427 | 760 | 368 | 563 | 0 | 0 |
| N.S. | 1 | 1.31 | 0.65 | 1.11 | 1.97 | 0.95 | 1.46 | 0.00 | 0.00 |
| time (sec) | N/A | 2.439 | 0.269 | 0.299 | 0.225 | 0.291 | 1.676 | 0.000 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 583 | 237 | 343 | 762 | 348 | 515 | 0 | 0 |
| N.S. | 1 | 1.97 | 0.80 | 1.16 | 2.57 | 1.18 | 1.74 | 0.00 | 0.00 |
| time (sec) | N/A | 2.443 | 0.240 | 0.217 | 0.251 | 0.280 | 1.320 | 0.000 | 0.000 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 303 | 380 | 227 | 345 | 619 | 327 | 483 | 0 | 0 |
| N.S. | 1 | 1.25 | 0.75 | 1.14 | 2.04 | 1.08 | 1.59 | 0.00 | 0.00 |
| time (sec) | N/A | 1.878 | 1.131 | 0.237 | 0.221 | 0.290 | 0.982 | 0.000 | 0.000 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 208 | 247 | 621 | 307 | 430 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.21 | 3.04 | 1.50 | 2.11 | 0.00 | 0.00 |
| time (sec) | N/A | 0.826 | 1.155 | 0.250 | 0.242 | 0.296 | 0.788 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 214 | 239 | 191 | 264 | 457 | 278 | 389 | 0 | 0 |
| N.S. | 1 | 1.12 | 0.89 | 1.23 | 2.14 | 1.30 | 1.82 | 0.00 | 0.00 |
| time (sec) | N/A | 0.918 | 0.926 | 0.202 | 0.234 | 0.285 | 0.461 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 378 | 326 | 573 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.47 | 1.27 | 2.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.492 | 0.411 | 0.310 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 302 | 306 | 364 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.32 | 1.34 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.079 | 0.878 | 0.243 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 272 | 375 | 329 | 584 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.38 | 1.21 | 2.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.611 | 0.754 | 0.277 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 348 | 357 | 363 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.40 | 1.44 | 1.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.162 | 0.662 | 0.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 465 | 692 | 299 | 519 | 1109 | 444 | 702 | 0 | 0 |
| N.S. | 1 | 1.49 | 0.64 | 1.12 | 2.38 | 0.95 | 1.51 | 0.00 | 0.00 |
| time (sec) | N/A | 3.823 | 0.284 | 0.306 | 0.231 | 0.284 | 3.485 | 0.000 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 376 | 899 | 285 | 414 | 1112 | 424 | 654 | 0 | 0 |
| N.S. | 1 | 2.39 | 0.76 | 1.10 | 2.96 | 1.13 | 1.74 | 0.00 | 0.00 |
| time (sec) | N/A | 3.902 | 0.280 | 0.295 | 0.275 | 0.269 | 2.552 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 382 | 526 | 275 | 437 | 922 | 403 | 626 | 0 | 0 |
| N.S. | 1 | 1.38 | 0.72 | 1.14 | 2.41 | 1.05 | 1.64 | 0.00 | 0.00 |
| time (sec) | N/A | 2.897 | 1.295 | 0.234 | 0.249 | 0.264 | 1.809 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 254 | 256 | 306 | 925 | 383 | 573 | 0 | 0 |
| N.S. | 1 | 0.97 | 0.98 | 1.17 | 3.54 | 1.47 | 2.20 | 0.00 | 0.00 |
| time (sec) | N/A | 1.064 | 1.376 | 0.220 | 0.247 | 0.264 | 1.392 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 342 | 239 | 354 | 712 | 354 | 524 | 0 | 0 |
| N.S. | 1 | 1.18 | 0.82 | 1.22 | 2.45 | 1.22 | 1.80 | 0.00 | 0.00 |
| time (sec) | N/A | 1.323 | 1.227 | 0.234 | 0.220 | 0.298 | 0.897 | 0.000 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 570 | 419 | 687 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.69 | 1.24 | 2.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.094 | 0.693 | 0.330 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 451 | 466 | 463 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.47 | 1.52 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.171 | 1.074 | 0.281 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 354 | 0 | 496 | 772 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 1.40 | 2.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.621 | 0.335 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 529 | 461 | 468 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.62 | 1.41 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.435 | 0.792 | 0.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 290 | 365 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.910 | 0.828 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 199 | 294 | 343 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.48 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.567 | 0.346 | 0.283 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 173 | 317 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.139 | 0.454 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 281 | 180 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.68 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.571 | 0.210 | 0.210 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 111 | 274 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 1.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.491 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 121 | 400 | 392 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 3.45 | 3.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.606 | 0.276 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 186 | 363 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.91 | 1.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.490 | 0.803 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 183 | 411 | 540 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 2.12 | 2.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.440 | 1.043 | 0.273 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 297 | 310 | 602 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 2.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.497 | 7.325 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 279 | 284 | 482 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.880 | 1.542 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 214 | 320 | 320 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.50 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.648 | 0.741 | 0.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 191 | 385 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.187 | 1.470 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 82 | 145 | 147 | 0 | 185 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 1.71 | 1.73 | 0.00 | 2.18 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.365 | 0.454 | 0.219 | 0.000 | 0.269 | 0.000 | 0.000 | 0.000 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 187 | 403 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.157 | 1.307 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 199 | 428 | 532 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 2.22 | 2.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.120 | 1.652 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 305 | 549 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 1.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.827 | 7.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 311 | 506 | 586 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 2.00 | 2.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.921 | 1.543 | 0.295 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 401 | 484 | 764 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.21 | 1.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.391 | 8.041 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 329 | 552 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.054 | 2.354 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 181 | 186 | 272 | 0 | 283 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 1.11 | 1.63 | 0.00 | 1.69 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.878 | 0.323 | 0.268 | 0.000 | 0.278 | 0.000 | 0.000 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 297 | 550 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 1.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.888 | 2.030 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 135 | 152 | 227 | 0 | 273 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 1.05 | 1.57 | 0.00 | 1.88 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.503 | 0.466 | 0.232 | 0.000 | 0.272 | 0.000 | 0.000 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 289 | 546 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 1.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.783 | 2.415 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 324 | 560 | 676 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 2.04 | 2.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.026 | 3.163 | 0.311 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 466 | 716 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.20 | 1.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.187 | 7.508 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 482 | 701 | 832 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.27 | 1.84 | 2.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.447 | 7.930 | 0.358 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 529 | 727 | 937 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.37 | 1.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 6.154 | 8.997 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 300 | 410 | 284 | 397 | 0 | 0 | 583 | 0 | 0 |
| N.S. | 1 | 1.37 | 0.95 | 1.32 | 0.00 | 0.00 | 1.94 | 0.00 | 0.00 |
| time (sec) | N/A | 1.426 | 1.871 | 0.197 | 0.000 | 0.000 | 28.503 | 0.000 | 0.000 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 253 | 202 | 289 | 0 | 0 | 405 | 0 | 0 |
| N.S. | 1 | 1.20 | 0.96 | 1.38 | 0.00 | 0.00 | 1.93 | 0.00 | 0.00 |
| time (sec) | N/A | 0.951 | 0.999 | 0.185 | 0.000 | 0.000 | 2.912 | 0.000 | 0.000 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 119 | 124 | 179 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 1.02 | 1.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.497 | 0.505 | 0.198 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 72 | 47 | 0 | 90 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.88 | 1.88 | 0.00 | 3.60 | 0.00 | 0.00 |
| time (sec) | N/A | 0.228 | 0.021 | 0.207 | 0.219 | 0.000 | 0.896 | 0.000 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 108 | 153 | 243 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.47 | 2.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.568 | 0.604 | 0.259 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 208 | 293 | 1729 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.44 | 8.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.203 | 0.798 | 0.256 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 358 | 351 | 222 | 1162 | 302 | 316 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.62 | 3.25 | 0.84 | 0.88 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.508 | 0.259 | 0.377 | 0.226 | 0.272 | 0.000 | 0.000 | 0.000 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 295 | 207 | 618 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.01 | 0.71 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.274 | 1.301 | 0.273 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 144 | 166 | 657 | 183 | 249 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 0.92 | 3.65 | 1.02 | 1.38 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.463 | 0.449 | 0.261 | 0.217 | 0.271 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 163 | 200 | 480 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 1.09 | 2.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.579 | 1.019 | 0.218 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 338 | 212 | 352 | 764 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.63 | 1.04 | 2.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.038 | 0.977 | 0.323 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 192 | 232 | 579 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 1.11 | 2.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.171 | 1.243 | 0.286 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 358 | 224 | 446 | 755 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.63 | 1.25 | 2.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.291 | 3.786 | 0.349 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 219 | 240 | 1729 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.74 | 0.82 | 5.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.010 | 0.681 | 0.335 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 482 | 541 | 251 | 1766 | 346 | 402 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 0.52 | 3.66 | 0.72 | 0.83 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.657 | 0.370 | 0.388 | 0.230 | 0.268 | 0.000 | 0.000 | 0.000 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 405 | 495 | 508 | 1552 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.22 | 1.25 | 3.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.308 | 2.111 | 0.361 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 183 | 198 | 1149 | 230 | 332 | 0 | 0 | 0 |
| N.S. | 1 | 0.69 | 0.74 | 4.30 | 0.86 | 1.24 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.568 | 1.239 | 0.374 | 0.227 | 0.274 | 0.000 | 0.000 | 0.000 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 320 | 329 | 959 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 1.12 | 3.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.066 | 2.341 | 0.262 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 498 | 351 | 520 | 1053 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.70 | 1.04 | 2.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.891 | 1.782 | 0.352 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 398 | 381 | 369 | 473 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.93 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.949 | 3.497 | 0.309 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 541 | 348 | 771 | 933 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.64 | 1.43 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.966 | 7.791 | 0.369 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 378 | 414 | 458 | 1929 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 1.21 | 5.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.580 | 1.322 | 0.368 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 625 | 771 | 277 | 2014 | 390 | 525 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 0.44 | 3.22 | 0.62 | 0.84 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.224 | 0.448 | 0.401 | 0.223 | 0.278 | 0.000 | 0.000 | 0.000 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 536 | 743 | 619 | 2280 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.39 | 1.15 | 4.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.734 | 2.836 | 0.404 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 366 | 220 | 224 | 1773 | 274 | 446 | 0 | 0 | 0 |
| N.S. | 1 | 0.60 | 0.61 | 4.84 | 0.75 | 1.22 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.702 | 1.393 | 0.370 | 0.228 | 0.276 | 0.000 | 0.000 | 0.000 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 420 | 502 | 499 | 1568 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.20 | 1.19 | 3.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.689 | 2.296 | 0.303 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 635 | 530 | 757 | 1321 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 1.19 | 2.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.227 | 3.560 | 0.323 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 530 | 622 | 550 | 589 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.17 | 1.04 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.475 | 2.788 | 0.345 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 687 | 526 | 990 | 720 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.77 | 1.44 | 1.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.355 | 8.037 | 0.331 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 561 | 0 | 616 | 2297 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 1.10 | 4.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 2.220 | 0.385 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 221 | 98 | 125 | 0 | 131 | 146 | 0 | 0 |
| N.S. | 1 | 1.44 | 0.64 | 0.82 | 0.00 | 0.86 | 0.95 | 0.00 | 0.00 |
| time (sec) | N/A | 1.025 | 0.055 | 0.216 | 0.000 | 0.279 | 0.592 | 0.000 | 0.000 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 154 | 79 | 113 | 101 | 98 | 121 | 0 | 0 |
| N.S. | 1 | 1.26 | 0.65 | 0.93 | 0.83 | 0.80 | 0.99 | 0.00 | 0.00 |
| time (sec) | N/A | 0.737 | 0.043 | 0.266 | 0.218 | 0.265 | 0.462 | 0.000 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 96 | 72 | 69 | 0 | 102 | 78 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.83 | 0.79 | 0.00 | 1.17 | 0.90 | 0.00 | 0.00 |
| time (sec) | N/A | 0.507 | 0.035 | 0.188 | 0.000 | 0.248 | 0.391 | 0.000 | 0.000 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 54 | 48 | 64 | 48 | 70 | 49 | 74 | 0 |
| N.S. | 1 | 1.04 | 0.92 | 1.23 | 0.92 | 1.35 | 0.94 | 1.42 | 0.00 |
| time (sec) | N/A | 0.300 | 0.027 | 0.276 | 0.196 | 0.248 | 0.333 | 0.298 | 0.000 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 11 | 23 | 10 | 0 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.77 | 0.77 | 0.00 | 0.85 |
| time (sec) | N/A | 0.197 | 0.007 | 0.202 | 0.189 | 0.275 | 0.234 | 0.000 | 3.065 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 80 | 100 | 144 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 1.47 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.487 | 0.117 | 0.276 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 83 | 65 | 132 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.26 | 0.98 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.536 | 0.264 | 0.190 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 141 | 188 | 233 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.39 | 1.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.950 | 0.919 | 0.302 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 397 | 230 | 1227 | 353 | 319 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.60 | 3.20 | 0.92 | 0.83 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.642 | 0.308 | 0.331 | 0.213 | 0.275 | 0.000 | 0.000 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 337 | 268 | 992 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.83 | 3.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.423 | 1.047 | 0.256 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 265 | 236 | 176 | 706 | 243 | 254 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 0.66 | 2.66 | 0.92 | 0.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.907 | 0.295 | 0.291 | 0.215 | 0.292 | 0.000 | 0.000 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 171 | 198 | 506 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 0.97 | 2.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.651 | 1.197 | 0.230 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 95 | 127 | 296 | 125 | 179 | 0 | 0 | 0 |
| N.S. | 1 | 0.69 | 0.92 | 2.14 | 0.91 | 1.30 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.337 | 0.571 | 0.236 | 0.202 | 0.275 | 0.000 | 0.000 | 0.000 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 115 | 120 | 47 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.45 | 2.55 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.236 | 0.146 | 0.211 | 0.202 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 125 | 266 | 546 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.56 | 1.19 | 2.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.631 | 0.762 | 0.281 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 150 | 168 | 478 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 1.01 | 2.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.744 | 0.687 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 360 | 227 | 455 | 780 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.63 | 1.26 | 2.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.369 | 3.807 | 0.307 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 263 | 278 | 1506 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 0.93 | 5.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.412 | 0.611 | 0.309 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 515 | 477 | 427 | 520 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.83 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.826 | 0.810 | 0.325 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 400 | 383 | 288 | 579 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.72 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.986 | 1.702 | 0.286 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 265 | 318 | 412 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.69 | 0.83 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.338 | 0.703 | 0.301 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 187 | 215 | 478 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 0.92 | 2.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.968 | 1.351 | 0.254 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 121 | 217 | 318 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.64 | 1.15 | 1.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.602 | 0.906 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 133 | 152 | 343 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.74 | 0.85 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.621 | 0.749 | 0.282 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 412 | 242 | 568 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.59 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.988 | 1.414 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 265 | 296 | 1117 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.97 | 3.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.020 | 1.332 | 0.298 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 573 | 404 | 884 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.71 | 1.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.180 | 7.416 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 452 | 446 | 438 | 2608 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 0.97 | 5.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.260 | 0.940 | 0.368 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 512 | 523 | 333 | 1042 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 0.65 | 2.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.557 | 1.594 | 0.405 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 398 | 408 | 359 | 766 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 0.90 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.825 | 1.224 | 0.307 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 315 | 301 | 704 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 0.98 | 2.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.995 | 1.124 | 0.204 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 312 | 209 | 280 | 2352 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.67 | 0.90 | 7.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.985 | 1.399 | 0.287 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 178 | 254 | 591 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.66 | 0.94 | 2.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.767 | 1.344 | 0.293 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 258 | 236 | 2131 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 0.81 | 7.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.359 | 1.449 | 0.279 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 518 | 419 | 547 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.81 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.060 | 3.211 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 421 | 436 | 408 | 3513 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.97 | 8.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.301 | 2.142 | 0.349 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | F | F | F | F | F | F(-1) |
| verified | N/A | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 687 | 0 | 983 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 7.775 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 506 | 694 | 417 | 3506 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.37 | 0.82 | 6.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 5.086 | 2.596 | 0.367 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 366 | 369 | 178 | 570 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.01 | 0.49 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.728 | 0.640 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 133 | 0 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 4.75 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 3.274 | 3.615 | 1.048 | 0.356 | 0.266 | 0.000 | 0.000 | 3.121 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 80 | 0 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.86 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 1.657 | 0.914 | 0.852 | 0.312 | 0.266 | 0.000 | 0.000 | 3.066 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 40 | 27 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.43 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.860 | 0.655 | 0.706 | 0.322 | 0.263 | 21.286 | 0.000 | 3.184 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 40 | 27 | 28 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.43 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.314 | 3.420 | 0.303 | 0.305 | 0.261 | 7.796 | 0.356 | 3.109 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 67 | 27 | 28 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.39 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.326 | 4.435 | 0.397 | 0.358 | 0.259 | 9.435 | 0.356 | 2.989 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 78 | 27 | 28 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.79 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.326 | 4.604 | 0.455 | 0.359 | 0.275 | 165.867 | 0.377 | 2.998 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 23 | 22 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.259 | 0.460 | 0.175 | 0.297 | 0.260 | 1.150 | 0.335 | 3.097 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 359 | 563 | 169 | 270 | 276 | 248 | 355 | 0 | 0 |
| N.S. | 1 | 1.57 | 0.47 | 0.75 | 0.77 | 0.69 | 0.99 | 0.00 | 0.00 |
| time (sec) | N/A | 2.568 | 0.191 | 0.215 | 0.207 | 0.280 | 1.189 | 0.000 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 265 | 367 | 137 | 200 | 210 | 204 | 262 | 0 | 0 |
| N.S. | 1 | 1.38 | 0.52 | 0.75 | 0.79 | 0.77 | 0.99 | 0.00 | 0.00 |
| time (sec) | N/A | 1.431 | 0.102 | 0.292 | 0.211 | 0.272 | 0.573 | 0.000 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 196 | 99 | 128 | 124 | 140 | 150 | 0 | 0 |
| N.S. | 1 | 1.28 | 0.65 | 0.84 | 0.81 | 0.92 | 0.98 | 0.00 | 0.00 |
| time (sec) | N/A | 0.816 | 0.093 | 0.046 | 0.199 | 0.265 | 0.300 | 0.000 | 0.000 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 141 | 223 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.81 | 1.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.596 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 254 | 568 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 1.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.688 | 1.790 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 409 | 414 | 654 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.01 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.859 | 3.736 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 509 | 591 | 177 | 802 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.16 | 0.35 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 4.202 | 0.603 | 0.242 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 348 | 354 | 136 | 484 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 0.39 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.224 | 0.278 | 0.212 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 164 | 86 | 231 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 0.42 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.810 | 0.162 | 0.218 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 41 | 39 | 14 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.98 | 0.35 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.219 | 0.047 | 0.237 | 0.203 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 218 | 138 | 133 | 262 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.63 | 0.61 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.718 | 0.255 | 0.278 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 363 | 273 | 195 | 550 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.75 | 0.54 | 1.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.655 | 0.474 | 0.297 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 515 | 458 | 297 | 888 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 0.58 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.977 | 0.590 | 0.294 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 23 | 22 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.250 | 0.470 | 0.178 | 0.299 | 0.264 | 2.539 | 0.363 | 2.549 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 271 | 121 | 156 | 0 | 166 | 185 | 0 | 0 |
| N.S. | 1 | 1.45 | 0.65 | 0.83 | 0.00 | 0.89 | 0.99 | 0.00 | 0.00 |
| time (sec) | N/A | 1.560 | 0.060 | 0.199 | 0.000 | 0.258 | 0.730 | 0.000 | 0.000 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 203 | 98 | 164 | 127 | 128 | 148 | 0 | 0 |
| N.S. | 1 | 1.33 | 0.64 | 1.07 | 0.83 | 0.84 | 0.97 | 0.00 | 0.00 |
| time (sec) | N/A | 1.283 | 0.049 | 0.260 | 0.204 | 0.264 | 0.518 | 0.000 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 114 | 83 | 84 | 0 | 128 | 100 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.79 | 0.80 | 0.00 | 1.22 | 0.95 | 0.00 | 0.00 |
| time (sec) | N/A | 0.770 | 0.058 | 0.214 | 0.000 | 0.251 | 0.406 | 0.000 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 69 | 58 | 90 | 61 | 92 | 61 | 101 | 0 |
| N.S. | 1 | 1.08 | 0.91 | 1.41 | 0.95 | 1.44 | 0.95 | 1.58 | 0.00 |
| time (sec) | N/A | 0.403 | 0.030 | 0.287 | 0.198 | 0.251 | 0.343 | 0.329 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 11 | 23 | 10 | 0 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.77 | 0.77 | 0.00 | 0.85 |
| time (sec) | N/A | 0.199 | 0.006 | 0.254 | 0.203 | 0.250 | 0.246 | 0.000 | 2.552 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 122 | 146 | 197 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.20 | 1.43 | 1.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.614 | 0.136 | 0.320 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | C | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 103 | 97 | 187 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.17 | 1.10 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.664 | 0.225 | 0.209 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 218 | 304 | 377 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.45 | 1.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.355 | 3.265 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 50 | 43 | 42 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.75 | 0.64 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.350 | 0.137 | 0.208 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 39 | 34 | 33 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.78 | 0.68 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.332 | 0.073 | 0.225 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 26 | 23 | 22 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.79 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.299 | 0.026 | 0.041 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 21 | 21 | 20 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 1.11 | 1.05 | 1.11 | 1.11 |
| time (sec) | N/A | 0.210 | 0.491 | 0.200 | 0.247 | 0.245 | 0.526 | 0.294 | 2.587 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 21 | 36 | 36 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 1.89 | 1.89 | 1.11 | 1.11 |
| time (sec) | N/A | 0.205 | 1.529 | 0.068 | 0.249 | 0.248 | 0.984 | 0.319 | 2.543 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 171 | 152 | 163 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.74 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.638 | 0.297 | 1.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 154 | 135 | 148 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 0.74 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.635 | 0.255 | 0.286 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 71 | 65 | 67 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.79 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.475 | 0.186 | 0.281 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 104 | 91 | 100 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 0.75 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.499 | 0.204 | 0.226 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 71 | 63 | 67 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.77 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.389 | 0.231 | 0.308 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 27 | 22 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.81 | 0.00 | 1.00 |
| time (sec) | N/A | 0.587 | 2.529 | 0.140 | 0.265 | 0.256 | 0.714 | 0.000 | 2.547 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 31 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.15 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.470 | 2.092 | 0.107 | 0.274 | 0.263 | 0.620 | 0.291 | 2.557 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 31 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.15 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.291 | 4.321 | 0.161 | 0.263 | 0.255 | 0.655 | 0.000 | 2.501 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 31 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.15 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.290 | 1.529 | 0.174 | 0.272 | 0.240 | 0.809 | 0.276 | 2.635 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 204 | 179 | 196 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.73 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.693 | 0.530 | 0.241 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 171 | 152 | 163 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.74 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.615 | 0.428 | 0.404 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 154 | 136 | 149 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 0.74 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.561 | 0.401 | 0.216 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 121 | 109 | 115 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 0.76 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.450 | 0.306 | 0.421 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 27 | 22 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.81 | 0.00 | 1.00 |
| time (sec) | N/A | 0.908 | 2.433 | 0.076 | 0.265 | 0.253 | 2.143 | 0.000 | 2.548 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 31 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.15 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.745 | 2.628 | 0.078 | 0.263 | 0.249 | 1.630 | 0.297 | 2.529 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 31 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.15 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.320 | 4.359 | 0.088 | 0.268 | 0.258 | 1.944 | 0.000 | 2.536 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 31 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.15 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.309 | 1.576 | 0.171 | 0.259 | 0.252 | 2.592 | 0.308 | 2.548 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 204 | 180 | 197 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.73 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.693 | 0.808 | 0.252 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 221 | 197 | 211 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.82 | 0.74 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.699 | 0.734 | 0.507 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 204 | 180 | 197 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.73 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.658 | 0.683 | 0.260 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 171 | 153 | 163 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.74 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.525 | 0.562 | 0.465 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 44 | 22 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.63 | 0.81 | 0.00 | 1.00 |
| time (sec) | N/A | 1.294 | 2.399 | 0.076 | 0.264 | 0.246 | 5.234 | 0.000 | 2.666 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 48 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.78 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 1.063 | 2.408 | 0.079 | 0.273 | 0.252 | 3.881 | 0.319 | 2.682 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 48 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.78 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.312 | 4.280 | 0.093 | 0.270 | 0.258 | 4.047 | 0.000 | 2.720 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 48 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.78 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.311 | 1.728 | 0.181 | 0.278 | 0.246 | 4.635 | 0.321 | 2.650 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 36 | 31 | 30 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 0.76 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.386 | 0.071 | 0.221 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 32 | 22 | 23 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.19 | 0.81 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.373 | 0.143 | 0.264 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 25 | 22 | 21 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.81 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.361 | 0.088 | 0.230 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 25 | 22 | 21 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.81 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.357 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 10 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.287 | 0.098 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 10 | 9 | 21 | 7 | 0 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 1.00 | 2.33 | 0.78 | 0.00 | 1.00 |
| time (sec) | N/A | 0.207 | 0.033 | 0.248 | 0.189 | 0.248 | 0.211 | 0.000 | 2.715 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 31 | 20 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.35 | 0.87 | 1.00 | 1.00 |
| time (sec) | N/A | 0.261 | 1.267 | 0.236 | 0.260 | 0.254 | 0.453 | 0.276 | 2.685 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 33 | 22 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.43 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.258 | 0.116 | 0.067 | 0.268 | 0.248 | 0.451 | 0.289 | 2.794 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 169 | 136 | 149 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 0.74 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.597 | 0.442 | 0.229 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 121 | 109 | 115 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 0.76 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.526 | 0.351 | 0.361 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 115 | 92 | 101 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 0.76 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.534 | 0.334 | 0.270 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 71 | 65 | 67 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.79 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.454 | 0.289 | 0.232 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 57 | 46 | 53 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 0.85 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.484 | 0.172 | 0.267 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 17 | 16 | 28 | 26 | 0 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 1.75 | 1.62 | 0.00 | 1.00 |
| time (sec) | N/A | 0.214 | 0.048 | 0.303 | 0.193 | 0.265 | 0.920 | 0.000 | 2.752 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 45 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.67 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.297 | 2.150 | 0.107 | 0.263 | 0.254 | 1.143 | 0.000 | 2.921 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 49 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.81 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.293 | 1.221 | 0.065 | 0.272 | 0.256 | 0.934 | 0.286 | 2.803 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 62 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.30 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.303 | 2.285 | 0.111 | 0.265 | 0.258 | 1.119 | 0.277 | 2.695 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 60 | 22 | 0 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.40 | 0.88 | 0.00 | 1.00 |
| time (sec) | N/A | 0.266 | 2.226 | 0.115 | 0.258 | 0.258 | 1.126 | 0.000 | 2.672 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 59 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.46 | 0.92 | 1.00 | 1.00 |
| time (sec) | N/A | 0.223 | 0.104 | 0.064 | 0.258 | 0.254 | 1.255 | 0.287 | 2.569 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 63 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.33 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.303 | 2.323 | 0.159 | 0.276 | 0.255 | 2.753 | 0.000 | 2.618 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 67 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.48 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.299 | 1.964 | 0.144 | 0.267 | 0.265 | 1.606 | 0.292 | 2.668 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 44 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.63 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.296 | 1.094 | 0.669 | 0.264 | 0.247 | 178.266 | 0.000 | 2.708 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 27 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.291 | 0.623 | 0.620 | 0.257 | 0.256 | 14.486 | 0.000 | 2.810 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 27 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.279 | 0.164 | 0.507 | 0.257 | 0.258 | 0.735 | 0.000 | 2.911 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 44 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.63 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.283 | 0.554 | 0.125 | 0.278 | 0.251 | 0.862 | 0.280 | 2.793 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 27 | 62 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.30 | 0.89 | 1.00 | 1.00 |
| time (sec) | N/A | 0.295 | 0.768 | 0.293 | 0.271 | 0.259 | 2.943 | 0.297 | 2.763 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 79 | 82 | 106 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 0.87 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.486 | 0.316 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 68 | 69 | 84 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 0.90 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.473 | 0.245 | 0.338 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 53 | 54 | 60 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 1.00 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.413 | 0.369 | 0.049 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 226 | 21 | 24 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 11.89 | 1.11 | 1.26 | 1.11 | 1.11 |
| time (sec) | N/A | 0.349 | 1.660 | 0.265 | 0.346 | 0.251 | 0.647 | 0.272 | 2.835 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 325 | 36 | 41 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 17.11 | 1.89 | 2.16 | 1.11 | 1.11 |
| time (sec) | N/A | 0.345 | 4.023 | 0.079 | 0.381 | 0.252 | 1.463 | 0.282 | 2.910 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 291 | 175 | 633 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.37 | 0.82 | 2.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.915 | 0.518 | 0.405 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 201 | 82 | 144 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 2.16 | 0.88 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.117 | 0.309 | 0.378 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 183 | 126 | 364 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 0.85 | 2.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.109 | 0.328 | 0.294 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 89 | 73 | 138 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.86 | 1.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.650 | 0.176 | 0.319 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 392 | 42 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 14.52 | 1.56 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 0.850 | 10.986 | 0.268 | 0.471 | 0.263 | 1.083 | 0.000 | 2.815 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 378 | 48 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 14.00 | 1.78 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.367 | 2.422 | 0.147 | 0.436 | 0.258 | 1.014 | 0.295 | 2.778 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 403 | 48 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 14.93 | 1.78 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.278 | 16.417 | 0.209 | 0.515 | 0.299 | 1.176 | 0.000 | 2.797 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 406 | 48 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 15.04 | 1.78 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.277 | 4.538 | 0.207 | 0.520 | 0.455 | 1.433 | 0.287 | 2.899 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 393 | 399 | 958 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.42 | 1.44 | 3.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.105 | 0.673 | 0.308 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 251 | 306 | 369 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 1.40 | 1.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.954 | 0.621 | 0.680 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 290 | 295 | 633 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.36 | 1.38 | 2.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.301 | 0.555 | 0.269 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 139 | 122 | 255 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.82 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.588 | 0.414 | 0.487 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 433 | 42 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 16.04 | 1.56 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 1.004 | 7.858 | 0.115 | 0.583 | 0.270 | 3.027 | 0.000 | 2.721 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 443 | 48 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 16.41 | 1.78 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.514 | 3.679 | 0.282 | 0.578 | 0.294 | 3.061 | 0.314 | 2.738 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 441 | 48 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 16.33 | 1.78 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.303 | 12.482 | 0.350 | 0.565 | 0.246 | 3.945 | 0.000 | 2.743 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 428 | 48 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 15.85 | 1.78 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.439 | 3.560 | 0.204 | 0.524 | 0.250 | 5.335 | 0.317 | 2.904 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 493 | 408 | 1070 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.78 | 1.47 | 3.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.352 | 1.108 | 0.344 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 401 | 413 | 484 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.43 | 1.47 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.228 | 1.024 | 0.655 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 390 | 404 | 958 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.42 | 1.47 | 3.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.625 | 0.874 | 0.284 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 189 | 311 | 374 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.44 | 1.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.659 | 0.601 | 0.560 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 480 | 59 | 24 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 17.78 | 2.19 | 0.89 | 0.00 | 1.00 |
| time (sec) | N/A | 1.219 | 9.485 | 0.128 | 0.620 | 0.257 | 7.711 | 0.000 | 2.681 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 491 | 65 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 18.19 | 2.41 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.569 | 3.275 | 0.125 | 0.686 | 0.253 | 8.279 | 0.331 | 2.903 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 492 | 65 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 18.22 | 2.41 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.301 | 12.584 | 0.141 | 0.691 | 0.247 | 7.788 | 0.000 | 2.737 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 491 | 65 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 18.19 | 2.41 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.302 | 3.826 | 0.523 | 0.666 | 0.257 | 11.445 | 0.372 | 2.722 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 177 | 158 | 633 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.77 | 3.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.680 | 0.311 | 0.266 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 131 | 117 | 243 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.83 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.631 | 0.240 | 0.326 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 127 | 113 | 364 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 0.80 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.609 | 0.225 | 0.259 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 83 | 70 | 135 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.89 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.716 | 0.167 | 0.283 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 69 | 60 | 151 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 0.82 | 2.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.557 | 0.151 | 0.284 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 19 | 18 | 30 | 36 | 0 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 1.67 | 2.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.212 | 0.014 | 0.254 | 0.197 | 0.258 | 2.024 | 0.000 | 2.653 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 416 | 75 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 15.41 | 2.78 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.371 | 6.476 | 0.147 | 0.485 | 0.255 | 1.696 | 0.000 | 2.808 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 427 | 81 | 27 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 15.81 | 3.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.371 | 2.017 | 0.080 | 0.486 | 0.245 | 1.479 | 0.284 | 2.721 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 475 | 105 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 17.59 | 3.89 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.299 | 53.718 | 0.302 | 0.564 | 0.252 | 1.831 | 0.000 | 2.607 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 447 | 105 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 16.56 | 3.89 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.442 | 3.146 | 0.123 | 0.464 | 0.252 | 1.798 | 0.314 | 2.600 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 461 | 103 | 24 | 0 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 18.44 | 4.12 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.262 | 54.144 | 0.145 | 0.482 | 0.255 | 1.836 | 0.000 | 2.638 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 446 | 102 | 24 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 18.58 | 4.25 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.364 | 2.189 | 0.063 | 0.452 | 0.268 | 2.045 | 0.297 | 2.537 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 477 | 107 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 17.67 | 3.96 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.291 | 35.978 | 0.203 | 0.556 | 0.248 | 4.461 | 0.000 | 2.560 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 485 | 113 | 27 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 17.96 | 4.19 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.304 | 15.906 | 0.186 | 0.543 | 0.259 | 2.952 | 0.297 | 2.587 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 0 | 25 | 584 | 137 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 0.00 | 0.93 | 21.63 | 5.07 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.297 | 0.000 | 0.194 | 0.646 | 0.264 | 3.910 | 0.000 | 2.702 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 585 | 137 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 21.67 | 5.07 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.294 | 10.246 | 0.198 | 0.562 | 0.248 | 3.739 | 0.340 | 2.625 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 0 | 23 | 568 | 135 | 24 | 0 | 25 |
| N.S. | 1 | 1.00 | 0.00 | 0.92 | 22.72 | 5.40 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.260 | 0.000 | 0.179 | 0.629 | 0.264 | 3.643 | 0.000 | 2.529 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 554 | 134 | 24 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 23.08 | 5.58 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.372 | 4.409 | 0.178 | 0.497 | 0.253 | 3.826 | 0.289 | 2.582 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 0 | 25 | 584 | 139 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 0.00 | 0.93 | 21.63 | 5.15 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.299 | 0.000 | 0.328 | 0.617 | 0.260 | 8.170 | 0.000 | 2.677 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 592 | 145 | 27 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 21.93 | 5.37 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.296 | 11.459 | 0.427 | 0.623 | 0.256 | 6.013 | 0.314 | 2.635 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 537 | 58 | 0 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 19.89 | 2.15 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.295 | 1.239 | 0.682 | 1.276 | 0.258 | 0.000 | 0.000 | 2.736 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 480 | 41 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 17.78 | 1.52 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.297 | 0.752 | 0.562 | 1.092 | 0.247 | 34.669 | 0.000 | 2.824 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 424 | 41 | 26 | 0 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 15.70 | 1.52 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.279 | 0.196 | 0.547 | 0.850 | 0.252 | 1.574 | 0.000 | 2.661 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 441 | 73 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 16.33 | 2.70 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.387 | 0.593 | 0.145 | 0.525 | 0.253 | 2.842 | 0.322 | 2.659 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 506 | 105 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 18.74 | 3.89 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.295 | 0.844 | 0.340 | 0.740 | 0.262 | 11.888 | 0.321 | 2.707 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 29 | 25 | 613 | 137 | 26 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 22.70 | 5.07 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.302 | 1.115 | 0.361 | 0.834 | 0.256 | 82.637 | 0.340 | 2.767 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 11 | 23 | 12 | 0 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.77 | 0.92 | 0.00 | 0.85 |
| time (sec) | N/A | 0.202 | 0.011 | 0.251 | 0.192 | 0.242 | 0.430 | 0.000 | 2.681 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 458 | 232 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.80 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.452 | 0.768 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 335 | 520 | 436 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.55 | 1.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.524 | 0.684 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 268 | 227 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.709 | 0.402 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 226 | 295 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.746 | 0.643 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 24 | 26 | 0 | 83 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 3.19 | 0.00 | 1.00 |
| time (sec) | N/A | 1.624 | 1.346 | 0.131 | 0.719 | 0.000 | 4.457 | 0.000 | 2.849 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 474 | 566 | 462 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.19 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.618 | 0.889 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 457 | 718 | 577 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.57 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.894 | 1.586 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 358 | 562 | 351 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.57 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.308 | 0.571 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 346 | 326 | 440 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.918 | 1.593 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 0 | 131 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 0.00 | 4.68 | 0.00 | 1.00 |
| time (sec) | N/A | 2.256 | 1.089 | 0.158 | 0.745 | 0.000 | 5.971 | 0.000 | 2.835 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 319 | 332 | 142 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.018 | 0.183 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 167 | 104 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.012 | 0.108 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 43 | 36 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.219 | 0.061 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 22 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 0.96 | 0.91 | 0.91 |
| time (sec) | N/A | 0.361 | 1.729 | 0.283 | 0.385 | 0.000 | 1.428 | 0.456 | 2.617 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 22 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 0.96 | 0.91 | 0.91 |
| time (sec) | N/A | 0.627 | 2.160 | 0.341 | 0.396 | 0.000 | 23.511 | 0.476 | 2.798 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 449 | 401 | 186 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 0.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.321 | 0.361 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 192 | 126 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.71 | 0.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.920 | 0.297 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 43 | 36 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.217 | 0.057 | 0.252 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 22 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 0.96 | 0.91 | 0.91 |
| time (sec) | N/A | 0.350 | 1.910 | 0.270 | 0.395 | 0.000 | 9.759 | 0.649 | 2.792 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | F | F | F(-2) | F(-1) | F(-2) | F(-1) |
| verified | N/A | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 514 | 0 | 201 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 0.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.373 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 238 | 135 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 0.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.601 | 0.319 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 43 | 36 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.224 | 0.057 | 0.261 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 0 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 0.00 | 0.91 | 0.91 |
| time (sec) | N/A | 0.351 | 1.882 | 0.253 | 0.399 | 0.000 | 0.000 | 0.656 | 2.623 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 332 | 156 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.946 | 0.145 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 162 | 110 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.025 | 0.068 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 39 | 34 | 0 | 20 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.00 | 0.51 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.208 | 0.022 | 0.266 | 0.000 | 0.241 | 0.000 | 0.000 | 0.000 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 18 | 20 | 0 | 19 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.09 | 0.82 | 0.91 | 0.00 | 0.86 | 0.91 | 0.91 |
| time (sec) | N/A | 0.339 | 1.312 | 0.293 | 0.392 | 0.000 | 1.392 | 0.562 | 2.888 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 18 | 20 | 0 | 19 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.09 | 0.82 | 0.91 | 0.00 | 0.86 | 0.91 | 0.91 |
| time (sec) | N/A | 0.588 | 1.365 | 0.348 | 0.384 | 0.000 | 23.343 | 0.560 | 2.761 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 433 | 410 | 210 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 0.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.407 | 0.300 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 195 | 133 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.75 | 0.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.981 | 0.252 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 39 | 34 | 0 | 20 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.00 | 0.51 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.204 | 0.032 | 0.256 | 0.000 | 0.249 | 0.000 | 0.000 | 0.000 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 18 | 20 | 0 | 19 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.09 | 0.82 | 0.91 | 0.00 | 0.86 | 0.91 | 0.91 |
| time (sec) | N/A | 0.337 | 1.344 | 0.263 | 0.386 | 0.000 | 9.844 | 0.854 | 2.661 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 41 | 34 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.24 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.352 | 0.061 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 396 | 199 | 197 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.50 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.510 | 0.278 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 264 | 141 | 141 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.53 | 0.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.438 | 0.173 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 96 | 101 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.62 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.378 | 0.108 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 41 | 36 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.219 | 0.055 | 0.255 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 24 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 1.04 | 0.91 | 0.91 |
| time (sec) | N/A | 0.215 | 1.832 | 0.265 | 0.410 | 0.000 | 3.699 | 0.498 | 2.743 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 24 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 1.04 | 0.91 | 0.91 |
| time (sec) | N/A | 0.213 | 2.068 | 0.342 | 0.393 | 0.000 | 58.041 | 0.495 | 2.722 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 391 | 229 | 399 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.59 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.637 | 0.893 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 171 | 225 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.67 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.560 | 0.376 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 136 | 115 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.621 | 0.225 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 41 | 36 | 0 | 57 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.90 | 0.00 | 1.42 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.213 | 0.057 | 0.263 | 0.000 | 0.241 | 0.000 | 0.000 | 0.000 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 24 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 1.04 | 0.91 | 0.91 |
| time (sec) | N/A | 0.364 | 1.914 | 0.260 | 0.412 | 0.000 | 25.933 | 0.347 | 2.693 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 0 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 0.00 | 0.91 | 0.91 |
| time (sec) | N/A | 0.364 | 1.994 | 0.334 | 0.400 | 0.000 | 0.000 | 0.359 | 2.766 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 232 | 262 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.78 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.274 | 0.354 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 165 | 122 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.91 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.582 | 0.154 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 43 | 36 | 0 | 57 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.86 | 0.00 | 1.36 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.224 | 0.056 | 0.268 | 0.000 | 0.249 | 0.000 | 0.000 | 0.000 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 0 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 0.00 | 0.91 | 0.91 |
| time (sec) | N/A | 0.367 | 1.854 | 0.267 | 0.405 | 0.000 | 0.000 | 0.353 | 2.541 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 21 | 0 | 0 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.91 | 0.00 | 0.00 | 0.91 | 0.91 |
| time (sec) | N/A | 0.353 | 2.024 | 0.307 | 0.407 | 0.000 | 0.000 | 0.384 | 2.655 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 178 | 171 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.76 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.562 | 0.750 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 273 | 229 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.77 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.634 | 0.810 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 178 | 160 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.76 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.512 | 0.497 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 28 | 26 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.93 | 0.00 | 1.00 |
| time (sec) | N/A | 0.771 | 0.298 | 0.222 | 0.406 | 0.262 | 2.233 | 0.000 | 2.618 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 28 | 27 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.96 | 0.00 | 1.00 |
| time (sec) | N/A | 0.556 | 0.334 | 0.226 | 0.406 | 0.264 | 4.094 | 0.000 | 2.600 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 616 | 437 | 429 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.71 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.878 | 2.342 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 542 | 397 | 390 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.73 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.760 | 1.498 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 420 | 301 | 288 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.72 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.611 | 1.092 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 28 | 26 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.93 | 0.00 | 1.00 |
| time (sec) | N/A | 1.285 | 0.362 | 0.211 | 0.405 | 0.290 | 125.890 | 0.000 | 2.557 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 28 | 0 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.00 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.979 | 0.946 | 0.221 | 0.403 | 0.262 | 0.000 | 0.000 | 2.562 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 816 | 561 | 667 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.69 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.030 | 5.616 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 745 | 526 | 685 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.71 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.898 | 2.329 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 632 | 441 | 529 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.70 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.753 | 3.996 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 53 | 0 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.89 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 2.067 | 0.398 | 0.189 | 0.426 | 0.260 | 0.000 | 0.000 | 2.440 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 26 | 28 | 53 | 0 | 0 | 28 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 1.89 | 0.00 | 0.00 | 1.00 |
| time (sec) | N/A | 1.539 | 0.654 | 0.225 | 0.432 | 0.260 | 0.000 | 0.000 | 2.557 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 23 | 22 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.272 | 0.650 | 0.129 | 0.374 | 0.254 | 3.799 | 0.412 | 2.616 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 116 | 100 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.471 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 75 | 86 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.428 | 0.212 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 54 | 43 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.345 | 0.101 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 18 | 17 | 83 | 34 | 29 | 33 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 4.88 | 2.00 | 1.71 | 1.94 |
| time (sec) | N/A | 0.208 | 0.011 | 0.272 | 0.194 | 0.268 | 0.357 | 0.275 | 2.933 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 31 | 20 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.35 | 0.87 | 1.00 | 1.00 |
| time (sec) | N/A | 0.269 | 5.709 | 0.108 | 0.365 | 0.265 | 0.678 | 0.428 | 2.683 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 33 | 22 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.43 | 0.96 | 1.00 | 1.00 |
| time (sec) | N/A | 0.268 | 1.858 | 0.108 | 0.395 | 0.266 | 1.044 | 0.422 | 2.732 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 416 | 192 | 361 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.46 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.833 | 2.791 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 148 | 273 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.599 | 2.825 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 96 | 233 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.65 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.517 | 1.261 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 96 | 227 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.61 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.572 | 1.520 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 115 | 283 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.64 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.639 | 2.550 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 123 | 141 | 0 | 219 | 548 | 0 | 0 | 0 |
| N.S. | 1 | 0.66 | 0.75 | 0.00 | 1.17 | 2.93 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.570 | 1.510 | 0.000 | 0.230 | 0.323 | 0.000 | 0.000 | 0.000 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 198 | 683 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.43 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.703 | 3.170 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 151 | 352 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.61 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.706 | 2.377 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 148 | 273 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.585 | 2.940 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 133 | 344 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.50 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.697 | 5.473 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 284 | 149 | 514 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.702 | 7.011 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 364 | 189 | 706 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.643 | 9.809 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 344 | 201 | 481 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.58 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.875 | 2.762 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 198 | 683 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.43 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.671 | 3.162 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 416 | 192 | 565 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.46 | 1.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.801 | 2.569 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 181 | 465 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.48 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.821 | 7.175 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 518 | 200 | 779 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.39 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.664 | 9.864 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 472 | 230 | 1005 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 2.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.692 | 12.468 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 181 | 465 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.48 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.855 | 7.342 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 133 | 344 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.50 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.688 | 5.552 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 96 | 227 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.61 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.540 | 1.760 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 113 | 0 | 32 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.92 | 0.00 | 0.54 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.394 | 1.225 | 0.000 | 0.211 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 89 | 113 | 0 | 98 | 443 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 1.02 | 0.00 | 0.88 | 3.99 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.475 | 0.874 | 0.000 | 0.304 | 0.305 | 0.000 | 0.000 | 0.000 |

| Problem 557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 170 | 143 | 0 | 233 | 576 | 0 | 0 | 0 |
| N.S. | 1 | 0.58 | 0.48 | 0.00 | 0.79 | 1.95 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.565 | 1.020 | 0.000 | 0.305 | 0.341 | 0.000 | 0.000 | 0.000 |

| Problem 558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 517 | 199 | 781 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.38 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.679 | 10.367 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 148 | 515 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.701 | 7.292 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 114 | 285 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.63 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.621 | 2.711 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 90 | 94 | 0 | 98 | 443 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 0.84 | 0.00 | 0.88 | 3.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.481 | 1.094 | 0.000 | 0.298 | 0.302 | 0.000 | 0.000 | 0.000 |

| Problem 562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 82 | 118 | 0 | 82 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 1.15 | 0.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.446 | 1.193 | 0.000 | 0.210 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 282 | 164 | 201 | 0 | 237 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.58 | 0.71 | 0.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.549 | 1.493 | 0.000 | 0.229 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 470 | 228 | 1083 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 2.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.674 | 15.416 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 362 | 187 | 706 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.657 | 10.023 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 121 | 131 | 0 | 220 | 548 | 0 | 0 | 0 |
| N.S. | 1 | 0.65 | 0.71 | 0.00 | 1.19 | 2.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.560 | 1.749 | 0.000 | 0.213 | 0.326 | 0.000 | 0.000 | 0.000 |

| Problem 567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 171 | 139 | 0 | 232 | 576 | 0 | 0 | 0 |
| N.S. | 1 | 0.58 | 0.47 | 0.00 | 0.79 | 1.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.594 | 1.059 | 0.000 | 0.319 | 0.332 | 0.000 | 0.000 | 0.000 |

| Problem 568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 282 | 165 | 202 | 0 | 237 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.59 | 0.72 | 0.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.577 | 1.431 | 0.000 | 0.235 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 132 | 193 | 0 | 159 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.65 | 0.95 | 0.00 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.571 | 1.260 | 0.000 | 0.227 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 680 | 341 | 890 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.50 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.267 | 3.597 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 508 | 261 | 705 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.51 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.877 | 3.234 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 244 | 148 | 352 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.61 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.777 | 1.877 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 134 | 315 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.704 | 2.712 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 544 | 243 | 594 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.45 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.165 | 5.325 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 518 | 267 | 783 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.376 | 9.855 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 774 | 383 | 1084 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.037 | 4.435 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 396 | 275 | 524 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.69 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.259 | 3.191 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 508 | 261 | 705 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.51 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.834 | 3.091 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 436 | 225 | 532 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.844 | 8.523 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 752 | 313 | 1174 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.42 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.267 | 14.851 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-1) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 580 | 287 | 1609 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 2.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.426 | 17.155 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 548 | 425 | 735 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.78 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.706 | 3.945 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 774 | 383 | 1084 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.076 | 4.634 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 680 | 341 | 890 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.50 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.238 | 3.725 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 312 | 723 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.51 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.003 | 14.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 972 | 395 | 2492 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.41 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.528 | 23.484 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-1) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 790 | 359 | 2622 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.45 | 3.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.546 | 24.699 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 312 | 723 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.51 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.960 | 13.876 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 436 | 225 | 529 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.827 | 8.546 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 134 | 315 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.721 | 2.638 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 168 | 0 | 53 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.85 | 0.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.486 | 2.258 | 0.000 | 0.228 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 464 | 219 | 508 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.47 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.943 | 3.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 942 | 442 | 524 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.47 | 0.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.514 | 6.914 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 972 | 395 | 2143 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.41 | 2.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.493 | 25.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 752 | 313 | 1346 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.42 | 1.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.286 | 17.842 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 544 | 243 | 530 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.45 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.124 | 6.688 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 464 | 219 | 511 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.47 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.962 | 3.163 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 139 | 488 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.62 | 2.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.904 | 3.046 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 743 | 365 | 754 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.153 | 9.913 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-1) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 794 | 363 | 2552 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.46 | 3.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.551 | 24.350 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F(-1) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 584 | 291 | 1617 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.50 | 2.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.374 | 17.776 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 522 | 271 | 788 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.52 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.327 | 10.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 942 | 442 | 528 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.47 | 0.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.467 | 6.957 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 604 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 743 | 365 | 757 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.49 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.121 | 10.009 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 605 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 386 | 227 | 642 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.59 | 1.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.531 | 8.665 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 606 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 312 | 312 | 260 | 425 | 415 | 333 | 593 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.36 | 1.33 | 1.07 | 1.90 | 0.00 | 0.00 |
| time (sec) | N/A | 0.701 | 0.213 | 0.295 | 0.199 | 0.270 | 1.202 | 0.000 | 0.000 |

| Problem 607 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 222 | 187 | 295 | 287 | 241 | 389 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.33 | 1.30 | 1.09 | 1.76 | 0.00 | 0.00 |
| time (sec) | N/A | 0.581 | 0.179 | 0.290 | 0.189 | 0.260 | 0.632 | 0.000 | 0.000 |

| Problem 608 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 149 | 125 | 188 | 180 | 163 | 240 | 0 | 0 |
| N.S. | 1 | 1.01 | 0.85 | 1.28 | 1.22 | 1.11 | 1.63 | 0.00 | 0.00 |
| time (sec) | N/A | 0.392 | 0.118 | 0.366 | 0.186 | 0.266 | 0.370 | 0.000 | 0.000 |

| Problem 609 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 84 | 71 | 97 | 91 | 94 | 109 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.88 | 1.20 | 1.12 | 1.16 | 1.35 | 0.00 | 0.00 |
| time (sec) | N/A | 0.282 | 0.046 | 0.015 | 0.184 | 0.260 | 0.222 | 0.000 | 0.000 |

| Problem 610 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 31 | 30 | 43 | 26 | 41 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 1.00 | 1.43 | 0.87 | 1.37 | 0.93 |
| time (sec) | N/A | 0.160 | 0.008 | 0.051 | 0.177 | 0.253 | 0.070 | 0.271 | 2.584 |

| Problem 611 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 485 | 485 | 434 | 224 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.202 | 0.333 | 7.909 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 612 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 707 | 707 | 622 | 848 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.485 | 1.317 | 7.960 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 613 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 559 | 559 | 443 | 683 | 684 | 586 | 989 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 1.22 | 1.22 | 1.05 | 1.77 | 0.00 | 0.00 |
| time (sec) | N/A | 1.235 | 0.336 | 0.617 | 0.201 | 0.272 | 1.024 | 0.000 | 0.000 |

| Problem 614 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 329 | 289 | 431 | 429 | 380 | 595 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.31 | 1.30 | 1.16 | 1.81 | 0.00 | 0.00 |
| time (sec) | N/A | 0.832 | 0.238 | 0.439 | 0.206 | 0.261 | 0.587 | 0.000 | 0.000 |

| Problem 615 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 164 | 215 | 218 | 209 | 279 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 1.41 | 1.42 | 1.37 | 1.82 | 0.00 | 0.00 |
| time (sec) | N/A | 0.474 | 0.144 | 0.221 | 0.198 | 0.276 | 0.315 | 0.000 | 0.000 |

| Problem 616 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 50 | 74 | 72 | 72 | 96 | 82 | 111 | 0 |
| N.S. | 1 | 1.09 | 1.61 | 1.57 | 1.57 | 2.09 | 1.78 | 2.41 | 0.00 |
| time (sec) | N/A | 0.281 | 0.087 | 0.112 | 0.182 | 0.260 | 0.108 | 0.403 | 0.000 |

| Problem 617 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 739 | 739 | 985 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.705 | 0.551 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 618 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 670 | 670 | 444 | 654 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.484 | 0.754 | 1.757 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 619 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 388 | 388 | 253 | 380 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.929 | 0.431 | 0.824 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 620 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 126 | 178 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.548 | 0.185 | 0.584 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 621 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 49 | 45 | 56 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.91 | 0.83 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.394 | 0.057 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 622 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 29 | 17 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.45 | 0.85 | 1.10 | 1.10 |
| time (sec) | N/A | 0.205 | 0.689 | 0.477 | 0.245 | 0.265 | 3.577 | 0.290 | 2.619 |

| Problem 623 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 53 | 19 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 2.65 | 0.95 | 1.10 | 1.10 |
| time (sec) | N/A | 0.213 | 2.876 | 0.499 | 0.257 | 0.242 | 65.608 | 0.293 | 2.553 |

| Problem 624 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 495 | 495 | 356 | 1036 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 2.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.075 | 2.179 | 0.971 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 625 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 190 | 438 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.672 | 0.913 | 0.651 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 626 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 89 | 71 | 118 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.84 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.602 | 0.190 | 0.095 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 627 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 766 | 57 | 19 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 38.30 | 2.85 | 0.95 | 1.10 | 1.10 |
| time (sec) | N/A | 0.213 | 12.000 | 0.480 | 0.906 | 0.255 | 33.777 | 0.291 | 2.663 |

| Problem 628 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 1027 | 98 | 0 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 51.35 | 4.90 | 0.00 | 1.10 | 1.10 |
| time (sec) | N/A | 0.217 | 25.139 | 0.750 | 1.177 | 0.252 | 0.000 | 0.298 | 2.549 |

| Problem 629 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 672 | 672 | 535 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.028 | 4.818 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 630 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 322 | 319 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.098 | 2.099 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 631 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 110 | 101 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.609 | 0.162 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 632 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | F(-2) | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 0 | 20 | 0 | 0 | 19 | 22 | 22 |
| N.S. | 1 | 1.00 | 0.00 | 0.91 | 0.00 | 0.00 | 0.86 | 1.00 | 1.00 |
| time (sec) | N/A | 0.229 | 0.000 | 0.632 | 0.000 | 0.000 | 0.822 | 1.022 | 2.663 |

| Problem 633 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 0 | 20 | 22 | 0 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 0.00 | 0.91 | 1.00 | 0.00 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.232 | 0.000 | 1.016 | 0.401 | 0.000 | 14.300 | 1.043 | 2.697 |

| Problem 634 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 427 | 427 | 770 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.421 | 2.287 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 635 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 140 | 251 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.711 | 0.390 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 636 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | F(-2) | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 0 | 20 | 0 | 0 | 19 | 22 | 22 |
| N.S. | 1 | 1.00 | 0.00 | 0.91 | 0.00 | 0.00 | 0.86 | 1.00 | 1.00 |
| time (sec) | N/A | 0.248 | 0.000 | 0.560 | 0.000 | 0.000 | 9.086 | 1.526 | 2.596 |

| Problem 637 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 0 | 20 | 22 | 0 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 0.00 | 0.91 | 1.00 | 0.00 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.243 | 0.000 | 0.837 | 0.497 | 0.000 | 95.972 | 1.533 | 2.617 |

| Problem 638 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 608 | 608 | 530 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.398 | 0.793 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 639 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 218 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.765 | 0.428 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 640 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 89 | 101 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.01 | 1.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.426 | 0.067 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 641 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 0 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 0.00 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.237 | 0.165 | 0.559 | 0.379 | 0.000 | 2.101 | 1.172 | 2.643 |

| Problem 642 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 0 | 22 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.228 | 0.216 | 0.895 | 0.396 | 0.000 | 41.199 | 1.130 | 2.634 |

| Problem 643 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 303 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.926 | 0.949 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 644 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 130 | 137 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.631 | 0.181 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 645 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 0 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 0.00 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.233 | 0.172 | 0.581 | 0.376 | 0.000 | 7.802 | 0.395 | 2.709 |

| Problem 646 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 0 | 22 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.238 | 0.227 | 0.796 | 0.403 | 0.000 | 139.898 | 0.409 | 2.763 |

| Problem 647 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 20 | 19 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 1.00 | 0.95 | 1.00 | 1.00 |
| time (sec) | N/A | 0.199 | 3.629 | 0.552 | 0.000 | 0.247 | 1.439 | 0.327 | 2.969 |

| Problem 648 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 20 | 19 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 1.00 | 0.95 | 1.00 | 1.00 |
| time (sec) | N/A | 0.200 | 2.796 | 0.297 | 0.000 | 0.263 | 0.878 | 0.298 | 2.864 |

| Problem 649 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 75 | 0 | 0 | 326 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 0.00 | 0.00 | 4.66 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.276 | 0.084 | 0.000 | 0.000 | 0.280 | 0.000 | 0.000 | 0.000 |

| Problem 650 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 147 | 139 | 0 | 0 | 738 | 0 | 0 | 0 |
| N.S. | 1 | 1.01 | 0.95 | 0.00 | 0.00 | 5.05 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.358 | 0.244 | 0.000 | 0.000 | 0.318 | 0.000 | 0.000 | 0.000 |

| Problem 651 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 247 | 191 | 0 | 0 | 1354 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.84 | 0.00 | 0.00 | 5.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.015 | 0.294 | 0.000 | 0.000 | 0.389 | 0.000 | 0.000 | 0.000 |

| Problem 652 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 34 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.55 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.215 | 13.099 | 0.178 | 0.000 | 0.254 | 0.998 | 0.374 | 2.684 |

| Problem 653 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 34 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.55 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.219 | 8.238 | 0.199 | 0.000 | 0.276 | 0.828 | 0.325 | 2.724 |

| Problem 654 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 54 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.45 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.227 | 2.760 | 0.683 | 0.000 | 0.267 | 4.451 | 0.330 | 2.641 |

| Problem 655 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 99 | 65 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 4.50 | 2.95 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.218 | 5.009 | 0.828 | 0.497 | 0.263 | 59.513 | 0.331 | 2.670 |

| Problem 656 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 19 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.86 | 1.00 | 1.00 |
| time (sec) | N/A | 0.217 | 1.017 | 0.595 | 0.253 | 0.248 | 0.368 | 0.297 | 2.612 |

| Problem 657 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 39 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.77 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.220 | 0.809 | 0.645 | 0.284 | 0.244 | 0.619 | 0.300 | 2.729 |

| Problem 658 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 63 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 2.86 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.221 | 1.396 | 0.662 | 0.258 | 0.243 | 1.698 | 0.313 | 2.530 |

| Problem 659 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 87 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 3.95 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.223 | 2.780 | 0.671 | 0.257 | 0.243 | 7.797 | 0.314 | 2.619 |

| Problem 660 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 575 | 36 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 26.14 | 1.64 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.216 | 3.566 | 0.654 | 0.660 | 0.274 | 0.649 | 0.353 | 2.653 |

| Problem 661 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 591 | 67 | 22 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 26.86 | 3.05 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.218 | 6.732 | 0.663 | 0.783 | 0.265 | 1.233 | 0.303 | 2.655 |

| Problem 662 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 864 | 108 | 22 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 39.27 | 4.91 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.220 | 13.987 | 0.702 | 1.328 | 0.260 | 4.716 | 0.329 | 2.792 |

| Problem 663 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 1123 | 149 | 22 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 51.05 | 6.77 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.217 | 22.713 | 0.668 | 1.908 | 0.272 | 27.889 | 0.340 | 2.627 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [221] had the largest ratio of [1.15385000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 6 | 5 | 0.98 | 22 | 0.227 |
| 2 | A | 6 | 6 | 1.10 | 22 | 0.273 |
| 3 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 4 | A | 4 | 4 | 1.03 | 20 | 0.200 |
| 5 | A | 6 | 5 | 1.03 | 19 | 0.263 |
| 6 | C | 12 | 11 | 1.38 | 22 | 0.500 |
| 7 | A | 8 | 7 | 1.03 | 22 | 0.318 |
| 8 | C | 12 | 11 | 1.20 | 22 | 0.500 |
| 9 | A | 7 | 6 | 0.99 | 22 | 0.273 |
| 10 | A | 6 | 5 | 0.93 | 24 | 0.208 |
| 11 | A | 8 | 8 | 1.04 | 24 | 0.333 |
| 12 | A | 6 | 5 | 0.95 | 24 | 0.208 |
| 13 | A | 5 | 5 | 0.98 | 22 | 0.227 |
| 14 | A | 6 | 5 | 0.97 | 21 | 0.238 |
| 15 | C | 17 | 16 | 1.41 | 24 | 0.667 |
| 16 | A | 8 | 7 | 0.95 | 24 | 0.292 |
| 17 | C | 17 | 16 | 1.34 | 24 | 0.667 |
| 18 | A | 11 | 10 | 1.06 | 24 | 0.417 |
| 19 | A | 6 | 5 | 0.93 | 24 | 0.208 |
| 20 | A | 8 | 8 | 0.96 | 24 | 0.333 |
| 21 | A | 6 | 5 | 0.94 | 24 | 0.208 |
| 22 | A | 6 | 6 | 0.97 | 22 | 0.273 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 23 | A | 6 | 5 | 0.95 | 21 | 0.238 |
| 24 | C | 22 | 21 | 1.57 | 24 | 0.875 |
| 25 | A | 6 | 5 | 0.91 | 24 | 0.208 |
| 26 | C | 22 | 21 | 1.43 | 24 | 0.875 |
| 27 | A | 10 | 9 | 0.90 | 24 | 0.375 |
| 28 | A | 13 | 12 | 1.09 | 24 | 0.500 |
| 29 | C | 12 | 11 | 1.04 | 24 | 0.458 |
| 30 | A | 9 | 8 | 0.92 | 24 | 0.333 |
| 31 | C | 8 | 7 | 1.03 | 22 | 0.318 |
| 32 | A | 6 | 5 | 0.84 | 21 | 0.238 |
| 33 | C | 8 | 7 | 1.07 | 24 | 0.292 |
| 34 | A | 11 | 10 | 0.95 | 24 | 0.417 |
| 35 | C | 11 | 10 | 0.99 | 24 | 0.417 |
| 36 | A | 16 | 15 | 1.03 | 24 | 0.625 |
| 37 | A | 13 | 12 | 1.06 | 24 | 0.500 |
| 38 | C | 12 | 11 | 1.01 | 24 | 0.458 |
| 39 | A | 9 | 8 | 0.92 | 24 | 0.333 |
| 40 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 41 | A | 9 | 8 | 0.92 | 21 | 0.381 |
| 42 | C | 11 | 10 | 1.05 | 24 | 0.417 |
| 43 | A | 14 | 13 | 1.08 | 24 | 0.542 |
| 44 | C | 14 | 13 | 1.31 | 24 | 0.542 |
| 45 | A | 20 | 19 | 1.17 | 24 | 0.792 |
| 46 | A | 13 | 12 | 1.08 | 24 | 0.500 |
| 47 | A | 4 | 4 | 0.98 | 24 | 0.167 |
| 48 | A | 11 | 10 | 0.92 | 24 | 0.417 |
| 49 | A | 3 | 3 | 0.98 | 22 | 0.136 |
| 50 | A | 11 | 10 | 0.92 | 21 | 0.476 |
| 51 | C | 14 | 13 | 1.15 | 24 | 0.542 |
| 52 | A | 17 | 16 | 1.12 | 24 | 0.667 |
| 53 | C | 18 | 17 | 1.21 | 24 | 0.708 |
| 54 | A | 24 | 23 | 1.23 | 24 | 0.958 |
| 55 | A | 3 | 3 | 0.94 | 26 | 0.115 |
| 56 | A | 5 | 5 | 0.96 | 26 | 0.192 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 57 | A | 2 | 2 | 0.95 | 24 | 0.083 |
| 58 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 59 | C | 9 | 8 | 1.02 | 26 | 0.308 |
| 60 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 61 | C | 9 | 8 | 0.94 | 26 | 0.308 |
| 62 | A | 3 | 3 | 0.94 | 26 | 0.115 |
| 63 | A | 4 | 4 | 0.90 | 26 | 0.154 |
| 64 | A | 8 | 8 | 1.08 | 26 | 0.308 |
| 65 | A | 3 | 3 | 0.88 | 24 | 0.125 |
| 66 | A | 6 | 6 | 1.16 | 23 | 0.261 |
| 67 | C | 11 | 10 | 1.07 | 26 | 0.385 |
| 68 | A | 6 | 6 | 1.13 | 26 | 0.231 |
| 69 | C | 12 | 11 | 0.97 | 26 | 0.423 |
| 70 | A | 6 | 6 | 1.05 | 26 | 0.231 |
| 71 | A | 4 | 4 | 0.85 | 26 | 0.154 |
| 72 | A | 13 | 12 | 1.18 | 26 | 0.462 |
| 73 | A | 3 | 3 | 0.81 | 24 | 0.125 |
| 74 | A | 8 | 8 | 1.13 | 23 | 0.348 |
| 75 | C | 14 | 13 | 1.15 | 26 | 0.500 |
| 76 | A | 11 | 10 | 1.25 | 26 | 0.385 |
| 77 | C | 14 | 13 | 1.04 | 26 | 0.500 |
| 78 | A | 11 | 10 | 1.16 | 26 | 0.385 |
| 79 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 80 | A | 6 | 6 | 1.07 | 26 | 0.231 |
| 81 | A | 5 | 5 | 1.06 | 26 | 0.192 |
| 82 | A | 4 | 4 | 1.03 | 26 | 0.154 |
| 83 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 84 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 85 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 86 | C | 7 | 6 | 1.02 | 26 | 0.231 |
| 87 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 88 | C | 9 | 8 | 0.95 | 26 | 0.308 |
| 89 | A | 4 | 4 | 0.98 | 26 | 0.154 |
| 90 | A | 4 | 4 | 0.94 | 26 | 0.154 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 91 | A | 8 | 7 | 1.17 | 26 | 0.269 |
| 92 | A | 4 | 4 | 0.93 | 26 | 0.154 |
| 93 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 94 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 95 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 96 | C | 9 | 8 | 1.02 | 26 | 0.308 |
| 97 | A | 7 | 6 | 0.95 | 26 | 0.231 |
| 98 | C | 12 | 11 | 0.99 | 26 | 0.423 |
| 99 | A | 6 | 5 | 0.93 | 26 | 0.192 |
| 100 | A | 12 | 11 | 1.31 | 26 | 0.423 |
| 101 | A | 6 | 6 | 0.95 | 26 | 0.231 |
| 102 | A | 8 | 7 | 1.19 | 26 | 0.269 |
| 103 | A | 4 | 4 | 0.99 | 26 | 0.154 |
| 104 | A | 5 | 4 | 0.92 | 26 | 0.154 |
| 105 | A | 3 | 3 | 0.99 | 24 | 0.125 |
| 106 | A | 4 | 4 | 1.05 | 23 | 0.174 |
| 107 | C | 12 | 11 | 1.14 | 26 | 0.423 |
| 108 | A | 6 | 5 | 0.93 | 26 | 0.192 |
| 109 | C | 16 | 15 | 1.04 | 26 | 0.577 |
| 110 | A | 6 | 5 | 0.90 | 26 | 0.192 |
| 111 | A | 6 | 6 | 1.06 | 19 | 0.316 |
| 112 | A | 5 | 5 | 1.09 | 21 | 0.238 |
| 113 | A | 4 | 4 | 1.04 | 21 | 0.190 |
| 114 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 115 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 116 | A | 1 | 1 | 1.00 | 18 | 0.056 |
| 117 | C | 7 | 6 | 1.35 | 21 | 0.286 |
| 118 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 119 | C | 9 | 8 | 1.05 | 21 | 0.381 |
| 120 | A | 3 | 3 | 0.71 | 26 | 0.115 |
| 121 | A | 5 | 5 | 0.87 | 26 | 0.192 |
| 122 | A | 2 | 2 | 0.76 | 24 | 0.083 |
| 123 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 124 | C | 9 | 8 | 0.76 | 26 | 0.308 |

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## 2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 125 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 126 | C | 9 | 8 | 0.75 | 26 | 0.308 |
| 127 | A | 3 | 3 | 0.75 | 26 | 0.115 |
| 128 | A | 4 | 4 | 0.62 | 26 | 0.154 |
| 129 | A | 8 | 8 | 0.96 | 26 | 0.308 |
| 130 | A | 3 | 3 | 0.62 | 24 | 0.125 |
| 131 | A | 6 | 6 | 1.09 | 23 | 0.261 |
| 132 | C | 11 | 10 | 0.84 | 26 | 0.385 |
| 133 | A | 6 | 6 | 1.07 | 26 | 0.231 |
| 134 | C | 12 | 11 | 0.81 | 26 | 0.423 |
| 135 | A | 6 | 6 | 1.02 | 26 | 0.231 |
| 136 | A | 4 | 4 | 0.55 | 26 | 0.154 |
| 137 | A | 13 | 12 | 1.02 | 26 | 0.462 |
| 138 | A | 3 | 3 | 0.52 | 24 | 0.125 |
| 139 | A | 8 | 8 | 1.05 | 23 | 0.348 |
| 140 | C | 14 | 13 | 0.90 | 26 | 0.500 |
| 141 | A | 11 | 10 | 1.12 | 26 | 0.385 |
| 142 | C | 14 | 13 | 0.86 | 26 | 0.500 |
| 143 | A | 11 | 10 | 1.07 | 26 | 0.385 |
| 144 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 145 | A | 6 | 6 | 1.05 | 26 | 0.231 |
| 146 | A | 5 | 5 | 1.04 | 26 | 0.192 |
| 147 | A | 4 | 4 | 1.02 | 26 | 0.154 |
| 148 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 149 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 150 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 151 | C | 7 | 6 | 0.65 | 26 | 0.231 |
| 152 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 153 | C | 9 | 8 | 0.75 | 26 | 0.308 |
| 154 | A | 4 | 4 | 0.99 | 26 | 0.154 |
| 155 | A | 4 | 4 | 0.73 | 26 | 0.154 |
| 156 | A | 8 | 7 | 1.08 | 26 | 0.269 |
| 157 | A | 4 | 4 | 0.77 | 26 | 0.154 |
| 158 | A | 3 | 3 | 1.00 | 26 | 0.115 |

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## 2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 159 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 160 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 161 | C | 9 | 8 | 0.75 | 26 | 0.308 |
| 162 | A | 7 | 6 | 0.79 | 26 | 0.231 |
| 163 | C | 12 | 11 | 0.82 | 26 | 0.423 |
| 164 | A | 6 | 5 | 0.73 | 26 | 0.192 |
| 165 | A | 12 | 11 | 1.23 | 26 | 0.423 |
| 166 | A | 6 | 6 | 0.78 | 26 | 0.231 |
| 167 | A | 8 | 7 | 1.19 | 26 | 0.269 |
| 168 | A | 4 | 4 | 0.90 | 26 | 0.154 |
| 169 | A | 5 | 4 | 0.83 | 26 | 0.154 |
| 170 | A | 3 | 3 | 0.87 | 24 | 0.125 |
| 171 | A | 4 | 4 | 1.03 | 23 | 0.174 |
| 172 | C | 12 | 11 | 0.93 | 26 | 0.423 |
| 173 | A | 6 | 5 | 0.77 | 26 | 0.192 |
| 174 | C | 16 | 15 | 0.89 | 26 | 0.577 |
| 175 | A | 6 | 5 | 0.72 | 26 | 0.192 |
| 176 | A | 6 | 6 | 1.06 | 19 | 0.316 |
| 177 | A | 5 | 5 | 1.09 | 21 | 0.238 |
| 178 | A | 4 | 4 | 1.04 | 21 | 0.190 |
| 179 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 180 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 181 | A | 1 | 1 | 1.00 | 18 | 0.056 |
| 182 | C | 7 | 6 | 1.35 | 21 | 0.286 |
| 183 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 184 | C | 9 | 8 | 1.05 | 21 | 0.381 |
| 185 | A | 7 | 7 | 1.02 | 24 | 0.292 |
| 186 | A | 6 | 6 | 0.99 | 24 | 0.250 |
| 187 | A | 4 | 4 | 0.98 | 22 | 0.182 |
| 188 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 189 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 190 | N/A | 6 | 0 | 1.00 | 24 | 0.000 |
| 191 | A | 9 | 9 | 0.72 | 26 | 0.346 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 192 | A | 6 | 6 | 0.82 | 26 | 0.231 |
| 193 | A | 3 | 3 | 0.90 | 26 | 0.115 |
| 194 | A | 1 | 1 | 1.00 | 26 | 0.038 |
| 195 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 196 | A | 5 | 5 | 0.96 | 26 | 0.192 |
| 197 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 198 | A | 11 | 11 | 1.14 | 24 | 0.458 |
| 199 | A | 9 | 9 | 1.66 | 24 | 0.375 |
| 200 | A | 9 | 9 | 1.15 | 24 | 0.375 |
| 201 | A | 7 | 7 | 1.11 | 22 | 0.318 |
| 202 | A | 5 | 5 | 1.10 | 21 | 0.238 |
| 203 | C | 14 | 13 | 1.37 | 24 | 0.542 |
| 204 | C | 13 | 12 | 1.27 | 24 | 0.500 |
| 205 | C | 14 | 13 | 1.26 | 24 | 0.542 |
| 206 | C | 11 | 10 | 1.32 | 24 | 0.417 |
| 207 | A | 17 | 17 | 1.31 | 26 | 0.654 |
| 208 | A | 13 | 13 | 1.97 | 26 | 0.500 |
| 209 | A | 15 | 15 | 1.25 | 26 | 0.577 |
| 210 | A | 9 | 9 | 1.00 | 24 | 0.375 |
| 211 | A | 8 | 8 | 1.12 | 23 | 0.348 |
| 212 | C | 22 | 21 | 1.47 | 26 | 0.808 |
| 213 | C | 18 | 17 | 1.32 | 26 | 0.654 |
| 214 | C | 22 | 21 | 1.38 | 26 | 0.808 |
| 215 | C | 17 | 16 | 1.40 | 26 | 0.615 |
| 216 | A | 22 | 22 | 1.49 | 26 | 0.846 |
| 217 | B | 18 | 17 | 2.39 | 26 | 0.654 |
| 218 | A | 20 | 20 | 1.38 | 26 | 0.769 |
| 219 | A | 11 | 11 | 0.97 | 24 | 0.458 |
| 220 | A | 9 | 9 | 1.18 | 23 | 0.391 |
| 221 | C | 31 | 30 | 1.69 | 26 | 1.154 |
| 222 | C | 23 | 22 | 1.47 | 26 | 0.846 |
| 223 | F | 0 | 0 | N/A | 0.000 | N/A |
| 224 | C | 21 | 20 | 1.62 | 26 | 0.769 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 225 | A | 13 | 12 | 1.05 | 26 | 0.462 |
| 226 | C | 14 | 13 | 1.00 | 26 | 0.500 |
| 227 | A | 11 | 10 | 0.87 | 26 | 0.385 |
| 228 | C | 9 | 8 | 1.00 | 24 | 0.333 |
| 229 | A | 7 | 6 | 0.80 | 23 | 0.261 |
| 230 | C | 9 | 8 | 1.04 | 26 | 0.308 |
| 231 | A | 15 | 14 | 0.91 | 26 | 0.538 |
| 232 | C | 13 | 12 | 0.94 | 26 | 0.462 |
| 233 | A | 17 | 16 | 1.04 | 26 | 0.615 |
| 234 | A | 16 | 15 | 1.02 | 26 | 0.577 |
| 235 | C | 14 | 13 | 1.00 | 26 | 0.500 |
| 236 | A | 11 | 10 | 0.90 | 26 | 0.385 |
| 237 | A | 3 | 3 | 0.96 | 24 | 0.125 |
| 238 | A | 11 | 10 | 0.89 | 23 | 0.435 |
| 239 | C | 13 | 12 | 1.03 | 26 | 0.462 |
| 240 | A | 20 | 19 | 1.06 | 26 | 0.731 |
| 241 | C | 19 | 18 | 1.23 | 26 | 0.692 |
| 242 | A | 23 | 22 | 1.21 | 26 | 0.846 |
| 243 | A | 16 | 15 | 1.03 | 26 | 0.577 |
| 244 | A | 9 | 8 | 1.08 | 26 | 0.308 |
| 245 | A | 13 | 12 | 0.93 | 26 | 0.462 |
| 246 | A | 5 | 5 | 0.93 | 24 | 0.208 |
| 247 | A | 13 | 12 | 0.94 | 23 | 0.522 |
| 248 | C | 18 | 17 | 1.18 | 26 | 0.654 |
| 249 | A | 25 | 24 | 1.20 | 26 | 0.923 |
| 250 | C | 24 | 23 | 1.27 | 26 | 0.885 |
| 251 | A | 29 | 28 | 1.37 | 26 | 1.077 |
| 252 | A | 12 | 12 | 1.37 | 25 | 0.480 |
| 253 | A | 10 | 10 | 1.20 | 25 | 0.400 |
| 254 | A | 5 | 5 | 0.98 | 25 | 0.200 |
| 255 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 256 | C | 9 | 8 | 1.04 | 25 | 0.320 |
| 257 | C | 12 | 11 | 1.02 | 25 | 0.440 |
| 258 | A | 13 | 12 | 0.98 | 28 | 0.429 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 259 | A | 10 | 10 | 1.01 | 28 | 0.357 |
| 260 | A | 7 | 6 | 0.80 | 26 | 0.231 |
| 261 | A | 5 | 5 | 0.89 | 25 | 0.200 |
| 262 | C | 10 | 9 | 0.63 | 28 | 0.321 |
| 263 | C | 11 | 10 | 0.92 | 28 | 0.357 |
| 264 | C | 13 | 12 | 0.63 | 28 | 0.429 |
| 265 | C | 13 | 12 | 0.74 | 28 | 0.429 |
| 266 | A | 19 | 18 | 1.12 | 28 | 0.643 |
| 267 | A | 17 | 17 | 1.22 | 28 | 0.607 |
| 268 | A | 7 | 6 | 0.69 | 26 | 0.231 |
| 269 | A | 10 | 10 | 1.09 | 25 | 0.400 |
| 270 | C | 16 | 15 | 0.70 | 28 | 0.536 |
| 271 | C | 18 | 17 | 0.96 | 28 | 0.607 |
| 272 | C | 17 | 16 | 0.64 | 28 | 0.571 |
| 273 | C | 23 | 22 | 1.10 | 28 | 0.786 |
| 274 | A | 25 | 24 | 1.23 | 28 | 0.857 |
| 275 | A | 26 | 26 | 1.39 | 28 | 0.929 |
| 276 | A | 7 | 6 | 0.60 | 26 | 0.231 |
| 277 | A | 12 | 12 | 1.20 | 25 | 0.480 |
| 278 | C | 22 | 21 | 0.83 | 28 | 0.750 |
| 279 | C | 27 | 26 | 1.17 | 28 | 0.929 |
| 280 | C | 24 | 23 | 0.77 | 28 | 0.821 |
| 281 | F | 0 | 0 | N/A | 0.000 | N/A |
| 282 | A | 10 | 10 | 1.44 | 23 | 0.435 |
| 283 | A | 9 | 8 | 1.26 | 23 | 0.348 |
| 284 | A | 5 | 5 | 1.10 | 23 | 0.217 |
| 285 | A | 3 | 3 | 1.04 | 21 | 0.143 |
| 286 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 287 | C | 8 | 7 | 1.18 | 23 | 0.304 |
| 288 | C | 10 | 9 | 1.26 | 23 | 0.391 |
| 289 | C | 13 | 12 | 1.04 | 23 | 0.522 |
| 290 | A | 13 | 12 | 1.04 | 28 | 0.429 |
| 291 | A | 10 | 10 | 1.04 | 28 | 0.357 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 292 | A | 8 | 7 | 0.89 | 28 | 0.250 |
| 293 | A | 5 | 5 | 0.84 | 28 | 0.179 |
| 294 | A | 2 | 2 | 0.69 | 26 | 0.077 |
| 295 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 296 | C | 8 | 7 | 0.56 | 28 | 0.250 |
| 297 | C | 10 | 9 | 0.90 | 28 | 0.321 |
| 298 | C | 13 | 12 | 0.63 | 28 | 0.429 |
| 299 | C | 13 | 12 | 0.88 | 28 | 0.429 |
| 300 | A | 19 | 18 | 0.93 | 28 | 0.643 |
| 301 | C | 16 | 15 | 0.96 | 28 | 0.536 |
| 302 | A | 11 | 10 | 0.69 | 28 | 0.357 |
| 303 | C | 10 | 9 | 0.80 | 28 | 0.321 |
| 304 | A | 7 | 6 | 0.64 | 26 | 0.231 |
| 305 | C | 9 | 8 | 0.74 | 25 | 0.320 |
| 306 | A | 14 | 13 | 0.59 | 28 | 0.464 |
| 307 | C | 17 | 16 | 0.87 | 28 | 0.571 |
| 308 | A | 24 | 23 | 0.71 | 28 | 0.821 |
| 309 | C | 19 | 18 | 0.99 | 28 | 0.643 |
| 310 | A | 15 | 14 | 1.02 | 28 | 0.500 |
| 311 | C | 13 | 12 | 1.03 | 28 | 0.429 |
| 312 | A | 15 | 14 | 1.03 | 28 | 0.500 |
| 313 | C | 12 | 11 | 0.67 | 28 | 0.393 |
| 314 | A | 9 | 8 | 0.66 | 26 | 0.308 |
| 315 | C | 12 | 11 | 0.88 | 25 | 0.440 |
| 316 | A | 22 | 21 | 0.81 | 28 | 0.750 |
| 317 | C | 22 | 21 | 1.04 | 28 | 0.750 |
| 318 | F | 0 | 0 | N/A | 0.000 | N/A |
| 319 | C | 25 | 24 | 1.37 | 28 | 0.857 |
| 320 | C | 15 | 14 | 1.01 | 21 | 0.667 |
| 321 | N/A | 14 | 0 | 1.00 | 28 | 0.000 |
| 322 | N/A | 8 | 0 | 1.00 | 28 | 0.000 |
| 323 | N/A | 4 | 0 | 1.00 | 28 | 0.000 |
| 324 | N/A | 1 | 0 | 1.00 | 28 | 0.000 |

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## 2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 325 | N/A | 1 | 0 | 1.00 | 28 | 0.000 |
| 326 | N/A | 1 | 0 | 1.00 | 28 | 0.000 |
| 327 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 328 | A | 19 | 18 | 1.57 | 19 | 0.947 |
| 329 | A | 15 | 14 | 1.38 | 19 | 0.737 |
| 330 | A | 11 | 10 | 1.28 | 17 | 0.588 |
| 331 | A | 8 | 7 | 0.81 | 19 | 0.368 |
| 332 | A | 16 | 15 | 0.86 | 19 | 0.789 |
| 333 | A | 19 | 18 | 1.01 | 19 | 0.947 |
| 334 | A | 22 | 22 | 1.16 | 21 | 1.048 |
| 335 | A | 14 | 14 | 1.02 | 21 | 0.667 |
| 336 | A | 6 | 6 | 0.80 | 21 | 0.286 |
| 337 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 338 | C | 10 | 9 | 0.63 | 21 | 0.429 |
| 339 | C | 14 | 13 | 0.75 | 21 | 0.619 |
| 340 | C | 19 | 18 | 0.89 | 21 | 0.857 |
| 341 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 342 | A | 9 | 9 | 1.45 | 23 | 0.391 |
| 343 | A | 10 | 10 | 1.33 | 23 | 0.435 |
| 344 | A | 6 | 6 | 1.09 | 23 | 0.261 |
| 345 | A | 4 | 4 | 1.08 | 21 | 0.190 |
| 346 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 347 | C | 9 | 8 | 1.20 | 23 | 0.348 |
| 348 | C | 11 | 10 | 1.17 | 23 | 0.435 |
| 349 | C | 13 | 12 | 1.04 | 23 | 0.522 |
| 350 | A | 5 | 4 | 0.75 | 19 | 0.211 |
| 351 | A | 5 | 4 | 0.78 | 19 | 0.211 |
| 352 | A | 5 | 4 | 0.90 | 17 | 0.235 |
| 353 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 354 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 355 | A | 4 | 3 | 0.83 | 27 | 0.111 |
| 356 | A | 5 | 4 | 0.84 | 27 | 0.148 |
| 357 | A | 4 | 3 | 0.87 | 27 | 0.111 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 358 | A | 5 | 4 | 0.86 | 25 | 0.160 |
| 359 | A | 5 | 4 | 0.87 | 24 | 0.167 |
| 360 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 361 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 362 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 363 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 364 | A | 5 | 4 | 0.83 | 27 | 0.148 |
| 365 | A | 4 | 3 | 0.83 | 27 | 0.111 |
| 366 | A | 5 | 4 | 0.84 | 25 | 0.160 |
| 367 | A | 5 | 4 | 0.84 | 24 | 0.167 |
| 368 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 369 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 370 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 371 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 372 | A | 5 | 4 | 0.83 | 27 | 0.148 |
| 373 | A | 4 | 3 | 0.82 | 27 | 0.111 |
| 374 | A | 5 | 4 | 0.83 | 25 | 0.160 |
| 375 | A | 5 | 4 | 0.83 | 24 | 0.167 |
| 376 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 377 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 378 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 379 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 380 | A | 5 | 4 | 0.88 | 23 | 0.174 |
| 381 | C | 6 | 5 | 1.19 | 23 | 0.217 |
| 382 | A | 6 | 5 | 0.93 | 23 | 0.217 |
| 383 | A | 6 | 5 | 0.93 | 23 | 0.217 |
| 384 | A | 5 | 4 | 1.00 | 21 | 0.190 |
| 385 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 386 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 387 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 388 | C | 7 | 6 | 0.92 | 27 | 0.222 |
| 389 | A | 5 | 4 | 0.84 | 27 | 0.148 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 390 | C | 7 | 6 | 0.95 | 27 | 0.222 |
| 391 | A | 6 | 5 | 0.87 | 27 | 0.185 |
| 392 | C | 11 | 10 | 1.06 | 25 | 0.400 |
| 393 | A | 1 | 1 | 1.00 | 24 | 0.042 |
| 394 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 395 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 396 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 397 | N/A | 1 | 0 | 1.00 | 25 | 0.000 |
| 398 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 399 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 400 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 401 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 402 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 403 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 404 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 405 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 406 | A | 5 | 4 | 0.84 | 19 | 0.211 |
| 407 | A | 5 | 4 | 0.88 | 19 | 0.211 |
| 408 | A | 5 | 4 | 0.98 | 17 | 0.235 |
| 409 | N/A | 2 | 0 | 1.00 | 19 | 0.000 |
| 410 | N/A | 2 | 0 | 1.00 | 19 | 0.000 |
| 411 | A | 5 | 4 | 1.37 | 27 | 0.148 |
| 412 | C | 15 | 14 | 2.16 | 27 | 0.519 |
| 413 | A | 13 | 12 | 1.23 | 25 | 0.480 |
| 414 | C | 14 | 13 | 1.05 | 24 | 0.542 |
| 415 | N/A | 11 | 0 | 1.00 | 27 | 0.000 |
| 416 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 417 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 418 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 419 | A | 5 | 4 | 1.42 | 27 | 0.148 |
| 420 | A | 6 | 5 | 1.15 | 27 | 0.185 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 421 | A | 9 | 8 | 1.36 | 25 | 0.320 |
| 422 | A | 6 | 5 | 0.93 | 24 | 0.208 |
| 423 | N/A | 7 | 0 | 1.00 | 27 | 0.000 |
| 424 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 425 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 426 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 427 | A | 5 | 4 | 1.78 | 27 | 0.148 |
| 428 | A | 6 | 5 | 1.43 | 27 | 0.185 |
| 429 | A | 9 | 8 | 1.42 | 25 | 0.320 |
| 430 | A | 6 | 5 | 0.88 | 24 | 0.208 |
| 431 | N/A | 7 | 0 | 1.00 | 27 | 0.000 |
| 432 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 433 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 434 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 435 | A | 5 | 4 | 0.87 | 27 | 0.148 |
| 436 | A | 6 | 5 | 0.93 | 27 | 0.185 |
| 437 | A | 5 | 4 | 0.89 | 27 | 0.148 |
| 438 | C | 14 | 13 | 1.05 | 27 | 0.481 |
| 439 | A | 10 | 9 | 0.95 | 25 | 0.360 |
| 440 | A | 1 | 1 | 1.00 | 24 | 0.042 |
| 441 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 442 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 443 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 444 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 445 | N/A | 1 | 0 | 1.00 | 25 | 0.000 |
| 446 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 447 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 448 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 449 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 450 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 451 | N/A | 1 | 0 | 1.00 | 25 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 452 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 453 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 454 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 455 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 456 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 457 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 458 | N/A | 2 | 0 | 1.00 | 27 | 0.000 |
| 459 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 460 | N/A | 1 | 0 | 1.00 | 27 | 0.000 |
| 461 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 462 | A | 5 | 4 | 1.80 | 26 | 0.154 |
| 463 | A | 6 | 5 | 1.55 | 26 | 0.192 |
| 464 | A | 9 | 8 | 1.14 | 24 | 0.333 |
| 465 | A | 6 | 5 | 0.99 | 23 | 0.217 |
| 466 | N/A | 8 | 0 | 1.00 | 26 | 0.000 |
| 467 | A | 5 | 4 | 1.19 | 28 | 0.143 |
| 468 | A | 6 | 5 | 1.57 | 28 | 0.179 |
| 469 | A | 9 | 8 | 1.57 | 26 | 0.308 |
| 470 | A | 6 | 5 | 0.94 | 25 | 0.200 |
| 471 | N/A | 8 | 0 | 1.00 | 28 | 0.000 |
| 472 | C | 16 | 15 | 1.04 | 23 | 0.652 |
| 473 | C | 12 | 11 | 0.90 | 23 | 0.478 |
| 474 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 475 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 476 | N/A | 3 | 0 | 1.00 | 23 | 0.000 |
| 477 | A | 15 | 14 | 0.89 | 23 | 0.609 |
| 478 | A | 9 | 8 | 0.71 | 23 | 0.348 |
| 479 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 480 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 481 | F | 0 | 0 | N/A | 0.000 | N/A |
| 482 | C | 15 | 14 | 0.80 | 23 | 0.609 |
| 483 | A | 1 | 1 | 1.00 | 23 | 0.043 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 484 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 485 | C | 17 | 16 | 1.07 | 22 | 0.727 |
| 486 | C | 12 | 11 | 0.92 | 22 | 0.500 |
| 487 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 488 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 489 | N/A | 5 | 0 | 1.00 | 22 | 0.000 |
| 490 | A | 16 | 15 | 0.95 | 22 | 0.682 |
| 491 | A | 9 | 8 | 0.75 | 22 | 0.364 |
| 492 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 493 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 494 | C | 8 | 7 | 1.24 | 17 | 0.412 |
| 495 | A | 5 | 4 | 0.50 | 23 | 0.174 |
| 496 | A | 5 | 4 | 0.53 | 23 | 0.174 |
| 497 | A | 5 | 4 | 0.62 | 23 | 0.174 |
| 498 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 499 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 500 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 501 | A | 5 | 4 | 0.59 | 23 | 0.174 |
| 502 | A | 5 | 4 | 0.67 | 23 | 0.174 |
| 503 | C | 11 | 10 | 0.89 | 23 | 0.435 |
| 504 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 505 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 506 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 507 | A | 10 | 9 | 0.78 | 23 | 0.391 |
| 508 | A | 9 | 8 | 0.91 | 23 | 0.348 |
| 509 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 510 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 511 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 512 | A | 4 | 3 | 0.76 | 28 | 0.107 |
| 513 | A | 5 | 4 | 0.77 | 26 | 0.154 |
| 514 | A | 5 | 4 | 0.76 | 25 | 0.160 |
| 515 | N/A | 2 | 0 | 1.00 | 28 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 516 | N/A | 2 | 0 | 1.00 | 28 | 0.000 |
| 517 | A | 4 | 3 | 0.71 | 28 | 0.107 |
| 518 | A | 5 | 4 | 0.73 | 26 | 0.154 |
| 519 | A | 5 | 4 | 0.72 | 25 | 0.160 |
| 520 | N/A | 2 | 0 | 1.00 | 28 | 0.000 |
| 521 | N/A | 2 | 0 | 1.00 | 28 | 0.000 |
| 522 | A | 4 | 3 | 0.69 | 28 | 0.107 |
| 523 | A | 5 | 4 | 0.71 | 26 | 0.154 |
| 524 | A | 5 | 4 | 0.70 | 25 | 0.160 |
| 525 | N/A | 2 | 0 | 1.00 | 28 | 0.000 |
| 526 | N/A | 2 | 0 | 1.00 | 28 | 0.000 |
| 527 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 528 | C | 6 | 5 | 1.03 | 23 | 0.217 |
| 529 | A | 6 | 5 | 0.94 | 23 | 0.217 |
| 530 | C | 6 | 5 | 1.10 | 21 | 0.238 |
| 531 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 532 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 533 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 534 | A | 4 | 4 | 0.46 | 35 | 0.114 |
| 535 | A | 4 | 4 | 0.49 | 35 | 0.114 |
| 536 | A | 4 | 4 | 0.65 | 35 | 0.114 |
| 537 | A | 4 | 4 | 0.61 | 35 | 0.114 |
| 538 | A | 4 | 4 | 0.64 | 35 | 0.114 |
| 539 | A | 7 | 7 | 0.66 | 35 | 0.200 |
| 540 | A | 4 | 4 | 0.43 | 35 | 0.114 |
| 541 | A | 7 | 7 | 0.61 | 35 | 0.200 |
| 542 | A | 4 | 4 | 0.49 | 35 | 0.114 |
| 543 | A | 4 | 4 | 0.50 | 35 | 0.114 |
| 544 | A | 4 | 4 | 0.52 | 35 | 0.114 |
| 545 | A | 4 | 4 | 0.52 | 35 | 0.114 |
| 546 | A | 9 | 9 | 0.58 | 35 | 0.257 |
| 547 | A | 4 | 4 | 0.43 | 35 | 0.114 |
| 548 | A | 4 | 4 | 0.46 | 35 | 0.114 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 549 | A | 4 | 4 | 0.48 | 35 | 0.114 |
| 550 | A | 4 | 4 | 0.39 | 35 | 0.114 |
| 551 | A | 4 | 4 | 0.49 | 35 | 0.114 |
| 552 | A | 4 | 4 | 0.48 | 35 | 0.114 |
| 553 | A | 4 | 4 | 0.50 | 35 | 0.114 |
| 554 | A | 4 | 4 | 0.61 | 35 | 0.114 |
| 555 | A | 2 | 2 | 1.00 | 35 | 0.057 |
| 556 | A | 6 | 6 | 0.80 | 35 | 0.171 |
| 557 | A | 4 | 4 | 0.58 | 35 | 0.114 |
| 558 | A | 4 | 4 | 0.38 | 35 | 0.114 |
| 559 | A | 4 | 4 | 0.52 | 35 | 0.114 |
| 560 | A | 4 | 4 | 0.63 | 35 | 0.114 |
| 561 | A | 7 | 7 | 0.80 | 35 | 0.200 |
| 562 | A | 3 | 3 | 0.80 | 35 | 0.086 |
| 563 | A | 4 | 4 | 0.58 | 35 | 0.114 |
| 564 | A | 4 | 4 | 0.49 | 35 | 0.114 |
| 565 | A | 4 | 4 | 0.52 | 35 | 0.114 |
| 566 | A | 7 | 7 | 0.65 | 35 | 0.200 |
| 567 | A | 4 | 4 | 0.58 | 35 | 0.114 |
| 568 | A | 4 | 4 | 0.59 | 35 | 0.114 |
| 569 | A | 5 | 5 | 0.65 | 35 | 0.143 |
| 570 | A | 4 | 4 | 0.50 | 37 | 0.108 |
| 571 | A | 4 | 4 | 0.51 | 37 | 0.108 |
| 572 | A | 6 | 6 | 0.61 | 37 | 0.162 |
| 573 | A | 4 | 4 | 0.52 | 37 | 0.108 |
| 574 | A | 4 | 4 | 0.45 | 37 | 0.108 |
| 575 | A | 4 | 4 | 0.52 | 37 | 0.108 |
| 576 | A | 4 | 4 | 0.49 | 37 | 0.108 |
| 577 | A | 11 | 11 | 0.69 | 37 | 0.297 |
| 578 | A | 4 | 4 | 0.51 | 37 | 0.108 |
| 579 | A | 7 | 6 | 0.52 | 37 | 0.162 |
| 580 | A | 4 | 4 | 0.42 | 37 | 0.108 |
| 581 | A | 4 | 4 | 0.49 | 37 | 0.108 |
| 582 | A | 13 | 13 | 0.78 | 37 | 0.351 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 583 | A | 4 | 4 | 0.49 | 37 | 0.108 |
| 584 | A | 4 | 4 | 0.50 | 37 | 0.108 |
| 585 | A | 7 | 6 | 0.51 | 37 | 0.162 |
| 586 | A | 4 | 4 | 0.41 | 37 | 0.108 |
| 587 | A | 4 | 4 | 0.45 | 37 | 0.108 |
| 588 | A | 7 | 6 | 0.51 | 37 | 0.162 |
| 589 | A | 7 | 6 | 0.52 | 37 | 0.162 |
| 590 | A | 4 | 4 | 0.52 | 37 | 0.108 |
| 591 | A | 2 | 2 | 1.00 | 37 | 0.054 |
| 592 | A | 4 | 4 | 0.47 | 37 | 0.108 |
| 593 | A | 4 | 4 | 0.47 | 37 | 0.108 |
| 594 | A | 4 | 4 | 0.41 | 37 | 0.108 |
| 595 | A | 4 | 4 | 0.42 | 37 | 0.108 |
| 596 | A | 4 | 4 | 0.45 | 37 | 0.108 |
| 597 | A | 4 | 4 | 0.47 | 37 | 0.108 |
| 598 | A | 10 | 9 | 0.62 | 37 | 0.243 |
| 599 | A | 4 | 4 | 0.49 | 37 | 0.108 |
| 600 | A | 4 | 4 | 0.46 | 37 | 0.108 |
| 601 | A | 4 | 4 | 0.50 | 37 | 0.108 |
| 602 | A | 4 | 4 | 0.52 | 37 | 0.108 |
| 603 | A | 4 | 4 | 0.47 | 37 | 0.108 |
| 604 | A | 4 | 4 | 0.49 | 37 | 0.108 |
| 605 | A | 13 | 12 | 0.59 | 37 | 0.324 |
| 606 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 607 | A | 6 | 5 | 1.00 | 18 | 0.278 |
| 608 | A | 6 | 5 | 1.01 | 18 | 0.278 |
| 609 | A | 6 | 5 | 1.04 | 16 | 0.312 |
| 610 | A | 1 | 1 | 1.00 | 8 | 0.125 |
| 611 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 612 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 613 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 614 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 615 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 616 | A | 3 | 3 | 1.09 | 10 | 0.300 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 617 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 618 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 619 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 620 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 621 | A | 9 | 8 | 0.91 | 10 | 0.800 |
| 622 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 623 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 624 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 625 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 626 | C | 12 | 11 | 1.05 | 10 | 1.100 |
| 627 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 628 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 629 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 630 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 631 | C | 10 | 9 | 1.08 | 12 | 0.750 |
| 632 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 633 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 634 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 635 | A | 10 | 9 | 1.04 | 12 | 0.750 |
| 636 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 637 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 638 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 639 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 640 | A | 8 | 7 | 1.01 | 12 | 0.583 |
| 641 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 642 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 643 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 644 | C | 10 | 9 | 1.12 | 12 | 0.750 |
| 645 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 646 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 647 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 648 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 649 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 650 | A | 7 | 6 | 1.01 | 20 | 0.300 |
| 651 | A | 9 | 8 | 1.09 | 20 | 0.400 |
| 652 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 653 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 654 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 655 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 656 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 657 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 658 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 659 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 660 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 661 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 662 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 663 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |

# CHAPTER 3

## LISTING OF INTEGRALS

# 3.1 $\int x^4(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$



## 3.1.1 Optimal result

Integrand size = 22, antiderivative size = 124

$$\int x^4(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = -\frac{2bd\sqrt{1+c^2x^2}}{35c^5}-\frac{bd(1+c^2x^2)^{3/2}}{105c^5}+\frac{8bd(1+c^2x^2)^{5/2}}{175c^5}-\frac{bd(1+c^2x^2)^{7/2}}{49c^5}+\frac{1}{5}dx^5(a+b\text{arcsinh}(cx))+\frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx))$$

output
```
-1/105*b*d*(c^2*x^2+1)^(3/2)/c^5+8/175*b*d*(c^2*x^2+1)^(5/2)/c^5-1/49*b*d*
(c^2*x^2+1)^(7/2)/c^5+1/5*d*x^5*(a+b*arcsinh(c*x))+1/7*c^2*d*x^7*(a+b*arcs
inh(c*x))-2/35*b*d*(c^2*x^2+1)^(1/2)/c^5
```

## 3.1.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int x^4(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{d\left(105ax^5(7+5c^2x^2)-\frac{b\sqrt{1+c^2x^2}(152-76c^2x^2+57c^4x^4+75c^6x^6)}{c^5}+105bx^5(7+5c^2x^2)\,\text{arcsinh}(cx)\right)}{3675}$$

input
```
Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(105*a*x^5*(7 + 5*c^2*x^2) - (b*Sqrt[1 + c^2*x^2]*(152 - 76*c^2*x^2 + 5
7*c^4*x^4 + 75*c^6*x^6))/c^5 + 105*b*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]))/36
75
```

### 3.1.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6218, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int \frac{dx^5\left(5c^2x^2+7\right)}{35\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx))+\frac{1}{5}dx^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35}bcd\int \frac{x^5\left(5c^2x^2+7\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx))+\frac{1}{5}dx^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{70}bcd\int \frac{x^4\left(5c^2x^2+7\right)}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx))+\frac{1}{5}dx^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{70}bcd\int \left(\frac{5\left(c^2x^2+1\right)^{5/2}}{c^4}-\frac{8\left(c^2x^2+1\right)^{3/2}}{c^4}+\frac{\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx))+\frac{1}{5}dx^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}c^2dx^7(a+b\text{arcsinh}(cx))+\frac{1}{5}dx^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bcd\left(\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^6}-\frac{16\left(c^2x^2+1\right)^{5/2}}{5c^6}+\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^6}+\frac{4\sqrt{c^2x^2+1}}{c^6}\right)$$

input
```
Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/70*(b*c*d*((4*Sqrt[1 + c^2*x^2])/c^6 + (2*(1 + c^2*x^2)^(3/2))/(3*c^6)
- (16*(1 + c^2*x^2)^(5/2))/(5*c^6) + (10*(1 + c^2*x^2)^(7/2))/(7*c^6))) +
(d*x^5*(a + b*ArcSinh[c*x]))/5 + (c^2*d*x^7*(a + b*ArcSinh[c*x]))/7
```

### 3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.1.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| parts | $da\left(\frac{1}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{19c^4x^4\sqrt{c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{c^2x^2+1}}{3675}-\frac{15}{}\right.}{c^5}$ |
| derivativedivides | $\frac{da\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{19c^4x^4\sqrt{c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{c^2x^2+1}}{3675}-\frac{152\sqrt{c^2x^2}}{3675}\right.}{c^5}$ |
| default | $\frac{da\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{19c^4x^4\sqrt{c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{c^2x^2+1}}{3675}-\frac{152\sqrt{c^2x^2}}{3675}\right.}{c^5}$ |

input
```
int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/7*c^2*x^7+1/5*x^5)+d*b/c^5*(1/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*
x)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-19/1225*c^4*x^4*(c^2*x^2+1)^(1/2
)+76/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-152/3675*(c^2*x^2+1)^(1/2))
```

### 3.1.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

$$\int x^4(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{525\,ac^7dx^7+735\,ac^5dx^5+105\,(5\,bc^7dx^7+7\,bc^5dx^5)\log\left(cx+\sqrt{c^2x^2+1}\right)-(75\,bc^6dx^6+57\,bc^4dx^4-76\,b}{3675\,c^5}$$

input
```
integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*d*x^7 + 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 + 7*b*c^5*d
*x^5)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d*x^6 + 57*b*c^4*d*x^4 - 76
*b*c^2*d*x^2 + 152*b*d)*sqrt(c^2*x^2 + 1))/c^5
```

### 3.1.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int x^4(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^2dx^7}{7}+\frac{adx^5}{5}+\frac{bc^2dx^7\operatorname{asinh}(cx)}{7}-\frac{bcdx^6\sqrt{c^2x^2+1}}{49}+\frac{bdx^5\operatorname{asinh}(cx)}{5}-\frac{19bdx^4\sqrt{c^2x^2+1}}{1225c}+\frac{76bdx^2\sqrt{c^2x^2+1}}{3675c^3}-\frac{152bd\sqrt{c^2x^2+1}}{3675c^5}\\ \frac{adx^5}{5}\end{cases}$$

input
```
integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**2*d*x**7/7 + a*d*x**5/5 + b*c**2*d*x**7*asinh(c*x)/7 - b*c
*d*x**6*sqrt(c**2*x**2 + 1)/49 + b*d*x**5*asinh(c*x)/5 - 19*b*d*x**4*sqrt(
c**2*x**2 + 1)/(1225*c) + 76*b*d*x**2*sqrt(c**2*x**2 + 1)/(3675*c**3) - 15
2*b*d*sqrt(c**2*x**2 + 1)/(3675*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

### 3.1.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int x^4(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx=\frac{1}{7}\,ac^2dx^7+\frac{1}{5}\,adx^5$$
$$+\frac{1}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d$$
$$+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd$$

input
```
integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x
^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^
6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arcsinh(c*x) - (3*
sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
 1)/c^6)*c)*b*d
```

### 3.1.8 Giac [F(-2)]

Exception generated.

$$\int x^4 (d + c^2 dx^2) (a + b\text{arcsinh}(cx)) \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (d + c^2 dx^2) (a + b\text{arcsinh}(cx)) \, dx = \int x^4 (a + b \,\text{asinh}(c\,x)) \left(d\,c^2\,x^2 + d\right) \, dx$$

input
```
int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

output
```
int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

# 3.2 $\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$



## 3.2.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}-\frac{bd\text{arcsinh}(cx)}{24c^4}+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))$$

output
```
-1/24*b*d*arcsinh(c*x)/c^4+1/4*d*x^4*(a+b*arcsinh(c*x))+1/6*c^2*d*x^6*(a+b
*arcsinh(c*x))+1/24*b*d*x*(c^2*x^2+1)^(1/2)/c^3-1/36*b*d*x^3*(c^2*x^2+1)^(
1/2)/c-1/36*b*c*d*x^5*(c^2*x^2+1)^(1/2)
```

## 3.2.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{d(6ac^4x^4(3+2c^2x^2)+bcx\sqrt{1+c^2x^2}(3-2c^2x^2-2c^4x^4)+3b(-1+6c^4x^4+4c^6x^6)\text{arcsinh}(cx))}{72c^4}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(6*a*c^4*x^4*(3 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 - 2*c^2*x^2 -
 2*c^4*x^4) + 3*b*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]))/(72*c^4)
```

### 3.2.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6218, 27, 363, 262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int \frac{dx^4\left(2c^2x^2+3\right)}{12\sqrt{c^2x^2+1}}dx+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bcd\int \frac{x^4\left(2c^2x^2+3\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{363}$$

$$-\frac{1}{12}bcd\left(\frac{4}{3}\int \frac{x^4}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{262}$$

$$-\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int \frac{x^2}{\sqrt{c^2x^2+1}}dx}{4c^2}\right)+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{262}$$

$$-\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{4c^2}\right)+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)+\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{6}c^2dx^6(a+b\text{arcsinh}(cx))+\frac{1}{4}dx^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)$$

input
```
Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcSinh[c*x]))/4 + (c^2*d*x^6*(a + b*ArcSinh[c*x]))/6 - (b*c
*d*((x^5*Sqrt[1 + c^2*x^2])/3 + (4*((x^3*Sqrt[1 + c^2*x^2])/(4*c^2) - (3*(
(x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/(4*c^2)))/3))/12
```

### 3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.2.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| parts | $da\left(\frac{1}{6}c^2x^6+\frac{1}{4}x^4\right)+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{c^3x^3\sqrt{c^2x^2+1}}{36}+\frac{cx\sqrt{c^2x^2+1}}{24}-\frac{\operatorname{arcsinh}(c}{24}\right.}{c^4}$ |
| derivativedivides | $\frac{da\left(\frac{1}{6}c^6x^6+\frac{1}{4}c^4x^4\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{c^3x^3\sqrt{c^2x^2+1}}{36}+\frac{cx\sqrt{c^2x^2+1}}{24}-\frac{\operatorname{arcsinh}(cx)}{24}\right)}{c^4}$ |
| default | $\frac{da\left(\frac{1}{6}c^6x^6+\frac{1}{4}c^4x^4\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{c^3x^3\sqrt{c^2x^2+1}}{36}+\frac{cx\sqrt{c^2x^2+1}}{24}-\frac{\operatorname{arcsinh}(cx)}{24}\right)}{c^4}$ |

input
```
int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/6*c^2*x^6+1/4*x^4)+d*b/c^4*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*
x)*c^4*x^4-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1/2)+1
/24*c*x*(c^2*x^2+1)^(1/2)-1/24*arcsinh(c*x))
```

## 3.2.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\int x^3\left(d+c^2dx^2\right)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{12\,ac^6dx^6+18\,ac^4dx^4+3\left(4\,bc^6dx^6+6\,bc^4dx^4-bd\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(2\,bc^5dx^5+2\,bc^3dx^3-3\,bcd\right.}{72\,c^4}$$

input
```
integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/72*(12*a*c^6*d*x^6 + 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 + 6*b*c^4*d*x^4 -
 b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^5*d*x^5 + 2*b*c^3*d*x^3 - 3*b*
c*d*x)*sqrt(c^2*x^2 + 1))/c^4
```

### 3.2.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15

$$\int x^3(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^2dx^6}{6} + \frac{adx^4}{4} + \frac{bc^2dx^6\,\text{asinh}\,(cx)}{6} - \frac{bcdx^5\sqrt{c^2x^2+1}}{36} + \frac{bdx^4\,\text{asinh}\,(cx)}{4} - \frac{bdx^3\sqrt{c^2x^2+1}}{36c} + \frac{bdx\sqrt{c^2x^2+1}}{24c^3} - \frac{bd\,\text{asinh}\,(cx)}{24c^4} & \text{for } c \\ \frac{adx^4}{4} & \text{othe} \end{cases}$$

input
```
integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**2*d*x**6/6 + a*d*x**4/4 + b*c**2*d*x**6*asinh(c*x)/6 - b*c
*d*x**5*sqrt(c**2*x**2 + 1)/36 + b*d*x**4*asinh(c*x)/4 - b*d*x**3*sqrt(c**
2*x**2 + 1)/(36*c) + b*d*x*sqrt(c**2*x**2 + 1)/(24*c**3) - b*d*asinh(c*x)/
(24*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

### 3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.38

$$\int x^3(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx = \frac{1}{6}\,ac^2dx^6 + \frac{1}{4}\,adx^4$$
$$+ \frac{1}{288}\left(48\,x^6\,\text{arsinh}\,(cx) - \left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\,\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\,\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\text{arsinh}\,(cx)}{c^7}\right)c\right)bc$$
$$+ \frac{1}{32}\left(8\,x^4\,\text{arsinh}\,(cx) - \left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\,(cx)}{c^5}\right)c\right)bd$$

input
```
integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x
^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^
6 - 15*arcsinh(c*x)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c
^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b
*d
```

### 3.2.8 Giac [F(-2)]

Exception generated.

$$\int x^3(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))\,dx = \int x^3\,(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

## 3.3 $\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx$



### 3.3.1 Optimal result

Integrand size = 22, antiderivative size = 102

$$\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx = \frac{2bd\sqrt{1+c^2x^2}}{15c^3}+\frac{bd(1+c^2x^2)^{3/2}}{45c^3}-\frac{bd(1+c^2x^2)^{5/2}}{25c^3}+\frac{1}{3}dx^3(a+b\text{arcsinh}(cx))+\frac{1}{5}c^2dx^5(a+b\text{arcsinh}(cx))$$

output
```
1/45*b*d*(c^2*x^2+1)^(3/2)/c^3-1/25*b*d*(c^2*x^2+1)^(5/2)/c^3+1/3*d*x^3*(a
+b*arcsinh(c*x))+1/5*c^2*d*x^5*(a+b*arcsinh(c*x))+2/15*b*d*(c^2*x^2+1)^(1/
2)/c^3
```

### 3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx = \frac{1}{225}d\left(15ax^3(5+3c^2x^2)+\frac{b\sqrt{1+c^2x^2}(26-13c^2x^2-9c^4x^4)}{c^3}+15bx^3(5+3c^2x^2)\,\text{arcsinh}(cx)\right)$$

input
```
Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(15*a*x^3*(5 + 3*c^2*x^2) + (b*Sqrt[1 + c^2*x^2]*(26 - 13*c^2*x^2 - 9*c
^4*x^4))/c^3 + 15*b*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]))/225
```

### 3.3.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6218, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2 d x^2+d\right) (a+b \text{arcsinh}(c x)) \, dx$$

$$\Big\downarrow \text{6218}$$

$$-b c \int \frac{d x^3 \left(3 c^2 x^2+5\right)}{15 \sqrt{c^2 x^2+1}}dx+\frac{1}{5} c^2 d x^5 (a+b \text{arcsinh}(c x))+\frac{1}{3} d x^3 (a+b \text{arcsinh}(c x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15} b c d \int \frac{x^3 \left(3 c^2 x^2+5\right)}{\sqrt{c^2 x^2+1}}dx+\frac{1}{5} c^2 d x^5 (a+b \text{arcsinh}(c x))+\frac{1}{3} d x^3 (a+b \text{arcsinh}(c x))$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{30} b c d \int \frac{x^2 \left(3 c^2 x^2+5\right)}{\sqrt{c^2 x^2+1}}dx^2+\frac{1}{5} c^2 d x^5 (a+b \text{arcsinh}(c x))+\frac{1}{3} d x^3 (a+b \text{arcsinh}(c x))$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{30} b c d \int \left(\frac{3 \left(c^2 x^2+1\right)^{3/2}}{c^2}-\frac{\sqrt{c^2 x^2+1}}{c^2}-\frac{2}{c^2 \sqrt{c^2 x^2+1}}\right)dx^2+\frac{1}{5} c^2 d x^5 (a+b \text{arcsinh}(c x))+\frac{1}{3} d x^3 (a+b \text{arcsinh}(c x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5} c^2 d x^5 (a+b \text{arcsinh}(c x))+\frac{1}{3} d x^3 (a+b \text{arcsinh}(c x))-\frac{1}{30} b c d \left(\frac{6 \left(c^2 x^2+1\right)^{5/2}}{5 c^4}-\frac{2 \left(c^2 x^2+1\right)^{3/2}}{3 c^4}-\frac{4 \sqrt{c^2 x^2+1}}{c^4}\right)$$

input
```
Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/30*(b*c*d*((-4*Sqrt[1 + c^2*x^2])/c^4 - (2*(1 + c^2*x^2)^(3/2))/(3*c^4)
 + (6*(1 + c^2*x^2)^(5/2))/(5*c^4))) + (d*x^3*(a + b*ArcSinh[c*x]))/3 + (c
^2*d*x^5*(a + b*ArcSinh[c*x]))/5
```

### 3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.3.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

| method | result | siz |
|---|---|---|
| parts | $da\left(\frac{1}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{13c^2x^2\sqrt{c^2x^2+1}}{225}+\frac{26\sqrt{c^2x^2+1}}{225}\right)}{c^3}$ | 10 |
| derivativedivides | $\frac{da\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{13c^2x^2\sqrt{c^2x^2+1}}{225}+\frac{26\sqrt{c^2x^2+1}}{225}\right)}{c^3}$ | 10 |
| default | $\frac{da\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{13c^2x^2\sqrt{c^2x^2+1}}{225}+\frac{26\sqrt{c^2x^2+1}}{225}\right)}{c^3}$ | 10 |

input
```
int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/5*c^2*x^5+1/3*x^3)+d*b/c^3*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*
x)*c^3*x^3-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^(1/2)
+26/225*(c^2*x^2+1)^(1/2))
```

### 3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{45\,ac^5dx^5+75\,ac^3dx^3+15\left(3\,bc^5dx^5+5\,bc^3dx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4dx^4+13\,bc^2dx^2-26\,bd\right)\sqrt{}}{225\,c^3}$$

input
```
integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*d*x^5 + 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 + 5*b*c^3*d*x^3
)*log(c*x + sqrt(c^2*x^2 + 1)) - (9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 26*b*d)
*sqrt(c^2*x^2 + 1))/c^3
```

### 3.3.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} \frac{ac^2dx^5}{5}+\frac{adx^3}{3}+\frac{bc^2dx^5\,\text{asinh}\,(cx)}{5}-\frac{bcdx^4\sqrt{c^2x^2+1}}{25}+\frac{bdx^3\,\text{asinh}\,(cx)}{3}-\frac{13bdx^2\sqrt{c^2x^2+1}}{225c}+\frac{26bd\sqrt{c^2x^2+1}}{225c^3} & \text{for } c\neq 0 \\ \frac{adx^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**2*d*x**5/5 + a*d*x**3/3 + b*c**2*d*x**5*asinh(c*x)/5 - b*c
*d*x**4*sqrt(c**2*x**2 + 1)/25 + b*d*x**3*asinh(c*x)/3 - 13*b*d*x**2*sqrt(
c**2*x**2 + 1)/(225*c) + 26*b*d*sqrt(c**2*x**2 + 1)/(225*c**3), Ne(c, 0)),
 (a*d*x**3/3, True))
```

### 3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.42

$$\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{5}\,ac^2dx^5$$
$$+\frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d$$
$$+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bd$$

input
```
integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^2*d*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2
 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^2*d + 1/3
*a*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt
(c^2*x^2 + 1)/c^4))*b*d
```

### 3.3.8 Giac [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 d x^2) (a + b \text{arcsinh}(cx)) \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 d x^2) (a + b \text{arcsinh}(cx)) \, dx = \int x^2 \left(a + b \operatorname{asinh}(c\,x)\right) \left(d\,c^2\,x^2 + d\right) \, dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

# 3.4 $\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$



## 3.4.1 Optimal result

Integrand size = 20, antiderivative size = 87

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = -\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx(1+c^2x^2)^{3/2}}{16c}-\frac{3bd\text{arcsinh}(cx)}{32c^2}+\frac{d(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{4c^2}$$

output
```
-1/16*b*d*x*(c^2*x^2+1)^(3/2)/c-3/32*b*d*arcsinh(c*x)/c^2+1/4*d*(c^2*x^2+1
)^2*(a+b*arcsinh(c*x))/c^2-3/32*b*d*x*(c^2*x^2+1)^(1/2)/c
```

## 3.4.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{d\left(cx\left(8acx(2+c^2x^2)-b\sqrt{1+c^2x^2}(5+2c^2x^2)\right)+b(5+16c^2x^2+8c^4x^4)\,\text{arcsinh}(cx)\right)}{32c^2}$$

input
```
Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(c*x*(8*a*c*x*(2 + c^2*x^2) - b*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)) + b*
(5 + 16*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x]))/(32*c^2)
```

### 3.4.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{d\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bd\int \left(c^2x^2+1\right)^{3/2}dx}{4c}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bd\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}$$

$$\Big\downarrow \text{222}$$

$$\frac{d\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bd\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}$$

input
```
Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*d*((x*(1 + c^2*x^2)^
(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)
```

### 3.4.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.4.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{da(c^2x^2+1)^2}{4}+db\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx(c^2x^2+1)^{\frac{3}{2}}}{16}-\frac{3cx\sqrt{c^2x^2+1}}{32}\right)}{c^2}$ | 85 |
| default | $\frac{\frac{da(c^2x^2+1)^2}{4}+db\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx(c^2x^2+1)^{\frac{3}{2}}}{16}-\frac{3cx\sqrt{c^2x^2+1}}{32}\right)}{c^2}$ | 85 |
| parts | $\frac{da(c^2x^2+1)^2}{4c^2}+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{5\operatorname{arcsinh}(cx)}{32}-\frac{cx(c^2x^2+1)^{\frac{3}{2}}}{16}-\frac{3cx\sqrt{c^2x^2+1}}{32}\right)}{c^2}$ | 87 |

input
```
int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4*d*a*(c^2*x^2+1)^2+d*b*(1/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x
)*c^2*x^2+5/32*arcsinh(c*x)-1/16*c*x*(c^2*x^2+1)^(3/2)-3/32*c*x*(c^2*x^2+1
)^(1/2)))
```

### 3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{8\,ac^4dx^4+16\,ac^2dx^2+(8\,bc^4dx^4+16\,bc^2dx^2+5\,bd)\log\left(cx+\sqrt{c^2x^2+1}\right)-(2\,bc^3dx^3+5\,bcdx)\sqrt{c^2x^2+}}{32\,c^2}$$

input
```
integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/32*(8*a*c^4*d*x^4 + 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 + 16*b*c^2*d*x^2 + 5
*b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(c^2*
x^2 + 1))/c^2
```

### 3.4.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^2dx^4}{4}+\frac{adx^2}{2}+\frac{bc^2dx^4\operatorname{asinh}(cx)}{4}-\frac{bcdx^3\sqrt{c^2x^2+1}}{16}+\frac{bdx^2\operatorname{asinh}(cx)}{2}-\frac{5bdx\sqrt{c^2x^2+1}}{32c}+\frac{5bd\operatorname{asinh}(cx)}{32c^2} & \text{for } c\neq 0\\ \frac{adx^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**2*d*x**4/4 + a*d*x**2/2 + b*c**2*d*x**4*asinh(c*x)/4 - b*c
*d*x**3*sqrt(c**2*x**2 + 1)/16 + b*d*x**2*asinh(c*x)/2 - 5*b*d*x*sqrt(c**2
*x**2 + 1)/(32*c) + 5*b*d*asinh(c*x)/(32*c**2), Ne(c, 0)), (a*d*x**2/2, Tr
ue))
```

### 3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.46

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{4}ac^2dx^4$$
$$+\frac{1}{32}\left(8x^4\text{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\,\text{arsinh}(cx)}{c^5}\right)c\right)bc^2d$$
$$+\frac{1}{2}adx^2+\frac{1}{4}\left(2x^2\text{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\text{arsinh}(cx)}{c^3}\right)\right)bd$$

input
```
integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*c^2*d*x^4 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2
- 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*c^2*d + 1/2*a*d*x^2
 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3
))*b*d
```

### 3.4.8 Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.4.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+c^2dx^2\right)(a+b\operatorname{arcsinh}(cx))\,dx=\int x\,(a+b\,\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

output
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

# 3.5 $\int (d+c^2dx^2) (a+b\text{arcsinh}(cx)) \, dx$



## 3.5.1 Optimal result

Integrand size = 19, antiderivative size = 75

$$\int (d+c^2dx^2) (a+b\text{arcsinh}(cx)) \, dx = -\frac{2bd\sqrt{1+c^2x^2}}{3c} - \frac{bd(1+c^2x^2)^{3/2}}{9c} + dx(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))$$

output
```
-1/9*b*d*(c^2*x^2+1)^(3/2)/c+d*x*(a+b*arcsinh(c*x))+1/3*c^2*d*x^3*(a+b*arc
sinh(c*x))-2/3*b*d*(c^2*x^2+1)^(1/2)/c
```

## 3.5.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int (d+c^2dx^2) (a+b\text{arcsinh}(cx)) \, dx = adx + \frac{1}{3}ac^2dx^3 - \frac{7bd\sqrt{1+c^2x^2}}{9c} - \frac{1}{9}bcdx^2\sqrt{1+c^2x^2} + bdx\text{arcsinh}(cx) + \frac{1}{3}bc^2dx^3\text{arcsinh}(cx)$$

input
```
Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
a*d*x + (a*c^2*d*x^3)/3 - (7*b*d*Sqrt[1 + c^2*x^2])/(9*c) - (b*c*d*x^2*Sqr
t[1 + c^2*x^2])/9 + b*d*x*ArcSinh[c*x] + (b*c^2*d*x^3*ArcSinh[c*x])/3
```

### 3.5.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6199, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6199}$$

$$-bc\int \frac{dx\left(c^2x^2+3\right)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bcd\int \frac{x\left(c^2x^2+3\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{353}$$

$$-\frac{1}{6}bcd\int \frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{6}bcd\int \left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))-\frac{1}{6}bcd\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)$$

input
```
Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/6*(b*c*d*((4*Sqrt[1 + c^2*x^2])/c^2 + (2*(1 + c^2*x^2)^(3/2))/(3*c^2)))
 + d*x*(a + b*ArcSinh[c*x]) + (c^2*d*x^3*(a + b*ArcSinh[c*x]))/3
```

#### 3.5.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## 3.5.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

| method | result | size |
|---|---|---|
| parts | $da\left(\frac{1}{3}x^3c^2+x\right)+\frac{db\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$ | 73 |
| derivativedivides | $\frac{da\left(\frac{1}{3}c^3x^3+cx\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$ | 76 |
| default | $\frac{da\left(\frac{1}{3}c^3x^3+cx\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$ | 76 |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/3*x^3*c^2+x)+d*b/c*(1/3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/9*c
^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2))
```

### 3.5.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int (d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))\,dx = \frac{3\,ac^3dx^3+9\,acdx+3\,(bc^3dx^3+3\,bcdx)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^2dx^2+7\,bd)\sqrt{c^2x^2+1}}{9\,c}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*d*x^3 + 9*a*c*d*x + 3*(b*c^3*d*x^3 + 3*b*c*d*x)*log(c*x + sqr
t(c^2*x^2 + 1)) - (b*c^2*d*x^2 + 7*b*d)*sqrt(c^2*x^2 + 1))/c
```

### 3.5.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int (d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))\,dx = \begin{cases} \frac{ac^2dx^3}{3}+adx+\frac{bc^2dx^3\operatorname{asinh}(cx)}{3}-\frac{bcdx^2\sqrt{c^2x^2+1}}{9}+bdx\operatorname{asinh}(cx)-\frac{7bd\sqrt{c^2x^2+1}}{9c} & \text{for } c\neq 0 \\ adx & \text{otherwise} \end{cases}$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**2*d*x**3/3 + a*d*x + b*c**2*d*x**3*asinh(c*x)/3 - b*c*d*x*
*2*sqrt(c**2*x**2 + 1)/9 + b*d*x**asinh(c*x) - 7*b*d*sqrt(c**2*x**2 + 1)/(9
*c), Ne(c, 0)), (a*d*x, True))
```

### 3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{3}ac^2dx^3+\frac{1}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d$$
$$+adx+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*c^2*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*
x^2 + 1))*b*d/c
```

### 3.5.8 Giac [F(-2)]

Exception generated.

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.5.9 Mupad [F(-1)]

Timed out.

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

# 3.6 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x} \, dx$



## 3.6.1 Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x} \, dx = -\frac{1}{4}bcdx\sqrt{1+c^2x^2} - \frac{1}{4}bd\text{arcsinh}(cx) + \frac{1}{2}d(1+c^2x^2)(a+b\text{arcsinh}(cx)) + \frac{d(a+b\text{arcsinh}(cx))^2}{2b} + d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - \frac{1}{2}bd\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(cx)}\right)$$

output
```
-1/4*b*d*arcsinh(c*x)+1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/2*d*(a+b*arcs
inh(c*x))^2/b+d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b
*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)
```

## 3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x} \, dx = \frac{1}{2}ac^2dx^2 - \frac{1}{4}bcdx\sqrt{1+c^2x^2} + \frac{1}{4}bd\text{arcsinh}(cx) + \frac{1}{2}bc^2dx^2\text{arcsinh}(cx) - \frac{1}{2}bd\text{arcsinh}(cx)^2 + bd\text{arcsinh}(cx)\log\left(1-e^{2\text{arcsinh}(cx)}\right) + ad\log(x) + \frac{1}{2}bd\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(cx)}\right)$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(a*c^2*d*x^2)/2 - (b*c*d*x*Sqrt[1 + c^2*x^2])/4 + (b*d*ArcSinh[c*x])/4 + (
b*c^2*d*x^2*ArcSinh[c*x])/2 - (b*d*ArcSinh[c*x]^2)/2 + b*d*ArcSinh[c*x]*Lo
g[1 - E^(2*ArcSinh[c*x])] + a*d*Log[x] + (b*d*PolyLog[2, E^(2*ArcSinh[c*x]
)])/2
```

### 3.6.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6216, 211, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))}{x}dx$$

$$\Big\downarrow \text{ 6216}$$

$$d\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bcd\int \sqrt{c^2x^2+1}dx+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 211}$$

$$d\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bcd\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 222}$$

$$d\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$$\Big\downarrow \text{ 6190}$$

$$\frac{d\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{d\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$\downarrow$ 3042

$$-\frac{d\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$\downarrow$ 26

$$\frac{id\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$\downarrow$ 4201

$$\frac{id\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$\downarrow$ 2620

$$\frac{id\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx}{b}}\right.\right.\right.}{b}$$
$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$\downarrow$ 2715

$$\frac{id\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+\text{barcsinh}\right.\right.}{b}$$
$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

$\downarrow$ 2838

---

3.6. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx$

$$\frac{id\left(2i\left(\frac{1}{4}b^2\,\mathrm{PolyLog}(2,-a-b\mathrm{arcsinh}(cx))-\frac{1}{2}b(a+b\mathrm{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\mathrm{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b}{b}$$

$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\mathrm{arcsinh}(cx))-\frac{1}{2}bcd\left(\frac{\mathrm{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)$$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 - (b*c*d*((x*Sqrt[1 + c^2*x^2])/2
 + ArcSinh[c*x]/(2*c)))/2 + (I*d*((-1/2*I)*(a + b*ArcSinh[c*x])^2 + (2*I)*
(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSin
h[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/b
```

### 3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## 3.6.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| parts | $da\left(\frac{c^2x^2}{2}+\ln(x)\right)-\frac{db\,\mathrm{arcsinh}(cx)^2}{2}+\frac{db\,\mathrm{arcsinh}(cx)c^2x^2}{2}-\frac{bcdx\sqrt{c^2x^2+1}}{4}+\frac{bd\,\mathrm{arcsinh}(cx)}{4}+db\,\mathrm{arcs}$ |
| derivativedivides | $da\left(\frac{c^2x^2}{2}+\ln(cx)\right)-\frac{db\,\mathrm{arcsinh}(cx)^2}{2}+\frac{db\,\mathrm{arcsinh}(cx)c^2x^2}{2}-\frac{bcdx\sqrt{c^2x^2+1}}{4}+\frac{bd\,\mathrm{arcsinh}(cx)}{4}+db\,\mathrm{ar}$ |
| default | $da\left(\frac{c^2x^2}{2}+\ln(cx)\right)-\frac{db\,\mathrm{arcsinh}(cx)^2}{2}+\frac{db\,\mathrm{arcsinh}(cx)c^2x^2}{2}-\frac{bcdx\sqrt{c^2x^2+1}}{4}+\frac{bd\,\mathrm{arcsinh}(cx)}{4}+db\,\mathrm{ar}$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/2*c^2*x^2+ln(x))-1/2*d*b*arcsinh(c*x)^2+1/2*d*b*arcsinh(c*x)*c^2*x^
2-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)+1/4*b*d*arcsinh(c*x)+d*b*arcsinh(c*x)*ln(1
+c*x+(c^2*x^2+1)^(1/2))+d*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d*b*arcsinh(
c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+d*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

### 3.6.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x, x)
```

### 3.6.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x}\,dx=d\left(\int \frac{a}{x}\,dx+\int ac^2x\,dx+\int \frac{b\,\text{asinh}\,(cx)}{x}\,dx+\int bc^2x\,\text{asinh}\,(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x,x)
```

output
```
d*(Integral(a/x, x) + Integral(a*c**2*x, x) + Integral(b*asinh(c*x)/x, x)
+ Integral(b*c**2*x*asinh(c*x), x))
```

### 3.6.7 Maxima [F]

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))}{x} dx = \int \frac{(c^2dx^2 + d)(b\,\text{arsinh}\,(cx) + a)}{x} dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*c^2*d*x^2 + a*d*log(x) + integrate(b*c^2*d*x*log(c*x + sqrt(c^2*x^2
+ 1)) + b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.6.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))}{x} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2 + d)}{x} dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x, x)
```

# 3.7 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.7.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = -bcd\sqrt{1+c^2x^2}-\frac{d(a+b\text{arcsinh}(cx))}{x}+c^2dx(a+b\text{arcsinh}(cx))-bcd\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

output
```
-d*(a+b*arcsinh(c*x))/x+c^2*d*x*(a+b*arcsinh(c*x))-b*c*d*arctanh((c^2*x^2+
1)^(1/2))-b*c*d*(c^2*x^2+1)^(1/2)
```

## 3.7.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{ad}{x}+ac^2dx-bcd\sqrt{1+c^2x^2}-\frac{bd\text{arcsinh}(cx)}{x}+bc^2dx\text{arcsinh}(cx)-bcd\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*c^2*d*x - b*c*d*Sqrt[1 + c^2*x^2] - (b*d*ArcSinh[c*x])/x +
b*c^2*d*x*ArcSinh[c*x] - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

### 3.7.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6218, 25, 27, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)(a+b\text{arcsinh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{ 6218}$$

$$-bc\int -\frac{d(1-c^2x^2)}{x\sqrt{c^2x^2+1}}dx+c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 25}$$

$$bc\int \frac{d(1-c^2x^2)}{x\sqrt{c^2x^2+1}}dx+c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 27}$$

$$bcd\int \frac{1-c^2x^2}{x\sqrt{c^2x^2+1}}dx+c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}bcd\int \frac{1-c^2x^2}{x^2\sqrt{c^2x^2+1}}dx^2+c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}bcd\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-2\sqrt{c^2x^2+1}\right)+c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}bcd\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}-2\sqrt{c^2x^2+1}\right)+c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 221}$$

$$c^2dx(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}bcd\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)$$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcSinh[c*x]))/x) + c^2*d*x*(a + b*ArcSinh[c*x]) + (b*c*d*(-2*
Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/2
```

### 3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.7.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| parts | $da\left(c^2x-\frac{1}{x}\right)+dbc\left(\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)$ |
| derivativedivides | $c\left(da\left(cx-\frac{1}{cx}\right)+db\left(\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)\right)$ |
| default | $c\left(da\left(cx-\frac{1}{cx}\right)+db\left(\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)\right)$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*a*(c^2*x-1/x)+d*b*c*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)
-arctanh(1/(c^2*x^2+1)^(1/2)))
```

### 3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.36

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx = \frac{ac^2dx^2-bcdx\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+bcdx\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-\sqrt{c^2x^2+1}bcdx-(bc^2-}{x}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

output
```
(a*c^2*d*x^2 - b*c*d*x*log(-c*x + sqrt(c^2*x^2 + 1) + 1) + b*c*d*x*log(-c*
x + sqrt(c^2*x^2 + 1) - 1) - sqrt(c^2*x^2 + 1)*b*c*d*x - (b*c^2 - b)*d*x*l
og(-c*x + sqrt(c^2*x^2 + 1)) - a*d + (b*c^2*d*x^2 - (b*c^2 - b)*d*x - b*d)
*log(c*x + sqrt(c^2*x^2 + 1)))/x
```

### 3.7.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = d\left(\int ac^2\,dx + \int \frac{a}{x^2}\,dx + \int bc^2\,\text{asinh}\,(cx)\,dx + \int \frac{b\,\text{asinh}\,(cx)}{x^2}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**2,x)
```

output
```
d*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(b*c**2*asinh(c*x),
 x) + Integral(b*asinh(c*x)/x**2, x))
```

### 3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = ac^2dx + \left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bcd - \left(c\,\text{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\text{arsinh}\,(cx)}{x}\right)bd - \frac{ad}{x}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

output
```
a*c^2*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c*d - (c*arcsinh(1/(c
*abs(x))) + arcsinh(c*x)/x)*b*d - a*d/x
```

### 3.7.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\ (d\,c^2\,x^2+d)}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2, x)
```

# 3.8 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx$



## 3.8.1 Optimal result

Integrand size = 22, antiderivative size = 128

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx =& -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\text{arcsinh}(cx)\\&-\frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2x^2}\\&+\frac{c^2d(a+b\text{arcsinh}(cx))^2}{2b}\\&+c^2d(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\&-\frac{1}{2}bc^2d\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
1/2*b*c^2*d*arcsinh(c*x)-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/x^2+1/2*c^2*
d*(a+b*arcsinh(c*x))^2/b+c^2*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^
(1/2))^2)-1/2*b*c^2*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*(c^
2*x^2+1)^(1/2)/x
```

### 3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{ad}{2x^2}-\frac{bcd\sqrt{1+c^2x^2}}{2x} -\frac{bd\text{arcsinh}(cx)}{2x^2}-\frac{1}{2}bc^2d\text{arcsinh}(cx)^2 +bc^2d\text{arcsinh}(cx)\log\left(1-e^{2\text{arcsinh}(cx)}\right) +ac^2d\log(x)+\frac{1}{2}bc^2d\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 - (b*c*d*Sqrt[1 + c^2*x^2])/(2*x) - (b*d*ArcSinh[c*x])/(2*x
^2) - (b*c^2*d*ArcSinh[c*x]^2)/2 + b*c^2*d*ArcSinh[c*x]*Log[1 - E^(2*ArcSi
nh[c*x])] + a*c^2*d*Log[x] + (b*c^2*d*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

### 3.8.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6217, 247, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)(a+b\text{arcsinh}(cx))}{x^3}dx$$

$$\Big\downarrow \text{6217}$$

$$c^2d\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}bcd\int \frac{\sqrt{c^2x^2+1}}{x^2}dx-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{247}$$

$$c^2d\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}bcd\left(c^2\int \frac{1}{\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}}{x}\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{222}$$

$$c^2d\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{6190}$$

$$\frac{c^2d\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^2d\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^2d\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{ic^2d\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{4201}$$

$$\frac{ic^2d\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}-\frac{d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{2620}$$

---

3.8. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx$

$$\frac{ic^2d\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}}{b}}\right.\right.\right.}{b}$$

$$\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$\big\downarrow$ 2715

$$\frac{ic^2d\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+b\text{arcsi}\right.\right.}{b}$$

$$\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

$\big\downarrow$ 2838

$$\frac{ic^2d\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+\right.}{b}$$

$$\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)$$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
-1/2*(d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/x^2 + (b*c*d*(-(Sqrt[1 + c^2*x
^2]/x) + c*ArcSinh[c*x]))/2 + (I*c^2*d*((-1/2*I)*(a + b*ArcSinh[c*x])^2 +
(2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*
ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/b
```

### 3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

## 3.8. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx$

rule 6217
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1
 + c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

## 3.8.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| parts | $da\left(-\frac{1}{2x^2}+c^2\ln\left(x\right)\right)+db\,c^2\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}+\operatorname{arcsinh}\left(cx\right)\ln\left(\right.\right.$ |
| derivativedivides | $c^2\left(da\left(\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}+\operatorname{arcsinh}\left(cx\right)\ln\left(1\right.\right.\right.$ |
| default | $c^2\left(da\left(\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}+\operatorname{arcsinh}\left(cx\right)\ln\left(1\right.\right.\right.$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d*a*(-1/2/x^2+c^2*ln(x))+d*b*c^2*(-1/2*arcsinh(c*x)^2-1/2*(c*x*(c^2*x^2+1)
^(1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1
/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

## 3.8.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x^3, x)
```

### 3.8.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^3}\,dx = d\left(\int \frac{a}{x^3}\,dx+\int \frac{ac^2}{x}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{x^3}\,dx + \int \frac{bc^2\,\text{asinh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**3,x)
```

output
```
d*(Integral(a/x**3, x) + Integral(a*c**2/x, x) + Integral(b*asinh(c*x)/x**
3, x) + Integral(b*c**2*asinh(c*x)/x, x))
```

### 3.8.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

output
```
b*c^2*d*integrate(log(c*x + sqrt(c^2*x^2 + 1))/x, x) + a*c^2*d*log(x) - 1/
2*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d/x^2
```

### 3.8.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3, x)
```

# 3.9 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.9.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d(a+b\text{arcsinh}(cx))}{3x^3}-\frac{c^2d(a+b\text{arcsinh}(cx))}{x}-\frac{5}{6}bc^3d\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

output
```
-1/3*d*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))/x-5/6*b*c^3*d*arcta
nh((c^2*x^2+1)^(1/2))-1/6*b*c*d*(c^2*x^2+1)^(1/2)/x^2
```

## 3.9.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ac^2d}{x}-\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{bd\text{arcsinh}(cx)}{3x^3}-\frac{bc^2d\text{arcsinh}(cx)}{x}-\frac{5}{6}bc^3d\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*(a*d)/x^3 - (a*c^2*d)/x - (b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (b*d*Ar
cSinh[c*x])/(3*x^3) - (b*c^2*d*ArcSinh[c*x])/x - (5*b*c^3*d*ArcTanh[Sqrt[1
 + c^2*x^2]])/6
```

### 3.9.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6218, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)\,(a+b\text{arcsinh}(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int -\frac{d(3c^2x^2+1)}{3x^3\sqrt{c^2x^2+1}}dx-\frac{c^2d(a+b\text{arcsinh}(cx))}{x}-\frac{d(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd\int \frac{3c^2x^2+1}{x^3\sqrt{c^2x^2+1}}dx-\frac{c^2d(a+b\text{arcsinh}(cx))}{x}-\frac{d(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{6}bcd\int \frac{3c^2x^2+1}{x^4\sqrt{c^2x^2+1}}dx^2-\frac{c^2d(a+b\text{arcsinh}(cx))}{x}-\frac{d(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{6}bcd\left(\frac{5}{2}c^2\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-\frac{\sqrt{c^2x^2+1}}{x^2}\right)-\frac{c^2d(a+b\text{arcsinh}(cx))}{x}-\frac{d(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}bcd\left(5\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}-\frac{\sqrt{c^2x^2+1}}{x^2}\right)-\frac{c^2d(a+b\text{arcsinh}(cx))}{x}-\frac{d(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2d(a+b\text{arcsinh}(cx))}{x}-\frac{d(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}bcd\left(-5c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)$$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcSinh[c*x]))/x^3 - (c^2*d*(a + b*ArcSinh[c*x]))/x + (b*c*
d*(-(Sqrt[1 + c^2*x^2]/x^2) - 5*c^2*ArcTanh[Sqrt[1 + c^2*x^2]]))/6
```

### 3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.9.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| parts | $da\left(-\frac{c^2}{x}-\frac{1}{3x^3}\right)+db\,c^3\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}-\frac{5\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}\right)$ |
| derivativedivides | $c^3\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}-\frac{5\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}\right)\right)$ |
| default | $c^3\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}-\frac{5\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{6}\right)\right)$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d*a*(-c^2/x-1/3/x^3)+d*b*c^3*(-1/3*arcsinh(c*x)/c^3/x^3-arcsinh(c*x)/c/x-1
/6/c^2/x^2*(c^2*x^2+1)^(1/2)-5/6*arctanh(1/(c^2*x^2+1)^(1/2)))
```

### 3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(70) = 140.

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.11

$$\int\frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx=$$
$$-\frac{5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)-5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)+6\,ac^2dx^2-2\,(3\,bc^2+b)dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(5*b*c^3*d*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) - 5*b*c^3*d*x^3*log(
-c*x + sqrt(c^2*x^2 + 1) - 1) + 6*a*c^2*d*x^2 - 2*(3*b*c^2 + b)*d*x^3*log(
-c*x + sqrt(c^2*x^2 + 1)) + sqrt(c^2*x^2 + 1)*b*c*d*x + 2*a*d + 2*(3*b*c^2
*d*x^2 - (3*b*c^2 + b)*d*x^3 + b*d)*log(c*x + sqrt(c^2*x^2 + 1)))/x^3
```

### 3.9.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = d\left(\int \frac{a}{x^4}\,dx + \int \frac{ac^2}{x^2}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{bc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**4,x)
```

output
```
d*(Integral(a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(b*asinh(c*x)/
x**4, x) + Integral(b*c**2*asinh(c*x)/x**2, x))
```

### 3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = -\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d + \frac{1}{6}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2+1}}{x^2}\right)c - \frac{2\operatorname{arsinh}(cx)}{x^3}\right)bd - \frac{ac^2d}{x} - \frac{ad}{3\,x^3}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

output
```
-(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d + 1/6*((c^2*arcsinh(1/
(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d - a*c^2*d
/x - 1/3*a*d/x^3
```

### 3.9.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4, x)
```

# 3.10 $\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$



## 3.10.1 Optimal result

Integrand size = 24, antiderivative size = 181

$$\begin{aligned}\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx &= -\frac{8bd^2\sqrt{1+c^2x^2}}{315c^5}-\frac{4bd^2(1+c^2x^2)^{3/2}}{945c^5}-\frac{bd^2(1+c^2x^2)^{5/2}}{525c^5}\\ &\quad+\frac{10bd^2(1+c^2x^2)^{7/2}}{441c^5}-\frac{bd^2(1+c^2x^2)^{9/2}}{81c^5}\\ &\quad+\frac{1}{5}d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2d^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}c^4d^2x^9(a+b\text{arcsinh}(cx))\end{aligned}$$

output
```
-4/945*b*d^2*(c^2*x^2+1)^(3/2)/c^5-1/525*b*d^2*(c^2*x^2+1)^(5/2)/c^5+10/44
1*b*d^2*(c^2*x^2+1)^(7/2)/c^5-1/81*b*d^2*(c^2*x^2+1)^(9/2)/c^5+1/5*d^2*x^5
*(a+b*arcsinh(c*x))+2/7*c^2*d^2*x^7*(a+b*arcsinh(c*x))+1/9*c^4*d^2*x^9*(a+
b*arcsinh(c*x))-8/315*b*d^2*(c^2*x^2+1)^(1/2)/c^5
```

## 3.10.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(315ac^5x^5(63+90c^2x^2+35c^4x^4)-b\sqrt{1+c^2x^2}(2104-1052c^2x^2+789c^4x^4+2650c^6x^6+1225c^8x^8)+\right.}{99225c^5}$$

input
```
Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(
2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) + 315*b*c
^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSinh[c*x]))/(99225*c^5)
```

### 3.10.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6218, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(c^2 d x^2+d\right)^2 (a+b \text{arcsinh}(c x)) \, dx$$

$$\Big\downarrow \text{ 6218}$$

$$-bc \int \frac{d^2 x^5 \left(35 c^4 x^4+90 c^2 x^2+63\right)}{315 \sqrt{c^2 x^2+1}}dx+\frac{1}{9} c^4 d^2 x^9 (a+b \text{arcsinh}(c x))+\frac{2}{7} c^2 d^2 x^7 (a+b \text{arcsinh}(c x))+\frac{1}{5} d^2 x^5 (a+b \text{arcsinh}(c x))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{315} b c d^2 \int \frac{x^5 \left(35 c^4 x^4+90 c^2 x^2+63\right)}{\sqrt{c^2 x^2+1}}dx+\frac{1}{9} c^4 d^2 x^9 (a+b \text{arcsinh}(c x))+\frac{2}{7} c^2 d^2 x^7 (a+b \text{arcsinh}(c x))+\frac{1}{5} d^2 x^5 (a+b \text{arcsinh}(c x))$$

$$\Big\downarrow \text{ 1578}$$

$$-\frac{1}{630} b c d^2 \int \frac{x^4 \left(35 c^4 x^4+90 c^2 x^2+63\right)}{\sqrt{c^2 x^2+1}}dx^2+\frac{1}{9} c^4 d^2 x^9 (a+b \text{arcsinh}(c x))+\frac{2}{7} c^2 d^2 x^7 (a+b \text{arcsinh}(c x))+\frac{1}{5} d^2 x^5 (a+b \text{arcsinh}(c x))$$

$$\Big\downarrow \text{ 1195}$$

$$-\frac{1}{630} b c d^2 \int \left(\frac{35 \left(c^2 x^2+1\right)^{7/2}}{c^4}-\frac{50 \left(c^2 x^2+1\right)^{5/2}}{c^4}+\frac{3 \left(c^2 x^2+1\right)^{3/2}}{c^4}+\frac{4 \sqrt{c^2 x^2+1}}{c^4}+\frac{8}{c^4 \sqrt{c^2 x^2+1}}\right)dx^2+\frac{1}{9} c^4 d^2 x^9 (a+b \text{arcsinh}(c x))+\frac{2}{7} c^2 d^2 x^7 (a+b \text{arcsinh}(c x))+\frac{1}{5} d^2 x^5 (a+b \text{arcsinh}(c x))$$

$$\downarrow \text{2009}$$

$$\frac{1}{9}c^4d^2x^9(a+b\operatorname{arcsinh}(cx))+\frac{2}{7}c^2d^2x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{5}d^2x^5(a+b\operatorname{arcsinh}(cx))-\frac{1}{630}bcd^2\left(\frac{70(c^2x^2+1)^{9/2}}{9c^6}-\frac{100(c^2x^2+1)^{7/2}}{7c^6}+\frac{6(c^2x^2+1)^{5/2}}{5c^6}+\frac{8(c^2x^2+1)^{3/2}}{3c^6}+\frac{16\sqrt{c^2x^2+1}}{c^6}\right)$$

input
```
Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/630*(b*c*d^2*((16*Sqrt[1 + c^2*x^2])/c^6 + (8*(1 + c^2*x^2)^(3/2))/(3*c
^6) + (6*(1 + c^2*x^2)^(5/2))/(5*c^6) - (100*(1 + c^2*x^2)^(7/2))/(7*c^6)
+ (70*(1 + c^2*x^2)^(9/2))/(9*c^6))) + (d^2*x^5*(a + b*ArcSinh[c*x]))/5 +
(2*c^2*d^2*x^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcSinh[c*x])
)/9
```

### 3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.10.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| parts | $d^2a\left(\frac{1}{9}c^4x^9+\frac{2}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^2b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{2\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-106c^6\right.}{c^5}$ |
| derivativedivides | $\frac{d^2a\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{2\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6x^6\sqrt{c^2x^2+}}{3969}\right.}{c^5}$ |
| default | $\frac{d^2a\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{2\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6x^6\sqrt{c^2x^2+}}{3969}\right.}{c^5}$ |

input
```
int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/9*c^4*x^9+2/7*c^2*x^7+1/5*x^5)+d^2*b/c^5*(1/9*arcsinh(c*x)*c^9*x^
9+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*(c^2*x^2+
1)^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2*x^2+1)^
(1/2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(1/2))
```

### 3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91

$$\int x^4\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{11025\,ac^9d^2x^9+28350\,ac^7d^2x^7+19845\,ac^5d^2x^5+315\left(35\,bc^9d^2x^9+90\,bc^7d^2x^7+63\,bc^5d^2x^5\right)\log\left(cx+\sqrt{\right.}{99225\,c^5}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*d^2*x^9 + 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
 315*(35*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sq
rt(c^2*x^2 + 1)) - (1225*b*c^8*d^2*x^8 + 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^
2*x^4 - 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

### 3.10.6 Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.27

$$\int x^4(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^4d^2x^9}{9}+\frac{2ac^2d^2x^7}{7}+\frac{ad^2x^5}{5}+\frac{bc^4d^2x^9\operatorname{asinh}(cx)}{9}-\frac{bc^3d^2x^8\sqrt{c^2x^2+1}}{81}+\frac{2bc^2d^2x^7\operatorname{asinh}(cx)}{7}-\frac{106bcd^2x^6\sqrt{c^2x^2+1}}{3969}+\frac{bd^2x^5\operatorname{asin}}{5}\\ \frac{ad^2x^5}{5}\end{cases}$$

input
```
integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**9/9 + 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c
**4*d**2*x**9*asinh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 + 1)/81 + 2*b
*c**2*d**2*x**7*asinh(c*x)/7 - 106*b*c*d**2*x**6*sqrt(c**2*x**2 + 1)/3969
+ b*d**2*x**5*asinh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 + 1)/(33075*c)
 + 1052*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**3) - 2104*b*d**2*sqrt(c*
*2*x**2 + 1)/(99225*c**5), Ne(c, 0)), (a*d**2*x**5/5, True))
```

### 3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(155) = 310$.

Time = 0.19 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.76

$$\int x^4(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))\,dx=\frac{1}{9}ac^4d^2x^9+\frac{2}{7}ac^2d^2x^7$$
$$+\frac{1}{2835}\left(315x^9\operatorname{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}x^2}{c^8}\right.\right.$$
$$+\frac{1}{5}ad^2x^5$$
$$+\frac{2}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d$$
$$+\frac{1}{75}\left(15x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^2$$

input
```
integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*c^4*d^2*x^9 + 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35
*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^
2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10
)*c)*b*c^4*d^2 + 1/5*a*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*
x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c
^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arcsinh(c*x) -
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 + 1)/c^6)*c)*b*d^2
```

### 3.10.8 Giac [F(-2)]

Exception generated.

$$\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.10.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \int x^4\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

output
```
int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

# 3.11 $\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$



## 3.11.1 Optimal result

Integrand size = 24, antiderivative size = 180

$$\begin{aligned}\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx &= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3}-\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}\\ &\quad-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}\\ &\quad-\frac{73bd^2\text{arcsinh}(cx)}{3072c^4}+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))\\ &\quad+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))\\ &\quad+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))\end{aligned}$$

output
```
-73/3072*b*d^2*arcsinh(c*x)/c^4+1/4*d^2*x^4*(a+b*arcsinh(c*x))+1/3*c^2*d^2
*x^6*(a+b*arcsinh(c*x))+1/8*c^4*d^2*x^8*(a+b*arcsinh(c*x))+73/3072*b*d^2*x
*(c^2*x^2+1)^(1/2)/c^3-73/4608*b*d^2*x^3*(c^2*x^2+1)^(1/2)/c-43/1152*b*c*d
^2*x^5*(c^2*x^2+1)^(1/2)-1/64*b*c^3*d^2*x^7*(c^2*x^2+1)^(1/2)
```

### 3.11.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{d^2\left(384ac^4x^4(6+8c^2x^2+3c^4x^4)-bcx\sqrt{1+c^2x^2}(-219+146c^2x^2+344c^4x^4+144c^6x^6)+3b(-73+768\ldots\right.}{9216c^4}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(384*a*c^4*x^4*(6 + 8*c^2*x^2 + 3*c^4*x^4) - b*c*x*Sqrt[1 + c^2*x^2]*
(-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6) + 3*b*(-73 + 768*c^4*x^4
+ 1024*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]))/(9216*c^4)
```

### 3.11.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6218, 27, 1590, 27, 363, 262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(c^2dx^2+d\right)^2(a+b\text{arcsinh}(cx))\,dx$$
$$\Big\downarrow \; 6218$$
$$-bc\int \frac{d^2x^4\left(3c^4x^4+8c^2x^2+6\right)}{24\sqrt{c^2x^2+1}}dx+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))$$
$$\Big\downarrow \; 27$$
$$-\frac{1}{24}bcd^2\int \frac{x^4\left(3c^4x^4+8c^2x^2+6\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))$$
$$\Big\downarrow \; 1590$$

$$-\frac{1}{24}bcd^2\left(\frac{\int \frac{c^2x^4(43c^2x^2+48)}{\sqrt{c^2x^2+1}}dx}{8c^2}+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{1}{24}bcd^2\left(\frac{1}{8}\int \frac{x^4(43c^2x^2+48)}{\sqrt{c^2x^2+1}}dx+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \textbf{363}$$

$$-\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\int \frac{x^4}{\sqrt{c^2x^2+1}}dx+\frac{43}{6}x^5\sqrt{c^2x^2+1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \textbf{262}$$

$$-\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int \frac{x^2}{\sqrt{c^2x^2+1}}dx}{4c^2}\right)+\frac{43}{6}x^5\sqrt{c^2x^2+1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \textbf{262}$$

$$-\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{4c^2}\right)+\frac{43}{6}x^5\sqrt{c^2x^2+1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \textbf{222}$$

$$\frac{1}{8}c^4d^2x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2d^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)+\frac{43}{6}x^5\sqrt{c^2x^2+1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)$$

input
```
Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

3.11. $\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$

output
```
(d^2*x^4*(a + b*ArcSinh[c*x]))/4 + (c^2*d^2*x^6*(a + b*ArcSinh[c*x]))/3 +
(c^4*d^2*x^8*(a + b*ArcSinh[c*x]))/8 - (b*c*d^2*((3*c^2*x^7*Sqrt[1 + c^2*x
^2])/8 + ((43*x^5*Sqrt[1 + c^2*x^2])/6 + (73*((x^3*Sqrt[1 + c^2*x^2])/(4*c
^2) - (3*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/(4*c^2)))
/6)/8))/24
```

#### 3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.11.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| parts | $d^2a\left(\frac{1}{8}c^4x^8+\frac{1}{3}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^2b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{3}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^7x^7\sqrt{c^2x^2+1}}{64}-\frac{43c^5x^5}{1}\right.}{c^4}$ |
| derivativedivides | $\frac{d^2a\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{3}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^7x^7\sqrt{c^2x^2+1}}{64}-\frac{43c^5x^5\sqrt{c^2x^2+1}}{1152}-\right.}{c^4}$ |
| default | $\frac{d^2a\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{3}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^7x^7\sqrt{c^2x^2+1}}{64}-\frac{43c^5x^5\sqrt{c^2x^2+1}}{1152}-\right.}{c^4}$ |

input
```
int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/8*c^4*x^8+1/3*c^2*x^6+1/4*x^4)+d^2*b/c^4*(1/8*arcsinh(c*x)*c^8*x^
8+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/64*c^7*x^7*(c^2*x^2+
1)^(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*c^3*x^3*(c^2*x^2+1)^(1/
2)+73/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x))
```

## 3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.89

$$\int x^3\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{1152\,ac^8d^2x^8+3072\,ac^6d^2x^6+2304\,ac^4d^2x^4+3\left(384\,bc^8d^2x^8+1024\,bc^6d^2x^6+768\,bc^4d^2x^4-73\,bd^2\right)\log\left(\right.}{9216\,c^4}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/9216*(1152*a*c^8*d^2*x^8 + 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(
384*b*c^8*d^2*x^8 + 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log
(c*x + sqrt(c^2*x^2 + 1)) - (144*b*c^7*d^2*x^7 + 344*b*c^5*d^2*x^5 + 146*b
*c^3*d^2*x^3 - 219*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

## 3.11.6 Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.21

$$\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} \frac{ac^4d^2x^8}{8}+\frac{ac^2d^2x^6}{3}+\frac{ad^2x^4}{4}+\frac{bc^4d^2x^8\operatorname{asinh}(cx)}{8}-\frac{bc^3d^2x^7\sqrt{c^2x^2+1}}{64}+\frac{bc^2d^2x^6\operatorname{asinh}(cx)}{3}-\frac{43bcd^2x^5\sqrt{c^2x^2+1}}{1152}+\frac{bd^2x^4\operatorname{asinh}(\ldots}{4} \\ \frac{ad^2x^4}{4} \end{cases}$$

input
```
integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**8/8 + a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**
4*d**2*x**8*asinh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)/64 + b*c**
2*d**2*x**6*asinh(c*x)/3 - 43*b*c*d**2*x**5*sqrt(c**2*x**2 + 1)/1152 + b*d
**2*x**4*asinh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 + 1)/(4608*c) + 73*b
*d**2*x*sqrt(c**2*x**2 + 1)/(3072*c**3) - 73*b*d**2*asinh(c*x)/(3072*c**4)
, Ne(c, 0)), (a*d**2*x**4/4, True))
```

## 3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.62

$$\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx=\frac{1}{8}ac^4d^2x^8+\frac{1}{3}ac^2d^2x^6$$
$$+\frac{1}{3072}\left(384x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2+1}x}{c^8}\ldots\right.\right.$$
$$+\frac{1}{4}ad^2x^4$$
$$+\frac{1}{144}\left(48x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc\ldots$$
$$+\frac{1}{32}\left(8x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bd^2$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*c^4*d^2*x^8 + 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48
*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^
2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*
c^4*d^2 + 1/4*a*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1
)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15
*arcsinh(c*x)/c^7)*c)*b*c^2*d^2 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x
^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*d^2
```

### 3.11.8 Giac [F(-2)]

Exception generated.

$$\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.11.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \int x^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

# 3.12 $\int x^2(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\, dx$



## 3.12.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\begin{aligned}\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx &= \frac{8bd^2\sqrt{1+c^2x^2}}{105c^3}+\frac{4bd^2(1+c^2x^2)^{3/2}}{315c^3}+\frac{bd^2(1+c^2x^2)^{5/2}}{175c^3}-\frac{bd^2(1+c^2x^2)^{7/2}}{49c^3}\\ &\quad+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))\end{aligned}$$

output
```
4/315*b*d^2*(c^2*x^2+1)^(3/2)/c^3+1/175*b*d^2*(c^2*x^2+1)^(5/2)/c^3-1/49*b
*d^2*(c^2*x^2+1)^(7/2)/c^3+1/3*d^2*x^3*(a+b*arcsinh(c*x))+2/5*c^2*d^2*x^5*
(a+b*arcsinh(c*x))+1/7*c^4*d^2*x^7*(a+b*arcsinh(c*x))+8/105*b*d^2*(c^2*x^2
+1)^(1/2)/c^3
```

## 3.12.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(105ac^3x^3(35+42c^2x^2+15c^4x^4)-b\sqrt{1+c^2x^2}(-818+409c^2x^2+612c^4x^4+225c^6x^6)+105bc^3x^3(35\right.}{11025c^3}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(105*a*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(
-818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(35 + 42*c
^2*x^2 + 15*c^4*x^4)*ArcSinh[c*x]))/(11025*c^3)
```

### 3.12.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6218, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c^2dx^2+d\right)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6218}$$

$$-bc\int \frac{d^2x^3\left(15c^4x^4+42c^2x^2+35\right)}{105\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{105}bcd^2\int \frac{x^3\left(15c^4x^4+42c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 1578}$$

$$-\frac{1}{210}bcd^2\int \frac{x^2\left(15c^4x^4+42c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 1195}$$

$$-\frac{1}{210}bcd^2\int \left(\frac{15\left(c^2x^2+1\right)^{5/2}}{c^2}-\frac{3\left(c^2x^2+1\right)^{3/2}}{c^2}-\frac{4\sqrt{c^2x^2+1}}{c^2}-\frac{8}{c^2\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}c^4d^2x^7(a+b\text{arcsinh}(cx))+\frac{2}{5}c^2d^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arcsinh}(cx))-\frac{1}{210}bcd^2\left(\frac{30(c^2x^2+1)^{7/2}}{7c^4}-\frac{6(c^2x^2+1)^{5/2}}{5c^4}-\frac{8(c^2x^2+1)^{3/2}}{3c^4}-\frac{16\sqrt{c^2x^2+1}}{c^4}\right)$$

input
```
Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/210*(b*c*d^2*((-16*Sqrt[1 + c^2*x^2])/c^4 - (8*(1 + c^2*x^2)^(3/2))/(3*
c^4) - (6*(1 + c^2*x^2)^(5/2))/(5*c^4) + (30*(1 + c^2*x^2)^(7/2))/(7*c^4))
) + (d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (2*c^2*d^2*x^5*(a + b*ArcSinh[c*x])
)/5 + (c^4*d^2*x^7*(a + b*ArcSinh[c*x]))/7
```

### 3.12.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.12.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| parts | $d^2a\left(\frac{1}{7}c^4x^7+\frac{2}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^2b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x}{}\right.}{c^3}$ |
| derivativedivides | $\frac{d^2a\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x^4\sqrt{c^2x^2+1}}{1225}\right.}{c^3}$ |
| default | $\frac{d^2a\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x^4\sqrt{c^2x^2+1}}{1225}\right.}{c^3}$ |

input
```
int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/7*c^4*x^7+2/5*c^2*x^5+1/3*x^3)+d^2*b/c^3*(1/7*arcsinh(c*x)*c^7*x^
7+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/49*c^6*x^6*(c^2*x^2+
1)^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-409/11025*c^2*x^2*(c^2*x^2+1)^(
1/2)+818/11025*(c^2*x^2+1)^(1/2))
```

### 3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int x^2\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{1575\,ac^7d^2x^7+4410\,ac^5d^2x^5+3675\,ac^3d^2x^3+105\left(15\,bc^7d^2x^7+42\,bc^5d^2x^5+35\,bc^3d^2x^3\right)\log\left(cx+\sqrt{c^2x}\right.}{11025\,c^3}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/11025*(1575*a*c^7*d^2*x^7 + 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 10
5*(15*b*c^7*d^2*x^7 + 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(
c^2*x^2 + 1)) - (225*b*c^6*d^2*x^6 + 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2
 - 818*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

### 3.12.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

$$\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \begin{cases} \frac{ac^4d^2x^7}{7}+\frac{2ac^2d^2x^5}{5}+\frac{ad^2x^3}{3}+\frac{bc^4d^2x^7\operatorname{asinh}(cx)}{7}-\frac{bc^3d^2x^6\sqrt{c^2x^2+1}}{49}+\frac{2bc^2d^2x^5\operatorname{asinh}(cx)}{5}-\frac{68bcd^2x^4\sqrt{c^2x^2+1}}{1225}+\frac{bd^2x^3\operatorname{asinh}}{3} \\ \frac{ad^2x^3}{3} \end{cases}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**7/7 + 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c
**4*d**2*x**7*asinh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 + 1)/49 + 2*b
*c**2*d**2*x**5*asinh(c*x)/5 - 68*b*c*d**2*x**4*sqrt(c**2*x**2 + 1)/1225 +
 b*d**2*x**3*asinh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(11025*c)
+ 818*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3), Ne(c, 0)), (a*d**2*x**3/3,
True))
```

### 3.12.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.66

$$\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx = \frac{1}{7}ac^4d^2x^7+\frac{2}{5}ac^2d^2x^5 + \frac{1}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^4d + \frac{2}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d^2 + \frac{1}{3}ad^2x^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^4*d^2*x^7 + 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sq
rt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1
)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^2 + 2/75*(15*x^5*arcsinh(
c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt
(c^2*x^2 + 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x)
- c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2
```

### 3.12.8 Giac [F(-2)]

Exception generated.

$$\int x^2 \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.12.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \int x^2\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

# 3.13 $\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx$



## 3.13.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int x\left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = -\frac{5bd^2x\sqrt{1+c^2x^2}}{96c}-\frac{5bd^2x(1+c^2x^2)^{3/2}}{144c} - \frac{bd^2x(1+c^2x^2)^{5/2}}{36c}-\frac{5bd^2\text{arcsinh}(cx)}{96c^2} + \frac{d^2(1+c^2x^2)^3 (a+b\text{arcsinh}(cx))}{6c^2}$$

output
```
-5/144*b*d^2*x*(c^2*x^2+1)^(3/2)/c-1/36*b*d^2*x*(c^2*x^2+1)^(5/2)/c-5/96*b
*d^2*arcsinh(c*x)/c^2+1/6*d^2*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c^2-5/96*b*
d^2*x*(c^2*x^2+1)^(1/2)/c
```

## 3.13.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int x\left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(cx\left(48acx(3+3c^2x^2+c^4x^4)-b\sqrt{1+c^2x^2}(33+26c^2x^2+8c^4x^4)\right)+3b(11+48c^2x^2+48c^4x^4+16c^6x^6\right.}{288c^2}$$

input
```
Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(c*x*(48*a*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(33 +
26*c^2*x^2 + 8*c^4*x^4)) + 3*b*(11 + 48*c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6)
*ArcSinh[c*x]))/(288*c^2)
```

### 3.13.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6213, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{d^2\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{bd^2\int\left(c^2x^2+1\right)^{5/2}dx}{6c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d^2\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{bd^2\left(\frac{5}{6}\int\left(c^2x^2+1\right)^{3/2}dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)}{6c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d^2\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)}{6c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d^2\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)}{6c}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{d^2\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)}{6c}$$

input
```
Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2) - (b*d^2*((x*(1 + c^2*x
^2)^(5/2))/6 + (5*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2
 + ArcSinh[c*x]/(2*c)))/4))/6))/(6*c)
```

### 3.13.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.13.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d^2 a \left(c^2 x^2+1\right)^3}{6}+d^2 b \left(\frac{\operatorname{arcsinh}(cx) c^6 x^6}{6}+\frac{\operatorname{arcsinh}(cx) c^4 x^4}{2}+\frac{\operatorname{arcsinh}(cx) c^2 x^2}{2}+\frac{11 \operatorname{arcsinh}(cx)}{96}-\frac{cx\left(c^2 x^2+1\right)^{\frac{5}{2}}}{36}-\frac{5cx\left(c^2 x^2+1\right)^{\frac{3}{2}}}{144}\right.}{c^2}$ |
| default | $\frac{\frac{d^2 a \left(c^2 x^2+1\right)^3}{6}+d^2 b \left(\frac{\operatorname{arcsinh}(cx) c^6 x^6}{6}+\frac{\operatorname{arcsinh}(cx) c^4 x^4}{2}+\frac{\operatorname{arcsinh}(cx) c^2 x^2}{2}+\frac{11 \operatorname{arcsinh}(cx)}{96}-\frac{cx\left(c^2 x^2+1\right)^{\frac{5}{2}}}{36}-\frac{5cx\left(c^2 x^2+1\right)^{\frac{3}{2}}}{144}\right.}{c^2}$ |
| parts | $\frac{d^2 a (c^2 x^2+1)^3}{6c^2}+\frac{d^2 b \left(\frac{\operatorname{arcsinh}(cx) c^6 x^6}{6}+\frac{\operatorname{arcsinh}(cx) c^4 x^4}{2}+\frac{\operatorname{arcsinh}(cx) c^2 x^2}{2}+\frac{11 \operatorname{arcsinh}(cx)}{96}-\frac{cx\left(c^2 x^2+1\right)^{\frac{5}{2}}}{36}-\frac{5cx\left(c^2 x^2+1\right)}{144}\right.}{c^2}$ |

input
```
int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/6*d^2*a*(c^2*x^2+1)^3+d^2*b*(1/6*arcsinh(c*x)*c^6*x^6+1/2*arcsinh
(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+11/96*arcsinh(c*x)-1/36*c*x*(c^2*x^
2+1)^(5/2)-5/144*c*x*(c^2*x^2+1)^(3/2)-5/96*c*x*(c^2*x^2+1)^(1/2)))
```

### 3.13.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{48\,ac^6d^2x^6+144\,ac^4d^2x^4+144\,ac^2d^2x^2+3\left(16\,bc^6d^2x^6+48\,bc^4d^2x^4+48\,bc^2d^2x^2+11\,bd^2\right)\log\left(cx+\sqrt{c^2}\right.}{288\,c^2}$$

input
```
integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/288*(48*a*c^6*d^2*x^6 + 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*
c^6*d^2*x^6 + 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 + 11*b*d^2)*log(c*x + sq
rt(c^2*x^2 + 1)) - (8*b*c^5*d^2*x^5 + 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqr
t(c^2*x^2 + 1))/c^2
```

### 3.13.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.58

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\begin{cases}\frac{ac^4d^2x^6}{6}+\frac{ac^2d^2x^4}{2}+\frac{ad^2x^2}{2}+\frac{bc^4d^2x^6\operatorname{asinh}(cx)}{6}-\frac{bc^3d^2x^5\sqrt{c^2x^2+1}}{36}+\frac{bc^2d^2x^4\operatorname{asinh}(cx)}{2}-\frac{13bcd^2x^3\sqrt{c^2x^2+1}}{144}+\frac{bd^2x^2\operatorname{asinh}(}{2} \\ \frac{ad^2x^2}{2}\end{cases}$$

input
```
integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**6/6 + a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**
4*d**2*x**6*asinh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)/36 + b*c**
2*d**2*x**4*asinh(c*x)/2 - 13*b*c*d**2*x**3*sqrt(c**2*x**2 + 1)/144 + b*d*
*2*x**2*asinh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 + 1)/(96*c) + 11*b*d**2*
asinh(c*x)/(96*c**2), Ne(c, 0)), (a*d**2*x**2/2, True))
```

### 3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(104) = 208.

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.95

$$\int x\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{6}\,ac^4d^2x^6 + \frac{1}{2}\,ac^2d^2x^4 + \frac{1}{288}\left(48\,x^6\operatorname{arsinh}(cx) - \left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\,\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\,\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc^4d^2 + \frac{1}{16}\left(8\,x^4\operatorname{arsinh}(cx) - \left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\operatorname{arsinh}(cx)}{c^5}\right)c\right)bc^2d^2 + \frac{1}{2}\,ad^2x^2 + \frac{1}{4}\left(2\,x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd^2$$

input
```
integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^4*d^2*x^6 + 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sq
rt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 +
 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^4*d^2 + 1/16*(8*x^4*arcsinh(c*x) -
 (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)
/c^5)*c)*b*c^2*d^2 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2
*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2
```

### 3.13.8 Giac [F(-2)]

Exception generated.

$$\int x\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.13.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx=\int x\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

output
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

# 3.14 $\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx$



## 3.14.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx = -\frac{8bd^2\sqrt{1+c^2x^2}}{15c}-\frac{4bd^2(1+c^2x^2)^{3/2}}{45c} -\frac{bd^2(1+c^2x^2)^{5/2}}{25c} +d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx)) +\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))$$

output
```
-4/45*b*d^2*(c^2*x^2+1)^(3/2)/c-1/25*b*d^2*(c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*
arcsinh(c*x))+2/3*c^2*d^2*x^3*(a+b*arcsinh(c*x))+1/5*c^4*d^2*x^5*(a+b*arcs
inh(c*x))-8/15*b*d^2*(c^2*x^2+1)^(1/2)/c
```

## 3.14.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(15acx(15+10c^2x^2+3c^4x^4)-b\sqrt{1+c^2x^2}(149+38c^2x^2+9c^4x^4)+15bcx(15+10c^2x^2+3c^4x^4)\,\text{arcsi}\right.}{225c}$$

input
```
Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(149 +
38*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c
*x]))/(225*c)
```

### 3.14.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6199, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^2 (a+b\text{arcsinh}(cx)) \, dx$$

$$\Big\downarrow \text{6199}$$

$$-bc\int \frac{d^2x\left(3c^4x^4+10c^2x^2+15\right)}{15\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bcd^2\int \frac{x\left(3c^4x^4+10c^2x^2+15\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}bcd^2\int \frac{3c^4x^4+10c^2x^2+15}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}bcd^2\int \left(3\left(c^2x^2+1\right)^{3/2}+4\sqrt{c^2x^2+1}+\frac{8}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))-\frac{1}{30}bcd^2\left(\frac{6(c^2x^2+1)^{5/2}}{5c^2}+\frac{8(c^2x^2+1)^{3/2}}{3c^2}+\frac{16\sqrt{c^2x^2+1}}{c^2}\right)$$

input
```
Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/30*(b*c*d^2*((16*Sqrt[1 + c^2*x^2])/c^2 + (8*(1 + c^2*x^2)^(3/2))/(3*c^
2) + (6*(1 + c^2*x^2)^(5/2))/(5*c^2))) + d^2*x*(a + b*ArcSinh[c*x]) + (2*c
^2*d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSinh[c*x]))/5
```

#### 3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

### 3.14.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| parts | $d^2 a\left(\frac{1}{5}c^4x^5+\frac{2}{3}x^3c^2+x\right)+\frac{d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{c^2x^2}}{25}\right.}{c}$ |
| derivativedivides | $\frac{d^2 a\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{38c^2x^2}{}\right.}{c}$ |
| default | $\frac{d^2 a\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2 b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)cx-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{38c^2x^2}{}\right.}{c}$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/5*c^4*x^5+2/3*x^3*c^2+x)+d^2*b/c*(1/5*arcsinh(c*x)*c^5*x^5+2/3*ar
csinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-149/225*(c^2*x^2+1)^(1/2)-1/25*c^4*x^4
*(c^2*x^2+1)^(1/2)-38/225*c^2*x^2*(c^2*x^2+1)^(1/2))
```

### 3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$\int \left(d+c^2dx^2\right)^2 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{45\,ac^5d^2x^5+150\,ac^3d^2x^3+225\,acd^2x+15\,(3\,bc^5d^2x^5+10\,bc^3d^2x^3+15\,bcd^2x)\log\left(cx+\sqrt{c^2x^2+1}\right)-(9}{225\,c}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*d^2*x^5 + 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*
d^2*x^5 + 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)) -
(9*b*c^4*d^2*x^4 + 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 + 1))/c
```

### 3.14.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^4d^2x^5}{5} + \frac{2ac^2d^2x^3}{3} + ad^2x + \frac{bc^4d^2x^5\operatorname{asinh}(cx)}{5} - \frac{bc^3d^2x^4\sqrt{c^2x^2+1}}{25} + \frac{2bc^2d^2x^3\operatorname{asinh}(cx)}{3} - \frac{38bcd^2x^2\sqrt{c^2x^2+1}}{225} + bd^2x\,\text{asi} \\ ad^2x \end{cases}$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**5/5 + 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d
**2*x**5*asinh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)/25 + 2*b*c**2
*d**2*x**3*asinh(c*x)/3 - 38*b*c*d**2*x**2*sqrt(c**2*x**2 + 1)/225 + b*d**
2*x*asinh(c*x) - 149*b*d**2*sqrt(c**2*x**2 + 1)/(225*c), Ne(c, 0)), (a*d**
2*x, True))
```

### 3.14.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.52

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{1}{5}ac^4d^2x^5$$
$$+ \frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^2$$
$$+ \frac{2}{3}ac^2d^2x^3 + \frac{2}{9}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^2$$
$$+ ad^2x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c
^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2 +
 2/3*a*c^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*b*d^2/c
```

### 3.14.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.14.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

# 3.15 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx$



## 3.15.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx = &-\frac{11}{32}bcd^2x\sqrt{1+c^2x^2} \\ &-\frac{1}{16}bcd^2x(1+c^2x^2)^{3/2}-\frac{11}{32}bd^2\text{arcsinh}(cx) \\ &+\frac{1}{2}d^2(1+c^2x^2)(a+b\text{arcsinh}(cx)) \\ &+\frac{1}{4}d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx)) \\ &+\frac{d^2(a+b\text{arcsinh}(cx))^2}{2b} \\ &+d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &-\frac{1}{2}bd^2\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
-1/16*b*c*d^2*x*(c^2*x^2+1)^(3/2)-11/32*b*d^2*arcsinh(c*x)+1/2*d^2*(c^2*x^
2+1)*(a+b*arcsinh(c*x))+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))+1/2*d^2*(
a+b*arcsinh(c*x))^2/b+d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2)
)^2)-1/2*b*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/32*b*c*d^2*x*(c^2
*x^2+1)^(1/2)
```

### 3.15.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx$$
$$= \frac{d^2\left(-16a^2+24ab+32abc^2x^2+8abc^4x^4-13b^2cx\sqrt{1+c^2x^2}-2b^2c^3x^3\sqrt{1+c^2x^2}-16b^2\text{arcsinh}(cx)^2+32\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(d^2*(-16*a^2 + 24*a*b + 32*a*b*c^2*x^2 + 8*a*b*c^4*x^4 - 13*b^2*c*x*Sqrt[
1 + c^2*x^2] - 2*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 16*b^2*ArcSinh[c*x]^2 + 3
2*a*b*Log[1 - E^(2*ArcSinh[c*x])] + b*ArcSinh[c*x]*(-32*a + b*(13 + 32*c^2
*x^2 + 8*c^4*x^4) + 32*b*Log[1 - E^(2*ArcSinh[c*x])]) + 16*b^2*PolyLog[2,
E^(2*ArcSinh[c*x])]))/(32*b)
```

### 3.15.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6216, 27, 211, 211, 222, 6216, 211, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2(a+b\text{arcsinh}(cx))}{x}\,dx$$

$$\Big\downarrow \text{6216}$$

$$d\int \frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bcd^2\int (c^2x^2+1)^{3/2}dx+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bcd^2\int (c^2x^2+1)^{3/2}dx+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{211}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bcd^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{211}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{222}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)$$

$$\Big\downarrow \text{6216}$$

$$d^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int \sqrt{c^2x^2+1}dx+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$d^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)$$

$$\Big\downarrow \text{222}$$

$$d^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)\right)+\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)$$

$$\Big\downarrow \text{6190}$$

$$d^2\left(\frac{\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))-\right.$$
$$\frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)$$

---

3.15. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx$

$$\Big\downarrow \text{25}$$

$$d^2\left(-\frac{\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(-\frac{\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{26}$$

$$d^2\left(\frac{i\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{4201}$$

$$d^2\left(\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{2620}$$

$$d^2\left(\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsin}}{b}}\right.\right.\right.}{b}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{2715}$$

---

3.15. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx$

$$d^2\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+b\text{arcs}\right.\right.}{b}\right.$$
$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)$$

$\downarrow$ 2838

$$d^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a\right.}{b}\right.$$
$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)$$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 - (b*c*d^2*((x*(1 + c^2*x^2)^
(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/4 + d^2*
(((1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/2 +
ArcSinh[c*x]/(2*c)))/2 + (I*((-1/2*I)*(a + b*ArcSinh[c*x])^2 + (2*I)*(-1/2
*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x
]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/b)
```

### 3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

## 3.15. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx$

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

### 3.15.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| parts | $d^2a\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(x\right)\right)-\frac{d^2b\,c^3x^3\sqrt{c^2x^2+1}}{16}-\frac{13bc\,d^2x\sqrt{c^2x^2+1}}{32}+\frac{d^2b\,\operatorname{arcsinh}(cx)c^4x^4}{4}+d^2b\,\text{arc}$ |
| derivativedivides | $d^2a\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(cx\right)\right)+\frac{13b\,d^2\operatorname{arcsinh}(cx)}{32}+d^2b\,\operatorname{arcsinh}\left(cx\right)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)$ |
| default | $d^2a\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(cx\right)\right)+\frac{13b\,d^2\operatorname{arcsinh}(cx)}{32}+d^2b\,\operatorname{arcsinh}\left(cx\right)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/4*c^4*x^4+c^2*x^2+ln(x))-1/16*d^2*b*c^3*x^3*(c^2*x^2+1)^(1/2)-13/
32*b*c*d^2*x*(c^2*x^2+1)^(1/2)+1/4*d^2*b*arcsinh(c*x)*c^4*x^4+d^2*b*arcsin
h(c*x)*c^2*x^2+d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^2*b*polylog(2,c*x
+(c^2*x^2+1)^(1/2))+13/32*b*d^2*arcsinh(c*x)+d^2*b*arcsinh(c*x)*ln(1+c*x+(
c^2*x^2+1)^(1/2))+d^2*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*d^2*b
*arcsinh(c*x)^2
```

### 3.15.5 Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^2\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x, x)
```

### 3.15.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx = d^2\left(\int \frac{a}{x}\,dx + \int 2ac^2x\,dx + \int ac^4x^3\,dx + \int \frac{b\,\text{asinh}\,(cx)}{x}\,dx + \int 2bc^2x\,\text{asinh}\,(cx)\,dx + \int bc^4x^3\,\text{asinh}\,(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x,x)
```

output
```
d**2*(Integral(a/x, x) + Integral(2*a*c**2*x, x) + Integral(a*c**4*x**3, x
) + Integral(b*asinh(c*x)/x, x) + Integral(2*b*c**2*x*asinh(c*x), x) + Int
egral(b*c**4*x**3*asinh(c*x), x))
```

### 3.15.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*c^4*d^2*x^4 + a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3
*log(c*x + sqrt(c^2*x^2 + 1)) + 2*b*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1))
 + b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.15.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x, x)
```

# 3.16 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.16.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{5}{3}bcd^2\sqrt{1+c^2x^2}-\frac{1}{9}bcd^2(1+c^2x^2)^{3/2} - \frac{d^2(a+b\text{arcsinh}(cx))}{x}+2c^2d^2x(a+b\text{arcsinh}(cx)) + \frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx)) - bcd^2\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

output
```
-1/9*b*c*d^2*(c^2*x^2+1)^(3/2)-d^2*(a+b*arcsinh(c*x))/x+2*c^2*d^2*x*(a+b*a
rcsinh(c*x))+1/3*c^4*d^2*x^3*(a+b*arcsinh(c*x))-b*c*d^2*arctanh((c^2*x^2+1
)^(1/2))-5/3*b*c*d^2*(c^2*x^2+1)^(1/2)
```

## 3.16.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{d^2\left(-9a+18ac^2x^2+3ac^4x^4-16bcx\sqrt{1+c^2x^2}-bc^3x^3\sqrt{1+c^2x^2}+3b(-3+6c^2x^2+c^4x^4)\,\text{arcsinh}(cx)+\right.}{9x}$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
(d^2*(-9*a + 18*a*c^2*x^2 + 3*a*c^4*x^4 - 16*b*c*x*Sqrt[1 + c^2*x^2] - b*c
^3*x^3*Sqrt[1 + c^2*x^2] + 3*b*(-3 + 6*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9
*b*c*x*Log[x] - 9*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(9*x)
```

### 3.16.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6218, 27, 1578, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^2(a+b\text{arcsinh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{ 6218}$$

$$-bc\int -\frac{d^2\left(-c^4x^4-6c^2x^2+3\right)}{3x\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx))-\frac{d^2(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}bcd^2\int \frac{-c^4x^4-6c^2x^2+3}{x\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx))-\frac{d^2(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{6}bcd^2\int \frac{-c^4x^4-6c^2x^2+3}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx))-\frac{d^2(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 1192}$$

$$\frac{bd^2\int -\frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{c^2x^2+1}}{3c^3}+\frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx))-\frac{d^2(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{bd^2\int \frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{c^2x^2+1}}{3c^3}+\frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx))-\frac{d^2(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{bd^2\int \left(x^4c^4+\frac{3c^4}{1-x^4}+5c^4\right)d\sqrt{c^2x^2+1}}{3c^3}+\frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx))-\frac{d^2(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^4d^2x^3(a+b\text{arcsinh}(cx))+2c^2d^2x(a+b\text{arcsinh}(cx))-\frac{d^2(a+b\text{arcsinh}(cx))}{x}+\frac{bd^2\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{1}{3}c^4x^6-5c^4\sqrt{c^2x^2+1}\right)}{3c^3}$$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcSinh[c*x]))/x) + 2*c^2*d^2*x*(a + b*ArcSinh[c*x]) + (c^4*
d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (b*d^2*(-1/3*(c^4*x^6) - 5*c^4*Sqrt[1 +
c^2*x^2] - 3*c^4*ArcTanh[Sqrt[1 + c^2*x^2]]))/(3*c^3)
```

### 3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.16.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| parts | $d^2a\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)+d^2bc\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+2\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{c^2x^2\sqrt{c^2x^2+}}{9}\right.$ |
| derivativedivides | $c\left(d^2a\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+2\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{c^2x^2\sqrt{c^2x^2}}{9}\right.\right.$ |
| default | $c\left(d^2a\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+2\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{c^2x^2\sqrt{c^2x^2}}{9}\right.\right.$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/3*c^4*x^3+2*c^2*x-1/x)+d^2*b*c*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsin
h(c*x)*c*x-arcsinh(c*x)/c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)
^(1/2)-arctanh(1/(c^2*x^2+1)^(1/2)))
```

### 3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(108) = 216$.

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))}{x^2}\,dx$$
$$= \frac{3\,ac^4d^2x^4 + 18\,ac^2d^2x^2 - 9\,bcd^2x\log\left(-cx + \sqrt{c^2x^2+1} + 1\right) + 9\,bcd^2x\log\left(-cx + \sqrt{c^2x^2+1} - 1\right) - 3\,(bc}{}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/9*(3*a*c^4*d^2*x^4 + 18*a*c^2*d^2*x^2 - 9*b*c*d^2*x*log(-c*x + sqrt(c^2*
x^2 + 1) + 1) + 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 3*(b*c^4 +
 6*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1)) - 9*a*d^2 + 3*(b*c^4*d
^2*x^4 + 6*b*c^2*d^2*x^2 - (b*c^4 + 6*b*c^2 - 3*b)*d^2*x - 3*b*d^2)*log(c*
x + sqrt(c^2*x^2 + 1)) - (b*c^3*d^2*x^3 + 16*b*c*d^2*x)*sqrt(c^2*x^2 + 1))
/x
```

### 3.16.6 Sympy [F]

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))}{x^2}\,dx = d^2\left(\int 2ac^2\,dx + \int \frac{a}{x^2}\,dx + \int ac^4x^2\,dx + \int 2bc^2\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx + \int bc^4x^2\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**2,x)
```

output
```
d**2*(Integral(2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2,
x) + Integral(2*b*c**2*asinh(c*x), x) + Integral(b*asinh(c*x)/x**2, x) + I
ntegral(b*c**4*x**2*asinh(c*x), x))
```

### 3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.19

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}{x^2}\,dx$$
$$= \frac{1}{3}\,ac^4d^2x^3 + \frac{1}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^2 + 2\,ac^2d^2x + 2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bcd^2 - \left(c\,\text{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\text{arsinh}\,(cx)}{x}\right)bd^2 - \frac{ad^2}{x}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^4*d^2 + 2*a*c^2*d^2*x + 2*(c*x*arcsinh(c*
x) - sqrt(c^2*x^2 + 1))*b*c*d^2 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/
x)*b*d^2 - a*d^2/x
```

### 3.16.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2, x)
```

# 3.17 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx$



## 3.17.1 Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx = \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2(1+c^2x^2)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\text{arcsinh}(cx) + c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx)) - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2x^2} + \frac{c^2d^2(a+b\text{arcsinh}(cx))^2}{b} + 2c^2d^2(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) - bc^2d^2\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)$$

output
```
-1/2*b*c*d^2*(c^2*x^2+1)^(3/2)/x+1/4*b*c^2*d^2*arcsinh(c*x)+c^2*d^2*(c^2*x
^2+1)*(a+b*arcsinh(c*x))-1/2*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/x^2+c^2*
d^2*(a+b*arcsinh(c*x))^2/b+2*c^2*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x
^2+1)^(1/2))^2)-b*c^2*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/4*b*c^3
*d^2*x*(c^2*x^2+1)^(1/2)
```

### 3.17.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \frac{(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = \frac{1}{4}d^2\left(2ac^4x^2-\frac{2bc\sqrt{1+c^2x^2}}{x}-bc^3x\sqrt{1+c^2x^2}+bc^2\operatorname{arcsinh}(cx)+2bc^4x^2\operatorname{arcsinh}(cx)-\frac{2(a+b\operatorname{arcsinh}(cx))}{x^2}-\frac{4c^2(a+b\operatorname{arcsinh}(cx))^2}{b}+4c^2\left(2(a+b\operatorname{arcsinh}(cx))\log\left(1-e^{2\operatorname{arcsinh}(cx)}\right)+b\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)\right)\right)$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(d^2*(2*a*c^4*x^2 - (2*b*c*Sqrt[1 + c^2*x^2])/x - b*c^3*x*Sqrt[1 + c^2*x^2
] + b*c^2*ArcSinh[c*x] + 2*b*c^4*x^2*ArcSinh[c*x] - (2*(a + b*ArcSinh[c*x]
))/x^2 - (4*c^2*(a + b*ArcSinh[c*x])^2)/b + 4*c^2*(2*(a + b*ArcSinh[c*x])*
Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])])))/4
```

### 3.17.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.34, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6217, 27, 247, 211, 222, 6216, 211, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{6217}$$

$$2c^2d\int \frac{d(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^2\int \frac{(c^2x^2+1)^{3/2}}{x^2}dx-\frac{d^2(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \ 27$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^2\int \frac{(c^2x^2+1)^{3/2}}{x^2}dx-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \ 247$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^2\left(3c^2\int \sqrt{c^2x^2+1}dx-\frac{(c^2x^2+1)^{3/2}}{x}\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \ 211$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^2\left(3c^2\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \ 222$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 6216$$

$$2c^2d^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int \sqrt{c^2x^2+1}dx+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 211$$

$$2c^2d^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 222$$

---

3.17. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx$

$$2c^2d^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)\right)-\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{\left(c^2x^2+1\right)^{3/2}}{x}\right)$$

$\downarrow$ 6190

$$2c^2d^2\left(\frac{\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{\left(c^2x^2+1\right)^{3/2}}{x}\right)$$

$\downarrow$ 25

$$2c^2d^2\left(-\frac{\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{\left(c^2x^2+1\right)^{3/2}}{x}\right)$$

$\downarrow$ 3042

$$2c^2d^2\left(-\frac{\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsin}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{\left(c^2x^2+1\right)^{3/2}}{x}\right)$$

$\downarrow$ 26

$$2c^2d^2\left(\frac{i\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsin}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{\left(c^2x^2+1\right)^{3/2}}{x}\right)$$

$\downarrow$ 4201

---

3.17. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx$

$$2c^2d^2\left(\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{2}(c^2x^2+1)\right.$$

$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)$$

$\downarrow$ 2620

$$2c^2d^2\left(\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+bar}{}}\right.\right.\right.}{b}\right.$$

$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)$$

$\downarrow$ 2715

$$2c^2d^2\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+ba\right.\right.}{b}\right.$$

$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)$$

$\downarrow$ 2838

$$2c^2d^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}\right.}{b}\right.$$

$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^2\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{(c^2x^2+1)^{3/2}}{x}\right)$$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

## 3.17. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx$

output
```
-1/2*(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/x^2 + (b*c*d^2*(-((1 + c^2
*x^2)^(3/2)/x) + 3*c^2*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c))))/2
+ 2*c^2*d^2*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^
2*x^2])/2 + ArcSinh[c*x]/(2*c)))/2 + (I*((-1/2*I)*(a + b*ArcSinh[c*x])^2 +
 (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b
*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/b)
```

### 3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6217
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1
 + c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

## 3.17. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^3}\,dx$

### 3.17.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.28

| method | result |
|---|---|
| parts | $d^2a\left(\frac{c^4x^2}{2}-\frac{1}{2x^2}+2c^2\ln\left(x\right)\right)+d^2b\,c^2\left(-\operatorname{arcsinh}\left(cx\right)^2+\frac{(-1+2\operatorname{arcsinh}(cx))\left(2c^2x^2+1+2cx\sqrt{c^2x\right.}{16}$ |
| derivativedivides | $c^2\left(d^2a\left(\frac{c^2x^2}{2}+2\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+d^2b\left(-\operatorname{arcsinh}\left(cx\right)^2+\frac{(-1+2\operatorname{arcsinh}(cx))\left(2c^2x^2+1+2cx\sqrt{\right.}{16}\right.\right.$ |
| default | $c^2\left(d^2a\left(\frac{c^2x^2}{2}+2\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+d^2b\left(-\operatorname{arcsinh}\left(cx\right)^2+\frac{(-1+2\operatorname{arcsinh}(cx))\left(2c^2x^2+1+2cx\sqrt{\right.}{16}\right.\right.$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/2*c^4*x^2-1/2/x^2+2*c^2*ln(x))+d^2*b*c^2*(-arcsinh(c*x)^2+1/16*(-
1+2*arcsinh(c*x))*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))+1/16*(-2*c*x*(c^2*
x^2+1)^(1/2)+2*c^2*x^2+1)*(1+2*arcsinh(c*x))-1/2*(c*x*(c^2*x^2+1)^(1/2)-c^
2*x^2+arcsinh(c*x))/c^2/x^2+2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2)
)+2*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

### 3.17.5 Fricas [F]

$$\int\frac{\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)}{x^3}\,dx=\int\frac{\left(c^2dx^2+d\right)^2\left(b\operatorname{arsinh}\left(cx\right)+a\right)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x^3, x)
```

### 3.17.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2\,(a+b\mathrm{arcsinh}(cx))}{x^3}\,dx = d^2\left(\int \frac{a}{x^3}\,dx + \int \frac{2ac^2}{x}\,dx + \int ac^4x\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x^3}\,dx + \int \frac{2bc^2\,\mathrm{asinh}\,(cx)}{x}\,dx + \int bc^4x\,\mathrm{asinh}\,(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**3,x)
```

output
```
d**2*(Integral(a/x**3, x) + Integral(2*a*c**2/x, x) + Integral(a*c**4*x, x
) + Integral(b*asinh(c*x)/x**3, x) + Integral(2*b*c**2*asinh(c*x)/x, x) +
Integral(b*c**4*x*asinh(c*x), x))
```

### 3.17.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^2\,(a+b\mathrm{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)^2(b\,\mathrm{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*c^4*d^2*x^2 + 2*a*c^2*d^2*log(x) - 1/2*b*d^2*(sqrt(c^2*x^2 + 1)*c/x
+ arcsinh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt
(c^2*x^2 + 1)) + 2*b*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.17.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\mathrm{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3, x)
```

# 3.18 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.18.1 Optimal result

Integrand size = 24, antiderivative size = 126

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^4}\,dx &= -bc^3d^2\sqrt{1+c^2x^2}-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}\\ &\quad -\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}\\ &\quad +c^4d^2x(a+b\text{arcsinh}(cx))\\ &\quad -\frac{11}{6}bc^3d^2\text{arctanh}\left(\sqrt{1+c^2x^2}\right)\end{aligned}$$

output
```
-1/3*d^2*(a+b*arcsinh(c*x))/x^3-2*c^2*d^2*(a+b*arcsinh(c*x))/x+c^4*d^2*x*(
a+b*arcsinh(c*x))-11/6*b*c^3*d^2*arctanh((c^2*x^2+1)^(1/2))-b*c^3*d^2*(c^2
*x^2+1)^(1/2)-1/6*b*c*d^2*(c^2*x^2+1)^(1/2)/x^2
```

## 3.18.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{d^2\left(-2a-12ac^2x^2+6ac^4x^4-bcx\sqrt{1+c^2x^2}-6bc^3x^3\sqrt{1+c^2x^2}+2b(-1-6c^2x^2+3c^4x^4)\,\text{arcsinh}(cx)+\right.}{6x^3}$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
(d^2*(-2*a - 12*a*c^2*x^2 + 6*a*c^4*x^4 - b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^
3*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-1 - 6*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] +
11*b*c^3*x^3*Log[x] - 11*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(6*x^3)
```

### 3.18.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6218, 27, 1578, 1192, 25, 1471, 25, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^2(a+b\text{arcsinh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int -\frac{d^2\left(-3c^4x^4+6c^2x^2+1\right)}{3x^3\sqrt{c^2x^2+1}}dx+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd^2\int \frac{-3c^4x^4+6c^2x^2+1}{x^3\sqrt{c^2x^2+1}}dx+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{6}bcd^2\int \frac{-3c^4x^4+6c^2x^2+1}{x^4\sqrt{c^2x^2+1}}dx^2+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{1192}$$

$$\frac{bd^2\int -\frac{3c^4x^8-12c^4x^4+8c^4}{\left(1-x^4\right)^2}d\sqrt{c^2x^2+1}}{3c}+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{bd^2\int \frac{3c^4x^8-12c^4x^4+8c^4}{(1-x^4)^2}d\sqrt{c^2x^2+1}}{3c}+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{1471}$$

$$\frac{bd^2\left(\frac{1}{2}\int -\frac{c^4(17-6x^4)}{1-x^4}d\sqrt{c^2x^2+1}+\frac{c^4\sqrt{c^2x^2+1}}{2(1-x^4)}\right)}{3c}+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{bd^2\left(\frac{c^4\sqrt{c^2x^2+1}}{2(1-x^4)}-\frac{1}{2}\int \frac{c^4(17-6x^4)}{1-x^4}d\sqrt{c^2x^2+1}\right)}{3c}+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^2\left(\frac{c^4\sqrt{c^2x^2+1}}{2(1-x^4)}-\frac{1}{2}c^4\int \frac{17-6x^4}{1-x^4}d\sqrt{c^2x^2+1}\right)}{3c}+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{299}$$

$$\frac{bd^2\left(\frac{c^4\sqrt{c^2x^2+1}}{2(1-x^4)}-\frac{1}{2}c^4\left(11\int \frac{1}{1-x^4}d\sqrt{c^2x^2+1}+6\sqrt{c^2x^2+1}\right)\right)}{3c}+c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$c^4d^2x(a+b\text{arcsinh}(cx))-\frac{2c^2d^2(a+b\text{arcsinh}(cx))}{x}-\frac{d^2(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bd^2\left(\frac{c^4\sqrt{c^2x^2+1}}{2(1-x^4)}-\frac{1}{2}c^4\left(11\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+6\sqrt{c^2x^2+1}\right)\right)}{3c}$$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcSinh[c*x]))/x^3 - (2*c^2*d^2*(a + b*ArcSinh[c*x]))/x +
 c^4*d^2*x*(a + b*ArcSinh[c*x]) + (b*d^2*((c^4*Sqrt[1 + c^2*x^2])/(2*(1 -
x^4)) - (c^4*(6*Sqrt[1 + c^2*x^2] + 11*ArcTanh[Sqrt[1 + c^2*x^2]]))/2))/(3
*c)
```

3.18. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^4}\,dx$

### 3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

## 3.18. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}{x^4}\,dx$

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.18.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| parts | $d^2a\left(c^4x-\frac{2c^2}{x}-\frac{1}{3x^3}\right)+d^2b\,c^3\left(\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{2\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\right.$ |
| derivativedivides | $c^3\left(d^2a\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b\left(\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{2\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}\right.\right.$ |
| default | $c^3\left(d^2a\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b\left(\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\frac{2\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}\right.\right.$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(c^4*x-2*c^2/x-1/3/x^3)+d^2*b*c^3*(arcsinh(c*x)*c*x-1/3*arcsinh(c*x)
/c^3/x^3-2*arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2
)-11/6*arctanh(1/(c^2*x^2+1)^(1/2)))
```

## 3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(114) = 228.

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.93

$$\int\frac{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx$$
$$=\frac{6\,ac^4d^2x^4-11\,bc^3d^2x^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+11\,bc^3d^2x^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-12\,ac^2d^2x^2}{}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a*c^4*d^2*x^4 - 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1)
+ 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 12*a*c^2*d^2*x^2 -
2*(3*b*c^4 - 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1)) - 2*a*d^2
+ 2*(3*b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (3*b*c^4 - 6*b*c^2 - b)*d^2*x^3 -
 b*d^2)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*b*c^3*d^2*x^3 + b*c*d^2*x)*sqrt(
c^2*x^2 + 1))/x^3
```

### 3.18.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = d^2\left(\int ac^4\,dx + \int \frac{a}{x^4}\,dx + \int \frac{2ac^2}{x^2}\,dx + \int bc^4\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{2bc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**4,x)
```

output
```
d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(2*a*c**2/x**2,
x) + Integral(b*c**4*asinh(c*x), x) + Integral(b*asinh(c*x)/x**4, x) + Int
egral(2*b*c**2*asinh(c*x)/x**2, x))
```

### 3.18.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = ac^4d^2x + \left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bc^3d^2 - 2\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d^2 + \frac{1}{6}\left(\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2+1}}{x^2}\right)c - \frac{2\operatorname{arsinh}(cx)}{x^3}\right)bd^2 - \frac{2\,ac^2d^2}{x} - \frac{ad^2}{3\,x^3}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

output
```
a*c^4*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c^3*d^2 - 2*(c*arcs
inh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d^2 + 1/6*((c^2*arcsinh(1/(c*abs
(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d^2 - 2*a*c^2*d^2
/x - 1/3*a*d^2/x^3
```

### 3.18.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^2}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4, x)
```

# 3.19 $\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$



## 3.19.1 Optimal result

Integrand size = 24, antiderivative size = 226

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx=-\frac{16bd^3\sqrt{1+c^2x^2}}{1155c^5}-\frac{8bd^3(1+c^2x^2)^{3/2}}{3465c^5}-\frac{2bd^3(1+c^2x^2)^{5/2}}{1925c^5}-\frac{bd^3(1+c^2x^2)^{7/2}}{1617c^5}+\frac{4bd^3(1+c^2x^2)^{9/2}}{297c^5}-\frac{bd^3(1+c^2x^2)^{11/2}}{121c^5}+\frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx))+\frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx))+\frac{1}{11}c^6d^3x^{11}(a+b\text{arcsin}$$

output
```
-8/3465*b*d^3*(c^2*x^2+1)^(3/2)/c^5-2/1925*b*d^3*(c^2*x^2+1)^(5/2)/c^5-1/1
617*b*d^3*(c^2*x^2+1)^(7/2)/c^5+4/297*b*d^3*(c^2*x^2+1)^(9/2)/c^5-1/121*b*
d^3*(c^2*x^2+1)^(11/2)/c^5+1/5*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^2*d^3*x^7*
(a+b*arcsinh(c*x))+1/3*c^4*d^3*x^9*(a+b*arcsinh(c*x))+1/11*c^6*d^3*x^11*(a
+b*arcsinh(c*x))-16/1155*b*d^3*(c^2*x^2+1)^(1/2)/c^5
```

## 3.19.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.63

$$\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx=\frac{d^3\left(3465ac^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)-b\sqrt{1+c^2x^2}(50488-25244c^2x^2+18933c^4x^4+117\right.}{400207}$$

input
```
Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6) - b*S
qrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 117625*c^6*x^6 +
 111475*c^8*x^8 + 33075*c^10*x^10) + 3465*b*c^5*x^5*(231 + 495*c^2*x^2 + 3
85*c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*x]))/(4002075*c^5)
```

### 3.19.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6218, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(c^2 d x^2+d\right)^3 (a+b\text{arcsinh}(cx)) \, dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int \frac{d^3 x^5 \left(105 c^6 x^6+385 c^4 x^4+495 c^2 x^2+231\right)}{1155 \sqrt{c^2 x^2+1}}dx+\frac{1}{11} c^6 d^3 x^{11} (a+b\text{arcsinh}(cx))+\frac{1}{3} c^4 d^3 x^9 (a+b\text{arcsinh}(cx))+\frac{3}{7} c^2 d^3 x^7 (a+b\text{arcsinh}(cx))+\frac{1}{5} d^3 x^5 (a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{b c d^3 \int \frac{x^5 \left(105 c^6 x^6+385 c^4 x^4+495 c^2 x^2+231\right)}{\sqrt{c^2 x^2+1}}dx}{1155}+\frac{1}{11} c^6 d^3 x^{11} (a+b\text{arcsinh}(cx))+\frac{1}{3} c^4 d^3 x^9 (a+b\text{arcsinh}(cx))+\frac{3}{7} c^2 d^3 x^7 (a+b\text{arcsinh}(cx))+\frac{1}{5} d^3 x^5 (a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{b c d^3 \int \frac{x^4 \left(105 c^6 x^6+385 c^4 x^4+495 c^2 x^2+231\right)}{\sqrt{c^2 x^2+1}}dx^2}{2310}+\frac{1}{11} c^6 d^3 x^{11} (a+b\text{arcsinh}(cx))+\frac{1}{3} c^4 d^3 x^9 (a+b\text{arcsinh}(cx))+\frac{3}{7} c^2 d^3 x^7 (a+b\text{arcsinh}(cx))+\frac{1}{5} d^3 x^5 (a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2123}$$

$$-\frac{bcd^3\int\left(\frac{105(c^2x^2+1)^{9/2}}{c^4}-\frac{140(c^2x^2+1)^{7/2}}{c^4}+\frac{5(c^2x^2+1)^{5/2}}{c^4}+\frac{6(c^2x^2+1)^{3/2}}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^4}+\frac{16}{c^4\sqrt{c^2x^2+1}}\right)dx^2}{2310}+$$
$$\frac{1}{11}c^6d^3x^{11}(a+b\text{arcsinh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx))+\frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{11}c^6d^3x^{11}(a+b\text{arcsinh}(cx))+\frac{1}{3}c^4d^3x^9(a+b\text{arcsinh}(cx))+\frac{3}{7}c^2d^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}d^3x^5(a+b\text{arcsinh}(cx))-$$
$$\frac{bcd^3\left(\frac{210(c^2x^2+1)^{11/2}}{11c^6}-\frac{280(c^2x^2+1)^{9/2}}{9c^6}+\frac{10(c^2x^2+1)^{7/2}}{7c^6}+\frac{12(c^2x^2+1)^{5/2}}{5c^6}+\frac{16(c^2x^2+1)^{3/2}}{3c^6}+\frac{32\sqrt{c^2x^2+1}}{c^6}\right)}{2310}$$

input
```
Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/2310*(b*c*d^3*((32*Sqrt[1 + c^2*x^2])/c^6 + (16*(1 + c^2*x^2)^(3/2))/(3
*c^6) + (12*(1 + c^2*x^2)^(5/2))/(5*c^6) + (10*(1 + c^2*x^2)^(7/2))/(7*c^6
) - (280*(1 + c^2*x^2)^(9/2))/(9*c^6) + (210*(1 + c^2*x^2)^(11/2))/(11*c^6
))) + (d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^2*d^3*x^7*(a + b*ArcSinh[c*x
]))/7 + (c^4*d^3*x^9*(a + b*ArcSinh[c*x]))/3 + (c^6*d^3*x^11*(a + b*ArcSin
h[c*x]))/11
```

### 3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.19.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| parts | $d^3a\left(\frac{1}{11}c^6x^{11}+\frac{1}{3}c^4x^9+\frac{3}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\operatorname{arcsinh}(cx)c^9x^9}{3}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\operatorname{arcsi}\right.}{}$ |
| derivativedivides | $\frac{d^3a\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\operatorname{arcsinh}(cx)c^9x^9}{3}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\right.}{c^5}$ |
| default | $\frac{d^3a\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\operatorname{arcsinh}(cx)c^9x^9}{3}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\right.}{c^5}$ |

input
```
int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/11*c^6*x^11+1/3*c^4*x^9+3/7*c^2*x^7+1/5*x^5)+d^3*b/c^5*(1/11*arcs
inh(c*x)*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+1/5*a
rcsinh(c*x)*c^5*x^5-91/3267*c^8*x^8*(c^2*x^2+1)^(1/2)-4705/160083*c^6*x^6*
(c^2*x^2+1)^(1/2)-6311/1334025*c^4*x^4*(c^2*x^2+1)^(1/2)+25244/4002075*c^2
*x^2*(c^2*x^2+1)^(1/2)-50488/4002075*(c^2*x^2+1)^(1/2)-1/121*c^10*x^10*(c^
2*x^2+1)^(1/2))
```

### 3.19.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.89

$$\int x^4 (d + c^2 dx^2)^3 (a + b\operatorname{arcsinh}(cx))\, dx$$
$$= \frac{363825\, ac^{11}d^3x^{11} + 1334025\, ac^9d^3x^9 + 1715175\, ac^7d^3x^7 + 800415\, ac^5d^3x^5 + 3465\, (105\, bc^{11}d^3x^{11} + 385\, bc^9 \ldots}{\ldots}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/4002075*(363825*a*c^11*d^3*x^11 + 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*
d^3*x^7 + 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 + 385*b*c^9*d^3
*x^9 + 495*b*c^7*d^3*x^7 + 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1))
 - (33075*b*c^10*d^3*x^10 + 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 +
18933*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 + 50488*b*d^3)*sqrt(c^2*x^2 + 1)
)/c^5
```

### 3.19.6 Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\int x^4 (d + c^2 dx^2)^3 (a + b\operatorname{arcsinh}(cx))\, dx$$
$$= \begin{cases} \frac{ac^6d^3x^{11}}{11} + \frac{ac^4d^3x^9}{3} + \frac{3ac^2d^3x^7}{7} + \frac{ad^3x^5}{5} + \frac{bc^6d^3x^{11}\operatorname{asinh}(cx)}{11} - \frac{bc^5d^3x^{10}\sqrt{c^2x^2+1}}{121} + \frac{bc^4d^3x^9\operatorname{asinh}(cx)}{3} - \frac{91bc^3d^3x^8\sqrt{c^2x^2+1}}{3267} \ldots \\ \frac{ad^3x^5}{5} \end{cases}$$

input
```
integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 + 3*a*c**2*d**3*x**7/
7 + a*d**3*x**5/5 + b*c**6*d**3*x**11*asinh(c*x)/11 - b*c**5*d**3*x**10*sq
rt(c**2*x**2 + 1)/121 + b*c**4*d**3*x**9*asinh(c*x)/3 - 91*b*c**3*d**3*x**
8*sqrt(c**2*x**2 + 1)/3267 + 3*b*c**2*d**3*x**7*asinh(c*x)/7 - 4705*b*c*d*
*3*x**6*sqrt(c**2*x**2 + 1)/160083 + b*d**3*x**5*asinh(c*x)/5 - 6311*b*d**
3*x**4*sqrt(c**2*x**2 + 1)/(1334025*c) + 25244*b*d**3*x**2*sqrt(c**2*x**2
+ 1)/(4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 + 1)/(4002075*c**5), Ne(
c, 0)), (a*d**3*x**5/5, True))
```

### 3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(194) = 388$.

Time = 0.20 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.06

$$\int x^4(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{11}ac^6d^3x^{11}+\frac{1}{3}ac^4d^3x^9+\frac{3}{7}ac^2d^3x^7$$
$$+\frac{1}{7623}\left(693\,x^{11}\operatorname{arsinh}(cx)-\left(\frac{63\sqrt{c^2x^2+1}x^{10}}{c^2}-\frac{70\sqrt{c^2x^2+1}x^8}{c^4}+\frac{80\sqrt{c^2x^2+1}x^6}{c^6}-\frac{96\sqrt{c^2x^2+1}x}{c^8}\right.\right.$$
$$+\frac{1}{945}\left(315\,x^9\operatorname{arsinh}(cx)-\left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\sqrt{c^2x^2+1}x^2}{c^8}+\right.\right.$$
$$+\frac{1}{5}ad^3x^5$$
$$+\frac{3}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d$$
$$+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^3$$

input
```
integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 + 3/7*a*c^2*d^3*x^7 + 1/7623*(693*
x^11*arcsinh(c*x) - (63*sqrt(c^2*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*
x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 12
8*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqrt(c^2*x^2 + 1)/c^12)*c)*b*c^6*d^3 +
1/945*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*
x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2
/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 + 3/245*(3
5*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^
4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d
^3 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2
*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^3
```

### 3.19.8 Giac [F(-2)]

Exception generated.

$$\int x^4 (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx)) dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.19.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx)) dx = \int x^4 (a + b\,\text{asinh}(c\,x)) \left(d\,c^2\,x^2 + d\right)^3 dx$$

input
```
int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

output
```
int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

# 3.20 $\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$



## 3.20.1 Optimal result

Integrand size = 24, antiderivative size = 199

$$\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx=\frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3}+\frac{49bd^3x(1+c^2x^2)^{3/2}}{7680c^3}+\frac{49bd^3x(1+c^2x^2)^{5/2}}{9600c^3}+\frac{7bd^3x(1+c^2x^2)^{7/2}}{1600c^3}-\frac{bd^3x(1+c^2x^2)^{9/2}}{100c^3}+\frac{49bd^3\text{arcsinh}(cx)}{5120c^4}-\frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))}{8c^4}+\frac{d^3(1+c^2x^2)^5(a+b\text{arcsinh}(cx))}{10c^4}$$

output
```
49/7680*b*d^3*x*(c^2*x^2+1)^(3/2)/c^3+49/9600*b*d^3*x*(c^2*x^2+1)^(5/2)/c^
3+7/1600*b*d^3*x*(c^2*x^2+1)^(7/2)/c^3-1/100*b*d^3*x*(c^2*x^2+1)^(9/2)/c^3
+49/5120*b*d^3*arcsinh(c*x)/c^4-1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))/c
^4+1/10*d^3*(c^2*x^2+1)^5*(a+b*arcsinh(c*x))/c^4+49/5120*b*d^3*x*(c^2*x^2+
1)^(1/2)/c^3
```

### 3.20.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{d^3\left(1920ac^4x^4(10+20c^2x^2+15c^4x^4+4c^6x^6)-bcx\sqrt{1+c^2x^2}(-1185+790c^2x^2+3208c^4x^4+2736c^6x^6\right.}{76800c^4}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(1920*a*c^4*x^4*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - b*c*x*Sq
rt[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c
^8*x^8) + 15*b*(-79 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^1
0*x^10)*ArcSinh[c*x]))/(76800*c^4)
```

### 3.20.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6218, 27, 299, 211, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(c^2dx^2+d\right)^3(a+b\text{arcsinh}(cx))\,dx$$
$$\Big\downarrow \text{6218}$$
$$-bc\int -\frac{d^3\left(1-4c^2x^2\right)\left(c^2x^2+1\right)^{7/2}}{40c^4}dx+\frac{d^3\left(c^2x^2+1\right)^5(a+b\text{arcsinh}(cx))}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))}{8c^4}$$
$$\Big\downarrow \text{27}$$
$$\frac{bd^3\int\left(1-4c^2x^2\right)\left(c^2x^2+1\right)^{7/2}dx}{40c^3}+\frac{d^3\left(c^2x^2+1\right)^5(a+b\text{arcsinh}(cx))}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))}{8c^4}$$
$$\Big\downarrow \text{299}$$

$$\frac{bd^3\left(\frac{7}{5}\int \left(c^2x^2+1\right)^{7/2}dx-\frac{2}{5}x\left(c^2x^2+1\right)^{9/2}\right)}{40c^3}+\frac{d^3\left(c^2x^2+1\right)^5\left(a+b\text{arcsinh}(cx)\right)}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\text{arcsinh}(cx)\right)}{8c^4}$$

$$\Big\downarrow \ 211$$

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\int \left(c^2x^2+1\right)^{5/2}dx+\frac{1}{8}x\left(c^2x^2+1\right)^{7/2}\right)-\frac{2}{5}x\left(c^2x^2+1\right)^{9/2}\right)}{40c^3}+\frac{d^3\left(c^2x^2+1\right)^5\left(a+b\text{arcsinh}(cx)\right)}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\text{arcsinh}(cx)\right)}{8c^4}$$

$$\Big\downarrow \ 211$$

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\int \left(c^2x^2+1\right)^{3/2}dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)+\frac{1}{8}x\left(c^2x^2+1\right)^{7/2}\right)-\frac{2}{5}x\left(c^2x^2+1\right)^{9/2}\right)}{40c^3}+\frac{d^3\left(c^2x^2+1\right)^5\left(a+b\text{arcsinh}(cx)\right)}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\text{arcsinh}(cx)\right)}{8c^4}$$

$$\Big\downarrow \ 211$$

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)+\frac{1}{8}x\left(c^2x^2+1\right)^{7/2}\right)-\frac{2}{5}x\left(c^2x^2+1\right)^{9/2}\right)}{40c^3}+\frac{d^3\left(c^2x^2+1\right)^5\left(a+b\text{arcsinh}(cx)\right)}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\text{arcsinh}(cx)\right)}{8c^4}$$

$$\Big\downarrow \ 211$$

$$\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)+\frac{1}{8}x\left(c^2x^2+1\right)^{7/2}\right)-\frac{2}{5}\cdots}{40c^3}$$
$$\frac{d^3\left(c^2x^2+1\right)^5\left(a+b\text{arcsinh}(cx)\right)}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\text{arcsinh}(cx)\right)}{8c^4}$$

$$\Big\downarrow \ 222$$

$$\frac{d^3\left(c^2x^2+1\right)^5\left(a+b\text{arcsinh}(cx)\right)}{10c^4}-\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\text{arcsinh}(cx)\right)}{8c^4}+$$
$$\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)+\frac{1}{8}x\left(c^2x^2+1\right)^{7/2}\right)-\frac{2}{5}x\left(c\cdots\right.}{40c^3}$$

input

```
Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

3.20. $\int x^3\left(d+c^2dx^2\right)^3\left(a+b\text{arcsinh}(cx)\right)dx$

output
```
-1/8*(d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x]))/c^4 + (d^3*(1 + c^2*x^2)^5
*(a + b*ArcSinh[c*x]))/(10*c^4) + (b*d^3*((-2*x*(1 + c^2*x^2)^(9/2))/5 + (
7*((x*(1 + c^2*x^2)^(7/2))/8 + (7*((x*(1 + c^2*x^2)^(5/2))/6 + (5*((x*(1 +
 c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4)
)/6))/8))/5))/(40*c^3)
```

### 3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.20.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| parts | $d^3a\left(\frac{1}{10}c^6x^{10}+\frac{3}{8}c^4x^8+\frac{1}{2}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10}+\frac{3\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\operatorname{arcsi}\right.}{}$ |
| derivativedivides | $\frac{d^3a\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10}+\frac{3\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\right.}{c^4}$ |
| default | $\frac{d^3a\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10}+\frac{3\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\right.}{c^4}$ |

input
```
int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/10*c^6*x^10+3/8*c^4*x^8+1/2*c^2*x^6+1/4*x^4)+d^3*b/c^4*(1/10*arcs
inh(c*x)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+1/4*a
rcsinh(c*x)*c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(c^2*x
^2+1)^(1/2)-401/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^2+1)
^(1/2)+79/5120*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x))
```

### 3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99

$$\int x^3\left(d+c^2dx^2\right)^3\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{7680\,ac^{10}d^3x^{10}+28800\,ac^8d^3x^8+38400\,ac^6d^3x^6+19200\,ac^4d^3x^4+15\left(512\,bc^{10}d^3x^{10}+1920\,bc^8d^3x^8+25\right.}{}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/76800*(7680*a*c^10*d^3*x^10 + 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6
+ 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 + 1920*b*c^8*d^3*x^8 + 256
0*b*c^6*d^3*x^6 + 1280*b*c^4*d^3*x^4 - 79*b*d^3)*log(c*x + sqrt(c^2*x^2 +
1)) - (768*b*c^9*d^3*x^9 + 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 + 790*b
*c^3*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

### 3.20.6 Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.41

$$\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^6d^3x^{10}}{10}+\frac{3ac^4d^3x^8}{8}+\frac{ac^2d^3x^6}{2}+\frac{ad^3x^4}{4}+\frac{bc^6d^3x^{10}\operatorname{asinh}(cx)}{10}-\frac{bc^5d^3x^9\sqrt{c^2x^2+1}}{100}+\frac{3bc^4d^3x^8\operatorname{asinh}(cx)}{8}-\frac{57bc^3d^3x^7\sqrt{c^2x^2+1}}{1600} \\ \frac{ad^3x^4}{4}\end{cases}$$

input
```
integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 + a*c**2*d**3*x**6/
2 + a*d**3*x**4/4 + b*c**6*d**3*x**10*asinh(c*x)/10 - b*c**5*d**3*x**9*sqr
t(c**2*x**2 + 1)/100 + 3*b*c**4*d**3*x**8*asinh(c*x)/8 - 57*b*c**3*d**3*x*
*7*sqrt(c**2*x**2 + 1)/1600 + b*c**2*d**3*x**6*asinh(c*x)/2 - 401*b*c*d**3
*x**5*sqrt(c**2*x**2 + 1)/9600 + b*d**3*x**4*asinh(c*x)/4 - 79*b*d**3*x**3
*sqrt(c**2*x**2 + 1)/(7680*c) + 79*b*d**3*x*sqrt(c**2*x**2 + 1)/(5120*c**3
) - 79*b*d**3*asinh(c*x)/(5120*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

### 3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(173) = 346$.

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.16

$$\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))\,dx=\frac{1}{10}ac^6d^3x^{10}+\frac{3}{8}ac^4d^3x^8+\frac{1}{2}ac^2d^3x^6$$
$$+\frac{1}{12800}\left(1280\,x^{10}\operatorname{arsinh}(cx)-\left(\frac{128\sqrt{c^2x^2+1}x^9}{c^2}-\frac{144\sqrt{c^2x^2+1}x^7}{c^4}+\frac{168\sqrt{c^2x^2+1}x^5}{c^6}-\frac{210\sqrt{c^2x^2}}{c^8}\right.\right.$$
$$+\frac{1}{1024}\left(384\,x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2+1}x}{c^8}\right.\right.$$
$$+\frac{1}{4}ad^3x^4$$
$$+\frac{1}{96}\left(48\,x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\operatorname{arsinh}(cx)}{c^7}\right)c\right)bc^2$$
$$+\frac{1}{32}\left(8\,x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)bd^3$$

input
```
integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 + 1/2*a*c^2*d^3*x^6 + 1/12800*(128
0*x^10*arcsinh(c*x) - (128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 +
1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8
 + 315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c)*b*c^6*d^3 + 1/
1024*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x
^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c
^8 + 105*arcsinh(c*x)/c^9)*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 + 1/96*(48*x^6*arc
sinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 +
15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x
^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4
 + 3*arcsinh(c*x)/c^5)*c)*b*d^3
```

### 3.20.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.20.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx)) \, dx = \int x^3 (a + b\,\text{asinh}(c\,x)) \left(d\,c^2\,x^2 + d\right)^3 dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

# 3.21 $\int x^2(d+c^2dx^2)^3(a+b\mathrm{arcsinh}(cx))\,dx$



## 3.21.1 Optimal result

Integrand size = 24, antiderivative size = 202

$$\int x^2(d+c^2dx^2)^3(a+b\mathrm{arcsinh}(cx))\,dx = \frac{16bd^3\sqrt{1+c^2x^2}}{315c^3}+\frac{8bd^3(1+c^2x^2)^{3/2}}{945c^3} + \frac{2bd^3(1+c^2x^2)^{5/2}}{525c^3}+\frac{bd^3(1+c^2x^2)^{7/2}}{441c^3}-\frac{bd^3(1+c^2x^2)^{9/2}}{81c^3} + \frac{1}{3}d^3x^3(a+b\mathrm{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\mathrm{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\mathrm{arcsinh}(cx))+\frac{1}{9}c^6d^3x^9(a+b\mathrm{arcsinh}($$

output
```
8/945*b*d^3*(c^2*x^2+1)^(3/2)/c^3+2/525*b*d^3*(c^2*x^2+1)^(5/2)/c^3+1/441*
b*d^3*(c^2*x^2+1)^(7/2)/c^3-1/81*b*d^3*(c^2*x^2+1)^(9/2)/c^3+1/3*d^3*x^3*(
a+b*arcsinh(c*x))+3/5*c^2*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^4*d^3*x^7*(a+b*
arcsinh(c*x))+1/9*c^6*d^3*x^9*(a+b*arcsinh(c*x))+16/315*b*d^3*(c^2*x^2+1)^
(1/2)/c^3
```

## 3.21.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.67

$$\int x^2(d+c^2dx^2)^3(a+b\mathrm{arcsinh}(cx))\,dx$$

$$= \frac{d^3\left(315ac^3x^3(105+189c^2x^2+135c^4x^4+35c^6x^6)-b\sqrt{1+c^2x^2}(-5258+2629c^2x^2+6297c^4x^4+4675c^6x\right.}{99225c^3}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(315*a*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - b*Sqr
t[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6 + 1225*
c^8*x^8) + 315*b*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*Ar
cSinh[c*x]))/(99225*c^3)
```

### 3.21.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6218, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2 d x^2+d\right)^3 (a+b\text{arcsinh}(cx)) \, dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int \frac{d^3x^3\left(35c^6x^6+135c^4x^4+189c^2x^2+105\right)}{315\sqrt{c^2x^2+1}}dx+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{315}bcd^3\int \frac{x^3\left(35c^6x^6+135c^4x^4+189c^2x^2+105\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{630}bcd^3\int \frac{x^2\left(35c^6x^6+135c^4x^4+189c^2x^2+105\right)}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{9}c^6d^3x^9(a+b\text{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arcsinh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2123}$$

$$-\frac{1}{630}bcd^3\int\left(\frac{35\left(c^2x^2+1\right)^{7/2}}{c^2}-\frac{5\left(c^2x^2+1\right)^{5/2}}{c^2}-\frac{6\left(c^2x^2+1\right)^{3/2}}{c^2}-\frac{8\sqrt{c^2x^2+1}}{c^2}-\frac{16}{c^2\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{9}c^6d^3x^9(a+b\operatorname{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\operatorname{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}d^3x^3(a+b\operatorname{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}c^6d^3x^9(a+b\operatorname{arcsinh}(cx))+\frac{3}{7}c^4d^3x^7(a+b\operatorname{arcsinh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\operatorname{arcsinh}(cx))+\frac{1}{3}d^3x^3(a+b\operatorname{arcsinh}(cx))-\frac{1}{630}bcd^3\left(\frac{70\left(c^2x^2+1\right)^{9/2}}{9c^4}-\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^4}-\frac{12\left(c^2x^2+1\right)^{5/2}}{5c^4}-\frac{16\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{32\sqrt{c^2x^2+1}}{c^4}\right)$$

input
```
Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/630*(b*c*d^3*((-32*Sqrt[1 + c^2*x^2])/c^4 - (16*(1 + c^2*x^2)^(3/2))/(3
*c^4) - (12*(1 + c^2*x^2)^(5/2))/(5*c^4) - (10*(1 + c^2*x^2)^(7/2))/(7*c^4
) + (70*(1 + c^2*x^2)^(9/2))/(9*c^4))) + (d^3*x^3*(a + b*ArcSinh[c*x]))/3
+ (3*c^2*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcSinh[c
*x]))/7 + (c^6*d^3*x^9*(a + b*ArcSinh[c*x]))/9
```

### 3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.21.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| parts | $d^3a\left(\frac{1}{9}c^6x^9+\frac{3}{7}c^4x^7+\frac{3}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^3b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(\right.}{}$ |
| derivativedivides | $\frac{d^3a\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-c^8x^8\right.}{c^3}$ |
| default | $\frac{d^3a\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-c^8x^8\right.}{c^3}$ |

input
```
int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/9*c^6*x^9+3/7*c^4*x^7+3/5*c^2*x^5+1/3*x^3)+d^3*b/c^3*(1/9*arcsinh
(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*arcsin
h(c*x)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^2+1)
^(1/2)-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2+1)
^(1/2)+5258/99225*(c^2*x^2+1)^(1/2))
```

## 3.21.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int x^2\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{11025\,ac^9d^3x^9+42525\,ac^7d^3x^7+59535\,ac^5d^3x^5+33075\,ac^3d^3x^3+315\,(35\,bc^9d^3x^9+135\,bc^7d^3x^7+189\,b}{}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*d^3*x^9 + 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 +
 33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 + 135*b*c^7*d^3*x^7 + 189*b*c
^5*d^3*x^5 + 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8
*d^3*x^8 + 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 + 2629*b*c^2*d^3*x^2 -
5258*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

### 3.21.6 Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.31

$$\int x^2\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^6d^3x^9}{9}+\frac{3ac^4d^3x^7}{7}+\frac{3ac^2d^3x^5}{5}+\frac{ad^3x^3}{3}+\frac{bc^6d^3x^9\operatorname{asinh}(cx)}{9}-\frac{bc^5d^3x^8\sqrt{c^2x^2+1}}{81}+\frac{3bc^4d^3x^7\operatorname{asinh}(cx)}{7}-\frac{187bc^3d^3x^6\sqrt{c^2x^2+}}{3969}\\ \frac{ad^3x^3}{3}\end{cases}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 + 3*a*c**2*d**3*x**5/
5 + a*d**3*x**3/3 + b*c**6*d**3*x**9*asinh(c*x)/9 - b*c**5*d**3*x**8*sqrt(
c**2*x**2 + 1)/81 + 3*b*c**4*d**3*x**7*asinh(c*x)/7 - 187*b*c**3*d**3*x**6
*sqrt(c**2*x**2 + 1)/3969 + 3*b*c**2*d**3*x**5*asinh(c*x)/5 - 2099*b*c*d**
3*x**4*sqrt(c**2*x**2 + 1)/33075 + b*d**3*x**3*asinh(c*x)/3 - 2629*b*d**3*
x**2*sqrt(c**2*x**2 + 1)/(99225*c) + 5258*b*d**3*sqrt(c**2*x**2 + 1)/(9922
5*c**3), Ne(c, 0)), (a*d**3*x**3/3, True))
```

### 3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(174) = 348$.

Time = 0.21 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.92

$$\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{1}{9}ac^6d^3x^9 + \frac{3}{7}ac^4d^3x^7 + \frac{1}{2835}\left(315\,x^9\,\text{arsinh}\,(cx) - \left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2} - \frac{40\sqrt{c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{c^2x^2+1}x^4}{c^6} - \frac{64\sqrt{c^2x^2+1}x^2}{c^8}\right.\right.$$
$$+ \frac{3}{5}ac^2d^3x^5$$
$$+ \frac{3}{245}\left(35\,x^7\,\text{arsinh}\,(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^4d$$
$$+ \frac{1}{25}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d^3$$
$$+ \frac{1}{3}ad^3x^3 + \frac{1}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^3$$

input
```
integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35
*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^
2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10
)*c)*b*c^6*d^3 + 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arcsinh(c*x) - (5*sqrt(
c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x
^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^3 + 1/25*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^
2*x^2 + 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c
*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^3
```

### 3.21.8 Giac [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 d x^2)^3 (a + b \text{arcsinh}(cx)) \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.21.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 d x^2)^3 (a + b \text{arcsinh}(cx)) \, dx = \int x^2 (a + b \, \text{asinh}(c \, x)) \left( d \, c^2 \, x^2 + d \right)^3 \, dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

# 3.22 $\int x(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx$



## 3.22.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx = -\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c} - \frac{35bd^3x(1+c^2x^2)^{3/2}}{1536c} - \frac{7bd^3x(1+c^2x^2)^{5/2}}{384c} - \frac{bd^3x(1+c^2x^2)^{7/2}}{64c} - \frac{35bd^3\text{arcsinh}(cx)}{1024c^2} + \frac{d^3(1+c^2x^2)^4 (a+b\text{arcsinh}(cx))}{8c^2}$$

output
```
-35/1536*b*d^3*x*(c^2*x^2+1)^(3/2)/c-7/384*b*d^3*x*(c^2*x^2+1)^(5/2)/c-1/6
4*b*d^3*x*(c^2*x^2+1)^(7/2)/c-35/1024*b*d^3*arcsinh(c*x)/c^2+1/8*d^3*(c^2*
x^2+1)^4*(a+b*arcsinh(c*x))/c^2-35/1024*b*d^3*x*(c^2*x^2+1)^(1/2)/c
```

## 3.22.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int x(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx = \frac{d^3\left(cx\left(384acx(4+6c^2x^2+4c^4x^4+c^6x^6)-b\sqrt{1+c^2x^2}(279+326c^2x^2+200c^4x^4+48c^6x^6)\right)+3b(93+5\right.}{3072c^2}$$

input
```
Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(c*x*(384*a*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) - b*Sqrt[1 + c^
2*x^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*b*(93 + 512*c^2
*x^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]))/(3072*c^2)
```

### 3.22.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6213, 211, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(c^2 d x^2+d\right)^3 (a+b \text{arcsinh}(c x)) \, dx$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{d^3 \left(c^2 x^2+1\right)^4 (a+b \text{arcsinh}(c x))}{8 c^2}-\frac{b d^3 \int \left(c^2 x^2+1\right)^{7/2}dx}{8 c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d^3 \left(c^2 x^2+1\right)^4 (a+b \text{arcsinh}(c x))}{8 c^2}-\frac{b d^3 \left(\frac{7}{8} \int \left(c^2 x^2+1\right)^{5/2}dx+\frac{1}{8} x \left(c^2 x^2+1\right)^{7/2}\right)}{8 c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d^3 \left(c^2 x^2+1\right)^4 (a+b \text{arcsinh}(c x))}{8 c^2}-\frac{b d^3 \left(\frac{7}{8} \left(\frac{5}{6} \int \left(c^2 x^2+1\right)^{3/2}dx+\frac{1}{6} x \left(c^2 x^2+1\right)^{5/2}\right)+\frac{1}{8} x \left(c^2 x^2+1\right)^{7/2}\right)}{8 c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d^3 \left(c^2 x^2+1\right)^4 (a+b \text{arcsinh}(c x))}{8 c^2}-\frac{b d^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{c^2 x^2+1}dx+\frac{1}{4} x \left(c^2 x^2+1\right)^{3/2}\right)+\frac{1}{6} x \left(c^2 x^2+1\right)^{5/2}\right)+\frac{1}{8} x \left(c^2 x^2+1\right)^{7/2}\right)}{8 c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d^3 (c^2x^2+1)^4 (a+b\text{arcsinh}(cx))}{8c^2} - \frac{bd^3 \left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)+\frac{1}{8}x(c^2x^2+1)^{7/2}\right)}{8c}$$

$$\Big\downarrow \text{222}$$

$$\frac{d^3 (c^2x^2+1)^4 (a+b\text{arcsinh}(cx))}{8c^2} - \frac{bd^3 \left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)+\frac{1}{8}x(c^2x^2+1)^{7/2}\right)}{8c}$$

input
```
Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x]))/(8*c^2) - (b*d^3*((x*(1 + c^2*x
^2)^(7/2))/8 + (7*((x*(1 + c^2*x^2)^(5/2))/6 + (5*((x*(1 + c^2*x^2)^(3/2))
/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/6))/8))/(8*c)
```

### 3.22.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.22.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d^3a(c^2x^2+1)^4}{8}+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{3\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{93\operatorname{arcsinh}(cx)}{1024}-\frac{cx(c^2x^2+1)^{\frac{7}{2}}}{64}\right.}{c^2}$ |
| default | $\frac{\frac{d^3a(c^2x^2+1)^4}{8}+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{3\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{93\operatorname{arcsinh}(cx)}{1024}-\frac{cx(c^2x^2+1)^{\frac{7}{2}}}{64}\right.}{c^2}$ |
| parts | $\frac{d^3a(c^2x^2+1)^4}{8c^2}+\frac{d^3b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{3\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}+\frac{93\operatorname{arcsinh}(cx)}{1024}-\frac{cx(c^2x^2+}{64}\right.}{c^2}$ |

input
```
int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/8*d^3*a*(c^2*x^2+1)^4+d^3*b*(1/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh
(c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+93/1024*ar
csinh(c*x)-1/64*c*x*(c^2*x^2+1)^(7/2)-7/384*c*x*(c^2*x^2+1)^(5/2)-35/1536*
c*x*(c^2*x^2+1)^(3/2)-35/1024*c*x*(c^2*x^2+1)^(1/2)))
```

### 3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int x\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{384\,ac^8d^3x^8+1536\,ac^6d^3x^6+2304\,ac^4d^3x^4+1536\,ac^2d^3x^2+3\left(128\,bc^8d^3x^8+512\,bc^6d^3x^6+768\,bc^4d^3x^4\right.}{}$$

input
```
integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/3072*(384*a*c^8*d^3*x^8 + 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 + 1536
*a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 + 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*
x^4 + 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (48*b*c
^7*d^3*x^7 + 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 + 279*b*c*d^3*x)*sqrt(c
^2*x^2 + 1))/c^2
```

### 3.22.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.74

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^6d^3x^8}{8} + \frac{ac^4d^3x^6}{2} + \frac{3ac^2d^3x^4}{4} + \frac{ad^3x^2}{2} + \frac{bc^6d^3x^8\operatorname{asinh}(cx)}{8} - \frac{bc^5d^3x^7\sqrt{c^2x^2+1}}{64} + \frac{bc^4d^3x^6\operatorname{asinh}(cx)}{2} - \frac{25bc^3d^3x^5\sqrt{c^2x^2+1}}{384} + \\ \frac{ad^3x^2}{2} \end{cases}$$

input
```
integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 + 3*a*c**2*d**3*x**4/4
+ a*d**3*x**2/2 + b*c**6*d**3*x**8*asinh(c*x)/8 - b*c**5*d**3*x**7*sqrt(c*
*2*x**2 + 1)/64 + b*c**4*d**3*x**6*asinh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt
(c**2*x**2 + 1)/384 + 3*b*c**2*d**3*x**4*asinh(c*x)/4 - 163*b*c*d**3*x**3*
sqrt(c**2*x**2 + 1)/1536 + b*d**3*x**2*asinh(c*x)/2 - 93*b*d**3*x*sqrt(c**
2*x**2 + 1)/(1024*c) + 93*b*d**3*asinh(c*x)/(1024*c**2), Ne(c, 0)), (a*d**
3*x**2/2, True))
```

### 3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(125) = 250$.

Time = 0.19 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.43

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \frac{1}{8}ac^6d^3x^8 + \frac{1}{2}ac^4d^3x^6$$
$$+ \frac{1}{3072}\left(384\,x^8\,\text{arsinh}\,(cx) - \left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2} - \frac{56\sqrt{c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{c^2x^2+1}x^3}{c^6} - \frac{105\sqrt{c^2x^2+1}x}{c^8}\right.\right.$$
$$+ \frac{3}{4}ac^2d^3x^4$$
$$+ \frac{1}{96}\left(48\,x^6\,\text{arsinh}\,(cx) - \left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\text{arsinh}\,(cx)}{c^7}\right)c\right)bc^4$$
$$+ \frac{3}{32}\left(8\,x^4\,\text{arsinh}\,(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\,(cx)}{c^5}\right)c\right)bc^2d^3$$
$$+ \frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2\,x^2\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\right)bd^3$$

input
```
integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48
*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^
2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*
c^6*d^3 + 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2
+ 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 -
 15*arcsinh(c*x)/c^7)*c)*b*c^4*d^3 + 3/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^
2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*
c^2*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x
/c^2 - arcsinh(c*x)/c^3))*b*d^3
```

### 3.22.8 Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.22.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx = \int x\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

output
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

# 3.23 $\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx$



## 3.23.1 Optimal result

Integrand size = 21, antiderivative size = 170

$$\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{35c} - \frac{8bd^3(1+c^2x^2)^{3/2}}{105c} - \frac{6bd^3(1+c^2x^2)^{5/2}}{175c} - \frac{bd^3(1+c^2x^2)^{7/2}}{49c}$$
$$+d^3x(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))$$

output
```
-8/105*b*d^3*(c^2*x^2+1)^(3/2)/c-6/175*b*d^3*(c^2*x^2+1)^(5/2)/c-1/49*b*d^
3*(c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arcsinh(c*x))+c^2*d^3*x^3*(a+b*arcsinh(c*
x))+3/5*c^4*d^3*x^5*(a+b*arcsinh(c*x))+1/7*c^6*d^3*x^7*(a+b*arcsinh(c*x))-
16/35*b*d^3*(c^2*x^2+1)^(1/2)/c
```

## 3.23.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

$$\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{d^3\big(105acx(35+35c^2x^2+21c^4x^4+5c^6x^6) - b\sqrt{1+c^2x^2}(2161+757c^2x^2+351c^4x^4+75c^6x^6)+105bcx(3}{3675c}$$

input
```
Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(105*a*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - b*Sqrt[1 + c^
2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(35 + 3
5*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]))/(3675*c)
```

### 3.23.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6199, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^3 (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6199}$$

$$-bc\int \frac{d^3x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{35\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{35}bcd^3\int \frac{x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2331}$$

$$-\frac{1}{70}bcd^3\int \frac{5c^6x^6+21c^4x^4+35c^2x^2+35}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2389}$$

$$-\frac{1}{70}bcd^3\int \left(5\left(c^2x^2+1\right)^{5/2}+6\left(c^2x^2+1\right)^{3/2}+8\sqrt{c^2x^2+1}+\frac{16}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))-\frac{1}{70}bcd^3\left(\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^2}+\frac{12\left(c^2x^2+1\right)^{5/2}}{5c^2}+\frac{16\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{32\sqrt{c^2x^2+1}}{c^2}\right)$$

input
```
Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/70*(b*c*d^3*((32*Sqrt[1 + c^2*x^2])/c^2 + (16*(1 + c^2*x^2)^(3/2))/(3*c
^2) + (12*(1 + c^2*x^2)^(5/2))/(5*c^2) + (10*(1 + c^2*x^2)^(7/2))/(7*c^2))
) + d^3*x*(a + b*ArcSinh[c*x]) + c^2*d^3*x^3*(a + b*ArcSinh[c*x]) + (3*c^4
*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*d^3*x^7*(a + b*ArcSinh[c*x]))/7
```

### 3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

### 3.23.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| parts | $d^3a\left(\frac{1}{7}c^6x^7+\frac{3}{5}c^4x^5+x^3c^2+x\right)+\frac{d^3b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)c^3x^3+\operatorname{arcsinh}(cx)cx\right.}{c}$ |
| derivativedivides | $\frac{d^3a\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)c^3x^3+\operatorname{arcsinh}(cx)cx-\frac{2161\sqrt{c^2x^2+}}{3675}\right.}{c}$ |
| default | $\frac{d^3a\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)c^3x^3+\operatorname{arcsinh}(cx)cx-\frac{2161\sqrt{c^2x^2+}}{3675}\right.}{c}$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/7*c^6*x^7+3/5*c^4*x^5+x^3*c^2+x)+d^3*b/c*(1/7*arcsinh(c*x)*c^7*x^
7+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-2161/3675
*(c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(c^2*x^
2+1)^(1/2)-757/3675*c^2*x^2*(c^2*x^2+1)^(1/2))
```

### 3.23.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \left(d+c^2dx^2\right)^3 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{525\,ac^7d^3x^7+2205\,ac^5d^3x^5+3675\,ac^3d^3x^3+3675\,acd^3x+105\,(5\,bc^7d^3x^7+21\,bc^5d^3x^5+35\,bc^3d^3x^3+35}{3675\,c}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*d^3*x^7 + 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 + 3675
*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 + 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 +
35*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d^3*x^6 + 351*b*c^4
*d^3*x^4 + 757*b*c^2*d^3*x^2 + 2161*b*d^3)*sqrt(c^2*x^2 + 1))/c
```

### 3.23.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.30

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} \frac{ac^6d^3x^7}{7}+\frac{3ac^4d^3x^5}{5}+ac^2d^3x^3+ad^3x+\frac{bc^6d^3x^7\operatorname{asinh}(cx)}{7}-\frac{bc^5d^3x^6\sqrt{c^2x^2+1}}{49}+\frac{3bc^4d^3x^5\operatorname{asinh}(cx)}{5}-\frac{117bc^3d^3x^4\sqrt{c^2x^2+}}{1225} \\ ad^3x \end{cases}$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 + a*c**2*d**3*x**3 +
a*d**3*x + b*c**6*d**3*x**7*asinh(c*x)/7 - b*c**5*d**3*x**6*sqrt(c**2*x**2
 + 1)/49 + 3*b*c**4*d**3*x**5*asinh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**
2*x**2 + 1)/1225 + b*c**2*d**3*x**3*asinh(c*x) - 757*b*c*d**3*x**2*sqrt(c*
*2*x**2 + 1)/3675 + b*d**3*x*asinh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 + 1)/
(3675*c), Ne(c, 0)), (a*d**3*x, True))
```

### 3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(150) = 300.

Time = 0.20 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.77

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=\frac{1}{7}\,ac^6d^3x^7+\frac{3}{5}\,ac^4d^3x^5$$
$$+\frac{1}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^6d$$
$$+\frac{1}{25}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^3$$
$$+ac^2d^3x^3+\frac{1}{3}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^3$$
$$+ad^3x+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sq
rt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1
)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcsinh(
c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt
(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^3 + a*c^2*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x)
- c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + a*d
^3*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

### 3.23.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^3 \left(a+b\text{arcsinh}(cx)\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.23.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^3 \left(a+b\text{arcsinh}(cx)\right) dx = \int \left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

# 3.24 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx$



## 3.24.1 Optimal result

Integrand size = 24, antiderivative size = 221

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx = & -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x(1+c^2x^2)^{3/2}\\ & -\frac{1}{36}bcd^3x(1+c^2x^2)^{5/2}-\frac{19}{48}bd^3\text{arcsinh}(cx)\\ & +\frac{1}{2}d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))\\ & +\frac{1}{4}d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))\\ & +\frac{1}{6}d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))\\ & +\frac{d^3(a+b\text{arcsinh}(cx))^2}{2b}\\ & +d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\ & -\frac{1}{2}bd^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
-7/72*b*c*d^3*x*(c^2*x^2+1)^(3/2)-1/36*b*c*d^3*x*(c^2*x^2+1)^(5/2)-19/48*b
*d^3*arcsinh(c*x)+1/2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/4*d^3*(c^2*x^2+
1)^2*(a+b*arcsinh(c*x))+1/6*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))+1/2*d^3*(
a+b*arcsinh(c*x))^2/b+d^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2)
)^2)-1/2*b*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-19/48*b*c*d^3*x*(c^2
*x^2+1)^(1/2)
```

### 3.24.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx$$
$$= \frac{d^3\left(-72a^2+132ab+216abc^2x^2+108abc^4x^4+24abc^6x^6-75b^2cx\sqrt{1+c^2x^2}-22b^2c^3x^3\sqrt{1+c^2x^2}-4b^2c\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(d^3*(-72*a^2 + 132*a*b + 216*a*b*c^2*x^2 + 108*a*b*c^4*x^4 + 24*a*b*c^6*x
^6 - 75*b^2*c*x*Sqrt[1 + c^2*x^2] - 22*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 4*b
^2*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*b^2*ArcSinh[c*x]^2 + 144*a*b*Log[1 - E^(
2*ArcSinh[c*x])] + 3*b*ArcSinh[c*x]*(-48*a + b*(25 + 72*c^2*x^2 + 36*c^4*x
^4 + 8*c^6*x^6) + 48*b*Log[1 - E^(2*ArcSinh[c*x])]) + 72*b^2*PolyLog[2, E^
(2*ArcSinh[c*x])]))/(144*b)
```

### 3.24.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.57, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6216, 27, 211, 211, 211, 222, 6216, 211, 211, 222, 6216, 211, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^3(a+b\text{arcsinh}(cx))}{x}\,dx$$

$$\Big\downarrow \text{6216}$$

$$d\int \frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{6}bcd^3\int \left(c^2x^2+1\right)^{5/2}dx+\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{6}bcd^3\int \left(c^2x^2+1\right)^{5/2}dx+\frac{1}{6}d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))$$

$\Big\downarrow$ 211

$$d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{6}bcd^3\left(\frac{5}{6}\int \left(c^2x^2+1\right)^{3/2}dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)+\frac{1}{6}d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))$$

$\Big\downarrow$ 211

$$d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)+\frac{1}{6}d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))$$

$\Big\downarrow$ 211

$$d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)+\frac{1}{6}d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))$$

$\Big\downarrow$ 222

$$d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{6}d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))-\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\Big\downarrow$ 6216

$$d^3\left(\int \frac{\left(c^2x^2+1\right) (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bc\int \left(c^2x^2+1\right)^{3/2}dx+\frac{1}{4}\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))\right)+\frac{1}{6}d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))-\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\Big\downarrow$ 211

3.24. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))}{x}\,dx$

$$d^3\left(\int \frac{(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{4}\left(c^2x^2+1\right)^2\left(a+b\text{arcs}\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$$\Big\downarrow \; 211$$

$$d^3\left(\int \frac{(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{4}\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$$\Big\downarrow \; 222$$

$$d^3\left(\int \frac{(c^2x^2+1)\,(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+}\right.\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$$\Big\downarrow \; 6216$$

$$d^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int \sqrt{c^2x^2+1}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\left(c^2x^2+1\right)\left(a+b\text{arcsinh}\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$$\Big\downarrow \; 211$$

$$d^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\left(c^2\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$$\Big\downarrow \; 222$$

---

3.24. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx$

$$d^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{\text{arcsinh}(c}{2c}\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\downarrow$ 6190

$$d^3\left(\frac{\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\downarrow$ 25

$$d^3\left(-\frac{\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\downarrow$ 3042

$$d^3\left(-\frac{\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\downarrow$ 26

$$d^3\left(\frac{i\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

---

3.24. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))}{x}\,dx$

$\Big\downarrow$ 4201

$$d^3\left(\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\Big\downarrow$ 2620

$$d^3\left(\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsin}}{b}}\right.\right.\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\Big\downarrow$ 2715

$$d^3\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+b\text{arcs}\right.\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

$\Big\downarrow$ 2838

$$d^3\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)$$

---

3.24. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x}\,dx$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/6 - (b*c*d^3*((x*(1 + c^2*x^2)^
(5/2))/6 + (5*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + A
rcSinh[c*x]/(2*c)))/4))/6))/6 + d^3*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/
2 + ((1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 - (b*c*((x*Sqrt[1 + c^2*x^2])
/2 + ArcSinh[c*x]/(2*c)))/2 - (b*c*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqr
t[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/4 + (I*((-1/2*I)*(a + b*ArcSi
nh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi
 - (2*(a + b*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4
)))/b)
```

### 3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

### 3.24.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| parts | $d^3a\left(\frac{c^6x^6}{6}+\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}+\ln\left(x\right)\right)-\frac{d^3b\,c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{11d^3b\,c^3x^3\sqrt{c^2x^2+1}}{72}-\frac{25bc\,d^3x\sqrt{c^2x^2+1}}{48}$ |
| derivativedivides | $d^3a\left(\frac{c^6x^6}{6}+\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}+\ln\left(cx\right)\right)+\frac{25b\,d^3\operatorname{arcsinh}(cx)}{48}+d^3b\operatorname{arcsinh}\left(cx\right)\ln\left(1+cx+\sqrt{c\right.$ |
| default | $d^3a\left(\frac{c^6x^6}{6}+\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}+\ln\left(cx\right)\right)+\frac{25b\,d^3\operatorname{arcsinh}(cx)}{48}+d^3b\operatorname{arcsinh}\left(cx\right)\ln\left(1+cx+\sqrt{c\right.$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/6*c^6*x^6+3/4*c^4*x^4+3/2*c^2*x^2+ln(x))-1/36*d^3*b*c^5*x^5*(c^2*
x^2+1)^(1/2)-11/72*d^3*b*c^3*x^3*(c^2*x^2+1)^(1/2)-25/48*b*c*d^3*x*(c^2*x^
2+1)^(1/2)+1/6*d^3*b*arcsinh(c*x)*c^6*x^6+3/4*d^3*b*arcsinh(c*x)*c^4*x^4+3
/2*d^3*b*arcsinh(c*x)*c^2*x^2+25/48*b*d^3*arcsinh(c*x)+d^3*b*polylog(2,-c*
x-(c^2*x^2+1)^(1/2))+d^3*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))-1/2*d^3*b*arcs
inh(c*x)^2+d^3*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+d^3*b*arcsinh(c*
x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
```

### 3.24.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x} \, dx = \int \frac{(c^2dx^2+d)^3 (b\,\text{arsinh}\,(cx)+a)}{x} \, dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c
^6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x, x
)
```

### 3.24.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x} \, dx = d^3 \left( \int \frac{a}{x} \, dx + \int 3ac^2x \, dx + \int 3ac^4x^3 \, dx + \int ac^6x^5 \, dx + \int \frac{b\,\text{asinh}\,(cx)}{x} \, dx + \int 3bc^2x\,\text{asinh}\,(cx) \, dx + \int 3bc^4x^3\,\text{asinh}\,(cx) \, dx + \int bc^6x^5\,\text{asinh}\,(cx) \, dx \right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x,x)
```

output
```
d**3*(Integral(a/x, x) + Integral(3*a*c**2*x, x) + Integral(3*a*c**4*x**3,
 x) + Integral(a*c**6*x**5, x) + Integral(b*asinh(c*x)/x, x) + Integral(3*
b*c**2*x*asinh(c*x), x) + Integral(3*b*c**4*x**3*asinh(c*x), x) + Integral
(b*c**6*x**5*asinh(c*x), x))
```

### 3.24.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))}{x}\,dx=\int \frac{(c^2dx^2+d)^3(b\,\mathrm{arsinh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

output
```
1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 + 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) +
 integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x^3*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^2*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) +
b*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.24.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3\,(a+b\mathrm{arcsinh}(cx))}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))}{x} \, dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^3}{x} \,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x, x)
```

# 3.25 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.25.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^2}\,dx = &-\frac{11}{5}bcd^3\sqrt{1+c^2x^2}-\frac{1}{5}bcd^3(1+c^2x^2)^{3/2}\\&-\frac{1}{25}bcd^3(1+c^2x^2)^{5/2}-\frac{d^3(a+b\text{arcsinh}(cx))}{x}\\&+3c^2d^3x(a+b\text{arcsinh}(cx))\\&+c^4d^3x^3(a+b\text{arcsinh}(cx))\\&+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))\\&-bcd^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)\end{aligned}$$

output
```
-1/5*b*c*d^3*(c^2*x^2+1)^(3/2)-1/25*b*c*d^3*(c^2*x^2+1)^(5/2)-d^3*(a+b*arc
sinh(c*x))/x+3*c^2*d^3*x*(a+b*arcsinh(c*x))+c^4*d^3*x^3*(a+b*arcsinh(c*x))
+1/5*c^6*d^3*x^5*(a+b*arcsinh(c*x))-b*c*d^3*arctanh((c^2*x^2+1)^(1/2))-11/
5*b*c*d^3*(c^2*x^2+1)^(1/2)
```

### 3.25.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.02

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^2}\,dx$$
$$= \frac{d^3\left(-25a+75ac^2x^2+25ac^4x^4+5ac^6x^6-61bcx\sqrt{1+c^2x^2}-7bc^3x^3\sqrt{1+c^2x^2}-bc^5x^5\sqrt{1+c^2x^2}+5b(-\right.}{25x}$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
(d^3*(-25*a + 75*a*c^2*x^2 + 25*a*c^4*x^4 + 5*a*c^6*x^6 - 61*b*c*x*Sqrt[1
+ c^2*x^2] - 7*b*c^3*x^3*Sqrt[1 + c^2*x^2] - b*c^5*x^5*Sqrt[1 + c^2*x^2] +
 5*b*(-5 + 15*c^2*x^2 + 5*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 25*b*c*x*Log[x
] - 25*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(25*x)
```

### 3.25.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6218, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3 (a+b\text{arcsinh}(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{6218}$$
$$-bc\int -\frac{d^3(-c^6x^6-5c^4x^4-15c^2x^2+5)}{5x\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx))+3c^2d^3x(a+b\text{arcsinh}(cx))-\frac{d^3(a+b\text{arcsinh}(cx))}{x}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{5}bcd^3\int \frac{-c^6x^6-5c^4x^4-15c^2x^2+5}{x\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^6d^3x^5(a+b\text{arcsinh}(cx))+c^4d^3x^3(a+b\text{arcsinh}(cx))+3c^2d^3x(a+b\text{arcsinh}(cx))-\frac{d^3(a+b\text{arcsinh}(cx))}{x}$$
$$\Big\downarrow \text{2331}$$

$$\frac{1}{10}bcd^3\int\frac{-c^6x^6-5c^4x^4-15c^2x^2+5}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{1}{5}c^6d^3x^5(a+\text{barcsinh}(cx))+c^4d^3x^3(a+\text{barcsinh}(cx))+3c^2d^3x(a+\text{barcsinh}(cx))-\frac{d^3(a+\text{barcsinh}(cx))}{x}$$

$\Big\downarrow$ 2123

$$\frac{1}{10}bcd^3\int\left(-\left(c^2x^2+1\right)^{3/2}c^2-3\sqrt{c^2x^2+1}c^2-\frac{11c^2}{\sqrt{c^2x^2+1}}+\frac{5}{x^2\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{5}c^6d^3x^5(a+\text{barcsinh}(cx))+c^4d^3x^3(a+\text{barcsinh}(cx))+3c^2d^3x(a+\text{barcsinh}(cx))-\frac{d^3(a+\text{barcsinh}(cx))}{x}$$

$\Big\downarrow$ 2009

$$\frac{1}{5}c^6d^3x^5(a+\text{barcsinh}(cx))+c^4d^3x^3(a+\text{barcsinh}(cx))+3c^2d^3x(a+\text{barcsinh}(cx))-\frac{d^3(a+\text{barcsinh}(cx))}{x}+\frac{1}{10}bcd^3\left(-10\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{2}{5}\left(c^2x^2+1\right)^{5/2}-2\left(c^2x^2+1\right)^{3/2}-22\sqrt{c^2x^2+1}\right)$$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-((d^3*(a + b*ArcSinh[c*x]))/x) + 3*c^2*d^3*x*(a + b*ArcSinh[c*x]) + c^4*d
^3*x^3*(a + b*ArcSinh[c*x]) + (c^6*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (b*c*
d^3*(-22*Sqrt[1 + c^2*x^2] - 2*(1 + c^2*x^2)^(3/2) - (2*(1 + c^2*x^2)^(5/2
))/5 - 10*ArcTanh[Sqrt[1 + c^2*x^2]]))/10
```

### 3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.25.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| parts | $d^3a\left(\frac{c^6x^5}{5}+c^4x^3+3c^2x-\frac{1}{x}\right)+d^3bc\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx)$ |
| derivativedivides | $c\left(d^3a\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx\right.\right.$ |
| default | $c\left(d^3a\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx\right.\right.$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/5*c^6*x^5+c^4*x^3+3*c^2*x-1/x)+d^3*b*c*(1/5*arcsinh(c*x)*c^5*x^5+
arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/25*c^4*x^4*(c^2
*x^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+1)^(1/2)-arcta
nh(1/(c^2*x^2+1)^(1/2)))
```

### 3.25.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.72

$$\int \frac{(d+c^2dx^2)^3\,(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx$$
$$=\frac{5\,ac^6d^3x^6+25\,ac^4d^3x^4+75\,ac^2d^3x^2-25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2}\right.}{}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/25*(5*a*c^6*d^3*x^6 + 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 25*b*c*d^3*x
*log(-c*x + sqrt(c^2*x^2 + 1) + 1) + 25*b*c*d^3*x*log(-c*x + sqrt(c^2*x^2
+ 1) - 1) - 5*(b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x*log(-c*x + sqrt(c^2
*x^2 + 1)) - 25*a*d^3 + 5*(b*c^6*d^3*x^6 + 5*b*c^4*d^3*x^4 + 15*b*c^2*d^3*
x^2 - (b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x - 5*b*d^3)*log(c*x + sqrt(c
^2*x^2 + 1)) - (b*c^5*d^3*x^5 + 7*b*c^3*d^3*x^3 + 61*b*c*d^3*x)*sqrt(c^2*x
^2 + 1))/x
```

### 3.25.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))}{x^2}\,dx = d^3\left(\int 3ac^2\,dx+\int \frac{a}{x^2}\,dx+\int 3ac^4x^2\,dx + \int ac^6x^4\,dx+\int 3bc^2\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx+\int 3bc^4x^2\operatorname{asinh}(cx)\,dx + \int bc^6x^4\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**2,x)
```

output
```
d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c**4*x**2
, x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*asinh(c*x), x) + Integ
ral(b*asinh(c*x)/x**2, x) + Integral(3*b*c**4*x**2*asinh(c*x), x) + Integr
al(b*c**6*x**4*asinh(c*x), x))
```

### 3.25.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.44

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{1}{5}\,ac^6d^3x^5 + \frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^6d^3 + ac^4d^3x^3 + \frac{1}{3}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^3 + 3\,ac^2d^3x + 3\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bcd^3 - \left(c\,\text{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\text{arsinh}\,(cx)}{x}\right)bd^3 - \frac{ad^3}{x}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/5*a*c^6*d^3*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c
^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^6*d^3 +
 a*c^4*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4))*b*c^4*d^3 + 3*a*c^2*d^3*x + 3*(c*x*arcsinh(c*x)
- sqrt(c^2*x^2 + 1))*b*c*d^3 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*
b*d^3 - a*d^3/x
```

### 3.25.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2, x)
```

# 3.26 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx$



## 3.26.1 Optimal result

Integrand size = 24, antiderivative size = 249

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx &= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2}+\frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} \\ &\quad -\frac{bcd^3(1+c^2x^2)^{5/2}}{2x}-\frac{3}{32}bc^2d^3\text{arcsinh}(cx) \\ &\quad +\frac{3}{2}c^2d^3(1+c^2x^2)(a+b\text{arcsinh}(cx)) \\ &\quad +\frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))}{2x^2} \\ &\quad +\frac{3c^2d^3(a+b\text{arcsinh}(cx))^2}{2b} \\ &\quad +3c^2d^3(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &\quad -\frac{3}{2}bc^2d^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
7/16*b*c^3*d^3*x*(c^2*x^2+1)^(3/2)-1/2*b*c*d^3*(c^2*x^2+1)^(5/2)/x-3/32*b*
c^2*d^3*arcsinh(c*x)+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+3/4*c^2*d^
3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))-1/2*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x)
)/x^2+3/2*c^2*d^3*(a+b*arcsinh(c*x))^2/b+3*c^2*d^3*(a+b*arcsinh(c*x))*ln(1
-1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b*c^2*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(
1/2))^2)-3/32*b*c^3*d^3*x*(c^2*x^2+1)^(1/2)
```

### 3.26.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^3} dx$$
$$= \frac{d^3\left(-16ab-48a^2c^2x^2+48abc^4x^4+8abc^6x^6-16b^2cx\sqrt{1+c^2x^2}-21b^2c^3x^3\sqrt{1+c^2x^2}-2b^2c^5x^5\sqrt{1+c^2x^2}\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(d^3*(-16*a*b - 48*a^2*c^2*x^2 + 48*a*b*c^4*x^4 + 8*a*b*c^6*x^6 - 16*b^2*c
*x*Sqrt[1 + c^2*x^2] - 21*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b^2*c^5*x^5*Sq
rt[1 + c^2*x^2] - 48*b^2*c^2*x^2*ArcSinh[c*x]^2 + 96*a*b*c^2*x^2*Log[1 - E
^(2*ArcSinh[c*x])] + b*ArcSinh[c*x]*(-96*a*c^2*x^2 + b*(-16 + 21*c^2*x^2 +
 48*c^4*x^4 + 8*c^6*x^6) + 96*b*c^2*x^2*Log[1 - E^(2*ArcSinh[c*x])]) + 48*
b^2*c^2*x^2*PolyLog[2, E^(2*ArcSinh[c*x])]))/(32*b*x^2)
```

### 3.26.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.43, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6217, 27, 247, 211, 211, 222, 6216, 211, 211, 222, 6216, 211, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3 (a+b\text{arcsinh}(cx))}{x^3} dx$$
$$\Big\downarrow \; 6217$$
$$3c^2d\int \frac{d^2(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^3\int \frac{(c^2x^2+1)^{5/2}}{x^2}dx-\frac{d^3(c^2x^2+1)^3 (a+b\text{arcsinh}(cx))}{2x^2}$$
$$\Big\downarrow \; 27$$

$$3c^2d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}}{x^2}dx-\frac{d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{247}$$

$$3c^2d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^3\left(5c^2\int \left(c^2x^2+1\right)^{3/2}dx-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)-\frac{d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{211}$$

$$3c^2d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)-\frac{d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{211}$$

$$3c^2d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)-\frac{d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{222}$$

$$3c^2d^3\int \frac{\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))}{x}dx-\frac{d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{6216}$$

$$3c^2d^3\left(\int \frac{\left(c^2x^2+1\right) (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bc\int \left(c^2x^2+1\right)^{3/2}dx+\frac{1}{4}\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))\right)-\frac{d^3\left(c^2x^2+1\right)^3 (a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

---

3.26. $\int \frac{\left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))}{x^3}\,dx$

$\downarrow$ 211

$$3c^2d^3\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{4}(c^2x^2+1)^2(a+ba$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)-\frac{(c^2x^2+1)^{5/2}}{x}\right)$$

$\downarrow$ 211

$$3c^2d^3\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)-\frac{(c^2x^2+1)^{5/2}}{x}\right)$$

$\downarrow$ 222

$$3c^2d^3\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{4}(c^2x^2+1)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2}\right.\right.\right.$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)-\frac{(c^2x^2+1)^{5/2}}{x}\right)$$

$\downarrow$ 6216

$$3c^2d^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int \sqrt{c^2x^2+1}dx+\frac{1}{4}(c^2x^2+1)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}(c^2x^2+1)(a+\text{barcs}\right.$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)-\frac{(c^2x^2+1)^{5/2}}{x}\right)$$

$\downarrow$ 211

3.26. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}dx$

$$3c^2d^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{222}$$

$$3c^2d^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{\text{arcsin}}{2c}\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{6190}$$

$$3c^2d^3\left(\frac{\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx)\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$3c^2d^3\left(-\frac{\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$
$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{3042}$$

3.26. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx$

$$3c^2d^3\left(-\frac{\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsi}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$3c^2d^3\left(\frac{i\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsi}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{4201}$$

$$3c^2d^3\left(\frac{i\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{2620}$$

$$3c^2d^3\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+bar}{}}\right.\right.\right.}{b}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{2715}$$

---

3.26. $\int\frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))}{x^3}dx$

$$3c^2d^3\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+ba\right.\right.}{b}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$3c^2d^3\left(\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}\right.}{b}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{2x^2}+$$

$$\frac{1}{2}bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)-\frac{\left(c^2x^2+1\right)^{5/2}}{x}\right)$$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
-1/2*(d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/x^2 + (b*c*d^3*(-((1 + c^2
*x^2)^(5/2)/x) + 5*c^2*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^
2])/2 + ArcSinh[c*x]/(2*c)))/4)))/2 + 3*c^2*d^3*(((1 + c^2*x^2)*(a + b*Arc
Sinh[c*x]))/2 + ((1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 - (b*c*((x*Sqrt[1
 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/2 - (b*c*((x*(1 + c^2*x^2)^(3/2))/4
+ (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/4 + (I*((-1/2*I)*
(a + b*ArcSinh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2
*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcS
inh[c*x]])/4)))/b)
```

## 3.26. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx$

#### 3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## 3.26. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6217
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1
 + c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

## 3.26.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| derivativedivides | $c^2\left(d^3a\left(\frac{c^4x^4}{4}+\frac{3c^2x^2}{2}+3\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+\frac{d^3b}{2}+\frac{21b\,d^3\,\mathrm{arcsinh}(cx)}{32}-\frac{3d^3b\,\mathrm{arcsinh}(cx)^2}{2}+3d^3b$ |
| default | $c^2\left(d^3a\left(\frac{c^4x^4}{4}+\frac{3c^2x^2}{2}+3\ln\left(cx\right)-\frac{1}{2c^2x^2}\right)+\frac{d^3b}{2}+\frac{21b\,d^3\,\mathrm{arcsinh}(cx)}{32}-\frac{3d^3b\,\mathrm{arcsinh}(cx)^2}{2}+3d^3b$ |
| parts | $d^3a\left(\frac{c^6x^4}{4}+\frac{3c^4x^2}{2}-\frac{1}{2x^2}+3c^2\ln\left(x\right)\right)+\frac{d^3b\,c^2}{2}+3d^3b\,c^2\,\mathrm{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^3*a*(1/4*c^4*x^4+3/2*c^2*x^2+3*ln(c*x)-1/2/c^2/x^2)+1/2*d^3*b+21/32
*b*d^3*arcsinh(c*x)-3/2*d^3*b*arcsinh(c*x)^2+3*d^3*b*polylog(2,c*x+(c^2*x^
2+1)^(1/2))+3*d^3*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-1/16*d^3*b*c^3*x^3*(
c^2*x^2+1)^(1/2)-21/32*b*c*d^3*x*(c^2*x^2+1)^(1/2)+1/4*d^3*b*arcsinh(c*x)*
c^4*x^4+3/2*d^3*b*arcsinh(c*x)*c^2*x^2-1/2*d^3*b*arcsinh(c*x)/c^2/x^2-1/2*
d^3*b/c/x*(c^2*x^2+1)^(1/2)+3*d^3*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2
))+3*d^3*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2)))
```

### 3.26.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c
^6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x^3,
 x)
```

### 3.26.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = d^3\left(\int \frac{a}{x^3}\,dx+\int \frac{3ac^2}{x}\,dx+\int 3ac^4x\,dx+\int ac^6x^3\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{3bc^2\operatorname{asinh}(cx)}{x}\,dx + \int 3bc^4x\operatorname{asinh}(cx)\,dx + \int bc^6x^3\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**3,x)
```

output
```
d**3*(Integral(a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(3*a*c**4*x,
 x) + Integral(a*c**6*x**3, x) + Integral(b*asinh(c*x)/x**3, x) + Integral
(3*b*c**2*asinh(c*x)/x, x) + Integral(3*b*c**4*x*asinh(c*x), x) + Integral
(b*c**6*x**3*asinh(c*x), x))
```

### 3.26.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 + 3*a*c^2*d^3*log(x) - 1/2*b*d^3*(sq
rt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d^3/x^2 + integrate(b*c^6*
d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x*log(c*x + sqrt(c^2*x^
2 + 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.26.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^3}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3, x)
```

# 3.27 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.27.1 Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2}-\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} -\frac{1}{9}bc^3d^3(1+c^2x^2)^{3/2}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3} -\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x} +3c^4d^3x(a+b\text{arcsinh}(cx)) +\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx)) -\frac{17}{6}bc^3d^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)$$

output
```
-1/9*b*c^3*d^3*(c^2*x^2+1)^(3/2)-1/3*d^3*(a+b*arcsinh(c*x))/x^3-3*c^2*d^3*
(a+b*arcsinh(c*x))/x+3*c^4*d^3*x*(a+b*arcsinh(c*x))+1/3*c^6*d^3*x^3*(a+b*a
rcsinh(c*x))-17/6*b*c^3*d^3*arctanh((c^2*x^2+1)^(1/2))-8/3*b*c^3*d^3*(c^2*
x^2+1)^(1/2)-1/6*b*c*d^3*(c^2*x^2+1)^(1/2)/x^2
```

### 3.27.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))}{x^4} dx = \frac{d^3(-6a-54ac^2x^2+54ac^4x^4+6ac^6x^6-3bcx\sqrt{1+c^2x^2}-50bc^3x^3\sqrt{1+c^2x^2}-2bc^5x^5\sqrt{1+c^2x^2}+6b(-}{18x^3}$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
(d^3*(-6*a - 54*a*c^2*x^2 + 54*a*c^4*x^4 + 6*a*c^6*x^6 - 3*b*c*x*Sqrt[1 +
c^2*x^2] - 50*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b*c^5*x^5*Sqrt[1 + c^2*x^2]
+ 6*b*(-1 - 9*c^2*x^2 + 9*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 51*b*c^3*x^3*L
og[x] - 51*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(18*x^3)
```

### 3.27.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6218, 27, 2331, 2124, 27, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3 (a+b\text{arcsinh}(cx))}{x^4} dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int -\frac{d^3(-c^6x^6-9c^4x^4+9c^2x^2+1)}{3x^3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd^3\int \frac{-c^6x^6-9c^4x^4+9c^2x^2+1}{x^3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{6}bcd^3\int \frac{-c^6x^6-9c^4x^4+9c^2x^2+1}{x^4\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{6}bcd^3\left(-\int -\frac{-2x^4c^6-18x^2c^4+17c^2}{2x^2\sqrt{c^2x^2+1}}dx^2-\frac{\sqrt{c^2x^2+1}}{x^2}\right)+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}bcd^3\left(\frac{1}{2}\int \frac{-2x^4c^6-18x^2c^4+17c^2}{x^2\sqrt{c^2x^2+1}}dx^2-\frac{\sqrt{c^2x^2+1}}{x^2}\right)+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{1192}$$

$$\frac{1}{6}bcd^3\left(\frac{\int -\frac{-2c^6x^8-14c^6x^4+33c^6}{1-x^4}d\sqrt{c^2x^2+1}}{c^4}-\frac{\sqrt{c^2x^2+1}}{x^2}\right)+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}bcd^3\left(-\frac{\int \frac{-2c^6x^8-14c^6x^4+33c^6}{1-x^4}d\sqrt{c^2x^2+1}}{c^4}-\frac{\sqrt{c^2x^2+1}}{x^2}\right)+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{1467}$$

$$\frac{1}{6}bcd^3\left(-\frac{\int \left(2x^4c^6+\frac{17c^6}{1-x^4}+16c^6\right)d\sqrt{c^2x^2+1}}{c^4}-\frac{\sqrt{c^2x^2+1}}{x^2}\right)+\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^6d^3x^3(a+b\text{arcsinh}(cx))+3c^4d^3x(a+b\text{arcsinh}(cx))-\frac{3c^2d^3(a+b\text{arcsinh}(cx))}{x}-\frac{d^3(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}bcd^3\left(\frac{-17c^6\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{2}{3}c^6x^6-16c^6\sqrt{c^2x^2+1}}{c^4}-\frac{\sqrt{c^2x^2+1}}{x^2}\right)$$

---

3.27. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))}{x^4}\,dx$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*(d^3*(a + b*ArcSinh[c*x]))/x^3 - (3*c^2*d^3*(a + b*ArcSinh[c*x]))/x +
 3*c^4*d^3*x*(a + b*ArcSinh[c*x]) + (c^6*d^3*x^3*(a + b*ArcSinh[c*x]))/3 +
 (b*c*d^3*(-(Sqrt[1 + c^2*x^2]/x^2) + ((-2*c^6*x^6)/3 - 16*c^6*Sqrt[1 + c^
2*x^2] - 17*c^6*ArcTanh[Sqrt[1 + c^2*x^2]])/c^4))/6
```

### 3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.27.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| parts | $d^3a\left(\frac{c^6x^3}{3}+3c^4x-\frac{3c^2}{x}-\frac{1}{3x^3}\right)+d^3b\,c^3\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+3\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\right.$ |
| derivativedivides | $c^3\left(d^3a\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+3\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\right.\right.$ |
| default | $c^3\left(d^3a\left(\frac{c^3x^3}{3}+3cx-\frac{1}{3c^3x^3}-\frac{3}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+3\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}-\right.\right.$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/3*c^6*x^3+3*c^4*x-3*c^2/x-1/3/x^3)+d^3*b*c^3*(1/3*arcsinh(c*x)*c^
3*x^3+3*arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-3*arcsinh(c*x)/c/x-1/9*c
^2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1
/2)-17/6*arctanh(1/(c^2*x^2+1)^(1/2)))
```

### 3.27.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.66

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))}{x^4}\,dx$$
$$= \frac{6\,ac^6d^3x^6 + 54\,ac^4d^3x^4 - 51\,bc^3d^3x^3 \log\left(-cx + \sqrt{c^2x^2+1} + 1\right) + 51\,bc^3d^3x^3 \log\left(-cx + \sqrt{c^2x^2+1} - 1\right)}{}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/18*(6*a*c^6*d^3*x^6 + 54*a*c^4*d^3*x^4 - 51*b*c^3*d^3*x^3*log(-c*x + sqr
t(c^2*x^2 + 1) + 1) + 51*b*c^3*d^3*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) -
 54*a*c^2*d^3*x^2 - 6*(b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3*log(-c*x + s
qrt(c^2*x^2 + 1)) - 6*a*d^3 + 6*(b*c^6*d^3*x^6 + 9*b*c^4*d^3*x^4 - 9*b*c^2
*d^3*x^2 - (b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3 - b*d^3)*log(c*x + sqrt
(c^2*x^2 + 1)) - (2*b*c^5*d^3*x^5 + 50*b*c^3*d^3*x^3 + 3*b*c*d^3*x)*sqrt(c
^2*x^2 + 1))/x^3
```

### 3.27.6 Sympy [F]

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))}{x^4}\,dx = d^3\left(\int 3ac^4\,dx + \int \frac{a}{x^4}\,dx + \int \frac{3ac^2}{x^2}\,dx + \int ac^6x^2\,dx + \int 3bc^4\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{3bc^2\operatorname{asinh}(cx)}{x^2}\,dx + \int bc^6x^2\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**4,x)
```

output
```
d**3*(Integral(3*a*c**4, x) + Integral(a/x**4, x) + Integral(3*a*c**2/x**2
, x) + Integral(a*c**6*x**2, x) + Integral(3*b*c**4*asinh(c*x), x) + Integ
ral(b*asinh(c*x)/x**4, x) + Integral(3*b*c**2*asinh(c*x)/x**2, x) + Integr
al(b*c**6*x**2*asinh(c*x), x))
```

### 3.27.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.20

$$\int \frac{(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx$$
$$=\frac{1}{3}ac^6d^3x^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^6d^3+3\,ac^4d^3x$$
$$+3\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bc^3d^3-3\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d^3$$
$$+\frac{1}{6}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)c-\frac{2\operatorname{arsinh}(cx)}{x^3}\right)bd^3-\frac{3\,ac^2d^3}{x}-\frac{ad^3}{3\,x^3}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/3*a*c^6*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arcsinh(c*
x) - sqrt(c^2*x^2 + 1))*b*c^3*d^3 - 3*(c*arcsinh(1/(c*abs(x))) + arcsinh(c
*x)/x)*b*c^2*d^3 + 1/6*((c^2*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2
)*c - 2*arcsinh(c*x)/x^3)*b*d^3 - 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

### 3.27.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4, x)
```

# 3.28 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$



## 3.28.1 Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b(1+c^2x^2)^{3/2}}{9c^5d} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d} + \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d}$$

output
```
-1/9*b*(c^2*x^2+1)^(3/2)/c^5/d-x*(a+b*arcsinh(c*x))/c^4/d+1/3*x^3*(a+b*arc
sinh(c*x))/c^2/d+2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d-
I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d+I*b*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/c^5/d+4/3*b*(c^2*x^2+1)^(1/2)/c^5/d
```

### 3.28.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$$
$$= \frac{-9acx + 3ac^3x^3 + 11b\sqrt{1+c^2x^2} - bc^2x^2\sqrt{1+c^2x^2} - 9bcx\text{arcsinh}(cx) + 3bc^3x^3\text{arcsinh}(cx) + 9a\arctan(c}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
(-9*a*c*x + 3*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] - b*c^2*x^2*Sqrt[1 + c^2*
x^2] - 9*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x] +
 (9*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*b*ArcSinh[c*x]*Log
[1 + I*E^ArcSinh[c*x]] - (9*I)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b
*PolyLog[2, I*E^ArcSinh[c*x]])/(9*c^5*d)
```

### 3.28.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6227, 27, 243, 53, 2009, 6227, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{c^2dx^2+d}\,dx$$
$$\Big\downarrow \text{6227}$$
$$-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{d(c^2x^2+1)}dx}{c^2} - \frac{b\int \frac{x^3}{\sqrt{c^2x^2+1}}dx}{3cd} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d} - \frac{b\int \frac{x^3}{\sqrt{c^2x^2+1}}dx}{3cd} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}$$
$$\Big\downarrow \text{243}$$
$$-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d} - \frac{b\int \frac{x^2}{\sqrt{c^2x^2+1}}dx^2}{6cd} + \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}$$

$$\Big\downarrow \ 53$$

$$-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d}-\frac{b\int \left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2}{6cd}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \ 6227$$

$$-\frac{-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \ 241$$

$$-\frac{-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \ 6204$$

$$-\frac{-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \ 4668$$

3.28. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}dx$

$$-\frac{-\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3}+\frac{x(a+b}{}}{c^2d}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

$\big\downarrow$ 2715

$$-\frac{-\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b}{c^3}}{c^2d}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

$\big\downarrow$ 2838

$$-\frac{-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x}}{c^3}}{c^2d}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6cd}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
-1/6*(b*((-2*Sqrt[1 + c^2*x^2])/c^4 + (2*(1 + c^2*x^2)^(3/2))/(3*c^4)))/(c
*d) + (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*d) - (-((b*Sqrt[1 + c^2*x^2])/c^3)
 + (x*(a + b*ArcSinh[c*x]))/c^2 - (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh
[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[
c*x]])/c^3)/(c^2*d)
```

## 3.28. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$

### 3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.28. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.28.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{c^3x^3}{3}-cx+\arctan(cx)\right)}{d}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\operatorname{arcsinh}(cx)cx+\operatorname{arcsinh}(cx)\arctan(cx)-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}+\frac{11\sqrt{c^2x^2+1}}{9}+\arctan(cx)\ldots\right.}{d}}{c^5}$ |
| default | $\frac{\frac{a\left(\frac{c^3x^3}{3}-cx+\arctan(cx)\right)}{d}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\operatorname{arcsinh}(cx)cx+\operatorname{arcsinh}(cx)\arctan(cx)-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}+\frac{11\sqrt{c^2x^2+1}}{9}+\arctan(cx)\ldots\right.}{d}}{c^5}$ |
| parts | $\frac{a\left(\frac{\frac{1}{3}x^3c^2-x}{c^4}+\frac{\arctan(cx)}{c^5}\right)}{d}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\operatorname{arcsinh}(cx)cx+\operatorname{arcsinh}(cx)\arctan(cx)-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}+\frac{11\sqrt{c^2x^2+1}}{9}+\ldots\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a/d*(1/3*c^3*x^3-c*x+arctan(c*x))+b/d*(1/3*arcsinh(c*x)*c^3*x^3-arc
sinh(c*x)*c*x+arcsinh(c*x)*arctan(c*x)-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)+11/9*
(c^2*x^2+1)^(1/2)+arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c
*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))+I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

### 3.28.5 Fricas [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{c^2dx^2+d}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^2*d*x^2 + d), x)
```

### 3.28.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\frac{\int \frac{ax^4}{c^2x^2+1}\,dx+\int \frac{bx^4\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

output
```
(Integral(a*x**4/(c**2*x**2 + 1), x) + Integral(b*x**4*asinh(c*x)/(c**2*x*
*2 + 1), x))/d
```

### 3.28.7 Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{c^2dx^2+d}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/3*a*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + b*integrate(x^4*
log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

### 3.28.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \int \frac{x^4\,(a + b\,\text{asinh}(c\,x))}{d\,c^2\,x^2 + d}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

# 3.29 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$



## 3.29.1 Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{bx\sqrt{1+c^2x^2}}{4c^3d}+\frac{b\text{arcsinh}(cx)}{4c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d}-\frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d}-\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d}$$

output
```
1/4*b*arcsinh(c*x)/c^4/d+1/2*x^2*(a+b*arcsinh(c*x))/c^2/d+1/2*(a+b*arcsinh
(c*x))^2/b/c^4/d-(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-
1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/4*b*x*(c^2*x^2+1)^(1/2
)/c^3/d
```

### 3.29.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx =$$
$$-\frac{-2ac^2x^2+bcx\sqrt{1+c^2x^2}-b\text{arcsinh}(cx)-2bc^2x^2\text{arcsinh}(cx)-2b\text{arcsinh}(cx)^2+4b\text{arcsinh}(cx)\log\left(1\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
-1/4*(-2*a*c^2*x^2 + b*c*x*Sqrt[1 + c^2*x^2] - b*ArcSinh[c*x] - 2*b*c^2*x^
2*ArcSinh[c*x] - 2*b*ArcSinh[c*x]^2 + 4*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] + 4*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])
/c] + 2*a*Log[1 + c^2*x^2] + 4*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
 + 4*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(c^4*d)
```

### 3.29.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6227, 27, 262, 222, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{c^2dx^2+d}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{d(c^2x^2+1)}dx}{c^2}-\frac{b\int \frac{x^2}{\sqrt{c^2x^2+1}}dx}{2cd}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d}-\frac{b\int \frac{x^2}{\sqrt{c^2x^2+1}}dx}{2cd}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{2cd}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

$$\Big\downarrow \text{6212}$$

$$-\frac{\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

$$\Big\downarrow \text{4201}$$

$$\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

$$\Big\downarrow \text{2715}$$

---

3.29. $\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arcs}}{2}\right.}{c^4d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

$\downarrow$ 2838

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4d}+$$

$$\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2cd}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
(x^2*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (b*((x*Sqrt[1 + c^2*x^2])/(2*c^2) -
 ArcSinh[c*x]/(2*c^3)))/(2*c*d) + (I*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b
+ (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog
[2, -E^(2*ArcSinh[c*x])])/4)))/(c^4*d)
```

### 3.29.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.29. $\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$

## 3.29.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\,\mathrm{arcsinh}(cx)^2}{2d}+\frac{b\,\mathrm{arcsinh}(cx)c^2x^2}{2d}-\frac{bcx\sqrt{c^2x^2+1}}{4d}+\frac{b\,\mathrm{arcsinh}(cx)}{4d}-\frac{b\,\mathrm{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.}{d}}{c^4}$ |
| default | $\frac{\frac{a\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\,\mathrm{arcsinh}(cx)^2}{2d}+\frac{b\,\mathrm{arcsinh}(cx)c^2x^2}{2d}-\frac{bcx\sqrt{c^2x^2+1}}{4d}+\frac{b\,\mathrm{arcsinh}(cx)}{4d}-\frac{b\,\mathrm{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.}{d}}{c^4}$ |
| parts | $\frac{a\left(\frac{x^2}{2c^2}-\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d}+\frac{b\,\mathrm{arcsinh}(cx)^2}{2d\,c^4}+\frac{b\,\mathrm{arcsinh}(cx)x^2}{2d\,c^2}-\frac{bx\sqrt{c^2x^2+1}}{4c^3d}+\frac{b\,\mathrm{arcsinh}(cx)}{4c^4d}-\frac{b\,\mathrm{arcsinh}(cx)\ln\left(1\right.}{}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d*(1/2*c^2*x^2-1/2*ln(c^2*x^2+1))+1/2*b/d*arcsinh(c*x)^2+1/2*b/d*
arcsinh(c*x)*c^2*x^2-1/4*b/d*c*x*(c^2*x^2+1)^(1/2)+1/4*b/d*arcsinh(c*x)-b/
d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*polylog(2,-(c*x+(c^2*
x^2+1)^(1/2))^2)/d)
```

## 3.29.5 Fricas [F]

$$\int \frac{x^3(a+b\mathrm{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^3}{c^2dx^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^2*d*x^2 + d), x)
```

### 3.29.6 Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{\int \frac{ax^3}{c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

output
```
(Integral(a*x**3/(c**2*x**2 + 1), x) + Integral(b*x**3*asinh(c*x)/(c**2*x*
*2 + 1), x))/d
```

### 3.29.7 Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^3}{c^2dx^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) - 1/8*b*((2*c^2*x^2 - log(c
^2*x^2 + 1))^2 - 4*(c^2*x^2 - log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) - 2*log(c^2*x^2 + 1))/(c^4*d) - 8*integrate(-1/2*(c^2*x^2 - log(c^2*x^2
+ 1))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqrt(c^2*x^2 + 1)), x))
```

### 3.29.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{x^3\,(a+b\,\text{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

# 3.30 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$



## 3.30.1 Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = -\frac{b\sqrt{1+c^2x^2}}{c^3d}+\frac{x(a+b\text{arcsinh}(cx))}{c^2d} - \frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} + \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} - \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d}$$

output
```
x*(a+b*arcsinh(c*x))/c^2/d-2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/
2))/c^3/d+I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-I*b*polylog(2,I*
(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-b*(c^2*x^2+1)^(1/2)/c^3/d
```

## 3.30.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \frac{acx-b\sqrt{1+c^2x^2}+bcx\text{arcsinh}(cx)-a\arctan(cx)-ib\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx)}\right)+ib\text{arcsinh}(cx)\,\text{l}}{c^3d}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
(a*c*x - b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - I*b*Ar
cSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] + I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*PolyLog[2, I*E^ArcSinh
[c*x]])/(c^3*d)
```

### 3.30.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6227, 27, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{d(c^2x^2+1)}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x(a+b\text{arcsinh}(cx))}{c^2d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2d}-\frac{b\int \frac{x}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x(a+b\text{arcsinh}(cx))}{c^2d}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2d}+\frac{x(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

$$\Big\downarrow \text{6204}$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3d}+\frac{x(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3d}+\frac{x(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+ba}{c^3d}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

$\big\downarrow$ 2715

$$-\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arcta}{c^3d}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

$\big\downarrow$ 2838

$$-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d}+$$
$$\frac{x(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}}{c^3d}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
-((b*Sqrt[1 + c^2*x^2])/(c^3*d)) + (x*(a + b*ArcSinh[c*x]))/(c^2*d) - (2*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh
[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d)
```

### 3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.30. $\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx$

## 3.30.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.57

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a(cx-\arctan(cx))}{d}+\frac{b\left(-\operatorname{arcsinh}(cx)\arctan(cx)+\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right.\right.}{d}}{c^3}$ |
| default | $\frac{\frac{a(cx-\arctan(cx))}{d}+\frac{b\left(-\operatorname{arcsinh}(cx)\arctan(cx)+\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right.\right.}{d}}{c^3}$ |
| parts | $\frac{a\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{d}+\frac{b\left(-\operatorname{arcsinh}(cx)\arctan(cx)+\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+\arctan(\right.}{d\,c^3}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d*(c*x-arctan(c*x))+b/d*(-arcsinh(c*x)*arctan(c*x)+arcsinh(c*x)*c
*x-(c^2*x^2+1)^(1/2)-arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+arcta
n(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*dilog(1+I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))-I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

## 3.30.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{c^2dx^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^2*d*x^2 + d), x)
```

## 3.30.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\frac{\int \frac{ax^2}{c^2x^2+1}\,dx+\int \frac{bx^2\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

output
```
(Integral(a*x**2/(c**2*x**2 + 1), x) + Integral(b*x**2*asinh(c*x)/(c**2*x*
*2 + 1), x))/d
```

### 3.30.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{c^2dx^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
a*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + b*integrate(x^2*log(c*x + sqrt(c^2*x
^2 + 1))/(c^2*d*x^2 + d), x)
```

### 3.30.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{c^2dx^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d), x)
```

### 3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{x^2\,(a+b\,\text{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

# 3.31 $\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$



## 3.31.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=-\frac{(a+b\text{arcsinh}(cx))^2}{2bc^2d}+\frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d}+\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$

output
```
-1/2*(a+b*arcsinh(c*x))^2/b/c^2/d+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)
^(1/2))^2)/c^2/d+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

## 3.31.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(73) = 146.

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.29

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx=-\frac{b\text{arcsinh}(cx)^2}{2c^2d}+\frac{b\text{arcsinh}(cx)\log\left(1-\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d}+\frac{b\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d}+\frac{a\log(1+c^2x^2)}{2c^2d}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)}{c^2d}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
-1/2*(b*ArcSinh[c*x]^2)/(c^2*d) + (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^Ar
cSinh[c*x])/c])/(c^2*d) + (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*
x])/c])/(c^2*d) + (a*Log[1 + c^2*x^2])/(2*c^2*d) + (b*PolyLog[2, -((Sqrt[-
c^2]*E^ArcSinh[c*x])/c)])/(c^2*d) + (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*
x])/c])/(c^2*d)
```

### 3.31.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \text{6212}$$

$$\frac{\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^2d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^2d}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^2d}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^2d}$$

$\downarrow$ 2715

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arc}}{}\right.}{c^2d}$$

$\downarrow$ 2838

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^2d}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
((-I)*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*
Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/
(c^2*d)
```

### 3.31.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.31.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2}\right)}{d}}{c^2}$ | 84 |
| default | $\frac{\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2}\right)}{d}}{c^2}$ | 84 |
| parts | $\frac{a\ln\left(c^2x^2+1\right)}{2d\,c^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2}\right)}{d\,c^2}$ | 86 |

input
```
int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*a/d*ln(c^2*x^2+1)+b/d*(-1/2*arcsinh(c*x)^2+arcsinh(c*x)*ln(1+(c
*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)))
```

### 3.31.5 Fricas [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x}{c^2dx^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arcsinh(c*x) + a*x)/(c^2*d*x^2 + d), x)
```

### 3.31.6 Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\frac{\int \frac{ax}{c^2x^2+1}\,dx+\int \frac{bx\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

output
```
(Integral(a*x/(c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(c**2*x**2 + 1
), x))/d
```

### 3.31.7 Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x}{c^2dx^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/8*b*((log(c^2*x^2 + 1)^2 - 4*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 +
1)))/(c^2*d) + 8*integrate(1/2*log(c^2*x^2 + 1)/(c^4*d*x^3 + c^2*d*x + (c^
3*d*x^2 + c*d)*sqrt(c^2*x^2 + 1)), x)) + 1/2*a*log(c^2*d*x^2 + d)/(c^2*d)
```

### 3.31.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{c^2dx^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d), x)
```

### 3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{x\,(a+b\,\text{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

output
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

# 3.32 $\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx$



## 3.32.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx=\frac{2(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

output
```
2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-I*b*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/d+I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

## 3.32.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.93

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx=-\frac{c\left(a\sqrt{-c^2}\arctan(cx)-bc\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)+bc\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\text{arcsinh}(cx)}}{c}\right)+b\right.}{\left(-c^2\right)^{3/2}d}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

output
```
-((c*(a*Sqrt[-c^2]*ArcTan[c*x] - b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x
])/Sqrt[-c^2]] + b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] +
 b*c*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*c*PolyLog[2, (Sqrt[-c^2
]*E^ArcSinh[c*x])/c]))/((-c^2)^(3/2)*d))
```

## 3.32.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \text{6204}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{cd}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+\text{barc}}{cd}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan}{cd}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

output
```
(2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^Arc
Sinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d)
```

#### 3.32.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

### 3.32.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\arctan(cx)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\arctan(cx)+\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+i\,\mathrm{dil}\ldots\right.}{d}}{c}$ |
| default | $\frac{\frac{a\arctan(cx)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\arctan(cx)+\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)+i\,\mathrm{dil}\ldots\right.}{d}}{c}$ |
| parts | $\frac{a\arctan(cx)}{dc}+\frac{b\left(\operatorname{arcsinh}(cx)\arctan(cx)+\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\ldots\right.\right.}{dc}$ |

input
```
int((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d*arctan(c*x)+b/d*(arcsinh(c*x)*arctan(c*x)+arctan(c*x)*ln(1+I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I
*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))))
```

### 3.32.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+c^2dx^2}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

### 3.32.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+c^2dx^2}\,dx=\frac{\int \frac{a}{c^2x^2+1}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

output
```
(Integral(a/(c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**2*x**2 + 1), x
))/d
```

### 3.32.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{c^2dx^2 + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x) + a*arctan(c*
x)/(c*d)
```

### 3.32.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{c^2dx^2 + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

### 3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{d + c^2dx^2}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{d\,c^2\,x^2 + d}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2), x)
```

# 3.33 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx$



## 3.33.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx = -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

output
```
-2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*polylog(2
,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/
d
```

### 3.33.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(61) = 122.

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.39

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx = -\frac{a\operatorname{arcsinh}(cx)}{d} - \frac{b\operatorname{arcsinh}(cx)\log\left(1-\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} - \frac{b\operatorname{arcsinh}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} + \frac{a\log\left(1-e^{2\operatorname{arcsinh}(cx)}\right)}{d} + \frac{b\operatorname{arcsinh}(cx)\log\left(1-e^{2\operatorname{arcsinh}(cx)}\right)}{d} - \frac{a\log(1+c^2x^2)}{2d} - \frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} - \frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\operatorname{arcsinh}(cx)}}{c}\right)}{d} + \frac{b\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(cx)}\right)}{2d}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

output
```
-((a*ArcSinh[c*x])/d) - (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c])/d - (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (a*L
og[1 - E^(2*ArcSinh[c*x])])/d + (b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])
])/d - (a*Log[1 + c^2*x^2])/(2*d) - (b*PolyLog[2, -((Sqrt[-c^2]*E^ArcSinh[
c*x])/c)])/d - (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (b*PolyLo
g[2, E^(2*ArcSinh[c*x])])/(2*d)
```

### 3.33.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x(c^2dx^2+d)}\,dx$$

$$\Big\downarrow \text{6214}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{2i\int (a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{4670}$$

$$\frac{2i\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a-}{d}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}+i\right.}{d}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right)}{d}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

output
```
((2*I)*(I*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyL
og[2, -E^(2*ArcSinh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])]))/d
```

---

3.33. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)}\,dx$

### 3.33.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

### 3.33.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.64

| method | result |
|---|---|
| parts | $\frac{a\left(-\frac{\ln\left(c^2x^2+1\right)}{2}+\ln(x)\right)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-\operatorname{arcsinh}(cx)\ln\left(1+(c\right.\right.}{}$ |
| derivativedivides | $\frac{a\left(\ln(cx)-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-\operatorname{arcsinh}(cx)\ln\left(1+(cx\right.\right.}{}$ |
| default | $\frac{a\left(\ln(cx)-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-\operatorname{arcsinh}(cx)\ln\left(1+(cx\right.\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*(-1/2*ln(c^2*x^2+1)+ln(x))+b/d*(arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2
))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*ln(1-c*x-(c
^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

### 3.33.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(d+c^2dx^2\right)}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

### 3.33.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\,(d + c^2dx^2)}\,dx = \frac{\int \frac{a}{c^2x^3+x}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^2x^3+x}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d),x)
```

output
```
(Integral(a/(c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**2*x**3 + x), x
))/d
```

### 3.33.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\,(d + c^2dx^2)}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c^2*
x^2 + 1))/(c^2*d*x^3 + d*x), x)
```

### 3.33.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\,(d + c^2dx^2)}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x), x)
```

### 3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(d+c^2dx^2)}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2+d\right)}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)), x)
```

# 3.34 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx$



## 3.34.1 Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx = -\frac{a+b\text{arcsinh}(cx)}{dx} - \frac{2c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} - \frac{bc\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d} + \frac{ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} - \frac{ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d}$$

output
```
(-a-b*arcsinh(c*x))/d/x-2*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))/d-b*c*arctanh((c^2*x^2+1)^(1/2))/d+I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))/d-I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d
```

## 3.34.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.80

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx = -\frac{a+b\text{arcsinh}(cx)+acx\arctan(cx)+bcx\text{arctanh}\left(\sqrt{1+c^2x^2}\right)+b\sqrt{-c^2}x\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

output
```
-((a + b*ArcSinh[c*x] + a*c*x*ArcTan[c*x] + b*c*x*ArcTanh[Sqrt[1 + c^2*x^2
]] + b*Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] -
b*Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*Sqr
t[-c^2]*x*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + b*Sqrt[-c^2]*x*PolyL
og[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(d*x))
```

### 3.34.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6224, 27, 243, 73, 221, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2dx^2+d)}dx$$

$$\Big\downarrow \text{6224}$$

$$c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{d(c^2x^2+1)}dx\right)+\frac{bc\int \frac{1}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{a+b\text{arcsinh}(cx)}{dx}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d}+\frac{bc\int \frac{1}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{a+b\text{arcsinh}(cx)}{dx}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d}+\frac{bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2}{2d}-\frac{a+b\text{arcsinh}(cx)}{dx}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d}+\frac{b\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{cd}-\frac{a+b\text{arcsinh}(cx)}{dx}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d}-\frac{a+b\text{arcsinh}(cx)}{dx}-\frac{b\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d}$$

$$\Big\downarrow \text{6204}$$

$$-\frac{c\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d}-\frac{a+b\text{arcsinh}(cx)}{dx}-\frac{b c\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d}$$

$\Big\downarrow$ 3042

$$-\frac{c\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{d}-\frac{a+b\text{arcsinh}(cx)}{dx}-\frac{b c\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d}$$

$\Big\downarrow$ 4668

$$-\frac{c\left(-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+\right.}{d}$$
$$\frac{a+b\text{arcsinh}(cx)}{dx}-\frac{b c\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d}$$

$\Big\downarrow$ 2715

$$-\frac{c\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\,\text{arc}\right.}{d}$$
$$\frac{a+b\text{arcsinh}(cx)}{dx}-\frac{b c\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d}$$

$\Big\downarrow$ 2838

$$-\frac{c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right)}{d}-$$
$$\frac{a+b\text{arcsinh}(cx)}{dx}-\frac{b c\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{d}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

output
```
-((a + b*ArcSinh[c*x])/(d*x)) - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d - (c*(2
*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSi
nh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]]))/d
```

3.34. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx$

#### 3.34.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.34. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx$

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.34.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.74

| method | result |
|---|---|
| parts | $\frac{a\left(-c\arctan(cx)-\frac{1}{x}\right)}{d}+\frac{bc\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\operatorname{arcsinh}(cx)\arctan(cx)-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)\right.}{d}$ |
| derivativedivides | $c\left(\frac{a\left(-\frac{1}{cx}-\arctan(cx)\right)}{d}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\operatorname{arcsinh}(cx)\arctan(cx)-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)\right.}{d}\right.$ |
| default | $c\left(\frac{a\left(-\frac{1}{cx}-\arctan(cx)\right)}{d}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\operatorname{arcsinh}(cx)\arctan(cx)-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)\right.}{d}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*(-c*arctan(c*x)-1/x)+b/d*c*(-arcsinh(c*x)/c/x-arcsinh(c*x)*arctan(c*x)
-arctanh(1/(c^2*x^2+1)^(1/2))-arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))+arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*dilog(1+I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))-I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))
```

### 3.34.5 Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2 dx^2)} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2 dx^2 + d)x^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^2*d*x^4 + d*x^2), x)
```

### 3.34.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2 dx^2)} \, dx = \frac{\int \frac{a}{c^2 x^4 + x^2} \, dx + \int \frac{b \operatorname{asinh}(cx)}{c^2 x^4 + x^2} \, dx}{d}$$

input
```
integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a/(c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c**2*x**4 + x*
*2), x))/d
```

### 3.34.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2 dx^2)} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2 dx^2 + d)x^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-a*(c*arctan(c*x)/d + 1/(d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/
(c^2*d*x^4 + d*x^2), x)
```

### 3.34.8 Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^2), x)
```

### 3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2(d+c^2dx^2)}\,dx = \int \frac{a+b\operatorname{asinh}(c\,x)}{x^2\,(d\,c^2\,x^2+d)}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)), x)
```

# 3.35 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx$



## 3.35.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{2dx}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}+\frac{2c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}+\frac{bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d}-\frac{bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d}$$

output
```
1/2*(-a-b*arcsinh(c*x))/d/x^2+2*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x
^2+1)^(1/2))^2)/d+1/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*
c^2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x
```

## 3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(113) = 226.

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.12

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx = \frac{-\frac{bc\sqrt{1+c^2x^2}}{x}-bc^2\text{arcsinh}(cx)^2-\frac{a+b\text{arcsinh}(cx)}{x^2}+\frac{c^2(a+b\text{arcsinh}(cx))^2}{b}+2bc^2\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

output
```
(-((b*c*Sqrt[1 + c^2*x^2])/x) - b*c^2*ArcSinh[c*x]^2 - (a + b*ArcSinh[c*x]
)/x^2 + (c^2*(a + b*ArcSinh[c*x])^2)/b + 2*b*c^2*ArcSinh[c*x]*Log[1 + (c*E
^ArcSinh[c*x])/Sqrt[-c^2]] + 2*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^Ar
cSinh[c*x])/c] + a*c^2*Log[1 + c^2*x^2] + 2*b*c^2*PolyLog[2, (c*E^ArcSinh[
c*x])/Sqrt[-c^2]] + 2*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - c^
2*(2*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*
ArcSinh[c*x])]))/(2*d)
```

### 3.35.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6224, 27, 242, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(c^2dx^2+d\right)}dx$$

$$\Big\downarrow \text{6224}$$

$$c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{dx\left(c^2x^2+1\right)}dx\right)+\frac{bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx}{2d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d}+\frac{bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx}{2d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}$$

$$\Big\downarrow \text{242}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

$$\Big\downarrow \text{6214}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

$$\Big\downarrow \text{5984}$$

$$-\frac{2c^2\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2c^2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2ic^2\int (a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{2ic^2\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)\right.}{d}$$
$$\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2ic^2\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right.}{d}$$
$$\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2ic^2\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right)}{d}-$$
$$\frac{a+b\text{arcsinh}(cx)}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2dx}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

output
```
-1/2*(b*c*Sqrt[1 + c^2*x^2])/(d*x) - (a + b*ArcSinh[c*x])/(2*d*x^2) - ((2*
I)*c^2*(I*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyL
og[2, -E^(2*ArcSinh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])]))/d
```

3.35. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx$

### 3.35.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.35. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.35.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| derivativedivides | $c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\left(-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\mathrm{pol}\right.}{}\right.$ |
| default | $c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b\left(-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\mathrm{pol}\right.}{}\right.$ |
| parts | $\frac{a\left(\frac{c^2\ln\left(c^2x^2+1\right)}{2}-\frac{1}{2x^2}-c^2\ln(x)\right)}{d}+\frac{bc^2\left(-\frac{cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)}{2c^2x^2}-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\mathrm{polylo}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/d*(-1/2/c^2/x^2-ln(c*x)+1/2*ln(c^2*x^2+1))+b/d*(-1/2*(c*x*(c^2*x^2+
1)^(1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2-arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))-polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)
^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh(c*x)*ln(1-c*x
-(c^2*x^2+1)^(1/2))-polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

### 3.35.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx = \int \frac{b\operatorname{arsinh}\left(cx\right)+a}{(c^2dx^2+d)x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

### 3.35.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx = \frac{\int \frac{a}{c^2x^5+x^3}\,dx + \int \frac{b\operatorname{asinh}\left(cx\right)}{c^2x^5+x^3}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d),x)
```

output
```
(Integral(a/(c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c**2*x**5 + x*
*3), x))/d
```

### 3.35.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx = \int \frac{b\operatorname{arsinh}\left(cx\right)+a}{(c^2dx^2+d)x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a + b*integrate(
log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

### 3.35.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^3), x)
```

### 3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\left(d\,c^2\,x^2+d\right)}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)), x)
```

# 3.36 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)}\,dx$



## 3.36.1 Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\text{arcsinh}(cx)}{3dx^3} + \frac{c^2(a+b\text{arcsinh}(cx))}{dx} + \frac{2c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} + \frac{7bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d} - \frac{ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} + \frac{ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d}$$

output
```
1/3*(-a-b*arcsinh(c*x))/d/x^3+c^2*(a+b*arcsinh(c*x))/d/x+2*c^3*(a+b*arcsin
h(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d+7/6*b*c^3*arctanh((c^2*x^2+1)^(1/2
))/d-I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+I*b*c^3*polylog(2,I*(
c*x+(c^2*x^2+1)^(1/2)))/d-1/6*b*c*(c^2*x^2+1)^(1/2)/d/x^2
```

### 3.36.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.58

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)}\,dx$$
$$=\frac{-2a+6ac^2x^2-bcx\sqrt{1+c^2x^2}-2b\text{arcsinh}(cx)+6bc^2x^2\text{arcsinh}(cx)+6ac^3x^3\arctan(cx)+7bc^3x^3\text{arctanh}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

output
```
(-2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + 6*b*c^2
*x^2*ArcSinh[c*x] + 6*a*c^3*x^3*ArcTan[c*x] + 7*b*c^3*x^3*ArcTanh[Sqrt[1 +
 c^2*x^2]] - 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/
Sqrt[-c^2]] + 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcS
inh[c*x])/c] + 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^
2]] - 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*d
*x^3)
```

### 3.36.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6224, 27, 243, 52, 73, 221, 6224, 243, 73, 221, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(c^2dx^2+d\right)}\,dx$$

$$\Big\downarrow \text{6224}$$

$$c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{dx^2\left(c^2x^2+1\right)}dx\right)+\frac{bc\int \frac{1}{x^3\sqrt{c^2x^2+1}}dx}{3d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(c^2x^2+1\right)}dx}{d}+\frac{bc\int \frac{1}{x^3\sqrt{c^2x^2+1}}dx}{3d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)}dx}{d}+\frac{bc\int \frac{1}{x^4\sqrt{c^2x^2+1}}dx^2}{6d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}$$

$$\Big\downarrow \; 52$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)}dx}{d}+\frac{bc\left(-\frac{1}{2}c^2\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)}dx}{d}+\frac{bc\left(-\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)}dx}{d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \; 6224$$

$$-\frac{c^2\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\sqrt{c^2x^2+1}}dx-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \; 243$$

$$-\frac{c^2\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^2\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)+\frac{b\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c}-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \; 221$$

$$-\frac{c^2\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

---

3.36. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)}\,dx$

$$\Big\downarrow \text{6204}$$

$$-\frac{c^2\left(-c\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^2\left(-c\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{d}-\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{c^2\left(-c\left(-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)\right.\right.}{d}$$

$$\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{c^2\left(-c\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.\right.}{d}$$

$$\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{c^2\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right)-\right.}{d}$$

$$\frac{a+b\text{arcsinh}(cx)}{3dx^3}+\frac{bc\left(c^2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)}{6d}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

---

## 3.36. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)}\,dx$

output
```
-1/3*(a + b*ArcSinh[c*x])/(d*x^3) + (b*c*(-(Sqrt[1 + c^2*x^2]/x^2) + c^2*A
rcTanh[Sqrt[1 + c^2*x^2]]))/(6*d) - (c^2*(-((a + b*ArcSinh[c*x])/x) - b*c*
ArcTanh[Sqrt[1 + c^2*x^2]] - c*(2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*
x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x
]])))/d
```

### 3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

### 3.36.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| derivativedivides | $c^3 \left( \frac{a\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}+\arctan(cx)\right)}{d} + \frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{\operatorname{arcsinh}(cx)}{cx}+\operatorname{arcsinh}(cx)\arctan(cx)-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}+\frac{7\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2}}\right.}{6}\right.}{}\right.$ |
| default | $c^3 \left( \frac{a\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}+\arctan(cx)\right)}{d} + \frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{\operatorname{arcsinh}(cx)}{cx}+\operatorname{arcsinh}(cx)\arctan(cx)-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}+\frac{7\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2}}\right.}{6}\right.}{}\right.$ |
| parts | $\frac{a\left(c^3\arctan(cx)-\frac{1}{3x^3}+\frac{c^2}{x}\right)}{d} + \frac{b\,c^3\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{\operatorname{arcsinh}(cx)}{cx}+\operatorname{arcsinh}(cx)\arctan(cx)-\frac{\sqrt{c^2x^2+1}}{6c^2x^2}+\frac{7\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+}}\right.}{6}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
c^3*(a/d*(-1/3/c^3/x^3+1/c/x+arctan(c*x))+b/d*(-1/3*arcsinh(c*x)/c^3/x^3+a
rcsinh(c*x)/c/x+arcsinh(c*x)*arctan(c*x)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)+7/6
*arctanh(1/(c^2*x^2+1)^(1/2))+arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))-arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

## 3.36.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{(c^2dx^2+d)x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^2*d*x^6 + d*x^4), x)
```

### 3.36.6 Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^4 (d + c^2dx^2)}\,dx = \frac{\int \frac{a}{c^2x^6+x^4}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^6+x^4}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d),x)
```

output
```
(Integral(a/(c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c**2*x**6 + x*
*4), x))/d
```

### 3.36.7 Maxima [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^4 (d + c^2dx^2)}\,dx = \int \frac{b\operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a + b*integrate(log(c*
x + sqrt(c^2*x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

### 3.36.8 Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^4 (d + c^2dx^2)}\,dx = \int \frac{b\operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^4), x)
```

### 3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)} \, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^4\,\left(d\,c^2\,x^2 + d\right)} \, dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)), x)
```

# 3.37 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$



## 3.37.1 Optimal result

Integrand size = 24, antiderivative size = 171

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx &= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}\\ &\quad+\frac{3x(a+b\text{arcsinh}(cx))}{2c^4d^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)}\\ &\quad-\frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d^2}\\ &\quad+\frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2c^5d^2}-\frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^5d^2}\end{aligned}$$

output
```
3/2*x*(a+b*arcsinh(c*x))/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x
^2+1)-3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^2+3/2*I*b*p
olylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3/2*I*b*polylog(2,I*(c*x+(c^2
*x^2+1)^(1/2)))/c^5/d^2+1/2*b/c^5/d^2/(c^2*x^2+1)^(1/2)-b*(c^2*x^2+1)^(1/2
)/c^5/d^2
```

### 3.37.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.57

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$$
$$= \frac{3acx+2ac^3x^3-b\sqrt{1+c^2x^2}-2bc^2x^2\sqrt{1+c^2x^2}+3bcx\text{arcsinh}(cx)+2bc^3x^3\text{arcsinh}(cx)-3a\arctan(cx)}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
(3*a*c*x + 2*a*c^3*x^3 - b*Sqrt[1 + c^2*x^2] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^
2] + 3*b*c*x*ArcSinh[c*x] + 2*b*c^3*x^3*ArcSinh[c*x] - 3*a*ArcTan[c*x] - 3
*a*c^2*x^2*ArcTan[c*x] - (3*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] -
(3*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)*b*ArcSinh[c
*x]*Log[1 + I*E^ArcSinh[c*x]] + (3*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + (3*I)*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (3*
I)*b*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^5*d^2*(1 + c^2*x^2))
```

### 3.37.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6225, 27, 243, 53, 2009, 6227, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^2}\,dx$$
$$\Big\downarrow \text{6225}$$
$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{d(c^2x^2+1)}dx}{2c^2d}+\frac{b\int \frac{x^3}{(c^2x^2+1)^{3/2}}dx}{2cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}$$
$$\Big\downarrow \text{27}$$
$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\int \frac{x^3}{(c^2x^2+1)^{3/2}}dx}{2cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}$$
$$\Big\downarrow \text{243}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\int \frac{x^2}{(c^2x^2+1)^{3/2}}dx^2}{4cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \ 53$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\int \left(\frac{1}{c^2\sqrt{c^2x^2+1}}-\frac{1}{c^2(c^2x^2+1)^{3/2}}\right)dx^2}{4cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \ 2009$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2d^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

$$\Big\downarrow \ 6227$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}\right)}{2c^2d^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

$$\Big\downarrow \ 241$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{2c^2d^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

$$\Big\downarrow \ 6204$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{2c^2d^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\left(-\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{2c^2d^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

---

3.37. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}dx$

$\Big\downarrow$ 4668

$$\frac{3\left(-\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3}+\frac{x(a+\dots}{}\right.}{2c^2d^2}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

$\Big\downarrow$ 2715

$$\frac{3\left(-\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+\dots}{c^3}\right.}{2c^2d^2}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

$\Big\downarrow$ 2838

$$\frac{3\left(-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2\dots}}{c\dots}\right.}{2c^2d^2}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4cd^2}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
(b*(2/(c^4*Sqrt[1 + c^2*x^2]) + (2*Sqrt[1 + c^2*x^2])/c^4))/(4*c*d^2) - (x
^3*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 + c^2*x^2)) + (3*(-((b*Sqrt[1 + c^2
*x^2])/c^3) + (x*(a + b*ArcSinh[c*x]))/c^2 - (2*(a + b*ArcSinh[c*x])*ArcTa
n[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I
*E^ArcSinh[c*x]])/c^3))/(2*c^2*d^2)
```

## 3.37. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$

### 3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.37. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.37.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| derivativedivides | $\frac{a\left(cx+\frac{cx}{2c^2x^2+2}-\frac{3\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\operatorname{arcsinh}(cx)cx+\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{1}{2\sqrt{c^2x^2+1}}-\frac{c^2x^2}{\sqrt{c^2x^2+1}}-\cdots\right.}{c^5}$ |
| default | $\frac{a\left(cx+\frac{cx}{2c^2x^2+2}-\frac{3\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\operatorname{arcsinh}(cx)cx+\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{1}{2\sqrt{c^2x^2+1}}-\frac{c^2x^2}{\sqrt{c^2x^2+1}}-\cdots\right.}{c^5}$ |
| parts | $\frac{a\left(\frac{x}{c^4}-\frac{-\frac{x}{2(c^2x^2+1)}+\frac{3\arctan(cx)}{2c}}{c^4}\right)}{d^2}+\frac{b\left(\operatorname{arcsinh}(cx)cx+\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{1}{2\sqrt{c^2x^2+1}}-\frac{c^2x^2}{\sqrt{c^2x^2+1}}\cdots\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a/d^2*(c*x+1/2*c*x/(c^2*x^2+1)-3/2*arctan(c*x))+b/d^2*(arcsinh(c*x)
*c*x+1/2*c*x/(c^2*x^2+1)*arcsinh(c*x)-3/2*arcsinh(c*x)*arctan(c*x)-1/2/(c^
2*x^2+1)^(1/2)-1/(c^2*x^2+1)^(1/2)*c^2*x^2-3/2*arctan(c*x)*ln(1+I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))+3/2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3
/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*dilog(1-I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))))
```

### 3.37.5 Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
 x)
```

### 3.37.6 Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{ax^4}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^4\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**4*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.37.7 Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2)
) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*
x^2 + d^2), x)
```

### 3.37.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{x^4\,(a+b\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

# 3.38 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx$



## 3.38.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}} + \frac{b\text{arcsinh}(cx)}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} - \frac{(a+b\text{arcsinh}(cx))^2}{2bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} + \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}$$

output
```
1/2*b*arcsinh(c*x)/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)-
1/2*(a+b*arcsinh(c*x))^2/b/c^4/d^2+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)/c^4/d^2+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/
2*b*x/c^3/d^2/(c^2*x^2+1)^(1/2)
```

### 3.38.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.66

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$$
$$= \frac{a-bcx\sqrt{1+c^2x^2}+b\operatorname{arcsinh}(cx)-b\operatorname{arcsinh}(cx)^2-bc^2x^2\operatorname{arcsinh}(cx)^2+2b\operatorname{arcsinh}(cx)\log\left(1-ie^{\operatorname{arcsinh}(c}\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
(a - b*c*x*Sqrt[1 + c^2*x^2] + b*ArcSinh[c*x] - b*ArcSinh[c*x]^2 - b*c^2*x
^2*ArcSinh[c*x]^2 + 2*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*c^2*x
^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + 2*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + a*Log[
1 + c^2*x^2] + a*c^2*x^2*Log[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)*PolyLog[2, (
-I)*E^ArcSinh[c*x]] + 2*b*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c
^4*d^2*(1 + c^2*x^2))
```

### 3.38.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6225, 27, 252, 222, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^2}\,dx$$
$$\Big\downarrow \text{6225}$$
$$\frac{\int \frac{x(a+b\operatorname{arcsinh}(cx))}{d(c^2x^2+1)}dx}{c^2d}+\frac{b\int \frac{x^2}{(c^2x^2+1)^{3/2}}dx}{2cd^2}-\frac{x^2(a+b\operatorname{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d^2}+\frac{b\int \frac{x^2}{(c^2x^2+1)^{3/2}}dx}{2cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d^2}+\frac{b\left(\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{c^2}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 6212}$$

$$\frac{\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 4201}$$

$$-\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

---

3.38. $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}dx$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{ar}}{}\right.}{c^4d^2}$$
$$\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4d^2}-$$
$$\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2cd^2}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*(x^2*(a + b*ArcSinh[c*x]))/(c^2*d^2*(1 + c^2*x^2)) + (b*(-(x/(c^2*Sqr
t[1 + c^2*x^2])) + ArcSinh[c*x]/c^3))/(2*c*d^2) - (I*(((-1/2*I)*(a + b*Arc
Sinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])]
)/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c^4*d^2)
```

### 3.38.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.38. $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.38.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\text{po}}{}\right.}{d^2}}{c^4}$ |
| default | $\frac{\frac{a\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{\text{po}}{}\right.}{d^2}}{c^4}$ |
| parts | $\frac{a\left(\frac{1}{2c^4\left(c^2x^2+1\right)}+\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)^2}{2}+\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2\right.\right.\right.}{d^2c^4}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d^2*(1/2/(c^2*x^2+1)+1/2*ln(c^2*x^2+1))+b/d^2*(-1/2*arcsinh(c*x)^
2+1/2*(-c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/(c^2*x^2+1)+arcsinh(
c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)))
```

### 3.38.5 Fricas [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^3}{(c^2dx^2 + d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
 x)
```

### 3.38.6 Sympy [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \frac{\int \frac{ax^3}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^3\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**3*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.38.7 Maxima [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^3}{(c^2dx^2 + d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/8*b*(((c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - 4*((c^2*x^2 + 1)*log(c^2*x^2 +
 1) + 1)*log(c*x + sqrt(c^2*x^2 + 1)) - 2)/(c^6*d^2*x^2 + c^4*d^2) + 8*int
egrate(1/2*((c^2*x^2 + 1)*log(c^2*x^2 + 1) + 1)/(c^8*d^2*x^5 + 2*c^6*d^2*x
^3 + c^4*d^2*x + (c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)
), x)) + 1/2*a*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2))
```

### 3.38.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^2}\,dx = \int \frac{x^3\,(a + b\,\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^2}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

# 3.39 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx$



## 3.39.1 Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2}$$

output
```
-1/2*x*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*
x+(c^2*x^2+1)^(1/2))/c^3/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))
/c^3/d^2+1/2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2-1/2*b/c^3/d^
2/(c^2*x^2+1)^(1/2)
```

## 3.39.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = -\frac{acx+b\sqrt{1+c^2x^2}+bcx\text{arcsinh}(cx)-a\arctan(cx)-ac^2x^2\arctan(cx)-ib\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}}\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*(a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - a
*c^2*x^2*ArcTan[c*x] - I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*c^
2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*
E^ArcSinh[c*x]] + I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + I*b
*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*(1 + c^2*x^2)*PolyLog
[2, I*E^ArcSinh[c*x]])/(c^3*d^2*(1 + c^2*x^2))
```

### 3.39.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6225, 27, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^2}dx$$

$$\Big\downarrow \text{ 6225}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{d(c^2x^2+1)}dx}{2c^2d}+\frac{b\int \frac{x}{(c^2x^2+1)^{3/2}}dx}{2cd^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\int \frac{x}{(c^2x^2+1)^{3/2}}dx}{2cd^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2c^2d^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6204}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c^3d^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c^3d^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2(c^2x^2+1)}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 4668$$

$$\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arc}}{2c^3d^2}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2715$$

$$\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan}{2c^3d^2}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2838$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^3d^2}-$$
$$\frac{x(a+b\text{arcsinh}(cx))}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b}{2c^3d^2\sqrt{c^2x^2+1}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*b/(c^3*d^2*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(
1 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyL
og[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^3*d^2)
```

### 3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.39. $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$

### 3.39.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.54

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(-\frac{cx}{2(c^2x^2+1)}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{cx\,\mathrm{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{2}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln\left(1-\frac{i(}{\sqrt{}}\right.}{2}\right.}{d^2}}{c^3}$ |
| default | $\frac{\frac{a\left(-\frac{cx}{2(c^2x^2+1)}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{cx\,\mathrm{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{2}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln\left(1-\frac{i(}{\sqrt{}}\right.}{2}\right.}{d^2}}{c^3}$ |
| parts | $\frac{a\left(-\frac{x}{2c^2(c^2x^2+1)}+\frac{\arctan(cx)}{2c^3}\right)}{d^2}+\frac{b\left(-\frac{cx\,\mathrm{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{2}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln}{}\right.}{d^2c^3}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d^2*(-1/2*c*x/(c^2*x^2+1)+1/2*arctan(c*x))+b/d^2*(-1/2*c*x/(c^2*x
^2+1)*arcsinh(c*x)+1/2*arcsinh(c*x)*arctan(c*x)+1/2*arctan(c*x)*ln(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))-1/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*dilog(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-1/2/(c^2*x^2+1)^(1/2)))
```

### 3.39.5 Fricas [F]

$$\int \frac{x^2(a+b\,\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
 x)
```

### 3.39.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{ax^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^2\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**2*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.39.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + b*integrate(x
^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.39.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^2, x)
```

### 3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

# 3.40 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx$



## 3.40.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = \frac{bx}{2cd^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{2c^2d^2(1+c^2x^2)}$$

output
```
1/2*(-a-b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+1/2*b*x/c/d^2/(c^2*x^2+1)^(1/2
)
```

## 3.40.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2} dx = -\frac{a}{2c^2d^2(1+c^2x^2)} + \frac{bx}{2cd^2\sqrt{1+c^2x^2}} - \frac{b\text{arcsinh}(cx)}{2c^2d^2(1+c^2x^2)}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*a/(c^2*d^2*(1 + c^2*x^2)) + (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (b*Ar
cSinh[c*x])/(2*c^2*d^2*(1 + c^2*x^2))
```

### 3.40.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6213, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^2}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\int \frac{1}{(c^2x^2+1)^{3/2}}dx}{2cd^2}-\frac{a+b\text{arcsinh}(cx)}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{208}$$

$$\frac{bx}{2cd^2\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2d^2(c^2x^2+1)}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
(b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*d^2*(1 + c
^2*x^2))
```

#### 3.40.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.40.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{a}{2d^2(c^2x^2+1)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2}}{c^2}$ | 61 |
| default | $\frac{-\frac{a}{2d^2(c^2x^2+1)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2}}{c^2}$ | 61 |
| parts | $-\frac{a}{2d^2c^2(c^2x^2+1)}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2c^2}$ | 63 |

input
```
int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*a/d^2/(c^2*x^2+1)+b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x
/(c^2*x^2+1)^(1/2)))
```

### 3.40.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\frac{ac^2x^2+\sqrt{c^2x^2+1}bcx-b\log\left(cx+\sqrt{c^2x^2+1}\right)}{2\left(c^4d^2x^2+c^2d^2\right)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
1/2*(a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x - b*log(c*x + sqrt(c^2*x^2 + 1)))
/(c^4*d^2*x^2 + c^2*d^2)
```

### 3.40.6 Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{ax}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/
(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.40.7 Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*b*((2*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^4*d^2*x^2 + c^2*d^2) - 4*i
ntegrate(1/2/(c^6*d^2*x^5 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c
^3*d^2*x^2 + c*d^2)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a/(c^4*d^2*x^2 + c^2*d^2
)
```

### 3.40.8 Giac [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^2, x)
```

### 3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

output
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

# 3.41 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx$



## 3.41.1 Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \frac{b}{2cd^2\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)}+\frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2cd^2}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2cd^2}$$

output
```
1/2*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*x+(c^
2*x^2+1)^(1/2))/c/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+
1/2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/2*b/c/d^2/(c^2*x^2+1)
^(1/2)
```

## 3.41.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \frac{acx+b\sqrt{1+c^2x^2}+bcx\text{arcsinh}(cx)+a\arctan(cx)+ac^2x^2\arctan(cx)+ib\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx)}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

output
```
(a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] + a*ArcTan[c*x] + a*c^2*
x^2*ArcTan[c*x] + I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*c^2*x^2
*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*ArcSinh[c*x]*Log[1 + I*E^Arc
Sinh[c*x]] - I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*b*(1 +
 c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*(1 + c^2*x^2)*PolyLog[2, I
*E^ArcSinh[c*x]])/(2*d^2*(c + c^3*x^2))
```

### 3.41.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6203, 27, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2dx^2+d\right)^2}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{d(c^2x^2+1)}dx}{2d}-\frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2d^2}-\frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6204}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arc}}{2cd^2}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan}{2cd^2}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2cd^2}+$$
$$\frac{x(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

output
```
b/(2*c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x
^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I
)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c*d^2)
```

### 3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

### 3.41.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.57

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{cx}{2c^2x^2+2}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{\operatorname{arcsinh}(cx)\arctan(cx)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln(}{}\right.}{d^2}}{c}$ |
| default | $\frac{\frac{a\left(\frac{cx}{2c^2x^2+2}+\frac{\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{\operatorname{arcsinh}(cx)\arctan(cx)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx)\ln(}{}\right.}{d^2}}{c}$ |
| parts | $\frac{a\left(\frac{x}{2c^2x^2+2}+\frac{\arctan(cx)}{2c}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{\operatorname{arcsinh}(cx)\arctan(cx)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(cx}{}\right.}{d^2c}$ |

input
```
int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d^2*(1/2*c*x/(c^2*x^2+1)+1/2*arctan(c*x))+b/d^2*(1/2*c*x/(c^2*x^2+1
)*arcsinh(c*x)+1/2*arcsinh(c*x)*arctan(c*x)+1/2/(c^2*x^2+1)^(1/2)+1/2*arct
an(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)*ln(1-I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))-1/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*d
ilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

## 3.41.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

## 3.41.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{a}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**
4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.41.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2} \, dx = \int \frac{b \,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + b*integrate(log(c*x
+ sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.41.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2} \, dx = \int \frac{b \,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

### 3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2} \, dx = \int \frac{a + b \,\text{asinh}(c\,x)}{(d\,c^2\,x^2 + d)^2} \, dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2, x)
```

# 3.42 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^2}\,dx$



## 3.42.1 Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = -\frac{bcx}{2d^2\sqrt{1+c^2x^2}}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^2}+\frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^2}$$

output
```
1/2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+(
c^2*x^2+1)^(1/2))^2)/d^2-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+1
/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-1/2*b*c*x/d^2/(c^2*x^2+1)^(1
/2)
```

## 3.42.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. 2(110) = 220.

Time = 0.31 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.13

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = -\frac{\frac{a^2}{b}-\frac{a}{1+c^2x^2}+\frac{bcx}{\sqrt{1+c^2x^2}}+2a\text{arcsinh}(cx)-\frac{b\text{arcsinh}(cx)}{1+c^2x^2}+2b\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)+2b\text{arcsinh}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

output
```
-1/2*(a^2/b - a/(1 + c^2*x^2) + (b*c*x)/Sqrt[1 + c^2*x^2] + 2*a*ArcSinh[c*
x] - (b*ArcSinh[c*x])/(1 + c^2*x^2) + 2*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] + 2*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])
/c] - 2*a*Log[1 - E^(2*ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Log[1 - E^(2*ArcS
inh[c*x])] + a*Log[1 + c^2*x^2] + 2*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-
c^2]] + 2*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*PolyLog[2, E^(2*
ArcSinh[c*x])])/d^2
```

### 3.42.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6226, 27, 208, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^2}dx$$

$$\Big\downarrow \text{ 6226}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{dx(c^2x^2+1)}dx}{d}-\frac{bc\int \frac{1}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d^2}-\frac{bc\int \frac{1}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d^2}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6214}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d^2}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d^2}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d^2}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{26}$$

$$\frac{2i\int (a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d^2}+\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{2i\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a-\right.}{d^2}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}+i\right.}{d^2}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right)}{d^2}+$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

output
```
-1/2*(b*c*x)/(d^2*Sqrt[1 + c^2*x^2]) + (a + b*ArcSinh[c*x])/(2*d^2*(1 + c^
2*x^2)) + ((2*I)*(I*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/
4)*b*PolyLog[2, -E^(2*ArcSinh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x
])]))/d^2
```

3.42. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^2}\,dx$

### 3.42.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.42. $\int \frac{a+b\operatorname{arcsinh}(cx)}{x(d+c^2dx^2)^2}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

## 3.42.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.97

| method | result |
|---|---|
| derivativedivides | $\frac{a\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\text{polylo}\right.}{}$ |
| default | $\frac{a\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b\left(\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+\text{polylo}\right.}{}$ |
| parts | $\frac{a}{2d^2(c^2x^2+1)}-\frac{a\ln(c^2x^2+1)}{2d^2}+\frac{a\ln(x)}{d^2}+\frac{b\left(\frac{-cx\sqrt{c^2x^2+1}+c^2x^2+\operatorname{arcsinh}(cx)+1}{2c^2x^2+2}+\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*(ln(c*x)+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b/d^2*(1/2*(-c*x*(c^2*x^
2+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/(c^2*x^2+1)+arcsinh(c*x)*ln(1+c*x+(c^2*
x^2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*
ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

### 3.42.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

### 3.42.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^5+2c^2x^3+x}\,dx+\int \frac{b\,\text{asinh}\left(cx\right)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**
4*x**5 + 2*c**2*x**3 + x), x))/d**2
```

### 3.42.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + b*in
tegrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x)
, x)
```

### 3.42.8 Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x), x)
```

### 3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2), x)
```

# 3.43 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx$



## 3.43.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx = -\frac{bc}{2d^2\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{d^2x(1+c^2x^2)}-\frac{3c^2x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)}-\frac{3c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}-\frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2}+\frac{3ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2d^2}-\frac{3ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2d^2}$$

output
```
(-a-b*arcsinh(c*x))/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^
2*x^2+1)-3*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b*c*arct
anh((c^2*x^2+1)^(1/2))/d^2+3/2*I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))
/d^2-3/2*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-1/2*b*c/d^2/(c^2*x
^2+1)^(1/2)
```

## 3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.51

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx =$$

$$-\frac{\frac{3a}{x}-\frac{a}{x+c^2x^3}+\frac{3b\text{arcsinh}(cx)}{x}-\frac{b\text{arcsinh}(cx)}{x+c^2x^3}+3ac\arctan(cx)+3b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)+\frac{bc\,\text{Hypergeometric2}\ldots}{\sqrt{1-\ldots}}}{\ldots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

output
```
-1/2*((3*a)/x - a/(x + c^2*x^3) + (3*b*ArcSinh[c*x])/x - (b*ArcSinh[c*x])/
(x + c^2*x^3) + 3*a*c*ArcTan[c*x] + 3*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (b*
c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 3*b*Sq
rt[-c^2]*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 3*b*Sqrt[-c
^2]*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 3*b*Sqrt[-c^2]*P
olyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 3*b*Sqrt[-c^2]*PolyLog[2, (Sqrt
[-c^2]*E^ArcSinh[c*x])/c])/d^2
```

### 3.43.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6224, 27, 243, 61, 73, 221, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(c^2dx^2+d\right)^2}dx$$

$$\Big\downarrow \text{6224}$$

$$-3c^2\int \frac{a+b\text{arcsinh}(cx)}{d^2\left(c^2x^2+1\right)^2}dx+\frac{bc\int \frac{1}{x\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\int \frac{1}{x\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{243}$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\int \frac{1}{x^2\left(c^2x^2+1\right)^{3/2}}dx^2}{2d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{61}$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{2}{\sqrt{c^2x^2+1}}\right)}{2d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{d^2}+\frac{bc\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+\frac{2}{\sqrt{c^2x^2+1}}\right)}{2d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

$$\Big\downarrow \textbf{6203}$$

$$-\frac{3c^2\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

$$\Big\downarrow \textbf{241}$$

$$-\frac{3c^2\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

$$\Big\downarrow \textbf{6204}$$

$$-\frac{3c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{3c^2\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

$$\Big\downarrow \textbf{4668}$$

3.43. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx$

$$-\frac{3c^2\left(\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{2c}+\frac{x(\ldots}{}\right.}{d^2}$$

$$\frac{a+b\text{arcsinh}(cx)}{d^2x\,(c^2x^2+1)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

$\downarrow$ 2715

$$-\frac{3c^2\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(\ldots}{2c}\right.}{d^2}$$

$$\frac{a+b\text{arcsinh}(cx)}{d^2x\,(c^2x^2+1)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

$\downarrow$ 2838

$$-\frac{3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{\ldots}{2c\ldots}\right.}{d^2}$$

$$\frac{a+b\text{arcsinh}(cx)}{d^2x\,(c^2x^2+1)}+\frac{bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{2d^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

output
```
-((a + b*ArcSinh[c*x])/(d^2*x*(1 + c^2*x^2))) + (b*c*(2/Sqrt[1 + c^2*x^2]
- 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/(2*d^2) - (3*c^2*(b/(2*c*Sqrt[1 + c^2*x^2
]) + (x*(a + b*ArcSinh[c*x]))/(2*(1 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])*
ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog
[2, I*E^ArcSinh[c*x]])/(2*c)))/d^2
```

## 3.43. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx$

### 3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

---

## 3.43. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.43. $\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2(d+c^2dx^2)^2}\,dx$

### 3.43.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.35

| method | result |
|---|---|
| derivativedivides | $c\left(\frac{a\left(-\frac{1}{cx}-\frac{cx}{2\left(c^{2}x^{2}+1\right)}-\frac{3\arctan(cx)}{2}\right)}{d^{2}}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{cx\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{3\arctan(cx)\ln\left(1+\cdots\right.}{2}\cdots\right.}{\cdots}\right.$ |
| default | $c\left(\frac{a\left(-\frac{1}{cx}-\frac{cx}{2\left(c^{2}x^{2}+1\right)}-\frac{3\arctan(cx)}{2}\right)}{d^{2}}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{cx\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{3\arctan(cx)\ln\left(1+\cdots\right.}{2}\cdots\right.}{\cdots}\right.$ |
| parts | $\frac{a\left(-c^{2}\left(\frac{x}{2c^{2}x^{2}+2}+\frac{3\arctan(cx)}{2c}\right)-\frac{1}{x}\right)}{d^{2}}+\frac{bc\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{cx\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{2}-\frac{3\arctan(cx)\ln\left(1+\frac{i(\cdots}{\sqrt{c\cdots}}\right.}{2}\cdots\right.}{\cdots}$ |

input
```
int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*(a/d^2*(-1/c/x-1/2*c*x/(c^2*x^2+1)-3/2*arctan(c*x))+b/d^2*(-arcsinh(c*x)
/c/x-1/2*c*x/(c^2*x^2+1)*arcsinh(c*x)-3/2*arcsinh(c*x)*arctan(c*x)-3/2*arc
tan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*arctan(c*x)*ln(1-I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))+3/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*
dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/(c^2*x^2+1)^(1/2)-arctanh(1/(c^
2*x^2+1)^(1/2))))
```

### 3.43.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

### 3.43.6 Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^2 (d + c^2dx^2)^2}\,dx = \frac{\int \frac{a}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{b\,\operatorname{asinh}(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(
c**4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

### 3.43.7 Maxima [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^2 (d + c^2dx^2)^2}\,dx = \int \frac{b\operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^2 x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + b*i
ntegrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x
^2), x)
```

### 3.43.8 Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^2 (d + c^2dx^2)^2}\,dx = \int \frac{b\operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^2 x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^2), x)
```

### 3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2), x)
```

# 3.44 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx$



## 3.44.1 Optimal result

Integrand size = 24, antiderivative size = 146

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx &= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}}-\frac{c^2(a+b\text{arcsinh}(cx))}{d^2\left(1+c^2x^2\right)}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(1+c^2x^2\right)}\\ &\quad+\frac{4c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2}\\ &\quad+\frac{bc^2\operatorname{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2}-\frac{bc^2\operatorname{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2}\end{aligned}$$

output
```
-c^2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+1/2*(-a-b*arcsinh(c*x))/d^2/x^2/(c
^2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^2+
b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c^2*polylog(2,(c*x+(c^2*
x^2+1)^(1/2))^2)/d^2-1/2*b*c/d^2/x/(c^2*x^2+1)^(1/2)
```

## 3.44.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 326 vs. 2(146) = 292.

Time = 0.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.23

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx$$

$$= \frac{\frac{2a^2c^2}{b}-\frac{2a}{x^2}+\frac{bc}{x\sqrt{1+c^2x^2}}+\frac{2bc^3x}{\sqrt{1+c^2x^2}}-\frac{2bc\sqrt{1+c^2x^2}}{x}+\frac{a}{x^2+c^2x^4}+4ac^2\text{arcsinh}(cx)-\frac{2b\text{arcsinh}(cx)}{x^2}+\frac{b\text{arcsinh}(cx)}{x^2+c^2x^4}+\cdots}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

output
```
((2*a^2*c^2)/b - (2*a)/x^2 + (b*c)/(x*Sqrt[1 + c^2*x^2]) + (2*b*c^3*x)/Sqr
t[1 + c^2*x^2] - (2*b*c*Sqrt[1 + c^2*x^2])/x + a/(x^2 + c^2*x^4) + 4*a*c^2
*ArcSinh[c*x] - (2*b*ArcSinh[c*x])/x^2 + (b*ArcSinh[c*x])/(x^2 + c^2*x^4)
+ 4*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*Ar
cSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 4*a*c^2*Log[1 - E^(2*A
rcSinh[c*x])] - 4*b*c^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a*c^2
*Log[1 + c^2*x^2] + 4*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*
b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b*c^2*PolyLog[2, E^(2*
ArcSinh[c*x])])/(2*d^2)
```

### 3.44.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.31, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6224, 27, 245, 208, 6226, 208, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(c^2dx^2+d\right)^2}dx$$

$$\Big\downarrow \text{6224}$$

$$-2c^2\int \frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)^2}dx+\frac{bc\int \frac{1}{x^2\left(c^2x^2+1\right)^{3/2}}dx}{2d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\int \frac{1}{x^2\left(c^2x^2+1\right)^{3/2}}dx}{2d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{245}$$

$$-\frac{2c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\left(-2c^2\int \frac{1}{\left(c^2x^2+1\right)^{3/2}}dx-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{208}$$

$$-\frac{2c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \ 6226$$

$$-\frac{2c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)}dx-\frac{1}{2}bc\int \frac{1}{\left(c^2x^2+1\right)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{2\left(c^2x^2+1\right)}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \ 208$$

$$-\frac{2c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)}dx+\frac{a+b\text{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \ 6214$$

$$-\frac{2c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \ 5984$$

$$-\frac{2c^2\left(2\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2c^2\left(2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{2c^2\left(2i\int (a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{a+b\text{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

---

3.44. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^2}dx$

$$\Big\downarrow \text{4670}$$

$$-\frac{2c^2\left(2i\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}\left(e^{2\text{arcsinh}(c\right.\right.\right.}{d^2}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2c^2\left(2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(\right.\right.}{d^2}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2c^2\left(2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right.\right.}{d^2}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)}{2d^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

output
```
(b*c*(-(1/(x*Sqrt[1 + c^2*x^2])) - (2*c^2*x)/Sqrt[1 + c^2*x^2]))/(2*d^2) -
 (a + b*ArcSinh[c*x])/(2*d^2*x^2*(1 + c^2*x^2)) - (2*c^2*(-1/2*(b*c*x)/Sqr
t[1 + c^2*x^2] + (a + b*ArcSinh[c*x])/(2*(1 + c^2*x^2)) + (2*I)*(I*(a + b*
ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcSi
nh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])])))/d^2
```

#### 3.44.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.44. $\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

## 3.44.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.66

| method | result |
|---|---|
| derivativedivides | $c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2\left(c^2x^2+1\right)}+\ln\left(c^2x^2+1\right)\right)}{d^2}+\frac{b\left(-\frac{2\,\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2c^2x^2\left(c^2x^2+1\right)}-2\,\mathrm{arcsinh}(cx\right.}{}\right.$ |
| default | $c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2\left(c^2x^2+1\right)}+\ln\left(c^2x^2+1\right)\right)}{d^2}+\frac{b\left(-\frac{2\,\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2c^2x^2\left(c^2x^2+1\right)}-2\,\mathrm{arcsinh}(cx\right.}{}\right.$ |
| parts | $\frac{a\left(\frac{c^4\left(-\frac{1}{c^2\left(c^2x^2+1\right)}+\frac{2\ln\left(c^2x^2+1\right)}{c^2}\right)}{2}-\frac{1}{2x^2}-2c^2\ln(x)\right)}{d^2}+\frac{b\,c^2\left(-\frac{2\,\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2c^2x^2\left(c^2x^2+1\right)}-2\,\mathrm{arcsin}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/d^2*(-1/2/c^2/x^2-2*ln(c*x)-1/2/(c^2*x^2+1)+ln(c^2*x^2+1))+b/d^2*(-
1/2*(2*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/c^2/x^2/(c
^2*x^2+1)-2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*polylog(2,-c*x-(c^2
*x^2+1)^(1/2))+2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+polylog(2,-(
c*x+(c^2*x^2+1)^(1/2))^2)-2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*pol
ylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

### 3.44.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

### 3.44.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^7+2c^2x^5+x^3}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(
c**4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

### 3.44.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c^
2*d^2*x^4 + d^2*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*
x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

### 3.44.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^2x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^3), x)
```

### 3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^2}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2), x)
```

# 3.45 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx$



## 3.45.1 Optimal result

Integrand size = 24, antiderivative size = 239

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx &= \frac{bc^3}{3d^2\sqrt{1+c^2x^2}}-\frac{bc}{6d^2x^2\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(1+c^2x^2)}\\ &+\frac{5c^2(a+b\text{arcsinh}(cx))}{3d^2x(1+c^2x^2)}+\frac{5c^4x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)}\\ &+\frac{5c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}\\ &+\frac{13bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^2}-\frac{5ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2d^2}\\ &+\frac{5ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2d^2}\end{aligned}$$

output
```
1/3*(-a-b*arcsinh(c*x))/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a+b*arcsinh(c*x))/d^2
/x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+5*c^3*(a+b*arc
sinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+13/6*b*c^3*arctanh((c^2*x^2+1
)^(1/2))/d^2-5/2*I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5/2*I*b
*c^3*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+1/3*b*c^3/d^2/(c^2*x^2+1)^(1
/2)-1/6*b*c/d^2/x^2/(c^2*x^2+1)^(1/2)
```

### 3.45.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.30

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4 \left(d+c^2dx^2\right)^2} \, dx = \frac{-\frac{5a}{3x^3}+\frac{5ac^2}{x}-\frac{5bc\sqrt{1+c^2x^2}}{6x^2}+\frac{a}{x^3+c^2x^5}-\frac{5b\text{arcsinh}(cx)}{3x^3}+\frac{5bc^2\text{arcsinh}(cx)}{x}+\frac{b\text{arcsinh}(cx)}{x^3+c^2x^5}+5ac^3\arctan(cx)+\frac{35}{6}\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

output
```
((-5*a)/(3*x^3) + (5*a*c^2)/x - (5*b*c*Sqrt[1 + c^2*x^2])/(6*x^2) + a/(x^3
 + c^2*x^5) - (5*b*ArcSinh[c*x])/(3*x^3) + (5*b*c^2*ArcSinh[c*x])/x + (b*A
rcSinh[c*x])/(x^3 + c^2*x^5) + 5*a*c^3*ArcTan[c*x] + (35*b*c^3*ArcTanh[Sqr
t[1 + c^2*x^2]])/6 + (b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^2])/
Sqrt[1 + c^2*x^2] - 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x
])/Sqrt[-c^2]] + 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSi
nh[c*x])/c] + 5*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] -
 5*b*(-c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(2*d^2)
```

### 3.45.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.17, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {6224, 27, 243, 52, 61, 73, 221, 6224, 243, 61, 73, 221, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4 \left(c^2dx^2+d\right)^2} \, dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{5}{3}c^2 \int \frac{a+b\text{arcsinh}(cx)}{d^2x^2 \left(c^2x^2+1\right)^2}dx+\frac{bc\int \frac{1}{x^3\left(c^2x^2+1\right)^{3/2}}dx}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3 \left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^2}dx}{3d^2}+\frac{bc\int \frac{1}{x^3(c^2x^2+1)^{3/2}}dx}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^2}dx}{3d^2}+\frac{bc\int \frac{1}{x^4(c^2x^2+1)^{3/2}}dx^2}{6d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^2}dx}{3d^2}+\frac{bc\left(-\frac{3}{2}c^2\int \frac{1}{x^2(c^2x^2+1)^{3/2}}dx^2-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^2}dx}{3d^2}+\frac{bc\left(-\frac{3}{2}c^2\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{2}{\sqrt{c^2x^2+1}}\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^2}dx}{3d^2}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+\frac{2}{\sqrt{c^2x^2+1}}\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^2}dx}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{5c^2\left(-3c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+bc\int \frac{1}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right)}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$$\Big\downarrow \text{ 243}$$

3.45. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}dx$

$$-\frac{5c^2\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)^{3/2}}dx^2-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right)}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 61

$$-\frac{5c^2\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{1}{2}bc\left(\int\frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{2}{\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right)}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 73

$$-\frac{5c^2\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{1}{2}bc\left(\frac{2\int\frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+\frac{2}{\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right)}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 221

$$-\frac{5c^2\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)\right)}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 6203

$$-\frac{5c^2\left(-3c^2\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int\frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)\right)}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 241

$$-\frac{5c^2\left(-3c^2\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)\right)}{3d^2}-\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 6204

---

3.45. $\int\frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}dx$

$$-\frac{5c^2\left(-3c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2a\right.}{3d^2}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 3042

$$-\frac{5c^2\left(-3c^2\left(\frac{\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right.}{3d^2}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 4668

$$-\frac{5c^2\left(-3c^2\left(\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx\right.}{2c}\right.}{3d^2}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 2715

$$-\frac{5c^2\left(-3c^2\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsi}}\right.}{2c}\right.}{}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

$\downarrow$ 2838

$$-\frac{5c^2\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(cx}{2(c^2x^2+1)}\right.\right.}{3d^2}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^2x^3(c^2x^2+1)}+\frac{bc\left(-\frac{3}{2}c^2\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)-\frac{1}{x^2\sqrt{c^2x^2+1}}\right)}{6d^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

## 3.45. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx$

output
```
-1/3*(a + b*ArcSinh[c*x])/(d^2*x^3*(1 + c^2*x^2)) + (b*c*(-(1/(x^2*Sqrt[1
+ c^2*x^2])) - (3*c^2*(2/Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]])
)/2))/(6*d^2) - (5*c^2*(-((a + b*ArcSinh[c*x])/(x*(1 + c^2*x^2))) + (b*c*(
2/Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/2 - 3*c^2*(b/(2*c*Sqr
t[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*(1 + c^2*x^2)) + (2*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]]
+ I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c))))/(3*d^2)
```

### 3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.45. $\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(d+c^2dx^2)^2}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.45.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| derivativedivides | $c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{2}{cx}+\frac{cx}{2c^2x^2+2}+\frac{5\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{2\,\text{arcsinh}(cx)}{cx}+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{5\,\text{arcsinh}(cx)\arctan(cx)}{2}\right.}{}\right.$ |
| default | $c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{2}{cx}+\frac{cx}{2c^2x^2+2}+\frac{5\arctan(cx)}{2}\right)}{d^2}+\frac{b\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{2\,\text{arcsinh}(cx)}{cx}+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{5\,\text{arcsinh}(cx)\arctan(cx)}{2}\right.}{}\right.$ |
| parts | $\frac{a\left(c^4\left(\frac{x}{2c^2x^2+2}+\frac{5\arctan(cx)}{2c}\right)-\frac{1}{3x^3}+\frac{2c^2}{x}\right)}{d^2}+\frac{b\,c^3\left(-\frac{\text{arcsinh}(cx)}{3c^3x^3}+\frac{2\,\text{arcsinh}(cx)}{cx}+\frac{\text{arcsinh}(cx)cx}{2c^2x^2+2}+\frac{5\,\text{arcsinh}(cx)\arctan(cx)}{2}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(a/d^2*(-1/3/c^3/x^3+2/c/x+1/2*c*x/(c^2*x^2+1)+5/2*arctan(c*x))+b/d^2*
(-1/3*arcsinh(c*x)/c^3/x^3+2*arcsinh(c*x)/c/x+1/2*c*x/(c^2*x^2+1)*arcsinh(
c*x)+5/2*arcsinh(c*x)*arctan(c*x)+1/3/(c^2*x^2+1)^(1/2)-1/6/c^2/x^2/(c^2*x
^2+1)^(1/2)+13/6*arctanh(1/(c^2*x^2+1)^(1/2))+5/2*arctan(c*x)*ln(1+I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))-5/2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-5/2*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+5/2*I*dilog(1-I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))))
```

### 3.45.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

### 3.45.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^8+2c^2x^6+x^4}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(
c**4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

### 3.45.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
 d^2*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8 + 2*c
^2*d^2*x^6 + d^2*x^4), x)
```

### 3.45.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^2}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^2x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^4), x)
```

### 3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^4\,\left(d\,c^2\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2), x)
```

# 3.46 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$



## 3.46.1 Optimal result

Integrand size = 24, antiderivative size = 186

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx &= \frac{b}{12c^5d^3(1+c^2x^2)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}\\ &\quad-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2}-\frac{3x(a+b\text{arcsinh}(cx))}{8c^4d^3(1+c^2x^2)}\\ &\quad+\frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\\ &\quad-\frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3}+\frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8c^5d^3}\end{aligned}$$

output
```
1/12*b/c^5/d^3/(c^2*x^2+1)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x
^2+1)^2-3/8*x*(a+b*arcsinh(c*x))/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x)
)*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c^5/d^3+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3-5
/8*b/c^5/d^3/(c^2*x^2+1)^(1/2)
```

### 3.46.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.83

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx =$$
$$-\frac{9acx+15ac^3x^3+13b\sqrt{1+c^2x^2}+15bc^2x^2\sqrt{1+c^2x^2}+9bcx\text{arcsinh}(cx)+15bc^3x^3\text{arcsinh}(cx)-9a\,\text{arc}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
-1/24*(9*a*c*x + 15*a*c^3*x^3 + 13*b*Sqrt[1 + c^2*x^2] + 15*b*c^2*x^2*Sqrt
[1 + c^2*x^2] + 9*b*c*x*ArcSinh[c*x] + 15*b*c^3*x^3*ArcSinh[c*x] - 9*a*Arc
Tan[c*x] - 18*a*c^2*x^2*ArcTan[c*x] - 9*a*c^4*x^4*ArcTan[c*x] - (9*I)*b*Ar
cSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1
 - I*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*
x]] + (9*I)*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*Ar
cSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] + (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*
x]] - (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3*(1 +
c^2*x^2)^2)
```

### 3.46.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6225, 27, 243, 53, 2009, 6225, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^3}\,dx$$
$$\Big\downarrow \text{ 6225}$$
$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{d^2(c^2x^2+1)^2}dx}{4c^2d}+\frac{b\int \frac{x^3}{(c^2x^2+1)^{5/2}}dx}{4cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\int \frac{x^3}{(c^2x^2+1)^{5/2}}dx}{4cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\int \frac{x^2}{(c^2x^2+1)^{5/2}}dx^2}{8cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\int \left(\frac{1}{c^2(c^2x^2+1)^{3/2}}-\frac{1}{c^2(c^2x^2+1)^{5/2}}\right)dx^2}{8cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{4c^2d^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

$$\Big\downarrow \text{6225}$$

$$\frac{3\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2c^2}+\frac{b\int \frac{x}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{4c^2d^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2c^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}-\frac{b}{2c^3\sqrt{c^2x^2+1}}\right)}{4c^2d^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

$$\Big\downarrow \text{6204}$$

$$\frac{3\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c^3}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}-\frac{b}{2c^3\sqrt{c^2x^2+1}}\right)}{4c^2d^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

$$\Big\downarrow \text{3042}$$

---

3.46. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}dx$

$$\frac{3\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c^3}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}-\frac{b}{2c^3\sqrt{c^2x^2+1}}\right)}{4c^2d^3}-\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

$\downarrow$ 4668

$$\frac{3\left(\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{2c^3}-\frac{x(a+ba}{2c^2}\right.}{4c^2d^3}$$
$$\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

$\downarrow$ 2715

$$\frac{3\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+ba}{2c^3}\right.}{4c^2d^3}$$
$$\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

$\downarrow$ 2838

$$\frac{3\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^3}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}-\frac{b}{2c^3\sqrt{c^2}}\right.}{4c^2d^3}$$
$$\frac{x^3(a+b\text{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\left(\frac{2}{3c^4(c^2x^2+1)^{3/2}}-\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{8cd^3}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
(b*(2/(3*c^4*(1 + c^2*x^2)^(3/2)) - 2/(c^4*Sqrt[1 + c^2*x^2])))/(8*c*d^3)
- (x^3*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) + (3*(-1/2*b/(c^3
*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/(2*c^2*(1 + c^2*x^2)) + (2*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSin
h[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^3)))/(4*c^2*d^3)
```

## 3.46. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$

#### 3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.46. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.46.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{-\frac{5}{8}c^3x^3-\frac{3}{8}cx}{\left(c^2x^2+1\right)^2}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{5c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{3cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{3\arctan(cx)\ln\left(1+\frac{i(icx-}{\sqrt{c^2x^2}}\right.}{8}\right.}{}}{c^5}$ |
| default | $\frac{\frac{a\left(\frac{-\frac{5}{8}c^3x^3-\frac{3}{8}cx}{\left(c^2x^2+1\right)^2}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{5c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{3cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{3\arctan(cx)\ln\left(1+\frac{i(icx-}{\sqrt{c^2x^2}}\right.}{8}\right.}{}}{c^5}$ |
| parts | $\frac{a\left(\frac{-\frac{5x^3}{8c^2}-\frac{3x}{8c^4}}{\left(c^2x^2+1\right)^2}+\frac{3\arctan(cx)}{8c^5}\right)}{d^3}+\frac{b\left(-\frac{5c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}-\frac{3cx\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{3\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{3\arctan(cx)\ln\left(1+\frac{i(i}{\sqrt{c}}\right.}{8}\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a/d^3*((-5/8*c^3*x^3-3/8*c*x)/(c^2*x^2+1)^2+3/8*arctan(c*x))+b/d^3*
(-5/8*c^3*x^3/(c^2*x^2+1)^2*arcsinh(c*x)-3/8*c*x/(c^2*x^2+1)^2*arcsinh(c*x
)+3/8*arcsinh(c*x)*arctan(c*x)+3/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-3/8*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I*dilog(1
+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))-13/24/(c^2*x^2+1)^(3/2)-5/8*c^2*x^2/(c^2*x^2+1)^(3/2)))
```

### 3.46.5 Fricas [F]

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^3}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{(c^2dx^2 + d)^3}\, dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 + d^3), x)
```

### 3.46.6 Sympy [F]

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^3}\, dx = \frac{\int \frac{ax^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\, dx + \int \frac{bx^4 \operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\, dx}{d^3}$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

### 3.46.7 Maxima [F]

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^3}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{(c^2dx^2 + d)^3}\, dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/8*a*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arct
an(c*x)/(c^5*d^3)) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3
*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.46.8 Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^3, x)
```

### 3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{x^4\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

# 3.47 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$



## 3.47.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{bx^3}{12cd^3(1+c^2x^2)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{b\text{arcsinh}(cx)}{4c^4d^3}+\frac{x^4(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}$$

output
```
1/12*b*x^3/c/d^3/(c^2*x^2+1)^(3/2)-1/4*b*arcsinh(c*x)/c^4/d^3+1/4*x^4*(a+b
*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/4*b*x/c^3/d^3/(c^2*x^2+1)^(1/2)
```

## 3.47.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{-3a(1+2c^2x^2)+bcx\sqrt{1+c^2x^2}(3+4c^2x^2)-3(b+2bc^2x^2)\,\text{arcsinh}(cx)}{12c^4d^3(1+c^2x^2)^2}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
(-3*a*(1 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2) - 3*(b + 2
*b*c^2*x^2)*ArcSinh[c*x])/(12*c^4*d^3*(1 + c^2*x^2)^2)
```

### 3.47.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6215, 252, 252, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^3}dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}-\frac{bc\int \frac{x^4}{(c^2x^2+1)^{5/2}}dx}{4d^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}-\frac{bc\left(\frac{\int \frac{x^2}{(c^2x^2+1)^{3/2}}dx}{c^2}-\frac{x^3}{3c^2(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}-\frac{bc\left(\frac{\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{c^2}-\frac{x}{c^2\sqrt{c^2x^2+1}}}{c^2}-\frac{x^3}{3c^2(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{222}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}-\frac{bc\left(\frac{\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}}{c^2}-\frac{x^3}{3c^2(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) - (b*c*(-1/3*x^3/(c^2*(
1 + c^2*x^2)^(3/2)) + (-(x/(c^2*Sqrt[1 + c^2*x^2])) + ArcSinh[c*x]/c^3)/c^
2))/(4*d^3)
```

#### 3.47.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.47.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{1}{2\left(c^{2}x^{2}+1\right)}\right)}{d^{3}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{cx}{12\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^{2}x^{2}+1}}\right)}{d^{3}}}{c^{4}}$ | 108 |
| default | $\frac{\frac{a\left(\frac{1}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{1}{2\left(c^{2}x^{2}+1\right)}\right)}{d^{3}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{cx}{12\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^{2}x^{2}+1}}\right)}{d^{3}}}{c^{4}}$ | 108 |
| parts | $\frac{a\left(-\frac{1}{2c^{4}\left(c^{2}x^{2}+1\right)}+\frac{1}{4c^{4}\left(c^{2}x^{2}+1\right)^{2}}\right)}{d^{3}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)}{4\left(c^{2}x^{2}+1\right)^{2}}-\frac{\operatorname{arcsinh}(cx)}{2\left(c^{2}x^{2}+1\right)}-\frac{cx}{12\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^{2}x^{2}+1}}\right)}{d^{3}c^{4}}$ | 113 |

input
```
int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d^3*(1/4/(c^2*x^2+1)^2-1/2/(c^2*x^2+1))+b/d^3*(1/4/(c^2*x^2+1)^2*
arcsinh(c*x)-1/2/(c^2*x^2+1)*arcsinh(c*x)-1/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c
*x/(c^2*x^2+1)^(1/2)))
```

### 3.47.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{x^3(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \frac{3\,ac^4x^4 - 3\,(2\,bc^2x^2 + b)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + (4\,bc^3x^3 + 3\,bcx)\sqrt{c^2x^2 + 1}}{12\,(c^8d^3x^4 + 2\,c^6d^3x^2 + c^4d^3)}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*x^4 - 3*(2*b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + (4*
b*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4
*d^3)
```

### 3.47.6 Sympy [F]

$$\int \frac{x^3(a + b\operatorname{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \frac{\int \frac{ax^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

### 3.47.7 Maxima [F]

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^3}{(c^2dx^2 + d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1)) + 3)/
(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 16*integrate(1/4*(2*c^2*x^2 + 1)
/(c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^3*x + (c^9*d^3*x^6
+ 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(
2*c^2*x^2 + 1)*a/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

### 3.47.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \int \frac{x^3\,(a + b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^3}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

# 3.48 $\int \frac{x^2(a+b\,\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$



## 3.48.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = -\frac{b}{12c^3d^3(1+c^2x^2)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(1+c^2x^2)^2}+\frac{x(a+b\text{arcsinh}(cx))}{8c^2d^3(1+c^2x^2)}+\frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3}-\frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3}+\frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8c^3d^3}$$

output
```
-1/12*b/c^3/d^3/(c^2*x^2+1)^(3/2)-1/4*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^
2+1)^2+1/8*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))
*arctan(c*x+(c^2*x^2+1)^(1/2))/c^3/d^3-1/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+
1)^(1/2)))/c^3/d^3+1/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/
8*b/c^3/d^3/(c^2*x^2+1)^(1/2)
```

## 3.48.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.85

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$$

$$= \frac{-3acx+3ac^3x^3+b\sqrt{1+c^2x^2}+3bc^2x^2\sqrt{1+c^2x^2}-3bcx\text{arcsinh}(cx)+3bc^3x^3\text{arcsinh}(cx)+3a\arctan(c}{}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
(-3*a*c*x + 3*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 3*b*c^2*x^2*Sqrt[1 + c^2*x
^2] - 3*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 3*a*ArcTan[c*x] +
6*a*c^2*x^2*ArcTan[c*x] + 3*a*c^4*x^4*ArcTan[c*x] + (3*I)*b*ArcSinh[c*x]*L
og[1 - I*E^ArcSinh[c*x]] + (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSin
h[c*x]] + (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*b
*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log
[1 + I*E^ArcSinh[c*x]] - (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] - (3*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*b*
(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c^3*d^3*(1 + c^2*x^2)^2)
```

### 3.48.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6225, 27, 241, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^3}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{d^2(c^2x^2+1)^2}dx}{4c^2d}+\frac{b\int \frac{x}{(c^2x^2+1)^{5/2}}dx}{4cd^3}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\int \frac{x}{(c^2x^2+1)^{5/2}}dx}{4cd^3}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}-\frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6203}$$

$$\frac{\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}-\frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}-\frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6204}$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}-\frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}-\frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{2c}+\frac{x(a+b\text{arcs}}{2(c^2x^2}}{4c^2d^3}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}-\frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcs}}{2c}}{4c^2d^3}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{4c^2d^3(c^2x^2+1)^2}-\frac{b}{12c^3d^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

3.48. $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}dx$

$$\frac{\frac{2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))-ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)+ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\operatorname{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}}{4c^2d^3}$$

$$\frac{x(a+b\operatorname{arcsinh}(cx))}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{b}{12c^3d^3\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
-1/12*b/(c^3*d^3*(1 + c^2*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(4*c^2*d^
3*(1 + c^2*x^2)^2) + (b/(2*c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))
/(2*(1 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*
PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c))
/(4*c^2*d^3)
```

### 3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.48. $\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$

### 3.48.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{\frac{1}{8}c^3x^3-\frac{1}{8}cx}{(c^2x^2+1)^2}+\frac{\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{cx\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{8}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{8}-\mathrm{arc}\cdots\right.}{\cdots}}{c^3}$ |
| default | $\frac{\frac{a\left(\frac{\frac{1}{8}c^3x^3-\frac{1}{8}cx}{(c^2x^2+1)^2}+\frac{\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{cx\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{8}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{8}-\mathrm{arc}\cdots\right.}{\cdots}}{c^3}$ |
| parts | $\frac{a\left(\frac{\frac{x^3}{8}-\frac{x}{8c^2}}{(c^2x^2+1)^2}+\frac{\arctan(cx)}{8c^3}\right)}{d^3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{cx\,\mathrm{arcsinh}(cx)}{8(c^2x^2+1)^2}+\frac{\mathrm{arcsinh}(cx)\arctan(cx)}{8}+\frac{\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{8}-\cdots\right.}{\cdots}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d^3*((1/8*c^3*x^3-1/8*c*x)/(c^2*x^2+1)^2+1/8*arctan(c*x))+b/d^3*(
1/8*c^3*x^3/(c^2*x^2+1)^2*arcsinh(c*x)-1/8*c*x/(c^2*x^2+1)^2*arcsinh(c*x)+
1/8*arcsinh(c*x)*arctan(c*x)+1/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))-1/8*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*I*dilog(1+I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
+1/8*c^2*x^2/(c^2*x^2+1)^(3/2)+1/24/(c^2*x^2+1)^(3/2)))
```

### 3.48.5 Fricas [F]

$$\int \frac{x^2(a+b\,\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 + d^3), x)
```

### 3.48.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{ax^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

### 3.48.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/8*a*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x)
/(c^3*d^3)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 +
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.48.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^3, x)
```

### 3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

# 3.49 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$



## 3.49.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\frac{bx}{12cd^3(1+c^2x^2)^{3/2}}+\frac{bx}{6cd^3\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(1+c^2x^2)^2}$$

output
```
1/12*b*x/c/d^3/(c^2*x^2+1)^(3/2)+1/4*(-a-b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+
1)^2+1/6*b*x/c/d^3/(c^2*x^2+1)^(1/2)
```

## 3.49.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\frac{-3a+bcx\sqrt{1+c^2x^2}(3+2c^2x^2)-3b\text{arcsinh}(cx)}{12d^3(c+c^3x^2)^2}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
(-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2) - 3*b*ArcSinh[c*x])/(12*d^
3*(c + c^3*x^2)^2)
```

### 3.49.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6213, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^3}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\int \frac{1}{(c^2x^2+1)^{5/2}}dx}{4cd^3}-\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{209}$$

$$\frac{b\left(\frac{2}{3}\int \frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4cd^3}-\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{b\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4cd^3}-\frac{a+b\text{arcsinh}(cx)}{4c^2d^3(c^2x^2+1)^2}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
(b*(x/(3*(1 + c^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + c^2*x^2])))/(4*c*d^3) -
(a + b*ArcSinh[c*x])/(4*c^2*d^3*(1 + c^2*x^2)^2)
```

#### 3.49.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.49.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{a}{4d^3\left(c^2x^2+1\right)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3}}{c^2}$ | 76 |
| default | $\frac{-\frac{a}{4d^3\left(c^2x^2+1\right)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3}}{c^2}$ | 76 |
| parts | $-\frac{a}{4d^3c^2\left(c^2x^2+1\right)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3c^2}$ | 78 |

input
```
int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/4*a/d^3/(c^2*x^2+1)^2+b/d^3*(-1/4/(c^2*x^2+1)^2*arcsinh(c*x)+1/1
2/(c^2*x^2+1)^(3/2)*c*x+1/6*c*x/(c^2*x^2+1)^(1/2)))
```

## 3.49.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{3\,ac^4x^4+6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(2\,bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}}{12\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*x^4 + 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 + 1)) + (2*b*
c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d
^3)
```

### 3.49.6 Sympy [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \frac{\int \frac{ax}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b
*x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

### 3.49.7 Maxima [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x}{(c^2dx^2 + d)^3}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/16*b*((4*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2
 + c^2*d^3) - 16*integrate(1/4/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^
3 + c^2*d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt
(c^2*x^2 + 1)), x)) - 1/4*a/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

### 3.49.8 Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x}{(c^2dx^2 + d)^3}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^3, x)
```

### 3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{x\,(a+b\,\text{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^3}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

output
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

# 3.50 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx$



## 3.50.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx &= \frac{b}{12cd^3(1+c^2x^2)^{3/2}}+\frac{3b}{8cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}\\&\quad+\frac{3x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)}+\frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}\\&\quad-\frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8cd^3}+\frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8cd^3}\end{aligned}$$

output
```
1/12*b/c/d^3/(c^2*x^2+1)^(3/2)+1/4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+
3/8*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))*arctan(c*x
+(c^2*x^2+1)^(1/2))/c/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/
d^3+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/8*b/c/d^3/(c^2*x^
2+1)^(1/2)
```

## 3.50.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.92

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{15acx+9ac^3x^3+11b\sqrt{1+c^2x^2}+9bc^2x^2\sqrt{1+c^2x^2}+15bcx\text{arcsinh}(cx)+9bc^3x^3\text{arcsinh}(cx)+9a\arctan\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

output
```
(15*a*c*x + 9*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] + 9*b*c^2*x^2*Sqrt[1 + c^
2*x^2] + 15*b*c*x*ArcSinh[c*x] + 9*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x
] + 18*a*c^2*x^2*ArcTan[c*x] + 9*a*c^4*x^4*ArcTan[c*x] + (9*I)*b*ArcSinh[c
*x]*Log[1 - I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^
ArcSinh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (
9*I)*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c
*x]*Log[1 + I*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] - (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (
9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c*d^3*(1 + c^2*x^
2)^2)
```

### 3.50.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6203, 27, 241, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^3}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{3\int \frac{a+b\text{arcsinh}(cx)}{d^2(c^2x^2+1)^2}dx}{4d}-\frac{bc\int \frac{x}{(c^2x^2+1)^{5/2}}dx}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{4d^3}-\frac{bc\int \frac{x}{(c^2x^2+1)^{5/2}}dx}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6203}$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{ 6204}$$

$$\frac{3\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{3\left(\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{2c}+\frac{x(a+ba}{2(}\right.}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{3\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\right.}{2c}}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

---

3.50. $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}dx$

$$\Big\downarrow \text{2838}$$

$$\frac{3\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\operatorname{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x}}\right.}{4d^3}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}+\frac{b}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

output
```
b/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^
2*x^2)^2) + (3*(b/(2*c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*(1
 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLo
g[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c)))/(4*d
^3)
```

### 3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

### 3.50.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| derivativedivides | $\frac{a\left(\frac{cx}{4\left(c^2x^2+1\right)^2}+\frac{3cx}{8\left(c^2x^2+1\right)}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{cx\,\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{3cx\,\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)}+\frac{3\,\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{11}{24\left(c^2x^2+1\right)^{\frac{3}{2}}}+\cdots\right.}{\cdots}$ |
| default | $\frac{a\left(\frac{cx}{4\left(c^2x^2+1\right)^2}+\frac{3cx}{8\left(c^2x^2+1\right)}+\frac{3\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(\frac{cx\,\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{3cx\,\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)}+\frac{3\,\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{11}{24\left(c^2x^2+1\right)^{\frac{3}{2}}}+\cdots\right.}{\cdots}$ |
| parts | $\frac{a\left(\frac{x}{4\left(c^2x^2+1\right)^2}+\frac{3x}{8\left(c^2x^2+1\right)}+\frac{3\arctan(cx)}{8c}\right)}{d^3}+\frac{b\left(\frac{cx\,\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{3cx\,\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)}+\frac{3\,\operatorname{arcsinh}(cx)\arctan(cx)}{8}+\frac{11}{24\left(c^2x^2+1\right)^{\cdots}}\right.}{\cdots}$ |

input
```
int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d^3*(1/4*c*x/(c^2*x^2+1)^2+3/8*c*x/(c^2*x^2+1)+3/8*arctan(c*x))+b/d
^3*(1/4*c*x/(c^2*x^2+1)^2*arcsinh(c*x)+3/8*c*x/(c^2*x^2+1)*arcsinh(c*x)+3/
8*arcsinh(c*x)*arctan(c*x)+11/24/(c^2*x^2+1)^(3/2)+3/8*c^2*x^2/(c^2*x^2+1)
^(3/2)+3/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*arctan(c*x)
*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))+3/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

### 3.50.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
 + d^3), x)
```

### 3.50.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*a
sinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

### 3.50.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/8*a*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c*
x)/(c*d^3)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^
4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.50.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^3, x)
```

### 3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3, x)
```

# 3.51 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3}\,dx$



## 3.51.1 Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3}\,dx = -\frac{bcx}{12d^3(1+c^2x^2)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\text{arcsinh}(cx)}{4d^3(1+c^2x^2)^2} + \frac{a+b\text{arcsinh}(cx)}{2d^3(1+c^2x^2)} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} + \frac{b\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^3}$$

output
```
-1/12*b*c*x/d^3/(c^2*x^2+1)^(3/2)+1/4*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2
+1/2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+
(c^2*x^2+1)^(1/2))^2)/d^3-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+
1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-2/3*b*c*x/d^3/(c^2*x^2+1)^(
1/2)
```

## 3.51.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.82

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3}\,dx = \frac{-\frac{2a^2}{b} + \frac{a}{(1+c^2x^2)^2} - \frac{bcx}{3(1+c^2x^2)^{3/2}} + \frac{2a}{1+c^2x^2} - \frac{8bcx}{3\sqrt{1+c^2x^2}} - 4a\text{arcsinh}(cx) + \frac{b\text{arcsinh}(cx)}{(1+c^2x^2)^2} + \frac{2b\text{arcsinh}(cx)}{1+c^2x^2} - 4b\text{arc}\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

output
```
((-2*a^2)/b + a/(1 + c^2*x^2)^2 - (b*c*x)/(3*(1 + c^2*x^2)^(3/2)) + (2*a)/
(1 + c^2*x^2) - (8*b*c*x)/(3*Sqrt[1 + c^2*x^2]) - 4*a*ArcSinh[c*x] + (b*Ar
cSinh[c*x])/(1 + c^2*x^2)^2 + (2*b*ArcSinh[c*x])/(1 + c^2*x^2) - 4*b*ArcSi
nh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*ArcSinh[c*x]*Log[1 +
(Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 4*a*Log[1 - E^(2*ArcSinh[c*x])] + 4*b*Arc
Sinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] - 2*a*Log[1 + c^2*x^2] - 4*b*PolyLog
[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[
c*x])/c] + 2*b*PolyLog[2, E^(2*ArcSinh[c*x])])/(4*d^3)
```

### 3.51.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6226, 27, 209, 208, 6226, 208, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^3}dx$$

$$\Big\downarrow \text{6226}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{d^2x\left(c^2x^2+1\right)^2}dx}{d}-\frac{bc\int \frac{1}{\left(c^2x^2+1\right)^{5/2}}dx}{4d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^3}-\frac{bc\int \frac{1}{\left(c^2x^2+1\right)^{5/2}}dx}{4d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{209}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^3}-\frac{bc\left(\frac{2}{3}\int \frac{1}{\left(c^2x^2+1\right)^{3/2}}dx+\frac{x}{3\left(c^2x^2+1\right)^{3/2}}\right)}{4d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 6226

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx-\frac{1}{2}bc\int \frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}}{d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 208

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}}{d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 6214

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}}{d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 5984

$$\frac{2\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}}{d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 3042

$$\frac{2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}}{d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 26

$$\frac{2i\int (a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}}{d^3}+\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 4670

3.51. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3}\,dx$

$$\frac{2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a-}{d^3}$$

$$\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\downarrow$ 2715

$$\frac{2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}+i}{d^3}$$

$$\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

$\downarrow$ 2838

$$\frac{2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right)+\frac{a+}{}}{d^3}$$

$$\frac{a+b\text{arcsinh}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{4d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

output
```
-1/4*(b*c*(x/(3*(1 + c^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + c^2*x^2])))/d^3 +
 (a + b*ArcSinh[c*x])/(4*d^3*(1 + c^2*x^2)^2) + (-1/2*(b*c*x)/Sqrt[1 + c^2
*x^2] + (a + b*ArcSinh[c*x])/(2*(1 + c^2*x^2)) + (2*I)*(I*(a + b*ArcSinh[c
*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcSinh[c*x])]
 - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])]))/d^3
```

### 3.51.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.51. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^3}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

## 3.51.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.76

| method | result |
|---|---|
| derivativedivides | $\frac{a\left(\ln(cx)+\frac{1}{4\left(c^2x^2+1\right)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^3}+\frac{b\left(\frac{-8c^3x^3\sqrt{c^2x^2+1}+8c^4x^4+6\operatorname{arcsinh}(cx)c^2x^2-9cx\sqrt{c^2x^2+1}+16c^2x^2}{12c^4x^4+24c^2x^2+12}\right.}{}$ |
| default | $\frac{a\left(\ln(cx)+\frac{1}{4\left(c^2x^2+1\right)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^3}+\frac{b\left(\frac{-8c^3x^3\sqrt{c^2x^2+1}+8c^4x^4+6\operatorname{arcsinh}(cx)c^2x^2-9cx\sqrt{c^2x^2+1}+16c^2x^2}{12c^4x^4+24c^2x^2+12}\right.}{}$ |
| parts | $\frac{a\left(-\frac{c^2\left(-\frac{1}{c^2\left(c^2x^2+1\right)}-\frac{1}{2c^2\left(c^2x^2+1\right)^2}+\frac{\ln\left(c^2x^2+1\right)}{c^2}\right)}{2}+\ln(x)\right)}{d^3}+\frac{b\left(\frac{-8c^3x^3\sqrt{c^2x^2+1}+8c^4x^4+6\operatorname{arcsinh}(cx)c^2x^2-9cx\sqrt{c}}{12c^4x^4+24c^2x^2+12}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d^3*(ln(c*x)+1/4/(c^2*x^2+1)^2+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b/d^3*
(1/12*(-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^4*x^4+6*arcsinh(c*x)*c^2*x^2-9*c*x
*(c^2*x^2+1)^(1/2)+16*c^2*x^2+9*arcsinh(c*x)+8)/(c^4*x^4+2*c^2*x^2+1)+arcs
inh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))-arc
sinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+
1)^(1/2)))
```

### 3.51.5 Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^3}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right) + a}{\left(c^2dx^2 + d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
 + d^3*x), x)
```

### 3.51.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^3}\,dx = \frac{\int \frac{a}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx + \int \frac{b\,\text{asinh}\left(cx\right)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b*a
sinh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

### 3.51.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\left(d + c^2dx^2\right)^3}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right) + a}{\left(c^2dx^2 + d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^2
 + 1)/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*
d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

### 3.51.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x\,(d + c^2dx^2)^3}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^3x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x), x)
```

### 3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x\,(d + c^2dx^2)^3}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x\,(d\,c^2\,x^2 + d)^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3), x)
```

# 3.52 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx$



## 3.52.1 Optimal result

Integrand size = 24, antiderivative size = 222

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx =& -\frac{bc}{12d^3(1+c^2x^2)^{3/2}}-\frac{7bc}{8d^3\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{d^3x(1+c^2x^2)^2}\\&-\frac{5c^2x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}-\frac{15c^2x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)}\\&-\frac{15c(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3}\\&-\frac{b\text{carctanh}\left(\sqrt{1+c^2x^2}\right)}{d^3}+\frac{15ibc\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8d^3}\\&-\frac{15ibc\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8d^3}\end{aligned}$$

output
```
-1/12*b*c/d^3/(c^2*x^2+1)^(3/2)+(-a-b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^2-5/
4*c^2*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))
/d^3/(c^2*x^2+1)-15/4*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d
^3-b*c*arctanh((c^2*x^2+1)^(1/2))/d^3+15/8*I*b*c*polylog(2,-I*(c*x+(c^2*x^
2+1)^(1/2)))/d^3-15/8*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/8*b
*c/d^3/(c^2*x^2+1)^(1/2)
```

### 3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.78 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.34

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx = -\frac{\frac{45(a+b\text{arcsinh}(cx))}{x}-\frac{6(a+b\text{arcsinh}(cx))}{x(1+c^2x^2)^2}-\frac{15(a+b\text{arcsinh}(cx))}{x+c^2x^3}+45ac\arctan(cx)+45bc\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

output
```
-1/24*((45*(a + b*ArcSinh[c*x]))/x - (6*(a + b*ArcSinh[c*x]))/(x*(1 + c^2*
x^2)^2) - (15*(a + b*ArcSinh[c*x]))/(x + c^2*x^3) + 45*a*c*ArcTan[c*x] + 4
5*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (2*b*c*Hypergeometric2F1[-3/2, 1, -1/2,
 1 + c^2*x^2])/(1 + c^2*x^2)^(3/2) + (15*b*c*Hypergeometric2F1[-1/2, 1, 1/
2, 1 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 +
(c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (Sqr
t[-c^2]*E^ArcSinh[c*x])/c] - 45*b*Sqrt[-c^2]*PolyLog[2, (c*E^ArcSinh[c*x])
/Sqrt[-c^2]] + 45*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/
d^3
```

### 3.52.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6224, 27, 243, 61, 61, 73, 221, 6203, 241, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2dx^2+d)^3}\,dx$$

$$\Big\downarrow \text{6224}$$

$$-5c^2\int \frac{a+b\text{arcsinh}(cx)}{d^3(c^2x^2+1)^3}dx+\frac{bc\int \frac{1}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x(c^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx}{d^3}+\frac{bc\int \frac{1}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx}{d^3}+\frac{bc\int \frac{1}{x^2(c^2x^2+1)^{5/2}}dx^2}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(\int \frac{1}{x^2(c^2x^2+1)^{3/2}}dx^2+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5c^2\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{6203}$$

$$-\frac{5c^2\left(\frac{3}{4}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx-\frac{1}{4}bc\int \frac{x}{(c^2x^2+1)^{5/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{241}$$

$$-\frac{5c^2\left(\frac{3}{4}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

---

3.52. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx$

$$\Big\downarrow \text{6203}$$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{241}$$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{6204}$$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\right.}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{2c}+\right.\right.}{d^3}-\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{2715}$$

---

3.52. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(c\ldots}\right.}{2c}\right.\right.}{d^3}$$

$$\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 2838

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\ldots\right.\right.}{d^3}$$

$$\frac{a+b\text{arcsinh}(cx)}{d^3x\left(c^2x^2+1\right)^2}+\frac{bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

output
```
-((a + b*ArcSinh[c*x])/(d^3*x*(1 + c^2*x^2)^2)) + (b*c*(2/(3*(1 + c^2*x^2)
^(3/2)) + 2/Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/(2*d^3) - (
5*c^2*(b/(12*c*(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSinh[c*x]))/(4*(1 + c^2
*x^2)^2) + (3*(b/(2*c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*(1
+ c^2*x^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog
[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c)))/4))/d
^3
```

### 3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.52. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^3}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.52.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| derivativedivides | $c \Bigg( \frac{a\left(-\frac{1}{cx}-\frac{\frac{7}{8}c^3x^3+\frac{9}{8}cx}{(c^2x^2+1)^2}-\frac{15\arctan(cx)}{8}\right)}{d^3} + \frac{b\Big(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{7c^3x^3\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{9cx\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{15\operatorname{arcsinh}(cx)\arctan(}{8}$ |
| default | $c \Bigg( \frac{a\left(-\frac{1}{cx}-\frac{\frac{7}{8}c^3x^3+\frac{9}{8}cx}{(c^2x^2+1)^2}-\frac{15\arctan(cx)}{8}\right)}{d^3} + \frac{b\Big(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{7c^3x^3\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{9cx\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{15\operatorname{arcsinh}(cx)\arctan(}{8}$ |
| parts | $\frac{a\left(-c^2\left(\frac{\frac{7}{8}x^3c^2+\frac{9}{8}x}{(c^2x^2+1)^2}+\frac{15\arctan(cx)}{8c}\right)-\frac{1}{x}\right)}{d^3} + \frac{bc\Big(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{7c^3x^3\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{9cx\operatorname{arcsinh}(cx)}{8(c^2x^2+1)^2}-\frac{15\operatorname{arcsinh}(cx)\arctan}{8}$ |

input
```
int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(a/d^3*(-1/c/x-(7/8*c^3*x^3+9/8*c*x)/(c^2*x^2+1)^2-15/8*arctan(c*x))+b/d
^3*(-arcsinh(c*x)/c/x-7/8*c^3*x^3/(c^2*x^2+1)^2*arcsinh(c*x)-9/8*c*x/(c^2*
x^2+1)^2*arcsinh(c*x)-15/8*arcsinh(c*x)*arctan(c*x)-15/8*arctan(c*x)*ln(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+15/8*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))+15/8*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-15/8*I*dilog(1-I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))-47/24/(c^2*x^2+1)^(3/2)+1/(c^2*x^2+1)^(1/2)-ar
ctanh(1/(c^2*x^2+1)^(1/2))-15/8*c^2*x^2/(c^2*x^2+1)^(3/2)))
```

### 3.52.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 + d^3*x^2), x)
```

### 3.52.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\frac{\int \frac{a}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx+\int \frac{b\operatorname{asinh}\left(cx\right)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integral(
b*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x))/d**3
```

### 3.52.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/8*a*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*x
) + 15*c*arctan(c*x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*
d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

### 3.52.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^2), x)
```

### 3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^2\,\left(d\,c^2\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3), x)
```

# 3.53 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx$



## 3.53.1 Optimal result

Integrand size = 24, antiderivative size = 232

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx = &-\frac{bc}{2d^3x(1+c^2x^2)^{3/2}}-\frac{5bc^3x}{12d^3(1+c^2x^2)^{3/2}}\\&+\frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}}-\frac{3c^2(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}\\&-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2(1+c^2x^2)^2}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2d^3(1+c^2x^2)}\\&+\frac{6c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3}\\&+\frac{3bc^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{2d^3}-\frac{3bc^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{2d^3}\end{aligned}$$

output
```
-1/2*b*c/d^3/x/(c^2*x^2+1)^(3/2)-5/12*b*c^3*x/d^3/(c^2*x^2+1)^(3/2)-3/4*c^
2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/2*(-a-b*arcsinh(c*x))/d^3/x^2/(c^
2*x^2+1)^2-3/2*c^2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c
*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b*c^2*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))^2)/d^3-3/2*b*c^2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+2
/3*b*c^3*x/d^3/(c^2*x^2+1)^(1/2)
```

### 3.53.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.52

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx$$
$$= \frac{-\frac{18bc\sqrt{1+c^2x^2}}{x}+\frac{9bc(1+2c^2x^2)}{x\sqrt{1+c^2x^2}}+\frac{bc(3+12c^2x^2+8c^4x^4)}{x(1+c^2x^2)^{3/2}}-18bc^2\text{arcsinh}(cx)^2-\frac{18(a+b\text{arcsinh}(cx))}{x^2}+\frac{3(a+b\text{arcsinh}(cx))}{(x+c^2x^3)^2}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

output
```
((-18*b*c*Sqrt[1 + c^2*x^2])/x + (9*b*c*(1 + 2*c^2*x^2))/(x*Sqrt[1 + c^2*x
^2]) + (b*c*(3 + 12*c^2*x^2 + 8*c^4*x^4))/(x*(1 + c^2*x^2)^(3/2)) - 18*b*c
^2*ArcSinh[c*x]^2 - (18*(a + b*ArcSinh[c*x]))/x^2 + (3*(a + b*ArcSinh[c*x]
))/(x + c^2*x^3)^2 + (9*(a + b*ArcSinh[c*x]))/(x^2 + c^2*x^4) + (18*c^2*(a
 + b*ArcSinh[c*x])^2)/b + 36*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])
/Sqrt[-c^2]] + 36*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c
] + 18*a*c^2*Log[1 + c^2*x^2] + 36*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqr
t[-c^2]] + 36*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 18*c^2*(2*
(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSi
nh[c*x])]))/(12*d^3)
```

### 3.53.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.21, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {6224, 27, 245, 209, 208, 6226, 209, 208, 6226, 208, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(c^2dx^2+d)^3}\,dx$$
$$\Big\downarrow \text{ 6224}$$
$$-3c^2\int \frac{a+b\text{arcsinh}(cx)}{d^3x(c^2x^2+1)^3}dx+\frac{bc\int \frac{1}{x^2(c^2x^2+1)^{5/2}}dx}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2(c^2x^2+1)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\int \frac{1}{x^2(c^2x^2+1)^{5/2}}dx}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 245$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(-4c^2\int \frac{1}{(c^2x^2+1)^{5/2}}dx-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 209$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(-4c^2\left(\frac{2}{3}\int \frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 208$$

$$-\frac{3c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^3}dx}{d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 6226$$

$$-\frac{3c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{1}{4}bc\int \frac{1}{(c^2x^2+1)^{5/2}}dx+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 209$$

$$-\frac{3c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{1}{4}bc\left(\frac{2}{3}\int \frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{x}{3(c^2x^2+1)^{3/2}}\right)+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 208$$

$$-\frac{3c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)\right)}{d^3}-\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \ 6226$$

---

3.53. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx$

$$-\frac{3c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx-\frac{1}{2}bc\int \frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 208

$$-\frac{3c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}-\frac{bcx}{2\sqrt{c^2x^2+1}}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)\right)}{d^3}-$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 6214

$$-\frac{3c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}-\frac{bcx}{2\sqrt{c^2x^2+1}}-\frac{1}{4}bc\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)}{d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 5984

$$-\frac{3c^2\left(2\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}-\frac{bcx}{2\sqrt{c^2x^2+1}}-\frac{1}{4}bc\left(\right.}{d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 3042

$$-\frac{3c^2\left(2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}-\frac{bcx}{2\sqrt{c^2x^2+1}}-\frac{1}{4}bc\right.}{d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 26

---

3.53. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx$

$$-\frac{3c^2\left(2i\int(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}+\frac{a+b\text{arcsinh}(cx)}{4(c^2x^2+1)^2}-\frac{bcx}{2\sqrt{c^2x^2+1}}-\frac{1}{4}bc(\right.}{d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \textbf{4670}$$

$$-\frac{3c^2\left(2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}\left(e^{2\text{arcsinh}(c\right.\right.\right.}{d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \textbf{2715}$$

$$-\frac{3c^2\left(2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(\right.\right.}{}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \textbf{2838}$$

$$-\frac{3c^2\left(2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right.\right.}{d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-4c^2\left(\frac{2x}{3\sqrt{c^2x^2+1}}+\frac{x}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

output
```
(b*c*(-(1/(x*(1 + c^2*x^2)^(3/2))) - 4*c^2*(x/(3*(1 + c^2*x^2)^(3/2)) + (2
*x)/(3*Sqrt[1 + c^2*x^2]))))/(2*d^3) - (a + b*ArcSinh[c*x])/(2*d^3*x^2*(1
+ c^2*x^2)^2) - (3*c^2*(-1/2*(b*c*x)/Sqrt[1 + c^2*x^2] - (b*c*(x/(3*(1 + c
^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + c^2*x^2])))/4 + (a + b*ArcSinh[c*x])/(4
*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])/(2*(1 + c^2*x^2)) + (2*I)*(I*(a +
 b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyLog[2, -E^(2*Ar
cSinh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])])))/d^3
```

## 3.53. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx$

### 3.53.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.53. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx$

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

### 3.53.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.48

| method | result |
|---|---|
| derivativedivides | $c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b\left(-\frac{-8c^5x^5\sqrt{c^2x^2+1}+8c^6x^6+18\,\mathrm{arcsinh}(cx)c^4x^4-\right.}{}$ |
| default | $c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b\left(-\frac{-8c^5x^5\sqrt{c^2x^2+1}+8c^6x^6+18\,\mathrm{arcsinh}(cx)c^4x^4-\right.}{}$ |
| parts | $\frac{a\left(\frac{c^4\left(-\frac{2}{c^2(c^2x^2+1)}-\frac{1}{2c^2(c^2x^2+1)^2}+\frac{3\ln(c^2x^2+1)}{c^2}\right)}{2}-\frac{1}{2x^2}-3c^2\ln(x)\right)}{d^3}+\frac{b\,c^2\left(-\frac{-8c^5x^5\sqrt{c^2x^2+1}+8c^6x^6+18\,\mathrm{arcsinh}(\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/d^3*(-1/2/c^2/x^2-3*ln(c*x)-1/4/(c^2*x^2+1)^2-1/(c^2*x^2+1)+3/2*ln(
c^2*x^2+1))+b/d^3*(-1/12/(c^4*x^4+2*c^2*x^2+1)/c^2/x^2*(-8*c^5*x^5*(c^2*x^
2+1)^(1/2)+8*c^6*x^6+18*arcsinh(c*x)*c^4*x^4-3*c^3*x^3*(c^2*x^2+1)^(1/2)+1
6*c^4*x^4+27*arcsinh(c*x)*c^2*x^2+6*c*x*(c^2*x^2+1)^(1/2)+8*c^2*x^2+6*arcs
inh(c*x))-3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-3*polylog(2,-c*x-(c^2
*x^2+1)^(1/2))+3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+3/2*polylog(
2,-(c*x+(c^2*x^2+1)^(1/2))^2)-3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-3
*polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

## 3.53.5 Fricas [F]

$$\int \frac{a+b\,\mathrm{arcsinh}(cx)}{x^3(d+c^2dx^2)^3}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(c^2dx^2+d)^3x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
 + d^3*x^3), x)
```

### 3.53.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx = \frac{\int \frac{a}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integral(
b*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x))/d**3
```

### 3.53.7 Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^2
) - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + b*integrate(log(c*x
+ sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^
3), x)
```

### 3.53.8 Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^3), x)
```

### 3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,\left(d\,c^2\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3), x)
```

# 3.54 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx$



## 3.54.1 Optimal result

Integrand size = 24, antiderivative size = 295

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx =& -\frac{bc^3}{12d^3(1+c^2x^2)^{3/2}}-\frac{bc}{6d^3x^2(1+c^2x^2)^{3/2}}+\frac{29bc^3}{24d^3\sqrt{1+c^2x^2}}\\&-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(1+c^2x^2)^2}+\frac{7c^2(a+b\text{arcsinh}(cx))}{3d^3x(1+c^2x^2)^2}\\&+\frac{35c^4x(a+b\text{arcsinh}(cx))}{12d^3(1+c^2x^2)^2}+\frac{35c^4x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)}\\&+\frac{35c^3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3}\\&+\frac{19bc^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{6d^3}-\frac{35ibc^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8d^3}\\&+\frac{35ibc^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8d^3}\end{aligned}$$

output
```
-1/12*b*c^3/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c/d^3/x^2/(c^2*x^2+1)^(3/2)+1/3*(-
a-b*arcsinh(c*x))/d^3/x^3/(c^2*x^2+1)^2+7/3*c^2*(a+b*arcsinh(c*x))/d^3/x/(
c^2*x^2+1)^2+35/12*c^4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(
a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+35/4*c^3*(a+b*arcsinh(c*x))*arctan(c*x+(
c^2*x^2+1)^(1/2))/d^3+19/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))/d^3-35/8*I*b*c
^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+35/8*I*b*c^3*polylog(2,I*(c*x
+(c^2*x^2+1)^(1/2)))/d^3+29/24*b*c^3/d^3/(c^2*x^2+1)^(1/2)
```

### 3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.06 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.27

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx$$

$$= \frac{\frac{3(a+b\operatorname{arcsinh}(cx))}{x^3(1+c^2x^2)^2}+\frac{21(a+b\operatorname{arcsinh}(cx))}{2(x^3+c^2x^5)}+\frac{bc^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},1+c^2x^2\right)}{(1+c^2x^2)^{3/2}}+\frac{21bc^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1+c^2x^2\right)}{2\sqrt{1+c^2x^2}}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

output
```
((3*(a + b*ArcSinh[c*x]))/(x^3*(1 + c^2*x^2)^2) + (21*(a + b*ArcSinh[c*x])
)/(2*(x^3 + c^2*x^5)) + (b*c^3*Hypergeometric2F1[-3/2, 2, -1/2, 1 + c^2*x^
2])/(1 + c^2*x^2)^(3/2) + (21*b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^
2*x^2])/(2*Sqrt[1 + c^2*x^2]) + (35*(-2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c
^2*x^2] - 2*b*ArcSinh[c*x] + 6*b*c^2*x^2*ArcSinh[c*x] + 6*a*c^3*x^3*ArcTan
[c*x] + 7*b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] - 6*b*(-c^2)^(3/2)*x^3*ArcS
inh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 6*b*(-c^2)^(3/2)*x^3*Arc
Sinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 6*b*(-c^2)^(3/2)*x^3*Po
lyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 6*b*(-c^2)^(3/2)*x^3*PolyLog[2,
(Sqrt[-c^2]*E^ArcSinh[c*x])/c]))/(4*x^3))/(12*d^3)
```

### 3.54.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.23, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.958$, Rules used = {6224, 27, 243, 52, 61, 61, 73, 221, 6224, 243, 61, 61, 73, 221, 6203, 241, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(c^2dx^2+d\right)^3}\,dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{7}{3}c^2\int \frac{a+b\operatorname{arcsinh}(cx)}{d^3x^2\left(c^2x^2+1\right)^3}dx+\frac{bc\int \frac{1}{x^3\left(c^2x^2+1\right)^{5/2}}dx}{3d^3}-\frac{a+b\operatorname{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{7c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^3}dx}{3d^3}+\frac{bc\int \frac{1}{x^3(c^2x^2+1)^{5/2}}dx}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 243$$

$$-\frac{7c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^3}dx}{3d^3}+\frac{bc\int \frac{1}{x^4(c^2x^2+1)^{5/2}}dx^2}{6d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 52$$

$$-\frac{7c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^3}dx}{3d^3}+\frac{bc\left(-\frac{5}{2}c^2\int \frac{1}{x^2(c^2x^2+1)^{5/2}}dx^2-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 61$$

$$-\frac{7c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^3}dx}{3d^3}+\frac{bc\left(-\frac{5}{2}c^2\left(\int \frac{1}{x^2(c^2x^2+1)^{3/2}}dx^2+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 61$$

$$-\frac{7c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^3}dx}{3d^3}+\frac{bc\left(-\frac{5}{2}c^2\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{7c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^3}dx}{3d^3}+\frac{bc\left(-\frac{5}{2}c^2\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 221$$

$$-\frac{7c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^3}dx}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \ 6224$$

---

3.54. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

$$-\frac{7c^2\left(-5c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx+bc\int\frac{1}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}\right)}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow 243$$

$$-\frac{7c^2\left(-5c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)^{5/2}}dx^2-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}\right)}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow 61$$

$$-\frac{7c^2\left(-5c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx+\frac{1}{2}bc\left(\int\frac{1}{x^2(c^2x^2+1)^{3/2}}dx^2+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}\right)}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow 61$$

$$-\frac{7c^2\left(-5c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx+\frac{1}{2}bc\left(\int\frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}\right)}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow 73$$

$$-\frac{7c^2\left(-5c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx+\frac{1}{2}bc\left(\frac{2\int\frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}\right)}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow 221$$

$$-\frac{7c^2\left(-5c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^3}dx-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)\right)}{3d^3}-\frac{a+b\text{arcsinh}(cx)}{3d^3x^3(c^2x^2+1)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

---

## 3.54. $\int\frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

$\downarrow$ 6203

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx-\frac{1}{4}bc\int\frac{x}{(c^2x^2+1)^{5/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\right.\right.\right.}{3d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 241

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2}\right.\right.\right.}{3d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 6203

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int\frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/}}\right.\right.}{3d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 241

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/2}}\right)-\frac{a+b}{x}\right.}{3d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 6204

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)+\frac{x(a+b\text{arcsinh}(cx))}{4(c^2x^2+1)^2}+\frac{b}{12c(c^2x^2+1)^{3/}}\right.\right.}{3d^3}$$
$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 3042

---

3.54. $\int\frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx$

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)+\frac{x(a+b\text{arcsinh}}{4(c^2x^2+1)}\right.\right.}{3d^3}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 4668

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}}{2c}\right.\right.\right.}{}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 2715

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e\right.}{2c}\right.\right.\right.}{}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 2838

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsin}}{2(c^2x^2+1}\right.\right.\right.}{}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{\sqrt{c^2x^2+1}}+\frac{2}{3(c^2x^2+1)^{3/2}}\right)-\frac{1}{x^2(c^2x^2+1)^{3/2}}\right)}{6d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

## 3.54. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

output
```
-1/3*(a + b*ArcSinh[c*x])/(d^3*x^3*(1 + c^2*x^2)^2) + (b*c*(-(1/(x^2*(1 +
c^2*x^2)^(3/2))) - (5*c^2*(2/(3*(1 + c^2*x^2)^(3/2)) + 2/Sqrt[1 + c^2*x^2]
 - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/2))/(6*d^3) - (7*c^2*(-((a + b*ArcSinh[c
*x])/(x*(1 + c^2*x^2)^2)) + (b*c*(2/(3*(1 + c^2*x^2)^(3/2)) + 2/Sqrt[1 + c
^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/2 - 5*c^2*(b/(12*c*(1 + c^2*x^2)^
(3/2)) + (x*(a + b*ArcSinh[c*x]))/(4*(1 + c^2*x^2)^2) + (3*(b/(2*c*Sqrt[1
+ c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*(1 + c^2*x^2)) + (2*(a + b*ArcSi
nh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*
b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c)))/4)))/(3*d^3)
```

### 3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.54. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.54.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{\frac{11}{8}c^3x^3+\frac{13}{8}cx}{\left(c^2x^2+1\right)^2}+\frac{35\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{3\operatorname{arcsinh}(cx)}{cx}+\frac{11c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{13cx\operatorname{arcsin}}{8\left(c^2x^2+\right.}\right.}{}\right.$ |
| default | $c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{\frac{11}{8}c^3x^3+\frac{13}{8}cx}{\left(c^2x^2+1\right)^2}+\frac{35\arctan(cx)}{8}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{3\operatorname{arcsinh}(cx)}{cx}+\frac{11c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{13cx\operatorname{arcsin}}{8\left(c^2x^2+\right.}\right.}{}\right.$ |
| parts | $\frac{a\left(c^4\left(\frac{\frac{11}{8}x^3c^2+\frac{13}{8}x}{\left(c^2x^2+1\right)^2}+\frac{35\arctan(cx)}{8c}\right)-\frac{1}{3x^3}+\frac{3c^2}{x}\right)}{d^3}+\frac{bc^3\left(-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}+\frac{3\operatorname{arcsinh}(cx)}{cx}+\frac{11c^3x^3\operatorname{arcsinh}(cx)}{8\left(c^2x^2+1\right)^2}+\frac{13cx\operatorname{arcsinh}}{8\left(c^2x^2+1\right.}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3*(a/d^3*(-1/3/c^3/x^3+3/c/x+(11/8*c^3*x^3+13/8*c*x)/(c^2*x^2+1)^2+35/8*
arctan(c*x))+b/d^3*(-1/3*arcsinh(c*x)/c^3/x^3+3*arcsinh(c*x)/c/x+11/8*c^3*
x^3/(c^2*x^2+1)^2*arcsinh(c*x)+13/8*c*x/(c^2*x^2+1)^2*arcsinh(c*x)+35/8*ar
csinh(c*x)*arctan(c*x)+103/24/(c^2*x^2+1)^(3/2)-1/6/c^2/x^2/(c^2*x^2+1)^(3
/2)-19/6/(c^2*x^2+1)^(1/2)+19/6*arctanh(1/(c^2*x^2+1)^(1/2))+35/8*c^2*x^2/
(c^2*x^2+1)^(3/2)+35/8*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-35/
8*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-35/8*I*dilog(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+35/8*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))))
```

### 3.54.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^3x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^
6 + d^3*x^4), x)
```

### 3.54.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx=\frac{\int \frac{a}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integral
(b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x))/d**3
```

### 3.54.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^3}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)^3x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/24*a*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2
- 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + b*integrate(log(c*x + sqrt
(c^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x
)
```

### 3.54.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^3x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^4), x)
```

### 3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^4\left(d\,c^2\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3), x)
```

# 3.55 $\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.55.1 Optimal result

Integrand size = 26, antiderivative size = 109

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=\frac{2b\sqrt{\pi}x}{15c^3}-\frac{b\sqrt{\pi}x^3}{45c}-\frac{1}{25}bc\sqrt{\pi}x^5-\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4\pi}+\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4\pi^2}$$

output
```
-1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/5*(Pi*c^2*x^2+Pi)^(
5/2)*(a+b*arcsinh(c*x))/c^4/Pi^2+2/15*b*x*Pi^(1/2)/c^3-1/45*b*x^3*Pi^(1/2)
/c-1/25*b*c*x^5*Pi^(1/2)
```

## 3.55.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{\pi}\left(15a\sqrt{1+c^2x^2}(-2+c^2x^2+3c^4x^4)+b(30cx-5c^3x^3-9c^5x^5)+15b\sqrt{1+c^2x^2}(-2+c^2x^2+3c^4x^4)\,\text{a}\right.}{225c^4}$$

input
```
Integrate[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[Pi]*(15*a*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4) + b*(30*c*x -
 5*c^3*x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4
)*ArcSinh[c*x]))/(225*c^4)
```

### 3.55.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6219, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{-3c^4x^4-c^2x^2+2}{15c^4}dx+\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi c^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\pi}b\int\left(-3c^4x^4-c^2x^2+2\right)dx}{15c^3}+\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi c^4}+\frac{\sqrt{\pi}b\left(-\frac{3}{5}c^4x^5-\frac{c^2x^3}{3}+2x\right)}{15c^3}$$

input
```
Int[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(b*Sqrt[Pi]*(2*x - (c^2*x^3)/3 - (3*c^4*x^5)/5))/(15*c^3) - ((Pi + c^2*Pi*
x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*Pi) + ((Pi + c^2*Pi*x^2)^(5/2)*(a
+ b*ArcSinh[c*x]))/(5*c^4*Pi^2)
```

### 3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.55.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| default | $a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{5\pi c^2}-\frac{2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{15\pi c^4}\right)+\frac{b\sqrt{\pi}\left(45\operatorname{arcsinh}(cx)c^6x^6+60\operatorname{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}-15\operatorname{arcsinh}(cx)c^2x^2\right.}{225c^4\sqrt{c^2x^2+1}}$ |
| parts | $a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{5\pi c^2}-\frac{2(\pi c^2x^2+\pi)^{\frac{3}{2}}}{15\pi c^4}\right)+\frac{b\sqrt{\pi}\left(45\operatorname{arcsinh}(cx)c^6x^6+60\operatorname{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}-15\operatorname{arcsinh}(cx)c^2x^2\right.}{225c^4\sqrt{c^2x^2+1}}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*x^2*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-2/15/Pi/c^4*(Pi*c^2*x^2+Pi)^(3/2))
+1/225*b/c^4*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6+60*arcsin
h(c*x)*c^4*x^4-9*c^5*x^5*(c^2*x^2+1)^(1/2)-15*arcsinh(c*x)*c^2*x^2-5*c^3*x
^3*(c^2*x^2+1)^(1/2)-30*arcsinh(c*x)+30*c*x*(c^2*x^2+1)^(1/2))
```

### 3.55.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.45

$$\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$$
$$= \frac{15\sqrt{\pi + \pi c^2x^2}(3\,bc^6x^6 + 4\,bc^4x^4 - bc^2x^2 - 2\,b)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \sqrt{\pi + \pi c^2x^2}\left(45\,ac^6x^6 + 60\,ac^4x^4\right.}{225\,(c^6x^2 + c^4)}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2
*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 + 6
0*a*c^4*x^4 - 15*a*c^2*x^2 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c
^2*x^2 + 1) - 30*a))/(c^6*x^2 + c^4)
```

### 3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(100) = 200$.

Time = 0.90 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.03

$$\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{\sqrt{\pi}ax^4\sqrt{c^2x^2+1}}{5} + \frac{\sqrt{\pi}ax^2\sqrt{c^2x^2+1}}{15c^2} - \frac{2\sqrt{\pi}a\sqrt{c^2x^2+1}}{15c^4} - \frac{\sqrt{\pi}bcx^5}{25} + \frac{\sqrt{\pi}bx^4\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{5} - \frac{\sqrt{\pi}bx^3}{45c} + \frac{\sqrt{\pi}bx^2\sqrt{c^2x^2+1}\,\text{asinh}\,(}{15c^2} \\ \frac{\sqrt{\pi}ax^4}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((sqrt(pi)*a*x**4*sqrt(c**2*x**2 + 1)/5 + sqrt(pi)*a*x**2*sqrt(c*
*2*x**2 + 1)/(15*c**2) - 2*sqrt(pi)*a*sqrt(c**2*x**2 + 1)/(15*c**4) - sqrt
(pi)*b*c*x**5/25 + sqrt(pi)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5 - sqrt
(pi)*b*x**3/(45*c) + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c*
*2) + 2*sqrt(pi)*b*x/(15*c**3) - 2*sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*
x)/(15*c**4), Ne(c, 0)), (sqrt(pi)*a*x**4/4, True))
```

### 3.55.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{15}b\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)\text{arsinh}\left(cx\right)$$
$$+\frac{1}{15}a\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)-\frac{\left(9\sqrt{\pi}c^4x^5+5\sqrt{\pi}c^2x^3-30\sqrt{\pi}x\right)b}{225\,c^3}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
1/15*b*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(3/2)
/(pi*c^4))*arcsinh(c*x) + 1/15*a*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) -
 2*(pi + pi*c^2*x^2)^(3/2)/(pi*c^4)) - 1/225*(9*sqrt(pi)*c^4*x^5 + 5*sqrt(
pi)*c^2*x^3 - 30*sqrt(pi)*x)*b/c^3
```

### 3.55.8 Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.55.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int x^3\,(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.56 $\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.56.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=-\frac{b\sqrt{\pi}x^2}{16c}-\frac{1}{16}bc\sqrt{\pi}x^4+\frac{\sqrt{\pi}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))-\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{16bc^3}$$

output
```
-1/16*b*x^2*Pi^(1/2)/c-1/16*b*c*x^4*Pi^(1/2)-1/16*(a+b*arcsinh(c*x))^2*Pi^
(1/2)/b/c^3+1/8*x*(a+b*arcsinh(c*x))*Pi^(1/2)*(c^2*x^2+1)^(1/2)/c^2+1/4*x^
3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.56.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{\sqrt{\pi}\left(16acx\sqrt{1+c^2x^2}(1+2c^2x^2)-8b\text{arcsinh}(cx)^2-b\cosh(4\text{arcsinh}(cx))+\text{arcsinh}(cx)(-16a+4b\sinh(\right.}{128c^3}$$

input
```
Integrate[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[Pi]*(16*a*c*x*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2) - 8*b*ArcSinh[c*x]^2
 - b*Cosh[4*ArcSinh[c*x]] + ArcSinh[c*x]*(-16*a + 4*b*Sinh[4*ArcSinh[c*x]]
)))/(128*c^3)
```

### 3.56.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6221, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6221}$$

$$\frac{1}{4}\sqrt{\pi}\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{4}\sqrt{\pi}bc\int x^3dx+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}\sqrt{\pi}\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{16}\sqrt{\pi}bcx^4$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{4}\sqrt{\pi}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{16}\sqrt{\pi}bcx^4$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}\sqrt{\pi}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{16}\sqrt{\pi}bcx^4$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+$$
$$\frac{1}{4}\sqrt{\pi}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)-\frac{1}{16}\sqrt{\pi}bcx^4$$

input
```
Int[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/16*(b*c*Sqrt[Pi]*x^4) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])
)/4 + (Sqrt[Pi]*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3)))/4
```

### 3.56.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.56.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| default | $\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4\pi c^2}-\frac{ax\sqrt{\pi c^2x^2+\pi}}{8c^2}-\frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8c^2\sqrt{\pi c^2}}-\frac{b\sqrt{\pi}\left(-4\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4-2\,\mathrm{arcsinh}(cx)c\right.}{16c^3}$ |
| parts | $\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4\pi c^2}-\frac{ax\sqrt{\pi c^2x^2+\pi}}{8c^2}-\frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8c^2\sqrt{\pi c^2}}-\frac{b\sqrt{\pi}\left(-4\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4-2\,\mathrm{arcsinh}(cx)c\right.}{16c^3}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-1/8*a/c^2*x*(Pi*c^2*x^2+Pi)^(1/2)-1/8
*a/c^2*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
-1/16*b*Pi^(1/2)*(-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+c^4*x^4-2*arcs
inh(c*x)*c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)^2)/c^3
```

### 3.56.5 Fricas [F]

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\mathrm{arcsinh}(cx))\,dx=\int\sqrt{\pi+\pi c^2x^2}(b\,\mathrm{arsinh}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

### 3.56.6 Sympy [F]

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\mathrm{arcsinh}(cx))\,dx=\sqrt{\pi}\left(\int ax^2\sqrt{c^2x^2+1}\,dx+\int bx^2\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)\,dx\right)$$

input
```
integrate(x**2*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
sqrt(pi)*(Integral(a*x**2*sqrt(c**2*x**2 + 1), x) + Integral(b*x**2*sqrt(c
**2*x**2 + 1)*asinh(c*x), x))
```

### 3.56.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.56.8 Giac [F]

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=\int\sqrt{\pi+\pi c^2x^2}(b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)*x^2, x)
```

### 3.56.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2+\Pi}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.57 $\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$



## 3.57.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = -\frac{b\sqrt{\pi}x}{3c} - \frac{1}{9}bc\sqrt{\pi}x^3 + \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{3c^2\pi}$$

output
```
1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^2/Pi-1/3*b*x*Pi^(1/2)/c-1/9
*b*c*x^3*Pi^(1/2)
```

## 3.57.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{\sqrt{\pi}\left(3a(1 + c^2x^2)^{3/2} - bcx(3 + c^2x^2) + 3b(1 + c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{9c^2}$$

input
```
Integrate[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[Pi]*(3*a*(1 + c^2*x^2)^(3/2) - b*c*x*(3 + c^2*x^2) + 3*b*(1 + c^2*x^
2)^(3/2)*ArcSinh[c*x]))/(9*c^2)
```

### 3.57.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{\sqrt{\pi}b\int \left(c^2x^2+1\right)dx}{3c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{\sqrt{\pi}b\left(\frac{c^2x^3}{3}+x\right)}{3c}$$

input
```
Int[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/3*(b*Sqrt[Pi]*(x + (c^2*x^3)/3))/c + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(3*c^2*Pi)
```

#### 3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.57.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(49) = 98.

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

| method | result | size |
|---|---|---|
| default | $\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi c^2}+\frac{b\sqrt{\pi}\left(3\operatorname{arcsinh}(cx)c^4x^4+6\operatorname{arcsinh}(cx)c^2x^2-c^3x^3\sqrt{c^2x^2+1}+3\operatorname{arcsinh}(cx)-3cx\sqrt{c^2x^2+1}\right)}{9c^2\sqrt{c^2x^2+1}}$ | 108 |
| parts | $\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi c^2}+\frac{b\sqrt{\pi}\left(3\operatorname{arcsinh}(cx)c^4x^4+6\operatorname{arcsinh}(cx)c^2x^2-c^3x^3\sqrt{c^2x^2+1}+3\operatorname{arcsinh}(cx)-3cx\sqrt{c^2x^2+1}\right)}{9c^2\sqrt{c^2x^2+1}}$ | 108 |

input
```
int(x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2+1/9*b/c^2*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3
*arcsinh(c*x)*c^4*x^4+6*arcsinh(c*x)*c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)+3*a
rcsinh(c*x)-3*c*x*(c^2*x^2+1)^(1/2))
```

### 3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.08

$$\int x\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{3\sqrt{\pi+\pi c^2x^2}(bc^4x^4+2bc^2x^2+b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3ac^4x^4+6ac^2x^2-(bc^3x^3+3bc\right.}{9(c^4x^2+c^2)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas"
)
```

output
```
1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(
c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*
x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1) + 3*a))/(c^4*x^2 + c^2)
```

### 3.57.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(53) = 106$.

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.31

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{\sqrt{\pi}ax^2\sqrt{c^2x^2+1}}{3} + \frac{\sqrt{\pi}a\sqrt{c^2x^2+1}}{3c^2} - \frac{\sqrt{\pi}bcx^3}{9} + \frac{\sqrt{\pi}bx^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{3} - \frac{\sqrt{\pi}bx}{3c} + \frac{\sqrt{\pi}b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{3c^2} & \text{for } c \neq 0 \\ \frac{\sqrt{\pi}ax^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((sqrt(pi)*a*x**2*sqrt(c**2*x**2 + 1)/3 + sqrt(pi)*a*sqrt(c**2*x*
*2 + 1)/(3*c**2) - sqrt(pi)*b*c*x**3/9 + sqrt(pi)*b*x**2*sqrt(c**2*x**2 +
1)*asinh(c*x)/3 - sqrt(pi)*b*x/(3*c) + sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asin
h(c*x)/(3*c**2), Ne(c, 0)), (sqrt(pi)*a*x**2/2, True))
```

### 3.57.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}b\,\text{arsinh}\,(cx)}{3\,\pi c^2} - \frac{\left(\pi^{\frac{3}{2}}c^2x^3 + 3\,\pi^{\frac{3}{2}}x\right)b}{9\,\pi c} + \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}a}{3\,\pi c^2}$$

input
```
integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima"
)
```

output
```
1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*c^2) - 1/9*(pi^(3/2)*c^2*x^
3 + 3*pi^(3/2)*x)*b/(pi*c) + 1/3*(pi + pi*c^2*x^2)^(3/2)*a/(pi*c^2)
```

### 3.57.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.57.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int x\,(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.58 $\int \sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.58.1 Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=-\frac{1}{4}bc\sqrt{\pi}x^2+\frac{1}{2}x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}$$

output
```
-1/4*b*c*x^2*Pi^(1/2)+1/4*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b/c+1/2*x*(a+b*arc
sinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.58.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int \sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{\pi}\left(4acx\sqrt{1+c^2x^2}+2b\text{arcsinh}(cx)^2-b\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(2a+b\sinh(2\text{arcsinh}(cx)))\right)}{8c}$$

input
```
Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[Pi]*(4*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSi
nh[c*x]] + 2*ArcSinh[c*x]*(2*a + b*Sinh[2*ArcSinh[c*x]])))/(8*c)
```

### 3.58.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\pi c^2 x^2+\pi}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6200}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}\sqrt{\pi}bcx^2$$

input
```
Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2
 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)
```

#### 3.58.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

### 3.58.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.49

| method | result | size |
|---|---|---|
| default | $\frac{ax\sqrt{\pi c^2x^2+\pi}}{2}+\frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}}+\frac{b\sqrt{\pi}\left(2\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)^2-1\right)}{4c}$ | 100 |
| parts | $\frac{ax\sqrt{\pi c^2x^2+\pi}}{2}+\frac{a\pi\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}}+\frac{b\sqrt{\pi}\left(2\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}-c^2x^2+\operatorname{arcsinh}(cx)^2-1\right)}{4c}$ | 100 |

input
```
int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*
x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b*Pi^(1/2)*(2*arcsinh(c*x)*c*x*(c^2*x^2+
1)^(1/2)-c^2*x^2+arcsinh(c*x)^2-1)/c
```

### 3.58.5 Fricas [F]

$$\int \sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))\,dx=\int \sqrt{\pi+\pi c^2x^2}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a), x)
```

### 3.58.6 Sympy [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx = \sqrt{\pi}\left(\int a\sqrt{c^2x^2+1}\,dx + \int b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx\right)$$

input
```
integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 +
 1)*asinh(c*x), x))
```

### 3.58.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.58.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.58.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

input
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.59 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx$



## 3.59.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\begin{aligned}\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx &= -bc\sqrt{\pi}x+\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) \\ &\quad -2\sqrt{\pi}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) \\ &\quad -b\sqrt{\pi}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) \\ &\quad +b\sqrt{\pi}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
-b*c*x*Pi^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*Pi^(1/
2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1
)^(1/2))*Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

### 3.59.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x}\,dx = \sqrt{\pi}\Big(a\sqrt{1+c^2x^2} + a\log(x) - a\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right) + b\Big(-cx + \sqrt{1+c^2x^2}\text{arcsinh}(cx) + \text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right) - \text{arcsinh}(cx)\log\left(1+e^{-\text{arcsinh}(cx)}\right) + \text{PolyLog}\left(2,-e^{-\text{arcsinh}(cx)}\right) - \text{PolyLog}\left(2,e^{-\text{arcsinh}(cx)}\right)\Big)\Big)$$

input
```
Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
Sqrt[Pi]*(a*Sqrt[1 + c^2*x^2] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2]
)] + b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(
-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^
(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))
```

### 3.59.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx$$

$$\Big\downarrow \text{6221}$$

$$\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx - \sqrt{\pi}bc\int 1dx + \sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{24}$$

$$\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx$$

$$\Big\downarrow \; 6231$$

$$\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\pi}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx$$

$$\Big\downarrow \; 26$$

$$i\sqrt{\pi}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx$$

$$\Big\downarrow \; 4670$$

$$i\sqrt{\pi}\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a$$
$$\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx$$

$$\Big\downarrow \; 2715$$

$$i\sqrt{\pi}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2iar\right.$$
$$\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx$$

$$\Big\downarrow \; 2838$$

$$i\sqrt{\pi}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)+$$
$$\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx$$

input
```
Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
-(b*c*Sqrt[Pi]*x) + Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) + I*Sqrt[Pi
]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^
ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]])
```

## 3.59. $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx$

#### 3.59.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.59. $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx$

### 3.59.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| default | $a\left(\sqrt{\pi\,c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)\right)+\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\,\sqrt{\pi}\,b+\operatorname{arcsinh}(cx)\ln(1$ |
| parts | $a\left(\sqrt{\pi\,c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)\right)+\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\,\sqrt{\pi}\,b+\operatorname{arcsinh}(cx)\ln(1$ |

input
```
int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
a*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))
+(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(1/2)*b+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+
1)^(1/2))*Pi^(1/2)*b-arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)*b-b
*c*x*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-b*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))*Pi^(1/2)
```

### 3.59.5 Fricas [F]

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\sqrt{\pi+\pi c^2x^2}(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x, x)
```

### 3.59.6 Sympy [F]

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x}\,dx =\sqrt{\pi}\left(\int \frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x,x)
```

output
```
sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2
 + 1)*asinh(c*x)/x, x))
```

### 3.59.7 Maxima [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x}\,dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="maxima"
)
```

output
```
-(sqrt(pi)*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2))*a + b*integrate(
sqrt(pi + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.59.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{arcsinh}(cx))}{x}\, dx = \int \frac{(a + b\operatorname{asinh}(c\, x))\, \sqrt{\Pi\, c^2\, x^2 + \Pi}}{x}\, dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x, x)
```

# 3.60 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.60.1 Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x} + \frac{c\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{2b} + bc\sqrt{\pi}\log(x)$$

output
```
1/2*c*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b+b*c*ln(x)*Pi^(1/2)-(a+b*arcsinh(c*x)
)*(Pi*c^2*x^2+Pi)^(1/2)/x
```

## 3.60.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{\sqrt{\pi}\left(-2a\sqrt{1+c^2x^2}+2\left(acx-b\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)+bcx\text{arcsinh}(cx)^2+2bcx\log(cx)\right)}{2x}$$

input
```
Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
(Sqrt[Pi]*(-2*a*Sqrt[1 + c^2*x^2] + 2*(a*c*x - b*Sqrt[1 + c^2*x^2])*ArcSin
h[c*x] + b*c*x*ArcSinh[c*x]^2 + 2*b*c*x*Log[c*x]))/(2*x)
```

## 3.60.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6220, 14, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{6220}$$

$$\sqrt{\pi}c^2\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\sqrt{\pi}bc\int \frac{1}{x}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{14}$$

$$\sqrt{\pi}c^2\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x}+\sqrt{\pi}bc\log(x)$$

$$\Big\downarrow \text{6198}$$

$$-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x}+\frac{\sqrt{\pi}c(a+b\text{arcsinh}(cx))^2}{2b}+\sqrt{\pi}bc\log(x)$$

input
```
Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[Pi]*(a + b*Arc
Sinh[c*x])^2)/(2*b) + b*c*Sqrt[Pi]*Log[x]
```

### 3.60.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

## 3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(53) = 106.

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.54

| method | result |
|---|---|
| default | $-\frac{a\left(\pi\,c^{2}x^{2}+\pi\right)^{\frac{3}{2}}}{\pi x}+a\,c^{2}x\sqrt{\pi\,c^{2}x^{2}+\pi}+\frac{a\,c^{2}\pi\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{\sqrt{\pi\,c^{2}}}+\frac{bc\sqrt{\pi}\,\operatorname{arcsinh}(cx)^{2}}{2}-bc\sqrt{\pi}\,\operatorname{arcsinh}\,(c$ |
| parts | $-\frac{a\left(\pi\,c^{2}x^{2}+\pi\right)^{\frac{3}{2}}}{\pi x}+a\,c^{2}x\sqrt{\pi\,c^{2}x^{2}+\pi}+\frac{a\,c^{2}\pi\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{\sqrt{\pi\,c^{2}}}+\frac{bc\sqrt{\pi}\,\operatorname{arcsinh}(cx)^{2}}{2}-bc\sqrt{\pi}\,\operatorname{arcsinh}\,(c$ |

input
```
int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/Pi/x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^2*Pi*ln(Pi
*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c*Pi^(1/
2)*arcsinh(c*x)^2-b*c*Pi^(1/2)*arcsinh(c*x)-b*Pi^(1/2)*arcsinh(c*x)/x*(c^2
*x^2+1)^(1/2)+b*c*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

## 3.60.5 Fricas [F]

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{\sqrt{\pi+\pi c^2x^2}(b\,\text{arsinh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^2, x)
```

### 3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(54) = 108.

Time = 1.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{\sqrt{\pi}ac^2x}{\sqrt{c^2x^2+1}} + \sqrt{\pi}ac\,\text{asinh}\,(cx) - \frac{\sqrt{\pi}a}{x\sqrt{c^2x^2+1}} + \sqrt{\pi}bc\log(x) + \frac{\sqrt{\pi}bc\,\text{asinh}^2(cx)}{2} - \frac{\sqrt{\pi}b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x}$$

input
```
integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**2,x)
```

output
```
-sqrt(pi)*a*c**2*x/sqrt(c**2*x**2 + 1) + sqrt(pi)*a*c*asinh(c*x) - sqrt(pi
)*a/(x*sqrt(c**2*x**2 + 1)) + sqrt(pi)*b*c*log(x) + sqrt(pi)*b*c*asinh(c*x
)**2/2 - sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
```

### 3.60.7 Maxima [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="maxim
a")
```

output
```
(sqrt(pi)*c*arcsinh(c*x) - sqrt(pi + pi*c^2*x^2)/x)*a + b*integrate(sqrt(p
i + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

### 3.60.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2, x)
```

# 3.61 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$



## 3.61.1 Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bc\sqrt{\pi}}{2x}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2x^2} - c^2\sqrt{\pi}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{1}{2}bc^2\sqrt{\pi}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{1}{2}bc^2\sqrt{\pi}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)$$

output
```
-1/2*b*c*Pi^(1/2)/x-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*
Pi^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+1/2*b*c^2*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2
+Pi)^(1/2)/x^2
```

### 3.61.2 Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\, dx = \frac{1}{8}\sqrt{\pi}\left(-\frac{4a\sqrt{1 + c^2x^2}}{x^2} + 4ac^2\log(x) - 4ac^2\log\left(\pi\left(1 + \sqrt{1 + c^2x^2}\right)\right) + bc^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - \text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 4\text{arcsinh}(cx)\log\left(1 - e^{-\text{arcsinh}(cx)}\right) - 4\text{arcsinh}(cx)\log\left(1 + e^{-\text{arcsinh}(cx)}\right) + 4\,\text{PolyLog}\left(2, -e^{-\text{arcsinh}(cx)}\right) - 4\,\text{PolyLog}\left(2, e^{-\text{arcsinh}(cx)}\right) - \text{arcsinh}(cx)\text{sech}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 2\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\right)$$

input
```
Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(Sqrt[Pi]*((-4*a*Sqrt[1 + c^2*x^2])/x^2 + 4*a*c^2*Log[x] - 4*a*c^2*Log[Pi*
(1 + Sqrt[1 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*C
sch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcS
inh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4
*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*T
anh[ArcSinh[c*x]/2])))/8
```

### 3.61.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6220, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x^3}dx$$

$$\Big\downarrow \text{6220}$$

$$\frac{1}{2}\sqrt{\pi}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{1}{2}\sqrt{\pi}bc\int \frac{1}{x^2}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\sqrt{\pi}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{\sqrt{\pi}bc}{2x}$$

$$\Big\downarrow \text{6231}$$

$$\frac{1}{2}\sqrt{\pi}c^2\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{\sqrt{\pi}bc}{2x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\sqrt{\pi}c^2\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{\sqrt{\pi}bc}{2x}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}i\sqrt{\pi}c^2\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{\sqrt{\pi}bc}{2x}$$

$$\Big\downarrow \text{4670}$$

$$\frac{1}{2}i\sqrt{\pi}c^2\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.$$
$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{\sqrt{\pi}bc}{2x}$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{2}i\sqrt{\pi}c^2\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.$$
$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{\sqrt{\pi}bc}{2x}$$
$$\Big\downarrow \text{2838}$$
$$\frac{1}{2}i\sqrt{\pi}c^2\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)-$$
$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{\sqrt{\pi}bc}{2x}$$

input
```
Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*Sqrt[Pi])/x - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*x^
2) + (I/2)*c^2*Sqrt[Pi]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]])
```

### 3.61.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.61.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| default | $a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{2\pi x^2}+\frac{c^2\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)}{2}\right)+b\left(-\frac{\sqrt{\pi}\left(\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(c\right.}{2\sqrt{c^2x^2+1}\,x^2}\right.$ |
| parts | $a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{2\pi x^2}+\frac{c^2\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)}{2}\right)+b\left(-\frac{\sqrt{\pi}\left(\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(c\right.}{2\sqrt{c^2x^2+1}\,x^2}\right.$ |

input
```
int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(3/2)+1/2*c^2*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/
2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+b*(-1/2*Pi^(1/2)/(c^2*x^2+1)^
(1/2)*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/x^2-1/2*c^
2*Pi^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/2*c^2*Pi^(1/2)*polyl
og(2,-c*x-(c^2*x^2+1)^(1/2))+1/2*c^2*Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x
^2+1)^(1/2))+1/2*c^2*Pi^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

### 3.61.5 Fricas [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\, dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x^3}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^3, x)
```

### 3.61.6 Sympy [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\, dx = \sqrt{\pi}\left(\int \frac{a\sqrt{c^2x^2 + 1}}{x^3}\, dx + \int \frac{b\sqrt{c^2x^2 + 1}\,\text{asinh}\,(cx)}{x^3}\, dx\right)$$

input
```
integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**3,x)
```

output
```
sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(b*sqrt(c**2*x
**2 + 1)*asinh(c*x)/x**3, x))
```

### 3.61.7 Maxima [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{\sqrt{\pi + \pi c^2x^2}(b\,\text{arsinh}\,(cx) + a)}{x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="maxim
a")
```

output
```
-1/2*(sqrt(pi)*c^2*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2)*c^2 + (pi
 + pi*c^2*x^2)^(3/2)/(pi*x^2))*a + b*integrate(sqrt(pi + pi*c^2*x^2)*log(c
*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

### 3.61.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3, x)
```

# 3.62 $\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.62.1 Optimal result

Integrand size = 26, antiderivative size = 62

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=-\frac{bc\sqrt{\pi}}{6x^2}-\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi x^3}+\frac{1}{3}bc^3\sqrt{\pi}\log(x)$$

output
```
-1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/Pi/x^3-1/6*b*c*Pi^(1/2)/x^2+
1/3*b*c^3*ln(x)*Pi^(1/2)
```

## 3.62.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\frac{\sqrt{\pi}\left(-bcx-3bc^3x^3-2a\sqrt{1+c^2x^2}-2ac^2x^2\sqrt{1+c^2x^2}-2b(1+c^2x^2)^{3/2}\text{arcsinh}(cx)+2bc^3x^3\log(x)\right)}{6x^3}$$

input
```
Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
(Sqrt[Pi]*(-(b*c*x) - 3*b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] - 2*a*c^2*x^2*Sq
rt[1 + c^2*x^2] - 2*b*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] + 2*b*c^3*x^3*Log[x
]))/(6*x^3)
```

### 3.62.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6215, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{1}{3}\sqrt{\pi}bc\int \frac{c^2x^2+1}{x^3}dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi x^3}$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{3}\sqrt{\pi}bc\int \left(\frac{c^2}{x}+\frac{1}{x^3}\right)dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\sqrt{\pi}bc\left(c^2\log(x)-\frac{1}{2x^2}\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi x^3}$$

input
```
Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(Pi*x^3) + (b*c*Sqrt[P
i]*(-1/2*1/x^2 + c^2*Log[x]))/3
```

#### 3.62.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(50) = 100$.

Time = 0.16 (sec) , antiderivative size = 501, normalized size of antiderivative = 8.08

| method | result |
|---|---|
| default | $-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi x^3}-\frac{2bc^3\sqrt{\pi}\,\operatorname{arcsinh}(cx)}{3}+\frac{b\sqrt{\pi}\,x^4\operatorname{arcsinh}(cx)c^7}{3c^4x^4+3c^2x^2+1}-\frac{b\sqrt{\pi}\,x^3\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)c^6}{3c^4x^4+3c^2x^2+1}+\frac{b\sqrt{\pi}\,x^4c^7}{18c^4x^4+18c^2x^2+6}-$ |
| parts | $-\frac{a(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3\pi x^3}-\frac{2bc^3\sqrt{\pi}\,\operatorname{arcsinh}(cx)}{3}+\frac{b\sqrt{\pi}\,x^4\operatorname{arcsinh}(cx)c^7}{3c^4x^4+3c^2x^2+1}-\frac{b\sqrt{\pi}\,x^3\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)c^6}{3c^4x^4+3c^2x^2+1}+\frac{b\sqrt{\pi}\,x^4c^7}{18c^4x^4+18c^2x^2+6}-$ |

input
```
int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(3/2)-2/3*b*c^3*Pi^(1/2)*arcsinh(c*x)+b*Pi^(
1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-b*Pi^(1/2)/(3*c^4*x^4+3*
c^2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^6+1/6*b*Pi^(1/2)/(3*c^4*x^
4+3*c^2*x^2+1)*x^4*c^7-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*(c^2*x^2
+1)*c^5+b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-2*b*Pi^(1/
2)/(3*c^4*x^4+3*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^4-1/3*b*Pi^(
1/2)/(3*c^4*x^4+3*c^2*x^2+1)*(c^2*x^2+1)*c^3+1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c
^2*x^2+1)*arcsinh(c*x)*c^3-4/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x*(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^2-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2*(c
^2*x^2+1)*c-1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3*(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)+1/3*b*c^3*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

### 3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(50) = 100$.

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx =$$
$$-\frac{2\sqrt{\pi + \pi c^2x^2}(bc^4x^4 + 2bc^2x^2 + b)\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \sqrt{\pi}(bc^5x^5 + bc^3x^3)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{}}{c^2}\right.}{6\left(c^2x^5 + x^3\right)}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="frica
s")
```

output
```
-1/6*(2*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt
(c^2*x^2 + 1)) - sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + p
i*c^2*x^2 + pi*x^4 + sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4
 - 1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(2*a*c^4*x^4 + 4*a*c^2*x^2
 - (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) + 2*a))/(c^2*x^5 + x^3)
```

### 3.62.6 Sympy [F]

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx$$
$$= \sqrt{\pi}\left(\int \frac{a\sqrt{c^2x^2 + 1}}{x^4}\,dx + \int \frac{b\sqrt{c^2x^2 + 1}\,\text{asinh}\,(cx)}{x^4}\,dx\right)$$

input
```
integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**4,x)
```

output
```
sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(b*sqrt(c**2*x
**2 + 1)*asinh(c*x)/x**4, x))
```

### 3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(50) = 100.

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx$$
$$= -\frac{\left(\pi^{\frac{3}{2}}(-1)^{2\,\pi+2\,\pi c^2x^2}\,c^2\log\left(2\,\pi c^2 + \frac{2\,\pi}{x^2}\right) - \pi^{\frac{3}{2}}c^2\log\left(x^2 + \frac{1}{c^2}\right) + \frac{\pi\sqrt{\pi+\pi c^4x^4+2\,\pi c^2x^2}}{x^2}\right)bc}{6\,\pi}$$
$$- \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}b\,\text{arsinh}\,(cx)}{3\,\pi x^3} - \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}a}{3\,\pi x^3}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="maxim
a")
```

output
```
-1/6*(pi^(3/2)*(-1)^(2*pi + 2*pi*c^2*x^2)*c^2*log(2*pi*c^2 + 2*pi/x^2) - p
i^(3/2)*c^2*log(x^2 + 1/c^2) + pi*sqrt(pi + pi*c^4*x^4 + 2*pi*c^2*x^2)/x^2
)*b*c/pi - 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*x^3) - 1/3*(pi +
 pi*c^2*x^2)^(3/2)*a/(pi*x^3)
```

### 3.62.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4, x)
```

# 3.63 $\int x^3(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx$



## 3.63.1 Optimal result

Integrand size = 26, antiderivative size = 125

$$\int x^3 \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \frac{2b\pi^{3/2}x}{35c^3} - \frac{b\pi^{3/2}x^3}{105c} - \frac{8}{175}bc\pi^{3/2}x^5 - \frac{1}{49}bc^3\pi^{3/2}x^7 - \frac{\left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))}{5c^4\pi} + \frac{\left(\pi + c^2\pi x^2\right)^{7/2} (a + b\text{arcsinh}(cx))}{7c^4\pi^2}$$

output
```
2/35*b*Pi^(3/2)*x/c^3-1/105*b*Pi^(3/2)*x^3/c-8/175*b*c*Pi^(3/2)*x^5-1/49*b
*c^3*Pi^(3/2)*x^7-1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/7*
(Pi*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

## 3.63.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int x^3 \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \frac{\pi^{3/2}\left(105a(1 + c^2x^2)^{5/2}(-2 + 5c^2x^2) - bcx(-210 + 35c^2x^2 + 168c^4x^4 + 75c^6x^6) + 105\right.}{3675c^4}$$

input
```
Integrate[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(3/2)*(105*a*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x^2) - b*c*x*(-210 + 35*c
^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2
*x^2)*ArcSinh[c*x]))/(3675*c^4)
```

### 3.63.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6219, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{\pi\left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2}{35c^4}dx+\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi c^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\pi^{3/2}b\int\left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2dx}{35c^3}+\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi c^4}$$

$$\Big\downarrow \text{290}$$

$$\frac{\pi^{3/2}b\int\left(-5c^6x^6-8c^4x^4-c^2x^2+2\right)dx}{35c^3}+\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi c^4}+\frac{\pi^{3/2}b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}$$

input
```
Int[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(b*Pi^(3/2)*(2*x - (c^2*x^3)/3 - (8*c^4*x^5)/5 - (5*c^6*x^7)/7))/(35*c^3)
- ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*Pi) + ((Pi + c^2*P
i*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi^2)
```

### 3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.63.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| default | $a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{7\pi c^2}-\frac{2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{35\pi c^4}\right)+\frac{b\pi^{\frac{3}{2}}\left(525\ \text{arcsinh}(cx)c^8x^8+1365\ \text{arcsinh}(cx)c^6x^6-75c^7x^7\sqrt{c^2x^2+1}+945\ \text{arcsinh}(c\right.}{}$ |
| parts | $a\left(\frac{x^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{7\pi c^2}-\frac{2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{35\pi c^4}\right)+\frac{b\pi^{\frac{3}{2}}\left(525\ \text{arcsinh}(cx)c^8x^8+1365\ \text{arcsinh}(cx)c^6x^6-75c^7x^7\sqrt{c^2x^2+1}+945\ \text{arcsinh}(c\right.}{}$ |

input
```
int(x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*x^2*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-2/35/Pi/c^4*(Pi*c^2*x^2+Pi)^(5/2))
+1/3675*b/c^4*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(525*arcsinh(c*x)*c^8*x^8+1365*ar
csinh(c*x)*c^6*x^6-75*c^7*x^7*(c^2*x^2+1)^(1/2)+945*arcsinh(c*x)*c^4*x^4-1
68*c^5*x^5*(c^2*x^2+1)^(1/2)-105*arcsinh(c*x)*c^2*x^2-35*c^3*x^3*(c^2*x^2+
1)^(1/2)-210*arcsinh(c*x)+210*c*x*(c^2*x^2+1)^(1/2))
```

### 3.63.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.59

$$\int x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{105\sqrt{\pi+\pi c^2x^2}(5\,\pi bc^8x^8+13\,\pi bc^6x^6+9\,\pi bc^4x^4-\pi bc^2x^2-2\,\pi b)\log\left(cx+\sqrt{c^2x^2+1}\right.}{}$$

input
```
integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="frica
s")
```

output
```
1/3675*(105*sqrt(pi + pi*c^2*x^2)*(5*pi*b*c^8*x^8 + 13*pi*b*c^6*x^6 + 9*pi
*b*c^4*x^4 - pi*b*c^2*x^2 - 2*pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi
 + pi*c^2*x^2)*(525*pi*a*c^8*x^8 + 1365*pi*a*c^6*x^6 + 945*pi*a*c^4*x^4 -
105*pi*a*c^2*x^2 - 210*pi*a - (75*pi*b*c^7*x^7 + 168*pi*b*c^5*x^5 + 35*pi*
b*c^3*x^3 - 210*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^4)
```

### 3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(117) = 234.

Time = 9.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.41

$$\int x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\begin{cases}\frac{\pi^{\frac{3}{2}}ac^2x^6\sqrt{c^2x^2+1}}{7}+\frac{8\pi^{\frac{3}{2}}ax^4\sqrt{c^2x^2+1}}{35}+\frac{\pi^{\frac{3}{2}}ax^2\sqrt{c^2x^2+1}}{35c^2}-\frac{2\pi^{\frac{3}{2}}a\sqrt{c^2x^2+1}}{35c^4}-\frac{\pi^{\frac{3}{2}}bc^3x^7}{49}+\frac{\pi^{\frac{3}{2}}bc^2x^6\sqrt{c^2x^2}}{7} \\ \frac{\pi^{\frac{3}{2}}ax^4}{4}\end{cases}$$

input
```
integrate(x**3*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(3/2)*a*c**2*x**6*sqrt(c**2*x**2 + 1)/7 + 8*pi**(3/2)*a*x**
4*sqrt(c**2*x**2 + 1)/35 + pi**(3/2)*a*x**2*sqrt(c**2*x**2 + 1)/(35*c**2)
- 2*pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(35*c**4) - pi**(3/2)*b*c**3*x**7/49 +
 pi**(3/2)*b*c**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - 8*pi**(3/2)*b*c*
x**5/175 + 8*pi**(3/2)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/35 - pi**(3/2
)*b*x**3/(105*c) + pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(35*c**
2) + 2*pi**(3/2)*b*x/(35*c**3) - 2*pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(35*c**4), Ne(c, 0)), (pi**(3/2)*a*x**4/4, True))
```

### 3.63.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

$$\int x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{1}{35}\left(\frac{5\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}{\pi c^4}\right)b\operatorname{arsinh}(cx)+\frac{1}{35}\left(\frac{5\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}{\pi c^4}\right)a-\frac{\left(75\,\pi^{\frac{3}{2}}c^6x^7+168\,\pi^{\frac{3}{2}}c^4x^5+35\,\pi^{\frac{3}{2}}c^2x^3-210\,\pi^{\frac{3}{2}}x\right)b}{3675\,c^3}$$

input
```
integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxim
a")
```

output
```
1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(5/2)/(
pi*c^4))*b*arcsinh(c*x) + 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2
*(pi + pi*c^2*x^2)^(5/2)/(pi*c^4))*a - 1/3675*(75*pi^(3/2)*c^6*x^7 + 168*p
i^(3/2)*c^4*x^5 + 35*pi^(3/2)*c^2*x^3 - 210*pi^(3/2)*x)*b/c^3
```

### 3.63.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (\pi + c^2 \pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.63.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (\pi + c^2 \pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int x^3 \left(a + b\,\text{asinh}(c\,x)\right) \left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2} dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.64 $\int x^2(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx$



## 3.64.1 Optimal result

Integrand size = 26, antiderivative size = 165

$$\int x^2\left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = -\frac{b\pi^{3/2}x^2}{32c} - \frac{7}{96}bc\pi^{3/2}x^4 - \frac{1}{36}bc^3\pi^{3/2}x^6 + \frac{\pi^{3/2}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c^2} + \frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{\pi^{3/2}(a+b\text{arcsinh}(cx))^2}{32bc^3}$$

output
```
-1/32*b*Pi^(3/2)*x^2/c-7/96*b*c*Pi^(3/2)*x^4-1/36*b*c^3*Pi^(3/2)*x^6+1/6*x
^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/32*Pi^(3/2)*(a+b*arcsinh(c*x
))^2/b/c^3+1/16*Pi^(3/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^2+1/8*Pi
*x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.64.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int x^2\left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \frac{\pi^{3/2}\left(144acx\sqrt{1+c^2x^2} + 672ac^3x^3\sqrt{1+c^2x^2} + 384ac^5x^5\sqrt{1+c^2x^2} - 72b\text{arcsinh}(cx)^2\right.}{}$$

input
```
Integrate[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(3/2)*(144*a*c*x*Sqrt[1 + c^2*x^2] + 672*a*c^3*x^3*Sqrt[1 + c^2*x^2] +
 384*a*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*b*ArcSinh[c*x]^2 + 18*b*Cosh[2*ArcSi
nh[c*x]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 12*ArcSin
h[c*x]*(12*a + 3*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4*ArcSinh[c*x]] - b*Sin
h[6*ArcSinh[c*x]])))/(2304*c^3)
```

### 3.64.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6223, 244, 2009, 6221, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(\pi c^2 x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx)) \, dx$$

$$\Big\downarrow \text{6223}$$

$$\frac{1}{2}\pi \int x^2 \sqrt{c^2 \pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}\pi^{3/2}bc\int x^3 \left(c^2x^2+1\right) dx+\frac{1}{6}x^3 \left(\pi c^2x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}\pi \int x^2 \sqrt{c^2 \pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}\pi^{3/2}bc\int \left(c^2x^5+x^3\right) dx+\frac{1}{6}x^3 \left(\pi c^2x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\pi \int x^2 \sqrt{c^2 \pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x^3 \left(\pi c^2x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))-\frac{1}{6}\pi^{3/2}bc\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)$$

$$\Big\downarrow \text{6221}$$

$$\frac{1}{2}\pi \left(\frac{1}{4}\sqrt{\pi}\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{4}\sqrt{\pi}bc\int x^3dx+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x^3 \left(\pi c^2x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))-\frac{1}{6}\pi^{3/2}bc\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\pi\left(\frac{1}{4}\sqrt{\pi}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{16}\sqrt{\pi}bcx^4\right)+\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{6}\pi^{3/2}bc\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}\pi\left(\frac{1}{4}\sqrt{\pi}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(\right.$$
$$\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{6}\pi^{3/2}bc\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\pi\left(\frac{1}{4}\sqrt{\pi}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right.$$
$$\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{6}\pi^{3/2}bc\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{1}{2}\pi\left(\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{1}{4}\sqrt{\pi}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)\right.$$
$$\frac{1}{6}\pi^{3/2}bc\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)$$

input
```
Int[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/6*(b*c*Pi^(3/2)*(x^4/4 + (c^2*x^6)/6)) + (x^3*(Pi + c^2*Pi*x^2)^(3/2)*(
a + b*ArcSinh[c*x]))/6 + (Pi*(-1/16*(b*c*Sqrt[Pi]*x^4) + (x^3*Sqrt[Pi + c^
2*Pi*x^2]*(a + b*ArcSinh[c*x]))/4 + (Sqrt[Pi]*(-1/4*(b*x^2)/c + (x*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3
)))/4))/2
```

## 3.64. $\int x^2(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

### 3.64.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.64.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30

| method | result |
|---|---|
| default | $\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6\pi c^2} - \frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24c^2} - \frac{a\pi x\sqrt{\pi c^2x^2+\pi}}{16c^2} - \frac{a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16c^2\sqrt{\pi c^2}} - \frac{b\pi^{\frac{3}{2}}\left(-48\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5\right.}{}$ |
| parts | $\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6\pi c^2} - \frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24c^2} - \frac{a\pi x\sqrt{\pi c^2x^2+\pi}}{16c^2} - \frac{a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16c^2\sqrt{\pi c^2}} - \frac{b\pi^{\frac{3}{2}}\left(-48\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5\right.}{}$ |

input
```
int(x^2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-1/24*a/c^2*x*(Pi*c^2*x^2+Pi)^(3/2)-1/
16*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)-1/16*a/c^2*Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(
1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/288*b*Pi^(3/2)*(-48*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)*x^5*c^5+8*c^6*x^6-84*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*
x^3*c^3+21*c^4*x^4-18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+9*c^2*x^2+9*arcsi
nh(c*x)^2-4)/c^3
```

## 3.64.5 Fricas [F]

$$\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^4 + pi*a*x^2 + (pi*b*c^2*x^4 +
pi*b*x^2)*arcsinh(c*x)), x)
```

### 3.64.6 Sympy [A] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.59

$$\int x^2(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \begin{cases} \frac{\pi^{\frac{3}{2}}ac^2x^5\sqrt{c^2x^2+1}}{6}+\frac{7\pi^{\frac{3}{2}}ax^3\sqrt{c^2x^2+1}}{24}+\frac{\pi^{\frac{3}{2}}ax\sqrt{c^2x^2+1}}{16c^2}-\frac{\pi^{\frac{3}{2}}a\operatorname{asinh}(cx)}{16c^3}-\frac{\pi^{\frac{3}{2}}bc^3x^6}{36}+\frac{\pi^{\frac{3}{2}}bc^2x^5\sqrt{c^2x^2+}}{6} \\ \frac{\pi^{\frac{3}{2}}ax^3}{3} \end{cases}$$

input
```
integrate(x**2*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(3/2)*a*c**2*x**5*sqrt(c**2*x**2 + 1)/6 + 7*pi**(3/2)*a*x**
3*sqrt(c**2*x**2 + 1)/24 + pi**(3/2)*a*x*sqrt(c**2*x**2 + 1)/(16*c**2) - p
i**(3/2)*a*asinh(c*x)/(16*c**3) - pi**(3/2)*b*c**3*x**6/36 + pi**(3/2)*b*c
**2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/6 - 7*pi**(3/2)*b*c*x**4/96 + 7*pi
**(3/2)*b*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/24 - pi**(3/2)*b*x**2/(32*c)
 + pi**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c**2) - pi**(3/2)*b*as
inh(c*x)**2/(32*c**3), Ne(c, 0)), (pi**(3/2)*a*x**3/3, True))
```

### 3.64.7 Maxima [F(-2)]

Exception generated.

$$\int x^2(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.64.8 Giac [F]

$$\int x^2 \left(\pi + c^2 \pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a) x^2\, dx$$

input
```
integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac"
)
```

output
```
integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

### 3.64.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(\pi + c^2 \pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int x^2 \left(a + b \operatorname{asinh}(c\, x)\right) \left(\Pi\, c^2\, x^2 + \Pi\right)^{3/2} dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.65 $\int x(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx$



## 3.65.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int x(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = -\frac{b\pi^{3/2}x}{5c} - \frac{2}{15}bc\pi^{3/2}x^3 - \frac{1}{25}bc^3\pi^{3/2}x^5 + \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{5c^2\pi}$$

output
```
-1/5*b*Pi^(3/2)*x/c-2/15*b*c*Pi^(3/2)*x^3-1/25*b*c^3*Pi^(3/2)*x^5+1/5*(Pi*
c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

## 3.65.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int x(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \frac{\pi^{3/2}\left(15a(1 + c^2x^2)^{5/2} - bcx(15 + 10c^2x^2 + 3c^4x^4) + 15b(1 + c^2x^2)^{5/2}\text{arcsinh}(cx)\right)}{75c^2}$$

input
```
Integrate[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(3/2)*(15*a*(1 + c^2*x^2)^(5/2) - b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)
+ 15*b*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]))/(75*c^2)
```

### 3.65.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6213, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6213$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{\pi^{3/2}b\int\left(c^2x^2+1\right)^2dx}{5c}$$

$$\Big\downarrow \; 210$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{\pi^{3/2}b\int\left(c^4x^4+2c^2x^2+1\right)dx}{5c}$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{\pi^{3/2}b\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)}{5c}$$

input
```
Int[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/5*(b*Pi^(3/2)*(x + (2*c^2*x^3)/3 + (c^4*x^5)/5))/c + ((Pi + c^2*Pi*x^2)
^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

#### 3.65.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(61) = 122.

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.81

| method | result |
|---|---|
| default | $\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5\pi c^2}+\frac{b\pi^{\frac{3}{2}}\left(15 \operatorname{arcsinh}(cx)c^6x^6+45 \operatorname{arcsinh}(cx)c^4x^4-3c^5x^5\sqrt{c^2x^2+1}+45 \operatorname{arcsinh}(cx)c^2x^2-10c^3x^3\sqrt{c^2x^2+1}+15 \operatorname{arcs}\right.}{75c^2\sqrt{c^2x^2+1}}$ |
| parts | $\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5\pi c^2}+\frac{b\pi^{\frac{3}{2}}\left(15 \operatorname{arcsinh}(cx)c^6x^6+45 \operatorname{arcsinh}(cx)c^4x^4-3c^5x^5\sqrt{c^2x^2+1}+45 \operatorname{arcsinh}(cx)c^2x^2-10c^3x^3\sqrt{c^2x^2+1}+15 \operatorname{arcs}\right.}{75c^2\sqrt{c^2x^2+1}}$ |

input
```
int(x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2+1/75*b/c^2*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(
15*arcsinh(c*x)*c^6*x^6+45*arcsinh(c*x)*c^4*x^4-3*c^5*x^5*(c^2*x^2+1)^(1/2
)+45*arcsinh(c*x)*c^2*x^2-10*c^3*x^3*(c^2*x^2+1)^(1/2)+15*arcsinh(c*x)-15*
c*x*(c^2*x^2+1)^(1/2))
```

### 3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(61) = 122.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.17

$$\int x(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{15\sqrt{\pi+\pi c^2x^2}(\pi bc^6x^6+3\pi bc^4x^4+3\pi bc^2x^2+\pi b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x}}{75\,(}$$

input
```
integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
1/75*(15*sqrt(pi + pi*c^2*x^2)*(pi*b*c^6*x^6 + 3*pi*b*c^4*x^4 + 3*pi*b*c^2
*x^2 + pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(15*pi*a
*c^6*x^6 + 45*pi*a*c^4*x^4 + 45*pi*a*c^2*x^2 + 15*pi*a - (3*pi*b*c^5*x^5 +
 10*pi*b*c^3*x^3 + 15*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

### 3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(70) = 140$.

Time = 2.93 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.87

$$\int x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \begin{cases} \frac{\pi^{\frac{3}{2}}ac^2x^4\sqrt{c^2x^2+1}}{5} + \frac{2\pi^{\frac{3}{2}}ax^2\sqrt{c^2x^2+1}}{5} + \frac{\pi^{\frac{3}{2}}a\sqrt{c^2x^2+1}}{5c^2} - \frac{\pi^{\frac{3}{2}}bc^3x^5}{25} + \frac{\pi^{\frac{3}{2}}bc^2x^4\sqrt{c^2x^2+1}\,\text{asinh}(cx)}{5} - \frac{2\pi^{\frac{3}{2}}\cdots}{1\cdots} \\ \frac{\pi^{\frac{3}{2}}ax^2}{2} \end{cases}$$

input
```
integrate(x*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(3/2)*a*c**2*x**4*sqrt(c**2*x**2 + 1)/5 + 2*pi**(3/2)*a*x**
2*sqrt(c**2*x**2 + 1)/5 + pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(5*c**2) - pi**(
3/2)*b*c**3*x**5/25 + pi**(3/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)
/5 - 2*pi**(3/2)*b*c*x**3/15 + 2*pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asin
h(c*x)/5 - pi**(3/2)*b*x/(5*c) + pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(5*c**2), Ne(c, 0)), (pi**(3/2)*a*x**2/2, True))
```

### 3.65.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int x\left(\pi + c^2\pi x^2\right)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \frac{\left(\pi + \pi c^2x^2\right)^{\frac{5}{2}}b\,\text{arsinh}\,(cx)}{5\,\pi c^2} + \frac{\left(\pi + \pi c^2x^2\right)^{\frac{5}{2}}a}{5\,\pi c^2} - \frac{\left(3\,\pi^{\frac{5}{2}}c^4x^5 + 10\,\pi^{\frac{5}{2}}c^2x^3 + 15\,\pi^{\frac{5}{2}}x\right)b}{75\,\pi c}$$

input
```
integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
1/5*(pi + pi*c^2*x^2)^(5/2)*b*arcsinh(c*x)/(pi*c^2) + 1/5*(pi + pi*c^2*x^2
)^(5/2)*a/(pi*c^2) - 1/75*(3*pi^(5/2)*c^4*x^5 + 10*pi^(5/2)*c^2*x^3 + 15*p
i^(5/2)*x)*b/(pi*c)
```

### 3.65.8 Giac [F(-2)]

Exception generated.

$$\int x(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.65.9 Mupad [F(-1)]

Timed out.

$$\int x(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \int x\,(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.66 $\int (\pi + c^2\pi x^2)^{3/2} (a + b\mathrm{arcsinh}(cx))\, dx$



## 3.66.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\mathrm{arcsinh}(cx))\, dx = -\frac{5}{16}bc\pi^{3/2}x^2 - \frac{1}{16}bc^3\pi^{3/2}x^4 + \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}(a+b\mathrm{arcsinh}(cx)) + \frac{1}{4}x(\pi+c^2\pi x^2)^{3/2}(a+b\mathrm{arcsinh}(cx)) + \frac{3\pi^{3/2}(a + b\mathrm{arcsinh}(cx))^2}{16bc}$$

output
```
-5/16*b*c*Pi^(3/2)*x^2-1/16*b*c^3*Pi^(3/2)*x^4+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)
*(a+b*arcsinh(c*x))+3/16*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b/c+3/8*Pi*x*(a+b*a
rcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.66.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\mathrm{arcsinh}(cx))\, dx = \frac{\pi^{3/2}\left(80acx\sqrt{1+c^2x^2} + 32ac^3x^3\sqrt{1+c^2x^2} + 24b\mathrm{arcsinh}(cx)^2 - 16b\cosh(2\mathrm{arcsinh}(cx)\right)}{}$$

input
```
Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(3/2)*(80*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 2
4*b*ArcSinh[c*x]^2 - 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] +
4*ArcSinh[c*x]*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))
)/(128*c)
```

### 3.66.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \ 6201$$

$$\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \ 244$$

$$\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int \left(c^2x^3+x\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \ 2009$$

$$\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

$$\Big\downarrow \ 6200$$

$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

$$\Big\downarrow \ 15$$

$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}\sqrt{\pi}bcx^2\right)-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

input
```
Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*Pi^(3/2)*(x^2/2 + (c^2*x^4)/4)) + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/4 + (3*Pi*(-1/4*(b*c*Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*P
i*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)
))/4
```

### 3.66.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

## 3.66.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

| method | result |
|---|---|
| default | $\frac{x(\pi c^2x^2+\pi)^{\frac{3}{2}}a}{4}+\frac{3a\pi x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{3a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}}+\frac{b\pi^{\frac{3}{2}}\left(4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4+10\operatorname{arcsinh}(\right.}{16c}$ |
| parts | $\frac{x(\pi c^2x^2+\pi)^{\frac{3}{2}}a}{4}+\frac{3a\pi x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{3a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}}+\frac{b\pi^{\frac{3}{2}}\left(4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4+10\operatorname{arcsinh}(\right.}{16c}$ |

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*a+3/8*a*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a*Pi^2*
ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/16*b*Pi
^(3/2)*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-c^4*x^4+10*arcsinh(c*x)*c
*x*(c^2*x^2+1)^(1/2)-5*c^2*x^2+3*arcsinh(c*x)^2-4)/c
```

### 3.66.5 Fricas [F]

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx) + a)\, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b
)*arcsinh(c*x)), x)
```

### 3.66.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \begin{cases} \frac{\pi^{\frac{3}{2}} a c^2 x^3 \sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} a x \sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}} a\,\text{asinh}\,(cx)}{8c} - \frac{\pi^{\frac{3}{2}} b c^3 x^4}{16} + \frac{\pi^{\frac{3}{2}} b c^2 x^3 \sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{4} - \frac{5\pi^{\frac{3}{2}}}{1} \\ \pi^{\frac{3}{2}} a x \end{cases}$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(3/2)*a*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*x*s
qrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a*asinh(c*x)/(8*c) - pi**(3/2)*b*c**3*x
**4/16 + pi**(3/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/4 - 5*pi**(3
/2)*b*c*x**2/16 + 5*pi**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/8 + 3*pi*
*(3/2)*b*asinh(c*x)**2/(16*c), Ne(c, 0)), (pi**(3/2)*a*x, True))
```

### 3.66.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.66.8 Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.66.9 Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int (a + b\,\text{asinh}(c\,x))\, \left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}\, dx$$

input
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.67 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\mathbf{arcsinh}(cx))}{x}\,dx$



## 3.67.1 Optimal result

Integrand size = 26, antiderivative size = 134

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{4}{3}bc\pi^{3/2}x-\frac{1}{9}bc^3\pi^{3/2}x^3$$
$$+\pi\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))-2\pi^{3/2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsi}}\right.$$

output
```
-4/3*b*c*Pi^(3/2)*x-1/9*b*c^3*Pi^(3/2)*x^3+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*
arcsinh(c*x))-2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))
-b*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*polylog(2,c*x+(c^
2*x^2+1)^(1/2))+Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.67.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.34

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \frac{1}{9}\pi^{3/2}\left(3a\sqrt{1+c^2x^2}(4+c^2x^2)-b\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsi}\right.\right.$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(Pi^(3/2)*(3*a*Sqrt[1 + c^2*x^2]*(4 + c^2*x^2) - b*(3*c*x + c^3*x^3 - 3*(1
 + c^2*x^2)^(3/2)*ArcSinh[c*x]) + 9*a*Log[x] - 9*a*Log[Pi*(1 + Sqrt[1 + c^
2*x^2])] + 9*b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLo
g[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])])))/9
```

### 3.67.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx$$

$$\Big\downarrow \text{6223}$$

$$\pi\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{3}\pi^{3/2}bc\int \left(c^2x^2+1\right)dx+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\pi\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{6221}$$

$$\pi\left(\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int 1dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{24}$$

$$\pi\left(\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{6231}$$

$$\pi\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{3042}$$

$$\pi\left(\sqrt{\pi}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{26}$$

$$\pi\left(i\sqrt{\pi}\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{4670}$$

$$\pi\left(i\sqrt{\pi}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.\right.$$
$$\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{2715}$$

$$\pi\left(i\sqrt{\pi}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\right.\right.$$
$$\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{2838}$$

$$\pi\left(i\sqrt{\pi}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)+\right.$$
$$\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)$$

input
```
Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

---

3.67. $\int\frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

output
```
-1/3*(b*c*Pi^(3/2)*(x + (c^2*x^3)/3)) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/3 + Pi*(-(b*c*Sqrt[Pi]*x) + Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]) + I*Sqrt[Pi]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]]
 + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))
```

#### 3.67.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.67.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| default | $a\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)+\frac{b\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)\pi^{\frac{3}{2}}x^2c^2}{3}+\frac{4b\sqrt{c^2}}{}$ |
| parts | $a\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)+\frac{b\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)\pi^{\frac{3}{2}}x^2c^2}{3}+\frac{4b\sqrt{c^2}}{}$ |

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi
^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+1/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3
/2)*x^2*c^2+4/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2)-4/3*b*c*Pi^(3/2)
*x-1/9*b*c^3*Pi^(3/2)*x^3+b*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/
2))-b*Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*polylog
(2,c*x+(c^2*x^2+1)^(1/2))-b*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
```

### 3.67.5 Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \int \frac{(\pi + \pi c^2 x^2)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)}{x} \, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b
)*arcsinh(c*x))/x, x)
```

### 3.67.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \pi^{\frac{3}{2}} \left( \int \frac{a\sqrt{c^2x^2+1}}{x} \, dx + \int ac^2x\sqrt{c^2x^2+1} \, dx + \int \frac{b\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{x} \, dx + \int bc^2x\sqrt{c^2x^2+1} \operatorname{asinh}(cx) \, dx \right)$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x,x)
```

output
```
pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(a*c**2*x*sqrt(c
**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Inte
gral(b*c**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x), x))
```

### 3.67.7 Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)}{x} \, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima"
)
```

output
```
-1/3*(3*pi^(3/2)*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2) - (pi
+ pi*c^2*x^2)^(3/2))*a + b*integrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqr
t(c^2*x^2 + 1))/x, x)
```

### 3.67.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x} \, dx = \int \frac{(a + b \operatorname{asinh}(c x)) \left(\Pi c^2 x^2 + \Pi\right)^{3/2}}{x} \, dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x, x)
```

# 3.68 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx$



## 3.68.1 Optimal result

Integrand size = 26, antiderivative size = 108

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx = -\frac{1}{4}bc^3\pi^{3/2}x^2 + \frac{3}{2}c^2\pi x\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x} + \frac{3c\pi^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{4b} + bc\pi^{3/2}\log(x)$$

output
```
-1/4*b*c^3*Pi^(3/2)*x^2-(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x+3/4*c*P
i^(3/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(3/2)*ln(x)+3/2*c^2*Pi*x*(a+b*arcsin
h(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.68.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx = \frac{\pi^{3/2}\left(-8a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+6bcx\operatorname{arcsinh}(cx)^2-bc\right.}{}$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
(Pi^(3/2)*(-8*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 6*b*c*
x*ArcSinh[c*x]^2 - b*c*x*Cosh[2*ArcSinh[c*x]] + 8*b*c*x*Log[c*x] + 2*ArcSi
nh[c*x]*(6*a*c*x - 4*b*Sqrt[1 + c^2*x^2] + b*c*x*Sinh[2*ArcSinh[c*x]])))/(
8*x)
```

### 3.68.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6222, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{6222}$$

$$3\pi c^2\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\pi^{3/2}bc\int \frac{c^2x^2+1}{x}dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{244}$$

$$3\pi c^2\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\pi^{3/2}bc\int \left(xc^2+\frac{1}{x}\right)dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$3\pi c^2\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\pi^{3/2}bc\left(\frac{c^2x^2}{2}+\log(x)\right)$$

$$\Big\downarrow \text{6200}$$

$$3\pi c^2\left(\frac{1}{2}\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\pi^{3/2}bc\left(\frac{c^2x^2}{2}+\log(x)\right)$$

$$\Big\downarrow \text{15}$$

$$3\pi c^2\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\pi^{3/2}bc\left(\frac{c^2x^2}{2}+\log(x)\right)$$

$$\Big\downarrow \text{6198}$$

$$3\pi c^2\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}\sqrt{\pi}bcx^2\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\pi^{3/2}bc\left(\frac{c^2x^2}{2}+\log(x)\right)$$

input
```
Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-(((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x) + 3*c^2*Pi*(-1/4*(b*c*
Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[P
i]*(a + b*ArcSinh[c*x])^2)/(4*b*c)) + b*c*Pi^(3/2)*((c^2*x^2)/2 + Log[x])
```

### 3.68.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(92) = 184$.

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.93

| method | result |
|---|---|
| default | $-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{\pi x}+a\,c^2x(\pi\,c^2x^2+\pi)^{\frac{3}{2}}+\frac{3a\,c^2\pi x\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{3a\,c^2\pi^2\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2\sqrt{\pi\,c^2}}+\frac{b\,\pi^{\frac{3}{2}}\left(4\,\mathrm{arcsinh}(c\right.}{}$ |
| parts | $-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{\pi x}+a\,c^2x(\pi\,c^2x^2+\pi)^{\frac{3}{2}}+\frac{3a\,c^2\pi x\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{3a\,c^2\pi^2\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2\sqrt{\pi\,c^2}}+\frac{b\,\pi^{\frac{3}{2}}\left(4\,\mathrm{arcsinh}(c\right.}{}$ |

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/2*a*c^2*Pi*x
*(Pi*c^2*x^2+Pi)^(1/2)+3/2*a*c^2*Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x
^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/8*b*Pi^(3/2)*(4*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)*x^2*c^2-2*c^3*x^3+6*arcsinh(c*x)^2*x*c-8*arcsinh(c*x)*c*x+8*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)*x*c-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)/x
```

### 3.68.5 Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)}{x^2}\,dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b
)*arcsinh(c*x))/x^2, x)
```

### 3.68.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \pi^{\frac{3}{2}}\left(\int ac^2\sqrt{c^2x^2+1}\,dx + \int \frac{a\sqrt{c^2x^2+1}}{x^2}\,dx + \int bc^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^2}\,dx\right)$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

output
```
pi**(3/2)*(Integral(a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*
x**2 + 1)/x**2, x) + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) +
Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

### 3.68.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.68.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,(\Pi\,c^2\,x^2 + \Pi)^{3/2}}{x^2} dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2, x)
```

# 3.69 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx$



## 3.69.1 Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = -\frac{bc\pi^{3/2}}{2x} - bc^3\pi^{3/2}x$$
$$+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx)) - \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2x^2} - 3c^2\pi^{3/2}(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}$$

output
```
-1/2*b*c*Pi^(3/2)/x-b*c^3*Pi^(3/2)*x-1/2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsin
h(c*x))/x^2-3*c^2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2
))-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^2*Pi^(3/2)
*polylog(2,c*x+(c^2*x^2+1)^(1/2))+3/2*c^2*Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^
2+Pi)^(1/2)
```

## 3.69.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.88

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = \frac{\pi^{3/2}\left(-8bc^3x^3 - 4a\sqrt{1+c^2x^2} + 8ac^2x^2\sqrt{1+c^2x^2} + 8bc^2x^2\sqrt{1+\right.}{}$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(Pi^(3/2)*(-8*b*c^3*x^3 - 4*a*Sqrt[1 + c^2*x^2] + 8*a*c^2*x^2*Sqrt[1 + c^2
*x^2] + 8*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*c^3*x^3*Csch[ArcSin
h[c*x]/2]^2 - b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 12*b*c^2*x^2
*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*ArcSinh[c*x]*Log[1
 + E^(-ArcSinh[c*x])] + 12*a*c^2*x^2*Log[x] - 12*a*c^2*x^2*Log[Pi*(1 + Sqr
t[1 + c^2*x^2])] + 12*b*c^2*x^2*PolyLog[2, -E^(-ArcSinh[c*x])] - 12*b*c^2*
x^2*PolyLog[2, E^(-ArcSinh[c*x])] + 4*b*c*x*Sinh[ArcSinh[c*x]/2]^2 - 4*b*A
rcSinh[c*x]*Sinh[ArcSinh[c*x]/2]^2))/(8*x^2)
```

### 3.69.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6222, 244, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}dx$$

$$\Big\downarrow \text{ 6222}$$

$$\frac{3}{2}\pi c^2\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}\pi^{3/2}bc\int \frac{c^2x^2+1}{x^2}dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{3}{2}\pi c^2\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}\pi^{3/2}bc\int \left(c^2+\frac{1}{x^2}\right)dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{2}\pi c^2\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 6221}$$

$$\frac{3}{2}\pi c^2\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int 1dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{3}{2}\pi c^2\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 6231}$$

$$\frac{3}{2}\pi c^2\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{2}\pi c^2\left(\sqrt{\pi}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{3}{2}\pi c^2\left(i\sqrt{\pi}\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{3}{2}\pi c^2\left(i\sqrt{\pi}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}}\right.\right.\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{3}{2}\pi c^2\left(i\sqrt{\pi}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}\right.\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

---

3.69. $\int\frac{\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

$$\Big\downarrow \text{2838}$$

$$\frac{3}{2}\pi c^2\left(i\sqrt{\pi}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right.\right.$$
$$\frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{3/2}bc\left(c^2x-\frac{1}{x}\right)$$

input
```
Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(b*c*Pi^(3/2)*(-x^(-1) + c^2*x))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSi
nh[c*x]))/(2*x^2) + (3*c^2*Pi*(-(b*c*Sqrt[Pi]*x) + Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]) + I*Sqrt[Pi]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^Arc
Sinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*
x]])))/2
```

### 3.69.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.69. $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

### 3.69.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| default | $a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{2\pi x^2}+\frac{3c^2\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)}{2}\right)+b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(c$ |
| parts | $a\left(-\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{2\pi x^2}+\frac{3c^2\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)}{2}\right)+b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(c$ |

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(5/2)+3/2*c^2*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi
*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))
)+b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(3/2)*c^2-b*c^3*Pi^(3/2)*x-1/2*b*Pi^
(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(3/2)/x-1/2*b*Pi^(3/2)
/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)-3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1+c
*x+(c^2*x^2+1)^(1/2))-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
+3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^2*Pi^
(3/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

### 3.69.5 Fricas [F]

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(\pi+\pi c^2x^2)^{\frac{3}{2}}\,(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b
)*arcsinh(c*x))/x^3, x)
```

### 3.69.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \pi^{\frac{3}{2}} \left( \int \frac{a\sqrt{c^2x^2+1}}{x^3} dx + \int \frac{ac^2\sqrt{c^2x^2+1}}{x} dx + \int \frac{b\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{x^3} dx + \int \frac{bc^2\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{x} dx \right)$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

output
```
pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(a*c**2*sqrt(
c**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x)
+ Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x))
```

### 3.69.7 Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)}{x^3} dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxim
a")
```

output
```
-1/2*(3*pi^(3/2)*c^2*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2)*c^
2 - (pi + pi*c^2*x^2)^(3/2)*c^2 + (pi + pi*c^2*x^2)^(5/2)/(pi*x^2))*a + b*
integrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

### 3.69.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^3} \, dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}}{x^3} \, dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3, x)
```

# 3.70 $\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.70.1 Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bc\pi^{3/2}}{6x^2} - \frac{c^2\pi\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{x} - \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{c^3\pi^{3/2}(a+b\text{arcsinh}(cx))^2}{2b} + \frac{4}{3}bc^3\pi^{3/2}\log(x)$$

output
```
-1/6*b*c*Pi^(3/2)/x^2-1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3+1/2
*c^3*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b+4/3*b*c^3*Pi^(3/2)*ln(x)-c^2*Pi*(a+b*
arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x
```

## 3.70.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{\pi^{3/2}\left(-bcx-2a\sqrt{1+c^2x^2}-8ac^2x^2\sqrt{1+c^2x^2}+\left(6ac^3x^3-2b\sqrt{1\right.\right.}{}$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
(Pi^(3/2)*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2] - 8*a*c^2*x^2*Sqrt[1 + c^2*x^2
] + (6*a*c^3*x^3 - 2*b*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2))*ArcSinh[c*x] + 3
*b*c^3*x^3*ArcSinh[c*x]^2 + 8*b*c^3*x^3*Log[c*x]))/(6*x^3)
```

### 3.70.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6222, 244, 2009, 6220, 14, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6222}$$

$$\pi c^2\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x^2}dx+\frac{1}{3}\pi^{3/2}bc\int \frac{c^2x^2+1}{x^3}dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{244}$$

$$\pi c^2\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x^2}dx+\frac{1}{3}\pi^{3/2}bc\int \left(\frac{c^2}{x}+\frac{1}{x^3}\right)dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\pi c^2\int \frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x^2}dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{3}\pi^{3/2}bc\left(c^2\log(x)-\frac{1}{2x^2}\right)$$

$$\Big\downarrow \text{6220}$$

$$\pi c^2\left(\sqrt{\pi}c^2\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\sqrt{\pi}bc\int \frac{1}{x}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x}\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{3}\pi^{3/2}bc\left(c^2\log(x)-\frac{1}{2x^2}\right)$$

$$\Big\downarrow \text{14}$$

$$\pi c^2\left(\sqrt{\pi}c^2\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x}+\sqrt{\pi}bc\log(x)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{3}\pi^{3/2}bc\left(c^2\log(x)-\frac{1}{2x^2}\right)$$

$$\Big\downarrow \text{6198}$$

$$\pi c^2\left(-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{x}+\frac{\sqrt{\pi}c(a+b\text{arcsinh}(cx))^2}{2b}+\sqrt{\pi}bc\log(x)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{3}\pi^{3/2}bc\left(c^2\log(x)-\frac{1}{2x^2}\right)$$

input
```
Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3 + (b*c*Pi^(3/2)*(-
1/2*1/x^2 + c^2*Log[x]))/3 + c^2*Pi*(-((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSi
nh[c*x]))/x) + (c*Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(2*b) + b*c*Sqrt[Pi]*Lo
g[x])
```

### 3.70.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(97) = 194$.

Time = 0.16 (sec) , antiderivative size = 622, normalized size of antiderivative = 5.41

| method | result |
|---|---|
| default | $-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x^3}-\frac{2ac^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x}+\frac{2ac^4x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+ac^4\pi x\sqrt{\pi c^2x^2+\pi}+\frac{ac^4\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}$ |
| parts | $-\frac{a(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x^3}-\frac{2ac^2(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3\pi x}+\frac{2ac^4x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+ac^4\pi x\sqrt{\pi c^2x^2+\pi}+\frac{ac^4\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}$ |

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(5/2)-2/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+2
/3*a*c^4*x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^4*Pi
^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*
c^3*Pi^(3/2)*arcsinh(c*x)^2-8/3*b*c^3*Pi^(3/2)*arcsinh(c*x)+32*b*Pi^(3/2)/
(24*c^4*x^4+9*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-32*b*Pi^(3/2)/(24*c^4*x^4+9*
c^2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^6+8/3*b*Pi^(3/2)/(24*c^4*x
^4+9*c^2*x^2+1)*x^4*c^7-8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*(c^2*x
^2+1)*c^5+12*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-20*b
*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^4-4/
3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*(c^2*x^2+1)*c^3+4/3*b*Pi^(3/2)/(24*c
^4*x^4+9*c^2*x^2+1)*arcsinh(c*x)*c^3-13/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2
+1)/x*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/6*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*
x^2+1)/x^2*(c^2*x^2+1)*c-1/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^3*(c^2*
x^2+1)^(1/2)*arcsinh(c*x)+4/3*b*c^3*Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-
1)
```

### 3.70.5 Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)}{x^4}\,dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b
)*arcsinh(c*x))/x^4, x)
```

### 3.70.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = \pi^{\frac{3}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^4}\,dx + \int \frac{ac^2\sqrt{c^2x^2+1}}{x^2}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^4}\,dx + \int \frac{bc^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^2}\,dx\right)$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

output
```
pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(a*c**2*sqrt(
c**2*x**2 + 1)/x**2, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**4,
x) + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

### 3.70.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.70.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,(\Pi\,c^2\,x^2 + \Pi)^{3/2}}{x^4} dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4, x)
```

# 3.71 $\int x^3(\pi + c^2\pi x^2)^{5/2} (a + b\,\mathrm{arcsinh}(cx))\,dx$



## 3.71.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int x^3 (\pi + c^2\pi x^2)^{5/2} (a + b\,\mathrm{arcsinh}(cx))\,dx = \frac{2b\pi^{5/2}x}{63c^3} - \frac{b\pi^{5/2}x^3}{189c} - \frac{1}{21}bc\pi^{5/2}x^5 - \frac{19}{441}bc^3\pi^{5/2}x^7 - \frac{1}{81}bc^5\pi^{5/2}x^9 - \frac{(\pi + c^2\pi x^2)^{7/2}(a + b\,\mathrm{arcsinh}(cx))}{7c^4\pi} + \frac{(\pi + c^2\pi x^2)^{9/2}(a + b\,\mathrm{arcsinh}(cx))}{9c^4\pi^2}$$

output
```
2/63*b*Pi^(5/2)*x/c^3-1/189*b*Pi^(5/2)*x^3/c-1/21*b*c*Pi^(5/2)*x^5-19/441*
b*c^3*Pi^(5/2)*x^7-1/81*b*c^5*Pi^(5/2)*x^9-1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*
arcsinh(c*x))/c^4/Pi+1/9*(Pi*c^2*x^2+Pi)^(9/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

## 3.71.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int x^3 (\pi + c^2\pi x^2)^{5/2} (a + b\,\mathrm{arcsinh}(cx))\,dx = \frac{\pi^{5/2}\Big(63a(1 + c^2x^2)^{7/2}(-2 + 7c^2x^2) - bcx(-126 + 21c^2x^2 + 189c^4x^4 + 171c^6x^6 + 49c^8}{3969c^4}$$

input
```
Integrate[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(5/2)*(63*a*(1 + c^2*x^2)^(7/2)*(-2 + 7*c^2*x^2) - b*c*x*(-126 + 21*c^
2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8) + 63*b*(1 + c^2*x^2)^(7/2)
*(-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4)
```

### 3.71.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{\pi^2\left(2-7c^2x^2\right)\left(c^2x^2+1\right)^3}{63c^4}dx+\frac{\left(\pi c^2x^2+\pi\right)^{9/2}(a+b\text{arcsinh}(cx))}{9\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\pi^{5/2}b\int\left(2-7c^2x^2\right)\left(c^2x^2+1\right)^3dx}{63c^3}+\frac{\left(\pi c^2x^2+\pi\right)^{9/2}(a+b\text{arcsinh}(cx))}{9\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^4}$$

$$\Big\downarrow \text{290}$$

$$\frac{\pi^{5/2}b\int\left(-7c^8x^8-19c^6x^6-15c^4x^4-c^2x^2+2\right)dx}{63c^3}+\frac{\left(\pi c^2x^2+\pi\right)^{9/2}(a+b\text{arcsinh}(cx))}{9\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{9/2}(a+b\text{arcsinh}(cx))}{9\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^4}+\frac{\pi^{5/2}b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}+2x\right)}{63c^3}$$

input
```
Int[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(b*Pi^(5/2)*(2*x - (c^2*x^3)/3 - 3*c^4*x^5 - (19*c^6*x^7)/7 - (7*c^8*x^9)/
9))/(63*c^3) - ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi) +
 ((Pi + c^2*Pi*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^4*Pi^2)
```

#### 3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.71.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| default | $a\left(\frac{x^{2}\left(\pi c^{2}x^{2}+\pi\right)^{\frac{7}{2}}}{9\pi c^{2}}-\frac{2\left(\pi c^{2}x^{2}+\pi\right)^{\frac{7}{2}}}{63\pi c^{4}}\right)+\frac{b\pi^{\frac{5}{2}}\left(441\,\text{arcsinh}(cx)c^{10}x^{10}+1638\,\text{arcsinh}(cx)c^{8}x^{8}-49c^{9}x^{9}\sqrt{c^{2}x^{2}+1}+2142\,\text{arcsinh}\right.}{}$ |
| parts | $a\left(\frac{x^{2}\left(\pi c^{2}x^{2}+\pi\right)^{\frac{7}{2}}}{9\pi c^{2}}-\frac{2\left(\pi c^{2}x^{2}+\pi\right)^{\frac{7}{2}}}{63\pi c^{4}}\right)+\frac{b\pi^{\frac{5}{2}}\left(441\,\text{arcsinh}(cx)c^{10}x^{10}+1638\,\text{arcsinh}(cx)c^{8}x^{8}-49c^{9}x^{9}\sqrt{c^{2}x^{2}+1}+2142\,\text{arcsinh}\right.}{}$ |

input
```
int(x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*x^2*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-2/63/Pi/c^4*(Pi*c^2*x^2+Pi)^(7/2))
+1/3969*b/c^4*Pi^(5/2)/(c^2*x^2+1)^(1/2)*(441*arcsinh(c*x)*c^10*x^10+1638*
arcsinh(c*x)*c^8*x^8-49*c^9*x^9*(c^2*x^2+1)^(1/2)+2142*arcsinh(c*x)*c^6*x^
6-171*c^7*x^7*(c^2*x^2+1)^(1/2)+1008*arcsinh(c*x)*c^4*x^4-189*c^5*x^5*(c^2
*x^2+1)^(1/2)-63*arcsinh(c*x)*c^2*x^2-21*c^3*x^3*(c^2*x^2+1)^(1/2)-126*arc
sinh(c*x)+126*c*x*(c^2*x^2+1)^(1/2))
```

### 3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(113) = 226$.

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.87

$$\int x^3\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \frac{63\sqrt{\pi + \pi c^2x^2}(7\pi^2bc^{10}x^{10} + 26\pi^2bc^8x^8 + 34\pi^2bc^6x^6 + 16\pi^2bc^4x^4 - \pi^2bc^2x^2 - 2\pi^2b)\,\mathrm{l}}{}$$

input
```
integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="frica
s")
```

output
```
1/3969*(63*sqrt(pi + pi*c^2*x^2)*(7*pi^2*b*c^10*x^10 + 26*pi^2*b*c^8*x^8 +
 34*pi^2*b*c^6*x^6 + 16*pi^2*b*c^4*x^4 - pi^2*b*c^2*x^2 - 2*pi^2*b)*log(c*
x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(441*pi^2*a*c^10*x^10 + 163
8*pi^2*a*c^8*x^8 + 2142*pi^2*a*c^6*x^6 + 1008*pi^2*a*c^4*x^4 - 63*pi^2*a*c
^2*x^2 - 126*pi^2*a - (49*pi^2*b*c^9*x^9 + 171*pi^2*b*c^7*x^7 + 189*pi^2*b
*c^5*x^5 + 21*pi^2*b*c^3*x^3 - 126*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^
2 + c^4)
```

### 3.71.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(133) = 266$.

Time = 90.31 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.69

$$\int x^3\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \begin{cases} \frac{\pi^{\frac{5}{2}}ac^4x^8\sqrt{c^2x^2+1}}{9} + \frac{19\pi^{\frac{5}{2}}ac^2x^6\sqrt{c^2x^2+1}}{63} + \frac{5\pi^{\frac{5}{2}}ax^4\sqrt{c^2x^2+1}}{21} + \frac{\pi^{\frac{5}{2}}ax^2\sqrt{c^2x^2+1}}{63c^2} - \frac{2\pi^{\frac{5}{2}}a\sqrt{c^2x^2+1}}{63c^4} - \frac{\pi^{\frac{5}{2}}}{} \\ \frac{\pi^{\frac{5}{2}}ax^4}{4} \end{cases}$$

input
```
integrate(x**3*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(5/2)*a*c**4*x**8*sqrt(c**2*x**2 + 1)/9 + 19*pi**(5/2)*a*c*
*2*x**6*sqrt(c**2*x**2 + 1)/63 + 5*pi**(5/2)*a*x**4*sqrt(c**2*x**2 + 1)/21
 + pi**(5/2)*a*x**2*sqrt(c**2*x**2 + 1)/(63*c**2) - 2*pi**(5/2)*a*sqrt(c**
2*x**2 + 1)/(63*c**4) - pi**(5/2)*b*c**5*x**9/81 + pi**(5/2)*b*c**4*x**8*s
qrt(c**2*x**2 + 1)*asinh(c*x)/9 - 19*pi**(5/2)*b*c**3*x**7/441 + 19*pi**(5
/2)*b*c**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/63 - pi**(5/2)*b*c*x**5/21
+ 5*pi**(5/2)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/21 - pi**(5/2)*b*x**3/
(189*c) + pi**(5/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(63*c**2) + 2*pi
**(5/2)*b*x/(63*c**3) - 2*pi**(5/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(63*c
**4), Ne(c, 0)), (pi**(5/2)*a*x**4/4, True))
```

### 3.71.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11

$$\int x^3(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{1}{63}\left(\frac{7\,(\pi+\pi c^2x^2)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\,(\pi+\pi c^2x^2)^{\frac{7}{2}}}{\pi c^4}\right)b\,\text{arsinh}\,(cx)+\frac{1}{63}\left(\frac{7\,(\pi+\pi c^2x^2)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\,(\pi+\pi c^2x^2)^{\frac{7}{2}}}{\pi c^4}\right)a-\frac{\left(49\,\pi^{\frac{5}{2}}c^8x^9+171\,\pi^{\frac{5}{2}}c^6x^7+189\,\pi^{\frac{5}{2}}c^4x^5+21\,\pi^{\frac{5}{2}}c^2x^3-126\,\pi^{\frac{5}{2}}x\right)b}{3969\,c^3}$$

input
```
integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxim
a")
```

output
```
1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(7/2)/(
pi*c^4))*b*arcsinh(c*x) + 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2
*(pi + pi*c^2*x^2)^(7/2)/(pi*c^4))*a - 1/3969*(49*pi^(5/2)*c^8*x^9 + 171*p
i^(5/2)*c^6*x^7 + 189*pi^(5/2)*c^4*x^5 + 21*pi^(5/2)*c^2*x^3 - 126*pi^(5/2
)*x)*b/c^3
```

### 3.71.8 Giac [F(-2)]

Exception generated.

$$\int x^3 \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.71.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int x^3 \left(a + b \operatorname{asinh}(c\, x)\right) \left(\Pi\, c^2\, x^2 + \Pi\right)^{5/2} dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.72 $\int x^2(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx)) dx$



## 3.72.1 Optimal result

Integrand size = 26, antiderivative size = 213

$$\int x^2(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx)) dx = -\frac{5b\pi^{5/2}x^2}{256c} - \frac{59}{768}bc\pi^{5/2}x^4 - \frac{17}{288}bc^3\pi^{5/2}x^6 - \frac{1}{64}bc^5\pi^{5/2}x^8 + \frac{5\pi^{5/2}x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{128c^2} + \frac{5}{64}\pi^2x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{5}{48}\pi x^3(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{8}x^3(\pi+c^2\pi x^2)^{5/2}(a+\text{bar}$$

output
```
-5/256*b*Pi^(5/2)*x^2/c-59/768*b*c*Pi^(5/2)*x^4-17/288*b*c^3*Pi^(5/2)*x^6-
1/64*b*c^5*Pi^(5/2)*x^8+5/48*Pi*x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x
))+1/8*x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))-5/256*Pi^(5/2)*(a+b*ar
csinh(c*x))^2/b/c^3+5/128*Pi^(5/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/
c^2+5/64*Pi^2*x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.72.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int x^2(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx)) dx = \frac{\pi^{5/2}\left(2880acx\sqrt{1+c^2x^2} + 22656ac^3x^3\sqrt{1+c^2x^2} + 26112ac^5x^5\sqrt{1+c^2x^2} + 9216ac^7x^7\right.}{}$$

input
```
Integrate[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(5/2)*(2880*a*c*x*Sqrt[1 + c^2*x^2] + 22656*a*c^3*x^3*Sqrt[1 + c^2*x^2
] + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2] -
 1440*b*ArcSinh[c*x]^2 + 576*b*Cosh[2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh
[c*x]] - 64*b*Cosh[6*ArcSinh[c*x]] - 9*b*Cosh[8*ArcSinh[c*x]] - 24*ArcSinh
[c*x]*(120*a + 48*b*Sinh[2*ArcSinh[c*x]] - 24*b*Sinh[4*ArcSinh[c*x]] - 16*
b*Sinh[6*ArcSinh[c*x]] - 3*b*Sinh[8*ArcSinh[c*x]])))/(73728*c^3)
```

### 3.72.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6223, 243, 49, 2009, 6223, 244, 2009, 6221, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(\pi c^2 x^2+\pi\right)^{5/2} (a+b\text{arcsinh}(cx)) \, dx$$

$$\Big\downarrow \text{6223}$$

$$\frac{5}{8}\pi \int x^2 \left(c^2\pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{1}{8}\pi^{5/2}bc\int x^3 \left(c^2x^2+1\right)^2dx+\frac{1}{8}x^3 \left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{243}$$

$$\frac{5}{8}\pi \int x^2 \left(c^2\pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{1}{16}\pi^{5/2}bc\int x^2 \left(c^2x^2+1\right)^2dx^2+\frac{1}{8}x^3 \left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{49}$$

$$\frac{5}{8}\pi \int x^2 \left(c^2\pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{1}{16}\pi^{5/2}bc\int \left(c^4x^6+2c^2x^4+x^2\right)dx^2+\frac{1}{8}x^3 \left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{8}\pi \int x^2 \left(c^2\pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx+\frac{1}{8}x^3 \left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \ 6223$$

$$\frac{5}{8}\pi\left(\frac{1}{2}\pi\int x^2\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}\pi^{3/2}bc\int x^3\left(c^2x^2+1\right)dx+\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(\right.$$
$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \ 244$$

$$\frac{5}{8}\pi\left(\frac{1}{2}\pi\int x^2\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}\pi^{3/2}bc\int\left(c^2x^5+x^3\right)dx+\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(c\right.$$
$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{5}{8}\pi\left(\frac{1}{2}\pi\int x^2\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{6}\pi^{3/2}bc\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\right.$$
$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \ 6221$$

$$\frac{5}{8}\pi\left(\frac{1}{2}\pi\left(\frac{1}{4}\sqrt{\pi}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{4}\sqrt{\pi}bc\int x^3dx+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x^3\left(\pi c^2x^2\right.\right.$$
$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \ 15$$

$$\frac{5}{8}\pi\left(\frac{1}{2}\pi\left(\frac{1}{4}\sqrt{\pi}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{16}\sqrt{\pi}bcx^4\right)+\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right.\right.$$
$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \ 6227$$

$$\frac{5}{8}\pi\left(\frac{1}{2}\pi\left(\frac{1}{4}\sqrt{\pi}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arc}\right.\right.$$
$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \ 15$$

---

3.72. $\int x^2(\pi+c^2\pi x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx$

$$\frac{5}{8}\pi\left(\frac{1}{2}\pi\left(\frac{1}{4}\sqrt{\pi}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}\right.\right.$$
$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{5}{8}\pi\left(\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{2}\pi\left(\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{1}{4}\sqrt{\pi}\left(-\frac{(a+b\text{arcsinh}(c}{4bc^3}\right.\right.\right.$$
$$\frac{1}{16}\pi^{5/2}bc\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)$$

input
```
Int[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/16*(b*c*Pi^(5/2)*(x^4/2 + (2*c^2*x^6)/3 + (c^4*x^8)/4)) + (x^3*(Pi + c^
2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8 + (5*Pi*(-1/6*(b*c*Pi^(3/2)*(x^4/4
 + (c^2*x^6)/6)) + (x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 +
(Pi*(-1/16*(b*c*Sqrt[Pi]*x^4) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[
c*x]))/4 + (Sqrt[Pi]*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3)))/4))/2))/8
```

### 3.72.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.72. $\int x^2(\pi + c^2\pi x^2)^{5/2}\,(a + b\text{arcsinh}(cx))\,dx$

### 3.72.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| default | $\frac{ax(\pi c^2x^2+\pi)^{\frac{7}{2}}}{8\pi c^2}-\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{48c^2}-\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{192c^2}-\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{128c^2}-\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{128c^2\sqrt{\pi c^2}}-\frac{b\pi^{\frac{5}{2}}(-$ |
| parts | $\frac{ax(\pi c^2x^2+\pi)^{\frac{7}{2}}}{8\pi c^2}-\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{48c^2}-\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{192c^2}-\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{128c^2}-\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{128c^2\sqrt{\pi c^2}}-\frac{b\pi^{\frac{5}{2}}(-$ |

input
```
int(x^2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*x*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-1/48*a/c^2*x*(Pi*c^2*x^2+Pi)^(5/2)-5/
192*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)-5/128*a/c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1
/2)-5/128*a/c^2*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi
*c^2)^(1/2)-1/2304*b*Pi^(5/2)*(-288*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^7*c^7
+36*c^8*x^8-816*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5*c^5+136*c^6*x^6-708*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+177*c^4*x^4-90*arcsinh(c*x)*c*x*(c^2*x
^2+1)^(1/2)+45*c^2*x^2+45*arcsinh(c*x)^2-32)/c^3
```

### 3.72.5 Fricas [F]

$$\int x^2\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^6 + 2*pi^2*a*c^2*x^4 + pi^2*a
*x^2 + (pi^2*b*c^4*x^6 + 2*pi^2*b*c^2*x^4 + pi^2*b*x^2)*arcsinh(c*x)), x)
```

### 3.72.6 Sympy [A] (verification not implemented)

Time = 49.52 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.64

$$\int x^2 (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \begin{cases} \frac{\pi^{\frac{5}{2}} ac^4x^7\sqrt{c^2x^2+1}}{8} + \frac{17\pi^{\frac{5}{2}} ac^2x^5\sqrt{c^2x^2+1}}{48} + \frac{59\pi^{\frac{5}{2}} ax^3\sqrt{c^2x^2+1}}{192} + \frac{5\pi^{\frac{5}{2}} ax\sqrt{c^2x^2+1}}{128c^2} - \frac{5\pi^{\frac{5}{2}} a\,\text{asinh}\,(cx)}{128c^3} - \pi \\ \frac{\pi^{\frac{5}{2}} ax^3}{3} \end{cases}$$

input
```
integrate(x**2*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(5/2)*a*c**4*x**7*sqrt(c**2*x**2 + 1)/8 + 17*pi**(5/2)*a*c*
*2*x**5*sqrt(c**2*x**2 + 1)/48 + 59*pi**(5/2)*a*x**3*sqrt(c**2*x**2 + 1)/1
92 + 5*pi**(5/2)*a*x*sqrt(c**2*x**2 + 1)/(128*c**2) - 5*pi**(5/2)*a*asinh(
c*x)/(128*c**3) - pi**(5/2)*b*c**5*x**8/64 + pi**(5/2)*b*c**4*x**7*sqrt(c*
*2*x**2 + 1)*asinh(c*x)/8 - 17*pi**(5/2)*b*c**3*x**6/288 + 17*pi**(5/2)*b*
c**2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/48 - 59*pi**(5/2)*b*c*x**4/768 +
59*pi**(5/2)*b*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/192 - 5*pi**(5/2)*b*x**
2/(256*c) + 5*pi**(5/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(128*c**2) - 5*
pi**(5/2)*b*asinh(c*x)**2/(256*c**3), Ne(c, 0)), (pi**(5/2)*a*x**3/3, True
))
```

### 3.72.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.72.8 Giac [F]

$$\int x^2 \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int (\pi + \pi c^2 x^2)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^2\, dx$$

input
```
integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac"
)
```

output
```
integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

### 3.72.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int x^2 \left(a + b \operatorname{asinh}(c\, x)\right) \left(\Pi\, c^2\, x^2 + \Pi\right)^{5/2} dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.73 $\int x(\pi + c^2\pi x^2)^{5/2}\,(a + b\,\mathrm{arcsinh}(cx))\,dx$



## 3.73.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int x\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\,\mathrm{arcsinh}(cx))\,dx = -\frac{b\pi^{5/2}x}{7c} - \frac{1}{7}bc\pi^{5/2}x^3 - \frac{3}{35}bc^3\pi^{5/2}x^5 - \frac{1}{49}bc^5\pi^{5/2}x^7 + \frac{\left(\pi + c^2\pi x^2\right)^{7/2}(a + b\,\mathrm{arcsinh}(cx))}{7c^2\pi}$$

output
```
-1/7*b*Pi^(5/2)*x/c-1/7*b*c*Pi^(5/2)*x^3-3/35*b*c^3*Pi^(5/2)*x^5-1/49*b*c^
5*Pi^(5/2)*x^7+1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

## 3.73.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int x\left(\pi + c^2\pi x^2\right)^{5/2}(a + b\,\mathrm{arcsinh}(cx))\,dx = \frac{\pi^{5/2}\Big(35a(1 + c^2x^2)^{7/2} - bcx(35 + 35c^2x^2 + 21c^4x^4 + 5c^6x^6) + 35b(1 + c^2x^2)^{7/2}\,\mathrm{arcsinh}}{245c^2}$$

input
```
Integrate[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(5/2)*(35*a*(1 + c^2*x^2)^(7/2) - b*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4
+ 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^(7/2)*ArcSinh[c*x]))/(245*c^2)
```

### 3.73.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6213, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^2}-\frac{\pi^{5/2}b\int\left(c^2x^2+1\right)^3dx}{7c}$$

$$\Big\downarrow \text{210}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^2}-\frac{\pi^{5/2}b\int\left(c^6x^6+3c^4x^4+3c^2x^2+1\right)dx}{7c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}(a+b\text{arcsinh}(cx))}{7\pi c^2}-\frac{\pi^{5/2}b\left(\frac{c^6x^7}{7}+\frac{3c^4x^5}{5}+c^2x^3+x\right)}{7c}$$

input
```
Int[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/7*(b*Pi^(5/2)*(x + c^2*x^3 + (3*c^4*x^5)/5 + (c^6*x^7)/7))/c + ((Pi + c
^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2*Pi)
```

#### 3.73.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(73) = 146$.

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.83

| method | result |
|---|---|
| default | $\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{7\pi c^2}+\frac{b\pi^{\frac{5}{2}}\left(35\operatorname{arcsinh}(cx)c^8x^8+140\operatorname{arcsinh}(cx)c^6x^6-5c^7x^7\sqrt{c^2x^2+1}+210\operatorname{arcsinh}(cx)c^4x^4-21c^5x^5\sqrt{c^2x^2+1}+140\right.}{245c^2\sqrt{c^2x^2+1}}$ |
| parts | $\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{7\pi c^2}+\frac{b\pi^{\frac{5}{2}}\left(35\operatorname{arcsinh}(cx)c^8x^8+140\operatorname{arcsinh}(cx)c^6x^6-5c^7x^7\sqrt{c^2x^2+1}+210\operatorname{arcsinh}(cx)c^4x^4-21c^5x^5\sqrt{c^2x^2+1}+140\right.}{245c^2\sqrt{c^2x^2+1}}$ |

input
```
int(x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/7*a*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2+1/245*b/c^2*Pi^(5/2)/(c^2*x^2+1)^(1/2)*
(35*arcsinh(c*x)*c^8*x^8+140*arcsinh(c*x)*c^6*x^6-5*c^7*x^7*(c^2*x^2+1)^(1
/2)+210*arcsinh(c*x)*c^4*x^4-21*c^5*x^5*(c^2*x^2+1)^(1/2)+140*arcsinh(c*x)
*c^2*x^2-35*c^3*x^3*(c^2*x^2+1)^(1/2)+35*arcsinh(c*x)-35*c*x*(c^2*x^2+1)^(
1/2))
```

### 3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(73) = 146$.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.42

$$\int x(\pi+c^2\pi x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{35\sqrt{\pi+\pi c^2x^2}(\pi^2bc^8x^8+4\pi^2bc^6x^6+6\pi^2bc^4x^4+4\pi^2bc^2x^2+\pi^2b)\log\left(cx+\sqrt{c^2x^2+1}\right)}{}$$

input
```
integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
1/245*(35*sqrt(pi + pi*c^2*x^2)*(pi^2*b*c^8*x^8 + 4*pi^2*b*c^6*x^6 + 6*pi^
2*b*c^4*x^4 + 4*pi^2*b*c^2*x^2 + pi^2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sq
rt(pi + pi*c^2*x^2)*(35*pi^2*a*c^8*x^8 + 140*pi^2*a*c^6*x^6 + 210*pi^2*a*c
^4*x^4 + 140*pi^2*a*c^2*x^2 + 35*pi^2*a - (5*pi^2*b*c^7*x^7 + 21*pi^2*b*c^
5*x^5 + 35*pi^2*b*c^3*x^3 + 35*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 +
c^2)
```

### 3.73.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(85) = 170$.

Time = 28.36 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.22

$$\int x(\pi + c^2\pi x^2)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \begin{cases} \frac{\pi^{\frac{5}{2}}ac^4x^6\sqrt{c^2x^2+1}}{7} + \frac{3\pi^{\frac{5}{2}}ac^2x^4\sqrt{c^2x^2+1}}{7} + \frac{3\pi^{\frac{5}{2}}ax^2\sqrt{c^2x^2+1}}{7} + \frac{\pi^{\frac{5}{2}}a\sqrt{c^2x^2+1}}{7c^2} - \frac{\pi^{\frac{5}{2}}bc^5x^7}{49} + \frac{\pi^{\frac{5}{2}}bc^4x^6\sqrt{c^2 \cdots}}{} \\ \frac{\pi^{\frac{5}{2}}ax^2}{2} \end{cases}$$

input
```
integrate(x*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise(((pi**(5/2)*a*c**4*x**6*sqrt(c**2*x**2 + 1)/7 + 3*pi**(5/2)*a*c**
2*x**4*sqrt(c**2*x**2 + 1)/7 + 3*pi**(5/2)*a*x**2*sqrt(c**2*x**2 + 1)/7 +
pi**(5/2)*a*sqrt(c**2*x**2 + 1)/(7*c**2) - pi**(5/2)*b*c**5*x**7/49 + pi**
(5/2)*b*c**4*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - 3*pi**(5/2)*b*c**3*x*
*5/35 + 3*pi**(5/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - pi**(5/
2)*b*c*x**3/7 + 3*pi**(5/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/7 - pi**
(5/2)*b*x/(7*c) + pi**(5/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(7*c**2), Ne(
c, 0)), (pi**(5/2)*a*x**2/2, True))
```

### 3.73.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int x(\pi + c^2\pi x^2)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \frac{(\pi + \pi c^2x^2)^{\frac{7}{2}}b\operatorname{arsinh}(cx)}{7\,\pi c^2} + \frac{(\pi + \pi c^2x^2)^{\frac{7}{2}}a}{7\,\pi c^2} - \frac{\left(5\,\pi^{\frac{7}{2}}c^6x^7 + 21\,\pi^{\frac{7}{2}}c^4x^5 + 35\,\pi^{\frac{7}{2}}c^2x^3 + 35\,\pi^{\frac{7}{2}}x\right)b}{245\,\pi c}$$

input
```
integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
1/7*(pi + pi*c^2*x^2)^(7/2)*b*arcsinh(c*x)/(pi*c^2) + 1/7*(pi + pi*c^2*x^2
)^(7/2)*a/(pi*c^2) - 1/245*(5*pi^(7/2)*c^6*x^7 + 21*pi^(7/2)*c^4*x^5 + 35*
pi^(7/2)*c^2*x^3 + 35*pi^(7/2)*x)*b/(pi*c)
```

### 3.73.8 Giac [F(-2)]

Exception generated.

$$\int x(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.73.9 Mupad [F(-1)]

Timed out.

$$\int x(\pi + c^2\pi x^2)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \int x\,(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.74 $\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx$



## 3.74.1 Optimal result

Integrand size = 23, antiderivative size = 165

$$\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = -\frac{25}{96} bc\pi^{5/2}x^2 - \frac{5}{96} bc^3\pi^{5/2}x^4 - \frac{b\pi^{5/2}(1 + c^2x^2)^3}{36c} + \frac{5}{16}\pi^2 x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{5}{24}\pi x(\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx)) + \frac{1}{6}x(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsi}$$

output
```
-25/96*b*c*Pi^(5/2)*x^2-5/96*b*c^3*Pi^(5/2)*x^4-1/36*b*Pi^(5/2)*(c^2*x^2+1
)^3/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(Pi*c^2*x^2
+Pi)^(5/2)*(a+b*arcsinh(c*x))+5/32*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b/c+5/16*
Pi^2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.74.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.93

$$\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \frac{\pi^{5/2}\left(1584acx\sqrt{1 + c^2x^2} + 1248ac^3x^3\sqrt{1 + c^2x^2} + 384ac^5x^5\sqrt{1 + c^2x^2} + 360b\text{arcsinh}(c\right.}{}$$

input
```
Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(5/2)*(1584*a*c*x*Sqrt[1 + c^2*x^2] + 1248*a*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 360*b*ArcSinh[c*x]^2 - 270*b*Cosh[2*A
rcSinh[c*x]] - 27*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] + 12*A
rcSinh[c*x]*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] +
 b*Sinh[6*ArcSinh[c*x]])))/(2304*c)
```

### 3.74.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{5}{6}\pi\int \left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2dx+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 241}$$

$$\frac{5}{6}\pi\int \left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int \left(c^2x^3+x\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)+$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$\big\downarrow$ 6200

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$\big\downarrow$ 15

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$\big\downarrow$ 6198

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{5}{6}\pi\left(\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}\right.\right.$$
$$\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

input
```
Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/36*(b*Pi^(5/2)*(1 + c^2*x^2)^3)/c + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*A
rcSinh[c*x]))/6 + (5*Pi*(-1/4*(b*c*Pi^(3/2)*(x^2/2 + (c^2*x^4)/4)) + (x*(P
i + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*Pi*(-1/4*(b*c*Sqrt[Pi]*
x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[Pi]*(a + b
*ArcSinh[c*x])^2)/(4*b*c)))/4))/6
```

## 3.74. $\int\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

### 3.74.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

## 3.74.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| default | $\frac{x(\pi c^2x^2+\pi)^{\frac{5}{2}}a}{6}+\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b\pi^{\frac{5}{2}}\left(48\operatorname{arcsinh}(cx)\sqrt{c^2x^2+\right.}{}$ |
| parts | $\frac{x(\pi c^2x^2+\pi)^{\frac{5}{2}}a}{6}+\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b\pi^{\frac{5}{2}}\left(48\operatorname{arcsinh}(cx)\sqrt{c^2x^2+\right.}{}$ |

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*a+5/24*a*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a*Pi^
2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x
^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/288*b*Pi^(5/2)*(48*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*x^5*c^5-8*c^6*x^6+156*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-39*c^4
*x^4+198*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-99*c^2*x^2+45*arcsinh(c*x)^2-6
8)/c
```

## 3.74.5 Fricas [F]

$$\int \left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int \left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}(cx)+a\right)dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
 + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x)), x)
```

### 3.74.6 Sympy [A] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.61

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \begin{cases} \frac{\pi^{\frac{5}{2}} a c^4 x^5 \sqrt{c^2x^2+1}}{6} + \frac{13\pi^{\frac{5}{2}} a c^2 x^3 \sqrt{c^2x^2+1}}{24} + \frac{11\pi^{\frac{5}{2}} a x \sqrt{c^2x^2+1}}{16} + \frac{5\pi^{\frac{5}{2}} a \operatorname{asinh}(cx)}{16c} - \frac{\pi^{\frac{5}{2}} b c^5 x^6}{36} + \frac{\pi^{\frac{5}{2}} b c^4 x^5 \sqrt{\ldots}}{\ldots} \\ \pi^{\frac{5}{2}} a x \end{cases}$$

input
```
integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(5/2)*a*c**4*x**5*sqrt(c**2*x**2 + 1)/6 + 13*pi**(5/2)*a*c*
*2*x**3*sqrt(c**2*x**2 + 1)/24 + 11*pi**(5/2)*a*x*sqrt(c**2*x**2 + 1)/16 +
 5*pi**(5/2)*a*asinh(c*x)/(16*c) - pi**(5/2)*b*c**5*x**6/36 + pi**(5/2)*b*
c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/6 - 13*pi**(5/2)*b*c**3*x**4/96 +
 13*pi**(5/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/24 - 11*pi**(5/2)
*b*c*x**2/32 + 11*pi**(5/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/16 + 5*pi**
(5/2)*b*asinh(c*x)**2/(32*c), Ne(c, 0)), (pi**(5/2)*a*x, True))
```

### 3.74.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.74.8 Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.74.9 Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int (a + b\,\text{asinh}(c\,x))\, \left(\Pi\, c^2\, x^2 + \Pi\right)^{5/2} dx$$

input
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.75 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$



## 3.75.1 Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{23}{15}bc\pi^{5/2}x-\frac{11}{45}bc^3\pi^{5/2}x^3-\frac{1}{25}bc^5\pi^{5/2}x^5 +\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{5}(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))$$

output
```
-23/15*b*c*Pi^(5/2)*x-11/45*b*c^3*Pi^(5/2)*x^3-1/25*b*c^5*Pi^(5/2)*x^5+1/3
*Pi*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+
b*arcsinh(c*x))-2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2
))-b*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*polylog(2,c*x+(
c^2*x^2+1)^(1/2))+Pi^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.75.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.44

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \frac{1}{225}\pi^{5/2}\Big(-345bcx-55bc^3x^3-9bc^5x^5+345a\sqrt{1+c^2x^2}+165ac^2x^2$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(Pi^(5/2)*(-345*b*c*x - 55*b*c^3*x^3 - 9*b*c^5*x^5 + 345*a*Sqrt[1 + c^2*x^
2] + 165*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 45*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 34
5*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 165*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x] + 45*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 225*b*ArcSinh[c*x]
*Log[1 - E^(-ArcSinh[c*x])] - 225*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])
] + 225*a*Log[x] - 225*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 225*b*PolyLog[2
, -E^(-ArcSinh[c*x])] - 225*b*PolyLog[2, E^(-ArcSinh[c*x])]))/225
```

### 3.75.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6223, 210, 2009, 6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\pi c^2x^2+\pi\right)^{5/2} (a+b\text{arcsinh}(cx))}{x} \, dx$$

$$\Big\downarrow \text{ 6223}$$

$$\pi \int \frac{\left(c^2\pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{5}\pi^{5/2}bc\int \left(c^2x^2+1\right)^2dx+\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 210}$$

$$\pi \int \frac{\left(c^2\pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{5}\pi^{5/2}bc\int \left(c^4x^4+2c^2x^2+1\right)dx+\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\pi \int \frac{\left(c^2\pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$$\Big\downarrow \text{ 6223}$$

$$\pi\left(\pi\int\frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{3}\pi^{3/2}bc\int\left(c^2x^2+1\right)dx+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\right)+$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$$\downarrow \text{2009}$$

$$\pi\left(\pi\int\frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)\right)+$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$$\downarrow \text{6221}$$

$$\pi\left(\pi\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int 1dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcs}\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$$\downarrow \text{24}$$

$$\pi\left(\pi\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsin}\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$$\downarrow \text{6231}$$

$$\pi\left(\pi\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$$\downarrow \text{3042}$$

$$\pi\left(\pi\left(\sqrt{\pi}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$$\downarrow \text{26}$$

---

3.75. $\int\frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx$

$$\pi\left(\pi\left(i\sqrt{\pi}\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$\downarrow$ 4670

$$\pi\left(\pi\left(i\sqrt{\pi}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(\right.\right.\right.\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$\downarrow$ 2715

$$\pi\left(\pi\left(i\sqrt{\pi}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}\right.\right.\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

$\downarrow$ 2838

$$\pi\left(\pi\left(i\sqrt{\pi}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right.\right.\right.$$
$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}\pi^{5/2}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)$$

input
```
Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
-1/5*(b*c*Pi^(5/2)*(x + (2*c^2*x^3)/3 + (c^4*x^5)/5)) + ((Pi + c^2*Pi*x^2)
^(5/2)*(a + b*ArcSinh[c*x]))/5 + Pi*(-1/3*(b*c*Pi^(3/2)*(x + (c^2*x^3)/3))
 + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + Pi*(-(b*c*Sqrt[Pi]*x
) + Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) + I*Sqrt[Pi]*((2*I)*(a + b*
ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] -
I*b*PolyLog[2, E^ArcSinh[c*x]])))
```

## 3.75. $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

#### 3.75.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

### 3.75. $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.75.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| default | $a\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5}+\pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right)-b\,\pi^{\frac{5}{2}}\operatorname{arcsinh}$ |
| parts | $a\left(\frac{(\pi c^2x^2+\pi)^{\frac{5}{2}}}{5}+\pi\left(\frac{(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^2x^2+\pi}-\sqrt{\pi}\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)\right)\right)\right)-b\,\pi^{\frac{5}{2}}\operatorname{arcsinh}$ |

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*(Pi*c^2*x^2+Pi)^(5/2)+Pi*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2
+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2)))))-b*Pi^(5/2)*
arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*arcsinh(c*x)*ln(1-c*x-
(c^2*x^2+1)^(1/2))-1/25*b*c^5*Pi^(5/2)*x^5-11/45*b*c^3*Pi^(5/2)*x^3+23/15*
b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)-23/15*b*c*Pi^(5/2)*x+b*Pi^(5/2)*
polylog(2,c*x+(c^2*x^2+1)^(1/2))-b*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/
2))+1/5*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*x^4*c^4+11/15*b*(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*x^2*c^2
```

### 3.75.5 Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x} dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x} dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
 + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x, x)
```

### 3.75.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x} dx = \pi^{\frac{5}{2}} \left( \int \frac{a\sqrt{c^2x^2 + 1}}{x} dx + \int 2ac^2x\sqrt{c^2x^2 + 1} dx + \int ac^4x^3\sqrt{c^2x^2 + 1} dx + \int \frac{b\sqrt{c^2x^2 + 1} \operatorname{asinh}(cx)}{x} dx + \int 2bc^2x\sqrt{c^2x^2 + 1} \operatorname{asinh}(cx) dx + \int bc^4x^3\sqrt{c^2x^2 + 1} \operatorname{asinh}(cx) dx \right)$$

input
```
integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x,x)
```

output
```
pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(2*a*c**2*x*sqrt
(c**2*x**2 + 1), x) + Integral(a*c**4*x**3*sqrt(c**2*x**2 + 1), x) + Integ
ral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integral(2*b*c**2*x*sqrt(c**2
*x**2 + 1)*asinh(c*x), x) + Integral(b*c**4*x**3*sqrt(c**2*x**2 + 1)*asinh
(c*x), x))
```

### 3.75.7 Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x} dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x} dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima"
)
```

output
```
-1/15*(15*pi^(5/2)*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2) -
 5*pi*(pi + pi*c^2*x^2)^(3/2) - 3*(pi + pi*c^2*x^2)^(5/2))*a + b*integrate
((pi + pi*c^2*x^2)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.75.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x} dx = \int \frac{(a + b \operatorname{asinh}(c\,x)) \, (\Pi\, c^2\, x^2 + \Pi)^{5/2}}{x} dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x, x)
```

# 3.76 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.76.1 Optimal result

Integrand size = 26, antiderivative size = 157

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{9}{16}bc^3\pi^{5/2}x^2-\frac{1}{16}bc^5\pi^{5/2}x^4 + \frac{15}{8}c^2\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{5}{4}c^2\pi x(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcs}}{x}$$

output
```
-9/16*b*c^3*Pi^(5/2)*x^2-1/16*b*c^5*Pi^(5/2)*x^4+5/4*c^2*Pi*x*(Pi*c^2*x^2+
Pi)^(3/2)*(a+b*arcsinh(c*x))-(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x+15
/16*c*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(5/2)*ln(x)+15/8*c^2*Pi^2*x*(
a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.76.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{\pi^{5/2}\left(-128a\sqrt{1+c^2x^2}+144ac^2x^2\sqrt{1+c^2x^2}+32ac^4x^4\sqrt{1+c^2x^2}\right.}{}$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
(Pi^(5/2)*(-128*a*Sqrt[1 + c^2*x^2] + 144*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 32
*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 120*b*c*x*ArcSinh[c*x]^2 - 32*b*c*x*Cosh[2*
ArcSinh[c*x]] - b*c*x*Cosh[4*ArcSinh[c*x]] + 128*b*c*x*Log[c*x] + 4*ArcSin
h[c*x]*(60*a*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]]
+ b*c*x*Sinh[4*ArcSinh[c*x]])))/(128*x)
```

### 3.76.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6222, 243, 49, 2009, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\pi c^2 x^2+\pi\right)^{5/2} (a+b\text{arcsinh}(cx))}{x^2} \, dx$$

$$\Big\downarrow \text{ 6222}$$

$$5\pi c^2 \int \left(c^2 \pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx+\pi^{5/2} b c \int \frac{\left(c^2 x^2+1\right)^2}{x}dx-\frac{\left(\pi c^2 x^2+\pi\right)^{5/2} (a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 243}$$

$$5\pi c^2 \int \left(c^2 \pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx+\frac{1}{2}\pi^{5/2} b c \int \frac{\left(c^2 x^2+1\right)^2}{x^2}dx^2-\frac{\left(\pi c^2 x^2+\pi\right)^{5/2} (a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 49}$$

$$5\pi c^2 \int \left(c^2 \pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx+\frac{1}{2}\pi^{5/2} b c \int \left(x^2 c^4+2 c^2+\frac{1}{x^2}\right)dx^2-\frac{\left(\pi c^2 x^2+\pi\right)^{5/2} (a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$5\pi c^2 \int \left(c^2 \pi x^2+\pi\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{\left(\pi c^2 x^2+\pi\right)^{5/2} (a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}\pi^{5/2} b c \left(\frac{c^4 x^4}{2}+2 c^2 x^2+\log \left(x^2\right)\right)$$

$$\Big\downarrow \text{ 6201}$$

$$5\pi c^2\left(\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx)\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 244}$$

$$5\pi c^2\left(\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int\left(c^2x^3+x\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$5\pi c^2\left(\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)-$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 6200}$$

$$5\pi c^2\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3}\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$5\pi c^2\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 6198}$$

$$-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+$$
$$5\pi c^2\left(\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\right.\right.$$
$$\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)$$

---

3.76. $\int\frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

input
```
Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-(((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x) + 5*c^2*Pi*(-1/4*(b*c*
Pi^(3/2)*(x^2/2 + (c^2*x^4)/4)) + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSin
h[c*x]))/4 + (3*Pi*(-1/4*(b*c*Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a
+ b*ArcSinh[c*x]))/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/4) + (b
*c*Pi^(5/2)*(2*c^2*x^2 + (c^4*x^4)/2 + Log[x^2]))/2
```

### 3.76.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.76.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.66

| method | result |
|---|---|
| default | $-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{\pi x}+a c^2x(\pi c^2x^2+\pi)^{\frac{5}{2}}+\frac{5a c^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4}+\frac{15a c^2\pi^2x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{15a c^2\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x}\right.}{8\sqrt{\pi c^2}}$ |
| parts | $-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{\pi x}+a c^2x(\pi c^2x^2+\pi)^{\frac{5}{2}}+\frac{5a c^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4}+\frac{15a c^2\pi^2x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{15a c^2\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x}\right.}{8\sqrt{\pi c^2}}$ |

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/4*a*c^2*Pi*x
*(Pi*c^2*x^2+Pi)^(3/2)+15/8*a*c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+15/8*a*c^2*
Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/12
8*b*Pi^(5/2)*(32*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-8*c^5*x^5+144*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-72*c^3*x^3+120*arcsinh(c*x)^2*x*c-128*a
rcsinh(c*x)*c*x+128*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x*c-128*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)-33*c*x)/x
```

## 3.76.5 Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^2}\, dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx) + a)}{x^2}\, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
 + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^2, x)
```

## 3.76.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^2}\, dx = \pi^{\frac{5}{2}} \left( \int 2ac^2\sqrt{c^2x^2+1}\, dx + \int \frac{a\sqrt{c^2x^2+1}}{x^2}\, dx + \int ac^4x^2\sqrt{c^2x^2+1}\, dx + \int 2bc^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\, dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^2}\, dx + \int bc^4x^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\, dx \right)$$

input
```
integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

output
```
pi**(5/2)*(Integral(2*a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**
2*x**2 + 1)/x**2, x) + Integral(a*c**4*x**2*sqrt(c**2*x**2 + 1), x) + Inte
gral(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**2, x) + Integral(b*c**4*x**2*sqrt(c**2*x**2 + 1)*asi
nh(c*x), x))
```

### 3.76.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.76.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^2} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,(\Pi\,c^2\,x^2 + \Pi)^{5/2}}{x^2} dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2, x)
```

# 3.77 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$



## 3.77.1 Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bc\pi^{5/2}}{2x}-\frac{7}{3}bc^3\pi^{5/2}x-\frac{1}{9}bc^5\pi^{5/2}x^3$$
$$+\frac{5}{2}c^2\pi^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))+\frac{5}{6}c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}}{2x^2}$$

output
```
-1/2*b*c*Pi^(5/2)/x-7/3*b*c^3*Pi^(5/2)*x-1/9*b*c^5*Pi^(5/2)*x^3+5/6*c^2*Pi
*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*a
rcsinh(c*x))/x^2-5*c^2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)
^(1/2))-5/2*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/2*b*c^2*Pi^
(5/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+5/2*c^2*Pi^2*(a+b*arcsinh(c*x))*(Pi
*c^2*x^2+Pi)^(1/2)
```

## 3.77.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.70

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \frac{\pi^{5/2}\left(-168bc^3x^3-8bc^5x^5-36a\sqrt{1+c^2x^2}+168ac^2x^2\sqrt{1+c^2x^2}\right.}{}$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(Pi^(5/2)*(-168*b*c^3*x^3 - 8*b*c^5*x^5 - 36*a*Sqrt[1 + c^2*x^2] + 168*a*c
^2*x^2*Sqrt[1 + c^2*x^2] + 24*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 168*b*c^2*x^2*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 24*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x] - 9*b*c^3*x^3*Csch[ArcSinh[c*x]/2]^2 - 9*b*c^2*x^2*ArcSinh[c*x]*Csch[Ar
cSinh[c*x]/2]^2 + 180*b*c^2*x^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] -
180*b*c^2*x^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 180*a*c^2*x^2*Log[
x] - 180*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 180*b*c^2*x^2*PolyLog
[2, -E^(-ArcSinh[c*x])] - 180*b*c^2*x^2*PolyLog[2, E^(-ArcSinh[c*x])] + 36
*b*c*x*Sinh[ArcSinh[c*x]/2]^2 - 36*b*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2]^2))
/(72*x^2)
```

### 3.77.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6222, 244, 2009, 6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\pi c^2x^2+\pi)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}dx$$

$$\Big\downarrow \ 6222$$

$$\frac{5}{2}\pi c^2\int \frac{(c^2\pi x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}\pi^{5/2}bc\int \frac{(c^2x^2+1)^2}{x^2}dx-\frac{(\pi c^2x^2+\pi)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \ 244$$

$$\frac{5}{2}\pi c^2\int \frac{(c^2\pi x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}\pi^{5/2}bc\int \left(x^2c^4+2c^2+\frac{1}{x^2}\right)dx-\frac{(\pi c^2x^2+\pi)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{5}{2}\pi c^2\int \frac{(c^2\pi x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{(\pi c^2x^2+\pi)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$$\downarrow 6223$$

$$\frac{5}{2}\pi c^2\left(\pi\int\frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{3}\pi^{3/2}bc\int\left(c^2x^2+1\right)dx+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\right)$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$$\downarrow 2009$$

$$\frac{5}{2}\pi c^2\left(\pi\int\frac{\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}\pi^{3/2}bc\left(\frac{c^2x^3}{3}+x\right)\right)-$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$$\downarrow 6221$$

$$\frac{5}{2}\pi c^2\left(\pi\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int 1dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+b$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$$\downarrow 24$$

$$\frac{5}{2}\pi c^2\left(\pi\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}(a+bar$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$$\downarrow 6231$$

$$\frac{5}{2}\pi c^2\left(\pi\left(\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/}$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$$\downarrow 3042$$

$$\frac{5}{2}\pi c^2\left(\pi\left(\sqrt{\pi}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right.\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$$\downarrow 26$$

---

3.77. $\int\frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

$$\frac{5}{2}\pi c^2\left(\pi\left(i\sqrt{\pi}\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\sqrt{\pi}(-b)cx\right.\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$\downarrow$ 4670

$$\frac{5}{2}\pi c^2\left(\pi\left(i\sqrt{\pi}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arc}}\right.\right.\right.\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$\downarrow$ 2715

$$\frac{5}{2}\pi c^2\left(\pi\left(i\sqrt{\pi}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}}\right.\right.\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

$\downarrow$ 2838

$$\frac{5}{2}\pi c^2\left(\pi\left(i\sqrt{\pi}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c}\right.\right.\right.\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\pi^{5/2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)$$

input
```
Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(b*c*Pi^(5/2)*(-x^(-1) + 2*c^2*x + (c^4*x^3)/3))/2 - ((Pi + c^2*Pi*x^2)^(5
/2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (5*c^2*Pi*(-1/3*(b*c*Pi^(3/2)*(x + (c^
2*x^3)/3)) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + Pi*(-(b*c*
Sqrt[Pi]*x) + Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) + I*Sqrt[Pi]*((2*
I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSin
h[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))))/2
```

## 3.77. $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

#### 3.77.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### 3.77. $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.77. $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

### 3.77.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| default | $a\left(-\frac{\left(\pi c^{2} x^{2}+\pi\right)^{\frac{7}{2}}}{2 \pi x^{2}}+\frac{5 c^{2}\left(\frac{\left(\pi c^{2} x^{2}+\pi\right)^{\frac{5}{2}}}{5}+\pi\left(\frac{\left(\pi c^{2} x^{2}+\pi\right)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^{2} x^{2}+\pi}-\sqrt{\pi}\, \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^{2} x^{2}+\pi}}\right)\right)\right)\right)}{2}\right)-\frac{b\,\pi^{\frac{5}{2}} \operatorname{arcsi}}{2\sqrt{c^{2}}}$ |
| parts | $a\left(-\frac{\left(\pi c^{2} x^{2}+\pi\right)^{\frac{7}{2}}}{2 \pi x^{2}}+\frac{5 c^{2}\left(\frac{\left(\pi c^{2} x^{2}+\pi\right)^{\frac{5}{2}}}{5}+\pi\left(\frac{\left(\pi c^{2} x^{2}+\pi\right)^{\frac{3}{2}}}{3}+\pi\left(\sqrt{\pi c^{2} x^{2}+\pi}-\sqrt{\pi}\, \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^{2} x^{2}+\pi}}\right)\right)\right)\right)}{2}\right)-\frac{b\,\pi^{\frac{5}{2}} \operatorname{arcsi}}{2\sqrt{c^{2}}}$ |

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(7/2)+5/2*c^2*(1/5*(Pi*c^2*x^2+Pi)^(5/2)+Pi
*(1/3*(Pi*c^2*x^2+Pi)^(3/2)+Pi*((Pi*c^2*x^2+Pi)^(1/2)-Pi^(1/2)*arctanh(Pi^
(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))))-1/2*b*Pi^(5/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*c^2+1/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/2)*x^2*c^4+5/2*b*c^2*P
i^(5/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/9*b*c^5*Pi^(5/2)*x^3-7/
3*b*c^3*Pi^(5/2)*x+5/2*b*c^2*Pi^(5/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-5/2
*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-5/2*b*c^2*Pi^(5/2)*arcsi
nh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/2*b*c*Pi^(5/2)/x-1/2*b*Pi^(5/2)/(c^2
*x^2+1)^(1/2)/x^2*arcsinh(c*x)+7/3*b*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*Pi^(5/
2)*c^2
```

### 3.77.5 Fricas [F]

$$\int \frac{\left(\pi+c^{2} \pi x^{2}\right)^{5/2}(a+b \operatorname{arcsinh}(c x))}{x^{3}}\, dx=\int \frac{\left(\pi+\pi c^{2} x^{2}\right)^{\frac{5}{2}}(b \operatorname{arsinh}(c x)+a)}{x^{3}}\, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
 + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^3, x)
```

### 3.77.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \pi^{\frac{5}{2}} \left( \int \frac{a\sqrt{c^2x^2+1}}{x^3} dx + \int \frac{2ac^2\sqrt{c^2x^2+1}}{x} dx + \int ac^4x\sqrt{c^2x^2+1}\, dx + \int \frac{b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x^3} dx + \int \frac{2bc^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x} dx + \int bc^4x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\, dx \right)$$

input
```
integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

output
```
pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(2*a*c**2*sqr
t(c**2*x**2 + 1)/x, x) + Integral(a*c**4*x*sqrt(c**2*x**2 + 1), x) + Integ
ral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) + Integral(2*b*c**2*sqrt(c**
2*x**2 + 1)*asinh(c*x)/x, x) + Integral(b*c**4*x*sqrt(c**2*x**2 + 1)*asinh
(c*x), x))
```

### 3.77.7 Maxima [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(\pi + \pi c^2x^2)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x^3} dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxim
a")
```

output
```
-1/6*(15*pi^(5/2)*c^2*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2
)*c^2 - 5*pi*(pi + pi*c^2*x^2)^(3/2)*c^2 - 3*(pi + pi*c^2*x^2)^(5/2)*c^2 +
 3*(pi + pi*c^2*x^2)^(7/2)/(pi*x^2))*a + b*integrate((pi + pi*c^2*x^2)^(5/
2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

### 3.77.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(a + b\,\text{asinh}(c\,x)) \, (\Pi\,c^2\,x^2 + \Pi)^{5/2}}{x^3} dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3, x)
```

# 3.78 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.78.1 Optimal result

Integrand size = 26, antiderivative size = 166

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bc\pi^{5/2}}{6x^2}-\frac{1}{4}bc^5\pi^{5/2}x^2 +\frac{5}{2}c^4\pi^2x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))-\frac{5c^2\pi(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x}-\frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsi}}{3x^3}$$

output
```
-1/6*b*c*Pi^(5/2)/x^2-1/4*b*c^5*Pi^(5/2)*x^2-5/3*c^2*Pi*(Pi*c^2*x^2+Pi)^(3
/2)*(a+b*arcsinh(c*x))/x-1/3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3+
5/4*c^3*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+7/3*b*c^3*Pi^(5/2)*ln(x)+5/2*c^4*P
i^2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.78.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08

$$\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{\pi^{5/2}\left(-4bcx-8a\sqrt{1+c^2x^2}-56ac^2x^2\sqrt{1+c^2x^2}+12ac^4x^4\sqrt{1+\right.}{}$$

input
```
Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
(Pi^(5/2)*(-4*b*c*x - 8*a*Sqrt[1 + c^2*x^2] - 56*a*c^2*x^2*Sqrt[1 + c^2*x^
2] + 12*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 30*b*c^3*x^3*ArcSinh[c*x]^2 - 3*b*c^
3*x^3*Cosh[2*ArcSinh[c*x]] + 56*b*c^3*x^3*Log[c*x] + ArcSinh[c*x]*(60*a*c^
3*x^3 - 8*b*Sqrt[1 + c^2*x^2]*(1 + 7*c^2*x^2) + 6*b*c^3*x^3*Sinh[2*ArcSinh
[c*x]])))/(24*x^3)
```

### 3.78.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6222, 243, 49, 2009, 6222, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6222}$$

$$\frac{5}{3}\pi c^2\int \frac{\left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx+\frac{1}{3}\pi^{5/2}bc\int \frac{\left(c^2x^2+1\right)^2}{x^3}dx-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{5}{3}\pi c^2\int \frac{\left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx+\frac{1}{6}\pi^{5/2}bc\int \frac{\left(c^2x^2+1\right)^2}{x^4}dx^2-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{49}$$

$$\frac{5}{3}\pi c^2\int \frac{\left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx+\frac{1}{6}\pi^{5/2}bc\int \left(c^4+\frac{2c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{3}\pi c^2\int \frac{\left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}\pi^{5/2}bc\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{6222}$$

$$\frac{5}{3}\pi c^2\left(3\pi c^2\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\pi^{3/2}bc\int\frac{c^2x^2+1}{x}dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}\pi^{5/2}bc\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{5}{3}\pi c^2\left(3\pi c^2\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\pi^{3/2}bc\int\left(xc^2+\frac{1}{x}\right)dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}\pi^{5/2}bc\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{3}\pi c^2\left(3\pi c^2\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\pi^{3/2}bc\left(\frac{c^2x^2}{2}+\log(x)\right)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}\pi^{5/2}bc\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{6200}$$

$$\frac{5}{3}\pi c^2\left(3\pi c^2\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\pi^{3/2}bc\left(\frac{c^2x^2}{2}+\log(x)\right)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}\pi^{5/2}bc\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{5}{3}\pi c^2\left(3\pi c^2\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\pi^{3/2}bc\left(\frac{c^2x^2}{2}+\log(x)\right)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}\pi^{5/2}bc\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{6198}$$

---

3.78. $\int\frac{\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

$$\frac{5}{3}\pi c^2\left(3\pi c^2\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}\sqrt{\pi}bcx^2\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}(a+ba\right.$$
$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{1}{6}\pi^{5/2}bc\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)$$

input
```
Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3 + (5*c^2*Pi*(-(((P
i + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x) + 3*c^2*Pi*(-1/4*(b*c*Sqrt[
Pi]*x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[Pi]*(a
 + b*ArcSinh[c*x])^2)/(4*b*c)) + b*c*Pi^(3/2)*((c^2*x^2)/2 + Log[x])))/3 +
 (b*c*Pi^(5/2)*(-x^(-2) + c^4*x^2 + 2*c^2*Log[x^2]))/6
```

### 3.78.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.78.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(138) = 276$.

Time = 0.22 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.17

| method | result |
|---|---|
| default | $-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x^3}-\frac{4a c^2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x}+\frac{4a c^4x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3}+\frac{5a c^4\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\frac{5a c^4\pi^2x\sqrt{\pi c^2x^2+\pi}}{2}+\frac{5a c^4\pi^3 \ln}{}$ |
| parts | $-\frac{a(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x^3}-\frac{4a c^2(\pi c^2x^2+\pi)^{\frac{7}{2}}}{3\pi x}+\frac{4a c^4x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{3}+\frac{5a c^4\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{3}+\frac{5a c^4\pi^2x\sqrt{\pi c^2x^2+\pi}}{2}+\frac{5a c^4\pi^3 \ln}{}$ |

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(7/2)-4/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+4
/3*a*c^4*x*(Pi*c^2*x^2+Pi)^(5/2)+5/3*a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/2*
a*c^4*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/2*a*c^4*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/
2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/8*b*c^3*Pi^(5/2)-14/3*b*c^3*Pi^
(5/2)*arcsinh(c*x)+5/4*b*c^3*Pi^(5/2)*arcsinh(c*x)^2+147*b*Pi^(5/2)/(63*c^
4*x^4+15*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^
2*x^2+1)*x^2*(c^2*x^2+1)*c^5+35*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*a
rcsinh(c*x)*c^5-7/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*(c^2*x^2+1)*c^3+7
/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*arcsinh(c*x)*c^3-1/6*b*Pi^(5/2)/(6
3*c^4*x^4+15*c^2*x^2+1)/x^2*(c^2*x^2+1)*c-1/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^
2*x^2+1)/x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/2*b*(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*Pi^(5/2)*x*c^4+7/3*b*c^3*Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-1
/4*b*c^5*Pi^(5/2)*x^2+49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^4*c^7-14
7*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
c^6-56*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*
x)*c^4-22/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x*(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)*c^2
```

### 3.78.5 Fricas [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} \, dx = \int \frac{(\pi + \pi c^2 x^2)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x^4} \, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
 + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^4, x)
```

### 3.78.6 Sympy [F]

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} \, dx = \pi^{\frac{5}{2}} \left( \int ac^4\sqrt{c^2x^2+1}\, dx + \int \frac{a\sqrt{c^2x^2+1}}{x^4}\, dx + \int \frac{2ac^2\sqrt{c^2x^2+1}}{x^2}\, dx + \int bc^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\, dx + \int \frac{b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x^4}\, dx + \int \frac{2bc^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x^2}\, dx \right)$$

input
```
integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

output
```
pi**(5/2)*(Integral(a*c**4*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*
x**2 + 1)/x**4, x) + Integral(2*a*c**2*sqrt(c**2*x**2 + 1)/x**2, x) + Inte
gral(b*c**4*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x**2
 + 1)*asinh(c*x)/x**4, x) + Integral(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*
x)/x**2, x))
```

### 3.78.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.78.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(\pi + c^2\pi x^2)^{5/2}\,(a + b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4, x)
```

# 3.79 $\int \sqrt{1+x^2}\,\text{arcsinh}(x)\,dx$



## 3.79.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{1+x^2}\,\text{arcsinh}(x)\,dx = -\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\,\text{arcsinh}(x)+\frac{\text{arcsinh}(x)^2}{4}$$

output
```
-1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)
```

## 3.79.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\,\text{arcsinh}(x)\,dx = \frac{1}{4}\left(-x^2+2x\sqrt{1+x^2}\,\text{arcsinh}(x)+\text{arcsinh}(x)^2\right)$$

input
```
Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

output
```
(-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

### 3.79.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+1}\text{arcsinh}(x)\,dx$$

$$\Big\downarrow \text{ 6200}$$

$$\frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{x^2+1}}dx-\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{x^2+1}\text{arcsinh}(x)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{x^2+1}}dx+\frac{1}{2}\sqrt{x^2+1}x\text{arcsinh}(x)-\frac{x^2}{4}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{1}{2}\sqrt{x^2+1}x\text{arcsinh}(x)+\frac{\text{arcsinh}(x)^2}{4}-\frac{x^2}{4}$$

input
```
Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

output
```
-1/4*x^2 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

#### 3.79.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

### 3.79.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $\frac{x\,\mathrm{arcsinh}(x)\sqrt{x^2+1}}{2}+\frac{\mathrm{arcsinh}(x)^2}{4}-\frac{x^2}{4}-\frac{1}{4}$ | 26 |

input
```
int(arcsinh(x)*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

### 3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \sqrt{1+x^2}\mathrm{arcsinh}(x)\,dx=\frac{1}{2}\sqrt{x^2+1}x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}x^2+\frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

input
```
integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^
2 + 1))^2
```

### 3.79.6 Sympy [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{asinh}\,(x)\,dx$$

input
```
integrate(asinh(x)*(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 1)*asinh(x), x)
```

### 3.79.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = -\frac{1}{4}\,x^2 + \frac{1}{2}\left(\sqrt{x^2+1}x + \text{arsinh}\,(x)\right)\text{arsinh}\,(x) - \frac{1}{4}\,\text{arsinh}\,(x)^2$$

input
```
integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^
2
```

### 3.79.8 Giac [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{arsinh}\,(x)\,dx$$

input
```
integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

### 3.79.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx = \int \operatorname{asinh}(x)\,\sqrt{x^2+1}\,dx$$

input
```
int(asinh(x)*(x^2 + 1)^(1/2),x)
```

output
```
int(asinh(x)*(x^2 + 1)^(1/2), x)
```

# 3.80 $\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.80.1 Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{8bx}{15c^5\sqrt{\pi}}+\frac{4bx^3}{45c^3\sqrt{\pi}}-\frac{bx^5}{25c\sqrt{\pi}} +\frac{8\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{15c^6\pi} -\frac{4x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{15c^4\pi} +\frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{5c^2\pi}$$

output
```
-8/15*b*x/c^5/Pi^(1/2)+4/45*b*x^3/c^3/Pi^(1/2)-1/25*b*x^5/c/Pi^(1/2)+8/15*
(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi-4/15*x^2*(a+b*arcsinh(c*x)
)*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/5*x^4*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^
(1/2)/c^2/Pi
```

### 3.80.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{15a\sqrt{1+c^2x^2}(8-4c^2x^2+3c^4x^4)+b(-120cx+20c^3x^3-9c^5x^5)+15b\sqrt{1+c^2x^2}(8-4c^2x^2+3c^4x^4)\,\text{arc}}{225c^6\sqrt{\pi}}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(15*a*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4) + b*(-120*c*x + 20*c^3
*x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4)*Arc
Sinh[c*x])/(225*c^6*Sqrt[Pi])
```

### 3.80.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6227, 15, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi c^2x^2+\pi}}\,dx$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{5c^2}-\frac{b\int x^4dx}{5\sqrt{\pi}c}+\frac{x^4\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{5\pi c^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{5c^2}+\frac{x^4\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{bx^5}{25\sqrt{\pi}c}$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{3c^2}-\frac{b\int x^2dx}{3\sqrt{\pi}c}+\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}\right)}{5c^2}+\frac{x^4\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{bx^5}{25\sqrt{\pi}c}$$

$$\Big\downarrow \; 15$$

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{3c^2}+\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{bx^3}{9\sqrt{\pi}c}\right)}{5c^2}+\frac{x^4\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{bx^5}{25\sqrt{\pi}c}$$

$$\Big\downarrow \; 6213$$

$$-\frac{4\left(-\frac{2\left(\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi c^2}-\frac{b\int 1dx}{\sqrt{\pi}c}\right)}{3c^2}+\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{bx^3}{9\sqrt{\pi}c}\right)}{5c^2}+\frac{x^4\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{bx^5}{25\sqrt{\pi}c}$$

$$\Big\downarrow \; 24$$

$$\frac{x^4\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{5\pi c^2}-\frac{4\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{2\left(\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi c^2}-\frac{bx}{\sqrt{\pi}c}\right)}{3c^2}-\frac{bx^3}{9\sqrt{\pi}c}\right)}{5c^2}-\frac{bx^5}{25\sqrt{\pi}c}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
-1/25*(b*x^5)/(c*Sqrt[Pi]) + (x^4*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x
]))/(5*c^2*Pi) - (4*(-1/9*(b*x^3)/(c*Sqrt[Pi]) + (x^2*Sqrt[Pi + c^2*Pi*x^2
]*(a + b*ArcSinh[c*x]))/(3*c^2*Pi) - (2*(-((b*x)/(c*Sqrt[Pi])) + (Sqrt[Pi
+ c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^2*Pi)))/(3*c^2)))/(5*c^2)
```

3.80. $\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

#### 3.80.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.80.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

| method | result |
|---|---|
| default | $a\left(\frac{x^4\sqrt{\pi c^2x^2+\pi}}{5\pi c^2}-\frac{4\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}}{3\pi c^4}\right)}{5c^2}\right)+\frac{b\left(45\,\mathrm{arcsinh}(cx)c^6x^6-15\,\mathrm{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}+60\,\mathrm{arcs}\right.}{225c^6\sqrt{\pi}\,\sqrt{}}$ |
| parts | $a\left(\frac{x^4\sqrt{\pi c^2x^2+\pi}}{5\pi c^2}-\frac{4\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}}{3\pi c^4}\right)}{5c^2}\right)+\frac{b\left(45\,\mathrm{arcsinh}(cx)c^6x^6-15\,\mathrm{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}+60\,\mathrm{arcs}\right.}{225c^6\sqrt{\pi}\,\sqrt{}}$ |

input
```
int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*x^4/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-4/5/c^2*(1/3*x^2/Pi/c^2*(Pi*c^2*x^
2+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2)))+1/225*b/c^6/Pi^(1/2)/(c^2*x
^2+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6-15*arcsinh(c*x)*c^4*x^4-9*c^5*x^5*(c^
2*x^2+1)^(1/2)+60*arcsinh(c*x)*c^2*x^2+20*c^3*x^3*(c^2*x^2+1)^(1/2)+120*ar
csinh(c*x)-120*c*x*(c^2*x^2+1)^(1/2))
```

### 3.80.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{15\sqrt{\pi+\pi c^2x^2}(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}(45\,ac^6x^6-15\,ac^4x^4}{225\,(\pi c^8x^2+\pi c^6)}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8
*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 - 1
5*a*c^4*x^4 + 60*a*c^2*x^2 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt
(c^2*x^2 + 1) + 120*a))/(pi*c^8*x^2 + pi*c^6)
```

### 3.80.6 Sympy [A] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{x^4\sqrt{c^2x^2+1}}{5c^2}-\frac{4x^2\sqrt{c^2x^2+1}}{15c^4}+\frac{8\sqrt{c^2x^2+1}}{15c^6} & \text{for } c^2\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x^5}{25c}+\frac{x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5c^2}+\frac{4x^3}{45c^3}-\frac{4x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{15c^4}-\frac{8x}{15c^5}+\frac{8\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{15c^6} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

input
```
integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
a*Piecewise((x**4*sqrt(c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(c**2*x**2 + 1
)/(15*c**4) + 8*sqrt(c**2*x**2 + 1)/(15*c**6), Ne(c**2, 0)), (x**6/6, True
))/sqrt(pi) + b*Piecewise((-x**5/(25*c) + x**4*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(5*c**2) + 4*x**3/(45*c**3) - 4*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
5*c**4) - 8*x/(15*c**5) + 8*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**6), Ne(c
, 0)), (0, True))/sqrt(pi)
```

### 3.80.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}x^4}{\pi c^2} - \frac{4\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^4} + \frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)b\,\text{arsinh}\,(cx) + \frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}x^4}{\pi c^2} - \frac{4\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^4} + \frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)a - \frac{(9\,c^4x^5-20\,c^2x^3+120\,x)b}{225\sqrt{\pi}c^5}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
1/15*(3*sqrt(pi + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(
pi*c^4) + 8*sqrt(pi + pi*c^2*x^2)/(pi*c^6))*b*arcsinh(c*x) + 1/15*(3*sqrt(
pi + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^4) + 8*s
qrt(pi + pi*c^2*x^2)/(pi*c^6))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*
b/(sqrt(pi)*c^5)
```

### 3.80.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{x^5\,(a+b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.81 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.81.1 Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{3bx^2}{16c^3\sqrt{\pi}} - \frac{bx^4}{16c\sqrt{\pi}} - \frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{8c^4\pi} + \frac{x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{4c^2\pi} + \frac{3(a+b\text{arcsinh}(cx))^2}{16bc^5\sqrt{\pi}}$$

output
```
3/16*b*x^2/c^3/Pi^(1/2)-1/16*b*x^4/c/Pi^(1/2)+3/16*(a+b*arcsinh(c*x))^2/b/
c^5/Pi^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/4*x^3
*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

## 3.81.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{-48acx\sqrt{1+c^2x^2}+32ac^3x^3\sqrt{1+c^2x^2}+24b\text{arcsinh}(cx)^2+16b\cosh(2\text{arcsinh}(cx))-b\cosh(4\text{arcsinh}(ca}{128c^5\sqrt{\pi}}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(-48*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24*b*ArcSi
nh[c*x]^2 + 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*ArcSinh
[c*x]*(12*a - 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))/(128*c^5
*Sqrt[Pi])
```

### 3.81.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{4c^2}-\frac{b\int x^3dx}{4\sqrt{\pi}c}+\frac{x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{4\pi c^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{4c^2}+\frac{x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{4\pi c^2}-\frac{bx^4}{16\sqrt{\pi}c}$$

$$\Big\downarrow \text{6227}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}-\frac{b\int xdx}{2\sqrt{\pi}c}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}\right)}{4c^2}+\frac{x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{4\pi c^2}-\frac{bx^4}{16\sqrt{\pi}c}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}\right)}{4c^2}+\frac{x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{4\pi c^2}-\frac{bx^4}{16\sqrt{\pi}c}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x^3\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{4\pi c^2}-\frac{3\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4\sqrt{\pi}bc^3}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}\right)}{4c^2}-\frac{bx^4}{16\sqrt{\pi}c}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
-1/16*(b*x^4)/(c*Sqrt[Pi]) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x
]))/(4*c^2*Pi) - (3*(-1/4*(b*x^2)/(c*Sqrt[Pi]) + (x*Sqrt[Pi + c^2*Pi*x^2]*
(a + b*ArcSinh[c*x]))/(2*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*Sqrt[Pi
])))/(4*c^2)
```

#### 3.81.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.81.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| default | $\frac{a\,x^{3}\sqrt{\pi\,c^{2}x^{2}+\pi}}{4\pi\,c^{2}}-\frac{3ax\sqrt{\pi\,c^{2}x^{2}+\pi}}{8c^{4}\pi}+\frac{3a\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{8c^{4}\sqrt{\pi\,c^{2}}}+\frac{b\left(4\,\mathrm{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}\,x^{3}c^{3}-c^{4}x^{4}-6\,\mathrm{arcsinh}(cx)cx\sqrt{c^{2}}\right.}{16c^{5}\sqrt{\pi}}$ |
| parts | $\frac{a\,x^{3}\sqrt{\pi\,c^{2}x^{2}+\pi}}{4\pi\,c^{2}}-\frac{3ax\sqrt{\pi\,c^{2}x^{2}+\pi}}{8c^{4}\pi}+\frac{3a\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{8c^{4}\sqrt{\pi\,c^{2}}}+\frac{b\left(4\,\mathrm{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}\,x^{3}c^{3}-c^{4}x^{4}-6\,\mathrm{arcsinh}(cx)cx\sqrt{c^{2}}\right.}{16c^{5}\sqrt{\pi}}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^3/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-3/8*a/c^4*x/Pi*(Pi*c^2*x^2+Pi)^(1/2
)+3/8*a/c^4*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/
2)+1/16*b*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-c^4*x^4-6*arcsinh(c*x)
*c*x*(c^2*x^2+1)^(1/2)+3*c^2*x^2+3*arcsinh(c*x)^2+3)/c^5/Pi^(1/2)
```

### 3.81.5 Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^4}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(pi + pi*c^2*x^2), x)
```

### 3.81.6 Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.47

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases} \frac{x^3\sqrt{c^2x^2+1}}{4c^2} - \frac{3x\sqrt{c^2x^2+1}}{8c^4} + \frac{3\log\left(2c^2x+2\sqrt{c^2x^2+1}\sqrt{c^2}\right)}{8c^4\sqrt{c^2}} & \text{for } c^2 \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases} -\frac{x^4}{16c} + \frac{x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{4c^2} + \frac{3x^2}{16c^3} - \frac{3x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{8c^4} + \frac{3\operatorname{asinh}^2(cx)}{16c^5} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{\sqrt{\pi}}$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
a*Piecewise((x**3*sqrt(c**2*x**2 + 1)/(4*c**2) - 3*x*sqrt(c**2*x**2 + 1)/(
8*c**4) + 3*log(2*c**2*x + 2*sqrt(c**2*x**2 + 1)*sqrt(c**2))/(8*c**4*sqrt(
c**2)), Ne(c**2, 0)), (x**5/5, True))/sqrt(pi) + b*Piecewise((-x**4/(16*c)
 + x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(4*c**2) + 3*x**2/(16*c**3) - 3*x*s
qrt(c**2*x**2 + 1)*asinh(c*x)/(8*c**4) + 3*asinh(c*x)**2/(16*c**5), Ne(c,
0)), (0, True))/sqrt(pi)
```

### 3.81.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{\sqrt{\pi + c^2\pi x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.81.8 Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)*x^4/sqrt(pi + pi*c^2*x^2), x)
```

### 3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{x^4\,(a+b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.82 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}} dx$



## 3.82.1 Optimal result

Integrand size = 26, antiderivative size = 98

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}} dx = \frac{2bx}{3c^3\sqrt{\pi}} - \frac{bx^3}{9c\sqrt{\pi}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3c^2\pi}$$

output
```
2/3*b*x/c^3/Pi^(1/2)-1/9*b*x^3/c/Pi^(1/2)-2/3*(a+b*arcsinh(c*x))*(Pi*c^2*x
^2+Pi)^(1/2)/c^4/Pi+1/3*x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/P
i
```

## 3.82.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}} dx = \frac{3a(-2+c^2x^2)\sqrt{1+c^2x^2}+b(6cx-c^3x^3)+3b(-2+c^2x^2)\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{9c^4\sqrt{\pi}}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(3*a*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2] + b*(6*c*x - c^3*x^3) + 3*b*(-2 + c^
2*x^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(9*c^4*Sqrt[Pi])
```

### 3.82.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{3c^2}-\frac{b\int x^2dx}{3\sqrt{\pi}c}+\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{3c^2}+\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{bx^3}{9\sqrt{\pi}c}$$

$$\Big\downarrow \text{ 6213}$$

$$-\frac{2\left(\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi c^2}-\frac{b\int 1dx}{\sqrt{\pi}c}\right)}{3c^2}+\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{bx^3}{9\sqrt{\pi}c}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x^2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi c^2}-\frac{2\left(\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi c^2}-\frac{bx}{\sqrt{\pi}c}\right)}{3c^2}-\frac{bx^3}{9\sqrt{\pi}c}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
-1/9*(b*x^3)/(c*Sqrt[Pi]) + (x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
))/(3*c^2*Pi) - (2*(-((b*x)/(c*Sqrt[Pi])) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*
ArcSinh[c*x]))/(c^2*Pi)))/(3*c^2)
```

#### 3.82.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.82.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.36

| method | result |
|---|---|
| default | $a\left(\frac{x^{2}\sqrt{\pi c^{2}x^{2}+\pi}}{3\pi c^{2}}-\frac{2\sqrt{\pi c^{2}x^{2}+\pi}}{3\pi c^{4}}\right)+\frac{b\left(3\,\mathrm{arcsinh}(cx)c^{4}x^{4}-3\,\mathrm{arcsinh}(cx)c^{2}x^{2}-c^{3}x^{3}\sqrt{c^{2}x^{2}+1}-6\,\mathrm{arcsinh}(cx)+6cx\sqrt{c^{2}x^{2}+1}\right)}{9c^{4}\sqrt{\pi}\,\sqrt{c^{2}x^{2}+1}}$ |
| parts | $a\left(\frac{x^{2}\sqrt{\pi c^{2}x^{2}+\pi}}{3\pi c^{2}}-\frac{2\sqrt{\pi c^{2}x^{2}+\pi}}{3\pi c^{4}}\right)+\frac{b\left(3\,\mathrm{arcsinh}(cx)c^{4}x^{4}-3\,\mathrm{arcsinh}(cx)c^{2}x^{2}-c^{3}x^{3}\sqrt{c^{2}x^{2}+1}-6\,\mathrm{arcsinh}(cx)+6cx\sqrt{c^{2}x^{2}+1}\right)}{9c^{4}\sqrt{\pi}\,\sqrt{c^{2}x^{2}+1}}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^2/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2))+
1/9*b/c^4/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3*arcsinh(c*x)*c^4*x^4-3*arcsinh(c*x
)*c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)-6*arcsinh(c*x)+6*c*x*(c^2*x^2+1)^(1/2)
)
```

### 3.82.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{3\sqrt{\pi+\pi c^2x^2}(bc^4x^4-bc^2x^2-2b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3ac^4x^4-3ac^2x^2-(bc^3x^3-6bc\right.}{9\left(\pi c^6x^2+\pi c^4\right)}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*log(c*x + sqrt(
c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*
x^3 - 6*b*c*x)*sqrt(c^2*x^2 + 1) - 6*a))/(pi*c^6*x^2 + pi*c^4)
```

### 3.82.6 Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{x^2\sqrt{c^2x^2+1}}{3c^2}-\frac{2\sqrt{c^2x^2+1}}{3c^4} & \text{for } c^2\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x^3}{9c}+\frac{x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^2}+\frac{2x}{3c^3}-\frac{2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^4} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
a*Piecewise((x**2*sqrt(c**2*x**2 + 1)/(3*c**2) - 2*sqrt(c**2*x**2 + 1)/(3*
c**4), Ne(c**2, 0)), (x**4/4, True))/sqrt(pi) + b*Piecewise((-x**3/(9*c) +
 x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**2) + 2*x/(3*c**3) - 2*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(3*c**4), Ne(c, 0)), (0, True))/sqrt(pi)
```

### 3.82.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.19

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{1}{3}\,b\left(\frac{\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^2} - \frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)\operatorname{arsinh}\left(cx\right) + \frac{1}{3}\,a\left(\frac{\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^2} - \frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right) - \frac{(c^2x^3-6\,x)b}{9\,\sqrt{\pi}c^3}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
1/3*b*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + pi*c^2*x^2)/(pi*c^
4))*arcsinh(c*x) + 1/3*a*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi +
 pi*c^2*x^2)/(pi*c^4)) - 1/9*(c^2*x^3 - 6*x)*b/(sqrt(pi)*c^3)
```

### 3.82.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{x^3\,(a+b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.83 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.83.1 Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{bx^2}{4c\sqrt{\pi}}+\frac{x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^2\pi}-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{\pi}}$$

output
```
-1/4*b*x^2/c/Pi^(1/2)-1/4*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(1/2)+1/2*x*(a+b*a
rcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

## 3.83.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{4acx\sqrt{1+c^2x^2}-2b\text{arcsinh}(cx)^2-b\cosh(2\text{arcsinh}(cx))+\text{arcsinh}(cx)(-4a+2b\sinh(2\text{arcsinh}(cx)))}{8c^3\sqrt{\pi}}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(4*a*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSinh[c*x]] +
 ArcSinh[c*x]*(-4*a + 2*b*Sinh[2*ArcSinh[c*x]]))/(8*c^3*Sqrt[Pi])
```

### 3.83.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \; 6227$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}-\frac{b\int xdx}{2\sqrt{\pi}c}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}$$

$$\Big\downarrow \; 6198$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{4\sqrt{\pi}bc^3}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
-1/4*(b*x^2)/(c*Sqrt[Pi]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))
/(2*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*Sqrt[Pi])
```

#### 3.83.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.83.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

| method | result | size |
|---|---|---|
| default | $\frac{ax\sqrt{\pi\, c^{2} x^{2}+\pi}}{2\pi\, c^{2}}-\frac{a \ln\left(\frac{\pi\, c^{2} x}{\sqrt{\pi\, c^{2}}}+\sqrt{\pi\, c^{2} x^{2}+\pi}\right)}{2 c^{2} \sqrt{\pi\, c^{2}}}-\frac{b\left(-2 \operatorname{arcsinh}\left(c x\right) c x \sqrt{c^{2} x^{2}+1}+c^{2} x^{2}+\operatorname{arcsinh}\left(c x\right)^{2}+1\right)}{4 \sqrt{\pi}\, c^{3}}$ | 107 |
| parts | $\frac{ax\sqrt{\pi\, c^{2} x^{2}+\pi}}{2\pi\, c^{2}}-\frac{a \ln\left(\frac{\pi\, c^{2} x}{\sqrt{\pi\, c^{2}}}+\sqrt{\pi\, c^{2} x^{2}+\pi}\right)}{2 c^{2} \sqrt{\pi\, c^{2}}}-\frac{b\left(-2 \operatorname{arcsinh}\left(c x\right) c x \sqrt{c^{2} x^{2}+1}+c^{2} x^{2}+\operatorname{arcsinh}\left(c x\right)^{2}+1\right)}{4 \sqrt{\pi}\, c^{3}}$ | 107 |

input
```
int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-1/2*a/c^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+
(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/4*b/Pi^(1/2)*(-2*arcsinh(c*x)*c*x*
(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)^2+1)/c^3
```

## 3.83.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(pi + pi*c^2*x^2), x)
```

### 3.83.6 Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\log\left(2c^2x+2\sqrt{c^2x^2+1}\sqrt{c^2}\right)}{2c^2\sqrt{c^2}} & \text{for } c^2\neq 0\\ \frac{x^3}{3} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x^2}{4c}+\frac{x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{2c^2}-\frac{\operatorname{asinh}^2(cx)}{4c^3} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
a*Piecewise((x*sqrt(c**2*x**2 + 1)/(2*c**2) - log(2*c**2*x + 2*sqrt(c**2*x
**2 + 1)*sqrt(c**2))/(2*c**2*sqrt(c**2)), Ne(c**2, 0)), (x**3/3, True))/sq
rt(pi) + b*Piecewise((-x**2/(4*c) + x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c*
*2) - asinh(c*x)**2/(4*c**3), Ne(c, 0)), (0, True))/sqrt(pi)
```

### 3.83.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.83.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/sqrt(pi + pi*c^2*x^2), x)
```

### 3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{x^2\,(a+b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.84 $\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.84.1 Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{bx}{c\sqrt{\pi}}+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{c^2\pi}$$

output
```
-b*x/c/Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

## 3.84.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{-bcx+a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c^2\sqrt{\pi}}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c^2*S
qrt[Pi])
```

### 3.84.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi c^2}-\frac{b\int 1dx}{\sqrt{\pi}c}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi c^2}-\frac{bx}{\sqrt{\pi}c}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
-((b*x)/(c*Sqrt[Pi])) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^2*
Pi)
```

#### 3.84.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.84.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.71

| method | result | size |
|---|---|---|
| default | $\frac{a\sqrt{\pi\,c^2x^2+\pi}}{\pi\,c^2}+\frac{b\left(\operatorname{arcsinh}(cx)c^2x^2+\operatorname{arcsinh}(cx)-cx\sqrt{c^2x^2+1}\right)}{c^2\sqrt{\pi}\,\sqrt{c^2x^2+1}}$ | 72 |
| parts | $\frac{a\sqrt{\pi\,c^2x^2+\pi}}{\pi\,c^2}+\frac{b\left(\operatorname{arcsinh}(cx)c^2x^2+\operatorname{arcsinh}(cx)-cx\sqrt{c^2x^2+1}\right)}{c^2\sqrt{\pi}\,\sqrt{c^2x^2+1}}$ | 72 |

input
```
int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)+b/c^2/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c
*x)*c^2*x^2+arcsinh(c*x)-c*x*(c^2*x^2+1)^(1/2))
```

### 3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.29

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\sqrt{\pi+\pi c^2x^2}(bc^2x^2+b)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(ac^2x^2-\sqrt{c^2x^2+1}bcx+a\right)}{\pi c^4x^2+\pi c^2}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas"
)
```

output
```
(sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt
(pi + pi*c^2*x^2)*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a))/(pi*c^4*x^2 +
 pi*c^2)
```

### 3.84.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{a\left(\begin{cases}\frac{\sqrt{c^2x^2+1}}{c^2} & \text{for } c^2\neq 0\\ \frac{x^2}{2} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases}-\frac{x}{c}+\frac{\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{c^2} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

input
```
integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
a*Piecewise((sqrt(c**2*x**2 + 1)/c**2, Ne(c**2, 0)), (x**2/2, True))/sqrt(
pi) + b*Piecewise((-x/c + sqrt(c**2*x**2 + 1)*asinh(c*x)/c**2, Ne(c, 0)),
(0, True))/sqrt(pi)
```

### 3.84.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{bx}{\sqrt{\pi}c} + \frac{\sqrt{\pi+\pi c^2x^2}b\,\text{arsinh}\,(cx)}{\pi c^2} + \frac{\sqrt{\pi+\pi c^2x^2}a}{\pi c^2}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima"
)
```

output
```
-b*x/(sqrt(pi)*c) + sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*c^2) + sqrt(p
i + pi*c^2*x^2)*a/(pi*c^2)
```

### 3.84.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/sqrt(pi + pi*c^2*x^2), x)
```

### 3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{x\,(a+b\,\text{asinh}(c\,x))}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.85 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.85.1 Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{\pi}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2/b/c/Pi^(1/2)
```

## 3.85.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{\pi}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

## 3.85.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi c^2x^2+\pi}}\,dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2\sqrt{\pi}bc}$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

### 3.85.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

| method | result | size |
|---|---|---|
| default | $\frac{a\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c\sqrt{\pi}}$ | 53 |
| parts | $\frac{a\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c\sqrt{\pi}}$ | 53 |

input
```
int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c
/Pi^(1/2)*arcsinh(c*x)^2
```

### 3.85.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

### 3.85.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(19) = 38$.

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.48

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\begin{cases} a\left(\begin{cases} \frac{\log\left(2\pi c^2x+2\sqrt{\pi}\sqrt{\pi c^2x^2+\pi}\sqrt{c^2}\right)}{\sqrt{\pi}\sqrt{c^2}} & \text{for } \pi c^2\neq 0\\ \frac{x}{\sqrt{\pi}} & \text{otherwise}\end{cases}\right) & \text{for } b=0\\ \frac{ax}{\sqrt{\pi}} & \text{for } c=0\\ \frac{(a+b\,\text{asinh}\,(cx))^2}{2\sqrt{\pi}bc} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((a*Piecewise((log(2*pi*c**2*x + 2*sqrt(pi)*sqrt(pi*c**2*x**2 + p
i)*sqrt(c**2))/(sqrt(pi)*sqrt(c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi), True))
, Eq(b, 0)), (a*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**2/(2*sqrt(pi)*
b*c), True))
```

### 3.85.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{\pi + c^2\pi x^2}}\,dx = \frac{b\,\text{arsinh}\,(cx)^2}{2\,\sqrt{\pi}c} + \frac{a\,\text{arsinh}\,(cx)}{\sqrt{\pi}c}$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a*arcsinh(c*x)/(sqrt(pi)*c)
```

### 3.85.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{\pi + \pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

### 3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{\pi + c^2\pi x^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.86 $\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.86.1 Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}}$$

output
```
-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-b*polylog(2,
-c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/
2)
```

## 3.86.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{b\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right) - b\text{arcsinh}(cx)\log\left(1+e^{-\text{arcsinh}(cx)}\right) + a\log(x) - a\log\left(\pi\left(1+\sqrt{1+c^2\right.\right.}{\sqrt{\pi}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
(b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c*x]*Log[1 + E^(-Ar
cSinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + b*PolyLog[2,
 -E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Sqrt[Pi]
```

### 3.86.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{ 6231}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arc}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arctanh}($$

$$\Big\downarrow \text{ 2838}$$

$$\frac{i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{\sqrt{\pi}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
(I*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E
^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt[Pi]
```

### 3.86.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.86.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.11

| method | result |
|---|---|
| default | $-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi}}+\frac{b\left(-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+\right.\right.}{\sqrt{\pi}}$ |
| parts | $-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi}}+\frac{b\left(-\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+\right.\right.}{\sqrt{\pi}}$ |

input
```
int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+b/Pi^(1/2)*(-arcsinh(c
*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(
c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

### 3.86.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{\pi+\pi c^2x^2}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^3 + pi*x), x
)
```

### 3.86.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\int \frac{a}{x\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{x\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

input
```
integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
(Integral(a/(x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x*sqrt(c*
*2*x**2 + 1)), x))/sqrt(pi)
```

### 3.86.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima"
)
```

output
```
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x), x) - a
*arcsinh(1/(c*abs(x)))/sqrt(pi)
```

### 3.86.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x), x)
```

### 3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

# 3.87 $\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.87.1 Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx=-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{\pi x}+\frac{bc\log(x)}{\sqrt{\pi}}$$

output
```
b*c*ln(x)/Pi^(1/2)-(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/Pi/x
```

## 3.87.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{-a\sqrt{1+c^2x^2}-b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+bcx\log(x)}{\sqrt{\pi}x}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
(-(a*Sqrt[1 + c^2*x^2]) - b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*c*x*Log[x])
/(Sqrt[Pi]*x)
```

### 3.87.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{bc\int \frac{1}{x}dx}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi x}$$

$$\Big\downarrow \text{14}$$

$$\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi x}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
-((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(Pi*x)) + (b*c*Log[x])/Sqrt
[Pi]
```

#### 3.87.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

### 3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05

| method | result | size |
|---|---|---|
| default | $-\frac{a\sqrt{\pi\,c^{2}x^{2}+\pi}}{\pi x}-\frac{bc\,\operatorname{arcsinh}(cx)}{\sqrt{\pi}}-\frac{b\,\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}}{\sqrt{\pi}\,x}+\frac{bc\ln\left(\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}-1\right)}{\sqrt{\pi}}$ | 84 |
| parts | $-\frac{a\sqrt{\pi\,c^{2}x^{2}+\pi}}{\pi x}-\frac{bc\,\operatorname{arcsinh}(cx)}{\sqrt{\pi}}-\frac{b\,\operatorname{arcsinh}(cx)\sqrt{c^{2}x^{2}+1}}{\sqrt{\pi}\,x}+\frac{bc\ln\left(\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}-1\right)}{\sqrt{\pi}}$ | 84 |

input
```
int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/Pi/x*(Pi*c^2*x^2+Pi)^(1/2)-b*c/Pi^(1/2)*arcsinh(c*x)-b/Pi^(1/2)*arcsinh
(c*x)/x*(c^2*x^2+1)^(1/2)+b*c/Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

### 3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.22

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx$$
$$= \frac{\sqrt{\pi}bcx\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}(x^4-1)}{c^2x^4+x^2}\right)-2\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{\pi+}}{2\,\pi x}$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
1/2*(sqrt(pi)*b*c*x*log((pi + pi*c^2*x^6 + pi*c^2*x^2 + pi*x^4 + sqrt(pi)*
sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 - 1))/(c^2*x^4 + x^2)) - 2*sq
rt(pi + pi*c^2*x^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^
2)*a)/(pi*x)
```

### 3.87.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\int \frac{a}{x^2\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{x^2\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

input
```
integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
(Integral(a/(x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**2*s
qrt(c**2*x**2 + 1)), x))/sqrt(pi)
```

### 3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(37) = 74$.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.46

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\left(\sqrt{\pi}(-1)^{2\pi+2\pi c^2x^2}\log\left(2\pi c^2+\frac{2\pi}{x^2}\right)-\sqrt{\pi}\log\left(x^2+\frac{1}{c^2}\right)\right)bc}{2\pi} - \frac{\sqrt{\pi+\pi c^2x^2}b\,\text{arsinh}\,(cx)}{\pi x} - \frac{\sqrt{\pi+\pi c^2x^2}a}{\pi x}$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/2*(sqrt(pi)*(-1)^(2*pi + 2*pi*c^2*x^2)*log(2*pi*c^2 + 2*pi/x^2) - sqrt(
pi)*log(x^2 + 1/c^2))*b*c/pi - sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*x)
 - sqrt(pi + pi*c^2*x^2)*a/(pi*x)
```

### 3.87.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^2), x)
```

### 3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^2\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

# 3.88 $\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.88.1 Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=-\frac{bc}{2\sqrt{\pi}x}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2\pi x^2}+\frac{c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{\pi}}+\frac{bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{\pi}}$$

output
```
-1/2*b*c/x/Pi^(1/2)+c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/
Pi^(1/2)+1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*b*c^2*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2
+Pi)^(1/2)/Pi/x^2
```

## 3.88.2 Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{-\frac{4a\sqrt{1+c^2x^2}}{x^2}-4ac^2\log(x)+4ac^2\log\left(\pi\left(1+\sqrt{1+c^2x^2}\right)\right)+bc^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\text{arcsinh}(cx)\text{csch}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
((-4*a*Sqrt[1 + c^2*x^2])/x^2 - 4*a*c^2*Log[x] + 4*a*c^2*Log[Pi*(1 + Sqrt[
1 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSin
h[c*x]/2]^2 - 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*L
og[1 + E^(-ArcSinh[c*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2
, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSin
h[c*x]/2]))/(8*Sqrt[Pi])
```

### 3.88.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6224, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx+\frac{bc\int \frac{1}{x^2}dx}{2\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}-\frac{bc}{2\sqrt{\pi}x}$$

$$\Big\downarrow \text{6231}$$

$$-\frac{c^2\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{2\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}-\frac{bc}{2\sqrt{\pi}x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^2\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}-\frac{bc}{2\sqrt{\pi}x}$$

$$\Big\downarrow \text{26}$$

$$-\frac{ic^2\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}-\frac{bc}{2\sqrt{\pi}x}$$

$\Big\downarrow$ 4670

$$-\frac{ic^2\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+\right.}{2\sqrt{\pi}}$$
$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}-\frac{bc}{2\sqrt{\pi}x}$$

$\Big\downarrow$ 2715

$$-\frac{ic^2\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arcta}\right.}{2\sqrt{\pi}}$$
$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}-\frac{bc}{2\sqrt{\pi}x}$$

$\Big\downarrow$ 2838

$$-\frac{ic^2\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{2\sqrt{\pi}}-$$
$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi x^2}-\frac{bc}{2\sqrt{\pi}x}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
-1/2*(b*c)/(Sqrt[Pi]*x) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*
Pi*x^2) - ((I/2)*c^2*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] +
 I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt[P
i]
```

#### 3.88.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.88. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx$

### 3.88.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| default | $a\left(-\frac{\sqrt{\pi c^2x^2+\pi}}{2\pi x^2}+\frac{c^2\,\mathrm{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{2\sqrt{\pi}}\right)+b\left(-\frac{\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2\sqrt{\pi}\sqrt{c^2x^2+1}\,x^2}+\frac{c^2\,\mathrm{arcsinh}(cx)\ln\left(1+c\right.}{2\sqrt{\pi}}\right.$ |
| parts | $a\left(-\frac{\sqrt{\pi c^2x^2+\pi}}{2\pi x^2}+\frac{c^2\,\mathrm{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{2\sqrt{\pi}}\right)+b\left(-\frac{\mathrm{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)}{2\sqrt{\pi}\sqrt{c^2x^2+1}\,x^2}+\frac{c^2\,\mathrm{arcsinh}(cx)\ln\left(1+c\right.}{2\sqrt{\pi}}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/Pi/x^2*(Pi*c^2*x^2+Pi)^(1/2)+1/2/Pi^(1/2)*c^2*arctanh(Pi^(1/2)/(Pi
*c^2*x^2+Pi)^(1/2)))+b*(-1/2/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*x)*c^2*
x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/x^2+1/2*c^2/Pi^(1/2)*arcsinh(c*x)*
ln(1+c*x+(c^2*x^2+1)^(1/2))+1/2*c^2/Pi^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))-1/2*c^2/Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*c^2/Pi^
(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

### 3.88.5 Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^5 + pi*x^3),
 x)
```

### 3.88.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\int \frac{a}{x^3\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^3\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

input
```
integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
(Integral(a/(x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**3*s
qrt(c**2*x**2 + 1)), x))/sqrt(pi)
```

### 3.88.7 Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{\pi+\pi c^2x^2}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(pi) - sqrt(pi + pi*c^2*x^2)/(pi*x^2))*
a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x^3),
x)
```

### 3.88.8 Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{\pi+\pi c^2x^2}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

### 3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

# 3.89 $\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx$



## 3.89.1 Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{bc}{6\sqrt{\pi}x^2}-\frac{\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{3\pi x}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}$$

output
```
-1/6*b*c/x^2/Pi^(1/2)-2/3*b*c^3*ln(x)/Pi^(1/2)-1/3*(a+b*arcsinh(c*x))*(Pi*
c^2*x^2+Pi)^(1/2)/Pi/x^3+2/3*c^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/
Pi/x
```

## 3.89.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{-bcx+6bc^3x^3-2a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(-1+2c^2x^2)\,\text{arcsinh}(cx)-4bc^3x^3\log(}{6\sqrt{\pi}x^3}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
(-(b*c*x) + 6*b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2
*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSinh[c*x] - 4*b*c^3*x^3*
Log[x])/(6*Sqrt[Pi]*x^3)
```

### 3.89.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6224, 15, 6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{2}{3}c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2\pi x^2+\pi}}dx+\frac{bc\int \frac{1}{x^3}dx}{3\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi x^3}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{3}c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2\pi x^2+\pi}}dx-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi x^3}-\frac{bc}{6\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6215}$$

$$-\frac{2}{3}c^2\left(\frac{bc\int \frac{1}{x}dx}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi x}\right)-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi x^3}-\frac{bc}{6\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{3}c^2\left(\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi x}\right)-\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{3\pi x^3}-\frac{bc}{6\sqrt{\pi}x^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

output
```
-1/6*(b*c)/(Sqrt[Pi]*x^2) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(
3*Pi*x^3) - (2*c^2*(-((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(Pi*x))
 + (b*c*Log[x])/Sqrt[Pi]))/3
```

#### 3.89.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

### 3.89.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(81) = 162$.

Time = 0.19 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.85

| method | result |
|---|---|
| default | $a\left(-\frac{\sqrt{\pi c^2x^2+\pi}}{3\pi x^3}+\frac{2c^2\sqrt{\pi c^2x^2+\pi}}{3\pi x}\right)+\frac{4b c^3 \operatorname{arcsinh}(cx)}{3\sqrt{\pi}}-\frac{2b x^4 c^7}{3\sqrt{\pi}\,(3c^2x^2-1)}+\frac{2b x^2 (c^2x^2+1) c^5}{3\sqrt{\pi}\,(3c^2x^2-1)}-\frac{2b x^2 \operatorname{arcsinh}(cx) c^5}{\sqrt{\pi}\,(3c^2x^2-1)}+$ |
| parts | $a\left(-\frac{\sqrt{\pi c^2x^2+\pi}}{3\pi x^3}+\frac{2c^2\sqrt{\pi c^2x^2+\pi}}{3\pi x}\right)+\frac{4b c^3 \operatorname{arcsinh}(cx)}{3\sqrt{\pi}}-\frac{2b x^4 c^7}{3\sqrt{\pi}\,(3c^2x^2-1)}+\frac{2b x^2 (c^2x^2+1) c^5}{3\sqrt{\pi}\,(3c^2x^2-1)}-\frac{2b x^2 \operatorname{arcsinh}(cx) c^5}{\sqrt{\pi}\,(3c^2x^2-1)}+$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/Pi/x^3*(Pi*c^2*x^2+Pi)^(1/2)+2/3/Pi*c^2/x*(Pi*c^2*x^2+Pi)^(1/2))+4
/3*b*c^3/Pi^(1/2)*arcsinh(c*x)-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*x^4*c^7+2/3*b/
Pi^(1/2)/(3*c^2*x^2-1)*x^2*(c^2*x^2+1)*c^5-2*b/Pi^(1/2)/(3*c^2*x^2-1)*x^2*
arcsinh(c*x)*c^5+2*b/Pi^(1/2)/(3*c^2*x^2-1)*x*(c^2*x^2+1)^(1/2)*arcsinh(c*
x)*c^4-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*(c^2*x^2+1)*c^3+2/3*b/Pi^(1/2)/(3*c^2*
x^2-1)*arcsinh(c*x)*c^3-5/3*b/Pi^(1/2)/(3*c^2*x^2-1)/x*(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*c^2+1/6*b/Pi^(1/2)/(3*c^2*x^2-1)/x^2*(c^2*x^2+1)*c+1/3*b/Pi^(1
/2)/(3*c^2*x^2-1)/x^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2/3*b*c^3/Pi^(1/2)*ln
((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

### 3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(81) = 162$.

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.29

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{\pi + c^2\pi x^2}}\,dx$$
$$= \frac{2\sqrt{\pi + \pi c^2x^2}(2\,bc^4x^4 + bc^2x^2 - b)\log\left(cx + \sqrt{c^2x^2+1}\right) + 2\sqrt{\pi}(bc^5x^5 + bc^3x^3)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4-\sqrt{\pi}\cdots}{c^2x\cdots}\right.\cdots}{6\left(\pi c^2x^5 + \pi x^3\right)\cdots}$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="frica
s")
```

output
```
1/6*(2*sqrt(pi + pi*c^2*x^2)*(2*b*c^4*x^4 + b*c^2*x^2 - b)*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 +
pi*c^2*x^2 + pi*x^4 - sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^
4 - 1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(4*a*c^4*x^4 + 2*a*c^2*x^
2 + (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) - 2*a))/(pi*c^2*x^5 + pi*x^3)
```

### 3.89.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{\pi + c^2\pi x^2}}\,dx = \frac{\int \frac{a}{x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{x^4\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

input
```
integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
(Integral(a/(x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**4*s
qrt(c**2*x**2 + 1)), x))/sqrt(pi)
```

### 3.89.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{1}{6}\left(\frac{4\,c^2\log\left(x\right)}{\sqrt{\pi}}+\frac{1}{\sqrt{\pi}x^2}\right)bc + \frac{1}{3}\,b\left(\frac{2\,\sqrt{\pi+\pi c^2x^2}c^2}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}}{\pi x^3}\right)\text{arsinh}\left(cx\right) + \frac{1}{3}\,a\left(\frac{2\,\sqrt{\pi+\pi c^2x^2}c^2}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}}{\pi x^3}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/6*(4*c^2*log(x)/sqrt(pi) + 1/(sqrt(pi)*x^2))*b*c + 1/3*b*(2*sqrt(pi + p
i*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))*arcsinh(c*x) + 1/3
*a*(2*sqrt(pi + pi*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))
```

### 3.89.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{\sqrt{\pi+\pi c^2x^2}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^4), x)
```

### 3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

# 3.90 $\int \frac{x^5(a+b\mathbf{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.90.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{5bx}{3c^5\pi^{3/2}} - \frac{bx^3}{9c^3\pi^{3/2}} - \frac{a+b\operatorname{arcsinh}(cx)}{c^6\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^6\pi^2} + \frac{(\pi+c^2\pi x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6\pi^3} + \frac{b\arctan(cx)}{c^6\pi^{3/2}}$$

output
```
5/3*b*x/c^5/Pi^(3/2)-1/9*b*x^3/c^3/Pi^(3/2)+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b
*arcsinh(c*x))/c^6/Pi^3+b*arctan(c*x)/c^6/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^6
/Pi/(Pi*c^2*x^2+Pi)^(1/2)-2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/P
i^2
```

## 3.90.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-24a-12ac^2x^2+3ac^4x^4+15bcx\sqrt{1+c^2x^2}-bc^3x^3\sqrt{1+c^2x^2}+3b(-8-4c}{9c^6\pi^{3/2}\sqrt{1+c^2x^2}}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(-24*a - 12*a*c^2*x^2 + 3*a*c^4*x^4 + 15*b*c*x*Sqrt[1 + c^2*x^2] - b*c^3*x
^3*Sqrt[1 + c^2*x^2] + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b*S
qrt[1 + c^2*x^2]*ArcTan[c*x])/(9*c^6*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

### 3.90.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \ 6219$$

$$-\sqrt{\pi}bc\int -\frac{-c^4x^4+4c^2x^2+8}{3c^6\pi^2(c^2x^2+1)}dx+\frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi^3c^6}-\frac{2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^6}-\frac{a+b\text{arcsinh}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{-c^4x^4+4c^2x^2+8}{c^2x^2+1}dx}{3\pi^{3/2}c^5}+\frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi^3c^6}-\frac{2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^6}-\frac{a+b\text{arcsinh}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \ 1467$$

$$\frac{b\int \left(-c^2x^2+\frac{3}{c^2x^2+1}+5\right)dx}{3\pi^{3/2}c^5}+\frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi^3c^6}-\frac{2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^6}-\frac{a+b\text{arcsinh}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \ 2009$$

$$\frac{(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))}{3\pi^3c^6}-\frac{2\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^6}-\frac{a+b\text{arcsinh}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(\frac{3\arctan(cx)}{c}-\frac{1}{3}c^2x^3+5x\right)}{3\pi^{3/2}c^5}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSinh[c*x])/(c^6*Pi*Sqrt[Pi + c^2*Pi*x^2])) - (2*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^6*Pi^2) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b
*ArcSinh[c*x]))/(3*c^6*Pi^3) + (b*(5*x - (c^2*x^3)/3 + (3*ArcTan[c*x])/c))
/(3*c^5*Pi^(3/2))
```

### 3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.90.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| default | $a\left(\frac{x^4}{3\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{4\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right)}{3c^2}\right)-\frac{ib\left(3i\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^4c^4-ix^5c^5-12i\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2}\right.}{}$ |
| parts | $a\left(\frac{x^4}{3\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{4\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right)}{3c^2}\right)-\frac{ib\left(3i\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^4c^4-ix^5c^5-12i\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2}\right.}{}$ |

input
```
int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-4/3/c^2*(x^2/Pi/c^2/(Pi*c^2*x^2+Pi
)^(1/2)+2/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2)))-1/9*I*b*(3*I*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)*x^4*c^4-I*x^5*c^5-12*I*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+1
4*I*x^3*c^3-9*ln(c*x+(c^2*x^2+1)^(1/2)+I)*x^2*c^2+9*ln(c*x+(c^2*x^2+1)^(1/
2)-I)*x^2*c^2-24*I*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+15*I*c*x-9*ln(c*x+(c^2*x
^2+1)^(1/2)+I)+9*ln(c*x+(c^2*x^2+1)^(1/2)-I))/Pi^(3/2)/c^6/(c^2*x^2+1)
```

### 3.90.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx =$$
$$-\frac{9\sqrt{\pi}(bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)-6\sqrt{\pi+\pi c^2x^2}(bc^4x^4-4bc^2x^2-8b)\log\left(cx+\sqrt{c^2x^2}\right.}{18\left(\pi^2c^8x^2+\pi^2c^6\right)}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="frica
s")
```

output
```
-1/18*(9*sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)
*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) - 6*sqrt(pi + pi*c^2*x^2)*(b*c^4
*x^4 - 4*b*c^2*x^2 - 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^
2*x^2)*(3*a*c^4*x^4 - 12*a*c^2*x^2 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 +
 1) - 24*a))/(pi^2*c^8*x^2 + pi^2*c^6)
```

### 3.90.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{ax^5}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^5\,\text{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a*x**5/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
 + Integral(b*x**5*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x
**2 + 1)), x))/pi**(3/2)
```

### 3.90.7 Maxima [F]

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^5}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxim
a")
```

output
```
1/3*a*(x^4/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - 4*x^2/(pi*sqrt(pi + pi*c^2*x^2
)*c^4) - 8/(pi*sqrt(pi + pi*c^2*x^2)*c^6)) + 1/3*b*((sqrt(pi)*c^4*x^4 - 4*
sqrt(pi)*c^2*x^2 - 8*sqrt(pi))*log(c*x + sqrt(c^2*x^2 + 1))/(pi^2*sqrt(c^2
*x^2 + 1)*c^6) - integrate((sqrt(pi)*c^4*x^4 - 4*sqrt(pi)*c^2*x^2 - 8*sqrt
(pi))/(sqrt(c^2*x^2 + 1)*x), x)/(pi^2*c^6) + 3*integrate(1/3*(sqrt(pi)*c^4
*x^4 - 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi))/(pi^2*c^9*x^4 + pi^2*c^7*x^2 + (pi
^2*c^8*x^3 + pi^2*c^6*x)*sqrt(c^2*x^2 + 1)), x))
```

### 3.90.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x^5\,(a+b\,\mathrm{asinh}(c\,x))}{{(\Pi\,c^2\,x^2+\Pi)}^{3/2}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.91 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.91.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{bx^2}{4c^3\pi^{3/2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}} + \frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^4\pi^2} - \frac{3(a+b\text{arcsinh}(cx))^2}{4bc^5\pi^{3/2}} - \frac{b\log(1+c^2x^2)}{2c^5\pi^{3/2}}$$

output
```
-1/4*b*x^2/c^3/Pi^(3/2)-3/4*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(3/2)-1/2*b*ln(c
^2*x^2+1)/c^5/Pi^(3/2)-x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
+3/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

## 3.91.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{12acx+4ac^3x^3-6b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2-b\sqrt{1+c^2x^2}\cosh(2\text{arcsinh}(cx))}{8c}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(12*a*c*x + 4*a*c^3*x^3 - 6*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[1
+ c^2*x^2]*Cosh[2*ArcSinh[c*x]] - 4*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] +
 ArcSinh[c*x]*(9*b*c*x - 12*a*Sqrt[1 + c^2*x^2] + b*Sinh[3*ArcSinh[c*x]]))
/(8*c^5*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

### 3.91.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6225, 243, 49, 2009, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \frac{x^3}{c^2x^2+1}dx}{\pi^{3/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{243}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \frac{x^2}{c^2x^2+1}dx^2}{2\pi^{3/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{2\pi^{3/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2\pi^{3/2}c}$$

$$\Big\downarrow \text{6227}$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}-\frac{b\int xdx}{2\sqrt{\pi}c}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}\right)}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2\pi^{3/2}c}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}\right)}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2\pi^{3/2}c}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4\sqrt{\pi}bc^3}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}\right)}{\pi c^2}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2\pi^{3/2}c}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
-((x^3*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (3*(-1/4*(b
*x^2)/(c*Sqrt[Pi]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2
*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*Sqrt[Pi])))/(c^2*Pi) + (b*(x^2/c^2
- Log[1 + c^2*x^2]/c^4))/(2*c*Pi^(3/2))
```

### 3.91.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(113) = 226$.

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.99

| method | result |
|---|---|
| default | $\frac{a\,x^3}{2\pi\,c^2\sqrt{\pi\,c^2x^2+\pi}}+\frac{3ax}{2c^4\pi\sqrt{\pi\,c^2x^2+\pi}}-\frac{3a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2c^4\pi\sqrt{\pi\,c^2}}-\frac{b\left(-4\,\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+2c^4x^4+6\,\operatorname{arcsinh}(cx)^2\right.}{}$ |
| parts | $\frac{a\,x^3}{2\pi\,c^2\sqrt{\pi\,c^2x^2+\pi}}+\frac{3ax}{2c^4\pi\sqrt{\pi\,c^2x^2+\pi}}-\frac{3a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2c^4\pi\sqrt{\pi\,c^2}}-\frac{b\left(-4\,\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+2c^4x^4+6\,\operatorname{arcsinh}(cx)^2\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+3/2*a/c^4*x/Pi/(Pi*c^2*x^2+Pi)^(1/2
)-3/2*a/c^4/Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^
(1/2)-1/8*b*(-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+2*c^4*x^4+6*arcsinh
(c*x)^2*x^2*c^2-8*arcsinh(c*x)*c^2*x^2+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x
^2*c^2-12*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+3*c^2*x^2+6*arcsinh(c*x)^2-8*
arcsinh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1)/Pi^(3/2)/c^5/(c^2*x^2+1)
```

## 3.91.5 Fricas [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^2*c^4*x^4
+ 2*pi^2*c^2*x^2 + pi^2), x)
```

## 3.91.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\int \frac{ax^4}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^4\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a*x**4/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
 + Integral(b*x**4*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x
**2 + 1)), x))/pi**(3/2)
```

### 3.91.7 Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxim
a")
```

output
```
1/2*a*(x^3/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 3*x/(pi*sqrt(pi + pi*c^2*x^2)*
c^4) - 3*arcsinh(c*x)/(pi^(3/2)*c^5)) + b*integrate(x^4*log(c*x + sqrt(c^2
*x^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

### 3.91.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x^4\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.92 $\int \frac{x^3(a+b\,\mathbf{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$



## 3.92.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=-\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\operatorname{arcsinh}(cx)}{c^4\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\operatorname{arcsinh}(cx))}{c^4\pi^2}-\frac{b\arctan(cx)}{c^4\pi^{3/2}}$$

output
```
-b*x/c^3/Pi^(3/2)-b*arctan(c*x)/c^4/Pi^(3/2)+(a+b*arcsinh(c*x))/c^4/Pi/(Pi
*c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

## 3.92.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\frac{2a+ac^2x^2-bcx\sqrt{1+c^2x^2}+b(2+c^2x^2)\operatorname{arcsinh}(cx)-b\sqrt{1+c^2x^2}\arctan(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(2*a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] + b*(2 + c^2*x^2)*ArcSinh[c*x]
- b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^4*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

### 3.92.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int \frac{c^2x^2+2}{c^4\pi^2(c^2x^2+1)}dx+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^4}+\frac{a+b\text{arcsinh}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{c^2x^2+2}{c^2x^2+1}dx}{\pi^{3/2}c^3}+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^4}+\frac{a+b\text{arcsinh}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{299}$$

$$-\frac{b\left(\int \frac{1}{c^2x^2+1}dx+x\right)}{\pi^{3/2}c^3}+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^4}+\frac{a+b\text{arcsinh}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^2c^4}+\frac{a+b\text{arcsinh}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{\pi^{3/2}c^3}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(a + b*ArcSinh[c*x])/(c^4*Pi*Sqrt[Pi + c^2*Pi*x^2]) + (Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(c^4*Pi^2) - (b*(x + ArcTan[c*x]/c))/(c^3*Pi^(3/2
))
```

#### 3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.92.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.90

| method | result |
|---|---|
| default | $a\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right)+\frac{b\left(\operatorname{arcsinh}(cx)c^2x^2+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right.\right.}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^4}$ |
| parts | $a\left(\frac{x^2}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{2}{\pi c^4\sqrt{\pi c^2x^2+\pi}}\right)+\frac{b\left(\operatorname{arcsinh}(cx)c^2x^2+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right.\right.}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^4}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+2/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2))+b/Pi^(
3/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*x)*c^2*x^2+I*(c^2*x^2+1)^(1/2)*ln(c*x+(c
^2*x^2+1)^(1/2)-I)-I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-c*x*(c^
2*x^2+1)^(1/2)+2*arcsinh(c*x))/c^4
```

### 3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\sqrt{\pi}(bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}(bc^2x^2+2b)\,\mathrm{l}}{2\left(\pi^2c^6x^2+\pi^{}\right.}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqr
t(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2
 + 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*(a*c^2*x^2
- sqrt(c^2*x^2 + 1)*b*c*x + 2*a))/(pi^2*c^6*x^2 + pi^2*c^4)
```

### 3.92.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{ax^3}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a*x**3/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
 + Integral(b*x**3*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x
**2 + 1)), x))/pi**(3/2)
```

### 3.92.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}} \, dx = -bc\left(\frac{x}{\pi^{\frac{3}{2}}c^4} + \frac{\arctan(cx)}{\pi^{\frac{3}{2}}c^5}\right) + b\left(\frac{x^2}{\pi\sqrt{\pi + \pi c^2x^2}c^2} + \frac{2}{\pi\sqrt{\pi + \pi c^2x^2}c^4}\right)\text{arsinh}(cx) + a\left(\frac{x^2}{\pi\sqrt{\pi + \pi c^2x^2}c^2} + \frac{2}{\pi\sqrt{\pi + \pi c^2x^2}c^4}\right)$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxim
a")
```

output
```
-b*c*(x/(pi^(3/2)*c^4) + arctan(c*x)/(pi^(3/2)*c^5)) + b*(x^2/(pi*sqrt(pi
+ pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))*arcsinh(c*x) + a*(x
^2/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))
```

### 3.92.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{arcsinh}(cx))}{(\pi + c^2\pi x^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.93 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.93.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{x(a+b\text{arcsinh}(cx))}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc^3\pi^{3/2}}+\frac{b\log(1+c^2x^2)}{2c^3\pi^{3/2}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(3/2)+1/2*b*ln(c^2*x^2+1)/c^3/Pi^(3/2)-x
*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.93.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-\frac{2acx}{\sqrt{1+c^2x^2}}+\left(2a-\frac{2bcx}{\sqrt{1+c^2x^2}}\right)\text{arcsinh}(cx)+b\text{arcsinh}(cx)^2+b\log(1+c^2x^2)}{2c^3\pi^{3/2}}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
((-2*a*c*x)/Sqrt[1 + c^2*x^2] + (2*a - (2*b*c*x)/Sqrt[1 + c^2*x^2])*ArcSin
h[c*x] + b*ArcSinh[c*x]^2 + b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

### 3.93.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6225, 240, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{3/2}} \, dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \frac{x}{c^2x^2+1}dx}{\pi^{3/2}c}-\frac{x(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}-\frac{x(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c^3}$$

$$\Big\downarrow \text{6198}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2\pi^{3/2}bc^3}-\frac{x(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c^3}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
-((x*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (a + b*ArcSin
h[c*x])^2/(2*b*c^3*Pi^(3/2)) + (b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

#### 3.93.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.93.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(70) = 140$.

Time $= 0.21$ (sec) , antiderivative size $= 168$, normalized size of antiderivative $= 2.10$

| method | result |
|---|---|
| default | $-\frac{ax}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\pi c^2\sqrt{\pi c^2}}+b\left(\frac{\operatorname{arcsinh}(cx)^2}{2c^3\pi^{\frac{3}{2}}}-\frac{2\operatorname{arcsinh}(cx)}{c^3\pi^{\frac{3}{2}}}+\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}c^3(c^2x^2+1)}\right.$ |
| parts | $-\frac{ax}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\pi c^2\sqrt{\pi c^2}}+b\left(\frac{\operatorname{arcsinh}(cx)^2}{2c^3\pi^{\frac{3}{2}}}-\frac{2\operatorname{arcsinh}(cx)}{c^3\pi^{\frac{3}{2}}}+\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}c^3(c^2x^2+1)}\right.$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+a/Pi/c^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*
c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+b*(1/2/c^3/Pi^(3/2)*arcsinh(c*x)^2-2/c^3
/Pi^(3/2)*arcsinh(c*x)+1/Pi^(3/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsin
h(c*x)/c^3/(c^2*x^2+1)+1/c^3/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2))
```

## 3.93.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^2*c^4*x^4
+ 2*pi^2*c^2*x^2 + pi^2), x)
```

### 3.93.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{ax^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^2\,\text{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a*x**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
 + Integral(b*x**2*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x
**2 + 1)), x))/pi**(3/2)
```

### 3.93.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxim
a")
```

output
```
-a*(x/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - arcsinh(c*x)/(pi^(3/2)*c^3)) + b*in
tegrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

### 3.93.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(3/2), x)
```

### 3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.94 $\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.94.1 Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\arctan(cx)}{c^2\pi^{3/2}}$$

output
```
b*arctan(c*x)/c^2/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2
)
```

## 3.94.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-a-b\text{arcsinh}(cx)+b\sqrt{1+c^2x^2}\arctan(cx)}{c^2\pi^{3/2}\sqrt{1+c^2x^2}}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(-a - b*ArcSinh[c*x] + b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^2*Pi^(3/2)*Sqrt
[1 + c^2*x^2])
```

### 3.94.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\int \frac{1}{c^2x^2+1}dx}{\pi^{3/2}c}-\frac{a+b\text{arcsinh}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\arctan(cx)}{\pi^{3/2}c^2}-\frac{a+b\text{arcsinh}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSinh[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (b*ArcTan[c*x])/(
c^2*Pi^(3/2))
```

#### 3.94.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.94.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

| method | result | size |
|---|---|---|
| default | $-\frac{a}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+b\left(-\frac{\operatorname{arcsinh}(cx)}{c^2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}+\frac{i\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{c^2\pi^{\frac{3}{2}}}-\frac{i\ln\left(cx+\sqrt{c^2x^2+1}-i\right)}{c^2\pi^{\frac{3}{2}}}\right)$ | 103 |
| parts | $-\frac{a}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+b\left(-\frac{\operatorname{arcsinh}(cx)}{c^2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}+\frac{i\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{c^2\pi^{\frac{3}{2}}}-\frac{i\ln\left(cx+\sqrt{c^2x^2+1}-i\right)}{c^2\pi^{\frac{3}{2}}}\right)$ | 103 |

input
```
int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+b*(-1/c^2/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1
)^(1/2)+I/c^2/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-I/c^2/Pi^(3/2)*ln(c*x+(
c^2*x^2+1)^(1/2)-I))
```

### 3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.82

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\pi}(bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi+\pi c^2x^2}a}{2(\pi^2c^4x^2+\pi^2c^2)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas"
)
```

output
```
-1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sq
rt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*b*log(c*x
 + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*a)/(pi^2*c^4*x^2 + pi^2*c^
2)
```

### 3.94.6 Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{ax}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a*x/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) +
Integral(b*x*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 +
1)), x))/pi**(3/2)
```

### 3.94.7 Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima"
)
```

output
```
b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(pi^(3/2)*c^2) - log(c*x + sqrt(c
^2*x^2 + 1))/(pi^(3/2)*sqrt(c^2*x^2 + 1)*c^2) - integrate(1/(pi^(3/2)*c^5*
x^4 + pi^(3/2)*c^3*x^2 + (pi^(3/2)*c^4*x^3 + pi^(3/2)*c^2*x)*sqrt(c^2*x^2
+ 1)), x)) - a/(pi*sqrt(pi + pi*c^2*x^2)*c^2)
```

### 3.94.8 Giac [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(3/2), x)
```

### 3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))}{\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.95 $\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}} dx$



## 3.95.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}} dx = \frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\log(1+c^2x^2)}{2c\pi^{3/2}}$$

output
```
-1/2*b*ln(c^2*x^2+1)/c/Pi^(3/2)+x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1
/2)
```

## 3.95.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}} dx = \frac{2acx+2bcx\text{arcsinh}(cx)-b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2c\pi^{3/2}\sqrt{1+c^2x^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2
*c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

### 3.95.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{bc\int \frac{x}{c^2x^2+1}dx}{\pi^{3/2}}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Log[1 + c^2*x^2])
/(2*c*Pi^(3/2))
```

#### 3.95.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

### 3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(45) = 90$.

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

| method | result | size |
|---|---|---|
| default | $\frac{ax}{\pi\sqrt{\pi c^2x^2+\pi}}+b\left(\frac{2\,\mathrm{arcsinh}(cx)}{c\,\pi^{\frac{3}{2}}}-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\mathrm{arcsinh}(cx)}{\pi^{\frac{3}{2}}c(c^2x^2+1)}-\frac{\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\,\pi^{\frac{3}{2}}}\right)$ | 110 |
| parts | $\frac{ax}{\pi\sqrt{\pi c^2x^2+\pi}}+b\left(\frac{2\,\mathrm{arcsinh}(cx)}{c\,\pi^{\frac{3}{2}}}-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\mathrm{arcsinh}(cx)}{\pi^{\frac{3}{2}}c(c^2x^2+1)}-\frac{\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\,\pi^{\frac{3}{2}}}\right)$ | 110 |

input
```
int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+b*(2/c/Pi^(3/2)*arcsinh(c*x)-1/Pi^(3/2)*(c^2*
x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/c/(c^2*x^2+1)-1/c/Pi^(3/2)*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2))
```

### 3.95.5 Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^4 + 2*pi^2
*c^2*x^2 + pi^2), x)
```

### 3.95.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + In
tegral(b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)),
 x))/pi**(3/2)
```

### 3.95.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \frac{bx\,\text{arsinh}\,(cx)}{\pi\sqrt{\pi + \pi c^2x^2}} + \frac{ax}{\pi\sqrt{\pi + \pi c^2x^2}} - \frac{b\log\left(x^2 + \frac{1}{c^2}\right)}{2\,\pi^{\frac{3}{2}}c}$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

output
```
b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a*x/(pi*sqrt(pi + pi*c^2*x^2
)) - 1/2*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

### 3.95.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(3/2), x)
```

### 3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(\Pi\,c^2\,x^2 + \Pi)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.96 $\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.96.1 Optimal result

Integrand size = 26, antiderivative size = 94

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\arctan(cx)}{\pi^{3/2}} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}}$$

output
```
-b*arctan(c*x)/Pi^(3/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2)
)/Pi^(3/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(3/2)+b*polylog(2,c*x+(c
^2*x^2+1)^(1/2))/Pi^(3/2)+(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.96.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\frac{a}{\sqrt{1+c^2x^2}} + \frac{b\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} - 2b\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right) + b\text{arcsinh}(cx)\log\left(1-\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
(a/Sqrt[1 + c^2*x^2] + (b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 2*b*ArcTan[Tan
h[ArcSinh[c*x]/2]] + b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh
[c*x]*Log[1 + E^(-ArcSinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x
^2])] + b*PolyLog[2, -E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])]
)/Pi^(3/2)
```

### 3.96.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(\pi c^2x^2+\pi\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6226}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}-\frac{bc\int \frac{1}{c^2x^2+1}dx}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}$$

$$\Big\downarrow \text{ 6231}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arc}\right.}{\pi^{3/2}}$$
$$\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}$$

$\Big\downarrow$ 2715

$$\frac{i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arctanh}\right.}{\pi^{3/2}}$$
$$\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}$$

$\Big\downarrow$ 2838

$$\frac{i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{\pi^{3/2}}+$$
$$\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
(a + b*ArcSinh[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*ArcTan[c*x])/Pi^(3/2)
 + (I*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2,
 -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Pi^(3/2)
```

### 3.96.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.96.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| default | $a\left(\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)+b\left(\frac{\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}-\frac{2\arctan\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}}-\frac{\operatorname{dilog}\left(1+cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}}-\right.$ |
| parts | $a\left(\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)+b\left(\frac{\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}-\frac{2\arctan\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}}-\frac{\operatorname{dilog}\left(1+cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}}-\right.$ |

input
```
int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^
(1/2)))+b*(1/Pi^(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2/Pi^(3/2)*arctan(c*x
+(c^2*x^2+1)^(1/2))-1/Pi^(3/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))-1/Pi^(3/2)*a
rcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(
1/2)))
```

### 3.96.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^5 + 2*pi^2
*c^2*x^3 + pi^2*x), x)
```

### 3.96.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\int \frac{a}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x) +
Integral(b*asinh(c*x)/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 +
1)), x))/pi**(3/2)
```

### 3.96.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(\pi + \pi c^2x^2)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima"
)
```

output
```
-a*(arcsinh(1/(c*abs(x)))/pi^(3/2) - 1/(pi*sqrt(pi + pi*c^2*x^2))) + b*int
egrate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

### 3.96.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(\pi + \pi c^2x^2)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

### 3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x\,(\Pi\,c^2\,x^2 + \Pi)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

# 3.97 $\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$



## 3.97.1 Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{bc\log(x)}{\pi^{3/2}} + \frac{bc\log\left(1+c^2x^2\right)}{2\pi^{3/2}}$$

output
```
b*c*ln(x)/Pi^(3/2)+1/2*b*c*ln(c^2*x^2+1)/Pi^(3/2)+(-a-b*arcsinh(c*x))/Pi/x
/(Pi*c^2*x^2+Pi)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.97.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{-2a-4ac^2x^2-2(b+2bc^2x^2)\,\text{arcsinh}(cx)+2bcx\sqrt{1+c^2x^2}\log(x)+bcx\sqrt{1+c^2}}{2\pi^{3/2}x\sqrt{1+c^2x^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
(-2*a - 4*a*c^2*x^2 - 2*(b + 2*b*c^2*x^2)*ArcSinh[c*x] + 2*b*c*x*Sqrt[1 +
c^2*x^2]*Log[x] + b*c*x*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*Pi^(3/2)*x*
Sqrt[1 + c^2*x^2])
```

### 3.97.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6219, 25, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\left(\pi c^2x^2+\pi\right)^{3/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{2c^2x^2+1}{\pi^2x\left(c^2x^2+1\right)}dx-\frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\operatorname{arcsinh}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\pi}bc\int \frac{2c^2x^2+1}{\pi^2x\left(c^2x^2+1\right)}dx-\frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\operatorname{arcsinh}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{2c^2x^2+1}{x(c^2x^2+1)}dx}{\pi^{3/2}}-\frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\operatorname{arcsinh}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{354}$$

$$\frac{bc\int \frac{2c^2x^2+1}{x^2(c^2x^2+1)}dx^2}{2\pi^{3/2}}-\frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\operatorname{arcsinh}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{86}$$

$$\frac{bc\int \left(\frac{c^2}{c^2x^2+1}+\frac{1}{x^2}\right)dx^2}{2\pi^{3/2}}-\frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\operatorname{arcsinh}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c^2x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\operatorname{arcsinh}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)}{2\pi^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcSinh[c*x])/(Pi*x*Sqrt[Pi + c^2*Pi*x^2])) - (2*c^2*x*(a + b*Arc
Sinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*(Log[x^2] + Log[1 + c^2*x^2]
))/(2*Pi^(3/2))
```

### 3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.97. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{3/2}}\,dx$

### 3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(85) = 170$.

Time = 0.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

| method | result |
|---|---|
| default | $a\left(-\frac{1}{\pi x\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{2c^{2}x}{\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{4}-1\right)x^{4}c^{4}-2\sqrt{c^{2}x^{2}+1}\,\ln\left(\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{4}-1\right)x^{3}c^{3}+2\ln\left(\right.}{}$ |
| parts | $a\left(-\frac{1}{\pi x\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{2c^{2}x}{\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{4}-1\right)x^{4}c^{4}-2\sqrt{c^{2}x^{2}+1}\,\ln\left(\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{4}-1\right)x^{3}c^{3}+2\ln\left(\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)-2/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2))-b*(2*ln
((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^4*c^4-2*(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2
+1)^(1/2))^4-1)*x^3*c^3+2*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^2*c^2-(c^2*x^2
+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x*c+arcsinh(c*x))*(2*c^3*x^3+2*c
^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)/x/(c^2*x^2+1)
```

### 3.97.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^{2}\left(\pi+c^{2}\pi x^{2}\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^{2}x^{2}\right)^{\frac{3}{2}}x^{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^6 + 2*pi^2
*c^2*x^4 + pi^2*x^2), x)
```

### 3.97.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{\int \frac{a}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1)), x)
 + Integral(b*asinh(c*x)/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x
**2 + 1)), x))/pi**(3/2)
```

### 3.97.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{1}{2}\,bc\left(\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{3}{2}}}+\frac{2\,\log\left(x\right)}{\pi^{\frac{3}{2}}}\right) - \left(\frac{2\,c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x}\right)b\,\text{arsinh}\,(cx) - \left(\frac{2\,c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x}\right)a$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxim
a")
```

output
```
1/2*b*c*(log(c^2*x^2 + 1)/pi^(3/2) + 2*log(x)/pi^(3/2)) - (2*c^2*x/(pi*sqr
t(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*b*arcsinh(c*x) - (2*
c^2*x/(pi*sqrt(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*a
```

### 3.97.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^2), x)
```

### 3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{asinh}(c\,x)}{x^2\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

# 3.98 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.98.1 Optimal result

Integrand size = 26, antiderivative size = 162

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx = &-\frac{bc}{2\pi^{3/2}x}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}\\ &+\frac{bc^2\arctan(cx)}{\pi^{3/2}}+\frac{3c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{3/2}}\\ &+\frac{3bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\pi^{3/2}}-\frac{3bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\pi^{3/2}}\end{aligned}$$

output
```
-1/2*b*c/Pi^(3/2)/x+b*c^2*arctan(c*x)/Pi^(3/2)+3*c^2*(a+b*arcsinh(c*x))*ar
ctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))/Pi^(3/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)-3/2*c
^2*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)+1/2*(-a-b*arcsinh(c*x))/Pi/
x^2/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.98.2 Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.66

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-\frac{8ac^2}{\sqrt{1+c^2x^2}}-\frac{4a\sqrt{1+c^2x^2}}{x^2}-\frac{8bc^2\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}+16bc^2\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)-2b}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
((-8*a*c^2)/Sqrt[1 + c^2*x^2] - (4*a*Sqrt[1 + c^2*x^2])/x^2 - (8*b*c^2*Arc
Sinh[c*x])/Sqrt[1 + c^2*x^2] + 16*b*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2*b
*c^2*Coth[ArcSinh[c*x]/2] - b*c^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12
*b*c^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*b*c^2*ArcSinh[c*x]*Log
[1 + E^(-ArcSinh[c*x])] - 12*a*c^2*Log[x] + 12*a*c^2*Log[Pi*(1 + Sqrt[1 +
c^2*x^2])] - 12*b*c^2*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*b*c^2*PolyLog[2,
 E^(-ArcSinh[c*x])] - b*c^2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*b*c^2*
Tanh[ArcSinh[c*x]/2])/(8*Pi^(3/2))
```

## 3.98.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6224, 264, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(\pi c^2x^2+\pi\right)^{3/2}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{3}{2}c^2\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(c^2\pi x^2+\pi\right)^{3/2}}dx+\frac{bc\int \frac{1}{x^2(c^2x^2+1)}dx}{2\pi^{3/2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{3}{2}c^2\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(c^2\pi x^2+\pi\right)^{3/2}}dx+\frac{bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)}{2\pi^{3/2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{3}{2}c^2\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(c^2\pi x^2+\pi\right)^{3/2}}dx-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$$\Big\downarrow \text{6226}$$

$$-\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}-\frac{bc\int \frac{1}{c^2x^2+1}dx}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$\Big\downarrow$ 216

$$-\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$\Big\downarrow$ 6231

$$-\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{3}{2}c^2\left(\frac{\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$\Big\downarrow$ 26

$$-\frac{3}{2}c^2\left(\frac{i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$\Big\downarrow$ 4670

$$-\frac{3}{2}c^2\left(\frac{i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a\right.}{\pi^{3/2}}\right.$$
$$\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$\Big\downarrow$ 2715

3.98. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx$

$$-\frac{3}{2}c^2\left(\frac{i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2ia\right.}{\pi^{3/2}}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{3}{2}c^2\left(\frac{i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{\pi^{3/2}}+\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-c\arctan(cx)-\frac{1}{x}\right)}{2\pi^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])/(Pi*x^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*(-x^(-1) -
 c*ArcTan[c*x]))/(2*Pi^(3/2)) - (3*c^2*((a + b*ArcSinh[c*x])/(Pi*Sqrt[Pi +
 c^2*Pi*x^2]) - (b*ArcTan[c*x])/Pi^(3/2) + (I*((2*I)*(a + b*ArcSinh[c*x])*
ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2,
 E^ArcSinh[c*x]]))/Pi^(3/2)))/2
```

### 3.98.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

## 3.98. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.98.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| default | $a\left(-\frac{1}{2\pi\,x^2\sqrt{\pi\,c^2x^2+\pi}}-\frac{3c^2\left(\frac{1}{\pi\sqrt{\pi\,c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{3\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)}{2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,x^2}\right.$ |
| parts | $a\left(-\frac{1}{2\pi\,x^2\sqrt{\pi\,c^2x^2+\pi}}-\frac{3c^2\left(\frac{1}{\pi\sqrt{\pi\,c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{3\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)}{2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,x^2}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/Pi/x^2/(Pi*c^2*x^2+Pi)^(1/2)-3/2*c^2*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1
/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+b*(-1/2/Pi^(3/2)/(c^2*
x^2+1)^(1/2)*(3*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/x
^2+2*c^2/Pi^(3/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+3/2*c^2/Pi^(3/2)*dilog(1+c
*x+(c^2*x^2+1)^(1/2))+3/2*c^2/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))+3/2*c^2/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2)))
```

## 3.98.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^7 + 2*pi^2
*c^2*x^5 + pi^2*x^3), x)
```

### 3.98.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1)), x)
 + Integral(b*asinh(c*x)/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x
**2 + 1)), x))/pi**(3/2)
```

### 3.98.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxim
a")
```

output
```
1/2*(3*c^2*arcsinh(1/(c*abs(x)))/pi^(3/2) - 3*c^2/(pi*sqrt(pi + pi*c^2*x^2
)) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^2))*a + b*integrate(log(c*x + sqrt(c^2*
x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

### 3.98.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

### 3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

# 3.99 $\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.99.1 Optimal result

Integrand size = 26, antiderivative size = 153

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = &-\frac{bc}{6\pi^{3/2}x^2}-\frac{a+b\operatorname{arcsinh}(cx)}{3\pi x^3\sqrt{\pi+c^2\pi x^2}}+\frac{4c^2(a+b\operatorname{arcsinh}(cx))}{3\pi x\sqrt{\pi+c^2\pi x^2}}\\&+\frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc^3\log(1+c^2x^2)}{2\pi^{3/2}}\end{aligned}$$

output
```
-1/6*b*c/Pi^(3/2)/x^2-5/3*b*c^3*ln(x)/Pi^(3/2)-1/2*b*c^3*ln(c^2*x^2+1)/Pi^
(3/2)+1/3*(-a-b*arcsinh(c*x))/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)+4/3*c^2*(a+b*ar
csinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi
*c^2*x^2+Pi)^(1/2)
```

## 3.99.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.10

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-2a+8ac^2x^2+16ac^4x^4-bcx\sqrt{1+c^2x^2}-16bc^3x^3\sqrt{1+c^2x^2}+2b(-1+4c^2x^2+\dots}{6\pi^{3/2}x^3\sqrt{\dots}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
(-2*a + 8*a*c^2*x^2 + 16*a*c^4*x^4 - b*c*x*Sqrt[1 + c^2*x^2] - 16*b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 2*b*(-1 + 4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] - 10*b
*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[x] - 3*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 +
c^2*x^2])/(6*Pi^(3/2)*x^3*Sqrt[1 + c^2*x^2])
```

### 3.99.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6219, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi c^2x^2+\pi\right)^{3/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{-8c^4x^4-4c^2x^2+1}{3\pi^2x^3\left(c^2x^2+1\right)}dx+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{-8c^4x^4-4c^2x^2+1}{x^3(c^2x^2+1)}dx}{3\pi^{3/2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{bc\int \frac{-8c^4x^4-4c^2x^2+1}{x^4(c^2x^2+1)}dx^2}{6\pi^{3/2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{bc\int \left(-\frac{3c^4}{c^2x^2+1}-\frac{5c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6\pi^{3/2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\sqrt{\pi c^2x^2+\pi}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c^2(a+b\operatorname{arcsinh}(cx))}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\operatorname{arcsinh}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\operatorname{arcsinh}(cx))}{3\pi\sqrt{\pi c^2x^2+\pi}}+\frac{bc\left(-5c^2\log\left(x^2\right)-3c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)}{6\pi^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(Pi*x^3*Sqrt[Pi + c^2*Pi*x^2]) + (4*c^2*(a + b*A
rcSinh[c*x]))/(3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x
]))/(3*Pi*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*(-x^(-2) - 5*c^2*Log[x^2] - 3*c^2*
Log[1 + c^2*x^2]))/(6*Pi^(3/2))
```

#### 3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

### 3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(132) = 264$.

Time = 0.17 (sec) , antiderivative size = 604, normalized size of antiderivative = 3.95

| method | result |
|---|---|
| default | $a\left(-\frac{1}{3\pi\,x^{3}\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{4c^{2}\left(-\frac{1}{\pi x\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{2c^{2}x}{\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)}{3}\right)+\frac{16b\,c^{3}\operatorname{arcsinh}(cx)}{3\pi^{\frac{3}{2}}}-\frac{32b\,x^{8}c^{11}}{3\pi^{\frac{3}{2}}\left(8c^{2}x^{2}-1\right)\left(c^{2}x^{2}+1\right)}+\frac{32b}{3\pi^{\frac{3}{2}}(8c}$ |
| parts | $a\left(-\frac{1}{3\pi\,x^{3}\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{4c^{2}\left(-\frac{1}{\pi x\sqrt{\pi\,c^{2}x^{2}+\pi}}-\frac{2c^{2}x}{\pi\sqrt{\pi\,c^{2}x^{2}+\pi}}\right)}{3}\right)+\frac{16b\,c^{3}\operatorname{arcsinh}(cx)}{3\pi^{\frac{3}{2}}}-\frac{32b\,x^{8}c^{11}}{3\pi^{\frac{3}{2}}\left(8c^{2}x^{2}-1\right)\left(c^{2}x^{2}+1\right)}+\frac{32b}{3\pi^{\frac{3}{2}}(8c}$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)-4/3*c^2*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(1/2
)-2/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2)))+16/3*b*c^3/Pi^(3/2)*arcsinh(c*x)-32/3
*b/Pi^(3/2)/(8*c^2*x^2-1)*x^8/(c^2*x^2+1)*c^11+32/3*b/Pi^(3/2)/(8*c^2*x^2-
1)*x^6*c^9-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^6/(c^2*x^2+1)*c^9+32/3*b/Pi^(3/
2)/(8*c^2*x^2-1)*x^4*c^7-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*arc
sinh(c*x)*c^7+64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^3/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*c^6-32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*c^7-56/3*b/Pi^(3/2)
/(8*c^2*x^2-1)*x^2/(c^2*x^2+1)*arcsinh(c*x)*c^5+8*b/Pi^(3/2)/(8*c^2*x^2-1)
*x/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^4-4/3*b/Pi^(3/2)/(8*c^2*x^2-1)*c^3+8/3
*b/Pi^(3/2)/(8*c^2*x^2-1)/(c^2*x^2+1)*arcsinh(c*x)*c^3-4*b/Pi^(3/2)/(8*c^2
*x^2-1)/x/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2+1/6*b/Pi^(3/2)/(8*c^2*x^2-1)/
x^2*c+1/3*b/Pi^(3/2)/(8*c^2*x^2-1)/x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-5/3*
b*c^3/Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-b*c^3/Pi^(3/2)*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)
```

### 3.99.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^8 + 2*pi^2
*c^2*x^6 + pi^2*x^4), x)
```

### 3.99.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\frac{\int \frac{a}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 + 1)), x)
 + Integral(b*asinh(c*x)/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x
**2 + 1)), x))/pi**(3/2)
```

### 3.99.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxim
a")
```

output
```
1/3*(8*c^4*x/(pi*sqrt(pi + pi*c^2*x^2)) + 4*c^2/(pi*sqrt(pi + pi*c^2*x^2)*
x) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^3))*a + b*integrate(log(c*x + sqrt(c^2*
x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

### 3.99.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

### 3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^4\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

# 3.100 $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.100.1 Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bx^2}{4c^5\pi^{5/2}}-\frac{b}{6c^7\pi^{5/2}(1+c^2x^2)} -\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} +\frac{5x\sqrt{\pi+c^2\pi x^2}(a+b\text{arcsinh}(cx))}{2c^6\pi^3}-\frac{5(a+b\text{arcsinh}(cx))^2}{4bc^7\pi^{5/2}}-\frac{7b\log(1+c^2x^2)}{6c^7\pi^{5/2}}$$

output
```
-1/4*b*x^2/c^5/Pi^(5/2)-1/6*b/c^7/Pi^(5/2)/(c^2*x^2+1)-1/3*x^5*(a+b*arcsin
h(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-5/4*(a+b*arcsinh(c*x))^2/b/c^7/Pi^(5/
2)-7/6*b*ln(c^2*x^2+1)/c^7/Pi^(5/2)-5/3*x^3*(a+b*arcsinh(c*x))/c^4/Pi^2/(P
i*c^2*x^2+Pi)^(1/2)+5/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi^
3
```

## 3.100.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{60acx+80ac^3x^3+12ac^5x^5-7b\sqrt{1+c^2x^2}-9bc^2x^2\sqrt{1+c^2x^2}-6bc^4x^4\sqrt{1+}}{}$$

input
```
Integrate[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(60*a*c*x + 80*a*c^3*x^3 + 12*a*c^5*x^5 - 7*b*Sqrt[1 + c^2*x^2] - 9*b*c^2*
x^2*Sqrt[1 + c^2*x^2] - 6*b*c^4*x^4*Sqrt[1 + c^2*x^2] + 4*(-15*a*(1 + c^2*
x^2)^(3/2) + b*c*x*(15 + 20*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] - 30*b*(1 +
 c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - 28*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2]
)/(24*c^7*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

### 3.100.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.31, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6225, 243, 49, 2009, 6225, 243, 49, 2009, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{5\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi c^2}+\frac{b\int \frac{x^5}{(c^2x^2+1)^2}dx}{3\pi^{5/2}c}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{5\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi c^2}+\frac{b\int \frac{x^4}{(c^2x^2+1)^2}dx^2}{6\pi^{5/2}c}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{5\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi c^2}+\frac{b\int \left(\frac{1}{c^4}-\frac{2}{c^4(c^2x^2+1)}+\frac{1}{c^4(c^2x^2+1)^2}\right)dx^2}{6\pi^{5/2}c}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi c^2}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \text{6225}$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \frac{x^3}{c^2x^2+1}dx}{\pi^{3/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}\right)}{3\pi c^2}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \; 243$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \frac{x^2}{c^2x^2+1}dx^2}{2\pi^{3/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}\right)}{3\pi c^2}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \; 49$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \left(\frac{1}{c^2}-\frac{1}{c^2\left(c^2x^2+1\right)}\right)dx^2}{2\pi^{3/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}\right)}{3\pi c^2}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \; 2009$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{2\pi^{3/2}c}\right)}{3\pi c^2}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \; 6227$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}-\frac{b\int xdx}{2\sqrt{\pi}c}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}\right)}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{2\pi^{3/2}c}\right)}{3\pi c^2}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

---

3.100. $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}dx$

$$\Big\downarrow \ 15$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{2c^2}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}\right)}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{2\pi^{3/2}c}\right)}{3\pi c^2}-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \ 6198$$

$$-\frac{x^5(a+b\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{5\left(-\frac{x^3(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4\sqrt{\pi}bc^3}+\frac{x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}c}\right)}{\pi c^2}+\frac{b\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{2\pi^{3/2}c}\right)}{3\pi c^2}+\frac{b\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6\pi^{5/2}c}$$

input
```
Int[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
-1/3*(x^5*(a + b*ArcSinh[c*x]))/(c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (b*(x^2
/c^4 - 1/(c^6*(1 + c^2*x^2)) - (2*Log[1 + c^2*x^2])/c^6))/(6*c*Pi^(5/2)) +
 (5*(-((x^3*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (3*(-1
/4*(b*x^2)/(c*Sqrt[Pi]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(
2*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*Sqrt[Pi])))/(c^2*Pi) + (b*(x^2
/c^2 - Log[1 + c^2*x^2]/c^4))/(2*c*Pi^(3/2))))/(3*c^2*Pi)
```

3.100. $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

### 3.100.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.100. $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

### 3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. 2(164) = 328.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 5.05

| method | result | size |
|---|---|---|
| default | Expression too large to display | 970 |
| parts | Expression too large to display | 970 |

input
```
int(x^6*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c*x^6+14*b/Pi^(5/2
)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c^3*x^4+6*b/Pi^(5/2)/(63*c^4*x^4
+111*c^2*x^2+49)/(c^2*x^2+1)/c^5*x^2+147*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^
2+49)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^7+1/2*b/c^6/Pi^(5/2)*(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*x-49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)
^2*c*x^8-98/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c*x^6-4
9*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^3*x^4-98/3*b/Pi^(
5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*x^2-343/3*b/Pi^(5/2)/(6
3*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7*arcsinh(c*x)+5/6*a/c^4*x^3/Pi/
(Pi*c^2*x^2+Pi)^(3/2)+5/2*a/c^6/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-5/2*a/c^6/Pi^
2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*a*x
^5/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-1/8*b/c^7/Pi^(5/2)-1/4*b*x^2/c^5/Pi^(5/2)-
5/4*b/c^7/Pi^(5/2)*arcsinh(c*x)^2-7/3*b/c^7/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)
^(1/2))^2)+14/3*b/c^7/Pi^(5/2)*arcsinh(c*x)-49/6*b/Pi^(5/2)/(63*c^4*x^4+11
1*c^2*x^2+49)/(c^2*x^2+1)^2/c^7-147*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)
/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^8-553*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+4
9)/(c^2*x^2+1)^2/c*arcsinh(c*x)*x^6-2338/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*
x^2+49)/(c^2*x^2+1)^2/c^3*arcsinh(c*x)*x^4-1463/3*b/Pi^(5/2)/(63*c^4*x^4+1
11*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*arcsinh(c*x)*x^2+385*b/Pi^(5/2)/(63*c^4*x
^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)*x^5+1009/3*b/Pi^(...
```

### 3.100.5 Fricas [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^6}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*x^6*arcsinh(c*x) + a*x^6)/(pi^3*c^6*x^6
+ 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

### 3.100.6 Sympy [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{ax^6}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^6\,\text{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate(x**6*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a*x**6/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x*
*2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**6*asinh(c*x)/(c**4*x**4
*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 +
1)), x))/pi**(5/2)
```

### 3.100.7 Maxima [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^6}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxim
a")
```

output
```
1/6*a*(3*x^5/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 5*x*(3*x^2/(pi*(pi + pi*c^
2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))/c^2 + 5*x/(pi^2*sq
rt(pi + pi*c^2*x^2)*c^6) - 15*arcsinh(c*x)/(pi^(5/2)*c^7)) + b*integrate(x
^6*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x)
```

### 3.100.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{x^6\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.101 $\int \frac{x^5(a+b\,\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.101.1 Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{x^5(a+b\,\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}(1+c^2x^2)}-\frac{a+b\,\mathrm{arcsinh}(cx)}{3c^6\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2(a+b\,\mathrm{arcsinh}(cx))}{c^6\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}(a+b\,\mathrm{arcsinh}(cx))}{c^6\pi^3}-\frac{11b\arctan(cx)}{6c^6\pi^{5/2}}$$

output
```
-b*x/c^5/Pi^(5/2)+1/6*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))
/c^6/Pi/(Pi*c^2*x^2+Pi)^(3/2)-11/6*b*arctan(c*x)/c^6/Pi^(5/2)+2*(a+b*arcsi
nh(c*x))/c^6/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)
^(1/2)/c^6/Pi^3
```

## 3.101.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \frac{x^5(a+b\,\mathrm{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{16a+24ac^2x^2+6ac^4x^4-5bcx\sqrt{1+c^2x^2}-6bc^3x^3\sqrt{1+c^2x^2}+2b(8+12c^2x^2}{6c^6\pi^{5/2}(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(16*a + 24*a*c^2*x^2 + 6*a*c^4*x^4 - 5*b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3*x
^3*Sqrt[1 + c^2*x^2] + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - 11*
b*(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^6*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

### 3.101.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6219, 27, 1471, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int \frac{3c^4x^4+12c^2x^2+8}{3c^6\pi^3(c^2x^2+1)^2}dx+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^3c^6}+\frac{2(a+b\text{arcsinh}(cx))}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^6(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{3c^4x^4+12c^2x^2+8}{(c^2x^2+1)^2}dx}{3\pi^{5/2}c^5}+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^3c^6}+\frac{2(a+b\text{arcsinh}(cx))}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^6(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{b\left(-\frac{1}{2}\int -\frac{6c^2x^2+17}{c^2x^2+1}dx-\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c^5}+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^3c^6}+\frac{2(a+b\text{arcsinh}(cx))}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^6(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(\frac{1}{2}\int \frac{6c^2x^2+17}{c^2x^2+1}dx-\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c^5}+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^3c^6}+\frac{2(a+b\text{arcsinh}(cx))}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^6(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{299}$$

$$-\frac{b\left(\frac{1}{2}\left(11\int\frac{1}{c^2x^2+1}dx+6x\right)-\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c^5}+\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^3c^6}+\frac{2(a+b\text{arcsinh}(cx))}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^6\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))}{\pi^3c^6}+\frac{2(a+b\text{arcsinh}(cx))}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^6\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{b\left(\frac{1}{2}\left(\frac{11\arctan(cx)}{c}+6x\right)-\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c^5}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(c^6*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*(a + b*Arc
Sinh[c*x]))/(c^6*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (Sqrt[Pi + c^2*Pi*x^2]*(a +
 b*ArcSinh[c*x]))/(c^6*Pi^3) - (b*(-1/2*x/(1 + c^2*x^2) + (6*x + (11*ArcTa
n[c*x])/c)/2))/(3*c^5*Pi^(5/2))
```

### 3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.101. $\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.101.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| default | $a\left(\frac{x^4}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{\pi c^2\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{2}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)}{c^6\pi^{\frac{5}{2}}}-\frac{bx}{c^5\pi^{\frac{5}{2}}}+\frac{2b\,\operatorname{arcsinh}(cx)x}{\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c}$ |
| parts | $a\left(\frac{x^4}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{\pi c^2\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{2}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)}{c^6\pi^{\frac{5}{2}}}-\frac{bx}{c^5\pi^{\frac{5}{2}}}+\frac{2b\,\operatorname{arcsinh}(cx)x}{\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c}$ |

input
```
int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-4/c^2*(-x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/
2)-2/3/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2)))+b/c^6/Pi^(5/2)*(c^2*x^2+1)^(1/2)*arc
sinh(c*x)-b*x/c^5/Pi^(5/2)+2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)
*x^2+1/6*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+5/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^6
*arcsinh(c*x)+11/6*I*b/c^6/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/6*I*b/c
^6/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

### 3.101.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.49

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{11\sqrt{\pi}(bc^4x^4+2\,bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}(\cdots}{\cdots}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
1/12*(11*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
 + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c
^2*x^2)*(3*b*c^4*x^4 + 12*b*c^2*x^2 + 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) +
2*sqrt(pi + pi*c^2*x^2)*(6*a*c^4*x^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c
*x)*sqrt(c^2*x^2 + 1) + 16*a))/(pi^3*c^10*x^4 + 2*pi^3*c^8*x^2 + pi^3*c^6)
```

### 3.101.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax^5}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^5\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a*x**5/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x*
*2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**5*asinh(c*x)/(c**4*x**4
*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 +
1)), x))/pi**(5/2)
```

### 3.101.7 Maxima [F]

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^5}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxim
a")
```

output
```
1/3*b*((3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))*log(c*x + s
qrt(c^2*x^2 + 1))/((pi^3*c^8*x^2 + pi^3*c^6)*sqrt(c^2*x^2 + 1)) + 3*integr
ate(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))/(pi^3*c^11
*x^6 + 2*pi^3*c^9*x^4 + pi^3*c^7*x^2 + (pi^3*c^10*x^5 + 2*pi^3*c^8*x^3 + p
i^3*c^6*x)*sqrt(c^2*x^2 + 1)), x) - 3*integrate(1/3*(3*sqrt(pi)*c^4*x^4 +
12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))/((pi^3*c^8*x^3 + pi^3*c^6*x)*sqrt(c^2*x^
2 + 1)), x)) + 1/3*a*(3*x^4/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 12*x^2/(pi*
(pi + pi*c^2*x^2)^(3/2)*c^4) + 8/(pi*(pi + pi*c^2*x^2)^(3/2)*c^6))
```

### 3.101.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{x^5\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.102 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.102.1 Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{b}{6c^5\pi^{5/2}(1+c^2x^2)} - \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2\pi(\pi+c^2\pi x^2)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{2bc^5\pi^{5/2}} + \frac{2b\log(1+c^2x^2)}{3c^5\pi^{5/2}}$$

output
```
1/6*b/c^5/Pi^(5/2)/(c^2*x^2+1)-1/3*x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x
^2+Pi)^(3/2)+1/2*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(5/2)+2/3*b*ln(c^2*x^2+1)/c
^5/Pi^(5/2)-x*(a+b*arcsinh(c*x))/c^4/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.102.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{b+bc^2x^2-6acx\sqrt{1+c^2x^2}-8ac^3x^3\sqrt{1+c^2x^2}+2\Big(3a(1+c^2x^2)^2-bcx\sqrt{1+}}{6c^5}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(b + b*c^2*x^2 - 6*a*c*x*Sqrt[1 + c^2*x^2] - 8*a*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 2*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2))*ArcSi
nh[c*x] + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]^2 + 4*b*(1 + c^2*x^2)^2*Log[1 +
 c^2*x^2])/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)^2)
```

### 3.102.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6225, 243, 49, 2009, 6225, 240, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{\pi c^2}+\frac{b\int \frac{x^3}{(c^2x^2+1)^2}dx}{3\pi^{5/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{\pi c^2}+\frac{b\int \frac{x^2}{(c^2x^2+1)^2}dx^2}{6\pi^{5/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{\pi c^2}+\frac{b\int \left(\frac{1}{c^2(c^2x^2+1)}-\frac{1}{c^2(c^2x^2+1)^2}\right)dx^2}{6\pi^{5/2}c}-\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2\pi x^2+\pi)^{3/2}}dx}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}+\frac{b\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \text{6225}$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}+\frac{b\int \frac{x}{c^2x^2+1}dx}{\pi^{3/2}c}-\frac{x(a+b\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}}{\pi c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}+\frac{b\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \; 240$$

$$\frac{\frac{\int \frac{a+b\,\text{arcsinh}(cx)}{\sqrt{c^2\pi x^2+\pi}}dx}{\pi c^2}-\frac{x(a+b\,\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\log(c^2x^2+1)}{2\pi^{3/2}c^3}}{\pi c^2}-\frac{x^3(a+b\,\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6\pi^{5/2}c}$$

$$\Big\downarrow \; 6198$$

$$-\frac{x^3(a+b\,\text{arcsinh}(cx))}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{\frac{(a+b\,\text{arcsinh}(cx))^2}{2\pi^{3/2}bc^3}-\frac{x(a+b\,\text{arcsinh}(cx))}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\log(c^2x^2+1)}{2\pi^{3/2}c^3}}{\pi c^2}+\frac{b\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6\pi^{5/2}c}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
-1/3*(x^3*(a + b*ArcSinh[c*x]))/(c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (b*(1/(
c^4*(1 + c^2*x^2)) + Log[1 + c^2*x^2]/c^4))/(6*c*Pi^(5/2)) + (-((x*(a + b*
ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (a + b*ArcSinh[c*x])^2/(2
*b*c^3*Pi^(3/2)) + (b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2)))/(c^2*Pi)
```

### 3.102.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.102. $\int \frac{x^4(a+b\,\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(121) = 242$.

Time = 0.17 (sec) , antiderivative size = 897, normalized size of antiderivative = 6.45

| method | result |
|---|---|
| default | $-\frac{a\,x^3}{3\pi\,c^2(\pi\,c^2x^2+\pi)^{\frac{3}{2}}}-\frac{ax}{\pi^2c^4\sqrt{\pi\,c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\pi^2c^4\sqrt{\pi\,c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c^5\pi^{\frac{5}{2}}}-\frac{8b\,\text{arcsinh}(cx)}{3c^5\pi^{\frac{5}{2}}}+\frac{32b\,c^3\,\text{a}}{\pi^{\frac{5}{2}}(24c^4x^4+39}$ |
| parts | $-\frac{a\,x^3}{3\pi\,c^2(\pi\,c^2x^2+\pi)^{\frac{3}{2}}}-\frac{ax}{\pi^2c^4\sqrt{\pi\,c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\pi^2c^4\sqrt{\pi\,c^2}}+\frac{b\,\text{arcsinh}(cx)^2}{2c^5\pi^{\frac{5}{2}}}-\frac{8b\,\text{arcsinh}(cx)}{3c^5\pi^{\frac{5}{2}}}+\frac{32b\,c^3\,\text{a}}{\pi^{\frac{5}{2}}(24c^4x^4+39}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-a/Pi^2/c^4*x/(Pi*c^2*x^2+Pi)^(1/2)
+a/Pi^2/c^4*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/
2)+1/2*b/c^5/Pi^(5/2)*arcsinh(c*x)^2-8/3*b/c^5/Pi^(5/2)*arcsinh(c*x)+32*b/
Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^8-32*
b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x
^7+8/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*x^8-8/3*b/P
i^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)*c*x^6+116*b/Pi^(5/2)/(24*c^
4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^6-76*b/Pi^(5/2)/(24*c^
4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5+32/3*b/Pi^(5/2)/(2
4*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c*x^6-4*b/Pi^(5/2)/(24*c^4*x^4+39*c
^2*x^2+16)/(c^2*x^2+1)/c*x^4+472/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(
c^2*x^2+1)^2/c*arcsinh(c*x)*x^4-181/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16
)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)*x^3+16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*
x^2+16)/(c^2*x^2+1)^2/c*x^4-3/2*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2
*x^2+1)/c^3*x^2+284/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/
c^3*arcsinh(c*x)*x^2-16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^
(3/2)/c^4*arcsinh(c*x)*x+32/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x
^2+1)^2/c^3*x^2+64/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c
^5*arcsinh(c*x)+8/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^
5+4/3*b/c^5/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

## 3.102.5 Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^3*c^6*x^6
+ 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

### 3.102.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax^4}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^4\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a*x**4/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x*
*2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**4*asinh(c*x)/(c**4*x**4
*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 +
1)), x))/pi**(5/2)
```

### 3.102.7 Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*(x*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^
(3/2)*c^4)) + x/(pi^2*sqrt(pi + pi*c^2*x^2)*c^4) - 3*arcsinh(c*x)/(pi^(5/2
)*c^5))*a + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)
^(5/2), x)
```

### 3.102.8 Giac [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)*x^4/(pi + pi*c^2*x^2)^(5/2), x)
```

### 3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{x^4\,(a+b\,\operatorname{asinh}(c\,x))}{{(\Pi\,c^2\,x^2+\Pi)}^{5/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.103 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.103.1 Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bx}{6c^3\pi^{5/2}(1+c^2x^2)} + \frac{a+b\text{arcsinh}(cx)}{3c^4\pi(\pi+c^2\pi x^2)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{5b\arctan(cx)}{6c^4\pi^{5/2}}$$

output
```
-1/6*b*x/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/c^4/Pi/(Pi*c^2*x^
2+Pi)^(3/2)+5/6*b*arctan(c*x)/c^4/Pi^(5/2)+(-a-b*arcsinh(c*x))/c^4/Pi^2/(P
i*c^2*x^2+Pi)^(1/2)
```

## 3.103.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{-4a-6ac^2x^2-bcx\sqrt{1+c^2x^2}-2b(2+3c^2x^2)\,\text{arcsinh}(cx)+5b(1+c^2x^2)^{3/2}}{6c^4\pi^{5/2}(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(-4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*(2 + 3*c^2*x^2)*ArcSin
h[c*x] + 5*b*(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^4*Pi^(5/2)*(1 + c^2*x^2
)^(3/2))
```

### 3.103.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{3c^2x^2+2}{3c^4\pi^3(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\text{arcsinh}(cx)}{3\pi c^4(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{3c^2x^2+2}{(c^2x^2+1)^2}dx}{3\pi^{5/2}c^3}-\frac{a+b\text{arcsinh}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\text{arcsinh}(cx)}{3\pi c^4(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{b\left(\frac{5}{2}\int \frac{1}{c^2x^2+1}dx-\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c^3}-\frac{a+b\text{arcsinh}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\text{arcsinh}(cx)}{3\pi c^4(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a+b\text{arcsinh}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\text{arcsinh}(cx)}{3\pi c^4(\pi c^2x^2+\pi)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c^3}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(a + b*ArcSinh[c*x])/(3*c^4*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (a + b*ArcSinh[c
*x])/(c^4*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*(-1/2*x/(1 + c^2*x^2) + (5*ArcT
an[c*x])/(2*c)))/(3*c^3*Pi^(5/2))
```

### 3.103.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.103.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| default | $a\left(-\frac{x^2}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{2}{3\pi c^4(\pi c^2x^2+\pi)^{\frac{3}{2}}}\right)+b\left(-\frac{6\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+4\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c^4}+\frac{5i\ln\left(cx+\sqrt{c^2x^2+1}\right.}{6c^4\pi^{\frac{5}{2}}}\right.$ |
| parts | $a\left(-\frac{x^2}{\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{2}{3\pi c^4(\pi c^2x^2+\pi)^{\frac{3}{2}}}\right)+b\left(-\frac{6\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+4\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}c^4}+\frac{5i\ln\left(cx+\sqrt{c^2x^2+1}\right.}{6c^4\pi^{\frac{5}{2}}}\right.$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-2/3/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2))+b*(
-1/6/Pi^(5/2)/(c^2*x^2+1)^(3/2)*(6*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1
/2)+4*arcsinh(c*x))/c^4+5/6*I/c^4/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6
*I/c^4/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I))
```

### 3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(91) = 182$.

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.78

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx =$$
$$-\frac{5\sqrt{\pi}(bc^4x^4+2\,bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}(3\,bc^2x^2+2\,b)\log\left(cx+\sqrt{c^2x^2}\right.}{12\,(\pi^3c^8x^4+2\,\pi^3c^6x^2+\pi^3c^4)}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
-1/12*(5*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
 + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c
^2*x^2)*(3*b*c^2*x^2 + 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*
c^2*x^2)*(6*a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x + 4*a))/(pi^3*c^8*x^4 + 2*
pi^3*c^6*x^2 + pi^3*c^4)
```

### 3.103.6 Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax^3}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^3\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a*x**3/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x*
*2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**3*asinh(c*x)/(c**4*x**4
*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 +
1)), x))/pi**(5/2)
```

### 3.103.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{1}{6}bc\left(\frac{x}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}-\frac{5\arctan(cx)}{\pi^{\frac{5}{2}}c^5}\right) - \frac{1}{3}b\left(\frac{3x^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}+\frac{2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^4}\right)\text{arsinh}(cx) - \frac{1}{3}a\left(\frac{3x^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}+\frac{2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^4}\right)$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/6*b*c*(x/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) - 5*arctan(c*x)/(pi^(5/2)*c^
5)) - 1/3*b*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x
^2)^(3/2)*c^4))*arcsinh(c*x) - 1/3*a*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^
2) + 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))
```

### 3.103.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{{(\Pi\,c^2\,x^2+\Pi)}^{5/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.104 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.104.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=-\frac{b}{6c^3\pi^{5/2}(1+c^2x^2)}+\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{b\log(1+c^2x^2)}{6c^3\pi^{5/2}}$$

output
```
-1/6*b/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*x^3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+
Pi)^(3/2)-1/6*b*ln(c^2*x^2+1)/c^3/Pi^(5/2)
```

## 3.104.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=-\frac{-2ac^3x^3+b\sqrt{1+c^2x^2}-2bc^3x^3\text{arcsinh}(cx)+b(1+c^2x^2)^{3/2}\log(1+c^2x^2)}{6c^3\pi^{5/2}(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
-1/6*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] - 2*b*c^3*x^3*ArcSinh[c*x] + b*(1
 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(c^3*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

### 3.104.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6215, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}-\frac{bc\int \frac{x^3}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}-\frac{bc\int \frac{x^2}{(c^2x^2+1)^2}dx^2}{6\pi^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}-\frac{bc\int \left(\frac{1}{c^2(c^2x^2+1)}-\frac{1}{c^2(c^2x^2+1)^2}\right)dx^2}{6\pi^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}-\frac{bc\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6\pi^{5/2}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (b*c*(1/(c^4*(
1 + c^2*x^2)) + Log[1 + c^2*x^2]/c^4))/(6*Pi^(5/2))
```

### 3.104.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(68) = 136$.

Time = 0.19 (sec) , antiderivative size = 729, normalized size of antiderivative = 9.11

| method | result |
|---|---|
| default | $a\left(-\frac{x}{2\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{x}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}}{2c^2}\right)+\frac{2b\operatorname{arcsinh}(cx)}{3c^3\pi^{\frac{5}{2}}}-\frac{b\,c^5\operatorname{arcsinh}(cx)x^8}{\pi^{\frac{5}{2}}(3c^4x^4+3c^2x^2+1)(c^2x^2+1)^2}+\frac{\ }{\pi^{\frac{5}{2}}(3}$ |
| parts | $a\left(-\frac{x}{2\pi c^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{x}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}}{2c^2}\right)+\frac{2b\operatorname{arcsinh}(cx)}{3c^3\pi^{\frac{5}{2}}}-\frac{b\,c^5\operatorname{arcsinh}(cx)x^8}{\pi^{\frac{5}{2}}(3c^4x^4+3c^2x^2+1)(c^2x^2+1)^2}+\frac{\ }{\pi^{\frac{5}{2}}(3}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+1/2/c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(
3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)))+2/3*b/c^3/Pi^(5/2)*arcsinh(c*x)-b/
Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^5*arcsinh(c*x)*x^8+b/Pi^(
5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*c^4*arcsinh(c*x)*x^7-1/6*b/
Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^5*x^8+1/6*b/Pi^(5/2)/(3*c
^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)*c^3*x^6-3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+
1)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^6+b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(
c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x^5-2/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+
1)/(c^2*x^2+1)^2*c^3*x^6-10/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+
1)^2*c*arcsinh(c*x)*x^4+1/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)
^(3/2)*arcsinh(c*x)*x^3-b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c
*x^4-5/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c*arcsinh(c*x)*x
^2-2/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c*x^2-1/3*b/Pi^(5/
2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3*arcsinh(c*x)-1/6*b/Pi^(5/2)/(
3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3-1/3*b/c^3/Pi^(5/2)*ln(1+(c*x+(c^2
*x^2+1)^(1/2))^2)
```

### 3.104.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^3*c^6*x^6
+ 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

### 3.104.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{ax^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^2\,\text{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a*x**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**2*asinh(c*x)/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(5/2)
```

### 3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(68) = 136$.

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}+\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^4}\right) - \frac{1}{3}\,b\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}c^2}\right)\text{arsinh}\left(cx\right) - \frac{1}{3}\,a\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}c^2}\right)$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b*c*(1/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) + log(c^2*x^2 + 1)/(pi^(5/2)*c^4)) - 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(pi^2*sqrt(pi + pi*c^2*x^2)*c^2))*arcsinh(c*x) - 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(pi^2*sqrt(pi + pi*c^2*x^2)*c^2))
```

### 3.104.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\left(cx\right)+a)x^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(5/2), x)
```

### 3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{x^2\,(a+b\,\operatorname{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.105 $\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.105.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{bx}{6c\pi^{5/2}(1+c^2x^2)}-\frac{a+b\text{arcsinh}(cx)}{3c^2\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{b\arctan(cx)}{6c^2\pi^{5/2}}$$

output
```
1/6*b*x/c/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+
Pi)^(3/2)+1/6*b*arctan(c*x)/c^2/Pi^(5/2)
```

## 3.105.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{-2a+bcx\sqrt{1+c^2x^2}-2b\text{arcsinh}(cx)+b(1+c^2x^2)^{3/2}\arctan(cx)}{6c^2\pi^{5/2}(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(-2*a + b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + b*(1 + c^2*x^2)^(3/2)
*ArcTan[c*x])/(6*c^2*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

### 3.105.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6213, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\int \frac{1}{(c^2x^2+1)^2}dx}{3\pi^{5/2}c}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b\left(\frac{1}{2}\int \frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}c}-\frac{a+b\text{arcsinh}(cx)}{3\pi c^2(\pi c^2x^2+\pi)^{3/2}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (b*(x/(2*(1 +
 c^2*x^2)) + ArcTan[c*x]/(2*c)))/(3*c*Pi^(5/2))
```

#### 3.105.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.105.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

| method | result | size |
|---|---|---|
| default | $-\frac{a}{3\pi c^{2}(\pi c^{2}x^{2}+\pi)^{\frac{3}{2}}}+b\left(-\frac{-cx\sqrt{c^{2}x^{2}+1}+2\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^{2}x^{2}+1)^{\frac{3}{2}}c^{2}}+\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}+i\right)}{6c^{2}\pi^{\frac{5}{2}}}-\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}-i\right)}{6c^{2}\pi^{\frac{5}{2}}}\right)$ | 121 |
| parts | $-\frac{a}{3\pi c^{2}(\pi c^{2}x^{2}+\pi)^{\frac{3}{2}}}+b\left(-\frac{-cx\sqrt{c^{2}x^{2}+1}+2\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^{2}x^{2}+1)^{\frac{3}{2}}c^{2}}+\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}+i\right)}{6c^{2}\pi^{\frac{5}{2}}}-\frac{i\ln\left(cx+\sqrt{c^{2}x^{2}+1}-i\right)}{6c^{2}\pi^{\frac{5}{2}}}\right)$ | 121 |

input
```
int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+b*(-1/6/Pi^(5/2)/(c^2*x^2+1)^(3/2)*(-c
*x*(c^2*x^2+1)^(1/2)+2*arcsinh(c*x))/c^2+1/6*I/c^2/Pi^(5/2)*ln(c*x+(c^2*x^
2+1)^(1/2)+I)-1/6*I/c^2/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I))
```

### 3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(63) = 126.

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.20

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx =$$

$$-\frac{\sqrt{\pi}(bc^4x^4+2\,bc^2x^2+b)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{\pi}}{12\left(\pi^3c^6x^4+2\,\pi^3c^4x^2+\pi^3c^2\right)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas"
)
```

output
```
-1/12*(sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi +
 pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*(sqrt(c^2*x
^2 + 1)*b*c*x - 2*a))/(pi^3*c^6*x^4 + 2*pi^3*c^4*x^2 + pi^3*c^2)
```

## 3.105.6 Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{ax}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx\,\text{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a*x/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2
+ 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x*asinh(c*x)/(c**4*x**4*sqrt(
c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x
))/pi**(5/2)
```

## 3.105.7 Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima"
)
```

output
```
b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x) - 1
/3*a/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
```

### 3.105.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(5/2), x)
```

### 3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{x\,(a+b\,\text{asinh}(c\,x))}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.106 $\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.106.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{b}{6c\pi^{5/2}(1+c^2x^2)} + \frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{b\log(1+c^2x^2)}{3c\pi^{5/2}}$$

output
```
1/6*b/c/Pi^(5/2)/(c^2*x^2+1)+1/3*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(
3/2)-1/3*b*ln(c^2*x^2+1)/c/Pi^(5/2)+2/3*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*
x^2+Pi)^(1/2)
```

## 3.106.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\,\text{arcsinh}(cx)-2b(1+c^2x^2)^{3/2}\log(}{6c\pi^{5/2}(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*c*x*(3 + 2*c^2*x^2)*Arc
Sinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(6*c*Pi^(5/2)*(1 + c
^2*x^2)^(3/2))
```

### 3.106.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi}-\frac{bc\int \frac{x}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}$$

$$\Big\downarrow \text{6202}$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{bc\int \frac{x}{c^2x^2+1}dx}{\pi^{3/2}}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log(c^2x^2+1)}{2\pi^{3/2}c}\right)}{3\pi}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
b/(6*c*Pi^(5/2)*(1 + c^2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*
Pi*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) -
 (b*Log[1 + c^2*x^2])/(2*c*Pi^(3/2))))/(3*Pi)
```

### 3.106.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

## 3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(92) = 184$.

Time = 0.20 (sec) , antiderivative size = 619, normalized size of antiderivative = 5.73

| method | result |
|---|---|
| default | $a\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)+\frac{4b\,\operatorname{arcsinh}(cx)}{3c\,\pi^{\frac{5}{2}}}+\frac{2b\,c^7x^8}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)^2}-\frac{2b\,c^5x^6}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)}-\frac{2b}{\pi^{\frac{5}{2}}(3}$ |
| parts | $a\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)+\frac{4b\,\operatorname{arcsinh}(cx)}{3c\,\pi^{\frac{5}{2}}}+\frac{2b\,c^7x^8}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)^2}-\frac{2b\,c^5x^6}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)}-\frac{2b}{\pi^{\frac{5}{2}}(3}$ |

input
```
int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))+4/3*b/
c/Pi^(5/2)*arcsinh(c*x)+2/3*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8
-2/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-2*b/Pi^(5/2)*c^5/(3*c^2*
x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+2*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*
x^2+1)^(3/2)*arcsinh(c*x)*x^5+8/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)
^2*x^6-2*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4-20/3*b/Pi^(5/2)*c^3/
(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4+17/3*b/Pi^(5/2)*c^2/(3*c^2*x^
2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3+4*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^
2*x^2+1)^2*x^4-3/2*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-22/3*b/Pi^(5
/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4*b/Pi^(5/2)/(3*c^2*x^2
+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x+8/3*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x
^2+1)^2*x^2-8/3*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)+2/3*
b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2-2/3*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)
```

### 3.106.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^6 + 3*pi^3
*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

### 3.106.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{a}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 +
1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**4*sqrt(c**2
*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/p
i**(5/2)
```

### 3.106.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{1}{6}bc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2}-\frac{2\log(c^2x^2+1)}{\pi^{\frac{5}{2}}c^2}\right) + \frac{1}{3}b\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{2x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\text{arsinh}(cx) + \frac{1}{3}a\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}}+\frac{2x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

output
```
1/6*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/2
)*c^2)) + 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi*c
^2*x^2)))*arcsinh(c*x) + 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2
*sqrt(pi + pi*c^2*x^2)))
```

### 3.106.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(5/2), x)
```

### 3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.107 $\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.107.1 Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bcx}{6\pi^{5/2}(1+c^2x^2)} + \frac{a+b\text{arcsinh}(cx)}{3\pi(\pi+c^2\pi x^2)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\arctan(cx)}{6\pi^{5/2}} - \frac{2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} - \frac{b\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} + \frac{b\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}}$$

output
```
-1/6*b*c*x/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^
(3/2)-7/6*b*arctan(c*x)/Pi^(5/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2
+1)^(1/2))/Pi^(5/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)+b*polylog
(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)
^(1/2)
```

## 3.107.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.41

$$\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\frac{2a}{(1+c^2x^2)^{3/2}} - \frac{bcx}{1+c^2x^2} + \frac{6a}{\sqrt{1+c^2x^2}} + \frac{8b\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}} + \frac{6bc^2x^2\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}} - 14b\arctan\left(\tan\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
((2*a)/(1 + c^2*x^2)^(3/2) - (b*c*x)/(1 + c^2*x^2) + (6*a)/Sqrt[1 + c^2*x^
2] + (8*b*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (6*b*c^2*x^2*ArcSinh[c*x])/(
1 + c^2*x^2)^(3/2) - 14*b*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*b*ArcSinh[c*x]*
Log[1 - E^(-ArcSinh[c*x])] - 6*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] +
 6*a*Log[x] - 6*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 6*b*PolyLog[2, -E^(-Ar
cSinh[c*x])] - 6*b*PolyLog[2, E^(-ArcSinh[c*x])])/(6*Pi^(5/2))
```

### 3.107.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6226, 215, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(\pi c^2x^2+\pi\right)^{5/2}}dx$$

$$\Big\downarrow \text{6226}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2\pi x^2+\pi)^{3/2}}dx}{\pi}-\frac{bc\int \frac{1}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2\pi x^2+\pi)^{3/2}}dx}{\pi}-\frac{bc\left(\frac{1}{2}\int \frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2\pi x^2+\pi)^{3/2}}dx}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \text{6226}$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}-\frac{bc\int \frac{1}{c^2x^2+1}dx}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \; 6231$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \; 4670$$

$$\frac{\frac{i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)}{\pi^{3/2}}+\frac{a+b\text{arc}}{\pi\sqrt{\pi}}}{\pi}$$
$$\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+ba}{\pi^{3/2}}}{\pi}$$
$$\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

$$\Big\downarrow \; 2838$$

---

3.107. $\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx$

$$\frac{\frac{i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(}{\pi^{3/2}}}{\pi}$$

$$\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
(a + b*ArcSinh[c*x])/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (b*c*(x/(2*(1 + c^2*
x^2)) + ArcTan[c*x]/(2*c)))/(3*Pi^(5/2)) + ((a + b*ArcSinh[c*x])/(Pi*Sqrt[
Pi + c^2*Pi*x^2]) - (b*ArcTan[c*x])/Pi^(3/2) + (I*((2*I)*(a + b*ArcSinh[c*
x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLo
g[2, E^ArcSinh[c*x]]))/Pi^(3/2))/Pi
```

### 3.107.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.107. $\int \frac{a+b\text{arcsinh}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.107.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| default | $a\left(\frac{1}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)+b\left(\frac{6\operatorname{arcsinh}(cx)c^2x^2-cx\sqrt{c^2x^2+1}+8\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}}-\frac{7\operatorname{arctan}}{}\right.$ |
| parts | $a\left(\frac{1}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)+b\left(\frac{6\operatorname{arcsinh}(cx)c^2x^2-cx\sqrt{c^2x^2+1}+8\operatorname{arcsinh}(cx)}{6\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}}-\frac{7\operatorname{arctan}}{}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3/Pi/(Pi*c^2*x^2+Pi)^(3/2)+1/Pi*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2
)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))))+b*(1/6/Pi^(5/2)/(c^2*x^2+1)^(3
/2)*(6*arcsinh(c*x)*c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x))-7/3/Pi^(
5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))-1/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2
))-1/Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/Pi^(5/2)*dilog(c*
x+(c^2*x^2+1)^(1/2)))
```

### 3.107.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^7 + 3*pi^3
*c^4*x^5 + 3*pi^3*c^2*x^3 + pi^3*x), x)
```

### 3.107.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\,(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{a}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a/(c**4*x**5*sqrt(c**2*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 +
1) + x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**5*sqrt(c*
*2*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x
))/pi**(5/2)
```

### 3.107.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*a*(3*arcsinh(1/(c*abs(x)))/pi^(5/2) - 1/(pi*(pi + pi*c^2*x^2)^(3/2))
- 3/(pi^2*sqrt(pi + pi*c^2*x^2))) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1
))/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

### 3.107.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

### 3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

# 3.108 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.108.1 Optimal result

Integrand size = 26, antiderivative size = 150

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{bc}{6\pi^{5/2}(1+c^2x^2)}-\frac{a+b\text{arcsinh}(cx)}{\pi x(\pi+c^2\pi x^2)^{3/2}} -\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{bc\log(x)}{\pi^{5/2}}+\frac{5bc\log(1+c^2x^2)}{6\pi^{5/2}}$$

output
```
-1/6*b*c/Pi^(5/2)/(c^2*x^2+1)+(-a-b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(3/
2)-4/3*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+b*c*ln(x)/Pi^(5/2
)+5/6*b*c*ln(c^2*x^2+1)/Pi^(5/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2
*x^2+Pi)^(1/2)
```

## 3.108.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{-bcx\sqrt{1+c^2x^2}-2a(3+12c^2x^2+8c^4x^4)-2b(3+12c^2x^2+8c^4x^4)\,\text{arcsinh}(cx)-}{6\pi^{5/2}x(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
(-(b*c*x*Sqrt[1 + c^2*x^2]) - 2*a*(3 + 12*c^2*x^2 + 8*c^4*x^4) - 2*b*(3 +
12*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + b*c*x*(1 + c^2*x^2)^(3/2)*(16 + 6*L
og[x] + 5*Log[1 + c^2*x^2]))/(6*Pi^(5/2)*x*(1 + c^2*x^2)^(3/2))
```

### 3.108.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6219, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi c^2x^2+\pi\right)^{5/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{8c^4x^4+12c^2x^2+3}{3\pi^3x\left(c^2x^2+1\right)^2}dx-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int\frac{8c^4x^4+12c^2x^2+3}{x\left(c^2x^2+1\right)^2}dx}{3\pi^{5/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{bc\int\frac{8c^4x^4+12c^2x^2+3}{x^2\left(c^2x^2+1\right)^2}dx^2}{6\pi^{5/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{bc\int\left(\frac{5c^2}{c^2x^2+1}+\frac{c^2}{\left(c^2x^2+1\right)^2}+\frac{3}{x^2}\right)dx^2}{6\pi^{5/2}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)}{6\pi^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcSinh[c*x])/(Pi*x*(Pi + c^2*Pi*x^2)^(3/2))) - (4*c^2*x*(a + b*A
rcSinh[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x
]))/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*(-(1 + c^2*x^2)^(-1) + 3*Log[x^2
] + 5*Log[1 + c^2*x^2]))/(6*Pi^(5/2))
```

### 3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

### 3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(132) = 264$.

Time = 0.18 (sec) , antiderivative size = 778, normalized size of antiderivative = 5.19

| method | result |
|---|---|
| default | $a\left(-\frac{1}{\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)\right)-\frac{16bc\,\mathrm{arcsinh}(cx)}{3\pi^{\frac{5}{2}}}+\frac{32b\,x^{10}c^{11}}{3\pi^{\frac{5}{2}}(8c^2x^2+9)(c^2x^2+1)^2}-$ |
| parts | $a\left(-\frac{1}{\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)\right)-\frac{16bc\,\mathrm{arcsinh}(cx)}{3\pi^{\frac{5}{2}}}+\frac{32b\,x^{10}c^{11}}{3\pi^{\frac{5}{2}}(8c^2x^2+9)(c^2x^2+1)^2}-$ |

input
```
int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)-4*c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3
/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)))-16/3*b*c/Pi^(5/2)*arcsinh(c*x)+32/3*b/Pi^(
5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^10*c^11-32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(
c^2*x^2+1)*x^8*c^9+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^8*c^9-32
*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^6*c^7+64/3*b/Pi^(5/2)/(8*c^2*x^2+9
)/(c^2*x^2+1)^2*x^6*arcsinh(c*x)*c^7-64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)*x^5*arcsinh(c*x)*c^6+64*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*
x^6*c^7-32*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^4*c^5+200/3*b/Pi^(5/2)/(
8*c^2*x^2+9)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^5-56*b/Pi^(5/2)/(8*c^2*x^2+9
)/(c^2*x^2+1)^(3/2)*x^3*arcsinh(c*x)*c^4+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c
^2*x^2+1)^2*x^4*c^5-12*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^2*c^3+208/3*
b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^3-44*b/Pi^(5/2)/
(8*c^2*x^2+9)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^2+32/3*b/Pi^(5/2)/(8*c^2*
x^2+9)/(c^2*x^2+1)^2*x^2*c^3-3/2*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*c+24
*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*arcsinh(c*x)*c-9*b/Pi^(5/2)/(8*c^2
*x^2+9)/(c^2*x^2+1)^(3/2)/x*arcsinh(c*x)+5/3*b*c/Pi^(5/2)*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)+b*c/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

### 3.108.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^8 + 3*pi^3
*c^4*x^6 + 3*pi^3*c^2*x^4 + pi^3*x^2), x)
```

### 3.108.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\frac{\int \frac{a}{c^4x^6\sqrt{c^2x^2+1}+2c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\left(cx\right)}{c^4x^6\sqrt{c^2x^2+1}+2c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a/(c**4*x**6*sqrt(c**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 +
1) + x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**6*sqrt
(c**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 +
1)), x))/pi**(5/2)
```

### 3.108.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a*(4*c^2*x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 8*c^2*x/(pi^2*sqrt(pi + pi*
c^2*x^2)) + 3/(pi*(pi + pi*c^2*x^2)^(3/2)*x)) + b*integrate(log(c*x + sqrt
(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

### 3.108.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (\pi + c^2\pi x^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(\pi + \pi c^2x^2)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

### 3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (\pi + c^2\pi x^2)^{5/2}} \, dx = \int \frac{a + b \operatorname{asinh}(c\, x)}{x^2 (\Pi\, c^2\, x^2 + \Pi)^{5/2}} \, dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

# 3.109 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.109.1 Optimal result

Integrand size = 26, antiderivative size = 247

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx &= -\frac{3bc}{4\pi^{5/2}x}+\frac{bc}{4\pi^{5/2}x(1+c^2x^2)}+\frac{5bc^3x}{12\pi^{5/2}(1+c^2x^2)} \\ &\quad -\frac{5c^2(a+b\text{arcsinh}(cx))}{6\pi(\pi+c^2\pi x^2)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2(\pi+c^2\pi x^2)^{3/2}}-\frac{5c^2(a+b\text{arcsinh}(cx))}{2\pi^2\sqrt{\pi+c^2\pi x^2}} \\ &\quad +\frac{13bc^2\arctan(cx)}{6\pi^{5/2}}+\frac{5c^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\pi^{5/2}} \\ &\quad +\frac{5bc^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}}-\frac{5bc^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\pi^{5/2}}\end{aligned}$$

output
```
-3/4*b*c/Pi^(5/2)/x+1/4*b*c/Pi^(5/2)/x/(c^2*x^2+1)+5/12*b*c^3*x/Pi^(5/2)/(
c^2*x^2+1)-5/6*c^2*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+1/2*(-a-b*a
rcsinh(c*x))/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)+13/6*b*c^2*arctan(c*x)/Pi^(5/2)+
5*c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+5/2*b*c^2
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x
^2+1)^(1/2))/Pi^(5/2)-5/2*c^2*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/2
)
```

### 3.109.2 Mathematica [A] (verified)

Time = 4.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.34

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{-\frac{8ac^2}{(1+c^2x^2)^{3/2}}+\frac{4bc^3x}{1+c^2x^2}-\frac{48ac^2}{\sqrt{1+c^2x^2}}-\frac{12a\sqrt{1+c^2x^2}}{x^2}-\frac{56bc^2\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}-\frac{48bc^4x^2\text{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
((-8*a*c^2)/(1 + c^2*x^2)^(3/2) + (4*b*c^3*x)/(1 + c^2*x^2) - (48*a*c^2)/S
qrt[1 + c^2*x^2] - (12*a*Sqrt[1 + c^2*x^2])/x^2 - (56*b*c^2*ArcSinh[c*x])/
(1 + c^2*x^2)^(3/2) - (48*b*c^4*x^2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + 10
4*b*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*b*c^2*Coth[ArcSinh[c*x]/2] - 3*b*
c^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*b*c^2*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] + 60*b*c^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*a
*c^2*Log[x] + 60*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] - 60*b*c^2*PolyLog[
2, -E^(-ArcSinh[c*x])] + 60*b*c^2*PolyLog[2, E^(-ArcSinh[c*x])] + 6*b*c^2*
Tanh[ArcSinh[c*x]/2] - (6*b*c*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2])/x)/(24*Pi
^(5/2))
```

### 3.109.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {6224, 253, 264, 216, 6226, 215, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi c^2x^2+\pi\right)^{5/2}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2\pi x^2+\pi\right)^{5/2}}dx+\frac{bc\int \frac{1}{x^2\left(c^2x^2+1\right)^2}dx}{2\pi^{5/2}}-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{5}{2}c^2\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(c^2\pi x^2+\pi\right)^{5/2}}dx+\frac{bc\left(\frac{3}{2}\int \frac{1}{x^2(c^2x^2+1)}dx+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 264

$$-\frac{5}{2}c^2\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(c^2\pi x^2+\pi\right)^{5/2}}dx+\frac{bc\left(\frac{3}{2}\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 216

$$-\frac{5}{2}c^2\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(c^2\pi x^2+\pi\right)^{5/2}}dx-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\Big\downarrow$ 6226

$$-\frac{5}{2}c^2\left(\frac{\int \frac{a+b\operatorname{arcsinh}(cx)}{x(c^2\pi x^2+\pi)^{3/2}}dx}{\pi}-\frac{bc\int \frac{1}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{a+b\operatorname{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}\right)-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\Big\downarrow$ 215

$$-\frac{5}{2}c^2\left(\frac{\int \frac{a+b\operatorname{arcsinh}(cx)}{x(c^2\pi x^2+\pi)^{3/2}}dx}{\pi}-\frac{bc\left(\frac{1}{2}\int \frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}+\frac{a+b\operatorname{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}\right)-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\Big\downarrow$ 216

$$-\frac{5}{2}c^2\left(\frac{\int \frac{a+b\operatorname{arcsinh}(cx)}{x(c^2\pi x^2+\pi)^{3/2}}dx}{\pi}+\frac{a+b\operatorname{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}\right)-\frac{a+b\operatorname{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\Big\downarrow$ 6226

3.109. $\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

$$-\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}-\frac{bc\int \frac{1}{c^2x^2+1}dx}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\downarrow$ 216

$$-\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2\pi x^2+\pi}}dx}{\pi}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\downarrow$ 6231

$$-\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\downarrow$ 3042

$$-\frac{5}{2}c^2\left(\frac{\frac{\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\downarrow$ 26

$$-\frac{5}{2}c^2\left(\frac{\frac{i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\arctan(cx)}{\pi^{3/2}}}{\pi}+\frac{a+b\text{arcsinh}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3\pi^{5/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

---

3.109. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

$\Big\downarrow$ 4670

$$-\frac{5}{2}c^2\left(\frac{\frac{i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)}{\pi^{3/2}}\right.}{\pi}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\Big\downarrow$ 2715

$$-\frac{5}{2}c^2\left(\frac{\frac{i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.}{\pi^{3/2}}}{\pi}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

$\Big\downarrow$ 2838

$$-\frac{5}{2}c^2\left(\frac{\frac{i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{\pi^{3/2}}+\frac{a+b\text{arcsinh}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-}{\pi}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2\pi^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])/(Pi*x^2*(Pi + c^2*Pi*x^2)^(3/2)) + (b*c*(1/(2*x*
(1 + c^2*x^2)) + (3*(-x^(-1) - c*ArcTan[c*x]))/2))/(2*Pi^(5/2)) - (5*c^2*(
(a + b*ArcSinh[c*x])/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (b*c*(x/(2*(1 + c^2*
x^2)) + ArcTan[c*x]/(2*c)))/(3*Pi^(5/2)) + ((a + b*ArcSinh[c*x])/(Pi*Sqrt[
Pi + c^2*Pi*x^2]) - (b*ArcTan[c*x])/Pi^(3/2) + (I*((2*I)*(a + b*ArcSinh[c*
x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLo
g[2, E^ArcSinh[c*x]]))/Pi^(3/2))/Pi))/2
```

3.109. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

### 3.109.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

## 3.109. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.109. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

## 3.109.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.27

| method | result |
|---|---|
| default | $a\left(-\frac{1}{2\pi x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{5c^2\left(\frac{1}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)}{2}\right)-\frac{5b\,x^2\operatorname{arcsinh}(cx)c^4}{2\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}}-\frac{b\,c^3}{3\pi^{\frac{5}{2}}(c^2}$ |
| parts | $a\left(-\frac{1}{2\pi x^2(\pi c^2x^2+\pi)^{\frac{3}{2}}}-\frac{5c^2\left(\frac{1}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{\frac{1}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{3}{2}}}}{\pi}\right)}{2}\right)-\frac{5b\,x^2\operatorname{arcsinh}(cx)c^4}{2\pi^{\frac{5}{2}}(c^2x^2+1)^{\frac{3}{2}}}-\frac{b\,c^3}{3\pi^{\frac{5}{2}}(c^2}$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)-5/2*c^2*(1/3/Pi/(Pi*c^2*x^2+Pi)^(3/2)
+1/Pi*(1/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+
Pi)^(1/2)))))-5/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*x^2*arcsinh(c*x)*c^4-1/3*b*
c^3*x/Pi^(5/2)/(c^2*x^2+1)-10/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*
c^2-1/2*b*c/Pi^(5/2)/x/(c^2*x^2+1)-1/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/x^2*ar
csinh(c*x)+13/3*b*c^2/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^
(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*dilog(c*x+(c^2*x^2+1)^(1/2))
```

## 3.109.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^9 + 3*pi^3
*c^4*x^7 + 3*pi^3*c^2*x^5 + pi^3*x^3), x)
```

### 3.109.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\frac{\int \frac{a}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a/(c**4*x**7*sqrt(c**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 +
1) + x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**7*sqrt
(c**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 +
1)), x))/pi**(5/2)
```

### 3.109.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxim
a")
```

output
```
1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/pi^(5/2) - 5*c^2/(pi*(pi + pi*c^2*x^2)
^(3/2)) - 15*c^2/(pi^2*sqrt(pi + pi*c^2*x^2)) - 3/(pi*(pi + pi*c^2*x^2)^(3
/2)*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5
/2)*x^3), x)
```

### 3.109.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^3), x)
```

### 3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

# 3.110 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.110.1 Optimal result

Integrand size = 26, antiderivative size = 208

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx &= -\frac{bc}{6\pi^{5/2}x^2}+\frac{bc^3}{6\pi^{5/2}(1+c^2x^2)} \\ &\quad -\frac{a+b\text{arcsinh}(cx)}{3\pi x^3(\pi+c^2\pi x^2)^{3/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x(\pi+c^2\pi x^2)^{3/2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}} \\ &\quad +\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{8bc^3\log(x)}{3\pi^{5/2}}-\frac{4bc^3\log(1+c^2x^2)}{3\pi^{5/2}}\end{aligned}$$

output
```
-1/6*b*c/Pi^(5/2)/x^2+1/6*b*c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x
))/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)+2*c^2*(a+b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+
Pi)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)-8/3*b*c^3*
ln(x)/Pi^(5/2)-4/3*b*c^3*ln(c^2*x^2+1)/Pi^(5/2)+16/3*c^4*x*(a+b*arcsinh(c*
x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.110.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4 (\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{-2a+12ac^2x^2+48ac^4x^4+32ac^6x^6-bcx\sqrt{1+c^2x^2}-32bc^3x^3\sqrt{1+c^2x^2}-32bc$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
(-2*a + 12*a*c^2*x^2 + 48*a*c^4*x^4 + 32*a*c^6*x^6 - b*c*x*Sqrt[1 + c^2*x^
2] - 32*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 32*b*c^5*x^5*Sqrt[1 + c^2*x^2] + 2*b
*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x] - 16*b*c^3*x^3*(1
 + c^2*x^2)^(3/2)*Log[x] - 8*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2]
- 8*b*c^5*x^5*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(6*Pi^(5/2)*x^3*(1 + c^2
*x^2)^(3/2))
```

## 3.110.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6219, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4 (\pi c^2x^2+\pi)^{5/2}}\,dx$$

$$\Big\downarrow \text{6219}$$

$$-\sqrt{\pi}bc\int -\frac{-16c^6x^6-24c^4x^4-6c^2x^2+1}{3\pi^3x^3(c^2x^2+1)^2}dx+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x(\pi c^2x^2+\pi)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3(\pi c^2x^2+\pi)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{-16c^6x^6-24c^4x^4-6c^2x^2+1}{x^3(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x(\pi c^2x^2+\pi)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3(\pi c^2x^2+\pi)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \; 2331$$

$$\frac{bc\int \frac{-16c^6x^6-24c^4x^4-6c^2x^2+1}{x^4(c^2x^2+1)^2}dx^2}{6\pi^{5/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \; 2123$$

$$\frac{bc\int \left(-\frac{8c^4}{c^2x^2+1}-\frac{c^4}{(c^2x^2+1)^2}-\frac{8c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6\pi^{5/2}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2c^2(a+b\text{arcsinh}(cx))}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc\left(\frac{c^2}{c^2x^2+1}-8c^2\log\left(x^2\right)-8c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)}{6\pi^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)) + (2*c^2*(a + b
*ArcSinh[c*x]))/(Pi*x*(Pi + c^2*Pi*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSinh[c
*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x]))/(3*
Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*(-x^(-2) + c^2/(1 + c^2*x^2) - 8*c^2*Lo
g[x^2] - 8*c^2*Log[1 + c^2*x^2]))/(6*Pi^(5/2))
```

### 3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

### 3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(181) = 362$.

Time = 0.17 (sec) , antiderivative size = 1155, normalized size of antiderivative = 5.55

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1155 |
| parts | Expression too large to display | 1155 |

input
```
int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*c^3+128/3*b/Pi^(5/2)/(
12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^4*c^7-2*b/Pi^(5/2)/(12*c^4*x^4+12*c
^2*x^2-1)/(c^2*x^2+1)*x^2*c^5+1/6*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^
2*x^2+1)/x^2*c+1/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/
x^3*arcsinh(c*x)+128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^
10*c^13+128*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^8*c^11+128*
b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^6*c^9+16/3*b/Pi^(5/2)/(
12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*arcsinh(c*x)*c^3-128/3*b/Pi^(5/2)/(
12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^4*c^7-128/3*b/Pi^(5/2)/(12*c^4*x^
4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^12*c^15-512/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^
2*x^2-1)/(c^2*x^2+1)^2*x^10*c^13-256*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/
(c^2*x^2+1)^2*x^8*c^11-512/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2
+1)^2*x^6*c^9+a*(-1/3/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)-2*c^2*(-1/Pi/x/(Pi*c^2*
x^2+Pi)^(3/2)-4*c^2*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2
+Pi)^(1/2))))+32/3*b*c^3/Pi^(5/2)*arcsinh(c*x)-8/3*b*c^3/Pi^(5/2)*ln((c*x+
(c^2*x^2+1)^(1/2))^4-1)-560/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^
2+1)^2*x^4*arcsinh(c*x)*c^7-160/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^
2*x^2+1)^2*x^2*arcsinh(c*x)*c^5-64*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c
^2*x^2+1)^2*x^8*arcsinh(c*x)*c^11-192*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)
/(c^2*x^2+1)^2*x^6*arcsinh(c*x)*c^9+64*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^...
```

### 3.110.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^10 + 3*pi^
3*c^4*x^8 + 3*pi^3*c^2*x^6 + pi^3*x^4), x)
```

## 3.110.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{\int \frac{a}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a/(c**4*x**8*sqrt(c**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 +
1) + x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**8*sqrt
(c**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 +
1)), x))/pi**(5/2)
```

## 3.110.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}} + \frac{16\,c^2\log\left(x\right)}{\pi^{\frac{5}{2}}} + \frac{1}{\pi^{\frac{5}{2}}c^2x^4+\pi^{\frac{5}{2}}x^2}\right)$$
$$+\frac{1}{3}\left(\frac{8\,c^4x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}} + \frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}} + \frac{6\,c^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x} - \frac{1}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x^3}\right)b\,\text{arsinh}\,(cx)$$
$$+\frac{1}{3}\left(\frac{8\,c^4x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}} + \frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}} + \frac{6\,c^2}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x} - \frac{1}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}x^3}\right)a$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/pi^(5/2) + 16*c^2*log(x)/pi^(5/2) + 1/(pi
^(5/2)*c^2*x^4 + pi^(5/2)*x^2)) + 1/3*(8*c^4*x/(pi*(pi + pi*c^2*x^2)^(3/2)
) + 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(pi*(pi + pi*c^2*x^2)^(3
/2)*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*b*arcsinh(c*x) + 1/3*(8*c^4*x
/(pi*(pi + pi*c^2*x^2)^(3/2)) + 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*
c^2/(pi*(pi + pi*c^2*x^2)^(3/2)*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*a
```

### 3.110.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^4), x)
```

### 3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^4\,(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

# 3.111 $\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$



## 3.111.1 Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}} - \frac{4\sqrt{1+a^2x^2}\log(1+a^2x^2)}{15ac^3\sqrt{c+a^2cx^2}}$$

output
```
1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^
2+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh
(a*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

## 3.111.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(4ax\sqrt{1+a^2x^2}(15+20a^2x^2+8a^4x^4)\,\text{arcsinh}(ax) - (1+a^2x^2)\left(-11-\right.\right.}{60ac^4(1+a^2x^2)^{7/2}}$$

input
```
Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4
)*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1
 + a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

### 3.111.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6203, 241, 6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{ 6203}$$

$$\frac{4\int \frac{\operatorname{arcsinh}(ax)}{(a^2cx^2+c)^{5/2}}dx}{5c}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\operatorname{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{4\int \frac{\operatorname{arcsinh}(ax)}{(a^2cx^2+c)^{5/2}}dx}{5c}+\frac{x\operatorname{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 6203}$$

$$\frac{4\left(\frac{2\int \frac{\operatorname{arcsinh}(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{x\operatorname{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\operatorname{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{4\left(\frac{2\int \frac{\operatorname{arcsinh}(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\operatorname{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{6ac^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)}{5c}+\frac{x\operatorname{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{4\left(\frac{2\left(\frac{x\text{arcsinh}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\int\frac{x}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{6ac^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)}{5c}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{240}$$

$$\frac{4\left(\frac{x\text{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\text{arcsinh}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{6ac^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)}{5c}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

output
```
1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5
*c*(c + a^2*c*x^2)^(5/2)) + (4*(1/(6*a*c^2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*
c*x^2]) + (x*ArcSinh[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*((x*ArcSinh[a*
x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c*
Sqrt[c + a^2*c*x^2])))/(3*c)))/(5*c)
```

### 3.111.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

3.111. $\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

### 3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(170) = 340$.

Time = 0.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| default | $\frac{16\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)}{15\sqrt{a^2x^2+1}\,a\,c^4} + \frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(-64a^8x^8-\right.}{}$ |

input
```
int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(
a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x^3-16*a^2
*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*a^8*x^8-64*(a^2*x^
2+1)^(1/2)*a^7*x^7-280*a^6*x^6-248*x^5*a^5*(a^2*x^2+1)^(1/2)+160*a^4*x^4*a
rcsinh(a*x)-456*a^4*x^4-340*a^3*x^3*(a^2*x^2+1)^(1/2)+380*a^2*x^2*arcsinh(
a*x)-328*a^2*x^2-165*a*x*(a^2*x^2+1)^(1/2)+256*arcsinh(a*x)-88)/(40*a^10*x
^10+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2
*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

### 3.111.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6
*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

## 3.111.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{(c(a^2x^2+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)
```

## 3.111.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{1}{60}a\left(\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}-\frac{16\log\left(x^2+\frac{1}{a^2}\right)}{a^2c^{\frac{7}{2}}}\right) + \frac{1}{15}\left(\frac{8x}{\sqrt{a^2cx^2+c}c^3}+\frac{4x}{(a^2cx^2+c)^{\frac{3}{2}}c^2}+\frac{3x}{(a^2cx^2+c)^{\frac{5}{2}}c}\right)\operatorname{arsinh}(ax)$$

input
```
integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^
4*c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1
/15*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x
/((a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

### 3.111.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{1}{60}\sqrt{c}\left(\frac{16\log(a^2x^2+1)}{ac^4} - \frac{24a^4x^4+56a^2x^2+35}{(a^2x^2+1)^2ac^4}\right) + \frac{\left(4\left(\frac{2a^4x^2}{c}+\frac{5a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\,(a^2cx^2+c)^{\frac{5}{2}}}$$

input
```
integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35
)/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x
*log(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

### 3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{(c\,a^2\,x^2+c)^{7/2}}\,dx$$

input
```
int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
```

output
```
int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

# 3.112 $\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx$



## 3.112.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} + \frac{3\operatorname{arcsinh}(ax)^2}{16a^5}$$

output
```
3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^
2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.112.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \frac{3a^2x^2 - a^4x^4 + 2ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\operatorname{arcsinh}(ax) + 3\operatorname{arcsinh}(ax)^2}{16a^5}$$

input
```
Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*
x] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

### 3.112.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int x^3dx}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{6227}$$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{3\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^4}{16a}$$

input
```
Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/16*x^4/a + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*(-1/4*x^2/
a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4*a^3)))/
(4*a^2)
```

#### 3.112.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.112.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{4a^3x^3 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}-a^4x^4-6 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\, ax+3a^2x^2+3 \operatorname{arcsinh}(ax)^2+3}{16a^5}$ | 74 |

input
```
int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-a^4*x^4-6*arcsinh(a*x)*(a^2
*x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+3)/a^5
```

## 3.112.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^4x^4 - 3\,a^2x^2 - 2\,(2\,a^3x^3 - 3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right) - 3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

input
```
integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a
*x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

## 3.112.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^4}{16a} + \frac{x^3\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{4a^2} + \frac{3x^2}{16a^3} - \frac{3x\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{8a^4} + \frac{3\,\text{asinh}^2\,(ax)}{16a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3
*x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*
x)**2/(16*a**5), Ne(a, 0)), (0, True))
```

## 3.112.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{16}\left(\frac{x^4}{a^2} - \frac{3\,x^2}{a^4} + \frac{3\,\text{arsinh}\,(ax)^2}{a^6}\right)a + \frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\,\sqrt{a^2x^2+1}x}{a^4} + \frac{3\,\text{arsinh}\,(ax)}{a^5}\right)\text{arsinh}\,(ax)$$

input
```
integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*
x)
```

### 3.112.8 Giac [F]

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.113 $\int \frac{x^3 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx$



## 3.113.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{x^3 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^2}$$

output
```
2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.113.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{x^3 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \frac{6ax - a^3x^3 + 3(-2+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^4}$$

input
```
Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4
)
```

### 3.113.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}$$

$$\Big\downarrow \text{ 6213}$$

$$-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}$$

input
```
Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/9*x^3/a + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2) - (2*(-(x/a) + (
Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2))/(3*a^2)
```

### 3.113.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.113.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

| method | result | size |
|---|---|---|
| default | $\frac{3a^4x^4 \operatorname{arcsinh}(ax)-3a^2x^2 \operatorname{arcsinh}(ax)-a^3x^3\sqrt{a^2x^2+1}-6 \operatorname{arcsinh}(ax)+6ax\sqrt{a^2x^2+1}}{9a^4\sqrt{a^2x^2+1}}$ | 82 |

input
```
int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9/a^4/(a^2*x^2+1)^(1/2)*(3*a^4*x^4*arcsinh(a*x)-3*a^2*x^2*arcsinh(a*x)-a
^3*x^3*(a^2*x^2+1)^(1/2)-6*arcsinh(a*x)+6*a*x*(a^2*x^2+1)^(1/2))
```

### 3.113.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^3x^3 - 3\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right) - 6\,ax}{9\,a^4}$$

input
```
integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
 1)) - 6*a*x)/a^4
```

### 3.113.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*
x/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, Tru
e))
```

### 3.113.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{9}a\left(\frac{x^3}{a^2} - \frac{6\,x}{a^4}\right) + \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}(ax)$$

input
```
integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x
^2 + 1)/a^4)*arcsinh(a*x)
```

### 3.113.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \int \frac{x^3 \, \text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}} \, dx$$

input
```
int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.114 $\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$



## 3.114.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x^2}{4a}+\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a^2}-\frac{\text{arcsinh}(ax)^2}{4a^3}$$

output
```
-1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.114.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^2x^2-2ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)+\text{arcsinh}(ax)^2}{4a^3}$$

input
```
Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

### 3.114.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}$$

input
```
Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(
4*a^3)
```

#### 3.114.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.114.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $-\frac{-2 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\, ax+a^2x^2+\operatorname{arcsinh}(ax)^2+1}{4a^3}$ | 40 |

input
```
int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

## 3.114.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{x^2 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^2x^2 - 2\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

input
```
integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log
(a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

### 3.114.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^2}{4a} + \frac{x\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{2a^2} - \frac{\text{asinh}^2\,(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh
(a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

### 3.114.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{4}\,a\left(\frac{x^2}{a^2} - \frac{\text{arsinh}\,(ax)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\text{arsinh}\,(ax)}{a^3}\right)\text{arsinh}\,(ax)$$

input
```
integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arc
sinh(a*x)/a^3)*arcsinh(a*x)
```

### 3.114.8 Giac [F]

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\mathrm{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.115 $\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$



## 3.115.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

output
```
-x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.115.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

input
```
Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

## 3.115.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}$$

input
```
Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

### 3.115.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.115.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

| method | result | size |
|---|---|---|
| default | $\frac{a^2x^2\,\mathrm{arcsinh}(ax)+\mathrm{arcsinh}(ax)-ax\sqrt{a^2x^2+1}}{a^2\sqrt{a^2x^2+1}}$ | 47 |

input
```
int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+arcsinh(a*x)-a*x*(a^2*x^2+1)
^(1/2))
```

### 3.115.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\mathrm{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{ax-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

input
```
integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

### 3.115.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x\mathrm{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\mathrm{asinh}\,(ax)}{a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True
))
```

### 3.115.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2}$$

input
```
integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

### 3.115.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

input
```
integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

### 3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.116 $\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx$



## 3.116.1 Optimal result

Integrand size = 18, antiderivative size = 13

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{\text{arcsinh}(ax)^2}{2a}$$

output
```
1/2*arcsinh(a*x)^2/a
```

## 3.116.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{\text{arcsinh}(ax)^2}{2a}$$

input
```
Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^2/(2*a)
```

## 3.116.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\text{arcsinh}(ax)^2}{2a}$$

input
```
Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^2/(2*a)
```

### 3.116.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.116.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{arcsinh}(ax)^2}{2a}$ | 12 |
| default | $\frac{\text{arcsinh}(ax)^2}{2a}$ | 12 |

input
```
int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(a*x)^2/a
```

### 3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,a}$$

input
```
integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

### 3.116.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

### 3.116.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arsinh}\,(ax)^2}{2\,a}$$

input
```
integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsinh(a*x)^2/a
```

### 3.116.8 Giac [F]

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.116.9 Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{asinh}(a\,x)^2}{2\,a}$$

input
```
int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
asinh(a*x)^2/(2*a)
```

# 3.117 $\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx$



## 3.117.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) + \text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

output
```
-2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^
(1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

## 3.117.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \text{arcsinh}(ax)\left(\log\left(1-e^{-\text{arcsinh}(ax)}\right) - \log\left(1+e^{-\text{arcsinh}(ax)}\right)\right) + \text{PolyLog}\left(2,-e^{-\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2,e^{-\text{arcsinh}(ax)}\right)$$

input
```
Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + P
olyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

### 3.117.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6231}$$

$$\int \frac{\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{3042}$$

$$\int i\text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{26}$$

$$i\int \text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{4670}$$

$$i\left(i\int \log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-i\int \log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$i\left(i\int e^{-\text{arcsinh}(ax)}\log\left(1-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}\log\left(1+e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}+2i\text{arcsin}\right.$$

$$\Big\downarrow \text{2838}$$

$$i\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\right)$$

input
```
Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
I*((2*I)*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + I*PolyLog[2, -E^ArcSinh[a*
x]] - I*PolyLog[2, E^ArcSinh[a*x]])
```

### 3.117.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.117.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.62

| method | result |
|---|---|
| default | $\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)-\operatorname{arcsinh}(ax)\ln\left(1+ax+\right.$ |

input
```
int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))-
arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

### 3.117.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

### 3.117.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

### 3.117.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.117.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.118 $\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$



## 3.118.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{x} + a\log(x)$$

output
```
a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

## 3.118.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{x} + a\log(ax)$$

input
```
Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

## 3.118.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{6215}$$
$$a\int \frac{1}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{x}$$
$$\Big\downarrow \text{14}$$
$$a\log(x)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{x}$$

input
```
Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

### 3.118.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

### 3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

| method | result | size |
|---|---|---|
| default | $-2a \operatorname{arcsinh}(ax) + \frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)}{x} + a\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$ | 56 |

input
```
int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^
2+1)^(1/2))^2-1)
```

### 3.118.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \frac{ax\log(x) - \sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

input
```
integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

### 3.118.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

### 3.118.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = a\log\left(x\right) - \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\left(ax\right)}{x}$$

input
```
integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

### 3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -a\log\left(-x|a|+\sqrt{a^2x^2+1}\right) + a\log\left(|x|\right) + \frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

input
```
integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
 sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

### 3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}(a\,x)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.119 $\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$



## 3.119.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - \frac{1}{2}a^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

output
```
-1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcs
inh(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

## 3.119.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(ax)\right) - \text{arcsinh}(ax)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) - 4\text{arcsinh}(ax)\log\left(1-e^{-\text{arcsinh}(ax)}\right) + 4\text{arcsinh}(ax)\log\left(1+e^{-\text{arcsinh}(ax)}\right) - 4\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(ax)}\right) + 4\,\text{PolyLog}\left(2,e^{-\text{arcsinh}(ax)}\right) - \text{arcsinh}(ax)\text{sech}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) + 2\tanh\left(\frac{1}{2}\text{arcsinh}(ax)\right)\right)$$

input
```
Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

output
```
(a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Ar
cSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh
[a*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x]
)] - ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

## 3.119.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6224, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx+\frac{1}{2}a\int \frac{1}{x^2}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 6231}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}a^2\int i\text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{2}ia^2\int \text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{1}{2}ia^2\left(i\int \log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-i\int \log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2i\text{arcsinh}(ax)\text{arctanh}\left(e\right.\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{1}{2}ia^2\left(i\int e^{-\text{arcsinh}(ax)}\log\left(1-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}\log\left(1+e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}+2ia\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{1}{2}ia^2\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\right)-$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

input
```
Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

output
```
-1/2*a/x - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) - (I/2)*a^2*((2*I)*Arc
Sinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + I*PolyLog[2, -E^ArcSinh[a*x]] - I*Poly
Log[2, E^ArcSinh[a*x]])
```

### 3.119.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.119.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| default | $-\frac{a^{2} x^{2} \operatorname{arcsinh}(ax)+ax\sqrt{a^{2}x^{2}+1}+\operatorname{arcsinh}(ax)}{2\sqrt{a^{2}x^{2}+1}\,x^{2}}-\frac{a^{2} \operatorname{arcsinh}(ax) \ln\left(1-ax-\sqrt{a^{2}x^{2}+1}\right)}{2}-\frac{a^{2} \operatorname{polylog}\left(2,ax+\sqrt{a^{2}x^{2}+1}\right)}{2}+\frac{a^{2} \mathrm{a}}{}$ |

input
```
int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+a*x*(a^2*x^2+1)^(1/2)+arcsinh
(a*x))/x^2-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylo
g(2,a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2)
)+1/2*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

## 3.119.5 Fricas [F]

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

## 3.119.6 Sympy [F]

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}\,(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

## 3.119.7 Maxima [F]

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.119.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.120 $\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.120.1 Optimal result

Integrand size = 26, antiderivative size = 175

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d}+\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2}$$

output
```
-1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^4/d+1/5*(c^2*d*x^2+d)^(5/2)*
(a+b*arcsinh(c*x))/c^4/d^2+2/15*b*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1
/2)-1/45*b*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/25*b*c*x^5*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

## 3.120.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$

$$=\frac{\sqrt{d+c^2dx^2}\left(15a(1+c^2x^2)^2(-2+3c^2x^2)+bcx\sqrt{1+c^2x^2}(30-5c^2x^2-9c^4x^4)+15b(1+c^2x^2)^2(-2+3\right.}{225c^4(1+c^2x^2)}$$

input
```
Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) + b*c*x*Sqrt[1
 + c^2*x^2]*(30 - 5*c^2*x^2 - 9*c^4*x^4) + 15*b*(1 + c^2*x^2)^2*(-2 + 3*c^
2*x^2)*ArcSinh[c*x]))/(225*c^4*(1 + c^2*x^2))
```

### 3.120.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6219, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{-3c^4x^4-c^2x^2+2}{15c^4}dx}{\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{c^2dx^2+d}\int \left(-3c^4x^4-c^2x^2+2\right)dx}{15c^3\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d}+\frac{b\left(-\frac{3}{5}c^4x^5-\frac{c^2x^3}{3}+2x\right)\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}$$

input
```
Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(b*Sqrt[d + c^2*d*x^2]*(2*x - (c^2*x^3)/3 - (3*c^4*x^5)/5))/(15*c^3*Sqrt[1
 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*d) + ((
d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d^2)
```

### 3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(149) = 298$.

Time = 0.27 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.30

| method | result |
|---|---|
| default | $a\left(\frac{x^2(c^2dx^2+d)^{\frac{3}{2}}}{5c^2d}-\frac{2(c^2dx^2+d)^{\frac{3}{2}}}{15dc^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx\right.}{800c^4(c^2x^2+1)}\right.$ |
| parts | $a\left(\frac{x^2(c^2dx^2+d)^{\frac{3}{2}}}{5c^2d}-\frac{2(c^2dx^2+d)^{\frac{3}{2}}}{15dc^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx\right.}{800c^4(c^2x^2+1)}\right.$ |

input
```
int(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b*(1/
800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*
x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1
+5*arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsi
nh(c*x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+
1)^(1/2)+1)*(-1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(
c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^4/(c^2*x^2+1)-1/288*(d
*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x
*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2
+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*
(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))
/c^4/(c^2*x^2+1))
```

### 3.120.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{15\,(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6+60\,ac^4x^4-15\,ac^2x^2-\right.}{225\,(c^6x^2+c^4)}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/225*(15*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 + 60*a*c^4*x^4 - 15*a*c^2*x
^2 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*x^2 + 1) - 30*a)*sqrt
(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

### 3.120.6 Sympy [F]

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int x^3\sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\,(cx))\,\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

### 3.120.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{15}\,b\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\text{arsinh}\,(cx)$$
$$+\frac{1}{15}\,a\left(\frac{3\,(c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)-\frac{\left(9\,c^4\sqrt{d}x^5+5\,c^2\sqrt{d}x^3-30\,\sqrt{d}x\right)b}{225\,c^3}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
1/15*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4
*d))*arcsinh(c*x) + 1/15*a*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d
*x^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x^3 -
30*sqrt(d)*x)*b/c^3
```

### 3.120.8 Giac [F(-2)]

Exception generated.

$$\int x^3 \sqrt{d + c^2 d x^2} (a + b\text{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.120.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{d + c^2 d x^2} (a + b\text{arcsinh}(cx)) \, dx = \int x^3 \left(a + b \operatorname{asinh}(c\,x)\right) \sqrt{d\,c^2\,x^2 + d} \, dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.121 $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.121.1 Optimal result

Integrand size = 26, antiderivative size = 181

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bx^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}$$

output
```
1/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)-1/16*b*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1
/16*b*c*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*(a+b*arcsinh(c*x))^
2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

## 3.121.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx =$$

$$-\frac{-16acx(1+2c^2x^2)\sqrt{d+c^2dx^2}+16a\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{b\sqrt{d+c^2dx^2}\left(8\text{arcsinh}(cx)^2+\cosh(4\text{arc}\right.}{}}{128c^3}$$

input
```
Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/128*(-16*a*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 16*a*Sqrt[d]*Log[c
*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*
x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1
 + c^2*x^2])/c^3
```

### 3.121.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6221, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6221}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^3dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6227}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

input
```
Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/16*(b*c*x^4*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x^3*Sqrt[d + c^2*
d*x^2]*(a + b*ArcSinh[c*x]))/4 + (Sqrt[d + c^2*d*x^2]*(-1/4*(b*x^2)/c + (x
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/
(4*b*c^3)))/(4*Sqrt[1 + c^2*x^2])
```

### 3.121.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.121.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(155) = 310$.

Time = 0.18 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.87

| method | result |
|---|---|
| default | $\frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{ax\sqrt{c^2dx^2+d}}{8c^2}-\frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c^3}+\frac{\sqrt{d(c^2x^2+1)}\,\left(8c^5x^5+8\right.}{}\right.$ |
| parts | $\frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{ax\sqrt{c^2dx^2+d}}{8c^2}-\frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c^3}+\frac{\sqrt{d(c^2x^2+1)}\,\left(8c^5x^5+8\right.}{}\right.$ |

input
```
int(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a/c^
2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/16*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/256*(d*(c^2*x^
2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c
^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+
1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c
^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c
*x))/c^3/(c^2*x^2+1))
```

### 3.121.5 Fricas [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

### 3.121.6 Sympy [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int x^2\sqrt{d(c^2x^2+1)}(a+b\operatorname{asinh}(cx))\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

### 3.121.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.121.8 Giac [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^2, x)
```

### 3.121.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.122 $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.122.1 Optimal result

Integrand size = 24, antiderivative size = 105

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d}$$

output
```
1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^2/d-1/3*b*x*(c^2*d*x^2+d)^(1/
2)/c/(c^2*x^2+1)^(1/2)-1/9*b*c*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

## 3.122.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{d+c^2dx^2}\left(3a(1+c^2x^2)^2-bcx\sqrt{1+c^2x^2}(3+c^2x^2)+3b(1+c^2x^2)^2\text{arcsinh}(cx)\right)}{9c^2(1+c^2x^2)}$$

input
```
Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + c
^2*x^2) + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(9*c^2*(1 + c^2*x^2))
```

## 3.122.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\sqrt{c^2dx^2+d}\int(c^2x^2+1)\,dx}{3c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{b\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}$$

input
```
Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/3*(b*Sqrt[d + c^2*d*x^2]*(x + (c^2*x^3)/3))/(c*Sqrt[1 + c^2*x^2]) + ((d
 + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2*d)
```

### 3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(89) = 178.

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.06

| method | result |
|---|---|
| default | $\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\,\mathrm{arcsinh}(cx))}{72c^2(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2\right.}{}\right.$ |
| parts | $\frac{a(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\,\mathrm{arcsinh}(cx))}{72c^2(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2\right.}{}\right.$ |

input
```
int(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(c^2*d*x^2+d)^(3/2)/c^2/d+b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4
*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcs
inh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+
1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/72*(d*(
c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(
c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^2/(c^2*x^2+1))
```

### 3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{3\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4+6\,ac^2x^2-\left(bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}\right.}{9\left(c^4x^2+c^2\right)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(3*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) + (3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^3 + 3*b*c*x)*sqrt(c^2*
x^2 + 1) + 3*a)*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

### 3.122.6 Sympy [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int x\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))\;dx$$

input
```
integrate(x*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

### 3.122.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{(c^2dx^2+d)^{\frac{3}{2}}b\,\text{arsinh}\,(cx)}{3\,c^2d}-\frac{\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)b}{9\,cd}+\frac{(c^2dx^2+d)^{\frac{3}{2}}a}{3\,c^2d}$$

input
```
integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(c^2*d) - 1/9*(c^2*d^(3/2)*x^3 +
3*d^(3/2)*x)*b/(c*d) + 1/3*(c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

### 3.122.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.122.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int x\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.123 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.123.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = -\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
1/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/4*b*c*x^2*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2
*x^2+1)^(1/2)
```

## 3.123.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \frac{1}{8}\left(4ax\sqrt{d+c^2dx^2}+\frac{4a\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{c} + \frac{b\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{c\sqrt{1+c^2x^2}}\right)$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(4*a*x*Sqrt[d + c^2*d*x^2] + (4*a*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2
*d*x^2]])/c + (b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*
x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2]))/8
```

## 3.123.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6200}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)
/(4*b*c*Sqrt[1 + c^2*x^2])
```

### 3.123.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

## 3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(95) = 190$.

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.31

| method | result |
|---|---|
| default | $\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+}{16c(c^2x^2+1)}$ |
| parts | $\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+}{16c(c^2x^2+1)}$ |

input
```
int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(c^2*d*x^2+d)^(1/2)+1/2*a*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)
^(1/2))/(c^2*d)^(1/2)+b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arc
sinh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1
/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/16*(d*(c^
2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(
1/2))*(1+2*arcsinh(c*x))/c/(c^2*x^2+1))
```

### 3.123.5 Fricas [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

### 3.123.6 Sympy [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))\,\,dx$$

input
```
integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

### 3.123.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.123.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.123.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.124 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx$



## 3.124.1 Optimal result

Integrand size = 26, antiderivative size = 177

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

output
```
(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-b*c*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.124.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx$$
$$= a\sqrt{d+c^2dx^2}+a\sqrt{d}\log(x)-a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)$$
$$+\frac{b\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\text{arcsinh}(cx)\log\left(1+e\right.\right.}{\sqrt{1+c^2x^2}}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
a*Sqrt[d + c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[
d + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E
^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSin
h[c*x])]))/Sqrt[1 + c^2*x^2]
```

### 3.124.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}\,dx$$
$$\Big\downarrow \; 6221$$
$$\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int 1dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))$$
$$\Big\downarrow \; 24$$
$$\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 6231$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 26$$

$$\frac{i\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 4670$$

$$\frac{i\sqrt{c^2dx^2+d}\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.}{\sqrt{c^2x^2+1}}$$
$$\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 2715$$

$$\frac{i\sqrt{c^2dx^2+d}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.}{\sqrt{c^2x^2+1}}$$
$$\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 2838$$

$$\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right.}{\sqrt{c^2x^2+1}}$$
$$\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

3.124. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx$

output
```
-((b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x]) + (I*Sqrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])*Arc
Tanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^
ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2]
```

### 3.124.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.124.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.87

| method | result |
|---|---|
| default | $-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)a+a\sqrt{c^2 d\,x^2+d}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x^2c^2}{c^2x^2+1}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{d(c^2x^2}{}$ |
| parts | $-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)a+a\sqrt{c^2 d\,x^2+d}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x^2c^2}{c^2x^2+1}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{d(c^2x^2}{}$ |

input
```
int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a+a*(c^2*d*x^2+d)^(1/2)
+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*c*x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(
c*x)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*
x^2+1)^(1/2))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2
*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1
+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))
```

### 3.124.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x, x)
```

### 3.124.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\operatorname{asinh}{(cx)})}{x}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x, x)
```

### 3.124.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
-(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a + b*integrate(sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.124.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x, x)
```

# 3.125 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.125.1 Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x} + \frac{c\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{1+c^2x^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}$$

output
```
-(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x+1/2*c*(a+b*arcsinh(c*x))^2*(c^2*
d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)
```

## 3.125.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{a\sqrt{d(1+c^2x^2)}}{x} + \frac{bc\sqrt{d(1+c^2x^2)}\left(-\frac{2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{cx}+\text{arcsinh}(cx)^2+2\log(cx)\right)}{2\sqrt{1+c^2x^2}} + ac\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-((a*Sqrt[d*(1 + c^2*x^2)])/x) + (b*c*Sqrt[d*(1 + c^2*x^2)]*((-2*Sqrt[1 +
c^2*x^2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/(2*Sqrt[1 + c
^2*x^2]) + a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]]
```

## 3.125.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6220, 14, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{6220}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\int \frac{1}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{14}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{c\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*
Log[x])/Sqrt[1 + c^2*x^2]
```

### 3.125.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

## 3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(93) = 186$.

Time = 0.21 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.39

| method | result |
|---|---|
| default | $-\frac{a(c^2 d x^2+d)^{\frac{3}{2}}}{dx}+a c^2 x\sqrt{c^2 d x^2+d}+\frac{a c^2 d \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{\sqrt{c^2 d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2 c}{2\sqrt{c^2x^2+1}}-\frac{2\sqrt{d(c^2x^2+}}{\sqrt{c}}\right.$ |
| parts | $-\frac{a(c^2 d x^2+d)^{\frac{3}{2}}}{dx}+a c^2 x\sqrt{c^2 d x^2+d}+\frac{a c^2 d \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{\sqrt{c^2 d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2 c}{2\sqrt{c^2x^2+1}}-\frac{2\sqrt{d(c^2x^2+}}{\sqrt{c}}\right.$ |

input
```
int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(c^2*d*x^2+d)^(3/2)+a*c^2*x*(c^2*d*x^2+d)^(1/2)+a*c^2*d*ln(c^2*d*x/
(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)*c-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)
+1)*arcsinh(c*x)/x/(c^2*x^2+1)+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln(
(c*x+(c^2*x^2+1)^(1/2))^2-1)*c)
```

### 3.125.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^2, x)
```

### 3.125.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))}{x^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**2, x)
```

### 3.125.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima"
)
```

output
```
(c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a + b*integrate(sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

### 3.125.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

# 3.126 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$



## 3.126.1 Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bc^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{bc^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

output
```
-1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2-1/2*b*c*(c^2*d*x^2+d)^(1/2
)/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2)
)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+
1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*b*c^2*polylog(2,c*x+(c
^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.126.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$$
$$= \frac{1}{8}\left(-\frac{4a\sqrt{d+c^2dx^2}}{x^2}+4ac^2\sqrt{d}\log(x)-4ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)\right.$$
$$+\frac{bc^2\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right)+4\text{arcsinh}(cx)\log\left(1-e^{-\text{ar}}\right.\right.}{}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
((-4*a*Sqrt[d + c^2*d*x^2])/x^2 + 4*a*c^2*Sqrt[d]*Log[x] - 4*a*c^2*Sqrt[d]
*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*Sqrt[d + c^2*d*x^2]*(-2*Cot
h[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*L
og[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*
PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh
[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])
/8
```

### 3.126.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6220, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{6220}$$
$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\int \frac{1}{x^2}dx}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}$$
$$\Big\downarrow \text{15}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6231}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{26}$$

$$\frac{ic^2\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{ic^2\sqrt{c^2dx^2+d}\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}}\right.\right.}{2\sqrt{c^2x^2+1}}$$
$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{ic^2\sqrt{c^2dx^2+d}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}\right.}{2\sqrt{c^2x^2+1}}$$
$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{ic^2\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx}\right.\right.}{2\sqrt{c^2x^2+1}}$$
$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc\sqrt{c^2dx^2+d}}{2x\sqrt{c^2x^2+1}}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

3.126. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

output
```
-1/2*(b*c*Sqrt[d + c^2*d*x^2])/(x*Sqrt[1 + c^2*x^2]) - (Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]))/(2*x^2) + ((I/2)*c^2*Sqrt[d + c^2*d*x^2]*((2*I)*(a
 + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x
]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2]
```

### 3.126.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.126.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.66

| method | result |
|---|---|
| default | $a\left(-\frac{(c^2 d\,x^2+d)^{\frac{3}{2}}}{2d\,x^2}+\frac{c^2\left(\sqrt{c^2 d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)\right)}{2}\right)+b\Bigg(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2x^2(c^2x^2+1)}$ |
| parts | $a\left(-\frac{(c^2 d\,x^2+d)^{\frac{3}{2}}}{2d\,x^2}+\frac{c^2\left(\sqrt{c^2 d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)\right)}{2}\right)+b\Bigg(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2x^2(c^2x^2+1)}$ |

input
```
int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(c^2*d*x^2+d)^(3/2)+1/2*c^2*((c^2*d*x^2+d)^(1/2)-d^(1/2)*ln(
(2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)))+b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x
*(c^2*x^2+1)^(1/2)+arcsinh(c*x))*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)-1/2
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)
^(1/2))*c^2-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*
x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2)
```

### 3.126.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^3, x)
```

### 3.126.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))}{x^3}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**3, x)
```

### 3.126.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima"
)
```

output
```
-1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d
*x^2 + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqr
t(c^2*x^2 + 1))/x^3, x)
```

### 3.126.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3, x)
```

# 3.127 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.127.1 Optimal result

Integrand size = 26, antiderivative size = 106

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bc\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3} + \frac{bc^3\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

output
```
-1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/d/x^3-1/6*b*c*(c^2*d*x^2+d)^(1
/2)/x^2/(c^2*x^2+1)^(1/2)+1/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)
```

## 3.127.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(bcx+3bc^3x^3+2a\sqrt{1+c^2x^2}+2ac^2x^2\sqrt{1+c^2x^2}+2b(1+c^2x^2)^{3/2}\text{arcsinh}(cx)-2bc^3x^3\,\mathrm{lo}\right.}{6x^3\sqrt{1+c^2x^2}}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + 3*b*c^3*x^3 + 2*a*Sqrt[1 + c^2*x^2] + 2
*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - 2*b*
c^3*x^3*Log[x]))/(x^3*Sqrt[1 + c^2*x^2])
```

## 3.127.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6215, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{c^2x^2+1}{x^3}dx}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{244}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \left(\frac{c^2}{x}+\frac{1}{x^3}\right)dx}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc\sqrt{c^2dx^2+d}\left(c^2\log(x)-\frac{1}{2x^2}\right)}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3dx^3}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(d*x^3) + (b*c*Sqrt[d +
c^2*d*x^2]*(-1/2*1/x^2 + c^2*Log[x]))/(3*Sqrt[1 + c^2*x^2])
```

### 3.127.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.127.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| default | $-\frac{a\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3 d\,x^{3}}-\frac{b\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(2\,\operatorname{arcsinh}\left(c x\right) c^{3} x^{3}-2 \ln\left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}-1\right) x^{3} c^{3}+2\,\operatorname{arcsinh}\left(c x\right) \sqrt{c^{2} x^{2}+1}\,x^{2} c^{2}+2\,\operatorname{arcsinh}\right.}{6\sqrt{c^{2} x^{2}+1}\,x^{3}}$ |
| parts | $-\frac{a\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3 d\,x^{3}}-\frac{b\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(2\,\operatorname{arcsinh}\left(c x\right) c^{3} x^{3}-2 \ln\left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}-1\right) x^{3} c^{3}+2\,\operatorname{arcsinh}\left(c x\right) \sqrt{c^{2} x^{2}+1}\,x^{2} c^{2}+2\,\operatorname{arcsinh}\right.}{6\sqrt{c^{2} x^{2}+1}\,x^{3}}$ |

input
```
int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(c^2*d*x^2+d)^(3/2)-1/6*b*(d*(c^2*x^2+1))^(1/2)*(2*arcsinh(c*
x)*c^3*x^3-2*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3+2*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x^2*c^2+2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+c*x)/(c^2*x^2+1)^(1/2
)/x^3
```

### 3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(90) = 180$.

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx =$$
$$-\frac{2\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx}}{c^2x^4}\right.}{6\left(c^2x^5+x^3\right)}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas"
)
```

output
```
-1/6*(2*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 + 1)) - (b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2
+ d*x^4 + sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^
2*x^4 + x^2)) + (2*a*c^4*x^4 + 4*a*c^2*x^2 - (b*c*x^3 - b*c*x)*sqrt(c^2*x^
2 + 1) + 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*x^5 + x^3)
```

### 3.127.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x^4}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**4, x)
```

### 3.127.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$$
$$= -\frac{\left((-1)^{2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(2\,c^2d+\frac{2\,d}{x^2}\right)-c^2d^{\frac{3}{2}}\log\left(x^2+\frac{1}{c^2}\right)+\frac{\sqrt{c^4dx^4+2\,c^2dx^2+d}d}{x^2}\right)bc}{6\,d}$$
$$-\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b\,\text{arsinh}\left(cx\right)}{3\,dx^3}-\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}a}{3\,dx^3}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima"
)
```

output
```
-1/6*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^
(3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*b*c/d -
1/3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/
2)*a/(d*x^3)
```

### 3.127.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4, x)
```

# 3.128 $\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.128.1 Optimal result

Integrand size = 26, antiderivative size = 217

$$\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}}-\frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}-\frac{bc^3dx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}+\frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}$$

output
```
-1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^4/d+1/7*(c^2*d*x^2+d)^(7/2)*
(a+b*arcsinh(c*x))/c^4/d^2+2/35*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^
(1/2)-1/105*b*d*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-8/175*b*c*d*x^
5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^3*d*x^7*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)
```

## 3.128.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.60

$$\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{d\sqrt{d+c^2dx^2}\left(105a(1+c^2x^2)^3(-2+5c^2x^2)-bcx\sqrt{1+c^2x^2}(-210+35c^2x^2+168c^4x^4\right.}{3675c^4(1+c^2x^2)}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - b*c*x*Sqr
t[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 +
 c^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]))/(3675*c^4*(1 + c^2*x^2))
```

## 3.128.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c^2 d x^2+d\right)^{3/2} (a+b\text{arcsinh}(cx)) \, dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{d\left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2}{35c^4}dx}{\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\sqrt{c^2dx^2+d}\int \left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2dx}{35c^3\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}$$

$$\Big\downarrow \text{290}$$

$$\frac{bd\sqrt{c^2dx^2+d}\int \left(-5c^6x^6-8c^4x^4-c^2x^2+2\right)dx}{35c^3\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c^2dx^2+d)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d^2}-\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d}+\frac{bd\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}$$

input
```
Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(b*d*Sqrt[d + c^2*d*x^2]*(2*x - (c^2*x^3)/3 - (8*c^4*x^5)/5 - (5*c^6*x^7)/
7))/(35*c^3*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x
]))/(5*c^4*d) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d^2)
```

### 3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.128.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(185) = 370$.

Time = 0.22 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.02

| method | result |
|---|---|
| default | $a\left(\frac{x^2(c^2d\,x^2+d)^{\frac{5}{2}}}{7c^2d}-\frac{2(c^2d\,x^2+d)^{\frac{5}{2}}}{35d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+}{6272c^4(c^2x^2+1)}\right.$ |
| parts | $a\left(\frac{x^2(c^2d\,x^2+d)^{\frac{5}{2}}}{7c^2d}-\frac{2(c^2d\,x^2+d)^{\frac{5}{2}}}{35d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+}{6272c^4(c^2x^2+1)}\right.$ |

input
```
int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b*(1/
6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^
6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/
2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^
2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)
+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2
)+1)*(-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*
c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(
-1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+
c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2
*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)*d/c^4/(c
^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/
2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)*d/c^4/(c^2*x^2+
1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+2
8*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+
1)*(1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c
^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1
/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^
(1/2)+1)*(1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1))
```

### 3.128.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.92

$$\int x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))\, dx = \frac{105\left(5\, bc^8dx^8+13\, bc^6dx^6+9\, bc^4dx^4-bc^2dx^2-2\, bd\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)}{}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
1/3675*(105*(5*b*c^8*d*x^8 + 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 - b*c^2*d*x^2
- 2*b*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (525*a*c^8*d*x
^8 + 1365*a*c^6*d*x^6 + 945*a*c^4*d*x^4 - 105*a*c^2*d*x^2 - 210*a*d - (75*
b*c^7*d*x^7 + 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 - 210*b*c*d*x)*sqrt(c^2*x^2
 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

### 3.128.6 Sympy [F]

$$\int x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))\, dx = \int x^3 \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\,\text{asinh}\,(cx))\; dx$$

input
```
integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

### 3.128.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\begin{aligned}\int x^3 \left(d+c^2 d x^2\right)^{3/2} (a &+ b\text{arcsinh}(cx))\, dx = \frac{1}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)b\,\text{arsinh}\,(cx)\\ &+\frac{1}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)a\\ &-\frac{\left(75\,c^6d^{\frac{3}{2}}x^7+168\,c^4d^{\frac{3}{2}}x^5+35\,c^2d^{\frac{3}{2}}x^3-210\,d^{\frac{3}{2}}x\right)b}{3675\,c^3}\end{aligned}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d
))*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x
^2 + d)^(5/2)/(c^4*d))*a - 1/3675*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^
5 + 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*b/c^3
```

### 3.128.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2 dx^2)^{3/2} (a + b\text{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.128.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (d + c^2 dx^2)^{3/2} (a + b\text{arcsinh}(cx)) \, dx = \int x^3 (a + b \,\text{asinh}(c\,x)) \left(d\,c^2\,x^2 + d\right)^{3/2} dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.129 $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.129.1 Optimal result

Integrand size = 26, antiderivative size = 254

$$\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bdx^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}}+\frac{dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c^2}+\frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{1}{6}x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}$$

output
```
1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/16*d*x*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-
1/32*b*d*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/96*b*c*d*x^4*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/36*b*c^3*d*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*
x^2+1)^(1/2)-1/32*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^
2+1)^(1/2)
```

### 3.129.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.99

$$\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{48acdx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(3+14c^2x^2+8c^4x^4)-144ad^{3/2}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\right.}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(48*a*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(3 + 14*c^2*x^2 + 8*c^4*
x^4) - 144*a*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*
x^2]] - 18*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]
] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + b*d*Sqrt[d + c^2*d*x^2]*(72*Arc
Sinh[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*
ArcSinh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSin
h[c*x]] + Sinh[6*ArcSinh[c*x]])))/(2304*c^3*Sqrt[1 + c^2*x^2])
```

### 3.129.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6223, 244, 2009, 6221, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6223}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int x^3\left(c^2x^2+1\right)dx}{6\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int\left(c^2x^5+x^3\right)dx}{6\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\sqrt{c^2dx^2+d}}{6\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6221}$$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^3dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\sqrt{c^2dx^2+d}}{6\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}\right)+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\sqrt{c^2dx^2+d}}{6\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)\right.$$
$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\sqrt{c^2dx^2+d}}{6\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{15}$$

---

3.129. $\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\sqrt{c^2dx^2+d}}{6\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))+$$

$$\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{c^2x^2+1}}-\frac{bc\cdots}{\cdots}\right.$$

$$\frac{bcd\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\sqrt{c^2dx^2+d}}{6\sqrt{c^2x^2+1}}$$

input
```
Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/6*(b*c*d*Sqrt[d + c^2*d*x^2]*(x^4/4 + (c^2*x^6)/6))/Sqrt[1 + c^2*x^2] +
 (x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 + (d*(-1/16*(b*c*x^4*S
qrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x]))/4 + (Sqrt[d + c^2*d*x^2]*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3)))/(4*S
qrt[1 + c^2*x^2])))/2
```

### 3.129.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.129. $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

### 3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(218) = 436$.

Time = 0.22 (sec) , antiderivative size = 799, normalized size of antiderivative = 3.15

| method | result |
|---|---|
| default | $\frac{ax(c^2d\,x^2+d)^{\frac{5}{2}}}{6c^2d} - \frac{ax(c^2d\,x^2+d)^{\frac{3}{2}}}{24c^2} - \frac{adx\sqrt{c^2d\,x^2+d}}{16c^2} - \frac{a\,d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2d}{32\sqrt{c^2x^2+1}\,c^3} + \right.$ |
| parts | $\frac{ax(c^2d\,x^2+d)^{\frac{5}{2}}}{6c^2d} - \frac{ax(c^2d\,x^2+d)^{\frac{3}{2}}}{24c^2} - \frac{adx\sqrt{c^2d\,x^2+d}}{16c^2} - \frac{a\,d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2d}{32\sqrt{c^2x^2+1}\,c^3} + \right.$ |

input
```
int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16*a/
c^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a/c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d
*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^3*arcsinh(c*x)^2*d+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x
^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18
*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d/
c^3/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+
1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(
-1+4*arcsinh(c*x))*d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^
3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))
*d/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x
^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+
1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*
x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x)
)*d/c^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c
^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*
x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d/c^3/(c
^2*x^2+1))
```

### 3.129.5 Fricas [F]

$$\int x^2 \left(d + c^2 d x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int (c^2 d x^2 + d)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a) x^2\, dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*c^2*d*x^4 + a*d*x^2 + (b*c^2*d*x^4 + b*d*x^2)*arcsinh(c*x))*sq
rt(c^2*d*x^2 + d), x)
```

### 3.129.6 Sympy [F]

$$\int x^2 \left(d + c^2 d x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int x^2 \left(d(c^2 x^2 + 1)\right)^{\frac{3}{2}} (a + b \operatorname{asinh}(cx))\, dx$$

input
```
integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

### 3.129.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(d + c^2 d x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.129.8 Giac [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

### 3.129.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int x^2\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.130 $\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.130.1 Optimal result

Integrand size = 24, antiderivative size = 146

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bdx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bcdx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3dx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}$$

output
```
1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^2/d-1/5*b*d*x*(c^2*d*x^2+d)^(
1/2)/c/(c^2*x^2+1)^(1/2)-2/15*b*c*d*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-1/25*b*c^3*d*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

## 3.130.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{d\sqrt{d+c^2dx^2}\left(15a(1+c^2x^2)^3-bcx\sqrt{1+c^2x^2}(15+10c^2x^2+3c^4x^4)+15b(1+c^2x^2)^3\text{ a}\right.}{75c^2\left(1+c^2x^2\right)}$$

input
```
Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15
 + 10*c^2*x^2 + 3*c^4*x^4) + 15*b*(1 + c^2*x^2)^3*ArcSinh[c*x]))/(75*c^2*(
1 + c^2*x^2))
```

### 3.130.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6213, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bd\sqrt{c^2dx^2+d}\int\left(c^2x^2+1\right)^2dx}{5c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bd\sqrt{c^2dx^2+d}\int\left(c^4x^4+2c^2x^2+1\right)dx}{5c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bd\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}$$

input
```
Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/5*(b*d*Sqrt[d + c^2*d*x^2]*(x + (2*c^2*x^3)/3 + (c^4*x^5)/5))/(c*Sqrt[1
 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*d)
```

### 3.130.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.130.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(124) = 248.

Time = 0.20 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.83

| method | result |
|---|---|
| default | $\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx\sqrt{c^2x^2+1}+1\right)(-1+5\ \text{arcsi}}{800c^2(c^2x^2+1)}$ |
| parts | $\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}+13c^2x^2+5cx\sqrt{c^2x^2+1}+1\right)(-1+5\ \text{arcsi}}{800c^2(c^2x^2+1)}$ |

input
```
int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*(c^2*d*x^2+d)^(5/2)/c^2/d+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6
+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c
^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/
96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+
3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(
c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^
2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)
*(arcsinh(c*x)+1)*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-
4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsin
h(c*x)+1)*d/c^2/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5
*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-
5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

### 3.130.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{15\left(bc^6dx^6+3\,bc^4dx^4+3\,bc^2dx^2+bd\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(15\,ac^6dx^6+\ldots\right.}{75\,(c^4\ldots}$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/75*(15*(b*c^6*d*x^6 + 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 + b*d)*sqrt(c^2*d*x^
2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (15*a*c^6*d*x^6 + 45*a*c^4*d*x^4 + 4
5*a*c^2*d*x^2 + 15*a*d - (3*b*c^5*d*x^5 + 10*b*c^3*d*x^3 + 15*b*c*d*x)*sqr
t(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

### 3.130.6 Sympy [F]

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

input
```
integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

### 3.130.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{(c^2dx^2+d)^{\frac{5}{2}}b\,\text{arsinh}\,(cx)}{5\,c^2d} + \frac{(c^2dx^2+d)^{\frac{5}{2}}a}{5\,c^2d} - \frac{\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)b}{75\,cd}$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/5*(c^2*d*x^2 + d)^(5/2)*b*arcsinh(c*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/
2)*a/(c^2*d) - 1/75*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x
)*b/(c*d)
```

### 3.130.8 Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.130.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int x\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.131 $\int (d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx)) \, dx$



## 3.131.1 Optimal result

Integrand size = 23, antiderivative size = 180

$$\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx)) \, dx = -\frac{5bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^3dx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}x(d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx)) + \frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

output
```
1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)-5/16*b*c*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/1
6*b*c^3*d*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/16*d*(a+b*arcsinh(c*
x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.131.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.11

$$\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \frac{1}{8}adx\left(5+2c^2x^2\right)\sqrt{d+c^2dx^2}$$
$$+\frac{3ad^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{8c}$$
$$+\frac{bd\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{8c\sqrt{1+c^2x^2}}$$
$$-\frac{bd\sqrt{d+c^2dx^2}(8\text{arcsinh}(cx)^2+\cosh(4\text{arcsinh}(cx))-4\text{arcsinh}(cx)\sinh(4\text{arcsinh}(cx)))}{128c\sqrt{1+c^2x^2}}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(a*d*x*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/8 + (3*a*d^(3/2)*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c) + (b*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*Arc
Sinh[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*S
qrt[1 + c^2*x^2]) - (b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*Ar
cSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^
2])
```

### 3.131.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 244}$$

$$\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2x^3+x\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6200}$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

input
```
Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*d*Sqrt[d + c^2*d*x^2]*(x^2/2 + (c^2*x^4)/4))/Sqrt[1 + c^2*x^2] +
 (x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sq
rt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
 c^2*x^2])))/4
```

3.131. $\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

### 3.131.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

## 3.131. $\int (d + c^2 d x^2)^{3/2} (a + b\operatorname{arcsinh}(cx))\,dx$

### 3.131.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(154) = 308$.

Time = 0.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.76

| method | result |
|---|---|
| default | $\frac{x(c^2dx^2+d)^{\frac{3}{2}}a}{4}+\frac{3adx\sqrt{c^2dx^2+d}}{8}+\frac{3a\,d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2d}{16\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(8c^5}{}\right.$ |
| parts | $\frac{x(c^2dx^2+d)^{\frac{3}{2}}a}{4}+\frac{3adx\sqrt{c^2dx^2+d}}{8}+\frac{3a\,d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^2d}{16\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(8c^5}{}\right.$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(c^2*d*x^2+d)^(3/2)*a+3/8*a*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*d^2*ln(c^2
*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(3/16*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)
*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(
1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*
(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1
)^(1/2))*(-1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2
*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh
(c*x))*d/c/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c
^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(
1/2))*(1+4*arcsinh(c*x))*d/c/(c^2*x^2+1))
```

### 3.131.5 Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d), x)
```

### 3.131.6 Sympy [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))\,\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

### 3.131.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.131.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.131.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(a+b\,\mathrm{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.132 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx$



## 3.132.1 Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bd\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} + \frac{bd\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

output
```
1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+d*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)-4/3*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d*x^
3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))*arctanh(c*x
+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*d*polylog(2,-c
*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*d*polylog(2,
c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.132.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx = \frac{1}{3}ad(4+c^2x^2)\sqrt{d+c^2dx^2} + \frac{bd\sqrt{d+c^2dx^2}\left(-cx(3+c^2x^2)+3(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)\right)}{9\sqrt{1+c^2x^2}} + ad^{3/2}\log(x) - ad^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bd\sqrt{d+c^2dx^2}(-cx+\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)+\operatorname{arcsinh}(cx)\log}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(a*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 + (b*d*Sqrt[d + c^2*d*x^2]*(-(c*
x*(3 + c^2*x^2)) + 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^
2]) + a*d^(3/2)*Log[x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] +
(b*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSin
h[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])
] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[
1 + c^2*x^2]
```

### 3.132.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x}\,dx$$

$$\Big\downarrow \text{6223}$$

$$d\int \frac{\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{x}dx - \frac{bcd\sqrt{c^2dx^2+d}\int (c^2x^2+1)\,dx}{3\sqrt{c^2x^2+1}} + \frac{1}{3}(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6221}$$

$$d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int 1dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{24}$$

$$d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6231}$$

$$d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx}}{\sqrt{c^2x^2}}\right.$$
$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{26}$$

$$d\left(\frac{i\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx}}{\sqrt{c^2x^2}}\right.$$
$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

---

3.132. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx$

$\Big\downarrow$ 4670

$$d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsin}}\right.\right.}{\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2715

$$d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(c}\right.}{\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2838

$$d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c}\right.\right.}{\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
-1/3*(b*c*d*Sqrt[d + c^2*d*x^2]*(x + (c^2*x^3)/3))/Sqrt[1 + c^2*x^2] + ((d
 + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + d*(-((b*c*x*Sqrt[d + c^2*d*x
^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]) + (I*Sq
rt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*
b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2])
```

## 3.132. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

### 3.132.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

## 3.132. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.132.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| default | $\frac{(c^2 d x^2+d)^{\frac{3}{2}} a}{3} - a d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right) + ad\sqrt{c^2 d x^2+d} + \frac{b\sqrt{d(c^2x^2+1)}\,\text{polylog}\left(2, cx+\sqrt{c^2x^2+1}\right) d}{\sqrt{c^2x^2+1}} + \frac{4b\sqrt{}}{}$ |
| parts | $\frac{(c^2 d x^2+d)^{\frac{3}{2}} a}{3} - a d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right) + ad\sqrt{c^2 d x^2+d} + \frac{b\sqrt{d(c^2x^2+1)}\,\text{polylog}\left(2, cx+\sqrt{c^2x^2+1}\right) d}{\sqrt{c^2x^2+1}} + \frac{4b\sqrt{}}{}$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))
/x)+a*d*(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*poly
log(2,c*x+(c^2*x^2+1)^(1/2))*d+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*a
rcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))*d-4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c*x-b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2
))*d+1/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-1/9*b*
(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c^3*x^3+5/3*b*(d*(c^2*x^2+1))^(1
/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d
```

### 3.132.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d)/x, x)
```

### 3.132.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))}{x}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x, x)
```

### 3.132.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

output
```
-1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2
*d*x^2 + d)*d)*a + b*integrate((c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^
2 + 1))/x, x)
```

### 3.132.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x, x)
```

# 3.133 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.133.1 Optimal result

Integrand size = 26, antiderivative size = 177

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = -\frac{bc^3dx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} + \frac{3cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4b\sqrt{1+c^2x^2}} + \frac{bcd\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}$$

output

```
-(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x+3/2*c^2*d*x*(a+b*arcsinh(c*x))*(
c^2*d*x^2+d)^(1/2)-1/4*b*c^3*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3
/4*c*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d*
ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

## 3.133.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.13

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{1}{8}\left(\frac{4ad(-2+c^2x^2)\sqrt{d+c^2dx^2}}{x} + \frac{4bd\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\text{arcsinh}(cx)+cx\text{arcsinh}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}} + 12acd^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bcd\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+}{\sqrt{1+c^2x^2}}\right.$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
((4*a*d*(-2 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/x + (4*b*d*Sqrt[d + c^2*d*x^2]
*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x])
)/(x*Sqrt[1 + c^2*x^2]) + 12*a*c*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]] + (b*c*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*
x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2])/8
```

## 3.133.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6222, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{6222}$$

$$3c^2d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{bcd\sqrt{c^2dx^2+d}\int \frac{c^2x^2+1}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{244}$$

$$3c^2d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{bcd\sqrt{c^2dx^2+d}\int \left(xc^2+\frac{1}{x}\right)dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$3c^2d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^2}{2}+\log(x)\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6200}$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^2}{2}+\log(x)\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 15}$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^2}{2}+\log(x)\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6198}$$

$$3c^2d\left(\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^2}{2}+\log(x)\right)}{\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-(((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x) + 3*c^2*d*(-1/4*(b*c*x^2
*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[
1 + c^2*x^2])) + (b*c*d*Sqrt[d + c^2*d*x^2]*((c^2*x^2)/2 + Log[x]))/Sqrt[1
 + c^2*x^2]
```

### 3.133.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

---

3.133. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

### 3.133.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.30

| method | result |
|---|---|
| default | $-\frac{a(c^2 d x^2+d)^{\frac{5}{2}}}{dx}+a c^2 x(c^2 d x^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2 d x^2+d}\, a c^2 d x}{2}+\frac{3 a c^2 d^2 \ln\left(\frac{c^2 d x}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{2\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\left(4 \text{ar}\right.}{}$ |
| parts | $-\frac{a(c^2 d x^2+d)^{\frac{5}{2}}}{dx}+a c^2 x(c^2 d x^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2 d x^2+d}\, a c^2 d x}{2}+\frac{3 a c^2 d^2 \ln\left(\frac{c^2 d x}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{2\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\left(4 \text{ar}\right.}{}$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(c^2*d*x^2+d)^(5/2)+a*c^2*x*(c^2*d*x^2+d)^(3/2)+3/2*(c^2*d*x^2+d)^(
1/2)*a*c^2*d*x+3/2*a*c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))
/(c^2*d)^(1/2)+1/8*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(4*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-2*c^3*x^3+6*arcsinh(c*x)^2*x*c-8*arcsinh(c*
x)*c*x+8*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x*c-8*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)-c*x)*d
```

### 3.133.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas"
)
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d)/x^2, x)
```

### 3.133.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))}{x^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**2, x)
```

### 3.133.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.133.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

# 3.134 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$



## 3.134.1 Optimal result

Integrand size = 26, antiderivative size = 270

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2} - \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{3bc^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} + \frac{3bc^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}$$

output
```
-1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2+3/2*c^2*d*(a+b*arcsinh(c*x
))*(c^2*d*x^2+d)^(1/2)-1/2*b*c*d*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-b
*c^3*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))*
arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b
*c^2*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)+3/2*b*c^2*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)
```

### 3.134.2 Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.30

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = a\left(c^2d-\frac{d}{2x^2}\right)\sqrt{d+c^2dx^2}+\frac{3}{2}ac^2d^{3/2}\log(x)$$
$$-\frac{3}{2}ac^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bc^2d\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)\log\left(1-\right.\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
a*(c^2*d - d/(2*x^2))*Sqrt[d + c^2*d*x^2] + (3*a*c^2*d^(3/2)*Log[x])/2 - (
3*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/2 + (b*c^2*d*Sqrt[d
+ c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1
 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[
2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
 + (b*c^2*d*Sqrt[d + c^2*d*x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Cs
ch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSi
nh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*
PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Ta
nh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

### 3.134.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6222, 244, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}dx$$
$$\Big\downarrow \; 6222$$
$$\frac{3}{2}c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\int \frac{c^2x^2+1}{x^2}dx}{2\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\downarrow \textbf{244}$$

$$\frac{3}{2}c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2+\frac{1}{x^2}\right)dx}{2\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\downarrow \textbf{2009}$$

$$\frac{3}{2}c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\downarrow \textbf{6221}$$

$$\frac{3}{2}c^2d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int 1dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\downarrow \textbf{24}$$

$$\frac{3}{2}c^2d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\downarrow \textbf{6231}$$

$$\frac{3}{2}c^2d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\downarrow \textbf{3042}$$

$$\frac{3}{2}c^2d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c}}{\sqrt{c^2}}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\downarrow \textbf{26}$$

---

3.134. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}dx$

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c}}{\sqrt{c^2}}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 4670

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{a}\right.\right.}{\sqrt{c^2x^2+1}}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 2715

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsi}}\right.}{\sqrt{c^2x^2+1}}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 2838

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsi}}\right.\right.}{\sqrt{c^2x^2+1}}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd\left(c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(b*c*d*(-x^(-1) + c^2*x)*Sqrt[d + c^2*d*x^2])/(2*Sqrt[1 + c^2*x^2]) - ((d
+ c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (3*c^2*d*(-((b*c*x*Sqrt
[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]) + (I*Sqrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSin
h[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]
))/Sqrt[1 + c^2*x^2]))/2
```

## 3.134. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

### 3.134.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.134. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.134.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| default | $a\left(-\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{2 d\,x^{2}}+\frac{3 c^{2}\left(\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right)\right)\right)}{2}\right)+\frac{b\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(2\,\mathrm{arcsinh}(cx)\sqrt{}\right.}{}$ |
| parts | $a\left(-\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{2 d\,x^{2}}+\frac{3 c^{2}\left(\frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right)\right)\right)}{2}\right)+\frac{b\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(2\,\mathrm{arcsinh}(cx)\sqrt{}\right.}{}$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(c^2*d*x^2+d)^(5/2)+3/2*c^2*(1/3*(c^2*d*x^2+d)^(3/2)+d*((c^2
*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x))))+1/2*b
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^2*(2*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)*x^2*c^2+3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-3*arcsinh(
c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-2*c^3*x^3+3*polylog(2,c*x+(c^2*x^
2+1)^(1/2))*x^2*c^2-3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-arcsinh(c*
x)*(c^2*x^2+1)^(1/2)-c*x)*d
```

### 3.134.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas"
)
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d)/x^3, x)
```

### 3.134.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))}{x^3}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**3, x)
```

### 3.134.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima"
)
```

output
```
-1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*
sqrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a + b*integrate
((c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

### 3.134.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^{3/2}}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3, x)
```

# 3.135 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.135.1 Optimal result

Integrand size = 26, antiderivative size = 184

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = -\frac{bcd\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x} - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3} + \frac{c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{1+c^2x^2}} + \frac{4bc^3d\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

output
```
-1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))*(
c^2*d*x^2+d)^(1/2)/x-1/6*b*c*d*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)+1
/2*c^3*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+4/3*
b*c^3*d*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.135.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.18

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \frac{1}{6}\left(-\frac{2ad(1+4c^2x^2)\sqrt{d+c^2dx^2}}{x^3} - \frac{2b(d+c^2dx^2)^{3/2}\text{arcsinh}(cx)}{x^3} + \frac{3bc^3d\sqrt{d+c^2dx^2}\left(-\frac{2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{cx}+\text{arcsinh}(cx)^2+2\log(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{bcd\sqrt{d+c^2dx^2}(-1+2c^2x^2\log(cx))}{x^2\sqrt{1+c^2x^2}} + 6ac^3d^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)\right)$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
((-2*a*d*(1 + 4*c^2*x^2)*Sqrt[d + c^2*d*x^2])/x^3 - (2*b*(d + c^2*d*x^2)^(
3/2)*ArcSinh[c*x])/x^3 + (3*b*c^3*d*Sqrt[d + c^2*d*x^2]*((-2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/Sqrt[1 + c^2*x^2]
 + (b*c*d*Sqrt[d + c^2*d*x^2]*(-1 + 2*c^2*x^2*Log[c*x]))/(x^2*Sqrt[1 + c^2
*x^2]) + 6*a*c^3*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/6
```

### 3.135.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6222, 244, 2009, 6220, 14, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{6222}$$

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x^2}dx + \frac{bcd\sqrt{c^2dx^2+d}\int \frac{c^2x^2+1}{x^3}dx}{3\sqrt{c^2x^2+1}} - \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{244}$$

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x^2}dx+\frac{bcd\sqrt{c^2dx^2+d}\int \left(\frac{c^2}{x}+\frac{1}{x^3}\right)dx}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd\sqrt{c^2dx^2+d}\left(c^2\log(x)-\frac{1}{2x^2}\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6220}$$

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\int \frac{1}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}\right)-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd\sqrt{c^2dx^2+d}\left(c^2\log(x)-\frac{1}{2x^2}\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{14}$$

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd\sqrt{c^2dx^2+d}\left(c^2\log(x)-\frac{1}{2x^2}\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$c^2d\left(\frac{c\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd\sqrt{c^2dx^2+d}\left(c^2\log(x)-\frac{1}{2x^2}\right)}{3\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

## 3.135. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

output
```
-1/3*((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3 + (b*c*d*Sqrt[d + c^
2*d*x^2]*(-1/2*1/x^2 + c^2*Log[x]))/(3*Sqrt[1 + c^2*x^2]) + c^2*d*(-((Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[x])
/Sqrt[1 + c^2*x^2])
```

### 3.135.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

---

3.135. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

### 3.135.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| default | $-\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{3dx^3}-\frac{2ac^2(c^2dx^2+d)^{\frac{5}{2}}}{3dx}+\frac{2ac^4x(c^2dx^2+d)^{\frac{3}{2}}}{3}+ac^4dx\sqrt{c^2dx^2+d}+\frac{ac^4d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+$ |
| parts | $-\frac{a(c^2dx^2+d)^{\frac{5}{2}}}{3dx^3}-\frac{2ac^2(c^2dx^2+d)^{\frac{5}{2}}}{3dx}+\frac{2ac^4x(c^2dx^2+d)^{\frac{3}{2}}}{3}+ac^4dx\sqrt{c^2dx^2+d}+\frac{ac^4d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(c^2*d*x^2+d)^(5/2)-2/3*a*c^2/d/x*(c^2*d*x^2+d)^(5/2)+2/3*a*c
^4*x*(c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(c^2*d*x^2+d)^(1/2)+a*c^4*d^2*ln(c^2*d*
x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/6*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/x^3*(3*arcsinh(c*x)^2*x^3*c^3-8*arcsinh(c*x)*c^3*x^
3+8*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-8*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*x^2*c^2-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)*d
```

### 3.135.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas"
)
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d)/x^4, x)
```

### 3.135.6 Sympy [F]

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \int \frac{(d(c^2x^2 + 1))^{\frac{3}{2}} (a + b \operatorname{asinh}(cx))}{x^4} dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**4, x)
```

### 3.135.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.135.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

# 3.136 $\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.136.1 Optimal result

Integrand size = 26, antiderivative size = 266

$$\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{2bd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{bd^2x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}} - \frac{bcd^2x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}} - \frac{19bc^3d^2x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d} + \frac{(d+c^2dx^2)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4d^2}$$

output
```
-1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^4/d+1/9*(c^2*d*x^2+d)^(9/2)*
(a+b*arcsinh(c*x))/c^4/d^2+2/63*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1
)^(1/2)-1/189*b*d^2*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/21*b*c*d
^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*x^7*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/81*b*c^5*d^2*x^9*(c^2*d*x^2+d)^(1/2)/(c^2*
x^2+1)^(1/2)
```

### 3.136.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.53

$$\int x^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \frac{d^2\sqrt{d+c^2dx^2}\Big(63a(1+c^2x^2)^4\left(-2+7c^2x^2\right)-bcx\sqrt{1+c^2x^2}(-126+21c^2x^2+189c^4x^4}{3969c^4\left(1+c^2x^2\right)}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - b*c*x*Sq
rt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^
8) + 63*b*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4*(1 + c
^2*x^2))
```

### 3.136.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c^2dx^2+d\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{d^2(2-7c^2x^2)(c^2x^2+1)^3}{63c^4}dx}{\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^2\sqrt{c^2dx^2+d}\int \left(2-7c^2x^2\right)\left(c^2x^2+1\right)^3dx}{63c^3\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4d}$$

$$\Big\downarrow \text{290}$$

$$\frac{bd^2\sqrt{c^2dx^2+d}\int\left(-7c^8x^8-19c^6x^6-15c^4x^4-c^2x^2+2\right)dx}{63c^3\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(c^2dx^2+d\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4d}+\frac{bd^2\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}+2x\right)\sqrt{c^2dx^2+d}}{63c^3\sqrt{c^2x^2+1}}$$

input
```
Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(b*d^2*Sqrt[d + c^2*d*x^2]*(2*x - (c^2*x^3)/3 - 3*c^4*x^5 - (19*c^6*x^7)/7
 - (7*c^8*x^9)/9))/(63*c^3*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(7/2)*(a
+ b*ArcSinh[c*x]))/(7*c^4*d) + ((d + c^2*d*x^2)^(9/2)*(a + b*ArcSinh[c*x])
)/(9*c^4*d^2)
```

### 3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

### 3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(228) = 456$.

Time = 0.23 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.74

| method | result |
|---|---|
| default | $a\left(\frac{x^{2}\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{9 c^{2} d}-\frac{2\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{63 d\,c^{4}}\right)+b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(256 c^{10} x^{10}+256 c^{9} x^{9} \sqrt{c^{2} x^{2}+1}+704 c^{8} x^{8}+576 c^{7} x^{7} \sqrt{c^{2} x^{2}+1}+688 c^{6}\right.}{}\right.$ |
| parts | $a\left(\frac{x^{2}\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{9 c^{2} d}-\frac{2\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{63 d\,c^{4}}\right)+b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\,\left(256 c^{10} x^{10}+256 c^{9} x^{9} \sqrt{c^{2} x^{2}+1}+704 c^{8} x^{8}+576 c^{7} x^{7} \sqrt{c^{2} x^{2}+1}+688 c^{6}\right.}{}\right.$ |

input
```
int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b*(1/
41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2)+7
04*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^2+
1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*x
^2+1)^(1/2)+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2
+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^
5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*
c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*
(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*
(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2
*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/c^4
/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)
*(arcsinh(c*x)+1)*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x
^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arc
sinh(c*x)+1)*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8
-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+10
4*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+
1)*(1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(2
56*c^10*x^10-256*c^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^
2+1)^(1/2)+688*c^6*x^6-432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^4-120*c^...
```

### 3.136.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99

$$\int x^3 (d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx)) \, dx = \frac{63 (7 bc^{10}d^2x^{10} + 26 bc^8d^2x^8 + 34 bc^6d^2x^6 + 16 bc^4d^2x^4 - bc^2d^2x^2 - 2 bd^2)\sqrt{c^2dx^2 + d} \, \text{lo}}{}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
1/3969*(63*(7*b*c^10*d^2*x^10 + 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 + 16*b
*c^4*d^2*x^4 - b*c^2*d^2*x^2 - 2*b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 + 1)) + (441*a*c^10*d^2*x^10 + 1638*a*c^8*d^2*x^8 + 2142*a*c^6*d^
2*x^6 + 1008*a*c^4*d^2*x^4 - 63*a*c^2*d^2*x^2 - 126*a*d^2 - (49*b*c^9*d^2*
x^9 + 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 + 21*b*c^3*d^2*x^3 - 126*b*c*d
^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

### 3.136.6 Sympy [F(-1)]

Timed out.

$$\int x^3 (d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx)) \, dx = \text{Timed out}$$

input
```
integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

### 3.136.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.59

$$\int x^3 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \frac{1}{63}\left(\frac{7\,(c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d} - \frac{2\,(c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)b\,\text{arsinh}\,(cx) + \frac{1}{63}\left(\frac{7\,(c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d} - \frac{2\,(c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)a - \frac{\left(49\,c^8d^{\frac{5}{2}}x^9 + 171\,c^6d^{\frac{5}{2}}x^7 + 189\,c^4d^{\frac{5}{2}}x^5 + 21\,c^2d^{\frac{5}{2}}x^3 - 126\,d^{\frac{5}{2}}x\right)b}{3969\,c^3}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d
))*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x
^2 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^
7 + 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*b/c^3
```

### 3.136.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.136.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int x^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.137 $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.137.1 Optimal result

Integrand size = 26, antiderivative size = 337

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=-\frac{5bd^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}}+\frac{5d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{128c^2}+\frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}$$

output
```
5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/8*x^3*(c^2*d*x^2+d)^(5
/2)*(a+b*arcsinh(c*x))+5/128*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
c^2+5/64*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-5/256*b*d^2*x^2*(c
^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*x^4*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)-17/288*b*c^3*d^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)-1/64*b*c^5*d^2*x^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/256*d^2*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

### 3.137.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.15

$$\int x^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \frac{d^2\left(2880acx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+22656ac^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+26112ac^5x^5\sqrt{\right.}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(2880*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 22656*a*c^3*x^3*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2]*S
qrt[d + c^2*d*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]
- 1440*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 + 576*b*Sqrt[d + c^2*d*x^2]*Co
sh[2*ArcSinh[c*x]] - 144*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 64*b
*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 9*b*Sqrt[d + c^2*d*x^2]*Cosh[8
*ArcSinh[c*x]] - 2880*a*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt
[d + c^2*d*x^2]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(-48*Sinh[2*ArcSi
nh[c*x]] + 24*Sinh[4*ArcSinh[c*x]] + 16*Sinh[6*ArcSinh[c*x]] + 3*Sinh[8*Ar
cSinh[c*x]])))/(73728*c^3*Sqrt[1 + c^2*x^2])
```

### 3.137.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6223, 243, 49, 2009, 6223, 244, 2009, 6221, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6223}$$

$$\frac{5}{8}d\int x^2 \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{bcd^2\sqrt{c^2dx^2+d}\int x^3 \left(c^2x^2+1\right)^2dx}{8\sqrt{c^2x^2+1}}+\frac{1}{8}x^3 \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 243}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{bcd^2\sqrt{c^2dx^2+d}\int x^2\left(c^2x^2+1\right)^2dx^2}{16\sqrt{c^2x^2+1}}+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{49}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{bcd^2\sqrt{c^2dx^2+d}\int\left(c^4x^6+2c^2x^4+x^2\right)dx^2}{16\sqrt{c^2x^2+1}}+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6223}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int x^3\left(c^2x^2+1\right)dx}{6\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+\text{barc}\right.$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{244}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int\left(c^2x^5+x^3\right)dx}{6\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+\text{barcs}\right.$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^6}{6}+\frac{x^4}{4}\right)\sqrt{c^2dx^2}}{6\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6221}$$

3.137. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^3dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}\right)+\frac{1}{6}x^3\left(c^2\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(c\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6198}$$

---

3.137. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{5}{8}d\left(\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcsin}}{4bc^3}\right.}{}\right.\right.$$
$$\frac{bcd^2\left(\frac{c^4x^8}{4}+\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

input
```
Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/16*(b*c*d^2*Sqrt[d + c^2*d*x^2]*(x^4/2 + (2*c^2*x^6)/3 + (c^4*x^8)/4))/
Sqrt[1 + c^2*x^2] + (x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8 + (
5*d*(-1/6*(b*c*d*Sqrt[d + c^2*d*x^2]*(x^4/4 + (c^2*x^6)/6))/Sqrt[1 + c^2*x
^2] + (x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 + (d*(-1/16*(b*c*
x^4*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x^3*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]))/4 + (Sqrt[d + c^2*d*x^2]*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3)))
/(4*Sqrt[1 + c^2*x^2])))/2))/8
```

### 3.137.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.137. $\int x^2(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx$

### 3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1164 vs. 2(291) = 582.

Time = 0.22 (sec) , antiderivative size = 1165, normalized size of antiderivative = 3.46

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1165 |
| parts | Expression too large to display | 1165 |

input
```
int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a/c^2*x*(c^2*d*x^2+d)^(5/2)-5/192*a
/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5/128*a
/c^2*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-5
/256*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+1/1638
4*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7
*x^7+256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/
2)+88*c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*(-1+8*
arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7
+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c
^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh
(c*x))*d^2/c^3/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x
^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2
+1)^(1/2))*(-1+4*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(
1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2
*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-
2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^
2/c^3/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x
^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2)
)*(1+4*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*
c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^...
```

### 3.137.5 Fricas [F]

$$\int x^2 (d + c^2 d x^2)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \int (c^2 d x^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^2\, dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^6 + 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 + 2
*b*c^2*d^2*x^4 + b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

### 3.137.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (d + c^2 d x^2)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \text{Timed out}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

### 3.137.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 d x^2)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.137.8 Giac [F]

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx)) \, dx = \int (c^2 dx^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^2 \, dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

### 3.137.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx)) \, dx = \int x^2 (a + b \operatorname{asinh}(c x)) \left(d c^2 x^2 + d\right)^{5/2} dx$$

input
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.138 $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.138.1 Optimal result

Integrand size = 24, antiderivative size = 193

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=-\frac{bd^2x\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{bcd^2x^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{3bc^3d^2x^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2d}$$

output
```
1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^2/d-1/7*b*d^2*x*(c^2*d*x^2+d)
^(1/2)/c/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)-3/35*b*c^3*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^5
*d^2*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

## 3.138.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.58

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{d^2\sqrt{d+c^2dx^2}\Big(35a(1+c^2x^2)^4-bcx\sqrt{1+c^2x^2}(35+35c^2x^2+21c^4x^4+5c^6x^6)+35b(1}{245c^2(1+c^2x^2)}$$

input
```
Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(35*a*(1 + c^2*x^2)^4 - b*c*x*Sqrt[1 + c^2*x^2]*(
35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^4*ArcSinh[c
*x]))/(245*c^2*(1 + c^2*x^2))
```

## 3.138.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6213, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2d}-\frac{bd^2\sqrt{c^2dx^2+d}\int\left(c^2x^2+1\right)^3dx}{7c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2d}-\frac{bd^2\sqrt{c^2dx^2+d}\int\left(c^6x^6+3c^4x^4+3c^2x^2+1\right)dx}{7c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2d}-\frac{bd^2\left(\frac{c^6x^7}{7}+\frac{3c^4x^5}{5}+c^2x^3+x\right)\sqrt{c^2dx^2+d}}{7c\sqrt{c^2x^2+1}}$$

input
```
Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/7*(b*d^2*Sqrt[d + c^2*d*x^2]*(x + c^2*x^3 + (3*c^4*x^5)/5 + (c^6*x^7)/7
))/(c*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7
*c^2*d)
```

### 3.138.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(165) = 330$.

Time = 0.22 (sec) , antiderivative size = 863, normalized size of antiderivative = 4.47

| method | result |
|---|---|
| default | $\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{7c^2d} + b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\,\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+56c^3x^3\sqrt{c^2x^2+1}+25c^2x^2+\right.}{6272c^2(c^2x^2+1)}$ |
| parts | $\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{7c^2d} + b\Bigg(\frac{\sqrt{d(c^2x^2+1)}\,\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+56c^3x^3\sqrt{c^2x^2+1}+25c^2x^2+\right.}{6272c^2(c^2x^2+1)}$ |

input
```
int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/7*a*(c^2*d*x^2+d)^(7/2)/c^2/d+b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^
8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+1
04*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)
+1)*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(1
6*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(
1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^
2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2
)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^
2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arc
sinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c
^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+
1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+
1)^(1/2)+1)*(3*arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(
1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x
^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d^2/c
^2/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^
2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^
3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x
))*d^2/c^2/(c^2*x^2+1))
```

## 3.138.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.17

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{35\left(bc^8d^2x^8+4\,bc^6d^2x^6+6\,bc^4d^2x^4+4\,bc^2d^2x^2+bd^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)}{}$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/245*(35*(b*c^8*d^2*x^8 + 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 + 4*b*c^2*d^2
*x^2 + b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (35*a*c^8
*d^2*x^8 + 140*a*c^6*d^2*x^6 + 210*a*c^4*d^2*x^4 + 140*a*c^2*d^2*x^2 + 35*
a*d^2 - (5*b*c^7*d^2*x^7 + 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 + 35*b*c*d^
2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

### 3.138.6 Sympy [F]

$$\int x(d + c^2dx^2)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \int x(d(c^2x^2 + 1))^{\frac{5}{2}}(a + b\operatorname{asinh}(cx))\,dx$$

input
```
integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(x*(d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x)), x)
```

### 3.138.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.50

$$\int x(d + c^2dx^2)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \frac{(c^2dx^2 + d)^{\frac{7}{2}}b\operatorname{arsinh}(cx)}{7\,c^2d} + \frac{(c^2dx^2 + d)^{\frac{7}{2}}a}{7\,c^2d} - \frac{\left(5\,c^6d^{\frac{7}{2}}x^7 + 21\,c^4d^{\frac{7}{2}}x^5 + 35\,c^2d^{\frac{7}{2}}x^3 + 35\,d^{\frac{7}{2}}x\right)b}{245\,cd}$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/7*(c^2*d*x^2 + d)^(7/2)*b*arcsinh(c*x)/(c^2*d) + 1/7*(c^2*d*x^2 + d)^(7/
2)*a/(c^2*d) - 1/245*(5*c^6*d^(7/2)*x^7 + 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7
/2)*x^3 + 35*d^(7/2)*x)*b/(c*d)
```

### 3.138.8 Giac [F(-2)]

Exception generated.

$$\int x(d + c^2dx^2)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.138.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int x\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.139 $\int (d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.139.1 Optimal result

Integrand size = 23, antiderivative size = 254

$$\int (d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{25bcd^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{5bc^3d^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}$$
$$-\frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))$$
$$+\frac{5}{24}dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{6}x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(c}{32bc\sqrt{1+c^2x^2}}$$

output
```
5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(c^2*d*x^2+d)^(5/2)*
(a+b*arcsinh(c*x))-1/36*b*d^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16
*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-25/96*b*c*d^2*x^2*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)+5/32*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+
1)^(1/2)
```

### 3.139.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.25

$$\int \left(d+c^2dx^2\right)^{5/2} (a +b\text{arcsinh}(cx))\,dx = \frac{d^2\Big(1584acx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+1248ac^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+384ac^5x^5\sqrt{1+}}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(1584*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 1248*a*c^3*x^3*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqrt
[d + c^2*d*x^2] + 360*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 - 270*b*Sqrt[d
+ c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 27*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSin
h[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 720*a*Sqrt[d]*Sqr
t[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 12*b*Sqrt[d + c^
2*d*x^2]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] +
Sinh[6*ArcSinh[c*x]])))/(2304*c*Sqrt[1 + c^2*x^2])
```

### 3.139.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6201$$

$$\frac{5}{6}d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2dx}{6\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \; 241$$

$$\frac{5}{6}d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$\Big\downarrow$ 6201

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$\Big\downarrow$ 244

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2x^3+x\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$\Big\downarrow$ 2009

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+c}}{4\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$\Big\downarrow$ 6200

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2d\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$\Big\downarrow$ 15

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)+\frac{1}{4}x(c^2dx^2+\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$\Big\downarrow$ 6198

3.139. $\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{5}{6}d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(c\right.\right.}{4bc\sqrt{c^2x^2+1}}$$
$$\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

input
```
Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/36*(b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/c + (x*(d + c^2*d*x^
2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*d*(-1/4*(b*c*d*Sqrt[d + c^2*d*x^2]*(
x^2/2 + (c^2*x^4)/4))/Sqrt[1 + c^2*x^2] + (x*(d + c^2*d*x^2)^(3/2)*(a + b*
ArcSinh[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x
^2] + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])))/4))/6
```

### 3.139.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

### 3.139.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(218) = 436$.

Time = 0.18 (sec) , antiderivative size = 801, normalized size of antiderivative = 3.15

| method | result |
|---|---|
| default | $\frac{x(c^2d\,x^2+d)^{\frac{5}{2}}a}{6}+\frac{5adx(c^2d\,x^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{c^2d\,x^2+d}}{16}+\frac{5a\,d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16\sqrt{c^2d}}+b\left(\frac{5\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{32\sqrt{c^2x^2+1}\,c}\right.$ |
| parts | $\frac{x(c^2d\,x^2+d)^{\frac{5}{2}}a}{6}+\frac{5adx(c^2d\,x^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{c^2d\,x^2+d}}{16}+\frac{5a\,d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16\sqrt{c^2d}}+b\left(\frac{5\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{32\sqrt{c^2x^2+1}\,c}\right.$ |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(c^2*d*x^2+d)^(5/2)*a+5/24*a*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(c
^2*d*x^2+d)^(1/2)+5/16*a*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))
/(c^2*d)^(1/2)+b*(5/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c
*x)^2*d^2+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^
(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x
^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1)
+3/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3
*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*
x))*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c
^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))*d^2/c/(c^2*x^
2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2
*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+3/512*(d*(c^2
*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2
*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))*d^2/c/(c^2*
x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/
2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+
1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1))
```

### 3.139.5 Fricas [F]

$$\int (d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \int (c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

### 3.139.6 Sympy [F]

$$\int (d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \int \left(d(c^2x^2+1)\right)^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x)), x)
```

### 3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.139.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.139.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.140 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$



## 3.140.1 Optimal result

Integrand size = 26, antiderivative size = 329

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx = -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}$$
$$-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{3}d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))$$
$$+\frac{1}{5}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx)) - \frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{bd^2\sqrt{d+c^2dx^2}}{}$$

output
```
1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/5*(c^2*d*x^2+d)^(5/2)*(a+b*
arcsinh(c*x))+d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-23/15*b*c*d^2*x*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-11/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)-1/25*b*c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)-b*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.140.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.03

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx = \frac{-40bcd^3x\sqrt{1+c^2x^2}(3+c^2x^2)-3bc^3d^3x^3\sqrt{1+c^2x^2}(5+3c^2x^2)+}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
(-40*b*c*d^3*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2) - 3*b*c^3*d^3*x^3*Sqrt[1 +
c^2*x^2]*(5 + 3*c^2*x^2) + 15*a*d^3*(1 + c^2*x^2)*(23 + 11*c^2*x^2 + 3*c^4
*x^4) + 150*b*d^3*(1 + c^2*x^2)^2*ArcSinh[c*x] + 15*b*d^3*(1 + c^2*x^2)^2*
(-2 + 3*c^2*x^2)*ArcSinh[c*x] + 225*a*d^(5/2)*Sqrt[d + c^2*d*x^2]*Log[x] -
 225*a*d^(5/2)*Sqrt[d + c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] +
225*b*d^3*Sqrt[1 + c^2*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*
x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/(2
25*Sqrt[d + c^2*d*x^2])
```

### 3.140.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6223, 210, 2009, 6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$$

$$\Big\downarrow \text{6223}$$

$$d\int \frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{bcd^2\sqrt{c^2dx^2+d}\int (c^2x^2+1)^2\,dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{210}$$

$$d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{bcd^2\sqrt{c^2dx^2+d}\int \left(c^4x^4+2c^2x^2+1\right)dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6223}$$

$$d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx-\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2x^2+1\right)dx}{3\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))\right)$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}\right)+$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6221}$$

$$d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int 1dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{3}\left(c^2dx^2+d\right)\right.$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{24}$$

$$d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a\right.$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

---

3.140. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

$\Big\downarrow$ 6231

$$d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)+\frac{1}{3}\left(c^2dx^2\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 3042

$$d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c}}{\sqrt{c^2}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 26

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2}}{\sqrt{c^2}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 4670

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{ar}}\right.\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2715

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsin}}\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2838

---

3.140. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx$

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))+ib\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)-ib\operatorname{PolyLog}\left(2,e^{\operatorname{arcsi}}\right.\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\operatorname{arcsinh}(cx))-\frac{bcd^2\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{5\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

output
```
-1/5*(b*c*d^2*Sqrt[d + c^2*d*x^2]*(x + (2*c^2*x^3)/3 + (c^4*x^5)/5))/Sqrt[
1 + c^2*x^2] + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/5 + d*(-1/3*(b
*c*d*Sqrt[d + c^2*d*x^2]*(x + (c^2*x^3)/3))/Sqrt[1 + c^2*x^2] + ((d + c^2*
d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + d*(-((b*c*x*Sqrt[d + c^2*d*x^2])/Sq
rt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]) + (I*Sqrt[d +
c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyL
og[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2
]))
```

### 3.140.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.140. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx$

## 3.140.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| default | $\frac{(c^2 d\,x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d (c^2 d\,x^2+d)^{\frac{3}{2}}}{3}-a\,d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)+a\,d^2\sqrt{c^2 d\,x^2+d}-\frac{b\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(2,-c\right.}{\sqrt{c^2x^2+1}}$ |
| parts | $\frac{(c^2 d\,x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d (c^2 d\,x^2+d)^{\frac{3}{2}}}{3}-a\,d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)+a\,d^2\sqrt{c^2 d\,x^2+d}-\frac{b\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(2,-c\right.}{\sqrt{c^2x^2+1}}$ |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*(c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2*
d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a*d^2*(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2+23/15
*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)+b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)
)*d^2+1/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^6*c^6-1/2
5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5+14/15*b*(d*(c^2*x^
2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-11/45*b*(d*(c^2*x^2+1))^(
1/2)*d^2/(c^2*x^2+1)^(1/2)*c^3*x^3+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*
x^2+1)*arcsinh(c*x)*x^2*c^2-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^
(1/2)*c*x
```

## 3.140.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{x}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

### 3.140.6 Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\left(d(c^2x^2+1)\right)^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))}{x}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x, x)
```

### 3.140.7 Maxima [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

output
```
-1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2
*d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a + b*integrate((c^2*d*x
^2 + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.140.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2 d x^2)^{5/2} (a + b \text{arcsinh}(cx))}{x} \, dx = \int \frac{(a + b \operatorname{asinh}(c x)) \, (d c^2 x^2 + d)^{5/2}}{x} \, dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x, x)
```

# 3.141 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$



## 3.141.1 Optimal result

Integrand size = 26, antiderivative size = 257

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx &= -\frac{9bc^3d^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}\\&-\frac{bc^5d^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\&+\frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))-\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\\&+\frac{15cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16b\sqrt{1+c^2x^2}}+\frac{bcd^2\sqrt{d+c^2dx^2}\log(x)}{\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-(c^2*d*x^2+d)^(5/2)*(a+
b*arcsinh(c*x))/x+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-9/
16*b*c^3*d^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^5*d^2*x^4*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+15/16*c*d^2*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d^2*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)
```

### 3.141.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \frac{1}{128}d^2\left(\frac{16a\sqrt{d+c^2dx^2}(-8+9c^2x^2+2c^4x^4)}{x} + \frac{64b\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\text{arcsinh}(cx)+cx\text{arcsinh}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}} + 240ac\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{32bc\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{\sqrt{1+c^2x^2}} - \frac{bc\sqrt{d+c^2dx^2}(8\text{arcsinh}(cx)^2+\cosh(4\text{arcsinh}(cx))-4\text{arcsinh}(cx)\sinh(4\text{arcsinh}(cx)))}{\sqrt{1+c^2x^2}}\right)$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
(d^2*((16*a*Sqrt[d + c^2*d*x^2]*(-8 + 9*c^2*x^2 + 2*c^4*x^4))/x + (64*b*Sq
rt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2
+ 2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x^2]) + 240*a*c*Sqrt[d]*Log[c*d*x + Sqr
t[d]*Sqrt[d + c^2*d*x^2]] + (32*b*c*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c
*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2
*x^2] - (b*c*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]]
- 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2]))/128
```

### 3.141.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6222, 243, 49, 2009, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{ 6222}$$

$$5c^2d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}$$

$\downarrow$ 243

$$5c^2d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2}{x^2}dx^2}{2\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}$$

$\downarrow$ 49

$$5c^2d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \left(x^2c^4+2c^2+\frac{1}{x^2}\right)dx^2}{2\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}$$

$\downarrow$ 2009

$$5c^2d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 6201

$$5c^2d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsin}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 244

$$5c^2d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2x^3+x\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

3.141. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}dx$

$$5c^2d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+}}{4\sqrt{c^2x^2+1}}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 6200

$$5c^2d\left(\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 15

$$5c^2d\left(\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)+\frac{1}{4}x(c^2dx^2\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 6198

$$-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+$$
$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(}{4bc\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^4}{2}+2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

output
```
-(((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x) + 5*c^2*d*(-1/4*(b*c*d*S
qrt[d + c^2*d*x^2]*(x^2/2 + (c^2*x^4)/4))/Sqrt[1 + c^2*x^2] + (x*(d + c^2*
d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sqrt[d + c^2*d*
x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 +
 (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])))/
4) + (b*c*d^2*Sqrt[d + c^2*d*x^2]*(2*c^2*x^2 + (c^4*x^4)/2 + Log[x^2]))/(2
*Sqrt[1 + c^2*x^2])
```

3.141. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

### 3.141.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

## 3.141. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx$

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.141.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| default | $-\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{dx}+a\,c^2x(c^2d\,x^2+d)^{\frac{5}{2}}+\frac{5(c^2d\,x^2+d)^{\frac{3}{2}}a\,c^2dx}{4}+\frac{15a\,d^2\sqrt{c^2d\,x^2+d}\,c^2x}{8}+\frac{15a\,c^2d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+\ldots}\right.}{8\sqrt{c^2d}}$ |
| parts | $-\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{dx}+a\,c^2x(c^2d\,x^2+d)^{\frac{5}{2}}+\frac{5(c^2d\,x^2+d)^{\frac{3}{2}}a\,c^2dx}{4}+\frac{15a\,d^2\sqrt{c^2d\,x^2+d}\,c^2x}{8}+\frac{15a\,c^2d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+\ldots}\right.}{8\sqrt{c^2d}}$ |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(c^2*d*x^2+d)^(7/2)+a*c^2*x*(c^2*d*x^2+d)^(5/2)+5/4*(c^2*d*x^2+d)^(
3/2)*a*c^2*d*x+15/8*a*d^2*(c^2*d*x^2+d)^(1/2)*c^2*x+15/8*a*c^2*d^3*ln(c^2*
d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/128*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/x*(32*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-8*
c^5*x^5+144*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-72*c^3*x^3+120*arcsinh(
c*x)^2*x*c-128*arcsinh(c*x)*c*x+128*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x*c-12
8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-33*c*x)*d^2
```

### 3.141.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}(cx)+a)}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

### 3.141.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{5}{2}}(a+b\,\text{asinh}(cx))}{x^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**2, x)
```

### 3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.141.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

# 3.142 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))}{x^3}\,dx$



## 3.142.1 Optimal result

Integrand size = 26, antiderivative size = 355

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))}{x^3}\,dx &= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} \\ &- \frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx)) \\ &+ \frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\,\mathrm{arcsinh}(cx)) - \frac{(d+c^2dx^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))}{2x^2} \\ &- \frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))\mathrm{arctanh}\left(e^{\mathrm{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &- \frac{5bc^2d^2\sqrt{d+c^2dx^2}\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}} \\ &+ \frac{5bc^2d^2\sqrt{d+c^2dx^2}\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(cx)}\right)}{2\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-1/2*(c^2*d*x^2+d)^(5/2)*(
a+b*arcsinh(c*x))/x^2+5/2*c^2*d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1
/2*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-7/3*b*c^3*d^2*x*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^5*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*
x^2+1)^(1/2)-5*c^2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/2*b*c^2*d^2*polylog(2,-c*x-(c^2*x^2
+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/2*b*c^2*d^2*polylog(2,c
*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.142.2 Mathematica [A] (verified)

Time = 6.41 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.19

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx = \frac{1}{72}d^2\left(\frac{12a\sqrt{d+c^2dx^2}(-3+14c^2x^2+2c^4x^4)}{x^2}\right.$$
$$-\frac{8bc^2\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{\sqrt{1+c^2x^2}}$$
$$+180ac^2\sqrt{d}\log(x)-180ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{144bc^2\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(d^2*((12*a*Sqrt[d + c^2*d*x^2]*(-3 + 14*c^2*x^2 + 2*c^4*x^4))/x^2 - (8*b*
c^2*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c
*x]))/Sqrt[1 + c^2*x^2] + 180*a*c^2*Sqrt[d]*Log[x] - 180*a*c^2*Sqrt[d]*Log
[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (144*b*c^2*Sqrt[d + c^2*d*x^2]*(-(c*x)
 + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])
] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x]
)] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (9*b*c^2*Sqrt[d +
 c^2*d*x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2
 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSi
nh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))
/Sqrt[1 + c^2*x^2]))/72
```

### 3.142.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6222, 244, 2009, 6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}dx$$

$$\Big\downarrow \text{ 6222}$$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2}{x^2}dx}{2\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \left(x^2c^4+2c^2+\frac{1}{x^2}\right)dx}{2\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6223}$$

$$\frac{5}{2}c^2d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx-\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2x^2+1\right)dx}{3\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(c\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5}{2}c^2d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^3}{3}+x\right)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}\right)$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6221}$$

3.142. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}dx$

$$\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int 1dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{3}(c^2dx^2+$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 24

$$\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)+\frac{1}{3}(c^2dx^2+d)^{3/}$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 6231

$$\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}\right)+\frac{1}{3}(c^2$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 3042

$$\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx}{}\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 26

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx}{}\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$\downarrow$ 4670

3.142. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 2715$$

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{a}}\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 2838$$

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e\right.\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{bcd^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\sqrt{c^2dx^2+d}}{2\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

output
```
(b*c*d^2*Sqrt[d + c^2*d*x^2]*(-x^(-1) + 2*c^2*x + (c^4*x^3)/3))/(2*Sqrt[1
+ c^2*x^2]) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (5*c^
2*d*(-1/3*(b*c*d*Sqrt[d + c^2*d*x^2]*(x + (c^2*x^3)/3))/Sqrt[1 + c^2*x^2]
+ ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + d*(-((b*c*x*Sqrt[d + c^
2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]) +
(I*Sqrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]]
 + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt
[1 + c^2*x^2])))/2
```

## 3.142. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

### 3.142.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.142. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.142. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx$

### 3.142.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| default | $a\left(-\frac{(c^2d\,x^2+d)^{\frac{7}{2}}}{2d\,x^2}+\frac{5c^2\left(\frac{(c^2d\,x^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2d\,x^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b\sqrt{d(c^2x^2}{}$ |
| parts | $a\left(-\frac{(c^2d\,x^2+d)^{\frac{7}{2}}}{2d\,x^2}+\frac{5c^2\left(\frac{(c^2d\,x^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2d\,x^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b\sqrt{d(c^2x^2}{}$ |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(c^2*d*x^2+d)^(7/2)+5/2*c^2*(1/5*(c^2*d*x^2+d)^(5/2)+d*(1/3*
(c^2*d*x^2+d)^(3/2)+d*((c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*
d*x^2+d)^(1/2))/x)))))+1/18*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^2*
(6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-2*c^5*x^5+42*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x^2*c^2+45*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-45
*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-42*c^3*x^3+45*polylog(2,
c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-45*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^
2-9*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-9*c*x)*d^2
```

### 3.142.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

### 3.142.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))}{x^3}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**3, x)
```

### 3.142.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima"
)
```

output
```
-1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
 5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d
*x^2 + d)^(7/2)/(d*x^2))*a + b*integrate((c^2*d*x^2 + d)^(5/2)*log(c*x + s
qrt(c^2*x^2 + 1))/x^3, x)
```

### 3.142.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^{5/2} (a + b\text{arcsinh}(cx))}{x^3} dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2 + d)^{5/2}}{x^3} dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3, x)
```

# 3.143 $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$



## 3.143.1 Optimal result

Integrand size = 26, antiderivative size = 266

$$\begin{aligned}\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx &= -\frac{bcd^2\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} \\ &- \frac{bc^5d^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\ &- \frac{5c^2d\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))}{3x} - \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3} \\ &+ \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4b\sqrt{1+c^2x^2}} + \frac{7bc^3d^2\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x-1/3*(c^2*d*x^2+d)^(5/2
)*(a+b*arcsinh(c*x))/x^3+5/2*c^4*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1
/2)-1/6*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)-1/4*b*c^5*d^2*x^
2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/4*c^3*d^2*(a+b*arcsinh(c*x))^2*(
c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*ln(x)*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)
```

### 3.143.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.08

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^4}\,dx = \frac{d^2\left(4a\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(-2-14c^2x^2+3c^4x^4)+24bc^2x^2\sqrt{d}\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
(d^2*(4*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-2 - 14*c^2*x^2 + 3*c^4*x
^4) + 24*b*c^2*x^2*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
+ c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]) - 4*b*Sqrt[d + c^2*d*x^2]*(c*x + 2*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - 2*c^3*x^3*Log[c*x]) + 60*a*c^3*Sqrt[d]*
x^3*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3*b*c^3*x
^3*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x
] + Sinh[2*ArcSinh[c*x]]))))/(24*x^3*Sqrt[1 + c^2*x^2])
```

### 3.143.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6222, 243, 49, 2009, 6222, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{5/2}(a+b\operatorname{arcsinh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{6222}$$

$$\frac{5}{3}c^2d\int \frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^2}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)^2}{x^3}dx}{3\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{5}{3}c^2d\int \frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))}{x^2}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)^2}{x^4}dx^2}{6\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{5/2}(a+b\operatorname{arcsinh}(cx))}{3x^3}$$

$\downarrow$ 49

$$\frac{5}{3}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\int \left(c^4+\frac{2c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}$$

$\downarrow$ 2009

$$\frac{5}{3}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{c^2x^2+1}}$$

$\downarrow$ 6222

$$\frac{5}{3}c^2d\left(3c^2d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{bcd\sqrt{c^2dx^2+d}\int \frac{c^2x^2+1}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$
$$-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{c^2x^2+1}}$$

$\downarrow$ 244

$$\frac{5}{3}c^2d\left(3c^2d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{bcd\sqrt{c^2dx^2+d}\int \left(xc^2+\frac{1}{x}\right)dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(\right.}{x}$$
$$-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$\frac{5}{3}c^2d\left(3c^2d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^2}{2}+\log\right.}{\sqrt{c^2x^2+1}}\right.$$
$$-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{c^2x^2+1}}$$

$\downarrow$ 6200

$$\frac{5}{3}c^2d\left(3c^2d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)-\frac{(c^2}{}\right.$$
$$-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{c^2x^2+1}}$$

---

3.143. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}dx$

$\big\downarrow$ 15

$$\frac{5}{3}c^2d\left(3c^2d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)-\frac{(c^2dx^2\ \right.$$
$$-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 6198

$$\frac{5}{3}c^2d\left(3c^2d\left(\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)-\frac{(c^2dx^2\ \right.$$
$$-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{3x^3}+\frac{bcd^2\sqrt{c^2dx^2+d}\left(c^4x^2+2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

output
```
-1/3*((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3 + (5*c^2*d*(-(((d +
c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x) + 3*c^2*d*(-1/4*(b*c*x^2*Sqrt[d
+ c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*
x^2])) + (b*c*d*Sqrt[d + c^2*d*x^2]*((c^2*x^2)/2 + Log[x]))/Sqrt[1 + c^2*x
^2]))/3 + (b*c*d^2*Sqrt[d + c^2*d*x^2]*(-x^(-2) + c^4*x^2 + 2*c^2*Log[x^2]
))/(6*Sqrt[1 + c^2*x^2])
```

### 3.143.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

3.143. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.143. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx$

### 3.143.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| default | $-\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{3dx^3}-\frac{4ac^2(c^2dx^2+d)^{\frac{7}{2}}}{3dx}+\frac{4ac^4x(c^2dx^2+d)^{\frac{5}{2}}}{3}+\frac{5ac^4dx(c^2dx^2+d)^{\frac{3}{2}}}{3}+\frac{5ac^4d^2x\sqrt{c^2dx^2+d}}{2}+\frac{5ac^4d^3\ln\left(\cdots\right.}{}$ |
| parts | $-\frac{a(c^2dx^2+d)^{\frac{7}{2}}}{3dx^3}-\frac{4ac^2(c^2dx^2+d)^{\frac{7}{2}}}{3dx}+\frac{4ac^4x(c^2dx^2+d)^{\frac{5}{2}}}{3}+\frac{5ac^4dx(c^2dx^2+d)^{\frac{3}{2}}}{3}+\frac{5ac^4d^2x\sqrt{c^2dx^2+d}}{2}+\frac{5ac^4d^3\ln\left(\cdots\right.}{}$ |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(c^2*d*x^2+d)^(7/2)-4/3*a*c^2/d/x*(c^2*d*x^2+d)^(7/2)+4/3*a*c
^4*x*(c^2*d*x^2+d)^(5/2)+5/3*a*c^4*d*x*(c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d^2*x
*(c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+1/24*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^3*(1
2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4-6*c^5*x^5+30*arcsinh(c*x)^2*x^3*c
^3-56*arcsinh(c*x)*c^3*x^3+56*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-56*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-3*c^3*x^3-8*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)-4*c*x)*d^2
```

### 3.143.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

### 3.143.6 Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{\left(d(c^2x^2+1)\right)^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))}{x^4}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**4, x)
```

### 3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.143.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

# 3.144 $\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx$



## 3.144.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\text{arcsinh}(x)+\frac{\text{arcsinh}(x)^2}{4}$$

output
```
-1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)
```

## 3.144.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx=\frac{1}{4}\left(-x^2+2x\sqrt{1+x^2}\text{arcsinh}(x)+\text{arcsinh}(x)^2\right)$$

input
```
Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

output
```
(-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

## 3.144.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+1}\text{arcsinh}(x)\,dx$$

$$\Big\downarrow \text{6200}$$

$$\frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{x^2+1}}dx-\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{x^2+1}\text{arcsinh}(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{x^2+1}}dx+\frac{1}{2}\sqrt{x^2+1}x\text{arcsinh}(x)-\frac{x^2}{4}$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}\sqrt{x^2+1}x\text{arcsinh}(x)+\frac{\text{arcsinh}(x)^2}{4}-\frac{x^2}{4}$$

input
```
Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

output
```
-1/4*x^2 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

### 3.144.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

### 3.144.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $\frac{x\,\mathrm{arcsinh}(x)\sqrt{x^2+1}}{2}+\frac{\mathrm{arcsinh}(x)^2}{4}-\frac{x^2}{4}-\frac{1}{4}$ | 26 |

input
```
int(arcsinh(x)*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

### 3.144.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \sqrt{1+x^2}\mathrm{arcsinh}(x)\,dx=\frac{1}{2}\sqrt{x^2+1}x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}x^2+\frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

input
```
integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^
2 + 1))^2
```

### 3.144.6 Sympy [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{asinh}\,(x)\,dx$$

input
```
integrate(asinh(x)*(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 1)*asinh(x), x)
```

### 3.144.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = -\frac{1}{4}\,x^2 + \frac{1}{2}\left(\sqrt{x^2+1}x + \text{arsinh}\,(x)\right)\text{arsinh}\,(x) - \frac{1}{4}\,\text{arsinh}\,(x)^2$$

input
```
integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^
2
```

### 3.144.8 Giac [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{arsinh}\,(x)\,dx$$

input
```
integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

### 3.144.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx = \int \operatorname{asinh}(x)\,\sqrt{x^2+1}\,dx$$

input
```
int(asinh(x)*(x^2 + 1)^(1/2),x)
```

output
```
int(asinh(x)*(x^2 + 1)^(1/2), x)
```

# 3.145 $\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$



## 3.145.1 Optimal result

Integrand size = 26, antiderivative size = 215

$$\begin{aligned}\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx =& -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}}+\frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}}\\ &-\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{15c^6d}\\ &-\frac{4x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{15c^4d}\\ &+\frac{x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2d}\end{aligned}$$

output
```
-8/15*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+4/45*b*x^3*(c^2*x^2+1)
^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/25*b*x^5*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d
)^(1/2)+8/15*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d-4/15*x^2*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+1/5*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^
2+d)^(1/2)/c^2/d
```

### 3.145.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.55

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$=\frac{bcx\sqrt{1+c^2x^2}(-120+20c^2x^2-9c^4x^4)+15a(8+4c^2x^2-c^4x^4+3c^6x^6)+15b(8+4c^2x^2-c^4x^4+3c^6x^6)}{225c^6\sqrt{d+c^2dx^2}}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4*x^4) + 15*a*(8 + 4*c^2
*x^2 - c^4*x^4 + 3*c^6*x^6) + 15*b*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*A
rcSinh[c*x])/(225*c^6*Sqrt[d + c^2*d*x^2])
```

### 3.145.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6227, 15, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \; 6227$$
$$-\frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{5c^2}-\frac{b\sqrt{c^2x^2+1}\int x^4dx}{5c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2d}$$
$$\Big\downarrow \; 15$$
$$-\frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bx^5\sqrt{c^2x^2+1}}{25c\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \; 6227$$

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{b\sqrt{c^2x^2+1}\int x^2dx}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}\right)}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bx^5\sqrt{c^2x^2+1}}{25c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}\right)}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bx^5\sqrt{c^2x^2+1}}{25c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6213}$$

$$-\frac{4\left(-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}\int 1dx}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}\right)}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{bx^5\sqrt{c^2x^2+1}}{25c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2d}-\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}\right)}{5c^2}-\frac{bx^5\sqrt{c^2x^2+1}}{25c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
-1/25*(b*x^5*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (x^4*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*d) - (4*(-1/9*(b*x^3*Sqrt[1 + c^2*x^
2])/(c*Sqrt[d + c^2*d*x^2]) + (x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(3*c^2*d) - (2*(-((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^2*d)))/(3*c^2)))/(5*c^2)
```

3.145. $\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}dx$

### 3.145.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.145.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(185) = 370$.

Time = 0.22 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.91

| method | result |
|---|---|
| default | $a\left(\frac{x^4\sqrt{c^2d\,x^2+d}}{5c^2d}-\frac{4\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)}{5c^2}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+}\right.}{800c^6d(c^2x^2+1)}\right.$ |
| parts | $a\left(\frac{x^4\sqrt{c^2d\,x^2+d}}{5c^2d}-\frac{4\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)}{5c^2}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+}\right.}{800c^6d(c^2x^2+1)}\right.$ |

input
```
int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^
(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c
^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2
)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2
+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/
16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x
))/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(
1/2)+1)*(arcsinh(c*x)+1)/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*
c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(
3*arcsinh(c*x)+1)/c^6/d/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^
6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*
c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

### 3.145.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{15\,(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6-15\,ac^4x^4+60\,ac^2x^2-\right.}{225\,(c^8dx^2+c^6d)}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/225*(15*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*sqrt(c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 - 15*a*c^4*x^4 + 60*a*c^2*x
^2 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^2*x^2 + 1) + 120*a)*s
qrt(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

### 3.145.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^5(a+b\,\text{asinh}\,(cx))}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.145.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)b\,\text{arsinh}\,(cx) + \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)a - \frac{(9\,c^4x^5-20\,c^2x^3+120\,x)b}{225\,c^5\sqrt{d}}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d
) + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2
 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 +
 d)/(c^6*d))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(c^5*sqrt(d))
```

### 3.145.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^5\,(a + b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.146 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$



## 3.146.1 Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{4c^2d} + \frac{3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{16bc^5\sqrt{d+c^2dx^2}}$$

output
```
3/16*b*x^2*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/16*b*x^4*(c^2*x^2+1
)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+3/16*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/
b/c^5/(c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4
/d+1/4*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

## 3.146.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$=\frac{\frac{16acx(-3+2c^2x^2)\sqrt{d+c^2dx^2}}{d}+\frac{48a\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}}+\frac{b\sqrt{1+c^2x^2}(16\cosh(2\text{arcsinh}(cx))-\cosh(4\text{arcsinh}(cx))+4\text{arcsinh}(cx}{\sqrt{d+c^2dx}}}{128c^5}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
((16*a*c*x*(-3 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/d + (48*a*Log[c*d*x + Sqr
t[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(16*Cosh[2*ArcSi
nh[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c*x]*(6*ArcSinh[c*x] - 8*Sinh[
2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/(128*c^5)
```

## 3.146.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$
$$\Big\downarrow \text{6227}$$
$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{4c^2}-\frac{b\sqrt{c^2x^2+1}\int x^3dx}{4c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{4c^2d}$$
$$\Big\downarrow \text{15}$$
$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{4c^2d}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \text{6227}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}-\frac{b\sqrt{c^2x^2+1}\int xdx}{2c\sqrt{c^2dx^2+d}}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}\right)}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{4c^2d}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}\right)}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{4c^2d}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{4c^2d}-\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}\right)}{4c^2}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
-1/16*(b*x^4*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (x^3*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*c^2*d) - (3*(-1/4*(b*x^2*Sqrt[1 + c^2*x^
2])/(c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))
/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^3*Sqrt[d +
c^2*d*x^2])))/(4*c^2)
```

### 3.146.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.146. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.146.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(166) = 332$.

Time = 0.20 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.70

| method | result |
|---|---|
| default | $\frac{a x^3 \sqrt{c^2 d\,x^2+d}}{4 c^2 d}-\frac{3 a x \sqrt{c^2 d\,x^2+d}}{8 c^4 d}+\frac{3 a \ln\left(\frac{c^2 d x}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{8 c^4 \sqrt{c^2 d}}+b\left(\frac{3 \sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c^5 d}+\frac{\sqrt{d(c^2x^2+1)}\,\left(8c^5x^5+8c\right.}{}\right.$ |
| parts | $\frac{a x^3 \sqrt{c^2 d\,x^2+d}}{4 c^2 d}-\frac{3 a x \sqrt{c^2 d\,x^2+d}}{8 c^4 d}+\frac{3 a \ln\left(\frac{c^2 d x}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{8 c^4 \sqrt{c^2 d}}+b\left(\frac{3 \sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{16\sqrt{c^2x^2+1}\,c^5 d}+\frac{\sqrt{d(c^2x^2+1)}\,\left(8c^5x^5+8c\right.}{}\right.$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3/8*
a/c^4*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(3/16*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+1/256*(d*(c^2
*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2
*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^5/d/(c^2
*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+
2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(d*(c^
2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(
1/2))*(1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*
c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)
+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(c*x))/c^5/d/(c^2*x^2+1))
```

### 3.146.5 Fricas [F]

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)x^4}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(c^2*d*x^2 + d), x)
```

### 3.146.6 Sympy [F]

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^4(a + b\operatorname{asinh}(cx))}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**4*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\operatorname{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.146.8 Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^4/sqrt(c^2*d*x^2 + d), x)
```

### 3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^4\,(a+b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.147 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$



## 3.147.1 Optimal result

Integrand size = 26, antiderivative size = 142

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2d}$$

output
```
2/3*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/9*b*x^3*(c^2*x^2+1)^(1
/2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d
+1/3*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

## 3.147.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{bcx(6-c^2x^2)\sqrt{1+c^2x^2}+3a(-2-c^2x^2+c^4x^4)+3b(-2-c^2x^2+c^4x^4)\,\text{arcsinh}(cx)}{9c^4\sqrt{d+c^2dx^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(b*c*x*(6 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-2 - c^2*x^2 + c^4*x^4) + 3*
b*(-2 - c^2*x^2 + c^4*x^4)*ArcSinh[c*x])/(9*c^4*Sqrt[d + c^2*d*x^2])
```

### 3.147.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}\,dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{b\sqrt{c^2x^2+1}\int x^2dx}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6213}$$

$$-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}\int 1dx}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
-1/9*(b*x^3*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (x^2*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2*d) - (2*(-((b*x*Sqrt[1 + c^2*x^2])/(c
*Sqrt[d + c^2*d*x^2])) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^2*d
)))/(3*c^2)
```

#### 3.147.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.147.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(122) = 244$.

Time = 0.20 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.52

| method | result |
|---|---|
| default | $a\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\,\operatorname{arcsinh}(cx))}{72c^4d(c^2x^2+1)}\right.$ |
| parts | $a\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)(-1+3\,\operatorname{arcsinh}(cx))}{72c^4d(c^2x^2+1)}\right.$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b*(1/7
2*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3
*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^4/d/
(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(a
rcsinh(c*x)+1)/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c
^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c
*x)+1)/c^4/d/(c^2*x^2+1))
```

## 3.147.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{3\left(bc^4x^4-bc^2x^2-2\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4-3\,ac^2x^2-\left(bc^3x^3-6\,bcx\right)\sqrt{c^2x^2+1}\right.}{9\left(c^6dx^2+c^4d\right)}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/9*(3*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) + (3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^3 - 6*b*c*x)*sqrt(c^2*
x^2 + 1) - 6*a)*sqrt(c^2*d*x^2 + d))/(c^6*d*x^2 + c^4*d)
```

## 3.147.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^3(a+b\,\text{asinh}\,(cx))}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.147.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.82

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{1}{3}b\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx) + \frac{1}{3}a\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)-\frac{(c^2x^3-6x)b}{9c^3\sqrt{d}}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
1/3*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*ar
csinh(c*x) + 1/3*a*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d
)/(c^4*d)) - 1/9*(c^2*x^3 - 6*x)*b/(c^3*sqrt(d))
```

### 3.147.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.148 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$



## 3.148.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}$$

output
```
-1/4*b*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*(a+b*arcsinh(c*x))^
2*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))*(c^
2*d*x^2+d)^(1/2)/c^2/d
```

## 3.148.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{-\frac{4acx\sqrt{d+c^2dx^2}}{d}+\frac{4a\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}}+\frac{b\sqrt{1+c^2x^2}(\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)-\sinh(2\text{arcsinh}(\ldots}{\sqrt{d+c^2dx^2}}}{8c^3}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
-1/8*((-4*a*c*x*Sqrt[d + c^2*d*x^2])/d + (4*a*Log[c*d*x + Sqrt[d]*Sqrt[d +
 c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[c*x]] + 2*Arc
Sinh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/c^3
```

### 3.148.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}-\frac{b\sqrt{c^2x^2+1}\int xdx}{2c\sqrt{c^2dx^2+d}}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
-1/4*(b*x^2*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

### 3.148.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.148.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(103) = 206$.

Time = 0.21 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.29

| method | result |
|---|---|
| default | $\frac{ax\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c^3d}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+\right.}{16c^3d(c^2x^2+1)}\right.$ |
| parts | $\frac{ax\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c^3d}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx+\right.}{16c^3d(c^2x^2+1)}\right.$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*
d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(-1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^3/d*arcsinh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(
c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x
^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*
c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1))
```

### 3.148.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(c^2*d*x^2 + d), x)
```

### 3.148.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^2(a+b\,\text{asinh}\,(cx))}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.148.8 Giac [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^2}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/sqrt(c^2*d*x^2 + d), x)
```

### 3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^2\,(a + b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.149 $\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$



## 3.149.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^2d}$$

output
```
-b*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)/c^2/d
```

## 3.149.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-bcx+a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\text{arcsinh}(cx)\right)}{c^2d\sqrt{1+c^2x^2}}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x]))/(c^2*d*Sqrt[1 + c^2*x^2])
```

### 3.149.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^2d}-\frac{b\sqrt{c^2x^2+1}\int 1dx}{c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
-((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x]))/(c^2*d)
```

#### 3.149.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(58) = 116.

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.31

| method | result |
|---|---|
| default | $\frac{a\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)(-1+\operatorname{arcsinh}(cx))}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)(\operatorname{arcsinh}(cx)}{2c^2d(c^2x^2+1)}\right.$ |
| parts | $\frac{a\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)(-1+\operatorname{arcsinh}(cx))}{2c^2d(c^2x^2+1)}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)(\operatorname{arcsinh}(cx)}{2c^2d(c^2x^2+1)}\right.$ |

input
```
int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a/c^2/d*(c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2
*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^2/d/(c^2*x^2+1))
```

### 3.149.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.50

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{(bc^2x^2+b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2x^2-\sqrt{c^2x^2+1}bcx+a\right)\sqrt{c^2dx^2+d}}{c^4dx^2+c^2d}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
((b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2
*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*
d)
```

### 3.149.6 Sympy [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x(a + b\,\text{asinh}\,(cx))}{\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.149.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = -\frac{bx}{c\sqrt{d}} + \frac{\sqrt{c^2dx^2 + d}b\,\text{arsinh}\,(cx)}{c^2d} + \frac{\sqrt{c^2dx^2 + d}a}{c^2d}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
-b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d
*x^2 + d)*a/(c^2*d)
```

### 3.149.8 Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/sqrt(c^2*d*x^2 + d), x)
```

### 3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.150 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx$



## 3.150.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

## 3.150.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d\,(1+c^2x^2)}}+\frac{a\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{c\sqrt{d}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d*(1 + c^2*x^2)]) + (a*ArcT
anh[(c*Sqrt[d]*x)/Sqrt[d + c^2*d*x^2]])/(c*Sqrt[d])
```

## 3.150.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{c^2dx^2 + d}}\,dx$$

$$\Big\downarrow \; 6198$$

$$\frac{\sqrt{c^2x^2 + 1}(a + b\text{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2 + d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

### 3.150.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.150.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

| method | result | size |
|---|---|---|
| default | $\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}$ | 77 |
| parts | $\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}$ | 77 |

input
```
int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

## 3.150.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

## 3.150.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

## 3.150.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{b\,\text{arsinh}\,(cx)^2}{2\,c\sqrt{d}}+\frac{a\,\text{arsinh}\,(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

### 3.150.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

### 3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.151 $\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx$



## 3.151.1 Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}$$

output
```
-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^
2*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^
2*d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2
*d*x^2+d)^(1/2)
```

## 3.151.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \frac{a\log(x)}{\sqrt{d}} - \frac{a\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)\left(\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\log\left(1+e^{-\text{arcsinh}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-\text{arcsinh}(cx)}\right)-\right.}{\sqrt{d(1+c^2x^2)}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/Sqrt[d] +
(b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E
^(-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSi
nh[c*x])]))/Sqrt[d*(1 + c^2*x^2)]
```

## 3.151.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{ 6231}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{i\sqrt{c^2x^2+1}\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{i\sqrt{c^2x^2+1}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.}{\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2838

$$\frac{i\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(I*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] +
 I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt[d
 + c^2*d*x^2]
```

### 3.151.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.151.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.91

| method | result |
|---|---|
| default | $-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{\sqrt{d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}+\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\right.$ |
| parts | $-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{\sqrt{d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}+\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\right.$ |

input
```
int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*((d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2
)))
```

## 3.151.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{c^2dx^2+d}\,x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

### 3.151.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{x\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.151.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x), x) - a*a
rcsinh(1/(c*abs(x)))/sqrt(d)
```

### 3.151.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x), x)
```

### 3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.152 $\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx$



## 3.152.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{dx}+\frac{bc\sqrt{1+c^2x^2}\log(x)}{\sqrt{d+c^2dx^2}}$$

output
```
b*c*ln(x)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))*(c^2*d*
x^2+d)^(1/2)/d/x
```

## 3.152.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-a\sqrt{1+c^2x^2}-b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+bcx\log(x)\right)}{dx\sqrt{1+c^2x^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-(a*Sqrt[1 + c^2*x^2]) - b*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + b*c*x*Log[x]))/(d*x*Sqrt[1 + c^2*x^2])
```

### 3.152.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{x}dx}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{dx}$$

$$\Big\downarrow \text{14}$$

$$\frac{bc\sqrt{c^2x^2+1}\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{dx}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
-((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[1 + c^2*x^
2]*Log[x])/Sqrt[d + c^2*d*x^2]
```

#### 3.152.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

### 3.152.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(57) = 114$.

Time = 0.24 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

| method | result |
|---|---|
| default | $-\frac{a\sqrt{c^2 d\,x^2+d}}{dx}+b\left(-\frac{2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c}{\sqrt{c^2x^2+1}\,d}-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)}{(c^2x^2+1)dx}+\frac{\sqrt{d(c^2x^2+1)}\,\ln\left(\left(\right.\right.}{\sqrt{c^2}}\right.$ |
| parts | $-\frac{a\sqrt{c^2 d\,x^2+d}}{dx}+b\left(-\frac{2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c}{\sqrt{c^2x^2+1}\,d}-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)}{(c^2x^2+1)dx}+\frac{\sqrt{d(c^2x^2+1)}\,\ln\left(\left(\right.\right.}{\sqrt{c^2}}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(c^2*d*x^2+d)^(1/2)+b*(-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
*arcsinh(c*x)*c-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*ar
csinh(c*x)/(c^2*x^2+1)/d/x+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c
*x+(c^2*x^2+1)^(1/2))^2-1)*c)
```

### 3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx = \frac{bc\sqrt{d}x\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}(x^4-1)\sqrt{d}+d}{c^2x^4+x^2}\right)-2\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2}}{2\,dx}$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/2*(b*c*sqrt(d)*x*log((c^2*d*x^6 + c^2*d*x^2 + d*x^4 + sqrt(c^2*d*x^2 + d
)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x^4 + x^2)) - 2*sqrt(c^2*d
*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*a)/(d*x)
```

### 3.152.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{x^2\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.152.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.60

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\sqrt{d + c^2dx^2}}\,dx = -\frac{\left((-1)^{2\,c^2dx^2+2\,d}\,\sqrt{d}\log\left(2\,c^2d + \frac{2\,d}{x^2}\right) - \sqrt{d}\log\left(x^2 + \frac{1}{c^2}\right)\right)bc}{2\,d} - \frac{\sqrt{c^2dx^2 + d}b\,\text{arsinh}\,(cx)}{dx} - \frac{\sqrt{c^2dx^2 + d}a}{dx}$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/2*((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*lo
g(x^2 + 1/c^2))*b*c/d - sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(d*x) - sqrt(c^
2*d*x^2 + d)*a/(d*x)
```

### 3.152.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

### 3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{x^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\operatorname{asinh}(c\,x)}{x^2\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.153 $\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$



## 3.153.1 Optimal result

Integrand size = 26, antiderivative size = 203

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx =& -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2dx^2}\\&+\frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}\\&+\frac{bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}}\\&-\frac{bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-1/2*b*c*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*arcsinh(c*x))*ar
ctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/2*b*c
^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)
-1/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d
)^(1/2)-1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x^2
```

### 3.153.2 Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-\frac{4a\sqrt{d+c^2dx^2}}{x^2}-4ac^2\sqrt{d}\log(x)+4ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bc^2d^2(1+c^2x^2)^{3/2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-a\right.}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

output
```
((-4*a*Sqrt[d + c^2*d*x^2])/x^2 - 4*a*c^2*Sqrt[d]*Log[x] + 4*a*c^2*Sqrt[d]
*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-2
*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*
x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])]
- 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])] - Arc
Sinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2
)^(3/2))/(8*d)
```

### 3.153.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6224, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \text{6224}$$
$$-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx+\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{x^2}dx}{2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}$$
$$\Big\downarrow \text{15}$$
$$-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6231}$$

$$-\frac{c^2\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^2\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{ic^2\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{ic^2\sqrt{c^2x^2+1}\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}}\right.\right.}{2\sqrt{c^2dx^2+d}}$$
$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{ic^2\sqrt{c^2x^2+1}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx}\right.}{2\sqrt{c^2dx^2+d}}$$
$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{ic^2\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c}\right.\right.}{2\sqrt{c^2dx^2+d}}$$
$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2dx^2}-\frac{bc\sqrt{c^2x^2+1}}{2x\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

---

3.153. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$

output
```
-1/2*(b*c*Sqrt[1 + c^2*x^2])/(x*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]))/(2*d*x^2) - ((I/2)*c^2*Sqrt[1 + c^2*x^2]*((2*I)*(a
 + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x
]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/Sqrt[d + c^2*d*x^2]
```

### 3.153.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.153. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.153.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| default | $-\frac{a\sqrt{c^2 d\,x^2+d}}{2d\,x^2}+\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)\sqrt{d(c^2x^2+1)}}{2x^2d(c^2x^2+1)}-\frac{\sqrt{d(c^2x^2}{}\right.$ |
| parts | $-\frac{a\sqrt{c^2 d\,x^2+d}}{2d\,x^2}+\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)\sqrt{d(c^2x^2+1)}}{2x^2d(c^2x^2+1)}-\frac{\sqrt{d(c^2x^2}{}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*
d*x^2+d)^(1/2))/x)+b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arc
sinh(c*x))*(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c
^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^
2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))*c^2)
```

### 3.153.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+dx^3}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

### 3.153.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{x^3\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.153.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+dx^3}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a +
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

### 3.153.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

### 3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.154 $\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx$



## 3.154.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3dx^3}+\frac{2c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3dx}-\frac{2bc^3\sqrt{1+c^2x^2}\log(x)}{3\sqrt{d+c^2dx^2}}$$

output
```
-1/6*b*c*(c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*ln(x)*(c^2*x^
2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
d/x^3+2/3*c^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x
```

## 3.154.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-bcx+6bc^3x^3-2a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(-1+2c^2x^2)\,\text{arcsinh}(cx)\right. -}{6dx^3\sqrt{1+c^2x^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-(b*c*x) + 6*b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 4*a
*c^2*x^2*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSin
h[c*x] - 4*b*c^3*x^3*Log[x]))/(6*d*x^3*Sqrt[1 + c^2*x^2])
```

### 3.154.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6224, 15, 6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{2}{3}c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2dx^2+d}}dx+\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{x^3}dx}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3dx^3}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{3}c^2\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2dx^2+d}}dx-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6215}$$

$$-\frac{2}{3}c^2\left(\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{x}dx}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{dx}\right)-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{3}c^2\left(\frac{bc\sqrt{c^2x^2+1}\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{dx}\right)-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

output
```
-1/6*(b*c*Sqrt[1 + c^2*x^2])/(x^2*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(3*d*x^3) - (2*c^2*(-((Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[1 + c^2*x^2]*Log[x])/Sqrt[d + c^2*d*x^
2]))/3
```

### 3.154.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.154.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| default | $a\left(-\frac{\sqrt{c^2 d\,x^2+d}}{3d\,x^3}+\frac{2c^2\sqrt{c^2 d\,x^2+d}}{3dx}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(4\,\mathrm{arcsinh}(cx)c^3x^3-4\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)x^3c^3+4\,\mathrm{arcsinh}(cx)\sqrt{c^2\cdots}\right.}{6\sqrt{c^2x^2+1}\,d\,x^3}$ |
| parts | $a\left(-\frac{\sqrt{c^2 d\,x^2+d}}{3d\,x^3}+\frac{2c^2\sqrt{c^2 d\,x^2+d}}{3dx}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(4\,\mathrm{arcsinh}(cx)c^3x^3-4\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)x^3c^3+4\,\mathrm{arcsinh}(cx)\sqrt{c^2\cdots}\right.}{6\sqrt{c^2x^2+1}\,d\,x^3}$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*c^2/d/x*(c^2*d*x^2+d)^(1/2))+1/6*b*(
d*(c^2*x^2+1))^(1/2)*(4*arcsinh(c*x)*c^3*x^3-4*ln((c*x+(c^2*x^2+1)^(1/2))^
2-1)*x^3*c^3+4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-2*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)-c*x)/(c^2*x^2+1)^(1/2)/d/x^3
```

### 3.154.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.57

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{d + c^2dx^2}}\,dx$$
$$= \frac{2\left(2\,bc^4x^4 + bc^2x^2 - b\right)\sqrt{c^2dx^2 + d}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + 2\left(bc^5x^5 + bc^3x^3\right)\sqrt{d}\log\left(\frac{c^2dx^6 + c^2dx^2 + dx^4 - \sqrt{c^2dx^2}}{c^2x^4 -}\right.}{6\left(c^2dx^5 + dx^3\right)}$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/6*(2*(2*b*c^4*x^4 + b*c^2*x^2 - b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) + 2*(b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2
 + d*x^4 - sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c
^2*x^4 + x^2)) + (4*a*c^4*x^4 + 2*a*c^2*x^2 + (b*c*x^3 - b*c*x)*sqrt(c^2*x
^2 + 1) - 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*d*x^5 + d*x^3)
```

### 3.154.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{x^4\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.154.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{1}{6}\left(\frac{4\,c^2\log\left(x\right)}{\sqrt{d}}+\frac{1}{\sqrt{d}x^2}\right)bc + \frac{1}{3}\,b\left(\frac{2\,\sqrt{c^2dx^2+d}c^2}{dx}-\frac{\sqrt{c^2dx^2+d}}{dx^3}\right)\text{arsinh}\left(cx\right) + \frac{1}{3}\,a\left(\frac{2\,\sqrt{c^2dx^2+d}c^2}{dx}-\frac{\sqrt{c^2dx^2+d}}{dx^3}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/6*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*b*c + 1/3*b*(2*sqrt(c^2*d*x^
2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a*(2*sq
rt(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))
```

### 3.154.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{\sqrt{c^2dx^2+dx^4}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

### 3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^4\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.155 $\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.155.1 Optimal result

Integrand size = 26, antiderivative size = 212

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{5bx\sqrt{d+c^2dx^2}}{3c^5d^2\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{9c^3d^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^2} + \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3} + \frac{b\sqrt{d+c^2dx^2}\arctan(cx)}{c^6d^2\sqrt{1+c^2x^2}}$$

output
```
1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^6/d^3+(-a-b*arcsinh(c*x))/c^6
/d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^2+5/
3*b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-1/9*b*x^3*(c^2*d*x^2+d
)^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)+b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^
2/(c^2*x^2+1)^(1/2)
```

### 3.155.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(15bcx+14bc^3x^3-bc^5x^5-24a\sqrt{1+c^2x^2}-12ac^2x^2\sqrt{1+c^2x^2}+\right.}{9}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(15*b*c*x + 14*b*c^3*x^3 - b*c^5*x^5 - 24*a*Sqrt[1 +
c^2*x^2] - 12*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 3*a*c^4*x^4*Sqrt[1 + c^2*x^2]
+ 3*b*Sqrt[1 + c^2*x^2]*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*(b + b
*c^2*x^2)*ArcTan[c*x]))/(9*c^6*d^2*(1 + c^2*x^2)^(3/2))
```

### 3.155.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{-c^4x^4+4c^2x^2+8}{3c^6d^2(c^2x^2+1)}dx}{\sqrt{c^2x^2+1}}+\frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6d^3}-\frac{2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{c^6d^2}-\frac{a+b\operatorname{arcsinh}(cx)}{c^6d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\sqrt{c^2dx^2+d}\int \frac{-c^4x^4+4c^2x^2+8}{c^2x^2+1}dx}{3c^5d^2\sqrt{c^2x^2+1}}+\frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c^6d^3}-\frac{2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{c^6d^2}-\frac{a+b\operatorname{arcsinh}(cx)}{c^6d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 1467}$$

$$\frac{b\sqrt{c^2dx^2+d}\int\left(-c^2x^2+\frac{3}{c^2x^2+1}+5\right)dx}{3c^5d^2\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3}-\frac{2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^2}-\frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6d^3}-\frac{2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^2}-\frac{a+b\text{arcsinh}(cx)}{c^6d\sqrt{c^2dx^2+d}}+\frac{b\left(\frac{3\arctan(cx)}{c}-\frac{1}{3}c^2x^3+5x\right)\sqrt{c^2dx^2+d}}{3c^5d^2\sqrt{c^2x^2+1}}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSinh[c*x])/(c^6*d*Sqrt[d + c^2*d*x^2])) - (2*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/(c^6*d^2) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/(3*c^6*d^3) + (b*Sqrt[d + c^2*d*x^2]*(5*x - (c^2*x^3)/3 + (3*ArcTa
n[c*x])/c))/(3*c^5*d^2*Sqrt[1 + c^2*x^2])
```

### 3.155.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

3.155. $\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

### 3.155.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| default | $a\left(\frac{x^4}{3c^2d\sqrt{c^2dx^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2}{dc^4\sqrt{c^2dx^2+d}}\right)}{3c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(3\operatorname{arcsinh}(cx)c^4x^4-c^3x^3\sqrt{c^2x^2+1}-12\operatorname{arcsinh}(c\right.}{}$ |
| parts | $a\left(\frac{x^4}{3c^2d\sqrt{c^2dx^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2}{dc^4\sqrt{c^2dx^2+d}}\right)}{3c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(3\operatorname{arcsinh}(cx)c^4x^4-c^3x^3\sqrt{c^2x^2+1}-12\operatorname{arcsinh}(c\right.}{}$ |

input
```
int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3/c^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2
)+2/d/c^4/(c^2*d*x^2+d)^(1/2)))+1/9*b*(d*(c^2*x^2+1))^(1/2)*(3*arcsinh(c*x
)*c^4*x^4-c^3*x^3*(c^2*x^2+1)^(1/2)-12*arcsinh(c*x)*c^2*x^2-9*I*(c^2*x^2+1
)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)+9*I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+
1)^(1/2)+I)+15*c*x*(c^2*x^2+1)^(1/2)-24*arcsinh(c*x))/c^6/d^2/(c^2*x^2+1)
```

### 3.155.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx =$$

$$-\frac{9\,(bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)-6\,(bc^4x^4-4\,bc^2x^2-8\,b)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right.}{18\,(c^8d^2x^2+c^6d^2)}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
-1/18*(9*(b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2
 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 6*(b*c^4*x^4 - 4*b*c^2*x^2 - 8*b)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(3*a*c^4*x^4 - 12*a*c^2*
x^2 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1) - 24*a)*sqrt(c^2*d*x^2 + d)
)/(c^8*d^2*x^2 + c^6*d^2)
```

### 3.155.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^5(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.155.7 Maxima [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^5}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
1/3*a*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*d)
 - 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*b*((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d
)*x^2 - 8*sqrt(d))*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^6*d^2
) - integrate((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^2 - 8*sqrt(d))/(sqrt(c^2*
x^2 + 1)*x), x)/(c^6*d^2) + 3*integrate(1/3*(c^4*sqrt(d)*x^4 - 4*c^2*sqrt(
d)*x^2 - 8*sqrt(d))/(c^9*d^2*x^4 + c^7*d^2*x^2 + (c^8*d^2*x^3 + c^6*d^2*x)
*sqrt(c^2*x^2 + 1)), x))
```

### 3.155.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^{3/2}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

# 3.156 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.156.1 Optimal result

Integrand size = 26, antiderivative size = 206

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^4d^2} - \frac{3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^5d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2c^5d\sqrt{d+c^2dx^2}}$$

output
```
-x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1
/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-3/4*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b
/c^5/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d/(c^
2*d*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

## 3.156.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.78

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{4ac\sqrt{d}x(3+c^2x^2) - 12a\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + b\sqrt{d}(8cxa}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(4*a*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt
[d]*Sqrt[d + c^2*d*x^2]] + b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^
2]*(6*ArcSinh[c*x]^2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSi
nh[c*x]*Sinh[2*ArcSinh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

### 3.156.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6225, 243, 49, 2009, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^3}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{243}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^2}{c^2x^2+1}dx^2}{2cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{2cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6227}$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}-\frac{b\sqrt{c^2x^2+1}\int xdx}{2c\sqrt{c^2dx^2+d}}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}\right)}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}\right)}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}\right)}{c^2d}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2cd\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((x^3*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2])) + (3*(-1/4*(b*x^
2*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c^3*Sqrt[d + c^2*d*x^2])))/(c^2*d) + (b*Sqrt[1 + c^2*x^2]*(x^2/c^2 - L
og[1 + c^2*x^2]/c^4))/(2*c*d*Sqrt[d + c^2*d*x^2])
```

### 3.156.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.156. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.156.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| default | $\frac{a\,x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3ax}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-4\,\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+2c^4x^4+6\,\text{ar}\right.}{}$ |
| parts | $\frac{a\,x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3ax}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-4\,\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+2c^4x^4+6\,\text{ar}\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3/2*
a/c^4/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/8*b/
(c^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)*(-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
*x^3*c^3+2*c^4*x^4+6*arcsinh(c*x)^2*x^2*c^2-8*arcsinh(c*x)*c^2*x^2+8*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-12*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+3
*c^2*x^2+6*arcsinh(c*x)^2-8*arcsinh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
+1)/c^5/d^2
```

## 3.156.5 Fricas [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2
*c^2*d^2*x^2 + d^2), x)
```

## 3.156.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^4(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.156.7 Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
1/2*a*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d) -
 3*arcsinh(c*x)/(c^5*d^(3/2))) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 +
1))/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.156.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^4\,(a+b\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

# 3.157 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.157.1 Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=-\frac{bx\sqrt{d+c^2dx^2}}{c^3d^2\sqrt{1+c^2x^2}}+\frac{a+b\text{arcsinh}(cx)}{c^4d\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^4d^2}-\frac{b\sqrt{d+c^2dx^2}\arctan(cx)}{c^4d^2\sqrt{1+c^2x^2}}$$

output
```
(a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)/c^4/d^2-b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)-b*arct
an(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^2/(c^2*x^2+1)^(1/2)
```

## 3.157.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3+2a\sqrt{1+c^2x^2}+ac^2x^2\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}(2\right.}{c^4d^2\left(1+c^2x^2\right)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 + 2*a*Sqrt[1 + c^2*x^2] + a*c^2
*x^2*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2)*ArcSinh[c*x] -
(b + b*c^2*x^2)*ArcTan[c*x]))/(c^4*d^2*(1 + c^2*x^2)^(3/2))
```

### 3.157.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6219, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int \frac{c^2x^2+2}{c^4d^2(c^2x^2+1)}dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^4d^2}+\frac{a+b\text{arcsinh}(cx)}{c^4d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{c^2dx^2+d}\int \frac{c^2x^2+2}{c^2x^2+1}dx}{c^3d^2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^4d^2}+\frac{a+b\text{arcsinh}(cx)}{c^4d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{299}$$

$$-\frac{b\sqrt{c^2dx^2+d}\left(\int \frac{1}{c^2x^2+1}dx+x\right)}{c^3d^2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^4d^2}+\frac{a+b\text{arcsinh}(cx)}{c^4d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^4d^2}+\frac{a+b\text{arcsinh}(cx)}{c^4d\sqrt{c^2dx^2+d}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)\sqrt{c^2dx^2+d}}{c^3d^2\sqrt{c^2x^2+1}}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcSinh[c*x])/(c^4*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x]))/(c^4*d^2) - (b*Sqrt[d + c^2*d*x^2]*(x + ArcTan[c*x]/c)
)/(c^3*d^2*Sqrt[1 + c^2*x^2])
```

#### 3.157.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.157.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| default | $a\left(\frac{x^{2}}{c^{2} d \sqrt{c^{2} d \,x^{2}+d}}+\frac{2}{d \,c^{4} \sqrt{c^{2} d \,x^{2}+d}}\right)+\frac{b \sqrt{d \left(c^{2} x^{2}+1\right)}\, \left(\operatorname{arcsinh}\left(c x \right) c^{2} x^{2}+i \sqrt{c^{2} x^{2}+1}\, \ln \left(c x +\sqrt{c^{2} x^{2}+1}-i\right)-i \sqrt{c^{2} x^{2}+1}\, \ln \left(c x +\right.\right.}{c^{4} d^{2} \left(c^{2} x^{2}+1\right)}$ |
| parts | $a\left(\frac{x^{2}}{c^{2} d \sqrt{c^{2} d \,x^{2}+d}}+\frac{2}{d \,c^{4} \sqrt{c^{2} d \,x^{2}+d}}\right)+\frac{b \sqrt{d \left(c^{2} x^{2}+1\right)}\, \left(\operatorname{arcsinh}\left(c x \right) c^{2} x^{2}+i \sqrt{c^{2} x^{2}+1}\, \ln \left(c x +\sqrt{c^{2} x^{2}+1}-i\right)-i \sqrt{c^{2} x^{2}+1}\, \ln \left(c x +\right.\right.}{c^{4} d^{2} \left(c^{2} x^{2}+1\right)}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2))+b*(d*(c^2*x^
2+1))^(1/2)*(arcsinh(c*x)*c^2*x^2+I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(
1/2)-I)-I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-c*x*(c^2*x^2+1)^(1
/2)+2*arcsinh(c*x))/c^4/d^2/(c^2*x^2+1)
```

### 3.157.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{(bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\left(bc^2x^2+2\,b\right)\sqrt{c^2dx^2+d}\log}{2\left(c^6d^2x^2+c^4d\right.}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1
)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*(b*c^2*x^2 + 2*b)*sqrt(c^2*d*x^2 + d)*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + 2*a
)*sqrt(c^2*d*x^2 + d))/(c^6*d^2*x^2 + c^4*d^2)
```

### 3.157.6 Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.157.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -bc\left(\frac{x}{c^4d^{\frac{3}{2}}}+\frac{\arctan(cx)}{c^5d^{\frac{3}{2}}}\right) + b\left(\frac{x^2}{\sqrt{c^2dx^2+d}c^2d}+\frac{2}{\sqrt{c^2dx^2+d}c^4d}\right)\text{arsinh}(cx) + a\left(\frac{x^2}{\sqrt{c^2dx^2+d}c^2d}+\frac{2}{\sqrt{c^2dx^2+d}c^4d}\right)$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + b*(x^2/(sqrt(c^2*d*x^
2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a*(x^2/(sqrt
(c^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))
```

### 3.157.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^{3/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

# 3.158 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.158.1 Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc^3d\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2c^3d\sqrt{d+c^2dx^2}}$$

output
```
-x*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(
c^2*x^2+1)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+1/2*b*ln(c^2*x^2+1)*(c^2*x^2+
1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

## 3.158.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{ax\sqrt{d(1+c^2x^2)}}{c^2d^2(1+c^2x^2)} + \frac{b\left(-2cx\text{arcsinh}(cx)+\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)^2+2\log\left(\sqrt{1+c^2x^2}\right)\right)\right)}{2c^3d\sqrt{d(1+c^2x^2)}} + \frac{a\log\left(cdx+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{c^3d^{3/2}}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((a*x*Sqrt[d*(1 + c^2*x^2)])/(c^2*d^2*(1 + c^2*x^2))) + (b*(-2*c*x*ArcSin
h[c*x] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]^2 + 2*Log[Sqrt[1 + c^2*x^2]])))/(
2*c^3*d*Sqrt[d*(1 + c^2*x^2)]) + (a*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^
2)]])/(c^3*d^(3/2))
```

### 3.158.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6225, 240, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc^3d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^3d\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((x*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2])) + (Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x])^2)/(2*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c
^2*x^2]*Log[1 + c^2*x^2])/(2*c^3*d*Sqrt[d + c^2*d*x^2])
```

#### 3.158.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

### 3.158.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

| method | result |
|---|---|
| default | $-\frac{ax}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^2d\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\mathrm{a}}{c^2d^2(c^2x}$ |
| parts | $-\frac{ax}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^2d\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b\sqrt{d(c^2x^2+1)}\,\mathrm{a}}{c^2d^2(c^2x}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a/c^2/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3
/d^2*arcsinh(c*x)^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcs
inh(c*x)-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2/(c^2*x^2+1)*x+b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2
)
```

### 3.158.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2
*c^2*d^2*x^2 + d^2), x)
```

### 3.158.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^2(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.158.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + b*integr
ate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.158.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^2\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

# 3.159 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.159.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=-\frac{a+b\text{arcsinh}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\arctan(cx)}{c^2d\sqrt{d+c^2dx^2}}$$

output
```
(-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+b*arctan(c*x)*(c^2*x^2+1)^(1
/2)/c^2/d/(c^2*d*x^2+d)^(1/2)
```

## 3.159.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.23

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(-a\sqrt{1+c^2x^2}-b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+(b+bc^2x^2)\arctan(cx)\right)}{c^2d^2\left(1+c^2x^2\right)^{3/2}}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-(a*Sqrt[1 + c^2*x^2]) - b*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + (b + b*c^2*x^2)*ArcTan[c*x]))/(c^2*d^2*(1 + c^2*x^2)^(3/2))
```

### 3.159.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\sqrt{c^2x^2+1}\int \frac{1}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSinh[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])) + (b*Sqrt[1 + c^2*x^2]
*ArcTan[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

#### 3.159.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.159.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.79

| method | result | size |
|---|---|---|
| default | $-\frac{a}{c^2 d\sqrt{c^2 d\,x^2+d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)+\operatorname{arcsinh}(cx)\right)}{c^2d^2(c^2x^2+1)}$ | 125 |
| parts | $-\frac{a}{c^2 d\sqrt{c^2 d\,x^2+d}}-\frac{b\sqrt{d(c^2x^2+1)}\left(-i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)+i\sqrt{c^2x^2+1}\,\ln\left(cx+\sqrt{c^2x^2+1}-i\right)+\operatorname{arcsinh}(cx)\right)}{c^2d^2(c^2x^2+1)}$ | 125 |

input
```
int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a/c^2/d/(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)*(-I*(c^2*x^2+1)^(1/2)
*ln(c*x+(c^2*x^2+1)^(1/2)+I)+I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)-
I)+arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)
```

### 3.159.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.83

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx =$$
$$-\frac{(bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{c^2dx^2+d}a}{2\,(c^4d^2x^2+c^2d^2)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 +
1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(c^2*d*x^2 + d)*b*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*sqrt(c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 + c^2*d^2)
```

### 3.159.6 Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.159.7 Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(c^2*d^(3/2)) - log(c*x + sqrt(c^
2*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^2*d^(3/2)) - integrate(1/(c^5*d^(3/2)*x^4
 + c^3*d^(3/2)*x^2 + (c^4*d^(3/2)*x^3 + c^2*d^(3/2)*x)*sqrt(c^2*x^2 + 1)),
 x)) - a/(sqrt(c^2*d*x^2 + d)*c^2*d)
```

### 3.159.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

# 3.160 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.160.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2cd\sqrt{d+c^2dx^2}}$$

output
```
x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)
^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)
```

## 3.160.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(2acx\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}\text{arcsinh}(cx)-(b+bc^2x^2)\log(1+c^2\right.}{2cd^2(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(2*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x] - (b + b*c^2*x^2)*Log[1 + c^2*x^2]))/(2*c*d^2*(1 + c^2*x^2)
^(3/2))
```

### 3.160.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Lo
g[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])
```

#### 3.160.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

### 3.160.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(68) = 136$.

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.88

| method | result | size |
|---|---|---|
| default | $\frac{ax}{d\sqrt{c^2 d\,x^2+d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c\,d^2}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{d^2(c^2x^2+1)}-\frac{b\sqrt{d(c^2x^2+1)}\,\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{\sqrt{c^2x^2+1}\,c\,d^2}$ | 143 |
| parts | $\frac{ax}{d\sqrt{c^2 d\,x^2+d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c\,d^2}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{d^2(c^2x^2+1)}-\frac{b\sqrt{d(c^2x^2+1)}\,\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{\sqrt{c^2x^2+1}\,c\,d^2}$ | 143 |

input
```
int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/d*x/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*
arcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^2*x^2+1)*x-b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

### 3.160.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2
*x^2 + d^2), x)
```

### 3.160.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.160.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx = \frac{bx\,\text{arsinh}\,(cx)}{\sqrt{c^2dx^2+d}d} + \frac{ax}{\sqrt{c^2dx^2+d}d} - \frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2\,cd^{\frac{3}{2}}}$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a*x/(sqrt(c^2*d*x^2 + d)*d) - 1
/2*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

### 3.160.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2), x)
```

# 3.161 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx$



## 3.161.1 Optimal result

Integrand size = 26, antiderivative size = 194

$$\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx=\frac{a+b\text{arcsinh}(cx)}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}}-\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}$$

output
```
(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-b*arctan(c*x)*(c^2*x^2+1)^(1/2)/d
/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(
c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))
*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

### 3.161.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.19

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{\frac{a\sqrt{d+c^2dx^2}}{1+c^2x^2}+a\sqrt{d}\log(x)-a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bd\left(\operatorname{arcsinh}(cx)-2\sqrt{1+c^2x^2}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
((a*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2) + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[
d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d*(ArcSinh[c*x] - 2*Sqrt[1 + c^2*x^2
]*ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x]
)] + Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*
PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2])/d^2
```

### 3.161.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6226}$$

$$\frac{\int \frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{a+b\operatorname{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\int \frac{a+b\operatorname{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{a+b\operatorname{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6231}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{i\sqrt{c^2x^2+1}\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\sqrt{c^2x^2+1}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{d\sqrt{c^2dx^2+d}}$$
$$\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

3.161. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx$

output
```
(a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan
[c*x])/(d*Sqrt[d + c^2*d*x^2]) + (I*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSin
h[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*Po
lyLog[2, E^ArcSinh[c*x]]))/(d*Sqrt[d + c^2*d*x^2])
```

### 3.161.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.161. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.161.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| default | $\frac{a}{d\sqrt{c^2 d\,x^2+d}} - \frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}} - \frac{b\sqrt{d(c^2x^2+1)}\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)x^2c^2+2\arctan\left(cx+\sqrt{c^2x^2+1}\right)x^2c\right.}{}$ |
| parts | $\frac{a}{d\sqrt{c^2 d\,x^2+d}} - \frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}} - \frac{b\sqrt{d(c^2x^2+1)}\left(\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)x^2c^2+2\arctan\left(cx+\sqrt{c^2x^2+1}\right)x^2c\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/d/(c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x
)-b/(c^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)/d^2*(arcsinh(c*x)*ln(1+c*x+(c^
2*x^2+1)^(1/2))*x^2*c^2+2*arctan(c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+dilog(1+c*
x+(c^2*x^2+1)^(1/2))*x^2*c^2+dilog(c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-arcsinh(
c*x)*(c^2*x^2+1)^(1/2)+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arctan(c
*x+(c^2*x^2+1)^(1/2))+dilog(1+c*x+(c^2*x^2+1)^(1/2))+dilog(c*x+(c^2*x^2+1)
^(1/2)))
```

### 3.161.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2
*x^3 + d^2*x), x)
```

### 3.161.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{x\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

### 3.161.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-a*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + b*integra
te(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

### 3.161.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x), x)
```

### 3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.162 $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx$



## 3.162.1 Optimal result

Integrand size = 26, antiderivative size = 143

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx = -\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(x)}{d^2\sqrt{1+c^2x^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{2d^2\sqrt{1+c^2x^2}}$$

output
```
(-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/d/(
c^2*d*x^2+d)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)+1/2
*b*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

## 3.162.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{3/2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(2a\sqrt{1+c^2x^2}+4ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(1+2c^2x^2)\text{arcsinh}(cx)+bcx(1+c^2x^2)\log(\right.}{2d^2x(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
-1/2*(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^
2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2)*ArcSinh[c*x] + b*c*x*(1 + c
^2*x^2)*Log[1 + 1/(c^2*x^2)] - 2*b*c*x*Log[1 + c^2*x^2] - 2*b*c^3*x^3*Log[
1 + c^2*x^2]))/(d^2*x*(1 + c^2*x^2)^(3/2))
```

### 3.162.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6219, 25, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2 (c^2dx^2+d)^{3/2}} \, dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{2c^2x^2+1}{d^2x(c^2x^2+1)}dx}{\sqrt{c^2x^2+1}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{25}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{2c^2x^2+1}{d^2x(c^2x^2+1)}dx}{\sqrt{c^2x^2+1}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{2c^2x^2+1}{x(c^2x^2+1)}dx}{d^2\sqrt{c^2x^2+1}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{354}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{2c^2x^2+1}{x^2(c^2x^2+1)}dx^2}{2d^2\sqrt{c^2x^2+1}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{86}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \left(\frac{c^2}{c^2x^2+1}+\frac{1}{x^2}\right)dx^2}{2d^2\sqrt{c^2x^2+1}}-\frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c^2x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{dx\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2dx^2+d}\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)}{2d^2\sqrt{c^2x^2+1}}$$

---

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcSinh[c*x])/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSin
h[c*x]))/(d*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*(Log[x^2] + Lo
g[1 + c^2*x^2]))/(2*d^2*Sqrt[1 + c^2*x^2])
```

### 3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

### 3.162.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| default | $a\left(-\frac{1}{dx\sqrt{c^2d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2d\,x^2+d}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^4c^4-2\sqrt{c^2x^2+1}\,\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^3c^3+2\ln\left(\right.\right.}{}$ |
| parts | $a\left(-\frac{1}{dx\sqrt{c^2d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2d\,x^2+d}}\right)-\frac{b\left(2\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^4c^4-2\sqrt{c^2x^2+1}\,\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^4-1\right)x^3c^3+2\ln\left(\right.\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2))-b*(2*ln((c*x+
(c^2*x^2+1)^(1/2))^4-1)*x^4*c^4-2*(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1
/2))^4-1)*x^3*c^3+2*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^2*c^2-(c^2*x^2+1)^(1
/2)*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x*c+arcsinh(c*x))*(2*c^2*x^2+1+2*c*x*(
c^2*x^2+1)^(1/2))*(d*(c^2*x^2+1))^(1/2)/x/d^2/(c^2*x^2+1)
```

### 3.162.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2
*x^4 + d^2*x^2), x)
```

### 3.162.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{3/2}}\, dx = \int \frac{a + b\,\text{asinh}\,(cx)}{x^2 \left(d\,(c^2x^2 + 1)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

### 3.162.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{3/2}}\, dx = \frac{1}{2}\,bc\left(\frac{\log\left(c^2x^2 + 1\right)}{d^{\frac{3}{2}}} + \frac{2\,\log\left(x\right)}{d^{\frac{3}{2}}}\right) - \left(\frac{2\,c^2x}{\sqrt{c^2dx^2 + d}d} + \frac{1}{\sqrt{c^2dx^2 + d}dx}\right)b\,\text{arsinh}\,(cx) - \left(\frac{2\,c^2x}{\sqrt{c^2dx^2 + d}d} + \frac{1}{\sqrt{c^2dx^2 + d}dx}\right)a$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
1/2*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - (2*c^2*x/(sqrt(c^2
*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*b*arcsinh(c*x) - (2*c^2*x/(s
qrt(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a
```

### 3.162.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{3/2}}\, dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^{\frac{3}{2}}x^2}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

### 3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{3/2}}\, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{x^2\,(d\,c^2\,x^2 + d)^{3/2}}\, dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.163 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx$



## 3.163.1 Optimal result

Integrand size = 26, antiderivative size = 287

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx &= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}}-\frac{3c^2(a+b\text{arcsinh}(cx))}{2d\sqrt{d+c^2dx^2}} \\ &\quad -\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{d+c^2dx^2}}+\frac{bc^2\sqrt{1+c^2x^2}\arctan(cx)}{d\sqrt{d+c^2dx^2}}+\frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &\quad +\frac{3bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}}-\frac{3bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-3/2*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)+1/2*(-a-b*arcsinh(c*x))/
d/x^2/(c^2*d*x^2+d)^(1/2)-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x/(c^2*d*x^2+d)^(1/2
)+b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arc
sinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d
)^(1/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c
^2*d*x^2+d)^(1/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(
1/2)/d/(c^2*d*x^2+d)^(1/2)
```

### 3.163.2 Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.29

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx = \frac{-\frac{4a(1+3c^2x^2)\sqrt{d+c^2dx^2}}{x^2+c^2x^4} - 12ac^2\sqrt{d}\log(x) + 12ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{bc^2}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
((-4*a*(1 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(x^2 + c^2*x^4) - 12*a*c^2*Sqr
t[d]*Log[x] + 12*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c
^2*d*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]]
- 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x
]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-A
rcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x]
)] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2
*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[
ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/Sqrt[d + c^
2*d*x^2])/(8*d^2)
```

### 3.163.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6224, 264, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{3}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2dx^2+d)^{3/2}}dx + \frac{bc\sqrt{c^2x^2+1}\int \frac{1}{x^2(c^2x^2+1)}dx}{2d\sqrt{c^2dx^2+d}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{3}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{3/2}}dx+\frac{bc\sqrt{c^2x^2+1}\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{3}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6226}$$

$$-\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}\right)-\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{3}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}\right)-\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6231}$$

$$-\frac{3}{2}c^2\left(\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}\right)-\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}c^2\left(\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(c}{d\sqrt{c^2dx^2+d}}\right. \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(c}{d\sqrt{c^2dx^2+d}}\right. \frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4670}$$

---

3.163. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx$

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{ar}}\right.\right.}{d\sqrt{c^2dx^2+d}}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2715

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsin}}\right.}{d\sqrt{c^2dx^2+d}}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2838

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsi}}\right.\right.}{d\sqrt{c^2dx^2+d}}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-c\arctan(cx)-\frac{1}{x}\right)}{2d\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])/(d*x^2*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*
x^2]*(-x^(-1) - c*ArcTan[c*x]))/(2*d*Sqrt[d + c^2*d*x^2]) - (3*c^2*((a + b
*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])
/(d*Sqrt[d + c^2*d*x^2]) + (I*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[
2, E^ArcSinh[c*x]]))/(d*Sqrt[d + c^2*d*x^2])))/2
```

### 3.163.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

---

3.163. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx$

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.163. $\int \frac{a+b\operatorname{arcsinh}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.163.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| default | $a\left(-\frac{1}{2d\,x^{2}\sqrt{c^{2}d\,x^{2}+d}}-\frac{3c^{2}\left(\frac{1}{d\sqrt{c^{2}d\,x^{2}+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^{2}d\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{\sqrt{d(c^{2}x^{2}+1)}\,\left(3\operatorname{arcsinh}(cx)c^{2}x^{2}+cx\sqrt{c^{2}x^{2}\cdot}\right.}{2d^{2}(c^{2}x^{2}+1)x^{2}}\right.$ |
| parts | $a\left(-\frac{1}{2d\,x^{2}\sqrt{c^{2}d\,x^{2}+d}}-\frac{3c^{2}\left(\frac{1}{d\sqrt{c^{2}d\,x^{2}+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^{2}d\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{\sqrt{d(c^{2}x^{2}+1)}\,\left(3\operatorname{arcsinh}(cx)c^{2}x^{2}+cx\sqrt{c^{2}x^{2}\cdot}\right.}{2d^{2}(c^{2}x^{2}+1)x^{2}}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2/(c^2*d*x^2+d)^(1/2)-3/2*c^2*(1/d/(c^2*d*x^2+d)^(1/2)-1/d^(3/
2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)))+b*(-1/2*(d*(c^2*x^2+1))^(1/
2)*(3*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/d^2/(c^2*x^
2+1)/x^2+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2
+1)^(1/2))*c^2+3/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c*x
+(c^2*x^2+1)^(1/2))*c^2+3/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ar
csinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2+3/2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^2+1)^(1/2))*c^2)
```

## 3.163.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2
*x^5 + d^2*x^3), x)
```

## 3.163.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

## 3.163.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) -
 1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1
))/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

### 3.163.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

### 3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.164 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx$



## 3.164.1 Optimal result

Integrand size = 26, antiderivative size = 228

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx=-\frac{bc\sqrt{d+c^2dx^2}}{6d^2x^2\sqrt{1+c^2x^2}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{d+c^2dx^2}}-\frac{5bc^3\sqrt{d+c^2dx^2}\log(x)}{3d^2\sqrt{1+c^2x^2}}-\frac{bc^3\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{2d^2\sqrt{1+c^2x^2}}$$

output
```
1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x
))/d/x/(c^2*d*x^2+d)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1
/2)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2+1)^(1/2)-5/3*b*c^3*ln(x)*
(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)-1/2*b*c^3*ln(c^2*x^2+1)*(c^2*d*x
^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

## 3.164.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3-2a\sqrt{1+c^2x^2}+8ac^2x^2\sqrt{1+c^2x^2}+16ac^4x^4\sqrt{1+c^2x}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 8*a*c
^2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c
^2*x^2]*(-1 + 4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 5*b*c^3*x^3*(1 + c^2*x
^2)*Log[1 + 1/(c^2*x^2)] - 8*b*c^3*x^3*Log[1 + c^2*x^2] - 8*b*c^5*x^5*Log[
1 + c^2*x^2]))/(6*d^2*x^3*(1 + c^2*x^2)^(3/2))
```

### 3.164.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6219, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(c^2dx^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{-8c^4x^4-4c^2x^2+1}{3d^2x^3(c^2x^2+1)}dx}{\sqrt{c^2x^2+1}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{-8c^4x^4-4c^2x^2+1}{x^3(c^2x^2+1)}dx}{3d^2\sqrt{c^2x^2+1}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{-8c^4x^4-4c^2x^2+1}{x^4(c^2x^2+1)}dx^2}{6d^2\sqrt{c^2x^2+1}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \left(-\frac{3c^4}{c^2x^2+1}-\frac{5c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6d^2\sqrt{c^2x^2+1}}+\frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}$$

$$\downarrow \text{2009}$$

$$\frac{4c^2(a+b\text{arcsinh}(cx))}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2dx^2+d}\left(-5c^2\log\left(x^2\right)-3c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)}{6d^2\sqrt{c^2x^2+1}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcS
inh[c*x]))/(3*d*x*Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3
*d*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*(-x^(-2) - 5*c^2*Log[x^
2] - 3*c^2*Log[1 + c^2*x^2]))/(6*d^2*Sqrt[1 + c^2*x^2])
```

### 3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.164. $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx$

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.164.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(201) = 402$.

Time = 0.22 (sec) , antiderivative size = 968, normalized size of antiderivative = 4.25

| method | result |
|---|---|
| default | $a\left(-\frac{1}{3d\,x^3\sqrt{c^2d\,x^2+d}}-\frac{4c^2\left(-\frac{1}{dx\sqrt{c^2d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2d\,x^2+d}}\right)}{3}\right)+\frac{16b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c^3}{3\sqrt{c^2x^2+1}\,d^2}+\frac{32b\sqrt{d(c^2x^2+1)}\,x^7c^{10}}{3(8c^4x^4+7c^2x^2-1)d^2}-$ |
| parts | $a\left(-\frac{1}{3d\,x^3\sqrt{c^2d\,x^2+d}}-\frac{4c^2\left(-\frac{1}{dx\sqrt{c^2d\,x^2+d}}-\frac{2c^2x}{d\sqrt{c^2d\,x^2+d}}\right)}{3}\right)+\frac{16b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)c^3}{3\sqrt{c^2x^2+1}\,d^2}+\frac{32b\sqrt{d(c^2x^2+1)}\,x^7c^{10}}{3(8c^4x^4+7c^2x^2-1)d^2}-$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3/(c^2*d*x^2+d)^(1/2)-4/3*c^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^
2/d*x/(c^2*d*x^2+d)^(1/2)))+16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/d^2*arcsinh(c*x)*c^3+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)
/d^2*x^7*c^10-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5
*(c^2*x^2+1)*c^8+16*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^
5*c^8-16/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^
2+1)*c^6+64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcs
inh(c*x)*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*
c^2*x^2-1)/d^2*x^3*c^6+4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)
/d^2*x*(c^2*x^2+1)*c^4+8*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d
^2*x*arcsinh(c*x)*c^4+8/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/
d^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*
x^4+7*c^2*x^2-1)/d^2*x*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^
2-1)/d^2*c^3*(c^2*x^2+1)^(1/2)-4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*
x^2-1)/d^2/x*arcsinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2
*x^2-1)/d^2/x^2*c*(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4
+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

### 3.164.5 Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}} \, dx = \int \frac{b \,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^{\frac{3}{2}}x^4} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2
*x^6 + d^2*x^4), x)
```

### 3.164.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{x^4 (d\,(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)
```

### 3.164.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1
/(sqrt(c^2*d*x^2 + d)*d*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))
/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

### 3.164.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^4 (d + c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

### 3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{x^4\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.165 $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.165.1 Optimal result

Integrand size = 26, antiderivative size = 281

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}}{4c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d(d+c^2dx^2)^{3/2}} - \frac{5x^3(a+b\text{arcsinh}(cx))}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^6d^3} - \frac{5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^7d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{6c^7d^2\sqrt{d+c^2dx^2}}$$

output
```
-1/3*x^5*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-5/3*x^3*(a+b*arcsinh
(c*x))/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/6*b/c^7/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*
x^2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-5/4*(
a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^7/d^2/(c^2*d*x^2+d)^(1/2)-7/6*b*
ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^7/d^2/(c^2*d*x^2+d)^(1/2)+5/2*x*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

### 3.165.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{4acdx(15+20c^2x^2+3c^4x^4)+bd\Big(4cx(15+20c^2x^2+3c^4x^4)\operatorname{arcsinh}(cx)-30$$

input
```
Integrate[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(4*a*c*d*x*(15 + 20*c^2*x^2 + 3*c^4*x^4) + b*d*(4*c*x*(15 + 20*c^2*x^2 + 3
*c^4*x^4)*ArcSinh[c*x] - 30*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - Sqrt[1 +
c^2*x^2]*(7 + 9*c^2*x^2 + 6*c^4*x^4 + 28*(1 + c^2*x^2)*Log[1 + c^2*x^2]))
- 60*a*Sqrt[d]*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[
d + c^2*d*x^2]])/(24*c^7*d^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

### 3.165.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6225, 243, 49, 2009, 6225, 243, 49, 2009, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{5\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{3c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^5}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^5(a+b\operatorname{arcsinh}(cx))}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{5\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{3c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^4}{(c^2x^2+1)^2}dx^2}{6cd^2\sqrt{c^2dx^2+d}}-\frac{x^5(a+b\operatorname{arcsinh}(cx))}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{5\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{3c^2d}+\frac{b\sqrt{c^2x^2+1}\int \left(\frac{1}{c^4}-\frac{2}{c^4(c^2x^2+1)}+\frac{1}{c^4(c^2x^2+1)^2}\right)dx^2}{6cd^2\sqrt{c^2dx^2+d}}-\frac{x^5(a+b\operatorname{arcsinh}(cx))}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{3c^2d}-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6225}$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^3}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{243}$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^2}{c^2x^2+1}dx^2}{2cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{49}$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{2cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{2cd\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6227}$$

3.165. $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}-\frac{b\sqrt{c^2x^2+1}\int xdx}{2c\sqrt{c^2dx^2+d}}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}\right)}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{2cd\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 15

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}\right)}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{2cd\sqrt{c^2dx^2+d}}\right)}{3c^2d}$$

$$\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 6198

$$-\frac{x^5(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+$$

$$\frac{5\left(-\frac{x^3(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}\right)}{c^2d}+\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right.}{c^4}\right.}{2cd\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{b\sqrt{c^2x^2+1}\left(\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}-\frac{2\log(c^2x^2+1)}{c^6}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

3.165. $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

output
```
-1/3*(x^5*(a + b*ArcSinh[c*x]))/(c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1
+ c^2*x^2]*(x^2/c^4 - 1/(c^6*(1 + c^2*x^2)) - (2*Log[1 + c^2*x^2])/c^6))/(
6*c*d^2*Sqrt[d + c^2*d*x^2]) + (5*(-((x^3*(a + b*ArcSinh[c*x]))/(c^2*d*Sqr
t[d + c^2*d*x^2])) + (3*(-1/4*(b*x^2*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*
x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])))/(c^2*d)
+ (b*Sqrt[1 + c^2*x^2]*(x^2/c^2 - Log[1 + c^2*x^2]/c^4))/(2*c*d*Sqrt[d + c
^2*d*x^2])))/(3*c^2*d)
```

### 3.165.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.165. $\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.165.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| default | $\frac{a\,x^5}{2c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,x^3}{6c^4d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5ax}{2c^6d^2\sqrt{c^2d\,x^2+d}}-\frac{5a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^6d^2\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(-12\,\mathrm{arc}\right.}{}$ |
| parts | $\frac{a\,x^5}{2c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,x^3}{6c^4d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5ax}{2c^6d^2\sqrt{c^2d\,x^2+d}}-\frac{5a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^6d^2\sqrt{c^2d}}-\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(-12\,\mathrm{arc}\right.}{}$ |

input
```
int(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^5/c^2/d/(c^2*d*x^2+d)^(3/2)+5/6*a/c^4*x^3/d/(c^2*d*x^2+d)^(3/2)+5/
2*a/c^6/d^2*x/(c^2*d*x^2+d)^(1/2)-5/2*a/c^6/d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(
c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/24*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)
^(1/2)*(-12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5*c^5+6*c^6*x^6+30*arcsinh(c*
x)^2*x^4*c^4-56*arcsinh(c*x)*c^4*x^4+56*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^
4*c^4-80*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+15*c^4*x^4+60*arcsinh(c*x)
^2*x^2*c^2-112*arcsinh(c*x)*c^2*x^2+112*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^
2*c^2-60*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+16*c^2*x^2+30*arcsinh(c*x)^2-5
6*arcsinh(c*x)+56*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+7)/(c^6*x^6+3*c^4*x^4+3*
c^2*x^2+1)/c^7/d^3
```

## 3.165.5 Fricas [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^6}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral((b*x^6*arcsinh(c*x) + a*x^6)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3
*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

## 3.165.6 Sympy [F]

$$\int \frac{x^6(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^6(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**6*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.165.7 Maxima [F]

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^6}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
1/6*a*(3*x^5/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 5*x*(3*x^2/((c^2*d*x^2 + d)^(
3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4*d))/c^2 + 5*x/(sqrt(c^2*d*x^2 +
 d)*c^6*d^2) - 15*arcsinh(c*x)/(c^7*d^(5/2))) + b*integrate(x^6*log(c*x +
sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.165.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^6\,(a+b\,\operatorname{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.166 $\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.166.1 Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{bx\sqrt{d+c^2dx^2}}{6c^5d^3(1+c^2x^2)^{3/2}} - \frac{bx\sqrt{d+c^2dx^2}}{c^5d^3\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3c^6d(d+c^2dx^2)^{3/2}} + \frac{2(a+b\text{arcsinh}(cx))}{c^6d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^6d^3} - \frac{11b\sqrt{d+c^2dx^2}\arctan(cx)}{6c^6d^3\sqrt{1+c^2x^2}}$$

output
```
1/3*(-a-b*arcsinh(c*x))/c^6/d/(c^2*d*x^2+d)^(3/2)+2*(a+b*arcsinh(c*x))/c^6
/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^3/(c^2*x^2+1)^(
3/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3-b*x*(c^2*d*x^2+d)^(1/2
)/c^5/d^3/(c^2*x^2+1)^(1/2)-11/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^3
/(c^2*x^2+1)^(1/2)
```

## 3.166.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\Big(-5bcx-11bc^3x^3-6bc^5x^5+16a\sqrt{1+c^2x^2}+24ac^2x^2\sqrt{1+c^2x^2}}{}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-5*b*c*x - 11*b*c^3*x^3 - 6*b*c^5*x^5 + 16*a*Sqrt[1
+ c^2*x^2] + 24*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 6*a*c^4*x^4*Sqrt[1 + c^2*x^2
] + 2*b*Sqrt[1 + c^2*x^2]*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - 11*b
*(1 + c^2*x^2)^2*ArcTan[c*x]))/(6*c^6*d^3*(1 + c^2*x^2)^(5/2))
```

## 3.166.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6219, 27, 1471, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int \frac{3c^4x^4+12c^2x^2+8}{3c^6d^3(c^2x^2+1)^2}dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^3}+\frac{2(a+b\text{arcsinh}(cx))}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^6d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{c^2dx^2+d}\int \frac{3c^4x^4+12c^2x^2+8}{(c^2x^2+1)^2}dx}{3c^5d^3\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^3}+\frac{2(a+b\text{arcsinh}(cx))}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^6d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{b\sqrt{c^2dx^2+d}\left(-\frac{1}{2}\int -\frac{6c^2x^2+17}{c^2x^2+1}dx-\frac{x}{2(c^2x^2+1)}\right)}{3c^5d^3\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^3}+\frac{2(a+b\text{arcsinh}(cx))}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^6d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\sqrt{c^2dx^2+d}\left(\frac{1}{2}\int \frac{6c^2x^2+17}{c^2x^2+1}dx-\frac{x}{2(c^2x^2+1)}\right)}{3c^5d^3\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{c^6d^3}+\frac{2(a+b\text{arcsinh}(cx))}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^6d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{299}$$

$$-\frac{b\sqrt{c^2dx^2+d}\left(\frac{1}{2}\left(11\int\frac{1}{c^2x^2+1}dx+6x\right)-\frac{x}{2(c^2x^2+1)}\right)}{3c^5d^3\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{c^6d^3}+\frac{2(a+b\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\operatorname{arcsinh}(cx)}{3c^6d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{c^6d^3}+\frac{2(a+b\operatorname{arcsinh}(cx))}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\operatorname{arcsinh}(cx)}{3c^6d\,(c^2dx^2+d)^{3/2}}-\frac{b\left(\frac{1}{2}\left(\frac{11\arctan(cx)}{c}+6x\right)-\frac{x}{2(c^2x^2+1)}\right)\sqrt{c^2dx^2+d}}{3c^5d^3\sqrt{c^2x^2+1}}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(c^6*d*(d + c^2*d*x^2)^(3/2)) + (2*(a + b*ArcSin
h[c*x]))/(c^6*d^2*Sqrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(c^6*d^3) - (b*Sqrt[d + c^2*d*x^2]*(-1/2*x/(1 + c^2*x^2) + (6*x
+ (11*ArcTan[c*x])/c)/2))/(3*c^5*d^3*Sqrt[1 + c^2*x^2])
```

### 3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.166. $\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.166.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.90

| method | result |
|---|---|
| default | $a\left(\frac{x^4}{c^2 d(c^2 d\,x^2+d)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{c^2 d\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2}{3 d\,c^4\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)x^2}{c^4d^3(c^2x^2+1)}-\frac{b\sqrt{d(c^2x^2+1)}\,x}{c^5d^3\sqrt{c^2x^2+1}}+$ |
| parts | $a\left(\frac{x^4}{c^2 d(c^2 d\,x^2+d)^{\frac{3}{2}}}-\frac{4\left(-\frac{x^2}{c^2 d\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2}{3 d\,c^4\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}\right)}{c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)x^2}{c^4d^3(c^2x^2+1)}-\frac{b\sqrt{d(c^2x^2+1)}\,x}{c^5d^3\sqrt{c^2x^2+1}}+$ |

input
```
int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(x^4/c^2/d/(c^2*d*x^2+d)^(3/2)-4/c^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3
/d/c^4/(c^2*d*x^2+d)^(3/2)))+b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*a
rcsinh(c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x+b*(d*(
c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)+2*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^2/c^4/d^3*arcsinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(3/2)/c^5/d^3*x+5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^6/d^3*
arcsinh(c*x)+11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c
*x+(c^2*x^2+1)^(1/2)-I)-11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

### 3.166.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{11\,(bc^4x^4+2\,bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\,\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\,(3\,bc^4x^4+12\,bc$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
1/12*(11*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d
)*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^4*x^4 + 12*b*c
^2*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^
4*x^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*sqrt(c^2*x^2 + 1) + 16*a)*s
qrt(c^2*d*x^2 + d))/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)
```

### 3.166.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^5(a+b\operatorname{asinh}(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.166.7 Maxima [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^5}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
1/3*b*((3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))*log(c*x + sqrt
(c^2*x^2 + 1))/((c^8*d^3*x^2 + c^6*d^3)*sqrt(c^2*x^2 + 1)) + 3*integrate(1
/3*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))/(c^11*d^3*x^6 + 2*
c^9*d^3*x^4 + c^7*d^3*x^2 + (c^10*d^3*x^5 + 2*c^8*d^3*x^3 + c^6*d^3*x)*sqr
t(c^2*x^2 + 1)), x) - 3*integrate(1/3*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*
x^2 + 8*sqrt(d))/((c^8*d^3*x^3 + c^6*d^3*x)*sqrt(c^2*x^2 + 1)), x)) + 1/3*
a*(3*x^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/2)*c^4
*d) + 8/((c^2*d*x^2 + d)^(3/2)*c^6*d))
```

### 3.166.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.167 $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.167.1 Optimal result

Integrand size = 26, antiderivative size = 203

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b}{6c^5d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\,(d+c^2dx^2)^{3/2}} - \frac{x(a+b\text{arcsinh}(cx))}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc^5d^2\sqrt{d+c^2dx^2}} + \frac{2b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3c^5d^2\sqrt{d+c^2dx^2}}$$

output
```
-1/3*x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-x*(a+b*arcsinh(c*x))
/c^4/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b/c^5/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)
^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^
(1/2)+2/3*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)
```

## 3.167.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{-2ac\sqrt{d}x(3+4c^2x^2)+b\sqrt{d}\Big(\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)-8cx(1+c^2x^2)\text{ar}}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(-2*a*c*Sqrt[d]*x*(3 + 4*c^2*x^2) + b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*x*A
rcSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*Ar
cSinh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 6*a*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2
]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(6*c^5*d^(5/2)*(1 + c^2*x^2)*S
qrt[d + c^2*d*x^2])
```

## 3.167.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {6225, 243, 49, 2009, 6225, 240, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{5/2}}\,dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^3}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x^2}{(c^2x^2+1)^2}dx^2}{6cd^2\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \left(\frac{1}{c^2(c^2x^2+1)}-\frac{1}{c^2(c^2x^2+1)^2}\right)dx^2}{6cd^2\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6225}$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{b\sqrt{c^2x^2+1}\int \frac{x}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{2c^3d\sqrt{c^2dx^2+d}}}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{-\frac{x(a+b\text{arcsinh}(cx))}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc^3d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{2c^3d\sqrt{c^2dx^2+d}}}{c^2d}+\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6cd^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
-1/3*(x^3*(a + b*ArcSinh[c*x]))/(c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1
+ c^2*x^2]*(1/(c^4*(1 + c^2*x^2)) + Log[1 + c^2*x^2]/c^4))/(6*c*d^2*Sqrt[d
 + c^2*d*x^2]) + (-((x*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]))
+ (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c^3*d*Sqrt[d + c^2*d*x^2
]) + (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^3*d*Sqrt[d + c^2*d*x^2]))
/(c^2*d)
```

### 3.167.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

3.167. $\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.167.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| default | $-\frac{a\,x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{ax}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(3\,\text{arcsinh}(cx)^2x^4c^4-8\,\text{arcsinh}\right.}{}$ |
| parts | $-\frac{a\,x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{ax}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(3\,\text{arcsinh}(cx)^2x^4c^4-8\,\text{arcsinh}\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a/c^4
/d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/6*b*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/c^5/d
^3*(3*arcsinh(c*x)^2*x^4*c^4-8*arcsinh(c*x)*c^4*x^4+8*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)*x^4*c^4-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+6*arcsinh(c*x
)^2*x^2*c^2-16*arcsinh(c*x)*c^2*x^2+16*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2
*c^2-6*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+3*arcsinh(c*x)^2-8*arcsi
nh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1)
```

## 3.167.5 Fricas [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3
*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

## 3.167.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^4(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.167.7 Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^
4*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a
+ b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.167.8 Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^4\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.168 $\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.168.1 Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{bx\sqrt{d+c^2dx^2}}{6c^3d^3(1+c^2x^2)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{3c^4d(d+c^2dx^2)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{5b\sqrt{d+c^2dx^2}\arctan(cx)}{6c^4d^3\sqrt{1+c^2x^2}}$$

output
```
1/3*(a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(3/2)+(-a-b*arcsinh(c*x))/c^4/d
^2/(c^2*d*x^2+d)^(1/2)-1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^3/(c^2*x^2+1)^(3/
2)+5/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^3/(c^2*x^2+1)^(1/2)
```

## 3.168.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(bcx+bc^3x^3+4a\sqrt{1+c^2x^2}+6ac^2x^2\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(2+3c^2x^2)\text{arcsinh}(cx)-5b(\right.}{6c^4d^3(1+c^2x^2)^{5/2}}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
-1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 + 4*a*Sqrt[1 + c^2*x^2] + 6*a
*c^2*x^2*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2)*ArcSinh
[c*x] - 5*b*(1 + c^2*x^2)^2*ArcTan[c*x]))/(c^4*d^3*(1 + c^2*x^2)^(5/2))
```

## 3.168.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6219, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \ 6219$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{3c^2x^2+2}{3c^4d^3(c^2x^2+1)^2}dx}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3c^4d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{b\sqrt{c^2dx^2+d}\int \frac{3c^2x^2+2}{(c^2x^2+1)^2}dx}{3c^3d^3\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3c^4d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \ 298$$

$$\frac{b\sqrt{c^2dx^2+d}\left(\frac{5}{2}\int \frac{1}{c^2x^2+1}dx-\frac{x}{2(c^2x^2+1)}\right)}{3c^3d^3\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3c^4d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \ 216$$

$$-\frac{a+b\text{arcsinh}(cx)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3c^4d\left(c^2dx^2+d\right)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)\sqrt{c^2dx^2+d}}{3c^3d^3\sqrt{c^2x^2+1}}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcSinh[c*x])/(3*c^4*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x]
)/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[d + c^2*d*x^2]*(-1/2*x/(1 + c^2*
x^2) + (5*ArcTan[c*x])/(2*c)))/(3*c^3*d^3*Sqrt[1 + c^2*x^2])
```

### 3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.168.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.83

| method | result |
|---|---|
| default | $a\left(-\frac{x^2}{c^2 d\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2}{3 d\,c^4\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}\right)-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)x^2}{\left(c^2x^2+1\right)^2 d^3c^2}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,x}{6\left(c^2x^2+1\right)^{\frac{3}{2}} d^3c^3}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}}{3\left(c^2x^2+1\right)^2 d^3c^4}$ |
| parts | $a\left(-\frac{x^2}{c^2 d\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2}{3 d\,c^4\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}\right)-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)x^2}{\left(c^2x^2+1\right)^2 d^3c^2}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,x}{6\left(c^2x^2+1\right)^{\frac{3}{2}} d^3c^3}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}}{3\left(c^2x^2+1\right)^2 d^3c^4}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(c^2*d*x^2+d)^(3/2))-b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)*x^2-1/6*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(3/2)/d^3/c^3*x-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^2/d^3/c^4*arcsinh(c*x)+5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

## 3.168.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx =$$
$$-\frac{5\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\,\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\left(3\,bc^2x^2+2\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+}\right.}{12\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
-1/12*(5*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d
)*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x^2 + 2*b)*s
qrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^2*x^2 + sqrt(c^
2*x^2 + 1)*b*c*x + 4*a)*sqrt(c^2*d*x^2 + d))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2
+ c^4*d^3)
```

## 3.168.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^3(a+b\operatorname{asinh}(cx))}{(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.168.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{x}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}-\frac{5\,\arctan(cx)}{c^5d^{\frac{5}{2}}}\right) - \frac{1}{3}\,b\left(\frac{3\,x^2}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}+\frac{2}{(c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)\operatorname{arsinh}(cx) - \frac{1}{3}\,a\left(\frac{3\,x^2}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}+\frac{2}{(c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/6*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2)))
 - 1/3*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c
^4*d))*arcsinh(c*x) - 1/3*a*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2
*d*x^2 + d)^(3/2)*c^4*d))
```

### 3.168.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))}{{(d\,c^2\,x^2+d)}^{5/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.169 $\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.169.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=-\frac{b}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x^3(a+b\text{arcsinh}(cx))}{3d\,(d+c^2dx^2)^{3/2}}-\frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{6c^3d^2\sqrt{d+c^2dx^2}}$$

output
```
1/3*x^3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)-1/6*b/c^3/d^2/(c^2*x^2+1)
^(1/2)/(c^2*d*x^2+d)^(1/2)-1/6*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^3/d^2/(
c^2*d*x^2+d)^(1/2)
```

## 3.169.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx=-\frac{\sqrt{d+c^2dx^2}\left(b+bc^2x^2-2ac^3x^3\sqrt{1+c^2x^2}-2bc^3x^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b(1+c^2x^2)^2\log(1+c^2x^2)\right)}{6c^3d^3(1+c^2x^2)^{5/2}}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
-1/6*(Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 - 2*a*c^3*x^3*Sqrt[1 + c^2*x^2] -
 2*b*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(1 + c^2*x^2)^2*Log[1 + c^
2*x^2]))/(c^3*d^3*(1 + c^2*x^2)^(5/2))
```

### 3.169.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6215, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{ 6215}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x^3}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x^2}{(c^2x^2+1)^2}dx^2}{6d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\int \left(\frac{1}{c^2(c^2x^2+1)}-\frac{1}{c^2(c^2x^2+1)^2}\right)dx^2}{6d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{1}{c^4(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{c^4}\right)}{6d^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*c*Sqrt[1 + c^2
*x^2]*(1/(c^4*(1 + c^2*x^2)) + Log[1 + c^2*x^2]/c^4))/(6*d^2*Sqrt[d + c^2*
d*x^2])
```

#### 3.169.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.169.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(103) = 206$.

Time = 0.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.47

| method | result |
|---|---|
| default | $a\left(-\frac{x}{2c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{\frac{x}{3d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2d\,x^2+d}}}{2c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(c^3x^3+c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)\left(-2\ln\left(1+\left(c\right.\right.\right.}{}$ |
| parts | $a\left(-\frac{x}{2c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{\frac{x}{3d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2d\,x^2+d}}}{2c^2}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(c^3x^3+c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)\left(-2\ln\left(1+\left(c\right.\right.\right.}{}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*x/c^2/d/(c^2*d*x^2+d)^(3/2)+1/2/c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2
/3/d^2*x/(c^2*d*x^2+d)^(1/2)))+1/6*b*(d*(c^2*x^2+1))^(1/2)*(c^3*x^3+c^2*x^
2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))*(-2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
*x^6*c^6+2*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^5*c^5+6*arc
sinh(c*x)*c^4*x^4-6*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+2*(c^2*x^2+1)^
(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3-c^4*x^4+c^3*x^3*(c^2*x^2+1)^
(1/2)+6*arcsinh(c*x)*c^2*x^2-6*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-2*c
^2*x^2+2*arcsinh(c*x)-2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1)/(3*c^8*x^8+9*c^
6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3
```

### 3.169.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3
*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.169.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^2(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.169.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}+\frac{\log\left(c^2x^2+1\right)}{c^4d^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{x}{\sqrt{c^2dx^2+d}c^2d^2}-\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}\right)\text{arsinh}\,(cx) + \frac{1}{3}\,a\left(\frac{x}{\sqrt{c^2dx^2+d}c^2d^2}-\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}c^2d}\right)$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/6*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/2
))) + 1/3*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*c^
2*d))*arcsinh(c*x) + 1/3*a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^
2 + d)^(3/2)*c^2*d))
```

### 3.169.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.170 $\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.170.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\text{arcsinh}(cx)}{3c^2d\,(d+c^2dx^2)^{3/2}} + \frac{b\sqrt{1+c^2x^2}\arctan(cx)}{6c^2d^2\sqrt{d+c^2dx^2}}$$

output
```
1/3*(-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*x/c/d^2/(c^2*x^2+1
)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/6*b*arctan(c*x)*(c^2*x^2+1)^(1/2)/c^2/d^2/(c
^2*d*x^2+d)^(1/2)
```

## 3.170.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(bcx+bc^3x^3-2a\sqrt{1+c^2x^2}-2b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b(1+c^2\right.}{6c^2d^3\,(1+c^2x^2)^{5/2}}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] - 2*b*Sqrt
[1 + c^2*x^2]*ArcSinh[c*x] + b*(1 + c^2*x^2)^2*ArcTan[c*x]))/(6*c^2*d^3*(1
 + c^2*x^2)^(5/2))
```

## 3.170.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6213, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\sqrt{c^2x^2+1}\int \frac{1}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}\int \frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1 + c^2*
x^2]*(x/(2*(1 + c^2*x^2)) + ArcTan[c*x]/(2*c)))/(3*c*d^2*Sqrt[d + c^2*d*x^
2])
```

### 3.170.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.170.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.74

| method | result |
|---|---|
| default | $-\frac{a}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,x}{6(c^2x^2+1)^{\frac{3}{2}}d^3c}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{3(c^2x^2+1)^2d^3c^2}+\frac{ib\sqrt{d(c^2x^2+1)}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{6\sqrt{c^2x^2+1}\,c^2d^3}-\frac{ib\sqrt{d(c^2x^2+1}}{6\sqrt{}}$ |
| parts | $-\frac{a}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,x}{6(c^2x^2+1)^{\frac{3}{2}}d^3c}-\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)}{3(c^2x^2+1)^2d^3c^2}+\frac{ib\sqrt{d(c^2x^2+1)}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{6\sqrt{c^2x^2+1}\,c^2d^3}-\frac{ib\sqrt{d(c^2x^2+1}}{6\sqrt{}}$ |

input
```
int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
3/2)/d^3/c*x-1/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x
)+1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+
1)^(1/2)+I)-1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x
+(c^2*x^2+1)^(1/2)-I)
```

### 3.170.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.46

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx =$$
$$-\frac{(bc^4x^4+2\,bc^2x^2+b)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2}}{12\,(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*((b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*
sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*sqrt(c^2*d*x^2 + d)*b*l
og(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*(sqrt(c^2*x^2 + 1)*b*c
*x - 2*a))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

### 3.170.6 Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.170.7 Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x) - 1/3
*a/((c^2*d*x^2 + d)^(3/2)*c^2*d)
```

### 3.170.8 Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x}{(c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{x\,(a + b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

input
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.171 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.171.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+c^2dx^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3cd^2\sqrt{d+c^2dx^2}}$$

output
```
1/3*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arcsinh(c*x))/d^
2/(c^2*d*x^2+d)^(1/2)+1/6*b/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/
3*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)
```

## 3.171.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(b+bc^2x^2+6acx\sqrt{1+c^2x^2}+4ac^3x^3\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}(3+\right.}{6cd^3(1+c^2x^2)^{5/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 + 6*a*c*x*Sqrt[1 + c^2*x^2] + 4*a*c^3*
x^3*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2)*ArcSinh[
c*x] - 2*b*(1 + c^2*x^2)^2*Log[1 + c^2*x^2]))/(6*c*d^3*(1 + c^2*x^2)^(5/2)
)
```

### 3.171.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2dx^2+d\right)^{3/2}}dx}{3d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{\left(c^2x^2+1\right)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2dx^2+d\right)^{3/2}}dx}{3d}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

output
```
b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x]
))/(3*d*(d + c^2*d*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d +
c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d
*x^2])))/(3*d)
```

### 3.171.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

## 3.171. $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx$

### 3.171.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(127) = 254$.

Time = 0.28 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.89

| method | result |
|---|---|
| default | $a\left(\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+3cx+2\sqrt{c^2x^2+1}\right)\left(-8\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.\right.}{}$ |
| parts | $a\left(\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+3cx+2\sqrt{c^2x^2+1}\right)\left(-8\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right.\right.\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))+1/6*b*(d*(c^
2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x+2*(c^2*x^2+1)
^(1/2))*(-8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^6*c^6+8*(c^2*x^2+1)^(1/2)*ln
(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^5*c^5-24*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x
^4*c^4+20*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3+2*c^4*
x^4-2*c^3*x^3*(c^2*x^2+1)^(1/2)+6*arcsinh(c*x)*c^2*x^2-24*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)*x^2*c^2+12*(c^2*x^2+1)^(1/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)*x*c+4*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)-8*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)+2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3
```

### 3.171.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3
*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.171.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.171.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}d}\right)\text{arsinh}\,(cx) + \frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\frac{x}{(c^2dx^2+d)^{\frac{3}{2}}d}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/6*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5/
2))) + 1/3*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d))
*arcsinh(c*x) + 1/3*a*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^
(3/2)*d))
```

### 3.171.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.172 $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{5/2}}\,dx$



## 3.172.1 Optimal result

Integrand size = 26, antiderivative size = 262

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx &= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\ &+ \frac{a+b\text{arcsinh}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\text{arcsinh}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} \\ &- \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\ &- \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
1/3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+(a+b*arcsinh(c*x))/d^2/(c^2*d
*x^2+d)^(1/2)-1/6*b*c*x/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-7/6*b*ar
ctan(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*a
rctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-b*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2
)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(
1/2)
```

### 3.172.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\frac{2a(4+3c^2x^2)\sqrt{d+c^2dx^2}}{(1+c^2x^2)^2}+6a\sqrt{d}\log(x)-6a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{bd^2(1+c^2x^2)^{3/}}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
((2*a*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 6*a*Sqrt[d]*L
og[x] - 6*a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d^2*(1 + c^2
*x^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2)
 + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]] +
6*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSinh[
c*x])]))/(d + c^2*d*x^2)^(3/2))/(6*d^3)
```

### 3.172.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6226, 215, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6226}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2dx^2+d)^{3/2}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2dx^2+d)^{3/2}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{1}{2}\int \frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{x(c^2dx^2+d)^{3/2}}dx}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6226}$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6231}$$

$$\frac{\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4670}$$

---

3.172. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{5/2}}dx$

$$\frac{\frac{i\sqrt{c^2x^2+1}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)}{d\sqrt{c^2dx^2+d}}-}{d}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2715

$$\frac{\frac{i\sqrt{c^2x^2+1}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.}{d\sqrt{c^2dx^2+d}}}{d}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2838

$$\frac{\frac{i\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-}{d}$$

$$\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
(a + b*ArcSinh[c*x])/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*c*Sqrt[1 + c^2*x^2]*
(x/(2*(1 + c^2*x^2)) + ArcTan[c*x]/(2*c)))/(3*d^2*Sqrt[d + c^2*d*x^2]) + (
(a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan
[c*x])/(d*Sqrt[d + c^2*d*x^2]) + (I*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSin
h[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*Po
lyLog[2, E^ArcSinh[c*x]]))/(d*Sqrt[d + c^2*d*x^2]))/d
```

## 3.172. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{5/2}}\,dx$

### 3.172.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.172. $\int \frac{a+b\text{arcsinh}(cx)}{x(d+c^2dx^2)^{5/2}}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.172.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| default | $\frac{a}{3d(c^2 d x^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{c^2 d x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right)}{d^{\frac{5}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x^2c^2}{(c^2x^2+1)^2d^3}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{6(c^2x^2+1)^{\frac{3}{2}}d^3}+\frac{4b\sqrt{d}}{}$ |
| parts | $\frac{a}{3d(c^2 d x^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{c^2 d x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right)}{d^{\frac{5}{2}}}+\frac{b\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x^2c^2}{(c^2x^2+1)^2d^3}-\frac{b\sqrt{d(c^2x^2+1)}\,cx}{6(c^2x^2+1)^{\frac{3}{2}}d^3}+\frac{4b\sqrt{d}}{}$ |

input
```
int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/d/(c^2*d*x^2+d)^(3/2)+a/d^2/(c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2*
d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3*
arcsinh(c*x)*x^2*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d^3*c*x
+4/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3*arcsinh(c*x)-7/3*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)
^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x+(c^2*x^2+1
)^(1/2))
```

### 3.172.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3
*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

### 3.172.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\operatorname{asinh}\left(cx\right)}{x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.172.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) - 1/
((c^2*d*x^2 + d)^(3/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^
2*d*x^2 + d)^(5/2)*x), x)
```

## 3.172.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x), x)
```

## 3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.173 $\int \frac{a+b\,\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx$



## 3.173.1 Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{a+b\,\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx = -\frac{bc\sqrt{d+c^2dx^2}}{6d^3(1+c^2x^2)^{3/2}} - \frac{a+b\,\text{arcsinh}(cx)}{dx(d+c^2dx^2)^{3/2}} - \frac{4c^2x(a+b\,\text{arcsinh}(cx))}{3d(d+c^2dx^2)^{3/2}} - \frac{8c^2x(a+b\,\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(x)}{d^3\sqrt{1+c^2x^2}} + \frac{5bc\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{6d^3\sqrt{1+c^2x^2}}$$

output

```
(-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))/d
/(c^2*d*x^2+d)^(3/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-
1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)+b*c*ln(x)*(c^2*d*x^2+d)^
(1/2)/d^3/(c^2*x^2+1)^(1/2)+5/6*b*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/
(c^2*x^2+1)^(1/2)
```

### 3.173.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.06

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx =$$
$$-\frac{\sqrt{d+c^2dx^2}\Big(bcx+bc^3x^3+6a\sqrt{1+c^2x^2}+24ac^2x^2\sqrt{1+c^2x^2}+16ac^4x^4\sqrt{1+c^2x^2}+2b\sqrt{1+c^2x^2}(3+12}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
-1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 + 6*a*Sqrt[1 + c^2*x^2] + 24*
a*c^2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1
+ c^2*x^2]*(3 + 12*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 3*b*c*x*(1 + c^2*x^
2)^2*Log[1 + 1/(c^2*x^2)] - 8*b*c*x*Log[1 + c^2*x^2] - 16*b*c^3*x^3*Log[1
+ c^2*x^2] - 8*b*c^5*x^5*Log[1 + c^2*x^2]))/(d^3*x*(1 + c^2*x^2)^(5/2))
```

### 3.173.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6219, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{8c^4x^4+12c^2x^2+3}{3d^3x\left(c^2x^2+1\right)^2}dx}{\sqrt{c^2x^2+1}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{8c^4x^4+12c^2x^2+3}{x(c^2x^2+1)^2}dx}{3d^3\sqrt{c^2x^2+1}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \; 1578$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{8c^4x^4+12c^2x^2+3}{x^2(c^2x^2+1)^2}dx^2}{6d^3\sqrt{c^2x^2+1}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \; 1195$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \left(\frac{5c^2}{c^2x^2+1}+\frac{c^2}{(c^2x^2+1)^2}+\frac{3}{x^2}\right)dx^2}{6d^3\sqrt{c^2x^2+1}}-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)}{6d^3\sqrt{c^2x^2+1}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcSinh[c*x])/(d*x*(d + c^2*d*x^2)^(3/2))) - (4*c^2*x*(a + b*ArcS
inh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3
*d^2*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*(-(1 + c^2*x^2)^(-1)
+ 3*Log[x^2] + 5*Log[1 + c^2*x^2]))/(6*d^3*Sqrt[1 + c^2*x^2])
```

## 3.173. $\int \frac{a+b\text{arcsinh}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx$

#### 3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

### 3.173.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(190) = 380$.

Time = 0.25 (sec) , antiderivative size = 1257, normalized size of antiderivative = 5.87

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1257 |
| parts | Expression too large to display | 1257 |

input
```
int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x/(c^2*d*x^2+d)^(3/2)-4*c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x
/(c^2*d*x^2+d)^(1/2)))-16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*
arcsinh(c*x)*c-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x
^2+9)/d^3*x^9*c^10+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c
^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-112/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6
+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8+80/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*
x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-64/3*b*(d*(c^2*x^2+1)
)^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6+64/3*
b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^5-140/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c
^4*x^4+26*c^2*x^2+9)/d^3*x^5*c^6+20*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*
c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-56*b*(d*(c^2*x^2+1))^(1/2)/(
8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4+136/3*b*(d*(c^
2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*c^3-24*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c
^2*x^2+9)/d^3*x^3*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26
*c^2*x^2+9)/d^3*x^2*c^3*(c^2*x^2+1)^(1/2)+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6
*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-44*b*(d*(c^2*x^2+1))^(
1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+24*b*(d*(c
^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1)^(1...
```

### 3.173.5 Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{5}{2}}x^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3
*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

### 3.173.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{a + b \operatorname{asinh}(cx)}{x^2 (d (c^2x^2 + 1))^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.173.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*a*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/2)
*d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2
+ 1))/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

### 3.173.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{x^2 (d + c^2dx^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

### 3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^2\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.174 $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx$



## 3.174.1 Optimal result

Integrand size = 26, antiderivative size = 400

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx = \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{6d(d+c^2dx^2)^{3/2}} - \frac{a+b\text{arcsinh}(cx)}{2dx^2(d+c^2dx^2)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))}{2d^2\sqrt{d+c^2dx^2}} + \frac{13bc^2\sqrt{1+c^2x^2}\arctan(cx)}{6d^2\sqrt{d+c^2dx^2}} + \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{5bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}} - \frac{5bc^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{2d^2\sqrt{d+c^2dx^2}}$$

output
```
-5/6*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/2*(-a-b*arcsinh(c*x))/
d/x^2/(c^2*d*x^2+d)^(3/2)-5/2*c^2*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/
2)+1/4*b*c/d^2/x/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+5/12*b*c^3*x/d^2/(c
^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-3/4*b*c*(c^2*x^2+1)^(1/2)/d^2/x/(c^2*d
*x^2+d)^(1/2)+13/6*b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(
1/2)+5*c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(
1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(
c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x^2+
1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

### 3.174.2 Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\frac{-\frac{4a\sqrt{d+c^2dx^2}(3+20c^2x^2+15c^4x^4)}{(x+c^2x^3)^2}-60ac^2\sqrt{d}\log(x)+60ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
((-4*a*Sqrt[d + c^2*d*x^2]*(3 + 20*c^2*x^2 + 15*c^4*x^4))/(x + c^2*x^3)^2
- 60*a*c^2*Sqrt[d]*Log[x] + 60*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*
d*x^2]] + (b*c^2*d*((4*c*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSi
nh[c*x])/(1 + c^2*x^2) + 104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]
] - 6*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E
^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] - 60*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 +
 c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
*Sech[ArcSinh[c*x]/2]^2 + 6*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/Sqrt[
d + c^2*d*x^2])/(24*d^3)
```

### 3.174.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {6224, 253, 264, 216, 6226, 215, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{5/2}}dx+\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{x^2\left(c^2x^2+1\right)^2}dx}{2d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{5/2}}dx+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\int \frac{1}{x^2\left(c^2x^2+1\right)}dx+\frac{1}{2x\left(c^2x^2+1\right)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{5/2}}dx+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)+\frac{1}{2x\left(c^2x^2+1\right)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x\left(c^2x^2+1\right)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6226}$$

$$-\frac{5}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{\left(c^2x^2+1\right)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x\left(c^2x^2+1\right)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 215}$$

$$-\frac{5}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{1}{2}\int \frac{1}{c^2x^2+1}dx+\frac{x}{2\left(c^2x^2+1\right)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x\left(c^2x^2+1\right)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{5}{2}c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{x}{2\left(c^2x^2+1\right)}\right)}{3d^2\sqrt{c^2dx^2+d}}\right)-\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x\left(c^2x^2+1\right)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6226}$$

---

3.174. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

$$-\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{1}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\cdots\right.}{3d^2\sqrt{c^2dx^2+\cdots}}\cdots\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(\frac{\arctan(cx)}{2c}+\frac{\cdots}{2\cdots}\right.}{3d^2\sqrt{c^2dx^2+d}}\cdots\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6231}$$

$$-\frac{5}{2}c^2\left(\frac{\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2\cdots}}{\cdots}\cdots\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{2}c^2\left(\frac{\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}\cdots}{3d\left(c^2dx^2+\cdots\right.}\cdots\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{5}{2}c^2\left(\frac{\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\arctan(cx)}{d\sqrt{c^2dx^2+d}}}{d}+\frac{a+b\text{arcsinh}\cdots}{3d\left(c^2dx^2+\cdots\right.}\cdots\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

---

3.174. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

$\Big\downarrow$ 4670

$$-\frac{5}{2}c^2\left(\frac{\frac{i\sqrt{c^2x^2+1}\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsin}}{d\sqrt{c^2dx^2+d}}}{d}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2715

$$-\frac{5}{2}c^2\left(\frac{\frac{i\sqrt{c^2x^2+1}\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arctanh}\left(e\right.\right.}{d\sqrt{c^2dx^2+d}}}{d}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2838

$$-\frac{5}{2}c^2\left(\frac{\frac{i\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\text{arcsin}}{d\sqrt{c^2dx}}}{d}\right.$$

$$\frac{a+b\text{arcsinh}(cx)}{2dx^2\left(c^2dx^2+d\right)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])/(d*x^2*(d + c^2*d*x^2)^(3/2)) + (b*c*Sqrt[1 + c^
2*x^2]*(1/(2*x*(1 + c^2*x^2)) + (3*(-x^(-1) - c*ArcTan[c*x]))/2))/(2*d^2*S
qrt[d + c^2*d*x^2]) - (5*c^2*((a + b*ArcSinh[c*x])/(3*d*(d + c^2*d*x^2)^(3
/2)) - (b*c*Sqrt[1 + c^2*x^2]*(x/(2*(1 + c^2*x^2)) + ArcTan[c*x]/(2*c)))/(
3*d^2*Sqrt[d + c^2*d*x^2]) + ((a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2])
 - (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(d*Sqrt[d + c^2*d*x^2]) + (I*Sqrt[1 +
 c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLo
g[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))/(d*Sqrt[d + c^2*d
*x^2]))/d))/2
```

## 3.174. $\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

### 3.174.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

## 3.174. $\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.174.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| default | $-\frac{a}{2d\,x^{2}(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{6d(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{2d^{2}\sqrt{c^{2}d\,x^{2}+d}}+\frac{5a\,c^{2}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^{2}d\,x^{2}+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{d(c^{2}x^{2}+1)}\left(15\operatorname{arcsinh}(\right.}{}\right.$ |
| parts | $-\frac{a}{2d\,x^{2}(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{6d(c^{2}d\,x^{2}+d)^{\frac{3}{2}}}-\frac{5a\,c^{2}}{2d^{2}\sqrt{c^{2}d\,x^{2}+d}}+\frac{5a\,c^{2}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^{2}d\,x^{2}+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{d(c^{2}x^{2}+1)}\left(15\operatorname{arcsinh}(\right.}{}\right.$ |

input
```
int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/x^2/(c^2*d*x^2+d)^(3/2)-5/6*a*c^2/d/(c^2*d*x^2+d)^(3/2)-5/2*a*c^2
/d^2/(c^2*d*x^2+d)^(1/2)+5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)
^(1/2))/x)+b*(-1/6*(d*(c^2*x^2+1))^(1/2)*(15*arcsinh(c*x)*c^4*x^4+2*c^3*x^
3*(c^2*x^2+1)^(1/2)+20*arcsinh(c*x)*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+3*arcs
inh(c*x))/(c^4*x^4+2*c^2*x^2+1)/d^3/x^2+13/3*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)/d^3*dilog(1+c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c
^2+5/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x+(c^2*x^2+1)^(
1/2))*c^2)
```

### 3.174.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3
*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

### 3.174.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{x^3\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.174.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d)*
d^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2))
+ b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

### 3.174.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

### 3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{x^3\,\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.175 $\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx$



## 3.175.1 Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx = \frac{bc^3\sqrt{d+c^2dx^2}}{6d^3(1+c^2x^2)^{3/2}} - \frac{bc\sqrt{d+c^2dx^2}}{6d^3x^2\sqrt{1+c^2x^2}} - \frac{a+b\text{arcsinh}(cx)}{3dx^3(d+c^2dx^2)^{3/2}} + \frac{2c^2(a+b\text{arcsinh}(cx))}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))}{3d(d+c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}} - \frac{8bc^3\sqrt{d+c^2dx^2}\log(x)}{3d^3\sqrt{1+c^2x^2}} - \frac{4bc^3\sqrt{d+c^2dx^2}\log(1+c^2x^2)}{3d^3\sqrt{1+c^2x^2}}$$

output
```
1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x))
/d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2
)+16/3*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*c^3*(c^2*d*x
^2+d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/x^2/(c^2
*x^2+1)^(1/2)-8/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)-4/
3*b*c^3*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)
```

### 3.175.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-bcx-bc^3x^3-2a\sqrt{1+c^2x^2}+12ac^2x^2\sqrt{1+c^2x^2}+48ac^4x^4\sqrt{1+c^2x^2}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 12*a*
c^2*x^2*Sqrt[1 + c^2*x^2] + 48*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 32*a*c^6*x^6*
Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 1
6*c^6*x^6)*ArcSinh[c*x] + 8*b*c^3*x^3*(1 + c^2*x^2)^2*Log[1 + 1/(c^2*x^2)]
 - 16*b*c^3*x^3*Log[1 + c^2*x^2] - 32*b*c^5*x^5*Log[1 + c^2*x^2] - 16*b*c^
7*x^7*Log[1 + c^2*x^2]))/(6*d^3*x^3*(1 + c^2*x^2)^(5/2))
```

### 3.175.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6219, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6219}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int -\frac{-16c^6x^6-24c^4x^4-6c^2x^2+1}{3d^3x^3\left(c^2x^2+1\right)^2}dx}{\sqrt{c^2x^2+1}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{-16c^6x^6-24c^4x^4-6c^2x^2+1}{x^3\left(c^2x^2+1\right)^2}dx}{3d^3\sqrt{c^2x^2+1}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{2331}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \frac{-16c^6x^6-24c^4x^4-6c^2x^2+1}{x^4(c^2x^2+1)^2}dx^2}{6d^3\sqrt{c^2x^2+1}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\,(c^2dx^2+d)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\,(c^2dx^2+d)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{2123}$$

$$\frac{bc\sqrt{c^2dx^2+d}\int \left(-\frac{8c^4}{c^2x^2+1}-\frac{c^4}{(c^2x^2+1)^2}-\frac{8c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6d^3\sqrt{c^2x^2+1}}+\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\,(c^2dx^2+d)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\,(c^2dx^2+d)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c^2(a+b\text{arcsinh}(cx))}{dx\,(c^2dx^2+d)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{3dx^3\,(c^2dx^2+d)^{3/2}}+\frac{16c^4x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x(a+b\text{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}+\frac{bc\sqrt{c^2dx^2+d}\left(\frac{c^2}{c^2x^2+1}-8c^2\log\left(x^2\right)-8c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)}{6d^3\sqrt{c^2x^2+1}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(d*x^3*(d + c^2*d*x^2)^(3/2)) + (2*c^2*(a + b*Ar
cSinh[c*x]))/(d*x*(d + c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/
(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[
d + c^2*d*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*(-x^(-2) + c^2/(1 + c^2*x^2) -
8*c^2*Log[x^2] - 8*c^2*Log[1 + c^2*x^2]))/(6*d^3*Sqrt[1 + c^2*x^2])
```

### 3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.175.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| default | $a\left(-\frac{1}{3d\,x^3(c^2d\,x^2+d)^{\frac{3}{2}}}-2c^2\left(-\frac{1}{dx(c^2d\,x^2+d)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2d\,x^2+d}}\right)\right)\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2\ldots}}{\ldots}$ |
| parts | $a\left(-\frac{1}{3d\,x^3(c^2d\,x^2+d)^{\frac{3}{2}}}-2c^2\left(-\frac{1}{dx(c^2d\,x^2+d)^{\frac{3}{2}}}-4c^2\left(\frac{x}{3d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2d\,x^2+d}}\right)\right)\right)+\frac{b\sqrt{d(c^2x^2+1)}\,\sqrt{c^2\ldots}}{\ldots}$ |

input
```
int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3/(c^2*d*x^2+d)^(3/2)-2*c^2*(-1/d/x/(c^2*d*x^2+d)^(3/2)-4*c^2*
(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))))+1/6*b*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*(32*arcsinh(c*x)*c^7*x^7-16*ln((c*x+(c^2
*x^2+1)^(1/2))^4-1)*x^7*c^7+32*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6*x^6+64*a
rcsinh(c*x)*c^5*x^5-32*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*x^5*c^5+48*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4+32*arcsinh(c*x)*c^3*x^3-16*ln((c*x+(c^2*x^2
+1)^(1/2))^4-1)*x^3*c^3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-c^3*x^3-
2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/d^3/
x^3
```

### 3.175.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^
3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

### 3.175.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{x^4\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(x**4*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.175.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.79

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx = -\frac{1}{6}bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{d^{\frac{5}{2}}}+\frac{16\,c^2\log\left(x\right)}{d^{\frac{5}{2}}}+\frac{1}{c^2d^{\frac{5}{2}}x^4+d^{\frac{5}{2}}x^2}\right) + \frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}d^2}+\frac{8\,c^4x}{(c^2dx^2+d)^{\frac{3}{2}}d}+\frac{6\,c^2}{(c^2dx^2+d)^{\frac{3}{2}}dx}-\frac{1}{(c^2dx^2+d)^{\frac{3}{2}}dx^3}\right)b\,\text{arsinh}\,(cx) + \frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}d^2}+\frac{8\,c^4x}{(c^2dx^2+d)^{\frac{3}{2}}d}+\frac{6\,c^2}{(c^2dx^2+d)^{\frac{3}{2}}dx}-\frac{1}{(c^2dx^2+d)^{\frac{3}{2}}dx^3}\right)a$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2*
d^(5/2)*x^4 + d^(5/2)*x^2)) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) + 8*
c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/((
c^2*d*x^2 + d)^(3/2)*d*x^3))*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^
2 + d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(
3/2)*d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a
```

### 3.175.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

### 3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{x^4\,\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.176 $\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$



## 3.176.1 Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{1}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)}{5c(c+a^2cx^2)^{5/2}} + \frac{4x\text{arcsinh}(ax)}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)}{15c^3\sqrt{c+a^2cx^2}} - \frac{4\sqrt{1+a^2x^2}\log(1+a^2x^2)}{15ac^3\sqrt{c+a^2cx^2}}$$

output
```
1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^
2+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh
(a*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

## 3.176.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(4ax\sqrt{1+a^2x^2}(15+20a^2x^2+8a^4x^4)\,\text{arcsinh}(ax) - (1+a^2x^2)\left(-11 - \right.\right.}{60ac^4(1+a^2x^2)^{7/2}}$$

input
```
Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4
)*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1
 + a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

### 3.176.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6203, 241, 6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{4\int \frac{\text{arcsinh}(ax)}{(a^2cx^2+c)^{5/2}}dx}{5c}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\int \frac{\text{arcsinh}(ax)}{(a^2cx^2+c)^{5/2}}dx}{5c}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{6203}$$

$$\frac{4\left(\frac{2\int \frac{\text{arcsinh}(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\left(\frac{2\int \frac{\text{arcsinh}(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\text{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{6ac^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)}{5c}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 6202$$

$$\frac{4\left(\frac{2\left(\frac{x\text{arcsinh}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{6ac^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)}{5c}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 240$$

$$\frac{4\left(\frac{x\text{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\text{arcsinh}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{6ac^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)}{5c}+\frac{x\text{arcsinh}(ax)}{5c\left(a^2cx^2+c\right)^{5/2}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

output
```
1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5
*c*(c + a^2*c*x^2)^(5/2)) + (4*(1/(6*a*c^2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*
c*x^2]) + (x*ArcSinh[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*((x*ArcSinh[a*
x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c*
Sqrt[c + a^2*c*x^2])))/(3*c)))/(5*c)
```

### 3.176.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

---

3.176. $\int \frac{\text{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

## 3.176.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(170) = 340$.

Time = 0.18 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| default | $\frac{16\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)}{15\sqrt{a^2x^2+1}\,a\,c^4} + \frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(-64a^8x^8-\right.}{}$ |

input
```
int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(
a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x^3-16*a^2
*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*a^8*x^8-64*(a^2*x^
2+1)^(1/2)*a^7*x^7-280*a^6*x^6-248*x^5*a^5*(a^2*x^2+1)^(1/2)+160*a^4*x^4*a
rcsinh(a*x)-456*a^4*x^4-340*a^3*x^3*(a^2*x^2+1)^(1/2)+380*a^2*x^2*arcsinh(
a*x)-328*a^2*x^2-165*a*x*(a^2*x^2+1)^(1/2)+256*arcsinh(a*x)-88)/(40*a^10*x
^10+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2
*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

## 3.176.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6
*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

### 3.176.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{(c(a^2x^2+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)
```

### 3.176.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \frac{1}{60}a\left(\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}-\frac{16\,\log\left(x^2+\frac{1}{a^2}\right)}{a^2c^{\frac{7}{2}}}\right) + \frac{1}{15}\left(\frac{8\,x}{\sqrt{a^2cx^2+c}c^3}+\frac{4\,x}{(a^2cx^2+c)^{\frac{3}{2}}c^2}+\frac{3\,x}{(a^2cx^2+c)^{\frac{5}{2}}c}\right)\operatorname{arsinh}(ax)$$

input
```
integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^
4*c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1
/15*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x
/((a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

### 3.176.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{1}{60}\sqrt{c}\left(\frac{16\,\log\left(a^2x^2+1\right)}{ac^4} - \frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right) + \frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

input
```
integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35
)/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x
*log(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

### 3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{7/2}}\,dx$$

input
```
int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
```

output
```
int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

# 3.177 $\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx$



## 3.177.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{4a^2} + \frac{3\operatorname{arcsinh}(ax)^2}{16a^5}$$

output
```
3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^
2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.177.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{x^4 \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{3a^2x^2 - a^4x^4 + 2ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\operatorname{arcsinh}(ax) + 3\operatorname{arcsinh}(ax)^2}{16a^5}$$

input
```
Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*
x] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

### 3.177.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int x^3dx}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{3\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^4}{16a}$$

input
```
Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/16*x^4/a + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*(-1/4*x^2/
a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4*a^3)))/
(4*a^2)
```

### 3.177.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.177.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{4a^{3}x^{3} \operatorname{arcsinh}(ax)\sqrt{a^{2}x^{2}+1}-a^{4}x^{4}-6 \operatorname{arcsinh}(ax)\sqrt{a^{2}x^{2}+1}\, ax+3a^{2}x^{2}+3 \operatorname{arcsinh}(ax)^{2}+3}{16a^{5}}$ | 74 |

input
```
int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-a^4*x^4-6*arcsinh(a*x)*(a^2
*x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+3)/a^5
```

### 3.177.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^4x^4 - 3\,a^2x^2 - 2\,(2\,a^3x^3 - 3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right) - 3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

input
```
integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a
*x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

### 3.177.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^4}{16a} + \frac{x^3\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{4a^2} + \frac{3x^2}{16a^3} - \frac{3x\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{8a^4} + \frac{3\,\text{asinh}^2\,(ax)}{16a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3
*x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*
x)**2/(16*a**5), Ne(a, 0)), (0, True))
```

### 3.177.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{16}\left(\frac{x^4}{a^2} - \frac{3\,x^2}{a^4} + \frac{3\,\text{arsinh}\,(ax)^2}{a^6}\right)a + \frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\,\sqrt{a^2x^2+1}x}{a^4} + \frac{3\,\text{arsinh}\,(ax)}{a^5}\right)\text{arsinh}\,(ax)$$

input
```
integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*
x)
```

### 3.177.8 Giac [F]

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4 \operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.178 $\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$



## 3.178.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{2x}{3a^3}-\frac{x^3}{9a}-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3a^2}$$

output
```
2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.178.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{6ax-a^3x^3+3(-2+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^4}$$

input
```
Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4
)
```

### 3.178.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \; 6227$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}$$

$$\Big\downarrow \; 6213$$

$$-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}$$

$$\Big\downarrow \; 24$$

$$\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}$$

input
```
Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/9*x^3/a + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2) - (2*(-(x/a) + (
Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2))/(3*a^2)
```

#### 3.178.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.178.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

| method | result | size |
|---|---|---|
| default | $\frac{3a^4x^4 \operatorname{arcsinh}(ax)-3a^2x^2 \operatorname{arcsinh}(ax)-a^3x^3\sqrt{a^2x^2+1}-6 \operatorname{arcsinh}(ax)+6ax\sqrt{a^2x^2+1}}{9a^4\sqrt{a^2x^2+1}}$ | 82 |

input
```
int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9/a^4/(a^2*x^2+1)^(1/2)*(3*a^4*x^4*arcsinh(a*x)-3*a^2*x^2*arcsinh(a*x)-a
^3*x^3*(a^2*x^2+1)^(1/2)-6*arcsinh(a*x)+6*a*x*(a^2*x^2+1)^(1/2))
```

### 3.178.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^3x^3 - 3\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right) - 6\,ax}{9\,a^4}$$

input
```
integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
 1)) - 6*a*x)/a^4
```

### 3.178.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{3a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*
x/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, Tru
e))
```

### 3.178.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{9}\,a\left(\frac{x^3}{a^2} - \frac{6\,x}{a^4}\right) + \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)$$

input
```
integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x
^2 + 1)/a^4)*arcsinh(a*x)
```

### 3.178.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\, dx = \int \frac{x^3 \,\text{asinh}(a\, x)}{\sqrt{a^2\, x^2+1}}\, dx$$

input
```
int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.179 $\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$



## 3.179.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=-\frac{x^2}{4a}+\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a^2}-\frac{\text{arcsinh}(ax)^2}{4a^3}$$

output
```
-1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.179.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=-\frac{a^2x^2-2ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)+\text{arcsinh}(ax)^2}{4a^3}$$

input
```
Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

## 3.179.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}$$

input
```
Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(
4*a^3)
```

### 3.179.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.179.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $-\frac{-2\,\mathrm{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+a^2x^2+\mathrm{arcsinh}(ax)^2+1}{4a^3}$ | 40 |

input
```
int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

### 3.179.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{x^2\mathrm{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{a^2x^2-2\,\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

input
```
integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log
(a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

### 3.179.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^2}{4a} + \frac{x\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{2a^2} - \frac{\text{asinh}^2\,(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh
(a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

### 3.179.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{4}a\left(\frac{x^2}{a^2} - \frac{\text{arsinh}\,(ax)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\text{arsinh}\,(ax)}{a^3}\right)\text{arsinh}\,(ax)$$

input
```
integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arc
sinh(a*x)/a^3)*arcsinh(a*x)
```

### 3.179.8 Giac [F]

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \int \frac{x^2 \, \text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}} \, dx$$

input
```
int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.180 $\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx$



## 3.180.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

output
```
-x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

## 3.180.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2}$$

input
```
Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

## 3.180.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}$$

input
```
Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

### 3.180.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.180.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

| method | result | size |
|---|---|---|
| default | $\frac{a^2x^2 \operatorname{arcsinh}(ax)+\operatorname{arcsinh}(ax)-ax\sqrt{a^2x^2+1}}{a^2\sqrt{a^2x^2+1}}$ | 47 |

input
```
int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+arcsinh(a*x)-a*x*(a^2*x^2+1)
^(1/2))
```

### 3.180.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = -\frac{ax-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

input
```
integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

### 3.180.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True
))
```

### 3.180.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2}$$

input
```
integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

### 3.180.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

input
```
integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

### 3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x\,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.181 $\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx$



## 3.181.1 Optimal result

Integrand size = 18, antiderivative size = 13

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{\operatorname{arcsinh}(ax)^2}{2a}$$

output
```
1/2*arcsinh(a*x)^2/a
```

## 3.181.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{\operatorname{arcsinh}(ax)^2}{2a}$$

input
```
Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^2/(2*a)
```

### 3.181.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$
$$\Big\downarrow \text{ 6198}$$
$$\frac{\text{arcsinh}(ax)^2}{2a}$$

input
```
Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^2/(2*a)
```

#### 3.181.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

### 3.181.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{arcsinh}(ax)^2}{2a}$ | 12 |
| default | $\frac{\text{arcsinh}(ax)^2}{2a}$ | 12 |

input
```
int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(a*x)^2/a
```

### 3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,a}$$

input
```
integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

### 3.181.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

### 3.181.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arsinh}\,(ax)^2}{2\,a}$$

input
```
integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsinh(a*x)^2/a
```

### 3.181.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.181.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}(a\,x)^2}{2\,a}$$

input
```
int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
asinh(a*x)^2/(2*a)
```

# 3.182 $\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx$



## 3.182.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = -2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) + \text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

output
```
-2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^
(1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

## 3.182.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \text{arcsinh}(ax)\left(\log\left(1-e^{-\text{arcsinh}(ax)}\right) - \log\left(1+e^{-\text{arcsinh}(ax)}\right)\right) + \text{PolyLog}\left(2,-e^{-\text{arcsinh}(ax)}\right) - \text{PolyLog}\left(2,e^{-\text{arcsinh}(ax)}\right)$$

input
```
Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + P
olyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

### 3.182.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6231}$$

$$\int \frac{\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{ 26}$$

$$i\int \text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{ 4670}$$

$$i\left(i\int \log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-i\int \log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}\right.$$

$$\Big\downarrow \text{ 2715}$$

$$i\left(i\int e^{-\text{arcsinh}(ax)}\log\left(1-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}\log\left(1+e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}+2i\text{arcsin}\right.$$

$$\Big\downarrow \text{ 2838}$$

$$i\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\right)$$

input
```
Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
I*((2*I)*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + I*PolyLog[2, -E^ArcSinh[a*
x]] - I*PolyLog[2, E^ArcSinh[a*x]])
```

### 3.182.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.182.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.62

| method | result |
|---|---|
| default | $\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)-\operatorname{arcsinh}(ax)\ln\left(1+ax+\right.$ |

input
```
int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))-
arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

### 3.182.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

### 3.182.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

### 3.182.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.182.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.183 $\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$



## 3.183.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{x} + a\log(x)$$

output
```
a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

## 3.183.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{x} + a\log(ax)$$

input
```
Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

### 3.183.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6215}$$

$$a\int \frac{1}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{x}$$

input
```
Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

#### 3.183.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

### 3.183.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

| method | result | size |
|---|---|---|
| default | $-2a \operatorname{arcsinh}(ax)+\frac{\left(ax-\sqrt{a^{2} x^{2}+1}\right) \operatorname{arcsinh}(ax)}{x}+a \ln \left(\left(ax+\sqrt{a^{2} x^{2}+1}\right)^{2}-1\right)$ | 56 |

input
```
int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^
2+1)^(1/2))^2-1)
```

### 3.183.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \frac{ax\log(x)-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

input
```
integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

### 3.183.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

### 3.183.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = a\log(x) - \frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)}{x}$$

input
```
integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

### 3.183.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -a\log\left(-x|a| + \sqrt{a^2x^2+1}\right) + a\log(|x|) + \frac{2\,|a|\log\left(ax + \sqrt{a^2x^2+1}\right)}{\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1}$$

input
```
integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
 sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

### 3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}(a\,x)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.184 $\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$



## 3.184.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{2x^2} + a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{2}a^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - \frac{1}{2}a^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

output
```
-1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcs
inh(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

## 3.184.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(ax)\right) - \text{arcsinh}(ax)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) - 4\text{arcsinh}(ax)\log\left(1-e^{-\text{arcsinh}(ax)}\right) + 4\text{arcsinh}(ax)\log\left(1+e^{-\text{arcsinh}(ax)}\right) - 4\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(ax)}\right) + 4\,\text{PolyLog}\left(2,e^{-\text{arcsinh}(ax)}\right) - \text{arcsinh}(ax)\text{sech}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) + 2\tanh\left(\frac{1}{2}\text{arcsinh}(ax)\right)\right)$$

input
```
Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

output
```
(a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Ar
cSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh
[a*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x]
)] - ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

### 3.184.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6224, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx+\frac{1}{2}a\int \frac{1}{x^2}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 6231}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}a^2\int i\text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{2}ia^2\int \text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{1}{2}ia^2\left(i\int \log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-i\int \log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2i\text{arcsinh}(ax)\text{arctanh}\left(e\right.\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$\big\downarrow$ 2715

$$-\frac{1}{2}ia^2\left(i\int e^{-\text{arcsinh}(ax)}\log\left(1-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}\log\left(1+e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}+2i\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

$\big\downarrow$ 2838

$$-\frac{1}{2}ia^2\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\right)-$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}$$

input
```
Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

output
```
-1/2*a/x - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) - (I/2)*a^2*((2*I)*Arc
Sinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + I*PolyLog[2, -E^ArcSinh[a*x]] - I*Poly
Log[2, E^ArcSinh[a*x]])
```

### 3.184.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.184.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| default | $-\frac{a^2x^2\operatorname{arcsinh}(ax)+ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)}{2\sqrt{a^2x^2+1}\,x^2}-\frac{a^2\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)}{2}-\frac{a^2\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)}{2}+\frac{a^2 a}{}$ |

input
```
int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+a*x*(a^2*x^2+1)^(1/2)+arcsinh
(a*x))/x^2-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylo
g(2,a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2)
)+1/2*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

### 3.184.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

### 3.184.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

### 3.184.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.184.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.185 $\int x^m (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx$



## 3.185.1 Optimal result

Integrand size = 24, antiderivative size = 313

$$\begin{aligned}
&\int x^m (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{bcd^3(2271+1329m+284m^2+27m^3+m^4)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} \\
&\quad -\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2}-\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} \\
&\quad +\frac{d^3x^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{3c^2d^3x^{3+m}(a+b\text{arcsinh}(cx))}{3+m} \\
&\quad +\frac{3c^4d^3x^{5+m}(a+b\text{arcsinh}(cx))}{5+m}+\frac{c^6d^3x^{7+m}(a+b\text{arcsinh}(cx))}{7+m} \\
&\quad -\frac{3bcd^3(2161+1813m+455m^2+35m^3)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2(7+m)^2}
\end{aligned}$$

output
```
d^3*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+3*c^2*d^3*x^(3+m)*(a+b*arcsinh(c*x))/
(3+m)+3*c^4*d^3*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)+c^6*d^3*x^(7+m)*(a+b*arcs
inh(c*x))/(7+m)-3*b*c*d^3*(35*m^3+455*m^2+1813*m+2161)*x^(2+m)*hypergeom([
1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^3+15*m^2+71*m+105)^2-b*c*
d^3*(m^4+27*m^3+284*m^2+1329*m+2271)*x^(2+m)*(c^2*x^2+1)^(1/2)/(7+m)^2/(m^
2+8*m+15)^2-b*c^3*d^3*(9+m)*(13+2*m)*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2/(7+
m)^2-b*c^5*d^3*x^(6+m)*(c^2*x^2+1)^(1/2)/(7+m)^2
```

## 3.185.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.82

$$\int x^m \left(d+c^2dx^2\right)^3 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{x^{1+m}\left(\left(d+c^2dx^2\right)^3 (a+b\operatorname{arcsinh}(cx)) - \frac{bcd^3x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1+\frac{m}{2},2+\frac{m}{2},-c^2x^2\right)}{2+m} + \frac{6d\left(\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}\right.}{}\right.}{}$$

input
```
Integrate[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(x^(1 + m)*((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]) - (b*c*d^3*x*Hypergeome
tric2F1[-5/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (6*d*((d + c^2*d*x^
2)^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeometric2F1[-3/2, 1 + m/2, 2
+ m/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2
)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2
+ m/2, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^
2*x^2)]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)))/(7 + m)
```

## 3.185.3 Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6218, 27, 2340, 1590, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(c^2dx^2+d\right)^3 (a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6218}$$

$$-bc\int \frac{d^3x^{m+1}\left(\frac{c^6x^6}{m+7}+\frac{3c^4x^4}{m+5}+\frac{3c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2x^2+1}}dx+\frac{c^6d^3x^{m+7}(a+b\operatorname{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{ 27}$$

$$-bcd^3\int \frac{x^{m+1}\left(\frac{c^6x^6}{m+7}+\frac{3c^4x^4}{m+5}+\frac{3c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2x^2+1}}dx+\frac{c^6d^3x^{m+7}(a+b\text{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\text{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\text{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{2340}$$

$$-bcd^3\left(\frac{\int \frac{x^{m+1}\left(\frac{(m+9)(2m+13)x^4c^6}{(m+5)(m+7)}+\frac{3(m+7)x^2c^4}{m+3}+\frac{(m+7)c^2}{m+1}\right)}{\sqrt{c^2x^2+1}}dx}{c^2(m+7)}+\frac{c^4\sqrt{c^2x^2+1}x^{m+6}}{(m+7)^2}\right)+\frac{c^6d^3x^{m+7}(a+b\text{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\text{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\text{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{1590}$$

$$-bcd^3\left(\frac{\frac{\int \frac{c^4x^{m+1}\left(\frac{c^2\left(m^4+27m^3+284m^2+1329m+2271\right)x^2}{(m+3)(m+5)(m+7)}+\frac{(m+5)(m+7)}{m+1}\right)}{\sqrt{c^2x^2+1}}dx}{c^2(m+5)}+\frac{c^4(m+9)(2m+13)\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)}+\frac{c^4\sqrt{c^2x^2+1}x^{m+6}}{(m+7)^2}\right)+\frac{c^6d^3x^{m+7}(a+b\text{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\text{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\text{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{27}$$

$$-bcd^3\left(\frac{\frac{c^2\int \frac{x^{m+1}\left(\frac{c^2\left(m^4+27m^3+284m^2+1329m+2271\right)x^2}{(m+3)(m+5)(m+7)}+\frac{(m+5)(m+7)}{m+1}\right)}{\sqrt{c^2x^2+1}}dx}{m+5}+\frac{c^4(m+9)(2m+13)\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)}+\frac{c^4\sqrt{c^2x^2+1}x^{m+6}}{(m+7)^2}\right)+\frac{c^6d^3x^{m+7}(a+b\text{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\text{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\text{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{363}$$

3.185. $\int x^m(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))\,dx$

$$-bcd^3\left(\frac{\frac{c^2\left(\frac{3(35m^3+455m^2+1813m+2161)\int\frac{x^{m+1}}{\sqrt{c^2x^2+1}}dx}{(m+1)(m+3)^2(m+5)(m+7)}+\frac{(m^4+27m^3+284m^2+1329m+2271)\sqrt{c^2x^2+1}x^{m+2}}{(m+3)^2(m+5)(m+7)}\right)}{m+5}+\frac{c^4(m+9)(2m+13)\sqrt{c^2x^2+1}x^{m-}}{(m+5)^2(m+7)}}{c^2(m+7)}\right.$$

$$\frac{c^6d^3x^{m+7}(a+b\text{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\text{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\text{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{c^6d^3x^{m+7}(a+b\text{arcsinh}(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\text{arcsinh}(cx))}{m+5}+\frac{3c^2d^3x^{m+3}(a+b\text{arcsinh}(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\text{arcsinh}(cx))}{m+1}-$$

$$bcd^3\left(\frac{\frac{c^2\left(\frac{3(35m^3+455m^2+1813m+2161)x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)(m+7)}+\frac{(m^4+27m^3+284m^2+1329m+2271)\sqrt{c^2x^2+1}x^{m+2}}{(m+3)^2(m+5)(m+7)}\right)}{m+5}+\frac{c^4(}{}}{c^2(m+7)}\right.$$

input
```
Int[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (3*c^2*d^3*x^(3 + m)*(a + b
*ArcSinh[c*x]))/(3 + m) + (3*c^4*d^3*x^(5 + m)*(a + b*ArcSinh[c*x]))/(5 +
m) + (c^6*d^3*x^(7 + m)*(a + b*ArcSinh[c*x]))/(7 + m) - b*c*d^3*((c^4*x^(6
 + m)*Sqrt[1 + c^2*x^2])/(7 + m)^2 + ((c^4*(9 + m)*(13 + 2*m)*x^(4 + m)*Sq
rt[1 + c^2*x^2])/((5 + m)^2*(7 + m)) + (c^2*(((2271 + 1329*m + 284*m^2 + 2
7*m^3 + m^4)*x^(2 + m)*Sqrt[1 + c^2*x^2])/((3 + m)^2*(5 + m)*(7 + m)) + (3
*(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2 + m)*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, -(c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 + m)*(7 + m)))
)/(5 + m))/(c^2*(7 + m)))
```

## 3.185. $\int x^m(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))\,dx$

### 3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

3.185. $\int x^m(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))\,dx$

### 3.185.4 Maple [F]

$$\int x^m \left(c^2 d\,x^2+d\right)^3 (a+b \operatorname{arcsinh}(cx))\,dx$$

input
```
int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

### 3.185.5 Fricas [F]

$$\int x^m \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^3 (b \operatorname{arsinh}(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c
^6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))*x^m,
 x)
```

### 3.185.6 Sympy [F]

$$\begin{aligned}\int x^m \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = d^3\Bigg(&\int ax^m\,dx+\int bx^m \operatorname{asinh}(cx)\,dx \\ &+\int 3ac^2x^2x^m\,dx+\int 3ac^4x^4x^m\,dx \\ &+\int ac^6x^6x^m\,dx+\int 3bc^2x^2x^m \operatorname{asinh}(cx)\,dx \\ &+\int 3bc^4x^4x^m \operatorname{asinh}(cx)\,dx \\ &+\int bc^6x^6x^m \operatorname{asinh}(cx)\,dx\Bigg)\end{aligned}$$

input
```
integrate(x**m*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
d**3*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(3*a*
c**2*x**2*x**m, x) + Integral(3*a*c**4*x**4*x**m, x) + Integral(a*c**6*x**
6*x**m, x) + Integral(3*b*c**2*x**2*x**m*asinh(c*x), x) + Integral(3*b*c**
4*x**4*x**m*asinh(c*x), x) + Integral(b*c**6*x**6*x**m*asinh(c*x), x))
```

### 3.185.7 Maxima [F]

$$\int x^m \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^3(b\,\text{arsinh}\,(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
a*c^6*d^3*x^(m + 7)/(m + 7) + 3*a*c^4*d^3*x^(m + 5)/(m + 5) + 3*a*c^2*d^3*
x^(m + 3)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) + ((m^3 + 9*m^2 + 23*m + 15)*b
*c^6*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^4*d^3*x^5 + 3*(m^3 + 13*m^
2 + 47*m + 35)*b*c^2*d^3*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*x)*x^m*lo
g(c*x + sqrt(c^2*x^2 + 1))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) - integra
te(((m^3 + 9*m^2 + 23*m + 15)*b*c^7*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)
*b*c^5*d^3*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^3*d^3*x^3 + (m^3 + 15*m^
2 + 71*m + 105)*b*c*d^3*x)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*
x^3 + (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2
+ 176*m + 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*sqrt(c^2*x^2
 + 1)), x) - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^8*d^3*x^8 + 3*(m^3 +
 11*m^2 + 31*m + 21)*b*c^6*d^3*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^4*d^
3*x^4 + (m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*x^2)*x^m/((m^4 + 16*m^3 + 86
*m^2 + 176*m + 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105), x)
```

### 3.185.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.185.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx=\int x^m\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

output
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

# 3.186 $\int x^m(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx$



## 3.186.1 Optimal result

Integrand size = 24, antiderivative size = 217

$$\begin{aligned}\int x^m(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))\,dx &= -\frac{bcd^2(38+13m+m^2)\,x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} \\ &\quad -\frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2}+\frac{d^2x^{1+m}(a+b\text{arcsinh}(cx))}{1+m} \\ &\quad +\frac{2c^2d^2x^{3+m}(a+b\text{arcsinh}(cx))}{3+m}+\frac{c^4d^2x^{5+m}(a+b\text{arcsinh}(cx))}{5+m} \\ &\quad -\frac{bcd^2(149+100m+15m^2)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2}\end{aligned}$$

output
```
d^2*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+2*c^2*d^2*x^(3+m)*(a+b*arcsinh(c*x))/
(3+m)+c^4*d^2*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)-b*c*d^2*(15*m^2+100*m+149)*
x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^2+8*m+
15)^2-b*c*d^2*(m^2+13*m+38)*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2/(5+m)^2-b*c^
3*d^2*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2
```

### 3.186.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.87

$$\int x^m (d+c^2dx^2)^2 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{x^{1+m}\left((d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx)) - \frac{bcd^2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1+\frac{m}{2},2+\frac{m}{2},-c^2x^2\right)}{2+m} + \frac{4d^2\left((2+m)(3+m+c^2x^2+c^2mx\right.}{}\right.}{5+m}$$

input
```
Integrate[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(x^(1 + m)*((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeome
tric2F1[-3/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3
+ m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeome
tric2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/
2, 1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)
```

### 3.186.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6218, 27, 1590, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (c^2dx^2+d)^2 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$\Big\downarrow \text{6218}$$
$$-bc\int \frac{d^2x^{m+1}\left(\frac{c^4x^4}{m+5}+\frac{2c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2x^2+1}}dx + \frac{c^4d^2x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5} + \frac{2c^2d^2x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3} + \frac{d^2x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}$$
$$\Big\downarrow \text{27}$$
$$-bcd^2\int \frac{x^{m+1}\left(\frac{c^4x^4}{m+5}+\frac{2c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2x^2+1}}dx + \frac{c^4d^2x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5} + \frac{2c^2d^2x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3} + \frac{d^2x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{1590}$$

$$-bcd^2\left(\frac{\int \frac{c^2x^{m+1}\left(\frac{c^2(m^2+13m+38)x^2}{(m+3)(m+5)}+\frac{m+5}{m+1}\right)}{\sqrt{c^2x^2+1}}dx}{c^2(m+5)}+\frac{c^2\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2}\right)+\frac{c^4d^2x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5}+\frac{2c^2d^2x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{27}$$

$$-bcd^2\left(\frac{\int \frac{x^{m+1}\left(\frac{c^2(m^2+13m+38)x^2}{(m+3)(m+5)}+\frac{m+5}{m+1}\right)}{\sqrt{c^2x^2+1}}dx}{m+5}+\frac{c^2\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2}\right)+\frac{c^4d^2x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5}+\frac{2c^2d^2x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{363}$$

$$-bcd^2\left(\frac{\frac{(15m^2+100m+149)\int \frac{x^{m+1}}{\sqrt{c^2x^2+1}}dx}{(m+1)(m+3)^2(m+5)}+\frac{(m^2+13m+38)\sqrt{c^2x^2+1}x^{m+2}}{(m+3)^2(m+5)}}{m+5}+\frac{c^2\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2}\right)+\frac{c^4d^2x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5}+\frac{2c^2d^2x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{c^4d^2x^{m+5}(a+b\operatorname{arcsinh}(cx))}{m+5}+\frac{2c^2d^2x^{m+3}(a+b\operatorname{arcsinh}(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\operatorname{arcsinh}(cx))}{m+1}-bcd^2\left(\frac{\frac{(15m^2+100m+149)x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)}+\frac{(m^2+13m+38)\sqrt{c^2x^2+1}x^{m+2}}{(m+3)^2(m+5)}}{m+5}+\frac{c^2\sqrt{c^2x^2+1}x^{m+4}}{(m+5)^2}\right)$$

input
```
Int[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (2*c^2*d^2*x^(3 + m)*(a + b
*ArcSinh[c*x]))/(3 + m) + (c^4*d^2*x^(5 + m)*(a + b*ArcSinh[c*x]))/(5 + m)
 - b*c*d^2*((c^2*x^(4 + m)*Sqrt[1 + c^2*x^2])/(5 + m)^2 + (((38 + 13*m + m
^2)*x^(2 + m)*Sqrt[1 + c^2*x^2])/((3 + m)^2*(5 + m)) + ((149 + 100*m + 15*
m^2)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(
(1 + m)*(2 + m)*(3 + m)^2*(5 + m)))/(5 + m))
```

3.186. $\int x^m(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))\,dx$

### 3.186.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

### 3.186.4 Maple [F]

$$\int x^{m} \left(c^{2} d \,x^{2}+d \right)^{2} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)d x$$

input
```
int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

### 3.186.5 Fricas [F]

$$\int x^m \left(d+c^2 d x^2\right)^2 (a+b\text{arcsinh}(c x)) \, dx=\int { {\left(c^{2} d x^{2} + d\right)}^{2} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)} x^{m} \,d x }$$

input
```
integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*x^m, x)
```

### 3.186.6 Sympy [F]

$$\int x^m \left(d+c^2 d x^2\right)^2 (a+b\text{arcsinh}(c x)) \, dx=d^{2} \left(\int a x^{m}\, dx + \int b x^{m} \operatorname{asinh}{\left(c x \right)}\, dx + \int 2 a c^{2} x^{2} x^{m}\, dx + \int a c^{4} x^{4} x^{m}\, dx + \int 2 b c^{2} x^{2} x^{m} \operatorname{asinh}{\left(c x \right)}\, dx + \int b c^{4} x^{4} x^{m} \operatorname{asinh}{\left(c x \right)}\, dx\right)$$

input
```
integrate(x**m*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
d**2*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(2*a*
c**2*x**2*x**m, x) + Integral(a*c**4*x**4*x**m, x) + Integral(2*b*c**2*x**
2*x**m*asinh(c*x), x) + Integral(b*c**4*x**4*x**m*asinh(c*x), x))
```

### 3.186.7 Maxima [F]

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^2 (b\,\text{arsinh}\,(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
a*c^4*d^2*x^(m + 5)/(m + 5) + 2*a*c^2*d^2*x^(m + 3)/(m + 3) + a*d^2*x^(m +
 1)/(m + 1) + ((m^2 + 4*m + 3)*b*c^4*d^2*x^5 + 2*(m^2 + 6*m + 5)*b*c^2*d^2
*x^3 + (m^2 + 8*m + 15)*b*d^2*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^3 + 9
*m^2 + 23*m + 15) - integrate(((m^2 + 4*m + 3)*b*c^5*d^2*x^5 + 2*(m^2 + 6*
m + 5)*b*c^3*d^2*x^3 + (m^2 + 8*m + 15)*b*c*d^2*x)*x^m/((m^3 + 9*m^2 + 23*
m + 15)*c^3*x^3 + (m^3 + 9*m^2 + 23*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 1
5)*c^2*x^2 + m^3 + 9*m^2 + 23*m + 15)*sqrt(c^2*x^2 + 1)), x) - integrate((
(m^2 + 4*m + 3)*b*c^6*d^2*x^6 + 2*(m^2 + 6*m + 5)*b*c^4*d^2*x^4 + (m^2 + 8
*m + 15)*b*c^2*d^2*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m
^2 + 23*m + 15), x)
```

### 3.186.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.186.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx=\int x^m\,\left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

output
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

# 3.187 $\int x^m(d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx$



## 3.187.1 Optimal result

Integrand size = 22, antiderivative size = 128

$$\int x^m\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx = -\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}(a+b\text{arcsinh}(cx))}{1+m}+\frac{c^2dx^{3+m}(a+b\text{arcsinh}(cx))}{3+m} - \frac{bcd(7+3m)x^{2+m}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{(1+m)(2+m)(3+m)^2}$$

output
```
d*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+c^2*d*x^(3+m)*(a+b*arcsinh(c*x))/(3+m)-
b*c*d*(7+3*m)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(3+m)^2
/(m^2+3*m+2)-b*c*d*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2
```

## 3.187.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int x^m\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))\,dx = \frac{dx^{1+m}\left((2+m)\left(3+m+c^2x^2+c^2mx^2\right)(a+b\text{arcsinh}(cx))-bc(1+m)x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1+\frac{m}{2}\right.\right.}{(1+m)(2+m)(3+m)}$$

input
```
Integrate[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*x^(1 + m)*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) -
 b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b
*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 +
m)*(3 + m))
```

## 3.187.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6218, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(c^2 d x^2+d\right) (a+b \operatorname{arcsinh}(c x)) \, dx$$

$$\Big\downarrow \text{6218}$$

$$-bc\int \frac{d x^{m+1} \left(\frac{c^2 x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2 x^2+1}}dx+\frac{c^2 d x^{m+3} (a+b \operatorname{arcsinh}(c x))}{m+3}+\frac{d x^{m+1} (a+b \operatorname{arcsinh}(c x))}{m+1}$$

$$\Big\downarrow \text{27}$$

$$-bcd\int \frac{x^{m+1} \left(\frac{c^2 x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{c^2 x^2+1}}dx+\frac{c^2 d x^{m+3} (a+b \operatorname{arcsinh}(c x))}{m+3}+\frac{d x^{m+1} (a+b \operatorname{arcsinh}(c x))}{m+1}$$

$$\Big\downarrow \text{363}$$

$$-bcd \left(\frac{(3m+7)\int \frac{x^{m+1}}{\sqrt{c^2x^2+1}}dx}{(m+1)(m+3)^2}+\frac{\sqrt{c^2 x^2+1} x^{m+2}}{(m+3)^2}\right)+\frac{c^2 d x^{m+3} (a+b \operatorname{arcsinh}(c x))}{m+3}+\frac{d x^{m+1} (a+b \operatorname{arcsinh}(c x))}{m+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{c^2 d x^{m+3} (a+b \operatorname{arcsinh}(c x))}{m+3}+\frac{d x^{m+1} (a+b \operatorname{arcsinh}(c x))}{m+1}-bcd \left(\frac{(3m+7) x^{m+2} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2 x^2\right)}{(m+1)(m+2)(m+3)^2}+\frac{\sqrt{c^2 x^2+1} x^{m+2}}{(m+3)^2}\right)$$

input
```
Int[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (c^2*d*x^(3 + m)*(a + b*ArcSi
nh[c*x]))/(3 + m) - b*c*d*((x^(2 + m)*Sqrt[1 + c^2*x^2])/(3 + m)^2 + ((7 +
 3*m)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/
((1 + m)*(2 + m)*(3 + m)^2))
```

### 3.187.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.187.4 Maple [F]

$$\int x^m \left(c^2 d\, x^2 + d\right) \left(a + b\, \mathrm{arcsinh}\left(cx\right)\right) dx$$

input
```
int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

### 3.187.5 Fricas [F]

$$\int x^m \left(d+c^2dx^2\right) (a+b\text{arcsinh}(cx))\,dx = \int (c^2dx^2+d)(b\operatorname{arsinh}(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*x^m, x)
```

### 3.187.6 Sympy [F]

$$\int x^m \left(d+c^2dx^2\right) (a+b\text{arcsinh}(cx))\,dx = d\left(\int ax^m\,dx + \int bx^m \operatorname{asinh}(cx)\,dx + \int ac^2x^2x^m\,dx + \int bc^2x^2x^m \operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate(x**m*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
d*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(a*c**2*
x**2*x**m, x) + Integral(b*c**2*x**2*x**m*asinh(c*x), x))
```

### 3.187.7 Maxima [F]

$$\int x^m \left(d+c^2dx^2\right) (a+b\text{arcsinh}(cx))\,dx = \int (c^2dx^2+d)(b\operatorname{arsinh}(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
a*c^2*d*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) + (b*c^2*d*(m + 1)*x^3 +
 b*d*(m + 3)*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^2 + 4*m + 3) - integra
te((b*c^3*d*(m + 1)*x^3 + b*c*d*(m + 3)*x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 +
(m^2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3)*sqrt(c^2*x
^2 + 1)), x) - integrate((b*c^4*d*(m + 1)*x^4 + b*c^2*d*(m + 3)*x^2)*x^m/(
(m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3), x)
```

### 3.187.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right) (a+b\operatorname{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.187.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right) (a+b\operatorname{arcsinh}(cx))\, dx = \int x^m\, (a+b\operatorname{asinh}(c\,x))\, \left(d\,c^2\,x^2+d\right)\, dx$$

input
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

output
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

# 3.188 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2} dx$



## 3.188.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2},x\right)$$

output
```
Unintegrable(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

## 3.188.2 Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

## 3.188.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2dx^2+d}\,dx$$
$$\Big\downarrow \ 6239$$
$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2dx^2+d}dx$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

output
```
$Aborted
```

### 3.188.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.188.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\,\operatorname{arcsinh}(cx))}{c^2d\,x^2+d}dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

### 3.188.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^m}{c^2dx^2 + d}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

### 3.188.6 Sympy [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \frac{\int \frac{ax^m}{c^2x^2+1}\,dx + \int \frac{bx^m\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

output
```
(Integral(a*x**m/(c**2*x**2 + 1), x) + Integral(b*x**m*asinh(c*x)/(c**2*x*
*2 + 1), x))/d
```

### 3.188.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{d + c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^m}{c^2dx^2 + d}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

### 3.188.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{c^2dx^2+d}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

### 3.188.9 Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

output
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

# 3.189 $\int \frac{x^m(a+b\,\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$



## 3.189.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(a+b\,\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \frac{x^{1+m}(a+b\,\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} - \frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2d^2(2+m)} + \frac{(1-m)\text{Int}\left(\frac{x^m(a+b\,\text{arcsinh}(cx))}{d+c^2dx^2},x\right)}{2d}$$

output
```
1/2*x^(1+m)*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-1/2*b*c*x^(2+m)*hypergeom([
3/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/d^2/(2+m)+1/2*(1-m)*Unintegrable(x^m*(a+
b*arcsinh(c*x))/(c^2*d*x^2+d),x)/d
```

## 3.189.2 Mathematica [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\,\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{x^m(a+b\,\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2, x]
```

## 3.189.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6226, 27, 278, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^2}dx$$

$$\Big\downarrow \text{6226}$$

$$\frac{(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{d(c^2x^2+1)}dx}{2d}-\frac{bc\int \frac{x^{m+1}}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2d^2}-\frac{bc\int \frac{x^{m+1}}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2d^2}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{2d^2(m+2)}$$

$$\Big\downarrow \text{6239}$$

$$\frac{(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2d^2}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{2d^2(c^2x^2+1)}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{2d^2(m+2)}$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

output
```
$Aborted
```

### 3.189.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.189.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a + b \operatorname{arcsinh}(cx))}{(c^2 d x^2 + d)^2} dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

### 3.189.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.189.6 Sympy [N/A]

Not integrable

Time = 16.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{ax^m}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{bx^m\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**m/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**m*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.189.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

### 3.189.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

### 3.189.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^2}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

output
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

# 3.190 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$



## 3.190.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx =& \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}+\frac{(3-m)x^{1+m}(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)}\\ &-\frac{bc(3-m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{8d^3(2+m)}\\ &-\frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{4d^3(2+m)}\\ &+\frac{(1-m)(3-m)\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))}{d+c^2dx^2},x\right)}{8d^2}\end{aligned}$$

output
```
1/4*x^(1+m)*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/8*(3-m)*x^(1+m)*(a+b*ar
csinh(c*x))/d^3/(c^2*x^2+1)-1/8*b*c*(3-m)*x^(2+m)*hypergeom([3/2, 1+1/2*m]
,[2+1/2*m],-c^2*x^2)/d^3/(2+m)-1/4*b*c*x^(2+m)*hypergeom([5/2, 1+1/2*m],[2
+1/2*m],-c^2*x^2)/d^3/(2+m)+1/8*(1-m)*(3-m)*Unintegrable(x^m*(a+b*arcsinh(
c*x))/(c^2*d*x^2+d),x)/d^2
```

### 3.190.2 Mathematica [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx=\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3, x]
```

### 3.190.3 Rubi [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6226, 27, 278, 6226, 278, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(c^2dx^2+d)^3}\,dx$$

$$\Big\downarrow \text{6226}$$

$$\frac{(3-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{d^2(c^2x^2+1)^2}dx}{4d}-\frac{bc\int \frac{x^{m+1}}{(c^2x^2+1)^{5/2}}dx}{4d^3}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{4d^3}-\frac{bc\int \frac{x^{m+1}}{(c^2x^2+1)^{5/2}}dx}{4d^3}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{278}$$

$$\frac{(3-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{4d^3}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}-\frac{bcx^{m+2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{4d^3(m+2)}$$

$$\Big\downarrow \; 6226$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x^{m+1}}{(c^2x^2+1)^{3/2}}dx+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)}{4d^3}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{4d^3(m+2)}$$

$$\Big\downarrow \; 278$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{2(m+2)}\right)}{4d^3}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{4d^3(m+2)}$$

$$\Big\downarrow \; 6239$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{2(m+2)}\right)}{4d^3}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{4d^3(m+2)}$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

output
```
$Aborted
```

### 3.190.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.190. $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.190.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a + b \operatorname{arcsinh}(cx))}{(c^2 d\, x^2 + d)^3} dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

### 3.190.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)x^m}{(c^2dx^2 + d)^3}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3
*x^2 + d^3), x)
```

### 3.190.6 Sympy [N/A]

Not integrable

Time = 117.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{ax^m}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^m\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a*x**m/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(b*x**m*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

### 3.190.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^m}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

### 3.190.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)x^m}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

### 3.190.9 Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a + b\text{arcsinh}(cx))}{(d + c^2dx^2)^3}\,dx = \int \frac{x^m\,(a + b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2 + d)^3}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

output
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

# 3.191 $\int x^m (d+c^2dx^2)^{5/2} (a+b\,\text{arcsinh}(cx))\,dx$



## 3.191.1 Optimal result

Integrand size = 26, antiderivative size = 618

$$\begin{aligned}\int x^m (d+c^2dx^2)^{5/2} (a+b\,\text{arcsinh}(cx))\,dx &= -\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}} \\ &- \frac{5bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(6+m)\,(8+6m+m^2)\,\sqrt{1+c^2x^2}} - \frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(12+8m+m^2)\,\sqrt{1+c^2x^2}} \\ &- \frac{5bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)\sqrt{1+c^2x^2}} - \frac{2bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\ &- \frac{bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}}{(6+m)^2\sqrt{1+c^2x^2}} + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\,\text{arcsinh}(cx))}{(6+m)\,(8+6m+m^2)} \\ &+ \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}\,(a+b\,\text{arcsinh}(cx))}{(4+m)(6+m)} + \frac{x^{1+m}(d+c^2dx^2)^{5/2}\,(a+b\,\text{arcsinh}(cx))}{6+m} \\ &+ \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\,\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(6+m)\,(8+14m+7m^2+m^3)\,\sqrt{1+c^2x^2}} \\ &- \frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)/(6+m)+x^(1+m)*(c^
2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(6+m)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)-15*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(
1/2)/(2+m)^2/(4+m)/(6+m)/(c^2*x^2+1)^(1/2)-5*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)
^(1/2)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(
1/2)/(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-5*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2
)/(4+m)^2/(6+m)/(c^2*x^2+1)^(1/2)-2*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2)/
(4+m)/(6+m)/(c^2*x^2+1)^(1/2)-b*c^5*d^2*x^(6+m)*(c^2*d*x^2+d)^(1/2)/(6+m)^
2/(c^2*x^2+1)^(1/2)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^3+7*m^2+14*m+8)/
(c^2*x^2+1)^(1/2)-15*b*c*d^2*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/
2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+5*m+4)/(c
^2*x^2+1)^(1/2)
```

### 3.191.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.54

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \frac{d^2x^{1+m}\sqrt{d+c^2dx^2}\left(-\frac{bcx((4+m)(6+m)+2c^2(2+m)(6+m)x^2+c^4(2+m)(4+m)x^4)}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}}+\left(1+c^2x^2\right)^2(a+\right.}{}$$

input
```
Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*((4 + m)*(6 + m) + 2*c^2*(2 +
 m)*(6 + m)*x^2 + c^4*(2 + m)*(4 + m)*x^4))/((2 + m)*(4 + m)*(6 + m)*Sqrt[
1 + c^2*x^2])) + (1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]) - (5*(b*c*(1 + m)*(2
 + m)*x*(4 + m + c^2*(2 + m)*x^2) - (1 + m)*(2 + m)^2*(4 + m)*(1 + c^2*x^2
)^(3/2)*(a + b*ArcSinh[c*x]) + 3*(4 + m)*(b*c*(1 + m)*x - (1 + m)*(2 + m)*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2 + m)*(a + b*ArcSinh[c*x])*Hype
rgeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] + b*c*x*Hypergeometri
cPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])))/((1 + m)*
(2 + m)^2*(4 + m)^2*Sqrt[1 + c^2*x^2])))/(6 + m)
```

### 3.191.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.72, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {6223, 244, 2009, 6223, 244, 2009, 6221, 15, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx)) \, dx$$

$$\Big\downarrow \text{6223}$$

$$\frac{5d\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))dx}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\int x^{m+1}\left(c^2x^2+1\right)^2dx}{(m+6)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}$$

$$\Big\downarrow \text{244}$$

$$\frac{5d\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))dx}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\int \left(x^{m+1}+2c^2x^{m+3}+c^4x^{m+5}\right)dx}{(m+6)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5d\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))dx}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}}{m+6}+\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6223}$$

$$\frac{5d\left(\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\int x^{m+1}\left(c^2x^2+1\right)dx}{(m+4)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}\right)}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}}{m+6}+\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{244}$$

$$\frac{5d\left(\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\int\left(x^{m+1}+c^2x^{m+3}\right)dx}{(m+4)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}\right)}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}}{m+6}+\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 2009

$$\frac{5d\left(\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}\right)}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}}{m+6}+\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 6221

$$\frac{5d\left(\frac{3d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{(m+2)\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^{m+1}dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{m+2}\right)}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsi}}{m+4}\right.}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}}{m+6}+\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 15

$$\frac{5d\left(\frac{3d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2\sqrt{c^2x^2+1}}\right)}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(}{m+4}\right.}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}}{m+6}+\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 6232

3.191. $\int x^m\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

$$5d\left(\frac{3d\left(\frac{\sqrt{c^2dx^2+d}\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{m+1}-\frac{bcx^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{m^2+3m+2}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c}\cdots}{\cdots}\right)}{m+4}\cdots\right.$$
$$\overline{\qquad\qquad m+6}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{m+6}-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}}{m+6}+\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{c^2x^2+1}}$$

input
```
Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-((b*c*d^2*Sqrt[d + c^2*d*x^2]*(x^(2 + m)/(2 + m) + (2*c^2*x^(4 + m))/(4 +
 m) + (c^4*x^(6 + m))/(6 + m)))/((6 + m)*Sqrt[1 + c^2*x^2])) + (x^(1 + m)*
(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(6 + m) + (5*d*(-((b*c*d*Sqrt[
d + c^2*d*x^2]*(x^(2 + m)/(2 + m) + (c^2*x^(4 + m))/(4 + m)))/((4 + m)*Sqr
t[1 + c^2*x^2])) + (x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/
(4 + m) + (3*d*(-((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*Sqrt[1 +
c^2*x^2])) + (x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2 + m)
+ (Sqrt[d + c^2*d*x^2]*((x^(1 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[
1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(1 + m) - (b*c*x^(2 + m)*Hypergeom
etricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(2 + 3*
m + m^2)))/((2 + m)*Sqrt[1 + c^2*x^2])))/(4 + m)))/(6 + m)
```

### 3.191.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

### 3.191.4 Maple [F]

$$\int x^m (c^2 d x^2 + d)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx)) \, dx$$

input
```
int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

### 3.191.5 Fricas [F]

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx=\int (c^2 d x^2+d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx)+a) x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

### 3.191.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

### 3.191.7 Maxima [F]

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx=\int (c^2 d x^2+d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx)+a) x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

### 3.191.8 Giac [F(-2)]

Exception generated.

$$\int x^m (d + c^2 d x^2)^{5/2} (a + b \operatorname{arcsinh}(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.191.9 Mupad [F(-1)]

Timed out.

$$\int x^m (d + c^2 d x^2)^{5/2} (a + b \operatorname{arcsinh}(cx)) \, dx = \int x^m (a + b \operatorname{asinh}(c x)) \left(d c^2 x^2 + d\right)^{5/2} dx$$

input
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.192 $\int x^m (d+c^2 d x^2)^{3/2} (a+b\text{arcsinh}(cx))\,dx$



## 3.192.1 Optimal result

Integrand size = 26, antiderivative size = 390

$$\begin{aligned}\int x^m (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))\,dx = &-\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)\sqrt{1+c^2x^2}}\\ &-\frac{bcdx^{2+m}\sqrt{d+c^2dx^2}}{(8+6m+m^2)\sqrt{1+c^2x^2}}-\frac{bc^3dx^{4+m}\sqrt{d+c^2dx^2}}{(4+m)^2\sqrt{1+c^2x^2}}\\ &+\frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8+6m+m^2}+\frac{x^{1+m}(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{4+m}\\ &+\frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(8+14m+7m^2+m^3)\sqrt{1+c^2x^2}}\\ &-\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2(4+m)\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)+3*d*x^(1+m)*(a+b*arcs
inh(c*x))*(c^2*d*x^2+d)^(1/2)/(m^2+6*m+8)-3*b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1
/2)/(2+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1/2)/(m^2
+6*m+8)/(c^2*x^2+1)^(1/2)-b*c^3*d*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2
*x^2+1)^(1/2)+3*d*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3
/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^3+7*m^2+14*m+8)/(c^2*x^2+1)^(1/
2)-3*b*c*d*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c
^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+5*m+4)/(c^2*x^2+1)^(1/2)
```

### 3.192.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.60

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \frac{dx^{1+m}\sqrt{d+c^2dx^2}\left(-\frac{bcx(4+m+c^2(2+m)x^2)}{(2+m)(4+m)\sqrt{1+c^2x^2}}+(1+c^2x^2)(a+b\text{arcsinh}(cx))-\frac{3\left(bc(1+m)x-(1\right.}{}\right.}{}$$

input
```
Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*(4 + m + c^2*(2 + m)*x^2))/((2
+ m)*(4 + m)*Sqrt[1 + c^2*x^2])) + (1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (3
*(b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) -
 (2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 -(c^2*x^2)] + b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
2 + m/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])))/(4 + m)
```

### 3.192.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6223, 244, 2009, 6221, 15, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6223}$$

$$\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\int x^{m+1}\left(c^2x^2+1\right)dx}{(m+4)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\int \left(x^{m+1}+c^2x^{m+3}\right)dx}{(m+4)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}$$

$\Big\downarrow$ 2009

$$\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 6221

$$\frac{3d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{(m+2)\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^{m+1}dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{m+2}\right)}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 15

$$\frac{3d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2\sqrt{c^2x^2+1}}\right)}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 6232

$$\frac{3d\left(\frac{\sqrt{c^2dx^2+d}\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{m+1}-\frac{bcx^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{m^2+3m+2}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2 \cdots}}{\cdots}\right)}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{m+4}-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}$$

input
```
Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

## 3.192. $\int x^m\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

output
```
-((b*c*d*Sqrt[d + c^2*d*x^2]*(x^(2 + m)/(2 + m) + (c^2*x^(4 + m))/(4 + m))
)/((4 + m)*Sqrt[1 + c^2*x^2])) + (x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a + b*A
rcSinh[c*x]))/(4 + m) + (3*d*(-((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 +
m)^2*Sqrt[1 + c^2*x^2])) + (x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/(2 + m) + (Sqrt[d + c^2*d*x^2]*((x^(1 + m)*(a + b*ArcSinh[c*x])*Hype
rgeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(1 + m) - (b*c*x^(2
+ m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*
x^2)])/(2 + 3*m + m^2)))/((2 + m)*Sqrt[1 + c^2*x^2])))/(4 + m)
```

### 3.192.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

### 3.192.4 Maple [F]

$$\int x^m \left(c^2 d\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)dx$$

input
```
int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

### 3.192.5 Fricas [F]

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d)*x^m, x)
```

### 3.192.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

### 3.192.7 Maxima [F]

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int (c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

### 3.192.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.192.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int x^m\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.193 $\int x^m \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.193.1 Optimal result

Integrand size = 26, antiderivative size = 240

$$\begin{aligned}&\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx\\&=-\frac{bcx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2+m}\\&\quad+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(2+3m+m^2)\sqrt{1+c^2x^2}}\\&\quad-\frac{bcx^{2+m}\sqrt{d+c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(1+m)(2+m)^2\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
x^(1+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)-b*c*x^(2+m)*(c^2*d*x^
2+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+x^(1+m)*(a+b*arcsinh(c*x))*hypergeom(
[1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^2+3*m+2)/(c^
2*x^2+1)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1
/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(1+m)/(2+m)^2/(c^2*x^2+1)^(1/2)
```

### 3.193.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.75

$$\int x^m\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{x^{1+m}\sqrt{d+c^2dx^2}\left((1+m)\left(-bcx+a(2+m)\sqrt{1+c^2x^2}+b(2+m)\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)\right)+(2+m)(a+\right.}{(1+m)(2+}$$

input
```
Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(x^(1 + m)*Sqrt[d + c^2*d*x^2]*((1 + m)*(-(b*c*x) + a*(2 + m)*Sqrt[1 + c^2
*x^2] + b*(2 + m)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]) + (2 + m)*(a + b*ArcSinh
[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*Hy
pergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))
/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

### 3.193.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6221, 15, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$\Big\downarrow \text{6221}$$
$$\frac{\sqrt{c^2dx^2+d}\int\frac{x^m(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{(m+2)\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^{m+1}dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{m+2}$$
$$\Big\downarrow \text{15}$$
$$\frac{\sqrt{c^2dx^2+d}\int\frac{x^m(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{m+2}-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{6232}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{m+1}-\frac{bcx^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{m^2+3m+2}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}}{(m+2)^2\sqrt{c^2x^2+1}}$$

input
```
Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*Sqrt[1 + c^2*x^2])) + (x^
(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2 + m) + (Sqrt[d + c^2*
d*x^2]*((x^(1 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, -(c^2*x^2)])/(1 + m) - (b*c*x^(2 + m)*HypergeometricPFQ[{1, 1 +
 m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(2 + 3*m + m^2)))/((2 +
 m)*Sqrt[1 + c^2*x^2])
```

### 3.193.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

### 3.193.4 Maple [F]

$$\int x^m \sqrt{c^2 d\, x^2 + d}\, (a + b \operatorname{arcsinh}(cx))\, dx$$

input
```
int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

### 3.193.5 Fricas [F]

$$\int x^m \sqrt{d + c^2 dx^2}(a + b\text{arcsinh}(cx))\, dx = \int \sqrt{c^2 dx^2 + d}(b \operatorname{arsinh}(cx) + a)x^m\, dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

### 3.193.6 Sympy [F]

$$\int x^m \sqrt{d + c^2 dx^2}(a + b\text{arcsinh}(cx))\, dx = \int x^m \sqrt{d\,(c^2 x^2 + 1)}(a + b \operatorname{asinh}(cx))\, dx$$

input
```
integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

### 3.193.7 Maxima [F]

$$\int x^m \sqrt{d + c^2dx^2}(a + b\text{arcsinh}(cx))\,dx = \int \sqrt{c^2dx^2 + d}(b\,\text{arsinh}\,(cx) + a)x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

### 3.193.8 Giac [F(-2)]

Exception generated.

$$\int x^m \sqrt{d + c^2dx^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.193.9 Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{d + c^2dx^2}(a + b\text{arcsinh}(cx))\,dx = \int x^m\,(a + b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2 + d}\,dx$$

input
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.194 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$



## 3.194.1 Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{(2+3m+m^2)\sqrt{d+c^2dx^2}}$$

output
```
x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2
)*(c^2*x^2+1)^(1/2)/(1+m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+
1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/(m^2+3*m+
2)/(c^2*d*x^2+d)^(1/2)
```

## 3.194.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^{1+m}\sqrt{1+c^2x^2}\left((2+m)(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)-bcx\,{}_3F_2\left(1,1+\frac{m}{2}\right.\right.}{(1+m)(2+m)\sqrt{d+c^2dx^2}}$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(x^(1 + m)*Sqrt[1 + c^2*x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1
+ m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)*Sqrt
[d + c^2*d*x^2])
```

### 3.194.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6232}$$

$$\frac{\sqrt{c^2x^2+1}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m^2+3m+2)\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (
1 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(
2 + m)*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m
/2, 2 + m/2}, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

### 3.194.3.1 Defintions of rubi rules used

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

## 3.194.4 Maple [F]

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2d\,x^2+d}}dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

## 3.194.5 Fricas [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)x^m}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

### 3.194.6 Sympy [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^m(a+b\,\text{asinh}\,(cx))}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**m*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.194.7 Maxima [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

### 3.194.8 Giac [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

### 3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.195 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.195.1 Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx &= \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}}\\
&- \frac{mx^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{d(1+m)\sqrt{d+c^2dx^2}}\\
&- \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{d(2+m)\sqrt{d+c^2dx^2}}\\
&+ \frac{bcmx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{d\left(2+3m+m^2\right)\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-m*x^(1+m)*(a+b*arcsinh(c*
x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d/(
1+m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2
*x^2)*(c^2*x^2+1)^(1/2)/d/(2+m)/(c^2*d*x^2+d)^(1/2)+b*c*m*x^(2+m)*hypergeo
m([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d
/(m^2+3*m+2)/(c^2*d*x^2+d)^(1/2)
```

### 3.195.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.77

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \frac{x^{1+m}\left(-m(2+m)\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\right.\right.}{}$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(x^(1 + m)*(-(m*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)]) + (1 + m)*((2 + m)*(a + b*A
rcSinh[c*x]) - b*c*x*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m
/2, -(c^2*x^2)]) + b*c*m*x*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2
, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/(d*(1 + m)*(2 + m)*Sqrt[d
+ c^2*d*x^2])
```

### 3.195.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6226, 278, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}\,dx$$

$$\Big\downarrow \text{6226}$$

$$-\frac{m\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x^{m+1}}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{x^{m+1}(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{m\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{d}+\frac{x^{m+1}(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\operatorname{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{d(m+2)\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6232}$$

$$-\frac{m\left(\frac{\sqrt{c^2x^2+1}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m^2+3m+2)\sqrt{c^2dx^2+d}}\right)}{d}$$

$$\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{d(m+2)\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(x^(1 + m)*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*
Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/
(d*(2 + m)*Sqrt[d + c^2*d*x^2]) - (m*((x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/((
1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeomet
ricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((2 + 3*m
 + m^2)*Sqrt[d + c^2*d*x^2])))/d
```

### 3.195.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

3.195. $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

### 3.195.4 Maple [F]

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}\,(cx))}{(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

### 3.195.5 Fricas [F]

$$\int \frac{x^m(a+b\mathrm{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2
*d^2*x^2 + d^2), x)
```

### 3.195.6 Sympy [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^m(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**m*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.195.7 Maxima [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.195.8 Giac [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^m\,(a+b\,\mathrm{asinh}(c\,x))}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

# 3.196 $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.196.1 Optimal result

Integrand size = 26, antiderivative size = 402

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x^{1+m}(a+b\text{arcsinh}(cx))}{3d\,(d+c^2dx^2)^{3/2}} + \frac{(2-m)x^{1+m}(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}$$
$$- \frac{(2-m)mx^{1+m}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{3d^2(1+m)\sqrt{d+c^2dx^2}}$$
$$- \frac{bc(2-m)x^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{3d^2(2+m)\sqrt{d+c^2dx^2}}$$
$$- \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{3d^2(2+m)\sqrt{d+c^2dx^2}}$$
$$+ \frac{bc(2-m)mx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-c^2x^2\right)}{3d^2(2+3m+m^2)\sqrt{d+c^2dx^2}}$$

output
```
1/3*x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/3*(2-m)*x^(1+m)*(a+
b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*x^(1+m)*(a+b*arcsinh(c
*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^
2/(1+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(2-m)*x^(2+m)*hypergeom([1, 1+1/2*m],[
2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(2+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c
*x^(2+m)*hypergeom([2, 1+1/2*m],[2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/
(2+m)/(c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*m*x^(2+m)*hypergeom([1, 1+1/2*m, 1
+1/2*m],[2+1/2*m, 3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(m^2+3*m+2)/(
c^2*d*x^2+d)^(1/2)
```

### 3.196.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.71

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x^{1+m}\Big((1+m)(2+m)(a+b\operatorname{arcsinh}(cx)) - bc(1+m)x(1+c^2x^2)^{3/2}\,\text{Hyperge}}{}$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(x^(1 + m)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*(1 + c^2*
x^2)^(3/2)*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, -(c^2*x^2)] + (2 - m)*(1
 + c^2*x^2)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Sqrt[1 +
 c^2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)] - m*Sqrt[1 +
c^2*x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/
2 + m/2, 2 + m/2}, -(c^2*x^2)]))))/(3*d^2*(1 + m)*(2 + m)*(1 + c^2*x^2)*Sq
rt[d + c^2*d*x^2])
```

### 3.196.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6226, 278, 6226, 278, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{5/2}}\,dx$$

$$\Big\downarrow \; 6226$$

$$\frac{(2-m)\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{3d} - \frac{bc\sqrt{c^2x^2+1}\int \frac{x^{m+1}}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}} + \frac{x^{m+1}(a+b\operatorname{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 278$$

$$\frac{(2-m)\int \frac{x^m(a+b\operatorname{arcsinh}(cx))}{(c^2dx^2+d)^{3/2}}dx}{3d} + \frac{x^{m+1}(a+b\operatorname{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}} - \frac{bc\sqrt{c^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 6226$$

$$\frac{(2-m)\left(-\frac{m\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x^{m+1}}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 278$$

$$\frac{(2-m)\left(-\frac{m\int \frac{x^m(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx}{d}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{d(m+2)\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 6232$$

$$\frac{(2-m)\left(-\frac{m\left(\frac{\sqrt{c^2x^2+1}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}\right)}{d}\right.}{3d}$$

$$\frac{x^{m+1}(a+b\text{arcsinh}(cx))}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}x^{m+2}\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{3d^2(m+2)\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(x^(1 + m)*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*c*x^(2 +
 m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -(c^2*x^2
)])/(3*d^2*(2 + m)*Sqrt[d + c^2*d*x^2]) + ((2 - m)*((x^(1 + m)*(a + b*ArcS
inh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hype
rgeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d*(2 + m)*Sqrt[d + c^
2*d*x^2]) - (m*((x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)*Sqrt[d + c^2*d*
x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[d + c^2*d
*x^2])))/d))/(3*d)
```

## 3.196. $\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

### 3.196.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] && !IntegerQ[m]
```

## 3.196.4 Maple [F]

$$\int \frac{x^m(a + b \operatorname{arcsinh}(cx))}{(c^2 d x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

### 3.196.5 Fricas [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4
*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.196.6 Sympy [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m(a+b\,\text{asinh}\,(cx))}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**m*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.196.7 Maxima [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

## 3.196.8 Giac [F]

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

## 3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

# 3.197 $\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx$



## 3.197.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \frac{x^{1+m} \operatorname{arcsinh}(ax) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2x^2\right)}{1+m} - \frac{a x^{2+m} {}_3F_2\left(1, 1+\frac{m}{2}, 1+\frac{m}{2}; \frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}; -a^2x^2\right)}{2+3m+m^2}$$

output
```
x^(1+m)*arcsinh(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m
)-a*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],-a^2*x^2)
/(m^2+3*m+2)
```

## 3.197.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}} \, dx = \frac{x^{1+m}\left((2+m) \operatorname{arcsinh}(ax) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2x^2\right) - ax\, {}_3F_2\left(1, 1+\frac{m}{2}, 1+\frac{m}{2}; \frac{3}{2}+\frac{m}{2}, 2 \right.\right.}{(1+m)(2+m)}$$

input
```
Integrate[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(x^(1 + m)*((2 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)
/2, -(a^2*x^2)] - a*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
 2 + m/2}, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

### 3.197.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{6232}$$
$$\frac{x^{m+1}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-a^2x^2\right)}{m^2+3m+2}$$

input
```
Int[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(x^(1 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2
*x^2)])/(1 + m) - (a*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3
/2 + m/2, 2 + m/2}, -(a^2*x^2)])/(2 + 3*m + m^2)
```

#### 3.197.3.1 Defintions of rubi rules used

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

### 3.197.4 Maple [F]

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

input
```
int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

### 3.197.5 Fricas [F]

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.197.6 Sympy [F]

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x**m*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*asinh(a*x)/sqrt(a**2*x**2 + 1), x)
```

### 3.197.7 Maxima [F]

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.197.8 Giac [F]

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

### 3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m \text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \,\text{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

# 3.198 $\int x^4(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$



## 3.198.1 Optimal result

Integrand size = 24, antiderivative size = 283

$$\begin{aligned}&\int x^4\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx\\&=\frac{304b^2dx}{3675c^4}-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7\\&\quad-\frac{32bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{525c^5}+\frac{16bdx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{525c^3}\\&\quad-\frac{4bdx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{175c}-\frac{2bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{21c^5}\\&\quad+\frac{4bd(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{35c^5}-\frac{2bd(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c^5}\\&\quad+\frac{2}{35}dx^5(a+b\text{arcsinh}(cx))^2+\frac{1}{7}dx^5\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
304/3675*b^2*d*x/c^4-152/11025*b^2*d*x^3/c^2+38/6125*b^2*d*x^5+2/343*b^2*c
^2*d*x^7-2/21*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/35*b*d*(c^2*x
^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/49*b*d*(c^2*x^2+1)^(7/2)*(a+b*arcsinh
(c*x))/c^5+2/35*d*x^5*(a+b*arcsinh(c*x))^2+1/7*d*x^5*(c^2*x^2+1)*(a+b*arcs
inh(c*x))^2-32/525*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+16/525*b*d
*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/175*b*d*x^4*(a+b*arcsinh(c
*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.198.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.71

$$\int x^4\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d\left(11025a^2c^5x^5(7+5c^2x^2)-210ab\sqrt{1+c^2x^2}(152-76c^2x^2+57c^4x^4+75c^6x^6)+b^2(31920cx-5320c^3x^3\right.}{}$$

input
```
Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(11025*a^2*c^5*x^5*(7 + 5*c^2*x^2) - 210*a*b*Sqrt[1 + c^2*x^2]*(152 - 7
6*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6) + b^2*(31920*c*x - 5320*c^3*x^3 + 239
4*c^5*x^5 + 2250*c^7*x^7) - 210*b*(-105*a*c^5*x^5*(7 + 5*c^2*x^2) + b*Sqrt
[1 + c^2*x^2]*(152 - 76*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] +
 11025*b^2*c^5*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^5)
```

### 3.198.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6223, 6191, 6219, 27, 2009, 6227, 15, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$\Big\downarrow \ 6223$$
$$-\frac{2}{7}bcd\int x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{7}d\int x^4(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$
$$\Big\downarrow \ 6191$$
$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{7}bcd\int x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$
$$\Big\downarrow \ 6219$$

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-$$
$$\frac{2}{7}bcd\left(-bc\int\frac{15c^6x^6+3c^4x^4-4c^2x^2+8}{105c^6}dx+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(c\right.}{5c^6}$$
$$\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 27

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-$$
$$\frac{2}{7}bcd\left(-\frac{b\int\left(15c^6x^6+3c^4x^4-4c^2x^2+8\right)dx}{105c^5}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(\right.}{5c^6}$$
$$\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+$$
$$b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}+\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6}\right.$$

$\Big\downarrow$ 6227

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{5c^2}-\frac{b\int x^4dx}{5c}+\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{5c^2}\right)\right)$$
$$\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}+\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6}\right.$$

$\Big\downarrow$ 15

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{5c^2}+\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{bx^5}{25c}\right)\right)+$$
$$\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}+\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6}\right.$$

$\Big\downarrow$ 6227

---

3.198. $\int x^4\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}\right)}{5c^2}+\frac{x^4\sqrt{c\ldots}}{}\right.\right.$$

$$\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{7}bcd\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}+\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6}-\right.$$

$\downarrow$ 15

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}+\frac{x^4\sqrt{c^2x^2\ldots}}{}\right.\right.$$

$$\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{7}bcd\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}+\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6}-\right.$$

$\downarrow$ 6213

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}\right.\right.$$

$$\frac{1}{7}dx^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{7}bcd\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}+\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6}-\right.$$

$\downarrow$ 24

3.198. $\int x^4(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{7}dx^5(c^2x^2+1)(a+b\text{arcsinh}(cx))^2+$$

$$\frac{2}{7}d\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{4\left(\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(\right.}{}\right.}{5c^2}\right.\right.$$

$$\frac{2}{7}bcd\left(\frac{(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}-\frac{2(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}+\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c^6}\right.$$

input
```
Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/7 - (2*b*c*d*(-1/105*(b*(8*x
- (4*c^2*x^3)/3 + (3*c^4*x^5)/5 + (15*c^6*x^7)/7))/c^5 + ((1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/(3*c^6) - (2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/(5*c^6) + ((1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^6)))/7 +
(2*d*((x^5*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*(-1/25*(b*x^5)/c + (x^4*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (4*(-1/9*(b*x^3)/c + (x^2*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) - (2*(-((b*x)/c) + (Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2))/(3*c^2)))/(5*c^2)))/5))/7
```

### 3.198.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.198. $\int x^4(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$

### 3.198.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| parts | $d\,a^2\left(\frac{1}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{2\operatorname{arcsinh}(cx)^2xc}{35}+\frac{\operatorname{arcsinh}(cx)^2}{}\right.}{}$ |
| derivativedivides | $\frac{d\,a^2\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{2\operatorname{arcsinh}(cx)^2xc}{35}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2\right.}{35}\right.}{}$ |
| default | $\frac{d\,a^2\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{2\operatorname{arcsinh}(cx)^2xc}{35}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2\right.}{35}\right.}{}$ |

input
```
int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/7*c^2*x^7+1/5*x^5)+d*b^2/c^5*(1/7*arcsinh(c*x)^2*c^3*x^3*(c^2*x^2
+1)^2-3/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+2/35*arcsinh(c*x)^2*x*c+1/35*a
rcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(5/2)+
62/1225*arcsinh(c*x)*(c^2*x^2+1)^(5/2)+2/343*c*x*(c^2*x^2+1)^3+37384/38587
5*c*x-484/42875*c*x*(c^2*x^2+1)^2-3358/385875*c*x*(c^2*x^2+1)-4/35*arcsinh
(c*x)*(c^2*x^2+1)^(1/2)-2/105*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d*a*b/c^5*
(1/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1
)^(1/2)-19/1225*c^4*x^4*(c^2*x^2+1)^(1/2)+76/3675*c^2*x^2*(c^2*x^2+1)^(1/2
)-152/3675*(c^2*x^2+1)^(1/2))
```

### 3.198.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92

$$\int x^4\left(d+c^2dx^2\right)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{1125\,(49\,a^2+2\,b^2)c^7dx^7+63\,(1225\,a^2+38\,b^2)c^5dx^5-5320\,b^2c^3dx^3+31920\,b^2cdx+11025\,(5\,b^2c^7dx^7+7}{}$$

input
```
integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d*x^7 + 63*(1225*a^2 + 38*b^2)*c^5*d*x
^5 - 5320*b^2*c^3*d*x^3 + 31920*b^2*c*d*x + 11025*(5*b^2*c^7*d*x^7 + 7*b^2
*c^5*d*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d*x^7 + 735*
a*b*c^5*d*x^5 - (75*b^2*c^6*d*x^6 + 57*b^2*c^4*d*x^4 - 76*b^2*c^2*d*x^2 +
152*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c
^6*d*x^6 + 57*a*b*c^4*d*x^4 - 76*a*b*c^2*d*x^2 + 152*a*b*d)*sqrt(c^2*x^2 +
 1))/c^5
```

### 3.198.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.37

$$\int x^4(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^2dx^7}{7} + \frac{a^2dx^5}{5} + \frac{2abc^2dx^7\operatorname{asinh}(cx)}{7} - \frac{2abcdx^6\sqrt{c^2x^2+1}}{49} + \frac{2abdx^5\operatorname{asinh}(cx)}{5} - \frac{38abdx^4\sqrt{c^2x^2+1}}{1225c} + \frac{152abdx^2\sqrt{c^2x^2+1}}{3675c^3} - 30 \\ \frac{a^2dx^5}{5} \end{cases}$$

input
```
integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**2*d*x**7/7 + a**2*d*x**5/5 + 2*a*b*c**2*d*x**7*asinh(c*
x)/7 - 2*a*b*c*d*x**6*sqrt(c**2*x**2 + 1)/49 + 2*a*b*d*x**5*asinh(c*x)/5 -
 38*a*b*d*x**4*sqrt(c**2*x**2 + 1)/(1225*c) + 152*a*b*d*x**2*sqrt(c**2*x**
2 + 1)/(3675*c**3) - 304*a*b*d*sqrt(c**2*x**2 + 1)/(3675*c**5) + b**2*c**2
*d*x**7*asinh(c*x)**2/7 + 2*b**2*c**2*d*x**7/343 - 2*b**2*c*d*x**6*sqrt(c*
*2*x**2 + 1)*asinh(c*x)/49 + b**2*d*x**5*asinh(c*x)**2/5 + 38*b**2*d*x**5/
6125 - 38*b**2*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1225*c) - 152*b**2*d
*x**3/(11025*c**2) + 152*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*
c**3) + 304*b**2*d*x/(3675*c**4) - 304*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*
x)/(3675*c**5), Ne(c, 0)), (a**2*d*x**5/5, True))
```

### 3.198.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.56

$$\int x^4(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{7}\,b^2c^2dx^7\,\mathrm{arsinh}\,(cx)^2 + \frac{1}{7}\,a^2c^2dx^7 + \frac{1}{5}\,b^2dx^5\,\mathrm{arsinh}\,(cx)^2 + \frac{1}{5}\,a^2dx^5$$
$$+ \frac{2}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx) - \left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\,\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^2$$
$$- \frac{2}{25725}\left(105\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\,\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\,\mathrm{arsinh}\,(cx) - \frac{75\,c^6\right.$$
$$+ \frac{2}{75}\left(15\,x^5\,\mathrm{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abd$$
$$- \frac{2}{1125}\left(15\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\,\mathrm{arsinh}\,(cx) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right)$$

input
```
integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*c^2*d*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^2*d*x^7 + 1/5*b^2*d*x^5*arcsi
nh(c*x)^2 + 1/5*a^2*d*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 +
 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 -
16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)
*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*
sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^
2*x^3 - 1680*x)/c^6)*b^2*c^2*d + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x
^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c
)*a*b*d - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^
2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3
+ 120*x)/c^4)*b^2*d
```

### 3.198.8 Giac [F(-2)]

Exception generated.

$$\int x^4(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.198.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))^2\,dx = \int x^4\,(a+b\,\mathrm{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

output
```
int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

# 3.199 $\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$



## 3.199.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = -\frac{b^2dx^2}{24c^2}+\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{12c^3}-\frac{bdx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{18c}-\frac{1}{18}bcdx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))-\frac{d(a+b\text{arcsinh}(cx))^2}{24c^4}+\frac{1}{12}dx^4(a+b\text{arcsinh}(cx))^2+\frac{1}{6}dx^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2$$

output
```
-1/24*b^2*d*x^2/c^2+1/72*b^2*d*x^4+1/108*b^2*c^2*d*x^6-1/24*d*(a+b*arcsinh
(c*x))^2/c^4+1/12*d*x^4*(a+b*arcsinh(c*x))^2+1/6*d*x^4*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2+1/12*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-1/18*b*d
*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/18*b*c*d*x^5*(a+b*arcsinh(c*
x))*(c^2*x^2+1)^(1/2)
```

### 3.199.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.94

$$\int x^3(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{d\left(cx\left(18a^2c^3x^3(3+2c^2x^2)-6ab\sqrt{1+c^2x^2}(-3+2c^2x^2+2c^4x^4)+b^2cx(-9+3c^2x^2+2c^4x^4)\right)+6b\left(bcx\sqrt{\right.\right.}{}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(c*x*(18*a^2*c^3*x^3*(3 + 2*c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-3 + 2*
c^2*x^2 + 2*c^4*x^4) + b^2*c*x*(-9 + 3*c^2*x^2 + 2*c^4*x^4)) + 6*b*(b*c*x*
Sqrt[1 + c^2*x^2]*(3 - 2*c^2*x^2 - 2*c^4*x^4) + 3*a*(-1 + 6*c^4*x^4 + 4*c^
6*x^6))*ArcSinh[c*x] + 9*b^2*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]^2))
/(216*c^4)
```

### 3.199.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.66, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6223, 6191, 6221, 15, 6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(c^2dx^2+d)(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{1}{3}bcd\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}d\int x^3(a+b\text{arcsinh}(cx))^2dx+\frac{1}{6}dx^4(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{3}bcd\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}dx^4(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6221}$$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-$$
$$\frac{1}{3}bcd\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{6}bc\int x^5dx+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+$$
$$\frac{1}{6}dx^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \ 15$$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-$$
$$\frac{1}{3}bcd\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{36}bcx^6\right)+$$
$$\frac{1}{6}dx^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \ 6227$$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}\right)\right)$$
$$\frac{1}{3}bcd\left(\frac{1}{6}\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}\right)+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsi}\right.$$
$$\frac{1}{6}dx^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \ 15$$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)\right)-$$
$$\frac{1}{3}bcd\left(\frac{1}{6}\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(c\right.$$
$$\frac{1}{6}dx^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \ 6227$$

3.199. $\int x^3\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2+}}{}\right.\right.$$

$$\frac{1}{3}bcd\left(\frac{1}{6}\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx}{16}\right.\right.$$

$$\frac{1}{6}dx^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(}{}\right.\right.$$

$$\frac{1}{3}bcd\left(\frac{1}{6}\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)\right.$$

$$\frac{1}{6}dx^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{6}dx^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+$$

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{3\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsi}}{2c^2}\right.}{4c^2}\right.\right.$$

$$\frac{1}{3}bcd\left(\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{1}{6}\left(\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{3\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+}}{}\right.}{4c^2}\right.\right.$$

input
```
Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

---

3.199. $\int x^3(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(d*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/6 - (b*c*d*(-1/36*(b*c*x^6) +
 (x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/6 + (-1/16*(b*x^4)/c + (x^3*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*c^2) - (3*(-1/4*(b*x^2)/c + (x*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(
4*b*c^3)))/(4*c^2))/6))/3 + (d*((x^4*(a + b*ArcSinh[c*x])^2)/4 - (b*c*(-1/
16*(b*x^4)/c + (x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*c^2) - (3*(
-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a +
 b*ArcSinh[c*x])^2/(4*b*c^3)))/(4*c^2)))/2))/3
```

### 3.199.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.199.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| parts | $d\,a^2\left(\frac{1}{6}c^2x^6+\frac{1}{4}x^4\right)+\frac{d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^2}{6}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^2}{12}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{18}+\ldots\right.}{\ldots}$ |
| derivativedivides | $\frac{d\,a^2\left(\frac{1}{6}c^6x^6+\frac{1}{4}c^4x^4\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^2}{6}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^2}{12}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{18}+\ldots\right.}{\ldots}$ |
| default | $\frac{d\,a^2\left(\frac{1}{6}c^6x^6+\frac{1}{4}c^4x^4\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^2}{6}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^2}{12}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{18}+\ldots\right.}{\ldots}$ |

input
```
int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/6*c^2*x^6+1/4*x^4)+d*b^2/c^4*(1/6*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2
+1)^2-1/12*arcsinh(c*x)^2*(c^2*x^2+1)^2-1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^
(5/2)+1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)+1/12*arcsinh(c*x)*c*x*(c^2*x
^2+1)^(1/2)+1/24*arcsinh(c*x)^2+1/108*(c^2*x^2+1)^3-1/72*(c^2*x^2+1)^2-1/2
4*c^2*x^2-1/24)+2*d*a*b/c^4*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4
*x^4-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1/2)+1/24*c*
x*(c^2*x^2+1)^(1/2)-1/24*arcsinh(c*x))
```

## 3.199.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.21

$$\int x^3(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{2\,(18\,a^2 + b^2)c^6dx^6 + 3\,(18\,a^2 + b^2)c^4dx^4 - 9\,b^2c^2dx^2 + 9\,(4\,b^2c^6dx^6 + 6\,b^2c^4dx^4 - b^2d)\log\left(cx + \sqrt{c^2x^2 +}\right.}{}$$

input
```
integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/216*(2*(18*a^2 + b^2)*c^6*d*x^6 + 3*(18*a^2 + b^2)*c^4*d*x^4 - 9*b^2*c^2
*d*x^2 + 9*(4*b^2*c^6*d*x^6 + 6*b^2*c^4*d*x^4 - b^2*d)*log(c*x + sqrt(c^2*
x^2 + 1))^2 + 6*(12*a*b*c^6*d*x^6 + 18*a*b*c^4*d*x^4 - 3*a*b*d - (2*b^2*c^
5*d*x^5 + 2*b^2*c^3*d*x^3 - 3*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt
(c^2*x^2 + 1)) - 6*(2*a*b*c^5*d*x^5 + 2*a*b*c^3*d*x^3 - 3*a*b*c*d*x)*sqrt(
c^2*x^2 + 1))/c^4
```

## 3.199.6 Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.68

$$\int x^3(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^2dx^6}{6} + \frac{a^2dx^4}{4} + \frac{abc^2dx^6\operatorname{asinh}(cx)}{3} - \frac{abcdx^5\sqrt{c^2x^2+1}}{18} + \frac{abdx^4\operatorname{asinh}(cx)}{2} - \frac{abdx^3\sqrt{c^2x^2+1}}{18c} + \frac{abdx\sqrt{c^2x^2+1}}{12c^3} - \frac{abd\operatorname{asinh}(cx)}{12c^4} \\ \frac{a^2dx^4}{4} \end{cases}$$

input
```
integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**2*d*x**6/6 + a**2*d*x**4/4 + a*b*c**2*d*x**6*asinh(c*x)
/3 - a*b*c*d*x**5*sqrt(c**2*x**2 + 1)/18 + a*b*d*x**4*asinh(c*x)/2 - a*b*d
*x**3*sqrt(c**2*x**2 + 1)/(18*c) + a*b*d*x*sqrt(c**2*x**2 + 1)/(12*c**3) -
 a*b*d*asinh(c*x)/(12*c**4) + b**2*c**2*d*x**6*asinh(c*x)**2/6 + b**2*c**2
*d*x**6/108 - b**2*c*d*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/18 + b**2*d*x**
4*asinh(c*x)**2/4 + b**2*d*x**4/72 - b**2*d*x**3*sqrt(c**2*x**2 + 1)*asinh
(c*x)/(18*c) - b**2*d*x**2/(24*c**2) + b**2*d*x*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(12*c**3) - b**2*d*asinh(c*x)**2/(24*c**4), Ne(c, 0)), (a**2*d*x**4/4
, True))
```

### 3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(174) = 348$.

Time = 0.22 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.23

$$\int x^3\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{6}\,b^2c^2dx^6\,\text{arsinh}\,(cx)^2 + \frac{1}{6}\,a^2c^2dx^6 + \frac{1}{4}\,b^2dx^4\,\text{arsinh}\,(cx)^2 + \frac{1}{4}\,a^2dx^4$$
$$+ \frac{1}{144}\left(48\,x^6\,\text{arsinh}\,(cx) - \left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\,\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\,\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\text{arsinh}\,(cx)}{c^7}\right)c\right)ab$$
$$+ \frac{1}{864}\left(\left(\left(\frac{8\,x^6}{c^2} - \frac{15\,x^4}{c^4} + \frac{45\,x^2}{c^6} - \frac{45\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2 - 6\left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\,\sqrt{c^2x^2+1}x^3}{c^4}\right.\right.\right.$$
$$+ \frac{1}{16}\left(8\,x^4\,\text{arsinh}\,(cx) - \left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\,(cx)}{c^5}\right)c\right)abd$$
$$+ \frac{1}{32}\left(\left(\left(\frac{x^4}{c^2} - \frac{3\,x^2}{c^4} + \frac{3\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2 - 2\left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\,(cx)}{c^5}\right.\right.\right.$$

input
```
integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*c^2*d*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^2*d*x^6 + 1/4*b^2*d*x^4*arcsi
nh(c*x)^2 + 1/4*a^2*d*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 +
 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 -
15*arcsinh(c*x)/c^7)*c)*a*b*c^2*d + 1/864*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^
2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1
)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15
*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2*d + 1/16*(8*x^4*arcsinh(c*x) -
(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/
c^5)*c)*a*b*d + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1)
)^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4
+ 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*d
```

## 3.199.8 Giac [F(-2)]

Exception generated.

$$\int x^3(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## 3.199.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx = \int x^3\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

# 3.200 $\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$



## 3.200.1 Optimal result

Integrand size = 24, antiderivative size = 206

$$\begin{aligned}\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = &-\frac{52b^2dx}{225c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5\\&+\frac{8bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c^3}\\&-\frac{4bdx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c}\\&+\frac{2bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{15c^3}\\&-\frac{2bd(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{25c^3}\\&+\frac{2}{15}dx^3(a+b\text{arcsinh}(cx))^2\\&+\frac{1}{5}dx^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
-52/225*b^2*d*x/c^2+26/675*b^2*d*x^3+2/125*b^2*c^2*d*x^5+2/15*b*d*(c^2*x^2
+1)^(3/2)*(a+b*arcsinh(c*x))/c^3-2/25*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c
*x))/c^3+2/15*d*x^3*(a+b*arcsinh(c*x))^2+1/5*d*x^3*(c^2*x^2+1)*(a+b*arcsin
h(c*x))^2+8/45*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/45*b*d*x^2*(
a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.200.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)^2dx$$
$$=\frac{d\left(225a^2c^3x^3(5+3c^2x^2)-30ab\sqrt{1+c^2x^2}(-26+13c^2x^2+9c^4x^4)+2b^2cx(-390+65c^2x^2+27c^4x^4)-30\cdots\right.}{\cdots}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(225*a^2*c^3*x^3*(5 + 3*c^2*x^2) - 30*a*b*Sqrt[1 + c^2*x^2]*(-26 + 13*c
^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(-390 + 65*c^2*x^2 + 27*c^4*x^4) - 30*b*(-
15*a*c^3*x^3*(5 + 3*c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^2*x^2 + 9*c
^4*x^4))*ArcSinh[c*x] + 225*b^2*c^3*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]^2))/(
3375*c^3)
```

### 3.200.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6223, 6191, 6219, 27, 2009, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c^2dx^2+d\right)\left(a+b\text{arcsinh}(cx)\right)^2dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2}{5}bcd\int x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{5}d\int x^2(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}dx^3\left(c^2x^2+1\right)\left(a+b\text{arcsinh}(cx)\right)^2$$

$$\Big\downarrow \text{6191}$$

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bcd\int x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{5}dx^3\left(c^2x^2+1\right)\left(a+b\text{arcsinh}(cx)\right)^2$$

$$\Big\downarrow \text{6219}$$

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bcd\left(-bc\int-\frac{-3c^4x^4-c^2x^2+2}{15c^4}dx+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4}\right)+\frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 27

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bcd\left(\frac{b\int\left(-3c^4x^4-c^2x^2+2\right)dx}{15c^3}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4}\right)+\frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bcd\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4}+\frac{b\left(-\frac{3}{5}c^4x^5-\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$\Big\downarrow$ 6227

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}\right)\right)+\frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bcd\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4}+\frac{b\left(-\frac{3}{5}c^4x^5-\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$\Big\downarrow$ 15

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)\right)+\frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bcd\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4}+\frac{b\left(-\frac{3}{5}c^4x^5-\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$\Big\downarrow$ 6213

---

3.200. $\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\right.\right.$$

$$\frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{5}bcd\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4}+\frac{b\left(-\frac{3}{5}c^4x^5-\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$$\Big\downarrow \; 24$$

$$\frac{1}{5}dx^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+$$

$$\frac{2}{5}d\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)\right)$$

$$\frac{2}{5}bcd\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4}+\frac{b\left(-\frac{3}{5}c^4x^5-\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

input
```
Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*d*((b*(2*x - (c^2*
x^3)/3 - (3*c^4*x^5)/5))/(15*c^3) - ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/(3*c^4) + ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4)))/5 + (2
*d*((x^3*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*(-1/9*(b*x^3)/c + (x^2*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) - (2*(-((b*x)/c) + (Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x]))/c^2))/(3*c^2)))/3))/5
```

### 3.200.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.200. $\int x^2(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.200. $\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$

### 3.200.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| parts | $d\,a^2\left(\frac{1}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}-\frac{2\operatorname{arcsinh}(cx)^2xc}{15}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{2\operatorname{arcsinh}(cx)\left(c^2\right.}{25}\right.}{}$ |
| derivativedivides | $\frac{d\,a^2\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}-\frac{2\operatorname{arcsinh}(cx)^2xc}{15}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{5}{2}}}{25}\right.}{}$ |
| default | $\frac{d\,a^2\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}-\frac{2\operatorname{arcsinh}(cx)^2xc}{15}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{5}{2}}}{25}\right.}{}$ |

input
```
int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/5*c^2*x^5+1/3*x^3)+d*b^2/c^3*(1/5*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^
2-2/15*arcsinh(c*x)^2*x*c-1/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/25*arcsinh
(c*x)*(c^2*x^2+1)^(5/2)-856/3375*c*x+2/125*c*x*(c^2*x^2+1)^2+22/3375*c*x*(
c^2*x^2+1)+4/15*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/45*arcsinh(c*x)*(c^2*x^2+
1)^(3/2))+2*d*a*b/c^3*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1
/25*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^(1/2)+26/225*(c^2
*x^2+1)^(1/2))
```

### 3.200.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)^2dx$$

$$=\frac{27\left(25\,a^2+2\,b^2\right)c^5dx^5+5\left(225\,a^2+26\,b^2\right)c^3dx^3-780\,b^2cdx+225\left(3\,b^2c^5dx^5+5\,b^2c^3dx^3\right)\log\left(cx+\sqrt{c^2x^2\right.}{}$$

input
```
integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 + 2*b^2)*c^5*d*x^5 + 5*(225*a^2 + 26*b^2)*c^3*d*x^3 - 7
80*b^2*c*d*x + 225*(3*b^2*c^5*d*x^5 + 5*b^2*c^3*d*x^3)*log(c*x + sqrt(c^2*
x^2 + 1))^2 + 30*(45*a*b*c^5*d*x^5 + 75*a*b*c^3*d*x^3 - (9*b^2*c^4*d*x^4 +
 13*b^2*c^2*d*x^2 - 26*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
1)) - 30*(9*a*b*c^4*d*x^4 + 13*a*b*c^2*d*x^2 - 26*a*b*d)*sqrt(c^2*x^2 + 1)
)/c^3
```

## 3.200.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.52

$$\int x^2(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^2dx^5}{5}+\frac{a^2dx^3}{3}+\frac{2abc^2dx^5\operatorname{asinh}(cx)}{5}-\frac{2abcdx^4\sqrt{c^2x^2+1}}{25}+\frac{2abdx^3\operatorname{asinh}(cx)}{3}-\frac{26abdx^2\sqrt{c^2x^2+1}}{225c}+\frac{52abd\sqrt{c^2x^2+1}}{225c^3}+\frac{b^2c^2dx}{} \\ \frac{a^2dx^3}{3}\end{cases}$$

input
```
integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**2*d*x**5/5 + a**2*d*x**3/3 + 2*a*b*c**2*d*x**5*asinh(c*
x)/5 - 2*a*b*c*d*x**4*sqrt(c**2*x**2 + 1)/25 + 2*a*b*d*x**3*asinh(c*x)/3 -
 26*a*b*d*x**2*sqrt(c**2*x**2 + 1)/(225*c) + 52*a*b*d*sqrt(c**2*x**2 + 1)/
(225*c**3) + b**2*c**2*d*x**5*asinh(c*x)**2/5 + 2*b**2*c**2*d*x**5/125 - 2
*b**2*c*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + b**2*d*x**3*asinh(c*x)*
*2/3 + 26*b**2*d*x**3/675 - 26*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/
(225*c) - 52*b**2*d*x/(225*c**2) + 52*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(225*c**3), Ne(c, 0)), (a**2*d*x**3/3, True))
```

### 3.200.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.68

$$
\begin{aligned}
&\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx \\
&= \frac{1}{5}\,b^2c^2dx^5\,\text{arsinh}\,(cx)^2 + \frac{1}{5}\,a^2c^2dx^5 + \frac{1}{3}\,b^2dx^3\,\text{arsinh}\,(cx)^2 \\
&\quad + \frac{2}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^2d \\
&\quad - \frac{2}{1125}\left(15\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right) \\
&\quad + \frac{1}{3}\,a^2dx^3 + \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abd \\
&\quad - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3 - 6\,x}{c^2}\right)b^2d
\end{aligned}
$$

input
```
integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*c^2*d*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^2*d*x^5 + 1/3*b^2*d*x^3*arcsi
nh(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*s
qrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d - 2/1125*
(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^
2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2
*c^2*d + 1/3*a^2*d*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2
/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d
```

### 3.200.8 Giac [F(-2)]

Exception generated.

$$\int x^2\left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.200.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 dx^2) (a + b\text{arcsinh}(cx))^2 \, dx = \int x^2 (a + b\,\text{asinh}(c\,x))^2 \left(d\,c^2\,x^2 + d\right) \, dx$$

input
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

# 3.201 $\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$



## 3.201.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = \frac{5}{32}b^2dx^2+\frac{1}{32}b^2c^2dx^4 - \frac{3bdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c} - \frac{bdx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{8c} - \frac{3d(a+b\text{arcsinh}(cx))^2}{32c^2} + \frac{d(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{4c^2}$$

output
```
5/32*b^2*d*x^2+1/32*b^2*c^2*d*x^4-1/8*b*d*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh
(c*x))/c-3/32*d*(a+b*arcsinh(c*x))^2/c^2+1/4*d*(c^2*x^2+1)^2*(a+b*arcsinh(
c*x))^2/c^2-3/16*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.201.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int x(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{d\left(cx\left(8a^2cx(2+c^2x^2)+b^2cx(5+c^2x^2)-2ab\sqrt{1+c^2x^2}(5+2c^2x^2)\right)+2b\left(-bcx\sqrt{1+c^2x^2}(5+2c^2x^2)+a\right.\right.}{32c^2}$$

input
```
Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(c*x*(8*a^2*c*x*(2 + c^2*x^2) + b^2*c*x*(5 + c^2*x^2) - 2*a*b*Sqrt[1 +
c^2*x^2]*(5 + 2*c^2*x^2)) + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)
) + a*(5 + 16*c^2*x^2 + 8*c^4*x^4))*ArcSinh[c*x] + b^2*(5 + 16*c^2*x^2 + 8
*c^4*x^4)*ArcSinh[c*x]^2))/(32*c^2)
```

### 3.201.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6213, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(c^2dx^2+d)(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{d(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))^2}{4c^2}-\frac{bd\int(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))dx}{2c}$$

$$\Big\downarrow \text{6201}$$

$$\frac{d(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))^2}{4c^2}-$$
$$\frac{bd\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx-\frac{1}{4}bc\int x(c^2x^2+1)\,dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))\right)}{2c}$$

$$\Big\downarrow \text{244}$$

$$\frac{d(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{4c^2}-$$
$$\frac{bd\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int(c^2x^3+x)\,dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\right)}{2c}$$

$\Big\downarrow$ 2009

$$\frac{d(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{4c^2}-$$
$$\frac{bd\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)}{2c}$$

$\Big\downarrow$ 6200

$$\frac{d(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{4c^2}-$$
$$\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\right.}{2c}$$

$\Big\downarrow$ 15

$$\frac{d(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{4c^2}-$$
$$\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\right.}{2c}$$

$\Big\downarrow$ 6198

$$\frac{d(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{4c^2}-$$
$$\frac{bd\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{1}{4}bc\left(\right.\right.}{2c}$$

input
```
Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(4*c^2) - (b*d*(-1/4*(b*c*(x^2/
2 + (c^2*x^4)/4)) + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*(-
1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcS
inh[c*x])^2/(4*b*c)))/4))/(2*c)
```

3.201. $\int x(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$

### 3.201.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.201. $\int x(d + c^2dx^2)\,(a + b\text{arcsinh}(cx))^2\,dx$

### 3.201.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d a^2 (c^2x^2+1)^2}{4} + d b^2 \left(\frac{\operatorname{arcsinh}(cx)^2 (c^2x^2+1)^2}{4} - \frac{\operatorname{arcsinh}(cx) cx (c^2x^2+1)^{\frac{3}{2}}}{8} - \frac{3 \operatorname{arcsinh}(cx) cx \sqrt{c^2x^2+1}}{16} - \frac{3 \operatorname{arcsinh}(cx)^2}{32} + \frac{(c^2x^2}{32}\right.}{c^2}$ |
| default | $\frac{\frac{d a^2 (c^2x^2+1)^2}{4} + d b^2 \left(\frac{\operatorname{arcsinh}(cx)^2 (c^2x^2+1)^2}{4} - \frac{\operatorname{arcsinh}(cx) cx (c^2x^2+1)^{\frac{3}{2}}}{8} - \frac{3 \operatorname{arcsinh}(cx) cx \sqrt{c^2x^2+1}}{16} - \frac{3 \operatorname{arcsinh}(cx)^2}{32} + \frac{(c^2x^2}{32}\right.}{c^2}$ |
| parts | $\frac{d a^2 (c^2x^2+1)^2}{4c^2} + \frac{d b^2 \left(\frac{\operatorname{arcsinh}(cx)^2 (c^2x^2+1)^2}{4} - \frac{\operatorname{arcsinh}(cx) cx (c^2x^2+1)^{\frac{3}{2}}}{8} - \frac{3 \operatorname{arcsinh}(cx) cx \sqrt{c^2x^2+1}}{16} - \frac{3 \operatorname{arcsinh}(cx)^2}{32} + \frac{(c^2\right.}{c^2}$ |

input
```
int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4*d*a^2*(c^2*x^2+1)^2+d*b^2*(1/4*arcsinh(c*x)^2*(c^2*x^2+1)^2-1/8
*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)-3/16*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2
)-3/32*arcsinh(c*x)^2+1/32*(c^2*x^2+1)^2+3/32*c^2*x^2+3/32)+2*d*a*b*(1/4*a
rcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+5/32*arcsinh(c*x)-1/16*c*x*(c
^2*x^2+1)^(3/2)-3/32*c*x*(c^2*x^2+1)^(1/2)))
```

### 3.201.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.51

$$\int x(d + c^2dx^2) (a + b\operatorname{arcsinh}(cx))^2 dx$$
$$= \frac{(8 a^2 + b^2)c^4dx^4 + (16 a^2 + 5 b^2)c^2dx^2 + (8 b^2c^4dx^4 + 16 b^2c^2dx^2 + 5 b^2d) \log \left(cx + \sqrt{c^2x^2 + 1}\right)^2 + 2 (8 abc}{}$$

input
```
integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/32*((8*a^2 + b^2)*c^4*d*x^4 + (16*a^2 + 5*b^2)*c^2*d*x^2 + (8*b^2*c^4*d*
x^4 + 16*b^2*c^2*d*x^2 + 5*b^2*d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(8*a*
b*c^4*d*x^4 + 16*a*b*c^2*d*x^2 + 5*a*b*d - (2*b^2*c^3*d*x^3 + 5*b^2*c*d*x)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(2*a*b*c^3*d*x^3 + 5*
a*b*c*d*x)*sqrt(c^2*x^2 + 1))/c^2
```

### 3.201.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(129) = 258$.

Time = 0.43 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.99

$$\int x(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^2dx^4}{4} + \frac{a^2dx^2}{2} + \frac{abc^2dx^4\operatorname{asinh}(cx)}{2} - \frac{abcdx^3\sqrt{c^2x^2+1}}{8} + abdx^2\operatorname{asinh}(cx) - \frac{5abdx\sqrt{c^2x^2+1}}{16c} + \frac{5abd\operatorname{asinh}(cx)}{16c^2} + \frac{b^2c^2dx^4}{} \\ \frac{a^2dx^2}{2} \end{cases}$$

input
```
integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**2*d*x**4/4 + a**2*d*x**2/2 + a*b*c**2*d*x**4*asinh(c*x)
/2 - a*b*c*d*x**3*sqrt(c**2*x**2 + 1)/8 + a*b*d*x**2*asinh(c*x) - 5*a*b*d*
x*sqrt(c**2*x**2 + 1)/(16*c) + 5*a*b*d*asinh(c*x)/(16*c**2) + b**2*c**2*d*
x**4*asinh(c*x)**2/4 + b**2*c**2*d*x**4/32 - b**2*c*d*x**3*sqrt(c**2*x**2
+ 1)*asinh(c*x)/8 + b**2*d*x**2*asinh(c*x)**2/2 + 5*b**2*d*x**2/32 - 5*b**
2*d*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c) + 5*b**2*d*asinh(c*x)**2/(32*c
**2), Ne(c, 0)), (a**2*d*x**2/2, True))
```

### 3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(119) = 238$.

Time = 0.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.57

$$\int x(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{4}b^2c^2dx^4\operatorname{arsinh}(cx)^2 + \frac{1}{4}a^2c^2dx^4 + \frac{1}{2}b^2dx^2\operatorname{arsinh}(cx)^2$$
$$+ \frac{1}{16}\left(8x^4\operatorname{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)abc^2d$$
$$+ \frac{1}{32}\left(\left(\frac{x^4}{c^2} - \frac{3x^2}{c^4} + \frac{3\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2 - 2\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right.\right.$$
$$+ \frac{1}{2}a^2dx^2 + \frac{1}{2}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)abd$$
$$+ \frac{1}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\operatorname{arsinh}(cx)\right)b^2d$$

input
```
integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*c^2*d*x^4*arcsinh(c*x)^2 + 1/4*a^2*c^2*d*x^4 + 1/2*b^2*d*x^2*arcsi
nh(c*x)^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sq
rt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d + 1/32*((x^4/c^2
- 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^
2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh
(c*x))*b^2*c^2*d + 1/2*a^2*d*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x
^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d + 1/4*(c^2*(x^2/c^2 - log(c*x + s
qrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3
)*arcsinh(c*x))*b^2*d
```

### 3.201.8 Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.201.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx=\int x\,(a+b\,\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

output
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

# 3.202 $\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^2 \, dx$



## 3.202.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\begin{aligned}\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^2 \, dx = {} & \frac{14}{9}b^2dx + \frac{2}{27}b^2c^2dx^3 \\ & - \frac{4bd\sqrt{1 + c^2x^2}(a + b\text{arcsinh}(cx))}{3c} \\ & - \frac{2bd(1 + c^2x^2)^{3/2} (a + b\text{arcsinh}(cx))}{9c} \\ & + \frac{2}{3}dx(a + b\text{arcsinh}(cx))^2 \\ & + \frac{1}{3}dx (1 + c^2x^2) (a + b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
14/9*b^2*d*x+2/27*b^2*c^2*d*x^3-2/9*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x
))/c+2/3*d*x*(a+b*arcsinh(c*x))^2+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2
-4/3*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.202.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d\left(9a^2cx(3+c^2x^2)-6ab\sqrt{1+c^2x^2}(7+c^2x^2)+2b^2cx(21+c^2x^2)-6b\left(-3acx(3+c^2x^2)+b\sqrt{1+c^2x^2}(7\right.\right.}{27c}$$

input
```
Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(9*a^2*c*x*(3 + c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 2*b^
2*c*x*(21 + c^2*x^2) - 6*b*(-3*a*c*x*(3 + c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(
7 + c^2*x^2))*ArcSinh[c*x] + 9*b^2*c*x*(3 + c^2*x^2)*ArcSinh[c*x]^2))/(27*
c)
```

### 3.202.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6201, 6187, 6213, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c^2dx^2+d)(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{2}{3}bcd\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}d\int (a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bcd\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{24}$$

$$-\frac{2}{3}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{2}{3}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{c^2x^3}{3}+x\right)}{3c}\right)$$

input
```
Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*d*(-1/3*(b*(x + (c^2
*x^3)/3))/c + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2)))/3 + (2*
d*(x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/c^2)))/3
```

### 3.202.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.202.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| derivativedivides | $\frac{d a^2\left(\frac{1}{3}c^3x^3+cx\right)+d b^2\left(\frac{2\operatorname{arcsinh}(cx)^2 xc}{3}+\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)}{3}-\frac{4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{3}+\frac{40cx}{27}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{3}{2}}}{9}\right.}{c}$ |
| default | $\frac{d a^2\left(\frac{1}{3}c^3x^3+cx\right)+d b^2\left(\frac{2\operatorname{arcsinh}(cx)^2 xc}{3}+\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)}{3}-\frac{4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{3}+\frac{40cx}{27}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{3}{2}}}{9}\right.}{c}$ |
| parts | $d a^2\left(\frac{1}{3}x^3c^2+x\right)+\frac{d b^2\left(\frac{2\operatorname{arcsinh}(cx)^2 xc}{3}+\frac{\operatorname{arcsinh}(cx)^2 cx\left(c^2x^2+1\right)}{3}-\frac{4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{3}+\frac{40cx}{27}-\frac{2\operatorname{arcsinh}(cx)\left(c^2\right.}{9}\right.}{c}$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d*a^2*(1/3*c^3*x^3+c*x)+d*b^2*(2/3*arcsinh(c*x)^2*x*c+1/3*arcsinh(c*x
)^2*c*x*(c^2*x^2+1)-4/3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+40/27*c*x-2/9*arcsi
nh(c*x)*(c^2*x^2+1)^(3/2)+2/27*c*x*(c^2*x^2+1))+2*d*a*b*(1/3*arcsinh(c*x)*
c^3*x^3+arcsinh(c*x)*c*x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/
2)))
```

### 3.202.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.42

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^2 dx$$
$$= \frac{(9\,a^2 + 2\,b^2)c^3dx^3 + 3\,(9\,a^2 + 14\,b^2)cdx + 9\,(b^2c^3dx^3 + 3\,b^2cdx)\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + 6\,(3\,abc^3dx^3 + 9\,\ldots}{27\,c}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 + 2*b^2)*c^3*d*x^3 + 3*(9*a^2 + 14*b^2)*c*d*x + 9*(b^2*c^3*d*
x^3 + 3*b^2*c*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^3*d*x^3 + 9
*a*b*c*d*x - (b^2*c^2*d*x^2 + 7*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c
^2*x^2 + 1)) - 6*(a*b*c^2*d*x^2 + 7*a*b*d)*sqrt(c^2*x^2 + 1))/c
```

### 3.202.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.79

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^2 dx$$
$$= \begin{cases} \frac{a^2c^2dx^3}{3} + a^2dx + \frac{2abc^2dx^3\operatorname{asinh}(cx)}{3} - \frac{2abcdx^2\sqrt{c^2x^2+1}}{9} + 2abdx\operatorname{asinh}(cx) - \frac{14abd\sqrt{c^2x^2+1}}{9c} + \frac{b^2c^2dx^3\operatorname{asinh}^2(cx)}{3} + \ldots \\ a^2dx \end{cases}$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**2*d*x**3/3 + a**2*d*x + 2*a*b*c**2*d*x**3*asinh(c*x)/3
- 2*a*b*c*d*x**2*sqrt(c**2*x**2 + 1)/9 + 2*a*b*d*x*asinh(c*x) - 14*a*b*d*s
qrt(c**2*x**2 + 1)/(9*c) + b**2*c**2*d*x**3*asinh(c*x)**2/3 + 2*b**2*c**2*
d*x**3/27 - 2*b**2*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/9 + b**2*d*x*as
inh(c*x)**2 + 14*b**2*d*x/9 - 14*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*
c), Ne(c, 0)), (a**2*d*x, True))
```

### 3.202.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(109) = 218.

Time = 0.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.84

$$
\begin{aligned}
&\int \left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)^2 dx \\
&= \frac{1}{3}b^2c^2dx^3\operatorname{arsinh}(cx)^2 + \frac{1}{3}a^2c^2dx^3 \\
&\quad + \frac{2}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d \\
&\quad - \frac{2}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3-6x}{c^2}\right)b^2c^2d \\
&\quad + b^2dx\operatorname{arsinh}(cx)^2 + 2b^2d\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) \\
&\quad + a^2dx + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd}{c}
\end{aligned}
$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*c^2*d*x^3*arcsinh(c*x)^2 + 1/3*a^2*c^2*d*x^3 + 2/9*(3*x^3*arcsinh(
c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d
- 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(
c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arcsinh(c*x)^2 + 2*b^2*d*(
x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*a*b*d/c
```

### 3.202.8 Giac [F(-2)]

Exception generated.

$$
\int \left(d+c^2dx^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)^2 dx = \text{Exception raised: TypeError}
$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.202.9 Mupad [F(-1)]

Timed out.

$$\int (d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx=\int (a+b\,\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

# 3.203 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx$



## 3.203.1 Optimal result

Integrand size = 24, antiderivative size = 166

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &\quad - \frac{1}{4}d(a+b\text{arcsinh}(cx))^2 \\ &\quad + \frac{1}{2}d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ &\quad + \frac{d(a+b\text{arcsinh}(cx))^3}{3b} \\ &\quad + d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &\quad - bd(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\ &\quad - \frac{1}{2}b^2d\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
1/4*b^2*c^2*d*x^2-1/4*d*(a+b*arcsinh(c*x))^2+1/2*d*(c^2*x^2+1)*(a+b*arcsin
h(c*x))^2+1/3*d*(a+b*arcsinh(c*x))^3/b+d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+
(c^2*x^2+1)^(1/2))^2)-b*d*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^
(1/2))^2)-1/2*b^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*x*(a+
b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.203.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.36

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{1}{2}d\Bigg(a^2c^2x^2+2abc^2x^2\text{arcsinh}(cx) + \frac{1}{4}b^2\left(1+2\text{arcsinh}(cx)^2\right)\cosh(2\text{arcsinh}(cx)) - 2ab\text{arcsinh}(cx)\left(\text{arcsinh}(cx) - 2\log\left(1-e^{2\text{arcsinh}(cx)}\right)\right) + 2a^2\log(x) - ab\left(cx\sqrt{1+c^2x^2}+\log\left(-cx+\sqrt{1+c^2x^2}\right)\right) + 2ab\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right) + 2b^2\left(-\frac{1}{3}\text{arcsinh}(cx)^3 + \text{arcsinh}(cx)^2\log\left(1-e^{2\text{arcsinh}(cx)}\right) + \text{arcsinh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)\right) - \frac{1}{2}b^2\text{arcsinh}(cx)\sinh(2\text{arcsinh}(cx))\Bigg)$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

output
```
(d*(a^2*c^2*x^2 + 2*a*b*c^2*x^2*ArcSinh[c*x] + (b^2*(1 + 2*ArcSinh[c*x]^2)
*Cosh[2*ArcSinh[c*x]])/4 - 2*a*b*ArcSinh[c*x]*(ArcSinh[c*x] - 2*Log[1 - E^
(2*ArcSinh[c*x])]) + 2*a^2*Log[x] - a*b*(c*x*Sqrt[1 + c^2*x^2] + Log[-(c*x
) + Sqrt[1 + c^2*x^2]]) + 2*a*b*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*(-1
/3*ArcSinh[c*x]^3 + ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + ArcSinh[c
*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - PolyLog[3, E^(2*ArcSinh[c*x])]/2) - (
b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]])/2))/2
```

### 3.203.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6223, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 6200, 15, 6198, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)(a+b\text{arcsinh}(cx))^2}{x}dx$$

$$\Big\downarrow \text{ 6223}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+d\int \frac{(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{2}d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6190}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{d\int -(a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 25}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{d\int (a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 3042}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{d\int -i(a+b\text{arcsinh}(cx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 26}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{id\int (a+b\text{arcsinh}(cx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+$$
$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{4201}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{id\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{3}i(a+b\text{arcsinh}(cx))^3\right)}{b}+$$
$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2620}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{id\left(2i\left(b\int (a+b\text{arcsinh}(cx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))^2\log\left(1\right.\right.\right.}{b}$$
$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{3011}$$

$$-bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{id\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)-\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right.\right.\right.\right.}{b}$$
$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2720}$$

$$\frac{id\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsinh}(\right.\right.\right.}{}$$
$$bcd\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6200}$$

3.203. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$\frac{id\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsinh}(\right.\right.\right.}{}$$

$$bcd\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+$$
$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 15

$$\frac{id\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsinh}(\right.\right.\right.}{}$$

$$bcd\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{2}d\left(c^2x^2+1\right)(a+$$
$$b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 6198

$$\frac{id\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsinh}(\right.\right.\right.}{}$$

$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$
$$bcd\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)$$

$\big\downarrow$ 7143

$$\frac{id\left(2i\left(b\left(\frac{1}{4}b^2\text{PolyLog}(3,-a-b\text{arcsinh}(cx))+\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)\right)-\frac{1}{2}\right.\right.}{b}$$

$$\frac{1}{2}d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-$$
$$bcd\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)$$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

## 3.203. $\int\frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx$

output
```
(d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 - b*c*d*(-1/4*(b*c*x^2) + (x*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*c))
+ (I*d*((-1/3*I)*(a + b*ArcSinh[c*x])^3 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*
x])^2*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)]) + b*((b*(a
 + b*ArcSinh[c*x])*PolyLog[2, -E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x])
)/b)])/2 + (b^2*PolyLog[3, -a - b*ArcSinh[c*x]])/4))))/b
```

### 3.203.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

## 3.203. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.203. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx$

### 3.203.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(179) = 358.

Time = 0.17 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.50

| method | result |
|---|---|
| derivativedivides | $d\,a^2\left(\frac{c^2x^2}{2}+\ln\left(cx\right)\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+1\right)\left(2c^2x^2+1+2cx\sqrt{c^2x^2+1}\right.}{16}\right.$ |
| default | $d\,a^2\left(\frac{c^2x^2}{2}+\ln\left(cx\right)\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+1\right)\left(2c^2x^2+1+2cx\sqrt{c^2x^2+1}\right.}{16}\right.$ |
| parts | $d\,a^2\left(\frac{c^2x^2}{2}+\ln\left(x\right)\right)-\frac{d\,b^2\operatorname{arcsinh}(cx)^3}{3}+\frac{d\,b^2\operatorname{arcsinh}(cx)^2x^2c^2}{2}-\frac{d\,b^2\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)xc}{2}+\frac{b^2c^2d\,x^2}{4}$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/2*c^2*x^2+ln(c*x))+d*b^2*(-1/3*arcsinh(c*x)^3+1/16*(2*arcsinh(c*x
)^2-2*arcsinh(c*x)+1)*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))+1/16*(-2*c*x*(
c^2*x^2+1)^(1/2)+2*c^2*x^2+1)*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)+arcsinh(
c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+
1)^(1/2))-2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)^2*ln(1-c*x-(c^2
*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3
,c*x+(c^2*x^2+1)^(1/2)))-d*a*b*arcsinh(c*x)^2+d*a*b*arcsinh(c*x)*c^2*x^2-1
/2*d*a*b*c*x*(c^2*x^2+1)^(1/2)+1/2*d*a*b*arcsinh(c*x)+2*d*a*b*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d
*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,c*x+(c^2*x
^2+1)^(1/2))
```

### 3.203.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx=\int \frac{(c^2dx^2+d)(b\operatorname{arsinh}\left(cx\right)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x, x)
```

### 3.203.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx = d\left(\int \frac{a^2}{x}\,dx + \int a^2c^2x\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x}\,dx + \int b^2c^2x\operatorname{asinh}^2(cx)\,dx + \int 2abc^2x\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x,x)
```

output
```
d*(Integral(a**2/x, x) + Integral(a**2*c**2*x, x) + Integral(b**2*asinh(c*
x)**2/x, x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(b**2*c**2*x*asinh
(c*x)**2, x) + Integral(2*a*b*c**2*x*asinh(c*x), x))
```

### 3.203.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/2*a^2*c^2*d*x^2 + a^2*d*log(x) + integrate(b^2*c^2*d*x*log(c*x + sqrt(c^
2*x^2 + 1))^2 + 2*a*b*c^2*d*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d*log(c*x
 + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.203.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x, x)
```

# 3.204 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$



## 3.204.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = {} & 2b^2c^2dx-2bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ & +2c^2dx(a+b\text{arcsinh}(cx))^2 \\ & -\frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{x} \\ & -4bcd(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) \\ & -2b^2cd\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) \\ & +2b^2cd\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
2*b^2*c^2*d*x+2*c^2*d*x*(a+b*arcsinh(c*x))^2-d*(c^2*x^2+1)*(a+b*arcsinh(c*
x))^2/x-4*b*c*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*
d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d*polylog(2,c*x+(c^2*x^2+1)^(1
/2))-2*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.204.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.47

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$$
$$= \frac{d\left(-a^2+a^2c^2x^2+2abcx\left(-\sqrt{1+c^2x^2}+cx\text{arcsinh}(cx)\right)+b^2cx\left(2cx-2\sqrt{1+c^2x^2}\text{arcsinh}(cx)+cx\text{arcsin}\right.\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
(d*(-a^2 + a^2*c^2*x^2 + 2*a*b*c*x*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x])
 + b^2*c*x*(2*c*x - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2)
 - 2*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]) - b^2*(ArcSinh[c*
x]*(ArcSinh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSin
h[c*x])])) - 2*c*x*PolyLog[2, -E^(-ArcSinh[c*x])] + 2*c*x*PolyLog[2, E^(-A
rcSinh[c*x])])))/x
```

### 3.204.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6222, 6187, 6213, 24, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \text{ 6222}$$
$$2bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+2c^2d\int (a+b\text{arcsinh}(cx))^2dx-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$
$$\Big\downarrow \text{ 6187}$$

$$2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+2bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6213}$$

$$2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)+2bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{24}$$

$$2bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6221}$$

$$2bcd\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int 1dx+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{24}$$

$$2bcd\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6231}$$

$$2bcd\left(\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

---

3.204. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$\downarrow$ 3042

$$2bcd\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$\downarrow$ 26

$$2bcd\left(i\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$\downarrow$ 4670

$$2bcd\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\right.\right.$$
$$2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$\downarrow$ 2715

$$2bcd\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\right.\right.$$
$$2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

$\downarrow$ 2838

$$2bcd\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)+\right.$$
$$2c^2d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}$$

## 3.204. $\int\frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
-((d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x) + 2*c^2*d*(x*(a + b*ArcSinh[
c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2
)) + 2*b*c*d*(-(b*c*x) + Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + I*((2*I)
*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[
c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))
```

### 3.204.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.204.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| derivativedivides | $c\Big(d\,a^2\big(cx-\frac{1}{cx}\big)+d\,b^2\operatorname{arcsinh}(cx)^2\,cx-2d\,b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2d\,b^2cx-\frac{d\,b^2\,a}{}$ |
| default | $c\Big(d\,a^2\big(cx-\frac{1}{cx}\big)+d\,b^2\operatorname{arcsinh}(cx)^2\,cx-2d\,b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2d\,b^2cx-\frac{d\,b^2\,a}{}$ |
| parts | $d\,a^2\big(c^2x-\frac{1}{x}\big)+d\,b^2c^2\operatorname{arcsinh}(cx)^2\,x-2d\,b^2c\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2b^2c^2dx-\frac{d\,b^2}{}$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(d*a^2*(c*x-1/c/x)+d*b^2*arcsinh(c*x)^2*c*x-2*d*b^2*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)+2*d*b^2*c*x-d*b^2*arcsinh(c*x)^2/c/x-2*d*b^2*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))-2*d*b^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d*b^2*a
rcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d*b^2*polylog(2,c*x+(c^2*x^2+1)^
(1/2))+2*d*a*b*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)-arctan
h(1/(c^2*x^2+1)^(1/2))))
```

### 3.204.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(c^2dx^2+d)(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^2, x)
```

### 3.204.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = d\left(\int a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int b^2c^2\operatorname{asinh}^2(cx)\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^2}\,dx + \int 2abc^2\operatorname{asinh}(cx)\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**2,x)
```

output
```
d*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(b**2*c**2*as
inh(c*x)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c*
*2*asinh(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x))
```

### 3.204.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
b^2*c^2*d*x*arcsinh(c*x)^2 + 2*b^2*c^2*d*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*
x)/c) + a^2*c^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d - 2
*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*d - b^2*d*(log(c*x + sqrt(
c^2*x^2 + 1))^2/x - integrate(2*(c^3*x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c)*lo
g(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 +
 1)), x)) - a^2*d/x
```

### 3.204.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2, x)
```

# 3.205 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^3} dx$



## 3.205.1 Optimal result

Integrand size = 24, antiderivative size = 180

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^3} dx = & -\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x} \\ & +\frac{1}{2}c^2d(a+b\text{arcsinh}(cx))^2 \\ & -\frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2x^2} \\ & +\frac{c^2d(a+b\text{arcsinh}(cx))^3}{3b} \\ & +c^2d(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ & +b^2c^2d\log(x) \\ & -bc^2d(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\ & -\frac{1}{2}b^2c^2d\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
1/2*c^2*d*(a+b*arcsinh(c*x))^2-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/x^2+
1/3*c^2*d*(a+b*arcsinh(c*x))^3/b+c^2*d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c
^2*x^2+1)^(1/2))^2)+b^2*c^2*d*ln(x)-b*c^2*d*(a+b*arcsinh(c*x))*polylog(2,1
/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*c^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/
2))^2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x
```

### 3.205.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.34

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = -\frac{a^2d}{2x^2}+2abc^2d\left(-\frac{\sqrt{1+c^2x^2}}{2cx}-\frac{\text{arcsinh}(cx)}{2c^2x^2}\right)+a^2c^2d\log(x)+b^2c^2d\left(-\frac{\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{cx}-\frac{\text{arcsinh}(cx)^2}{2c^2x^2}+\log(cx)\right)+2abc^2d\left(-\frac{1}{2}\text{arcsinh}(cx)^2+\text{arcsinh}(cx)\log\left(1-e^{2\text{arcsinh}(cx)}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right)+b^2c^2d\left(-\frac{1}{3}\text{arcsinh}(cx)^3+\text{arcsinh}(cx)^2\log\left(1-e^{2\text{arcsinh}(cx)}\right)+\text{arcsinh}(cx)\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)\right)$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
-1/2*(a^2*d)/x^2 + 2*a*b*c^2*d*(-1/2*Sqrt[1 + c^2*x^2]/(c*x) - ArcSinh[c*x
]/(2*c^2*x^2)) + a^2*c^2*d*Log[x] + b^2*c^2*d*(-((Sqrt[1 + c^2*x^2]*ArcSin
h[c*x])/(c*x)) - ArcSinh[c*x]^2/(2*c^2*x^2) + Log[c*x]) + 2*a*b*c^2*d*(-1/
2*ArcSinh[c*x]^2 + ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + PolyLog[2, E
^(2*ArcSinh[c*x])]/2) + b^2*c^2*d*(-1/3*ArcSinh[c*x]^3 + ArcSinh[c*x]^2*Lo
g[1 - E^(2*ArcSinh[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] -
PolyLog[3, E^(2*ArcSinh[c*x])]/2)
```

### 3.205.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6222, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 6220, 14, 6198, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2 d x^2+d\right) (a+b \text{arcsinh}(c x))^2}{x^3} \, dx$$

$$\Big\downarrow \text{6222}$$

$$b c d \int \frac{\sqrt{c^2 x^2+1} (a+b \text{arcsinh}(c x))}{x^2}dx+c^2 d \int \frac{(a+b \text{arcsinh}(c x))^2}{x}dx-\frac{d \left(c^2 x^2+1\right) (a+b \text{arcsinh}(c x))^2}{2 x^2}$$

$$\Big\downarrow \text{6190}$$

$$b c d \int \frac{\sqrt{c^2 x^2+1} (a+b \text{arcsinh}(c x))}{x^2}dx+\frac{c^2 d \int -(a+b \text{arcsinh}(c x))^2 \coth \left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)d(a+b \text{arcsinh}(c x))}{b}-\frac{d \left(c^2 x^2+1\right) (a+b \text{arcsinh}(c x))^2}{2 x^2}$$

$$\Big\downarrow \text{25}$$

$$b c d \int \frac{\sqrt{c^2 x^2+1} (a+b \text{arcsinh}(c x))}{x^2}dx-\frac{c^2 d \int (a+b \text{arcsinh}(c x))^2 \coth \left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)d(a+b \text{arcsinh}(c x))}{b}-\frac{d \left(c^2 x^2+1\right) (a+b \text{arcsinh}(c x))^2}{2 x^2}$$

$$\Big\downarrow \text{3042}$$

$$b c d \int \frac{\sqrt{c^2 x^2+1} (a+b \text{arcsinh}(c x))}{x^2}dx-\frac{c^2 d \int -i (a+b \text{arcsinh}(c x))^2 \tan \left(\frac{i a}{b}-\frac{i (a+b \text{arcsinh}(c x))}{b}+\frac{\pi}{2}\right)d(a+b \text{arcsinh}(c x))}{b}-\frac{d \left(c^2 x^2+1\right) (a+b \text{arcsinh}(c x))^2}{2 x^2}$$

$$\Big\downarrow \text{26}$$

$$bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x^2}dx+$$
$$\frac{ic^2d\int (a+b\text{arcsinh}(cx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}-$$
$$\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 4201

$$bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x^2}dx+$$
$$\frac{ic^2d\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{3}i(a+b\text{arcsinh}(cx))^3\right)}{b}-$$
$$\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 2620

$$bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x^2}dx+$$
$$\frac{ic^2d\left(2i\left(b\int (a+b\text{arcsinh}(cx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))^2\log\right.\right.}{b}$$
$$\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 3011

$$\frac{ic^2d\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)-\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}\right.\right.\right.\right.}{b}$$
$$bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x^2}dx-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 2720

$$ic^2d\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsin}\right.\right.\right.$$
$$bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x^2}dx-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 6220

3.205. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^3}dx$

$$\frac{ic^2d\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsin}\right.\right.\right.}{}$$

$$bcd\left(c^2\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+bc\int\frac{1}{x}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}\right)-\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{ic^2d\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsin}\right.\right.\right.}{}$$

$$bcd\left(c^2\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}+bc\log(x)\right)-\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6198}$$

$$\frac{ic^2d\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsin}\right.\right.\right.}{}$$

$$\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2x^2}+bcd\left(-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}+\frac{c(a+b\text{arcsinh}(cx))^2}{2b}+bc\log(x)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{ic^2d\left(2i\left(b\left(\frac{1}{4}b^2\text{PolyLog}(3,-a-b\text{arcsinh}(cx))+\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)\right)-\right.\right.}{b}$$

$$\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{2x^2}+bcd\left(-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}+\frac{c(a+b\text{arcsinh}(cx))^2}{2b}+bc\log(x)\right)$$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

3.205. $\int\frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

output
```
-1/2*(d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x^2 + b*c*d*(-((Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*(a + b*ArcSinh[c*x])^2)/(2*b) + b*c*Lo
g[x]) + (I*c^2*d*((-1/3*I)*(a + b*ArcSinh[c*x])^3 + (2*I)*(-1/2*(b*(a + b*
ArcSinh[c*x])^2*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)])
+ b*((b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^((2*a)/b - I*Pi - (2*(a + b*Arc
Sinh[c*x]))/b)])/2 + (b^2*PolyLog[3, -a - b*ArcSinh[c*x]])/4))))/b
```

### 3.205.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.205. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

### 3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(197) = 394$.

Time = 0.20 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.24

| method | result |
|---|---|
| derivativedivides | $c^2\left(d\,a^2\left(\ln(cx)-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\right.\right.$ |
| default | $c^2\left(d\,a^2\left(\ln(cx)-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\right.\right.$ |
| parts | $d\,a^2\left(-\frac{1}{2x^2}+c^2\ln(x)\right)+d\,b^2c^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\right.$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d*a^2*(ln(c*x)-1/2/c^2/x^2)+d*b^2*(-1/3*arcsinh(c*x)^3-1/2*arcsinh(c*
x)*(-2*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/c^2/x^2+ln(1+c*x+(c^2
*x^2+1)^(1/2))-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln(c*x+(c^2*x^2+1)^(1/2)-1)+arc
sinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2
*x^2+1)^(1/2))-2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)^2*ln(1-c*x
-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*poly
log(3,c*x+(c^2*x^2+1)^(1/2)))+2*d*a*b*(-1/2*arcsinh(c*x)^2-1/2*(c*x*(c^2*x
^2+1)^(1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1
)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+
1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

### 3.205.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^3, x)
```

### 3.205.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = d\left(\int \frac{a^2}{x^3}\,dx + \int \frac{a^2c^2}{x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{b^2c^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2abc^2\operatorname{asinh}(cx)}{x}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**3,x)
```

output
```
d*(Integral(a**2/x**3, x) + Integral(a**2*c**2/x, x) + Integral(b**2*asinh
(c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(b**2*c**
2*asinh(c*x)**2/x, x) + Integral(2*a*b*c**2*asinh(c*x)/x, x))
```

### 3.205.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
a^2*c^2*d*log(x) - a*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*
a^2*d/x^2 + integrate(b^2*c^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*c
^2*d*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2
/x^3, x)
```

### 3.205.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)(a + b\text{arcsinh}(cx))^2}{x^3} dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2 + d)}{x^3} dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3, x)
```

# 3.206 $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$



## 3.206.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\begin{aligned}\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = & -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2} \\ & -\frac{2c^2d(a+b\text{arcsinh}(cx))^2}{3x} \\ & -\frac{d(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x^3} \\ & -\frac{10}{3}bc^3d(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) \\ & -\frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) \\ & +\frac{5}{3}b^2c^3d\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d/x-2/3*c^2*d*(a+b*arcsinh(c*x))^2/x-1/3*d*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/x^3-10/3*b*c^3*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)
^(1/2))-5/3*b^2*c^3*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/3*b^2*c^3*d*poly
log(2,c*x+(c^2*x^2+1)^(1/2))-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2
)/x^2
```

### 3.206.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.55

$$\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = -\frac{d\left(a^2+3a^2c^2x^2+b^2c^2x^2+abcx\sqrt{1+c^2x^2}+2ab\text{arcsinh}(cx)+6abc^2x^2\text{arcsinh}(cx)+b^2cx\sqrt{1+c^2x^2}\text{arc}\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
-1/3*(d*(a^2 + 3*a^2*c^2*x^2 + b^2*c^2*x^2 + a*b*c*x*Sqrt[1 + c^2*x^2] + 2
*a*b*ArcSinh[c*x] + 6*a*b*c^2*x^2*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] + b^2*ArcSinh[c*x]^2 + 3*b^2*c^2*x^2*ArcSinh[c*x]^2 + 5*a*b*
c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] - 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] + 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] -
 5*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] + 5*b^2*c^3*x^3*PolyLog[2, E
^(-ArcSinh[c*x])]))/x^3
```

### 3.206.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6222, 6191, 6220, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)(a+b\text{arcsinh}(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{6222}$$

$$\frac{2}{3}c^2d\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2}dx+\frac{2}{3}bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x^3}dx-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6191}$$

$$\frac{2}{3}c^2d\left(2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)+\frac{2}{3}bcd\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x^3}dx-\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6220}$$

$$\frac{2}{3}c^2d\left(2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)+\frac{2}{3}bcd\left(\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{1}{2}bc\int \frac{1}{x^2}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}\right)-\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}c^2d\left(2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)+\frac{2}{3}bcd\left(\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)-\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6231}$$

$$\frac{2}{3}bcd\left(\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)+\frac{2}{3}c^2d\left(2bc\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)-\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}bcd\left(\frac{1}{2}c^2\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)+\frac{2}{3}c^2d\left(-\frac{(a+b\text{arcsinh}(cx))^2}{x}+2bc\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)\right)-\frac{d(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{26}$$

3.206. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$\frac{2}{3}bcd\left(\frac{1}{2}ic^2\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)+\frac{2}{3}c^2d\left(-\frac{(a+b\text{arcsinh}(cx))^2}{x}+2ibc\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)\right)-\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 4670

$$\frac{2}{3}bcd\left(\frac{1}{2}ic^2\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}}\right.\right.\right.$$
$$\frac{2}{3}c^2d\left(-\frac{(a+b\text{arcsinh}(cx))^2}{x}+2ibc\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(\right.\right.$$
$$\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2715

$$\frac{2}{3}bcd\left(\frac{1}{2}ic^2\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}\right.\right.$$
$$\frac{2}{3}c^2d\left(-\frac{(a+b\text{arcsinh}(cx))^2}{x}+2ibc\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+\right.\right.\right.$$
$$\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2838

$$\frac{2}{3}bcd\left(\frac{1}{2}ic^2\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right.\right.$$
$$\frac{2}{3}c^2d\left(-\frac{(a+b\text{arcsinh}(cx))^2}{x}+2ibc\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\right.\right.$$
$$\frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{3x^3}$$

input
```
Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

---

3.206. $\int \frac{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

output
```
-1/3*(d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x^3 + (2*c^2*d*(-((a + b*Arc
Sinh[c*x])^2/x) + (2*I)*b*c*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[
c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))
)/3 + (2*b*c*d*(-1/2*(b*c)/x - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
*x^2) + (I/2)*c^2*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*
b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]])))/3
```

### 3.206.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

## 3.206. $\int \frac{(d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx$

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.206.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| parts | $d\,a^2\left(-\frac{c^2}{x}-\frac{1}{3x^3}\right)+d\,b^2c^3\left(-\frac{3\operatorname{arcsinh}(cx)^2x^2c^2+\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)^2+c^2x^2}{3c^3x^3}-\frac{5\operatorname{arcsinh}}{}\right.$ |
| derivativedivides | $c^3\left(d\,a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+d\,b^2\left(-\frac{3\operatorname{arcsinh}(cx)^2x^2c^2+\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)^2+c^2x^2}{3c^3x^3}-\frac{5\operatorname{arc}}{}\right.\right.$ |
| default | $c^3\left(d\,a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+d\,b^2\left(-\frac{3\operatorname{arcsinh}(cx)^2x^2c^2+\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)^2+c^2x^2}{3c^3x^3}-\frac{5\operatorname{arc}}{}\right.\right.$ |

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(-c^2/x-1/3/x^3)+d*b^2*c^3*(-1/3*(3*arcsinh(c*x)^2*x^2*c^2+arcsinh(c
*x)*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2)/c^3/x^3-5/3*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))-5/3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/3*arc
sinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+5/3*polylog(2,c*x+(c^2*x^2+1)^(1/2))
)+2*d*a*b*c^3*(-1/3*arcsinh(c*x)/c^3/x^3-arcsinh(c*x)/c/x-1/6/c^2/x^2*(c^2
*x^2+1)^(1/2)-5/6*arctanh(1/(c^2*x^2+1)^(1/2)))
```

### 3.206.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(c^2dx^2+d)(b\operatorname{arsinh}(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^4, x)
```

### 3.206.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=d\bigg(\int \frac{a^2}{x^4}\,dx+\int \frac{a^2c^2}{x^2}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{x^4}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{x^4}\,dx+\int \frac{b^2c^2\operatorname{asinh}^2(cx)}{x^2}\,dx+\int \frac{2abc^2\operatorname{asinh}(cx)}{x^2}\,dx\bigg)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**4,x)
```

output
```
d*(Integral(a**2/x**4, x) + Integral(a**2*c**2/x**2, x) + Integral(b**2*as
inh(c*x)**2/x**4, x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(b**2*
c**2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2*asinh(c*x)/x**2, x))
```

### 3.206.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-2*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d + 1/3*((c^2*arcsin
h(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d - a
^2*c^2*d/x - 1/3*a^2*d/x^3 - 1/3*(3*b^2*c^2*d*x^2 + b^2*d)*log(c*x + sqrt(
c^2*x^2 + 1))^2/x^3 + integrate(2/3*(3*b^2*c^5*d*x^4 + 4*b^2*c^3*d*x^2 + b
^2*c*d + (3*b^2*c^4*d*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt
(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

### 3.206.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4, x)
```

# 3.207 $\int x^4 (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx$



## 3.207.1 Optimal result

Integrand size = 26, antiderivative size = 386

$$\begin{aligned}\int x^4 (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx &= \frac{4208b^2d^2x}{99225c^4} - \frac{2104b^2d^2x^3}{297675c^2} + \frac{526b^2d^2x^5}{165375} \\ &+ \frac{212b^2c^2d^2x^7}{27783} + \frac{2}{729}b^2c^4d^2x^9 - \frac{128bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{4725c^5} \\ &+ \frac{64bd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{4725c^3} - \frac{16bd^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{1575c} \\ &- \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{189c^5} + \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{315c^5} \\ &+ \frac{20bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{441c^5} - \frac{2bd^2(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{81c^5} \\ &+ \frac{8}{315}d^2x^5(a+b\text{arcsinh}(cx))^2 + \frac{4}{63}d^2x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{1}{9}d^2x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
4208/99225*b^2*d^2*x/c^4-2104/297675*b^2*d^2*x^3/c^2+526/165375*b^2*d^2*x^
5+212/27783*b^2*c^2*d^2*x^7+2/729*b^2*c^4*d^2*x^9-8/189*b*d^2*(c^2*x^2+1)^
(3/2)*(a+b*arcsinh(c*x))/c^5+2/315*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*
x))/c^5+20/441*b*d^2*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^5-2/81*b*d^2*(
c^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^5+8/315*d^2*x^5*(a+b*arcsinh(c*x))^2
+4/63*d^2*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/9*d^2*x^5*(c^2*x^2+1)^2*(
a+b*arcsinh(c*x))^2-128/4725*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^
5+64/4725*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/1575*b*d^2
*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.207.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.65

$$\int x^4(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{d^2\left(99225a^2c^5x^5(63+90c^2x^2+35c^4x^4)-630ab\sqrt{1+c^2x^2}(2104-1052c^2x^2+789c^4x^4+2650c^6x^6+1225\right.}{}$$

input
```
Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(99225*a^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - 630*a*b*Sqrt[1 +
c^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8)
 + 2*b^2*c*x*(662760 - 110460*c^2*x^2 + 49707*c^4*x^4 + 119250*c^6*x^6 + 4
2875*c^8*x^8) - 630*b*(-315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) + b*S
qrt[1 + c^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*
c^8*x^8))*ArcSinh[c*x] + 99225*b^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*
ArcSinh[c*x]^2))/(31255875*c^5)
```

### 3.207.3 Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.654, Rules used = {6223, 27, 6219, 27, 1467, 2009, 6223, 6191, 6219, 27, 2009, 6227, 15, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(c^2dx^2+d\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \ 6223$$

$$-\frac{2}{9}bcd^2\int x^5\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{4}{9}d\int dx^4\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \ 27$$

$$-\frac{2}{9}bcd^2\int x^5\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{4}{9}d^2\int x^4\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6219}$$

$$\frac{4}{9}d^2\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{9}bcd^2\left(-bc\int\frac{\left(c^2x^2+1\right)^2\left(35c^4x^4-20c^2x^2+8\right)}{315c^6}dx+\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+ba\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{9}d^2\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{9}bcd^2\left(-\frac{b\int\left(c^2x^2+1\right)^2\left(35c^4x^4-20c^2x^2+8\right)dx}{315c^5}+\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+barc\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{1467}$$

$$\frac{4}{9}d^2\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{9}bcd^2\left(-\frac{b\int\left(35c^8x^8+50c^6x^6+3c^4x^4-4c^2x^2+8\right)dx}{315c^5}+\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{9}d^2\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$$\Big\downarrow \text{6223}$$

$$\frac{4}{9}d^2\left(-\frac{2}{7}bc\int x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{7}\int x^4(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}x^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$$\Big\downarrow \text{6191}$$

---

3.207. $\int x^4\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{7}bc\int x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$\downarrow$ 6219

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{7}bc\left(-bc\int\frac{15c^6x^6+3c^4x^4-4c^2x^2+8}{105c^6}dx\right.\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$\downarrow$ 27

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{7}bc\left(-\frac{b\int\left(15c^6x^6+3c^4x^4-4c^2x^2+8\right)d}{105c^5}\right.\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$\downarrow$ 2009

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{7}x^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{2}{7}bc\right.$$
$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$\downarrow$ 6227

3.207. $\int x^4\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{5c^2}-\frac{b\int x^4dx}{5c}+\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{5c^2}\right.\right.\right.$$

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$\downarrow$ 15

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{5c^2}+\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{bx^5}{25c}\right)\right)$$

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$\downarrow$ 6227

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}\right)}{5c^2}+\right.\right.\right.$$

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$\downarrow$ 15

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}+\frac{x^4\sqrt{}}{}\right.\right.\right.$$

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

3.207. $\int x^4\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\Big\downarrow \; 6213$$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int x^2dx}{9\ldots}\right)}{5c^2}\right.\right.\right.$$

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

$$\Big\downarrow \; 24$$

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+$$

$$\frac{4}{9}d^2\left(\frac{1}{7}x^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{5c^2}-\right.\right.\right.$$

$$\frac{2}{9}bcd^2\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}-\frac{2\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^6}+\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^6}\right.$$

input
```
Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*x^5*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/9 - (2*b*c*d^2*(-1/315*(b
*(8*x - (4*c^2*x^3)/3 + (3*c^4*x^5)/5 + (50*c^6*x^7)/7 + (35*c^8*x^9)/9))/
c^5 + ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^6) - (2*(1 + c^2*x^2
)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^6) + ((1 + c^2*x^2)^(9/2)*(a + b*ArcSin
h[c*x]))/(9*c^6)))/9 + (4*d^2*((x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/
7 - (2*b*c*(-1/105*(b*(8*x - (4*c^2*x^3)/3 + (3*c^4*x^5)/5 + (15*c^6*x^7)/
7))/c^5 + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6) - (2*(1 + c^2
*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^6) + ((1 + c^2*x^2)^(7/2)*(a + b*Ar
cSinh[c*x]))/(7*c^6)))/7 + (2*((x^5*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*(-1
/25*(b*x^5)/c + (x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (4*
(-1/9*(b*x^3)/c + (x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) - (
2*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2))/(3*c^2)))/(
5*c^2)))/5))/7))/9
```

## 3.207. $\int x^4\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

### 3.207.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## 3.207. $\int x^4(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2\,dx$

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.207.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| parts | $d^2a^2\left(\frac{1}{9}c^4x^9+\frac{2}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\operatorname{arcsinh}(cx)^2xc}{315}\right.}{}$ |
| derivativedivides | $\frac{d^2a^2\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\operatorname{arcsinh}(cx)^2xc}{315}+\frac{\operatorname{arcsinh}(c}{}\right.}{}$ |
| default | $\frac{d^2a^2\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\operatorname{arcsinh}(cx)^2xc}{315}+\frac{\operatorname{arcsinh}(c}{}\right.}{}$ |

input
```
int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/9*c^4*x^9+2/7*c^2*x^7+1/5*x^5)+d^2*b^2/c^5*(1/9*arcsinh(c*x)^2*
c^3*x^3*(c^2*x^2+1)^3-1/21*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+8/315*arcsinh(
c*x)^2*x*c+1/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/315*arcsinh(c*x)^2*c*x
*(c^2*x^2+1)-2/81*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(7/2)+82/3969*arcsinh(c
*x)*(c^2*x^2+1)^(7/2)+2/729*c*x*(c^2*x^2+1)^4+1493104/31255875*c*x-836/250
047*c*x*(c^2*x^2+1)^3-33862/10418625*c*x*(c^2*x^2+1)^2-47248/31255875*c*x*
(c^2*x^2+1)-16/315*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/525*arcsinh(c*x)*(c^2*
x^2+1)^(5/2)-8/945*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^2*a*b/c^5*(1/9*arcs
inh(c*x)*c^9*x^9+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^
8*x^8*(c^2*x^2+1)^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x
^4*(c^2*x^2+1)^(1/2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*
x^2+1)^(1/2))
```

### 3.207.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.95

$$\int x^4 \left(d + c^2 d x^2\right)^2 (a + b\text{arcsinh}(cx))^2 \, dx$$

$$= \frac{42875\,(81\,a^2 + 2\,b^2)c^9d^2x^9 + 2250\,(3969\,a^2 + 106\,b^2)c^7d^2x^7 + 189\,(33075\,a^2 + 526\,b^2)c^5d^2x^5 - 220920\,b^2c^3}{}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^2*x^9 + 2250*(3969*a^2 + 106*b^2)
*c^7*d^2*x^7 + 189*(33075*a^2 + 526*b^2)*c^5*d^2*x^5 - 220920*b^2*c^3*d^2*
x^3 + 1325520*b^2*c*d^2*x + 99225*(35*b^2*c^9*d^2*x^9 + 90*b^2*c^7*d^2*x^7
 + 63*b^2*c^5*d^2*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9
*d^2*x^9 + 28350*a*b*c^7*d^2*x^7 + 19845*a*b*c^5*d^2*x^5 - (1225*b^2*c^8*d
^2*x^8 + 2650*b^2*c^6*d^2*x^6 + 789*b^2*c^4*d^2*x^4 - 1052*b^2*c^2*d^2*x^2
 + 2104*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(12
25*a*b*c^8*d^2*x^8 + 2650*a*b*c^6*d^2*x^6 + 789*a*b*c^4*d^2*x^4 - 1052*a*b
*c^2*d^2*x^2 + 2104*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

### 3.207.6 Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.46

$$\int x^4(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} \frac{a^2c^4d^2x^9}{9}+\frac{2a^2c^2d^2x^7}{7}+\frac{a^2d^2x^5}{5}+\frac{2abc^4d^2x^9\operatorname{asinh}(cx)}{9}-\frac{2abc^3d^2x^8\sqrt{c^2x^2+1}}{81}+\frac{4abc^2d^2x^7\operatorname{asinh}(cx)}{7}-\frac{212abcd^2x^6\sqrt{c^2x^2+1}}{3969}+ \\ \frac{a^2d^2x^5}{5} \end{cases}$$

input
```
integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**9/9 + 2*a**2*c**2*d**2*x**7/7 + a**2*d**2*x**
5/5 + 2*a*b*c**4*d**2*x**9*asinh(c*x)/9 - 2*a*b*c**3*d**2*x**8*sqrt(c**2*x
**2 + 1)/81 + 4*a*b*c**2*d**2*x**7*asinh(c*x)/7 - 212*a*b*c*d**2*x**6*sqrt
(c**2*x**2 + 1)/3969 + 2*a*b*d**2*x**5*asinh(c*x)/5 - 526*a*b*d**2*x**4*sq
rt(c**2*x**2 + 1)/(33075*c) + 2104*a*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(9922
5*c**3) - 4208*a*b*d**2*sqrt(c**2*x**2 + 1)/(99225*c**5) + b**2*c**4*d**2*
x**9*asinh(c*x)**2/9 + 2*b**2*c**4*d**2*x**9/729 - 2*b**2*c**3*d**2*x**8*s
qrt(c**2*x**2 + 1)*asinh(c*x)/81 + 2*b**2*c**2*d**2*x**7*asinh(c*x)**2/7 +
 212*b**2*c**2*d**2*x**7/27783 - 212*b**2*c*d**2*x**6*sqrt(c**2*x**2 + 1)*
asinh(c*x)/3969 + b**2*d**2*x**5*asinh(c*x)**2/5 + 526*b**2*d**2*x**5/1653
75 - 526*b**2*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(33075*c) - 2104*b*
*2*d**2*x**3/(297675*c**2) + 2104*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh
(c*x)/(99225*c**3) + 4208*b**2*d**2*x/(99225*c**4) - 4208*b**2*d**2*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(99225*c**5), Ne(c, 0)), (a**2*d**2*x**5/5, True)
)
```

### 3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(342) = 684$.

Time = 0.22 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.97

$$\int x^4 (d + c^2 d x^2)^2 (a + b \operatorname{arcsinh}(cx))^2 \, dx = \frac{1}{9} b^2 c^4 d^2 x^9 \operatorname{arsinh}(cx)^2$$
$$+ \frac{1}{9} a^2 c^4 d^2 x^9 + \frac{2}{7} b^2 c^2 d^2 x^7 \operatorname{arsinh}(cx)^2 + \frac{2}{7} a^2 c^2 d^2 x^7 + \frac{1}{5} b^2 d^2 x^5 \operatorname{arsinh}(cx)^2$$
$$+ \frac{2}{2835} \left( 315 x^9 \operatorname{arsinh}(cx) - \left( \frac{35 \sqrt{c^2x^2+1} x^8}{c^2} - \frac{40 \sqrt{c^2x^2+1} x^6}{c^4} + \frac{48 \sqrt{c^2x^2+1} x^4}{c^6} - \frac{64 \sqrt{c^2x^2+1} x^2}{c^8} \right. \right.$$
$$- \frac{2}{893025} \left( 315 \left( \frac{35 \sqrt{c^2x^2+1} x^8}{c^2} - \frac{40 \sqrt{c^2x^2+1} x^6}{c^4} + \frac{48 \sqrt{c^2x^2+1} x^4}{c^6} - \frac{64 \sqrt{c^2x^2+1} x^2}{c^8} + \frac{128 \sqrt{c^2x^2 -}}{c^{10}} \right. \right.$$
$$+ \frac{1}{5} a^2 d^2 x^5$$
$$+ \frac{4}{245} \left( 35 x^7 \operatorname{arsinh}(cx) - \left( \frac{5 \sqrt{c^2x^2+1} x^6}{c^2} - \frac{6 \sqrt{c^2x^2+1} x^4}{c^4} + \frac{8 \sqrt{c^2x^2+1} x^2}{c^6} - \frac{16 \sqrt{c^2x^2+1}}{c^8} \right) c \right) abc^2$$
$$- \frac{4}{25725} \left( 105 \left( \frac{5 \sqrt{c^2x^2+1} x^6}{c^2} - \frac{6 \sqrt{c^2x^2+1} x^4}{c^4} + \frac{8 \sqrt{c^2x^2+1} x^2}{c^6} - \frac{16 \sqrt{c^2x^2+1}}{c^8} \right) c \operatorname{arsinh}(cx) - \frac{75 c^6}{} \right.$$
$$+ \frac{2}{75} \left( 15 x^5 \operatorname{arsinh}(cx) - \left( \frac{3 \sqrt{c^2x^2+1} x^4}{c^2} - \frac{4 \sqrt{c^2x^2+1} x^2}{c^4} + \frac{8 \sqrt{c^2x^2+1}}{c^6} \right) c \right) abd^2$$
$$- \frac{2}{1125} \left( 15 \left( \frac{3 \sqrt{c^2x^2+1} x^4}{c^2} - \frac{4 \sqrt{c^2x^2+1} x^2}{c^4} + \frac{8 \sqrt{c^2x^2+1}}{c^6} \right) c \operatorname{arsinh}(cx) - \frac{9 c^4 x^5 - 20 c^2 x^3 + 120 x}{c^4} \right)$$

input
```
integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/9*b^2*c^4*d^2*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^4*d^2*x^9 + 2/7*b^2*c^2*d^2
*x^7*arcsinh(c*x)^2 + 2/7*a^2*c^2*d^2*x^7 + 1/5*b^2*d^2*x^5*arcsinh(c*x)^2
 + 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(
c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)
*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^4*d^2 - 2/893025*(315*(35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)
*c*arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x
^3 + 40320*x)/c^8)*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 + 4/245*(35*x^7*arcsinh(c
*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(
c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^2 - 4/25725*
(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c
^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x
^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^2 + 2/75*(15*x^5*a
rcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 +
 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4
/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*
x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d^2
```

### 3.207.8 Giac [F(-2)]

Exception generated.

$$\int x^4 (d + c^2 d x^2)^2 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.207.9 Mupad [F(-1)]

Timed out.

$$\int x^4\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx=\int x^4\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

output
```
int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

# 3.208 $\int x^3(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx$



## 3.208.1 Optimal result

Integrand size = 26, antiderivative size = 296

$$\begin{aligned}\int x^3(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx = &-\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216} \\ &+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8 \\ &+\frac{73bd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{1536c^3} \\ &-\frac{73bd^2x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2304c} \\ &-\frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &-\frac{1}{32}bcd^2x^5(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) \\ &-\frac{73d^2(a+b\text{arcsinh}(cx))^2}{3072c^4} \\ &+\frac{1}{24}d^2x^4(a+b\text{arcsinh}(cx))^2 \\ &+\frac{1}{12}d^2x^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ &+\frac{1}{8}d^2x^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
-73/3072*b^2*d^2*x^2/c^2+73/9216*b^2*d^2*x^4+43/3456*b^2*c^2*d^2*x^6+1/256
*b^2*c^4*d^2*x^8-1/32*b*c*d^2*x^5*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-73/
3072*d^2*(a+b*arcsinh(c*x))^2/c^4+1/24*d^2*x^4*(a+b*arcsinh(c*x))^2+1/12*d
^2*x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/8*d^2*x^4*(c^2*x^2+1)^2*(a+b*arc
sinh(c*x))^2+73/1536*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-73/2
304*b*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-25/576*b*c*d^2*x^5*(a
+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.208.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.80

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d^2\left(cx\left(1152a^2c^3x^3(6+8c^2x^2+3c^4x^4)+b^2cx(-657+219c^2x^2+344c^4x^4+108c^6x^6)-6ab\sqrt{1+c^2x^2}(-2\right.\right.}{}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(c*x*(1152*a^2*c^3*x^3*(6 + 8*c^2*x^2 + 3*c^4*x^4) + b^2*c*x*(-657 +
219*c^2*x^2 + 344*c^4*x^4 + 108*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(-219 +
 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2
]*(-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 3*a*(-73 + 768*c^4*x
^4 + 1024*c^6*x^6 + 384*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(-73 + 768*c^4*x^4
+ 1024*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]^2))/(27648*c^4)
```

### 3.208.3 Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6223, 27, 6223, 244, 2009, 6191, 6221, 15, 6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(c^2dx^2+d\right)^2(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{1}{4}bcd^2\int x^4\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{2}d\int dx^3\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}bcd^2\int x^4\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{2}d^2\int x^3\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6223}$$

$$-\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx-\frac{1}{8}bc\int x^5\left(c^2x^2+1\right)dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\right.$$
$$\frac{1}{2}d^2\left(-\frac{1}{3}bc\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{3}\int x^3(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{6}x^4\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(c\right.$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}d^2\left(-\frac{1}{3}bc\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{3}\int x^3(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{6}x^4\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(c\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx-\frac{1}{8}bc\int\left(c^2x^7+x^5\right)dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\right)+$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d^2\left(-\frac{1}{3}bc\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{3}\int x^3(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{6}x^4\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(c\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{1}{8}bc\left(\frac{c^2x^8}{8}+\frac{x^6}{6}\right)\right)+$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\operatorname{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{3}bc\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))-\frac{1}{8}bc\left(\frac{c^2x^8}{8}+\frac{x^6}{6}\right)\right)+$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6221}$$

3.208. $\int x^3\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{6}bc\int\right.\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{6}bc\int x^5dx+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}\right.$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 15

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}x^5\sqrt{}\right.\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{36}bcx^6\right)+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+\right.$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6227

$$-\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}\right)+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+\right.\right.$$
$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}\right.\right.\right.$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 15

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)\right)\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsi}\right.\right.$$
$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6227

3.208. $\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c\cdots}}{}\right.\right.\right.$$

$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}\right.\right.\right.$$

$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 15

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2\cdots}}{}\right.\right.\right.$$

$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\cdots\right.\right.\right.$$

$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6198

$$\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+$$

$$\frac{1}{2}d^2\left(\frac{1}{6}x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\cdots\right.\right.\right.$$

$$\frac{1}{4}bcd^2\left(\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}\left(\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{1}{6}\left(\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcs}\cdots}{4c^2}\right.\right.\right.$$

input
```
Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

3.208. $\int x^3\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(d^2*x^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/8 - (b*c*d^2*(-1/8*(b*c*(
x^6/6 + (c^2*x^8)/8)) + (x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/8 +
 (3*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/6 + (-
1/16*(b*x^4)/c + (x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*c^2) - (3
*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a
 + b*ArcSinh[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/8))/4 + (d^2*((x^4*(1 + c^2*
x^2)*(a + b*ArcSinh[c*x])^2)/6 - (b*c*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/6 + (-1/16*(b*x^4)/c + (x^3*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x]))/(4*c^2) - (3*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3)))/(4*c^2)
)/6))/3 + ((x^4*(a + b*ArcSinh[c*x])^2)/4 - (b*c*(-1/16*(b*x^4)/c + (x^3*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*c^2) - (3*(-1/4*(b*x^2)/c + (x*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4
*b*c^3)))/(4*c^2)))/2)/3))/2
```

#### 3.208.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.208. $\int x^3(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx$

### 3.208.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| parts | $d^2a^2\left(\frac{1}{8}c^4x^8+\frac{1}{3}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^3}{8}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^3}{24}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2\right.}{32}\right.}{}$ |
| derivativedivides | $\frac{d^2a^2\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^3}{8}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^3}{24}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}+\frac{11}{}\right.}{}$ |
| default | $\frac{d^2a^2\left(\frac{1}{8}c^8x^8+\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^3}{8}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^3}{24}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}+\frac{11}{}\right.}{}$ |

input
```
int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/8*c^4*x^8+1/3*c^2*x^6+1/4*x^4)+d^2*b^2/c^4*(1/8*arcsinh(c*x)^2*
c^2*x^2*(c^2*x^2+1)^3-1/24*arcsinh(c*x)^2*(c^2*x^2+1)^3-1/32*arcsinh(c*x)*
c*x*(c^2*x^2+1)^(7/2)+11/576*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+55/2304*ar
csinh(c*x)*c*x*(c^2*x^2+1)^(3/2)+55/1536*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2
)+55/3072*arcsinh(c*x)^2+1/256*(c^2*x^2+1)^4-11/3456*(c^2*x^2+1)^3-55/9216
*(c^2*x^2+1)^2-55/3072*c^2*x^2-55/3072)+2*d^2*a*b/c^4*(1/8*arcsinh(c*x)*c^
8*x^8+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/64*c^7*x^7*(c^2*
x^2+1)^(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*c^3*x^3*(c^2*x^2+1)
^(1/2)+73/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x))
```

### 3.208.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.18

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{108\left(32a^2+b^2\right)c^8d^2x^8+8\left(1152a^2+43b^2\right)c^6d^2x^6+3\left(2304a^2+73b^2\right)c^4d^2x^4-657b^2c^2d^2x^2+9\left(384b^2c\right.}{}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/27648*(108*(32*a^2 + b^2)*c^8*d^2*x^8 + 8*(1152*a^2 + 43*b^2)*c^6*d^2*x^
6 + 3*(2304*a^2 + 73*b^2)*c^4*d^2*x^4 - 657*b^2*c^2*d^2*x^2 + 9*(384*b^2*c
^8*d^2*x^8 + 1024*b^2*c^6*d^2*x^6 + 768*b^2*c^4*d^2*x^4 - 73*b^2*d^2)*log(
c*x + sqrt(c^2*x^2 + 1))^2 + 6*(1152*a*b*c^8*d^2*x^8 + 3072*a*b*c^6*d^2*x^
6 + 2304*a*b*c^4*d^2*x^4 - 219*a*b*d^2 - (144*b^2*c^7*d^2*x^7 + 344*b^2*c^
5*d^2*x^5 + 146*b^2*c^3*d^2*x^3 - 219*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*log(
c*x + sqrt(c^2*x^2 + 1)) - 6*(144*a*b*c^7*d^2*x^7 + 344*a*b*c^5*d^2*x^5 +
146*a*b*c^3*d^2*x^3 - 219*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

### 3.208.6 Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.74

$$\int x^3 \left(d + c^2 d x^2\right)^2 (a + b\text{arcsinh}(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^8}{8} + \frac{a^2c^2d^2x^6}{3} + \frac{a^2d^2x^4}{4} + \frac{abc^4d^2x^8 \operatorname{asinh}(cx)}{4} - \frac{abc^3d^2x^7\sqrt{c^2x^2+1}}{32} + \frac{2abc^2d^2x^6 \operatorname{asinh}(cx)}{3} - \frac{43abcd^2x^5\sqrt{c^2x^2+1}}{576} + abd \\ \frac{a^2d^2x^4}{4} \end{cases}$$

input
```
integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**8/8 + a**2*c**2*d**2*x**6/3 + a**2*d**2*x**4/
4 + a*b*c**4*d**2*x**8*asinh(c*x)/4 - a*b*c**3*d**2*x**7*sqrt(c**2*x**2 +
1)/32 + 2*a*b*c**2*d**2*x**6*asinh(c*x)/3 - 43*a*b*c*d**2*x**5*sqrt(c**2*x
**2 + 1)/576 + a*b*d**2*x**4*asinh(c*x)/2 - 73*a*b*d**2*x**3*sqrt(c**2*x**
2 + 1)/(2304*c) + 73*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(1536*c**3) - 73*a*b*d
**2*asinh(c*x)/(1536*c**4) + b**2*c**4*d**2*x**8*asinh(c*x)**2/8 + b**2*c*
*4*d**2*x**8/256 - b**2*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 +
 b**2*c**2*d**2*x**6*asinh(c*x)**2/3 + 43*b**2*c**2*d**2*x**6/3456 - 43*b*
*2*c*d**2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/576 + b**2*d**2*x**4*asinh(c
*x)**2/4 + 73*b**2*d**2*x**4/9216 - 73*b**2*d**2*x**3*sqrt(c**2*x**2 + 1)*
asinh(c*x)/(2304*c) - 73*b**2*d**2*x**2/(3072*c**2) + 73*b**2*d**2*x*sqrt(
c**2*x**2 + 1)*asinh(c*x)/(1536*c**3) - 73*b**2*d**2*asinh(c*x)**2/(3072*c
**4), Ne(c, 0)), (a**2*d**2*x**4/4, True))
```

### 3.208.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(264) = 528$.

Time = 0.25 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.57

$$\int x^3\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{1}{8}\,b^2c^4d^2x^8\operatorname{arsinh}\left(cx\right)^2$$
$$+\frac{1}{8}\,a^2c^4d^2x^8+\frac{1}{3}\,b^2c^2d^2x^6\operatorname{arsinh}\left(cx\right)^2+\frac{1}{3}\,a^2c^2d^2x^6+\frac{1}{4}\,b^2d^2x^4\operatorname{arsinh}\left(cx\right)^2$$
$$+\frac{1}{1536}\left(384\,x^8\operatorname{arsinh}\left(cx\right)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2+1}x}{c^8}\right.\right.$$
$$+\frac{1}{9216}\left(\left(\left(\frac{36\,x^8}{c^2}-\frac{56\,x^6}{c^4}+\frac{105\,x^4}{c^6}-\frac{315\,x^2}{c^8}+\frac{315\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^{10}}\right)c^2-6\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\right.\right.\right.$$
$$+\frac{1}{4}\,a^2d^2x^4$$
$$+\frac{1}{72}\left(48\,x^6\operatorname{arsinh}\left(cx\right)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\operatorname{arsinh}\left(cx\right)}{c^7}\right)c\right)abc$$
$$+\frac{1}{432}\left(\left(\left(\frac{8\,x^6}{c^2}-\frac{15\,x^4}{c^4}+\frac{45\,x^2}{c^6}-\frac{45\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2-6\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}\right.\right.\right.$$
$$+\frac{1}{16}\left(8\,x^4\operatorname{arsinh}\left(cx\right)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}\left(cx\right)}{c^5}\right)c\right)abd^2$$
$$+\frac{1}{32}\left(\left(\left(\frac{x^4}{c^2}-\frac{3\,x^2}{c^4}+\frac{3\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2-2\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}\left(cx\right)}{c^5}\right.\right.\right.$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*c^4*d^2*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^4*d^2*x^8 + 1/3*b^2*c^2*d^2
*x^6*arcsinh(c*x)^2 + 1/3*a^2*c^2*d^2*x^6 + 1/4*b^2*d^2*x^4*arcsinh(c*x)^2
 + 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(
c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1
)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^4*d^2 + 1/9216*((36*x^8/c^2 - 56*
x^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c
^10)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4
+ 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh
(c*x)/c^9)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/4*a^2*d^2*x^4 + 1/72*(48*x^6*ar
csinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 +
 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^2*d^2 + 1/432*
((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/
c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 +
 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2
*d^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^
2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*d^2 + 1/32*((x^4/c^2 - 3*x^2
/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*
x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*
b^2*d^2
```

### 3.208.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.208.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \int x^3\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

output
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

# 3.209 $\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx$



## 3.209.1 Optimal result

Integrand size = 26, antiderivative size = 303

$$\begin{aligned}\int x^2(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx = &-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}\\&+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{315c^3}\\&-\frac{16bd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{315c}+\frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{105c^3}\\&+\frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{175c^3}-\frac{2bd^2(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c^3}\\&+\frac{8}{105}d^2x^3(a+b\text{arcsinh}(cx))^2+\frac{4}{35}d^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2+\frac{1}{7}d^2x^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
-1636/11025*b^2*d^2*x/c^2+818/33075*b^2*d^2*x^3+136/6125*b^2*c^2*d^2*x^5+2
/343*b^2*c^4*d^2*x^7+8/105*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^3+
2/175*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^3-2/49*b*d^2*(c^2*x^2+1
)^(7/2)*(a+b*arcsinh(c*x))/c^3+8/105*d^2*x^3*(a+b*arcsinh(c*x))^2+4/35*d^2
*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/7*d^2*x^3*(c^2*x^2+1)^2*(a+b*arcsi
nh(c*x))^2+32/315*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/315*b*
d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.209.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.75

$$\int x^2(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{d^2\left(11025a^2c^3x^3(35+42c^2x^2+15c^4x^4)-210ab\sqrt{1+c^2x^2}(-818+409c^2x^2+612c^4x^4+225c^6x^6)+2b^2cx\right.}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(11025*a^2*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - 210*a*b*Sqrt[1 +
c^2*x^2]*(-818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 2*b^2*c*x*(-85
890 + 14315*c^2*x^2 + 12852*c^4*x^4 + 3375*c^6*x^6) - 210*b*(-105*a*c^3*x^
3*(35 + 42*c^2*x^2 + 15*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(-818 + 409*c^2*x^2
 + 612*c^4*x^4 + 225*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c^3*x^3*(35 + 42*c
^2*x^2 + 15*c^4*x^4)*ArcSinh[c*x]^2))/(1157625*c^3)
```

### 3.209.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {6223, 27, 6219, 27, 290, 2009, 6223, 6191, 6219, 27, 2009, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(c^2dx^2+d)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2}{7}bcd^2\int x^3(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{4}{7}d\int dx^2(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{7}d^2x^3(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{7}d^2\int x^2(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))^2dx-\frac{2}{7}bcd^2\int x^3(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{7}d^2x^3(c^2x^2+1)^2(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6219}$$

$$\frac{4}{7}d^2\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-$$
$$\frac{2}{7}bcd^2\left(-bc\int -\frac{\left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2}{35c^4}dx+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx)}{5c^4}\right.$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 27

$$\frac{4}{7}d^2\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-$$
$$\frac{2}{7}bcd^2\left(\frac{b\int\left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2dx}{35c^3}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}\right)+$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 290

$$\frac{4}{7}d^2\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-$$
$$\frac{2}{7}bcd^2\left(\frac{b\int\left(-5c^6x^6-8c^4x^4-c^2x^2+2\right)dx}{35c^3}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx)}{5c^4}\right.$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 2009

$$\frac{4}{7}d^2\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\big\downarrow$ 6223

$$\frac{4}{7}d^2\left(-\frac{2}{5}bc\int x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{5}\int x^2(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\big\downarrow$ 6191

---

3.209. $\int x^2\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bc\int x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\right.$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\downarrow$ 6219

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bc\left(-bc\int-\frac{-3c^4x^4-c^2x^2+2}{15c^4}dx+\frac{(c^2}{}\right.\right.$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\downarrow$ 27

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bc\left(\frac{b\int\left(-3c^4x^4-c^2x^2+2\right)dx}{15c^3}+\frac{(c^2x^2}{}\right.\right.$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\downarrow$ 2009

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{5}x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(\right.\right.$$
$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\downarrow$ 6227

---

3.209. $\int x^2\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}\right.\right.\right.$$

$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)\right)\right.$$

$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{6213}$$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)}{3c^2}\right.\right.\right.$$

$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{7}d^2x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+$$

$$\frac{4}{7}d^2\left(\frac{1}{5}x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\right.\right.\right.$$

$$\frac{2}{7}bcd^2\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4}+\frac{b\left(-\frac{5}{7}c^6x^7-\frac{8c^4x^5}{5}-\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

input
```
Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

## 3.209. $\int x^2\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(d^2*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/7 - (2*b*c*d^2*((b*(2*x -
 (c^2*x^3)/3 - (8*c^4*x^5)/5 - (5*c^6*x^7)/7))/(35*c^3) - ((1 + c^2*x^2)^(
5/2)*(a + b*ArcSinh[c*x]))/(5*c^4) + ((1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c
*x]))/(7*c^4)))/7 + (4*d^2*((x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/5 -
 (2*b*c*((b*(2*x - (c^2*x^3)/3 - (3*c^4*x^5)/5))/(15*c^3) - ((1 + c^2*x^2)
^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4) + ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh
[c*x]))/(5*c^4)))/5 + (2*((x^3*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*(-1/9*(b
*x^3)/c + (x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) - (2*(-((b*
x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2))/(3*c^2)))/3))/5))/7
```

### 3.209.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.209. $\int x^2(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2\,dx$

## 3.209.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| parts | $d^2a^2\left(\frac{1}{7}c^4x^7+\frac{2}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}-\frac{8\operatorname{arcsinh}(cx)^2xc}{105}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}-\frac{4}{}\right.}{}$ |
| derivativedivides | $\frac{d^2a^2\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}-\frac{8\operatorname{arcsinh}(cx)^2xc}{105}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}-\frac{4\operatorname{arcsinh}(cx}{1}\right.}{}$ |
| default | $\frac{d^2a^2\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}-\frac{8\operatorname{arcsinh}(cx)^2xc}{105}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}-\frac{4\operatorname{arcsinh}(cx}{1}\right.}{}$ |

input
```
int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/7*c^4*x^7+2/5*c^2*x^5+1/3*x^3)+d^2*b^2/c^3*(1/7*arcsinh(c*x)^2*
c*x*(c^2*x^2+1)^3-8/105*arcsinh(c*x)^2*x*c-1/35*arcsinh(c*x)^2*c*x*(c^2*x^
2+1)^2-4/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*(c^2*x^2+1)^
(7/2)-181456/1157625*c*x+2/343*c*x*(c^2*x^2+1)^3+202/42875*c*x*(c^2*x^2+1)
^2-2528/1157625*c*x*(c^2*x^2+1)+16/105*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/17
5*arcsinh(c*x)*(c^2*x^2+1)^(5/2)+8/315*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d
^2*a*b/c^3*(1/7*arcsinh(c*x)*c^7*x^7+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(
c*x)*c^3*x^3-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1
/2)-409/11025*c^2*x^2*(c^2*x^2+1)^(1/2)+818/11025*(c^2*x^2+1)^(1/2))
```

## 3.209.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.08

$$\int x^2\left(d+c^2dx^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)^2dx$$
$$=\frac{3375\,(49\,a^2+2\,b^2)c^7d^2x^7+378\,(1225\,a^2+68\,b^2)c^5d^2x^5+35\,(11025\,a^2+818\,b^2)c^3d^2x^3-171780\,b^2cd^2x+}{}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/1157625*(3375*(49*a^2 + 2*b^2)*c^7*d^2*x^7 + 378*(1225*a^2 + 68*b^2)*c^5
*d^2*x^5 + 35*(11025*a^2 + 818*b^2)*c^3*d^2*x^3 - 171780*b^2*c*d^2*x + 110
25*(15*b^2*c^7*d^2*x^7 + 42*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3)*log(c*x
+ sqrt(c^2*x^2 + 1))^2 + 210*(1575*a*b*c^7*d^2*x^7 + 4410*a*b*c^5*d^2*x^5
+ 3675*a*b*c^3*d^2*x^3 - (225*b^2*c^6*d^2*x^6 + 612*b^2*c^4*d^2*x^4 + 409*
b^2*c^2*d^2*x^2 - 818*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
 1)) - 210*(225*a*b*c^6*d^2*x^6 + 612*a*b*c^4*d^2*x^4 + 409*a*b*c^2*d^2*x^
2 - 818*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

## 3.209.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.59

$$\int x^2(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^7}{7} + \frac{2a^2c^2d^2x^5}{5} + \frac{a^2d^2x^3}{3} + \frac{2abc^4d^2x^7\,\text{asinh}\,(cx)}{7} - \frac{2abc^3d^2x^6\sqrt{c^2x^2+1}}{49} + \frac{4abc^2d^2x^5\,\text{asinh}\,(cx)}{5} - \frac{136abcd^2x^4\sqrt{c^2x^2+1}}{1225} + \\ \frac{a^2d^2x^3}{3} \end{cases}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**7/7 + 2*a**2*c**2*d**2*x**5/5 + a**2*d**2*x**
3/3 + 2*a*b*c**4*d**2*x**7*asinh(c*x)/7 - 2*a*b*c**3*d**2*x**6*sqrt(c**2*x
**2 + 1)/49 + 4*a*b*c**2*d**2*x**5*asinh(c*x)/5 - 136*a*b*c*d**2*x**4*sqrt
(c**2*x**2 + 1)/1225 + 2*a*b*d**2*x**3*asinh(c*x)/3 - 818*a*b*d**2*x**2*sq
rt(c**2*x**2 + 1)/(11025*c) + 1636*a*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**
3) + b**2*c**4*d**2*x**7*asinh(c*x)**2/7 + 2*b**2*c**4*d**2*x**7/343 - 2*b
**2*c**3*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/49 + 2*b**2*c**2*d**2*x*
*5*asinh(c*x)**2/5 + 136*b**2*c**2*d**2*x**5/6125 - 136*b**2*c*d**2*x**4*s
qrt(c**2*x**2 + 1)*asinh(c*x)/1225 + b**2*d**2*x**3*asinh(c*x)**2/3 + 818*
b**2*d**2*x**3/33075 - 818*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(
11025*c) - 1636*b**2*d**2*x/(11025*c**2) + 1636*b**2*d**2*sqrt(c**2*x**2 +
 1)*asinh(c*x)/(11025*c**3), Ne(c, 0)), (a**2*d**2*x**3/3, True))
```

### 3.209.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(269) = 538.

Time = 0.22 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.04

$$\int x^2\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{7}b^2c^4d^2x^7\operatorname{arsinh}(cx)^2 + \frac{1}{7}a^2c^4d^2x^7 + \frac{2}{5}b^2c^2d^2x^5\operatorname{arsinh}(cx)^2 + \frac{2}{5}a^2c^2d^2x^5$$
$$+ \frac{2}{245}\left(35\,x^7\operatorname{arsinh}(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^4$$
$$- \frac{2}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\operatorname{arsinh}(cx) - \frac{75\,c^6\right.$$
$$+ \frac{1}{3}b^2d^2x^3\operatorname{arsinh}(cx)^2$$
$$+ \frac{4}{75}\left(15\,x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^2d^2$$
$$- \frac{4}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\operatorname{arsinh}(cx) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right)$$
$$+ \frac{1}{3}a^2d^2x^3 + \frac{2}{9}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abd^2$$
$$- \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3 - 6\,x}{c^2}\right)b^2d^2$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*c^4*d^2*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^4*d^2*x^7 + 2/5*b^2*c^2*d^2
*x^5*arcsinh(c*x)^2 + 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (
5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2
 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^2 - 2/25725*(105*(5
*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2
+ 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 12
6*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^2 + 1/3*b^2*d^2*x^3*arcsi
nh(c*x)^2 + 4/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*s
qrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^2 - 4/112
5*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(
c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b
^2*c^2*d^2 + 1/3*a^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2 - 2/27*(3*c*(sqrt(c^2*x^2
+ 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2
)*b^2*d^2
```

### 3.209.8 Giac [F(-2)]

Exception generated.

$$\int x^2 (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.209.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx = \int x^2\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

output
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

# 3.210 $\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx$



## 3.210.1 Optimal result

Integrand size = 24, antiderivative size = 204

$$\begin{aligned}\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx &= \frac{25}{288}b^2d^2x^2+\frac{5}{288}b^2c^2d^2x^4+\frac{b^2d^2(1+c^2x^2)^3}{108c^2} \\ &\quad -\frac{5bd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{48c} \\ &\quad -\frac{5bd^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{72c} \\ &\quad -\frac{bd^2x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{18c} \\ &\quad -\frac{5d^2(a+b\text{arcsinh}(cx))^2}{96c^2} \\ &\quad +\frac{d^2(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{6c^2}\end{aligned}$$

output
```
25/288*b^2*d^2*x^2+5/288*b^2*c^2*d^2*x^4+1/108*b^2*d^2*(c^2*x^2+1)^3/c^2-5
/72*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-1/18*b*d^2*x*(c^2*x^2+1
)^(5/2)*(a+b*arcsinh(c*x))/c-5/96*d^2*(a+b*arcsinh(c*x))^2/c^2+1/6*d^2*(c^
2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c^2-5/48*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x
^2+1)^(1/2)/c
```

### 3.210.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{d^2\left(cx\left(144a^2cx(3+3c^2x^2+c^4x^4)-6ab\sqrt{1+c^2x^2}(33+26c^2x^2+8c^4x^4)+b^2cx(99+39c^2x^2+8c^4x^4)\right)+\right.}{}$$

input
```
Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(c*x*(144*a^2*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - 6*a*b*Sqrt[1 + c^2*x^2]
*(33 + 26*c^2*x^2 + 8*c^4*x^4) + b^2*c*x*(99 + 39*c^2*x^2 + 8*c^4*x^4)) +
6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + 3*a*(11 +
48*c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6))*ArcSinh[c*x] + 9*b^2*(11 + 48*c^2*x
^2 + 48*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x]^2))/(864*c^2)
```

### 3.210.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6213, 6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(c^2dx^2+d)^2 (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \; 6213$$

$$\frac{d^2(c^2x^2+1)^3 (a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\int (c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))dx}{3c}$$

$$\Big\downarrow \; 6201$$

$$\frac{d^2(c^2x^2+1)^3 (a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\left(\frac{5}{6}\int (c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}bc\int x(c^2x^2+1)^2dx+\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))\right)}{3c}$$

$$\Big\downarrow \; 241$$

$$\frac{d^2(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\left(\frac{5}{6}\int(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))-\frac{b(c^2x^2+1)^3}{36c}\right)}{3c}$$

$\downarrow$ 6201

$$\frac{d^2(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x(c^2x^2+1)\,dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x(c}{3c}$$

$\downarrow$ 244

$$\frac{d^2(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int(c^2x^3+x)\,dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x(c^2}{3c}$$

$\downarrow$ 2009

$$\frac{d^2(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)+\frac{1}{6}x(c^2x^2}{3c}$$

$\downarrow$ 6200

$$\frac{d^2(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(\right.\right.}{3c}$$

$\downarrow$ 15

$$\frac{d^2(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)\right.\right.}{3c}$$

$\downarrow$ 6198

3.210. $\int x(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{d^2(c^2x^2+1)^3(a+b\operatorname{arcsinh}(cx))^2}{6c^2}-$$
$$\frac{bd^2\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\operatorname{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}\right.\right.\right.}{3c}$$

input
```
Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(6*c^2) - (b*d^2*(-1/36*(b*(1
 + c^2*x^2)^3)/c + (x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*(-1
/4*(b*c*(x^2/2 + (c^2*x^4)/4)) + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
]))/4 + (3*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2
+ (a + b*ArcSinh[c*x])^2/(4*b*c)))/4))/6))/(3*c)
```

### 3.210.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.210.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d^2a^2(c^2x^2+1)^3}{6}+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^3}{6}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{18}-\frac{5\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{3}{2}}}{72}-\frac{5\operatorname{arcsinh}(cx)cx\sqrt{c^2}}{48}\right.}{}$ |
| default | $\frac{\frac{d^2a^2(c^2x^2+1)^3}{6}+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^3}{6}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{18}-\frac{5\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{3}{2}}}{72}-\frac{5\operatorname{arcsinh}(cx)cx\sqrt{c^2}}{48}\right.}{}$ |
| parts | $\frac{d^2a^2(c^2x^2+1)^3}{6c^2}+\frac{d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^3}{6}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{18}-\frac{5\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{3}{2}}}{72}-\frac{5\operatorname{arcsinh}(cx)cx}{48}\right.}{c^2}$ |

input
```
int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/6*d^2*a^2*(c^2*x^2+1)^3+d^2*b^2*(1/6*arcsinh(c*x)^2*(c^2*x^2+1)^3
-1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)-5/72*arcsinh(c*x)*c*x*(c^2*x^2+1)
^(3/2)-5/48*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-5/96*arcsinh(c*x)^2+1/108*(
c^2*x^2+1)^3+5/288*(c^2*x^2+1)^2+5/96*c^2*x^2+5/96)+2*d^2*a*b*(1/6*arcsinh
(c*x)*c^6*x^6+1/2*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2+11/96*arcs
inh(c*x)-1/36*c*x*(c^2*x^2+1)^(5/2)-5/144*c*x*(c^2*x^2+1)^(3/2)-5/96*c*x*(
c^2*x^2+1)^(1/2)))
```

### 3.210.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.50

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{8\,(18\,a^2+b^2)c^6d^2x^6+3\,(144\,a^2+13\,b^2)c^4d^2x^4+9\,(48\,a^2+11\,b^2)c^2d^2x^2+9\,(16\,b^2c^6d^2x^6+48\,b^2c^4d^2x^4+$$

input
```
integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/864*(8*(18*a^2 + b^2)*c^6*d^2*x^6 + 3*(144*a^2 + 13*b^2)*c^4*d^2*x^4 + 9
*(48*a^2 + 11*b^2)*c^2*d^2*x^2 + 9*(16*b^2*c^6*d^2*x^6 + 48*b^2*c^4*d^2*x^
4 + 48*b^2*c^2*d^2*x^2 + 11*b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(4
8*a*b*c^6*d^2*x^6 + 144*a*b*c^4*d^2*x^4 + 144*a*b*c^2*d^2*x^2 + 33*a*b*d^2
 - (8*b^2*c^5*d^2*x^5 + 26*b^2*c^3*d^2*x^3 + 33*b^2*c*d^2*x)*sqrt(c^2*x^2
+ 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(8*a*b*c^5*d^2*x^5 + 26*a*b*c^3*d^2
*x^3 + 33*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^2
```

### 3.210.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(196) = 392$.

Time = 0.79 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.11

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^6}{6}+\frac{a^2c^2d^2x^4}{2}+\frac{a^2d^2x^2}{2}+\frac{abc^4d^2x^6\operatorname{asinh}(cx)}{3}-\frac{abc^3d^2x^5\sqrt{c^2x^2+1}}{18}+abc^2d^2x^4\operatorname{asinh}(cx)-\frac{13abcd^2x^3\sqrt{c^2x^2+1}}{72}+ \\ \frac{a^2d^2x^2}{2} \end{cases}$$

input
```
integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**6/6 + a**2*c**2*d**2*x**4/2 + a**2*d**2*x**2/
2 + a*b*c**4*d**2*x**6*asinh(c*x)/3 - a*b*c**3*d**2*x**5*sqrt(c**2*x**2 +
1)/18 + a*b*c**2*d**2*x**4*asinh(c*x) - 13*a*b*c*d**2*x**3*sqrt(c**2*x**2
+ 1)/72 + a*b*d**2*x**2*asinh(c*x) - 11*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(48
*c) + 11*a*b*d**2*asinh(c*x)/(48*c**2) + b**2*c**4*d**2*x**6*asinh(c*x)**2
/6 + b**2*c**4*d**2*x**6/108 - b**2*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)*asi
nh(c*x)/18 + b**2*c**2*d**2*x**4*asinh(c*x)**2/2 + 13*b**2*c**2*d**2*x**4/
288 - 13*b**2*c*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/72 + b**2*d**2*x*
*2*asinh(c*x)**2/2 + 11*b**2*d**2*x**2/96 - 11*b**2*d**2*x*sqrt(c**2*x**2
+ 1)*asinh(c*x)/(48*c) + 11*b**2*d**2*asinh(c*x)**2/(96*c**2), Ne(c, 0)),
(a**2*d**2*x**2/2, True))
```

## 3.210.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(182) = 364$.

Time = 0.24 (sec) , antiderivative size = 621, normalized size of antiderivative = 3.04

$$\int x(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx$$
$$= \frac{1}{6} b^2c^4d^2x^6 \text{arsinh}(cx)^2 + \frac{1}{6} a^2c^4d^2x^6 + \frac{1}{2} b^2c^2d^2x^4 \text{arsinh}(cx)^2 + \frac{1}{2} a^2c^2d^2x^4$$
$$+ \frac{1}{144}\left(48\,x^6 \text{arsinh}(cx) - \left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\sqrt{c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{c^2x^2+1}x}{c^6} - \frac{15\,\text{arsinh}(cx)}{c^7}\right)c\right)ab$$
$$+ \frac{1}{864}\left(\left(\frac{8\,x^6}{c^2} - \frac{15\,x^4}{c^4} + \frac{45\,x^2}{c^6} - \frac{45\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^8}\right)c^2 - 6\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2} - \frac{10\sqrt{c^2x^2+1}x^3}{c^4}\right.\right.$$
$$+ \frac{1}{2} b^2d^2x^2 \text{arsinh}(cx)^2$$
$$+ \frac{1}{8}\left(8\,x^4 \text{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}(cx)}{c^5}\right)c\right)abc^2d^2$$
$$+ \frac{1}{16}\left(\left(\frac{x^4}{c^2} - \frac{3\,x^2}{c^4} + \frac{3\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2 - 2\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}(cx)}{c^5}\right.\right.$$
$$+ \frac{1}{2} a^2d^2x^2 + \frac{1}{2}\left(2\,x^2 \text{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}(cx)}{c^3}\right)\right)abd^2$$
$$+ \frac{1}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}(cx)}{c^3}\right)\text{arsinh}(cx)\right)b^2d^2$$

input
```
integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*c^4*d^2*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^4*d^2*x^6 + 1/2*b^2*c^2*d^2
*x^4*arcsinh(c*x)^2 + 1/2*a^2*c^2*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (
8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x
^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^2 + 1/864*((8*x^6/c^2 -
15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(
8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x
^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/2*b^2
*d^2*x^2*arcsinh(c*x)^2 + 1/8*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x
^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d^2 +
1/16*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2
*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)
/c^5)*c*arcsinh(c*x))*b^2*c^2*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arcsinh(c
*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2 + 1/4*(c^2*(
x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c
^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*d^2
```

### 3.210.8 Giac [F(-2)]

Exception generated.

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)^2dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.210.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+c^2dx^2\right)^2(a+b\operatorname{arcsinh}(cx))^2\,dx=\int x\,(a+b\,\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

output
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

# 3.211 $\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx$



## 3.211.1 Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 dx &= \frac{298}{225}b^2d^2x + \frac{76}{675}b^2c^2d^2x^3 + \frac{2}{125}b^2c^4d^2x^5 \\ &- \frac{16bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{15c} \\ &- \frac{8bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{45c} \\ &- \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{25c} \\ &+ \frac{8}{15}d^2x(a+b\text{arcsinh}(cx))^2 \\ &+ \frac{4}{15}d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ &+ \frac{1}{5}d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3+2/125*b^2*c^4*d^2*x^5-8/45*b*d^2*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-2/25*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*a
rcsinh(c*x))/c+8/15*d^2*x*(a+b*arcsinh(c*x))^2+4/15*d^2*x*(c^2*x^2+1)*(a+b
*arcsinh(c*x))^2+1/5*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-16/15*b*d^2*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.211.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.89

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{d^2\left(225a^2cx(15+10c^2x^2+3c^4x^4)-30ab\sqrt{1+c^2x^2}(149+38c^2x^2+9c^4x^4)+2b^2cx(2235+190c^2x^2+27c\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(225*a^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]
*(149 + 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 + 190*c^2*x^2 + 27*c^4*x
^4) - 30*b*(-15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*
(149 + 38*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*b^2*c*x*(15 + 10*c^2*x^
2 + 3*c^4*x^4)*ArcSinh[c*x]^2))/(3375*c)
```

### 3.211.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6201, 27, 6201, 6187, 6213, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}d\int d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}d^2\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6201}$$

$$-\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{4}{5}d^2\left(-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}\int(a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6187

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}x(\right.$$
$$\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6213

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arc}}{3c^2}\right.\right.$$
$$\frac{2}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\int\left(c^2x^2+1\right)^2dx}{5c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$
$$b\text{arcsinh}(cx))^2$$

$\downarrow$ 24

$$\frac{4}{5}d^2\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int\left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(\right.\right.$$
$$\frac{2}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\int\left(c^2x^2+1\right)^2dx}{5c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$
$$b\text{arcsinh}(cx))^2$$

$\downarrow$ 210

$$\frac{4}{5}d^2\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int\left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(\right.\right.$$
$$\frac{2}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\int\left(c^4x^4+2c^2x^2+1\right)dx}{5c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$
$$b\text{arcsinh}(cx))^2$$

$\downarrow$ 2009

---

3.211. $\int\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{5}d^2x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2+$$
$$\frac{4}{5}d^2\left(\frac{1}{3}x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)\right)$$
$$\frac{2}{5}bcd^2\left(\frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)}{5c}\right)$$

input
```
Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*d^2*(-1/5*(b*(x
+ (2*c^2*x^3)/3 + (c^4*x^5)/5))/c + ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x]))/(5*c^2)))/5 + (4*d^2*((x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2
*b*c*(-1/3*(b*(x + (c^2*x^3)/3))/c + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c
*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2)))/3))/5
```

### 3.211.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.211.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| derivativedivides | $d^2a^2\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{8\,\text{arcsinh}(cx)^2xc}{15}+\frac{\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}+\frac{4\,\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{16\,\text{arcsinh}(cx)\sqrt{}}{15}\right.$ |
| default | $d^2a^2\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{8\,\text{arcsinh}(cx)^2xc}{15}+\frac{\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}+\frac{4\,\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{16\,\text{arcsinh}(cx)\sqrt{}}{15}\right.$ |
| parts | $d^2a^2\left(\frac{1}{5}c^4x^5+\frac{2}{3}x^3c^2+x\right)+\frac{d^2b^2\left(\frac{8\,\text{arcsinh}(cx)^2xc}{15}+\frac{\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}+\frac{4\,\text{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{16\,\text{a}}{}\right.}{}$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^2*a^2*(1/5*c^5*x^5+2/3*c^3*x^3+c*x)+d^2*b^2*(8/15*arcsinh(c*x)^2*x*
c+1/5*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)
-16/15*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4144/3375*c*x-2/25*arcsinh(c*x)*(c^2
*x^2+1)^(5/2)+2/125*c*x*(c^2*x^2+1)^2+272/3375*c*x*(c^2*x^2+1)-8/45*arcsin
h(c*x)*(c^2*x^2+1)^(3/2))+2*d^2*a*b*(1/5*arcsinh(c*x)*c^5*x^5+2/3*arcsinh(
c*x)*c^3*x^3+arcsinh(c*x)*c*x-149/225*(c^2*x^2+1)^(1/2)-1/25*c^4*x^4*(c^2*
x^2+1)^(1/2)-38/225*c^2*x^2*(c^2*x^2+1)^(1/2)))
```

### 3.211.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.30

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{27\,(25\,a^2+2\,b^2)c^5d^2x^5+10\,(225\,a^2+38\,b^2)c^3d^2x^3+15\,(225\,a^2+298\,b^2)cd^2x+225\,(3\,b^2c^5d^2x^5+10\,b^2c^3d$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 + 2*b^2)*c^5*d^2*x^5 + 10*(225*a^2 + 38*b^2)*c^3*d^2*x^
3 + 15*(225*a^2 + 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 + 10*b^2*c^3*d
^2*x^3 + 15*b^2*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^5*d
^2*x^5 + 150*a*b*c^3*d^2*x^3 + 225*a*b*c*d^2*x - (9*b^2*c^4*d^2*x^4 + 38*b
^2*c^2*d^2*x^2 + 149*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
1)) - 30*(9*a*b*c^4*d^2*x^4 + 38*a*b*c^2*d^2*x^2 + 149*a*b*d^2)*sqrt(c^2*x
^2 + 1))/c
```

### 3.211.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.82

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^4d^2x^5}{5}+\frac{2a^2c^2d^2x^3}{3}+a^2d^2x+\frac{2abc^4d^2x^5\operatorname{asinh}(cx)}{5}-\frac{2abc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{4abc^2d^2x^3\operatorname{asinh}(cx)}{3}-\frac{76abcd^2x^2\sqrt{c^2x^2+1}}{225}+ \\ a^2d^2x\end{cases}$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**5/5 + 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
 2*a*b*c**4*d**2*x**5*asinh(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(c**2*x**2 +
 1)/25 + 4*a*b*c**2*d**2*x**3*asinh(c*x)/3 - 76*a*b*c*d**2*x**2*sqrt(c**2*
x**2 + 1)/225 + 2*a*b*d**2*x*asinh(c*x) - 298*a*b*d**2*sqrt(c**2*x**2 + 1)
/(225*c) + b**2*c**4*d**2*x**5*asinh(c*x)**2/5 + 2*b**2*c**4*d**2*x**5/125
 - 2*b**2*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + 2*b**2*c**2*d
**2*x**3*asinh(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 - 76*b**2*c*d**2*x**
2*sqrt(c**2*x**2 + 1)*asinh(c*x)/225 + b**2*d**2*x*asinh(c*x)**2 + 298*b**
2*d**2*x/225 - 298*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c), Ne(c,
 0)), (a**2*d**2*x, True))
```

### 3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(190) = 380$.

Time = 0.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.14

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{5}\,b^2c^4d^2x^5\,\text{arsinh}\,(cx)^2 + \frac{1}{5}\,a^2c^4d^2x^5 + \frac{2}{3}\,b^2c^2d^2x^3\,\text{arsinh}\,(cx)^2$$
$$+ \frac{2}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^4d^2$$
$$- \frac{2}{1125}\left(15\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right)$$
$$+ \frac{2}{3}\,a^2c^2d^2x^3 + \frac{4}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d^2$$
$$- \frac{4}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3 - 6\,x}{c^2}\right)b^2c^2d^2$$
$$+ b^2d^2x\,\text{arsinh}\,(cx)^2 + 2\,b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right)$$
$$+ a^2d^2x + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*c^4*d^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 + 2/3*b^2*c^2*d^2
*x^3*arcsinh(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4
/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d
^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4
 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120
*x)/c^4)*b^2*c^4*d^2 + 2/3*a^2*c^2*d^2*x^3 + 4/9*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 - 4/27*(
3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (
c^2*x^3 - 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arcsinh(c*x)^2 + 2*b^2*d^2*(x
- sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*a*b*d^2/c
```

### 3.211.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.211.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx=\int \left(a+b\,\text{asinh}(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

# 3.212 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$



## 3.212.1 Optimal result

Integrand size = 26, antiderivative size = 257

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4\\
&\quad-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\
&\quad-\frac{1}{8}bcd^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\\
&\quad-\frac{11}{32}d^2(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{1}{2}d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{1}{4}d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{d^2(a+b\text{arcsinh}(cx))^3}{3b}\\
&\quad+d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\
&\quad-bd^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\\
&\quad-\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4-1/8*b*c*d^2*x*(c^2*x^2+1)^(3/2)
*(a+b*arcsinh(c*x))-11/32*d^2*(a+b*arcsinh(c*x))^2+1/2*d^2*(c^2*x^2+1)*(a+
b*arcsinh(c*x))^2+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/3*d^2*(a+b*
arcsinh(c*x))^3/b+d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^
2)-b*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2
*d^2*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/16*b*c*d^2*x*(a+b*arcsinh(c
*x))*(c^2*x^2+1)^(1/2)
```

## 3.212.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.27

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x} \, dx = \frac{1}{768}d^2\Big(768a^2c^2x^2+192a^2c^4x^4-624abcx\sqrt{1+c^2x^2} -96abc^3x^3\sqrt{1+c^2x^2}+1536abc^2x^2\text{arcsinh}(cx) +384abc^4x^4\text{arcsinh}(cx)-768ab\text{arcsinh}(cx)^2 -256b^2\text{arcsinh}(cx)^3+144b^2\cosh(2\text{arcsinh}(cx)) +288b^2\text{arcsinh}(cx)^2\cosh(2\text{arcsinh}(cx)) +3b^2\cosh(4\text{arcsinh}(cx)) +24b^2\text{arcsinh}(cx)^2\cosh(4\text{arcsinh}(cx)) +1536ab\text{arcsinh}(cx)\log\left(1-e^{2\text{arcsinh}(cx)}\right) +768b^2\text{arcsinh}(cx)^2\log\left(1-e^{2\text{arcsinh}(cx)}\right) +768a^2\log(cx)-624ab\log\left(-cx+\sqrt{1+c^2x^2}\right) +768b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right) -384b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right) -288b^2\text{arcsinh}(cx)\sinh(2\text{arcsinh}(cx)) -12b^2\text{arcsinh}(cx)\sinh(4\text{arcsinh}(cx))\Big)$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

output
```
(d^2*(768*a^2*c^2*x^2 + 192*a^2*c^4*x^4 - 624*a*b*c*x*Sqrt[1 + c^2*x^2] -
96*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 1536*a*b*c^2*x^2*ArcSinh[c*x] + 384*a*b
*c^4*x^4*ArcSinh[c*x] - 768*a*b*ArcSinh[c*x]^2 - 256*b^2*ArcSinh[c*x]^3 +
144*b^2*Cosh[2*ArcSinh[c*x]] + 288*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]]
 + 3*b^2*Cosh[4*ArcSinh[c*x]] + 24*b^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]]
 + 1536*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 768*b^2*ArcSinh[c*x
]^2*Log[1 - E^(2*ArcSinh[c*x])] + 768*a^2*Log[c*x] - 624*a*b*Log[-(c*x) +
Sqrt[1 + c^2*x^2]] + 768*b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*
x])] - 384*b^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 288*b^2*ArcSinh[c*x]*Sinh[
2*ArcSinh[c*x]] - 12*b^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/768
```

### 3.212.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.47, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.808$, Rules used = {6223, 27, 6201, 244, 2009, 6200, 15, 6198, 6223, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 6200, 15, 6198, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^2 (a+b\text{arcsinh}(cx))^2}{x} \, dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{1}{2}bcd^2\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+d\int \frac{d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}bcd^2\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+d^2\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6201}$$

$$-\frac{1}{2}bcd^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)+d^2\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{244}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx - \frac{1}{2}bcd^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx - \frac{1}{4}bc\int (c^2x^3+x)\,dx + \frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\right) + \frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx - \frac{1}{2}bcd^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx + \frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) - \frac{1}{4}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right) + \frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6200}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx - \frac{1}{2}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx - \frac{1}{2}bc\int xdx + \frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right) + \frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{ar}\right. \frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{15}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx - \frac{1}{2}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx + \frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) - \frac{1}{4}bcx^2\right) + \frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsin}\right. \frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6198}$$

$$d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx + \frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}bcd^2\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) + \frac{(a+b\text{arcsinh}(cx))^2}{4bc} - \frac{1}{4}bcx^2\right.\right.$$

$$\Big\downarrow \text{6223}$$

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx + \int \frac{(a+b\text{arcsinh}(cx))^2}{x}dx + \frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))^2\right) + \frac{1}{4}d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}bcd^2\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) + \frac{(a+b\text{arcsinh}(cx))^2}{4bc} - \frac{1}{4}bcx^2\right.\right.$$

3.212. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$\Big\downarrow$ 6190

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{\int -(a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(c\right.}{b}\right.$$
$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$\Big\downarrow$ 25

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int (a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx)\right.}{b}\right.$$
$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$\Big\downarrow$ 3042

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int -i(a+b\text{arcsinh}(cx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arc}\right.}{b}\right.$$
$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$\Big\downarrow$ 26

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\int (a+b\text{arcsinh}(cx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\right.}{b}\right.$$
$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$\Big\downarrow$ 4201

---

3.212. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\right.}{b}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$$\Big\downarrow \text{2620}$$

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\int (a+b\text{arcsinh}(cx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+\right.\right.}{}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)\right.\right.\right.}{}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsi}\right.\right.\right.}{}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$$\Big\downarrow \text{6200}$$

3.212. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsi}\right.\right.\right.}{}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 15

$$d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsi}\right.\right.\right.}{}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 6198

$$d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsi}\right.\right.\right.}{}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 7143

$$d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\text{PolyLog}(3,-a-b\text{arcsinh}(cx))+\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)\right)\right.\right.}{b}\right.$$

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right.\right.$$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

3.212. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$

output
```
(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (b*c*d^2*(-1/4*(b*c*(x^2/
2 + (c^2*x^4)/4)) + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*(-
1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcS
inh[c*x])^2/(4*b*c)))/4))/2 + d^2*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/
2 - b*c*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (
a + b*ArcSinh[c*x])^2/(4*b*c)) + (I*((-1/3*I)*(a + b*ArcSinh[c*x])^3 + (2*
I)*(-1/2*(b*(a + b*ArcSinh[c*x])^2*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*A
rcSinh[c*x]))/b)]) + b*((b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^((2*a)/b - I
*Pi - (2*(a + b*ArcSinh[c*x]))/b)])/2 + (b^2*PolyLog[3, -a - b*ArcSinh[c*x
]])/4))))/b)
```

### 3.212.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

## 3.212. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

## 3.212. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.212.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(262) = 524$.

Time = 0.31 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.23

| method | result |
|---|---|
| parts | $d^2a^2\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(x\right)\right)-\frac{d^2ab\,c^3x^3\sqrt{c^2x^2+1}}{8}+\frac{13d^2b^2\operatorname{arcsinh}(cx)^2}{32}-\frac{d^2b^2\operatorname{arcsinh}(cx)^3}{3}-2d^2b^2$ |
| derivativedivides | $-\frac{d^2ab\,c^3x^3\sqrt{c^2x^2+1}}{8}+d^2a^2\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(cx\right)\right)+\frac{13d^2b^2\operatorname{arcsinh}(cx)^2}{32}-\frac{d^2b^2\operatorname{arcsinh}(cx)^3}{3}-2d$ |
| default | $-\frac{d^2ab\,c^3x^3\sqrt{c^2x^2+1}}{8}+d^2a^2\left(\frac{c^4x^4}{4}+c^2x^2+\ln\left(cx\right)\right)+\frac{13d^2b^2\operatorname{arcsinh}(cx)^2}{32}-\frac{d^2b^2\operatorname{arcsinh}(cx)^3}{3}-2d$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/4*c^4*x^4+c^2*x^2+ln(x))-1/8*d^2*a*b*c^3*x^3*(c^2*x^2+1)^(1/2)+
13/32*d^2*b^2*arcsinh(c*x)^2-1/3*d^2*b^2*arcsinh(c*x)^3-2*d^2*b^2*polylog(
3,-c*x-(c^2*x^2+1)^(1/2))-2*d^2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))+49/25
6*d^2*b^2+13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4+d^2*b^2*arcsinh(c*x)^
2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d^2*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^
2+1)^(1/2))+2*d^2*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^2*b
^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-d^2*a*b*arcsinh(c*x)^2+13/16
*d^2*a*b*arcsinh(c*x)+2*d^2*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d^2*a*
b*polylog(2,c*x+(c^2*x^2+1)^(1/2))-13/16*d^2*a*b*c*x*(c^2*x^2+1)^(1/2)+1/2
*d^2*a*b*arcsinh(c*x)*c^4*x^4+2*d^2*a*b*arcsinh(c*x)*c^2*x^2-1/8*d^2*b^2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-13/16*d^2*b^2*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*c*x+1/4*d^2*b^2*arcsinh(c*x)^2*c^4*x^4+d^2*b^2*arcsinh(c*x)^2*c^2
*x^2+2*d^2*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d^2*a*b*arcsinh(
c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
```

### 3.212.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x, x)
```

### 3.212.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx = d^2\left(\int \frac{a^2}{x}\,dx + \int 2a^2c^2x\,dx + \int a^2c^4x^3\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{x}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{x}\,dx + \int 2b^2c^2x\,\text{asinh}^2\,(cx)\,dx + \int b^2c^4x^3\,\text{asinh}^2\,(cx)\,dx + \int 4abc^2x\,\text{asinh}\,(cx)\,dx + \int 2abc^4x^3\,\text{asinh}\,(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x,x)
```

output
```
d**2*(Integral(a**2/x, x) + Integral(2*a**2*c**2*x, x) + Integral(a**2*c**
4*x**3, x) + Integral(b**2*asinh(c*x)**2/x, x) + Integral(2*a*b*asinh(c*x)
/x, x) + Integral(2*b**2*c**2*x*asinh(c*x)**2, x) + Integral(b**2*c**4*x**
3*asinh(c*x)**2, x) + Integral(4*a*b*c**2*x*asinh(c*x), x) + Integral(2*a*
b*c**4*x**3*asinh(c*x), x))
```

### 3.212.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/4*a^2*c^4*d^2*x^4 + a^2*c^2*d^2*x^2 + a^2*d^2*log(x) + integrate(b^2*c^4
*d^2*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x^3*log(c*x + sqrt
(c^2*x^2 + 1)) + 2*b^2*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 4*a*b*c^
2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1)
)^2/x + 2*a*b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.212.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^2}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x, x)
```

# 3.213 $\int \frac{(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx$



## 3.213.1 Optimal result

Integrand size = 26, antiderivative size = 229

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx = {}& \frac{32}{9}b^2c^2d^2x+\frac{2}{27}b^2c^4d^2x^3 \\ & -\frac{10}{3}bcd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) \\ & -\frac{2}{9}bcd^2\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) \\ & +\frac{8}{3}c^2d^2x(a+b\operatorname{arcsinh}(cx))^2 \\ & +\frac{4}{3}c^2d^2x\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2 \\ & -\frac{d^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{x} \\ & -4bcd^2(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right) \\ & -2b^2cd^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right) \\ & +2b^2cd^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)\end{aligned}$$

output
```
32/9*b^2*c^2*d^2*x+2/27*b^2*c^4*d^2*x^3-2/9*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b
*arcsinh(c*x))+8/3*c^2*d^2*x*(a+b*arcsinh(c*x))^2+4/3*c^2*d^2*x*(c^2*x^2+1
)*(a+b*arcsinh(c*x))^2-d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-4*b*c*d^2*
(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^2*polylog(2,-c
*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-10/3*b*
c*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.213.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.34

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2} dx$$
$$= \frac{1}{54}d^2 \Bigg( -\frac{54a^2}{x} + 108a^2c^2x + 18a^2c^4x^3 - 12abc(-2+c^2x^2)\sqrt{1+c^2x^2} + 36abc^4x^3\text{arcsinh}(cx) - 189b^2c\sqrt{1+c^2x^2}\text{arcsinh}(cx) + 216abc\left(-\sqrt{1+c^2x^2} + cx\text{arcsinh}(cx)\right) + 108b^2c^2x(2+\text{arcsinh}(cx)^2) + 2b^2c^2x(-12+2c^2x^2+9c^2x^2\text{arcsinh}(cx)^2) - \frac{108ab\left(\text{arcsinh}(cx) + cx\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)\right)}{x} - 3b^2c\,\text{arcsinh}(cx)\cosh(3\text{arcsinh}(cx)) - \frac{54b^2\text{arcsinh}(cx)\left(\text{arcsinh}(cx) + 2cx\left(-\log\left(1-e^{-\text{arcsinh}(cx)}\right) + \log\left(1+e^{-\text{arcsinh}(cx)}\right)\right)\right)}{x} + 108b^2c\,\text{PolyLog}\left(2, -e^{-\text{arcsinh}(cx)}\right) - 108b^2c\,\text{PolyLog}\left(2, e^{-\text{arcsinh}(cx)}\right) \Bigg)$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
(d^2*((-54*a^2)/x + 108*a^2*c^2*x + 18*a^2*c^4*x^3 - 12*a*b*c*(-2 + c^2*x^
2)*Sqrt[1 + c^2*x^2] + 36*a*b*c^4*x^3*ArcSinh[c*x] - 189*b^2*c*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x] + 216*a*b*c*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x]) +
108*b^2*c^2*x*(2 + ArcSinh[c*x]^2) + 2*b^2*c^2*x*(-12 + 2*c^2*x^2 + 9*c^2*
x^2*ArcSinh[c*x]^2) - (108*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^
2]]))/x - 3*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] - (54*b^2*ArcSinh[c*x]
*(ArcSinh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[
c*x])])))/x + 108*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 108*b^2*c*PolyLog
[2, E^(-ArcSinh[c*x])]))/54
```

### 3.213.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.32, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {6222, 27, 6201, 6187, 6213, 24, 2009, 6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

---

3.213. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^2} dx$

$$\int \frac{\left(c^2dx^2+d\right)^2(a+b\text{arcsinh}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{6222}$$

$$2bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+4c^2d\int d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{27}$$

$$2bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+4c^2d^2\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6201}$$

$$2bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+4c^2d^2\left(-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}\int (a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))\right. \ldots \frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6187}$$

$$4c^2d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}x \ldots\right.$$
$$2bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6213}$$

$$4c^2d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{ar} \ldots}{3c^2}\right.\right.$$
$$2bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{24}$$

$$4c^2d^2\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x \ldots\right.\right.$$
$$2bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}$$

---

3.213. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$$\Big\downarrow \text{ 2009}$$

$$2bcd^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$$\Big\downarrow \text{ 6223}$$

$$2bcd^2\left(\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{3}bc\int\left(c^2x^2+1\right)dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)-$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2bcd^2\left(\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\left(\frac{c^2x^3}{3}+x\right)\right)-$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$$\Big\downarrow \text{ 6221}$$

$$2bcd^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int 1dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$$\Big\downarrow \text{ 24}$$

---

3.213. $\int\frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$2bcd^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\left(\frac{c^2x^3}{3}+\right.\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$\downarrow$ 6231

$$2bcd^2\left(\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{3}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$\downarrow$ 3042

$$2bcd^2\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a-\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$\downarrow$ 26

$$2bcd^2\left(i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a-\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$\downarrow$ 4670

3.213. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$2bcd^2\left(i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$\downarrow$ 2715

$$2bcd^2\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

$\downarrow$ 2838

$$2bcd^2\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+$$
$$4c^2d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
-((d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/x) + 4*c^2*d^2*((x*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*(-1/3*(b*(x + (c^2*x^3)/3))/c + (
(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSi
nh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c^2)))/3) + 2*b*c*d^2*(-(b*c*x) - (b*c*(x + (c^2*x^3)/3))/3 + Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3
+ I*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -
E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]))
```

## 3.213. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

### 3.213.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

## 3.213. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

## 3.213. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.213.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| derivativedivides | $c\left(d^2a^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+\frac{2d^2b^2c^3x^3}{27}+\frac{32d^2b^2cx}{9}+2d^2b^2\,\mathrm{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)-2$ |
| default | $c\left(d^2a^2\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+\frac{2d^2b^2c^3x^3}{27}+\frac{32d^2b^2cx}{9}+2d^2b^2\,\mathrm{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)-2$ |
| parts | $d^2a^2\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)+2d^2b^2\,\mathrm{arcsinh}\,(cx)^2\,c^2x-\frac{d^2b^2\,\mathrm{arcsinh}(cx)^2}{x}+\frac{d^2b^2\,\mathrm{arcsinh}(cx)^2c^4x^3}{3}+$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(d^2*a^2*(1/3*c^3*x^3+2*c*x-1/c/x)+2/27*d^2*b^2*c^3*x^3+32/9*d^2*b^2*c*x
+2*d^2*b^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*d^2*b^2*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))-2/9*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+1/3*d^2*b
^2*arcsinh(c*x)^2*c^3*x^3+2*d^2*b^2*arcsinh(c*x)^2*c*x-32/9*d^2*b^2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)+2*d^2*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2)
)-2*d^2*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-d^2*b^2*arcsinh(c*x)^
2/c/x+2*d^2*a*b*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsinh(c*x)*c*x-arcsinh(c*x)/
c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)^(1/2)-arctanh(1/(c^2*x^
2+1)^(1/2))))
```

### 3.213.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^2\,(a+b\mathrm{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^2(b\,\mathrm{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^2, x)
```

### 3.213.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2} dx = d^2 \left( \int 2a^2c^2 dx + \int \frac{a^2}{x^2} dx + \int a^2c^4x^2 dx + \int 2b^2c^2 \operatorname{asinh}^2(cx) dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{x^2} dx + \int 4abc^2 \operatorname{asinh}(cx) dx + \int \frac{2ab \operatorname{asinh}(cx)}{x^2} dx + \int b^2c^4x^2 \operatorname{asinh}^2(cx) dx + \int 2abc^4x^2 \operatorname{asinh}(cx) dx \right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**2,x)
```

output
```
d**2*(Integral(2*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(a**2*c*
*4*x**2, x) + Integral(2*b**2*c**2*asinh(c*x)**2, x) + Integral(b**2*asinh
(c*x)**2/x**2, x) + Integral(4*a*b*c**2*asinh(c*x), x) + Integral(2*a*b*as
inh(c*x)/x**2, x) + Integral(b**2*c**4*x**2*asinh(c*x)**2, x) + Integral(2
*a*b*c**4*x**2*asinh(c*x), x))
```

### 3.213.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2} dx = \int \frac{(c^2dx^2+d)^2(b \operatorname{arsinh}(cx)+a)^2}{x^2} dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c^4*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c
^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^2 + 2*b^2*c^2*d^2*x*arcsinh(c*x)^
2 + 4*b^2*c^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 2*a^2*c^2*d^2*x
 + 4*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^2 - 2*(c*arcsinh(1/(c*
abs(x))) + arcsinh(c*x)/x)*a*b*d^2 - a^2*d^2/x + 1/3*(b^2*c^4*d^2*x^4 - 3*
b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/3*(b^2*c^7*d^2*x^6
 + b^2*c^5*d^2*x^4 - 3*b^2*c^3*d^2*x^2 - 3*b^2*c*d^2 + (b^2*c^6*d^2*x^5 -
3*b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4
+ c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.213.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^2}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2, x)
```

# 3.214 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$



## 3.214.1 Optimal result

Integrand size = 26, antiderivative size = 272

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx &= \frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{x} \\ &\quad +\frac{1}{4}c^2d^2(a+b\text{arcsinh}(cx))^2 \\ &\quad +c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\ &\quad -\frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2x^2} \\ &\quad +\frac{2c^2d^2(a+b\text{arcsinh}(cx))^3}{3b} \\ &\quad +2c^2d^2(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\ &\quad +b^2c^2d^2\log(x)-2bc^2d^2(a \\ &\quad +b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\ &\quad -b^2c^2d^2\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
1/4*b^2*c^4*d^2*x^2-b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/x+1/4*c^2
*d^2*(a+b*arcsinh(c*x))^2+c^2*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-1/2*d^2
*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^2+2/3*c^2*d^2*(a+b*arcsinh(c*x))^3/b
+2*c^2*d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*
d^2*ln(x)-2*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2
))^2)-b^2*c^2*d^2*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*c^3*d^2*x*(
a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.214.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.21

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3} dx$$
$$= \frac{1}{2}d^2\left(-\frac{a^2}{x^2}+a^2c^4x^2+2abc^4x^2\text{arcsinh}(cx)-\frac{2ab\left(cx\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{x^2}\right.$$
$$+\frac{1}{4}b^2c^2\left(1+2\text{arcsinh}(cx)^2\right)\cosh(2\text{arcsinh}(cx))+4a^2c^2\log(x)$$
$$-\frac{b^2\left(2cx\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)^2-2c^2x^2\log(cx)\right)}{x^2}$$
$$-abc^2\left(cx\sqrt{1+c^2x^2}+\log\left(-cx+\sqrt{1+c^2x^2}\right)\right)$$
$$-4abc^2\left(\text{arcsinh}(cx)\left(\text{arcsinh}(cx)-2\log\left(1-e^{2\text{arcsinh}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)\right)$$
$$-\frac{2}{3}b^2c^2\left(2\text{arcsinh}(cx)^2\left(\text{arcsinh}(cx)-3\log\left(1-e^{2\text{arcsinh}(cx)}\right)\right)\right.$$
$$\left.-6\text{arcsinh}(cx)\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)+3\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)\right)$$
$$\left.-\frac{1}{2}b^2c^2\text{arcsinh}(cx)\sinh(2\text{arcsinh}(cx))\right)$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
(d^2*(-(a^2/x^2) + a^2*c^4*x^2 + 2*a*b*c^4*x^2*ArcSinh[c*x] - (2*a*b*(c*x*
Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + (b^2*c^2*(1 + 2*ArcSinh[c*x]^2)*C
osh[2*ArcSinh[c*x]])/4 + 4*a^2*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x] + ArcSinh[c*x]^2 - 2*c^2*x^2*Log[c*x]))/x^2 - a*b*c^2*(c*x*Sq
rt[1 + c^2*x^2] + Log[-(c*x) + Sqrt[1 + c^2*x^2]]) - 4*a*b*c^2*(ArcSinh[c*
x]*(ArcSinh[c*x] - 2*Log[1 - E^(2*ArcSinh[c*x])]) - PolyLog[2, E^(2*ArcSin
h[c*x])]) - (2*b^2*c^2*(2*ArcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log[1 - E^(2*Ar
cSinh[c*x])]) - 6*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + 3*PolyLog[
3, E^(2*ArcSinh[c*x])]))/3 - (b^2*c^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]])/2
))/2
```

### 3.214.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.38, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.808, Rules used = {6222, 27, 6222, 244, 2009, 6200, 15, 6198, 6223, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 6200, 15, 6198, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^2 (a+b\operatorname{arcsinh}(cx))^2}{x^3} \, dx$$

$$\Big\downarrow \text{6222}$$

$$bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2} (a+b\operatorname{arcsinh}(cx))}{x^2}dx+2c^2d\int \frac{d\left(c^2x^2+1\right) (a+b\operatorname{arcsinh}(cx))^2}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2 (a+b\operatorname{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$bcd^2\int \frac{\left(c^2x^2+1\right)^{3/2} (a+b\operatorname{arcsinh}(cx))}{x^2}dx+2c^2d^2\int \frac{\left(c^2x^2+1\right) (a+b\operatorname{arcsinh}(cx))^2}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2 (a+b\operatorname{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6222}$$

$$bcd^2\left(3c^2\int \sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+bc\int \frac{c^2x^2+1}{x}dx-\frac{\left(c^2x^2+1\right)^{3/2} (a+b\operatorname{arcsinh}(cx))}{x}\right)+2c^2d^2\int \frac{\left(c^2x^2+1\right) (a+b\operatorname{arcsinh}(cx))^2}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2 (a+b\operatorname{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{244}$$

$$bcd^2\left(3c^2\int \sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+bc\int \left(xc^2+\frac{1}{x}\right)dx-\frac{\left(c^2x^2+1\right)^{3/2} (a+b\operatorname{arcsinh}(cx))}{x}\right)+2c^2d^2\int \frac{\left(c^2x^2+1\right) (a+b\operatorname{arcsinh}(cx))^2}{x}dx-\frac{d^2\left(c^2x^2+1\right)^2 (a+b\operatorname{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^2\left(3c^2\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{x}+bc\left(\frac{c^2x^2}{2}+\log(x)\right)\right)-$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 6200}$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsi}}{x}\right.$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 15}$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(c}{x}\right.$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 6198}$$

$$2c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$$\Big\downarrow \text{ 6223}$$

$$2c^2d^2\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\int \frac{(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))^2\right)-$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$$\Big\downarrow \text{ 6190}$$

---

3.214. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$2c^2d^2\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{\int -(a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}}{b}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 25

$$2c^2d^2\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int (a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(}{b}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 3042

$$2c^2d^2\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int -i(a+b\text{arcsinh}(cx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b}{b}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 26

$$2c^2d^2\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\int (a+b\text{arcsinh}(cx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a-}{b}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 4201

3.214. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$2c^2d^2\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))\right.}{b}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 2620

$$2c^2d^2\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\int(a+b\text{arcsinh}(cx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a\right.\right.}{}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 3011

$$2c^2d^2\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}\right.\right.\right.\right.}{}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 2720

$$2c^2d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+bar\right.\right.\right.}{}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

---

## 3.214. $\int\frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$\downarrow$ 6200

$$2c^2d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+\text{bar}\right.\right.\right.}{}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 15

$$2c^2d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+\text{bar}\right.\right.\right.}{}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 6198

$$2c^2d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+\text{bar}\right.\right.\right.}{}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

$\downarrow$ 7143

$$2c^2d^2\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\text{PolyLog}(3,-a-b\text{arcsinh}(cx))+\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)\right.\right.\right.}{b}\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^2\left(3c^2\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}\right.$$

---

3.214. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
-1/2*(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2 + b*c*d^2*(-(((1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x) + 3*c^2*(-1/4*(b*c*x^2) + (x*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*c)) + b*
c*((c^2*x^2)/2 + Log[x])) + 2*c^2*d^2*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x])
^2)/2 - b*c*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2
 + (a + b*ArcSinh[c*x])^2/(4*b*c)) + (I*((-1/3*I)*(a + b*ArcSinh[c*x])^3 +
 (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])^2*Log[1 + E^((2*a)/b - I*Pi - (2*(a +
 b*ArcSinh[c*x]))/b)]) + b*((b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^((2*a)/b
 - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)])/2 + (b^2*PolyLog[3, -a - b*ArcSinh
[c*x]])/4))))/b)
```

### 3.214.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.214. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.214.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(285) = 570.

Time = 0.28 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.15

| method | result |
|---|---|
| derivativedivides | $c^2\left(d^2a^2\left(\frac{c^2x^2}{2}+2\ln(cx)-\frac{1}{2c^2x^2}\right)+d^2b^2\left(-\frac{2\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+1\right)(2c}{16}\right.\right.$ |
| default | $c^2\left(d^2a^2\left(\frac{c^2x^2}{2}+2\ln(cx)-\frac{1}{2c^2x^2}\right)+d^2b^2\left(-\frac{2\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+1\right)(2c}{16}\right.\right.$ |
| parts | $d^2a^2\left(\frac{c^4x^2}{2}-\frac{1}{2x^2}+2c^2\ln(x)\right)+d^2b^2c^2\left(-\frac{2\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+1\right)(2c^2x^2}{16}\right.$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^2*a^2*(1/2*c^2*x^2+2*ln(c*x)-1/2/c^2/x^2)+d^2*b^2*(-2/3*arcsinh(c*x
)^3+1/16*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1
)^(1/2))+1/16*(-2*c*x*(c^2*x^2+1)^(1/2)+2*c^2*x^2+1)*(2*arcsinh(c*x)^2+2*a
rcsinh(c*x)+1)-1/2*arcsinh(c*x)*(-2*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsin
h(c*x))/c^2/x^2+ln(1+c*x+(c^2*x^2+1)^(1/2))-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln
(c*x+(c^2*x^2+1)^(1/2)-1)+2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+4*a
rcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-4*polylog(3,-c*x-(c^2*x^2+1)
^(1/2))+2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+4*arcsinh(c*x)*polylo
g(2,c*x+(c^2*x^2+1)^(1/2))-4*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*d^2*a*b*(
-arcsinh(c*x)^2+1/16*(-1+2*arcsinh(c*x))*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1
/2))+1/16*(-2*c*x*(c^2*x^2+1)^(1/2)+2*c^2*x^2+1)*(1+2*arcsinh(c*x))-1/2*(c
*x*(c^2*x^2+1)^(1/2)-c^2*x^2+arcsinh(c*x))/c^2/x^2+2*arcsinh(c*x)*ln(1+c*x
+(c^2*x^2+1)^(1/2))+2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*ln(
1-c*x-(c^2*x^2+1)^(1/2))+2*polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

## 3.214.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^3, x)
```

### 3.214.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3} dx = d^2 \left( \int \frac{a^2}{x^3} dx + \int \frac{2a^2c^2}{x} dx + \int a^2c^4x dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{x^3} dx + \int \frac{2ab \operatorname{asinh}(cx)}{x^3} dx + \int \frac{2b^2c^2 \operatorname{asinh}^2(cx)}{x} dx + \int b^2c^4x \operatorname{asinh}^2(cx) dx + \int \frac{4abc^2 \operatorname{asinh}(cx)}{x} dx + \int 2abc^4x \operatorname{asinh}(cx) dx \right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**3,x)
```

output
```
d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c**2/x, x) + Integral(a**2*
c**4*x, x) + Integral(b**2*asinh(c*x)**2/x**3, x) + Integral(2*a*b*asinh(c
*x)/x**3, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x, x) + Integral(b**2*c*
*4*x*asinh(c*x)**2, x) + Integral(4*a*b*c**2*asinh(c*x)/x, x) + Integral(2
*a*b*c**4*x*asinh(c*x), x))
```

### 3.214.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3} dx = \int \frac{(c^2dx^2+d)^2(b \operatorname{arsinh}(cx)+a)^2}{x^3} dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*a^2*c^4*d^2*x^2 + 2*a^2*c^2*d^2*log(x) - a*b*d^2*(sqrt(c^2*x^2 + 1)*c/
x + arcsinh(c*x)/x^2) - 1/2*a^2*d^2/x^2 + integrate(b^2*c^4*d^2*x*log(c*x
+ sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*
b^2*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 4*a*b*c^2*d^2*log(c*x + sqr
t(c^2*x^2 + 1))/x + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

### 3.214.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^3} dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^2}{x^3} dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3, x)
```

# 3.215 $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$



## 3.215.1 Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3x^2} + \frac{8}{3}c^4d^2x(a+b\text{arcsinh}(cx))^2 - \frac{4c^2d^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3x} - \frac{d^2(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3x^3} - \frac{22}{3}bc^3d^2(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) - \frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) + \frac{11}{3}b^2c^3d^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)$$

output
```
-1/3*b^2*c^2*d^2/x+2*b^2*c^4*d^2*x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcs
inh(c*x))/x^2+8/3*c^4*d^2*x*(a+b*arcsinh(c*x))^2-4/3*c^2*d^2*(c^2*x^2+1)*(
a+b*arcsinh(c*x))^2/x-1/3*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^3-22/3*
b*c^3*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*c^3*d
^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+11/3*b^2*c^3*d^2*polylog(2,c*x+(c^2*x
^2+1)^(1/2))-5/3*b*c^3*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.215.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.44

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^4}\,dx$$
$$= \frac{d^2\left(-a^2-6a^2c^2x^2-b^2c^2x^2+3a^2c^4x^4+6b^2c^4x^4-abcx\sqrt{1+c^2x^2}-6abc^3x^3\sqrt{1+c^2x^2}-2ab\text{arcsinh}(cx)\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
(d^2*(-a^2 - 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 3*a^2*c^4*x^4 + 6*b^2*c^4*x^4 -
 a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSi
nh[c*x] - 12*a*b*c^2*x^2*ArcSinh[c*x] + 6*a*b*c^4*x^4*ArcSinh[c*x] - b^2*c
*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x] - b^2*ArcSinh[c*x]^2 - 6*b^2*c^2*x^2*ArcSinh[c*x]^2 + 3*b^2*c^4*x^4
*ArcSinh[c*x]^2 - 11*a*b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] + 11*b^2*c^3*x
^3*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 11*b^2*c^3*x^3*ArcSinh[c*x]*L
og[1 + E^(-ArcSinh[c*x])] + 11*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])]
- 11*b^2*c^3*x^3*PolyLog[2, E^(-ArcSinh[c*x])]))/(3*x^3)
```

### 3.215.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.40, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6222, 27, 6222, 244, 2009, 6187, 6213, 24, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2 (a+b\text{arcsinh}(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \; 6222$$

$$\frac{2}{3}bcd^2\int \frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}dx+\frac{4}{3}c^2d\int \frac{d(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow 27$$

$$\frac{4}{3}c^2d^2\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x^2}dx+\frac{2}{3}bcd^2\int \frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{x^3}dx-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow 6222$$

$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bc\int \frac{c^2x^2+1}{x^2}dx-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}\right)+\frac{4}{3}c^2d^2\left(2bc\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+2c^2\int (a+b\text{arcsinh}(cx))^2dx-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow 244$$

$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bc\int \left(c^2+\frac{1}{x^2}\right)dx-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}\right)+\frac{4}{3}c^2d^2\left(2bc\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+2c^2\int (a+b\text{arcsinh}(cx))^2dx-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow 2009$$

$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(c^2x-\frac{1}{x}\right)\right)+\frac{4}{3}c^2d^2\left(2bc\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+2c^2\int (a+b\text{arcsinh}(cx))^2dx-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow 6187$$

$$\frac{4}{3}c^2d^2\left(2c^2\left(x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+2bc\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-(c\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(c^2x-\frac{1}{x}\right)\right)-\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

---

3.215. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$\big\downarrow$ 6213

$$\frac{4}{3}c^2d^2\left(2c^2\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)+2bc\int\frac{\sqrt{c^2x^2+1}(a+b\text{arc}}{x}\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(c^2x-\frac{1}{x}\right)\right)-$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\big\downarrow$ 24

$$\frac{4}{3}c^2d^2\left(2bc\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+2c^2\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}\right.\right.\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(c^2x-\frac{1}{x}\right)\right)-$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\big\downarrow$ 6221

$$\frac{4}{3}c^2d^2\left(2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int 1dx+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+2c^2\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int 1dx+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(c}{2x^2}\right.$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\big\downarrow$ 24

$$\frac{4}{3}c^2d^2\left(2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\right.\right.\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)-\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{2x^2}+\right.$$
$$\frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\big\downarrow$ 6231

3.215. $\int\frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$\frac{4}{3}c^2d^2\left(2bc\left(\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2\left(x(a+b\text{arcsinh}(cx))\right.\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)-\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsi}}{2x^2}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}c^2d^2\left(2bc\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2\left(\right.\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)-\frac{(c^2x}{}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{26}$$

$$\frac{4}{3}c^2d^2\left(2bc\left(i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)+2c^2\left(\right.\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-bcx\right)-\frac{(c^2x}{}\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{4670}$$

$$\frac{4}{3}c^2d^2\left(2bc\left(i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcs}}\right.\right.\right.\right.$$
$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcs}}\right.\right.\right.\right.$$
$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2715}$$

3.215. $\int \frac{\left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$\frac{4}{3}c^2d^2\left(2bc\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(\right.\right.\right.$$

$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}\right.\right.\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2838}$$

$$\frac{4}{3}c^2d^2\left(2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx}\right.\right.\right.\right.$$

$$\frac{2}{3}bcd^2\left(\frac{3}{2}c^2\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c}\right.\right.\right.\right.$$

$$\frac{d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{3x^3}$$

input
```
Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3 + (4*c^2*d^2*(-(((1
+ c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x) + 2*c^2*(x*(a + b*ArcSinh[c*x])^2 -
2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2)) + 2*b*c
*(-(b*c*x) + Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + I*((2*I)*(a + b*ArcS
inh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*
PolyLog[2, E^ArcSinh[c*x]]))))/3 + (2*b*c*d^2*((b*c*(-x^(-1) + c^2*x))/2 -
 ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (3*c^2*(-(b*c*x) + S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + I*((2*I)*(a + b*ArcSinh[c*x])*ArcT
anh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^A
rcSinh[c*x]])))/2))/3
```

### 3.215.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.215. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.215. $\int \frac{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.215.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| derivativedivides | $c^3\Bigg(d^2a^2\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b^2\operatorname{arcsinh}(cx)^2cx-2d^2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2d^2$ |
| default | $c^3\Bigg(d^2a^2\left(cx-\frac{1}{3c^3x^3}-\frac{2}{cx}\right)+d^2b^2\operatorname{arcsinh}(cx)^2cx-2d^2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2d^2$ |
| parts | $d^2a^2\left(c^4x-\frac{2c^2}{x}-\frac{1}{3x^3}\right)+d^2b^2c^4\operatorname{arcsinh}(cx)^2x-2d^2b^2c^3\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)+2b^2c$ |

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(d^2*a^2*(c*x-1/3/c^3/x^3-2/c/x)+d^2*b^2*arcsinh(c*x)^2*c*x-2*d^2*b^2*
arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*d^2*b^2*c*x-2*d^2*b^2*arcsinh(c*x)^2/c/x-
1/3*d^2*b^2/c^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-1/3*d^2*b^2/c^3/x^3*arc
sinh(c*x)^2-1/3*d^2*b^2/c/x-11/3*d^2*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)
^(1/2))-11/3*d^2*b^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+11/3*d^2*b^2*arcsin
h(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+11/3*d^2*b^2*polylog(2,c*x+(c^2*x^2+1)^
(1/2))+2*d^2*a*b*(arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-2*arcsinh(c*x)
/c/x-(c^2*x^2+1)^(1/2)-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)-11/6*arctanh(1/(c^2*x
^2+1)^(1/2))))
```

## 3.215.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^4, x)
```

## 3.215.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^4}\,dx = d^2\left(\int a^2c^4\,dx + \int \frac{a^2}{x^4}\,dx + \int \frac{2a^2c^2}{x^2}\,dx + \int b^2c^4\,\text{asinh}^2\,(cx)\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{x^4}\,dx + \int 2abc^4\,\text{asinh}\,(cx)\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{x^4}\,dx + \int \frac{2b^2c^2\,\text{asinh}^2\,(cx)}{x^2}\,dx + \int \frac{4abc^2\,\text{asinh}\,(cx)}{x^2}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**4,x)
```

output
```
d**2*(Integral(a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(2*a**2*c*
*2/x**2, x) + Integral(b**2*c**4*asinh(c*x)**2, x) + Integral(b**2*asinh(c
*x)**2/x**4, x) + Integral(2*a*b*c**4*asinh(c*x), x) + Integral(2*a*b*asin
h(c*x)/x**4, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x**2, x) + Integral(4
*a*b*c**2*asinh(c*x)/x**2, x))
```

### 3.215.7 Maxima [F]

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))^2}{x^4} \, dx = \int \frac{(c^2dx^2 + d)^2 (b \operatorname{arsinh}(cx) + a)^2}{x^4} \, dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
b^2*c^4*d^2*x*arcsinh(c*x)^2 + 2*b^2*c^4*d^2*(x - sqrt(c^2*x^2 + 1)*arcsin
h(c*x)/c) + a^2*c^4*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c
^3*d^2 - 4*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d^2 + 1/3*((
c^2*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)
*a*b*d^2 - 2*a^2*c^2*d^2/x - 1/3*a^2*d^2/x^3 - 1/3*(6*b^2*c^2*d^2*x^2 + b^
2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + integrate(2/3*(6*b^2*c^5*d^2*x
^4 + 7*b^2*c^3*d^2*x^2 + b^2*c*d^2 + (6*b^2*c^4*d^2*x^3 + b^2*c^2*d^2*x)*s
qrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5
 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

### 3.215.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^2 (a + b\text{arcsinh}(cx))^2}{x^4} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}{x^4} \, dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^2}{x^4} \, dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4, x)
```

# 3.216 $\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$



## 3.216.1 Optimal result

Integrand size = 26, antiderivative size = 465

$$\begin{aligned}\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx &= \frac{100976b^2d^3x}{4002075c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}\\&+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{182b^2c^4d^3x^9}{29403}+\frac{2b^2c^6d^3x^{11}}{1331}-\frac{256bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{17325c^5}\\&+\frac{128bd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{17325c^3}-\frac{32bd^3x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{5775c}\\&-\frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{693c^5}+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{1155c^5}\\&-\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{1617c^5}+\frac{8bd^3(1+c^2x^2)^{9/2}(a+b\text{arcsinh}(cx))}{297c^5}\\&-\frac{2bd^3(1+c^2x^2)^{11/2}(a+b\text{arcsinh}(cx))}{121c^5}+\frac{16d^3x^5(a+b\text{arcsinh}(cx))^2}{1155}\\&+\frac{8}{231}d^3x^5(1+c^2x^2)(a+b\text{arcsinh}(cx))^2+\frac{2}{33}d^3x^5(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2+\frac{1}{11}d^3x^5(1+c^2x^2)^3(a+ba\end{aligned}$$

output
```
100976/4002075*b^2*d^3*x/c^4-50488/12006225*b^2*d^3*x^3/c^2+12622/6670125*
b^2*d^3*x^5+9410/1120581*b^2*c^2*d^3*x^7+182/29403*b^2*c^4*d^3*x^9+2/1331*
b^2*c^6*d^3*x^11-16/693*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/1
155*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/1617*b*d^3*(c^2*x^2+1
)^(7/2)*(a+b*arcsinh(c*x))/c^5+8/297*b*d^3*(c^2*x^2+1)^(9/2)*(a+b*arcsinh(
c*x))/c^5-2/121*b*d^3*(c^2*x^2+1)^(11/2)*(a+b*arcsinh(c*x))/c^5+16/1155*d^
3*x^5*(a+b*arcsinh(c*x))^2+8/231*d^3*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+
2/33*d^3*x^5*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/11*d^3*x^5*(c^2*x^2+1)^3
*(a+b*arcsinh(c*x))^2-256/17325*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
/c^5+128/17325*b*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/5775*
b*d^3*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.216.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.64

$$\int x^4\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx = \frac{d^3\left(12006225a^2c^5x^5(231+495c^2x^2+385c^4x^4+105c^6x^6)-6930ab\sqrt{1+c^2x^2}(50488-25244c^2x^2+1893\right.}{}$$

input
```
Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*(12006225*a^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)
 - 6930*a*b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 117
625*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10) + 2*b^2*c*x*(174940920 - 2
9156820*c^2*x^2 + 13120569*c^4*x^4 + 58224375*c^6*x^6 + 42917875*c^8*x^8 +
 10418625*c^10*x^10) - 6930*b*(-3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^
4*x^4 + 105*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*
c^4*x^4 + 117625*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10))*ArcSinh[c*x]
 + 12006225*b^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)*Ar
cSinh[c*x]^2))/(13867189875*c^5)
```

### 3.216.3 Rubi [A] (verified)

Time = 3.82 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.49, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {6223, 27, 6219, 27, 1467, 2009, 6223, 6219, 27, 1467, 2009, 6223, 6191, 6219, 27, 2009, 6227, 15, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(c^2 d x^2+d\right)^3 (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2}{11} b c d^3 \int x^5 \left(c^2 x^2+1\right)^{5/2} (a+b\text{arcsinh}(cx))dx+\frac{6}{11} d \int d^2 x^4 \left(c^2 x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2dx+\frac{1}{11} d^3 x^5 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{11} b c d^3 \int x^5 \left(c^2 x^2+1\right)^{5/2} (a+b\text{arcsinh}(cx))dx+\frac{6}{11} d^3 \int x^4 \left(c^2 x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2dx+\frac{1}{11} d^3 x^5 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6219}$$

$$\frac{6}{11} d^3 \int x^4 \left(c^2 x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2dx-\frac{2}{11} b c d^3 \left(-b c \int \frac{\left(c^2 x^2+1\right)^3 \left(63 c^4 x^4-28 c^2 x^2+8\right)}{693 c^6}dx+\frac{\left(c^2 x^2+1\right)^{11/2} (a+b\text{arcsinh}(cx))}{11 c^6}-\frac{2 \left(c^2 x^2+1\right)^{9/2} (a+}{9 c^6}\right.$$
$$\frac{1}{11} d^3 x^5 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{11} d^3 \int x^4 \left(c^2 x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2dx-\frac{2}{11} b c d^3 \left(-\frac{b \int \left(c^2 x^2+1\right)^3 \left(63 c^4 x^4-28 c^2 x^2+8\right) dx}{693 c^5}+\frac{\left(c^2 x^2+1\right)^{11/2} (a+b\text{arcsinh}(cx))}{11 c^6}-\frac{2 \left(c^2 x^2+1\right)^{9/2} (a+ba}{9 c^6}\right.$$
$$\frac{1}{11} d^3 x^5 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{1467}$$

$$\frac{6}{11}d^3\int x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx-$$
$$\frac{2}{11}bcd^3\Bigg(-\frac{b\int\left(63c^{10}x^{10}+161c^8x^8+113c^6x^6+3c^4x^4-4c^2x^2+8\right)dx}{693c^5}+\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2(c$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 2009

$$\frac{6}{11}d^3\int x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx+\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\Bigg(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}$$

$\downarrow$ 6223

$$\frac{6}{11}d^3\Bigg(-\frac{2}{9}bc\int x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{9}\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{9}x^5\left(c^2x^2+1\right)$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\Bigg(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}$$

$\downarrow$ 6219

$$\frac{6}{11}d^3\Bigg(\frac{4}{9}\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{9}bc\Bigg(-bc\int\frac{\left(c^2x^2+1\right)^2\left(35c^4x^4-20c^2x^2+8\right)}{315c^6}dx+\frac{\left(c^2x^2+1\right)}{}$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\Bigg(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}$$

$\downarrow$ 27

$$\frac{6}{11}d^3\Bigg(\frac{4}{9}\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{9}bc\Bigg(-\frac{b\int\left(c^2x^2+1\right)^2\left(35c^4x^4-20c^2x^2+8\right)dx}{315c^5}+\frac{\left(c^2x^2+1\right)^9}{}$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\Bigg(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}$$

$\downarrow$ 1467

3.216. $\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{6}{11}d^3\left(\frac{4}{9}\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{9}bc\left(-\frac{b\int\left(35c^8x^8+50c^6x^6+3c^4x^4-4c^2x^2+8\right)dx}{315c^5}+\frac{(c^2x^2 -$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 2009

$$\frac{6}{11}d^3\left(\frac{4}{9}\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{9}x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-\frac{2}{9}bc\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+}{9c^6}\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 6223

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(-\frac{2}{7}bc\int x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{7}\int x^4(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}x^5\left(c^2x^2+1\right)(a+b\text{arcs}\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 6191

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{7}bc\int x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 6219

3.216. $\int x^4\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{7}bc\left(-bc\int\frac{15c^6x^6+3c^4x^4-4c^2x^2}{105c^6}\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 27

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{7}bc\left(-\frac{b\int\left(15c^6x^6+3c^4x^4-4c^2x^2+\right.}{105c^5}\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 2009

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{7}x^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 6227

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{5c^2}-\frac{b\int x^4dx}{5c}+\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}}{5c^2}\right.\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx}{7c^6}\right.$$

$\downarrow$ 15

3.216. $\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{5c^2}+\frac{x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{bx^5}{25c}\right.\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx\right.}{7c^6}\right.$$

$\Big\downarrow$ 6227

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}\right)}{5c^2}\right.\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx\right.}{7c^6}\right.$$

$\Big\downarrow$ 15

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}+\right.\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx\right.}{7c^6}\right.$$

$\Big\downarrow$ 6213

3.216. $\int x^4(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2-\frac{2}{5}bc\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}\right.}{5c^2}\right.\right.\right.\right.$$

$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx\right.}{7c^6}\right.$$

$\downarrow$ 24

$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2+$$

$$\frac{6}{11}d^3\left(\frac{1}{9}x^5\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{9}\left(\frac{1}{7}x^5\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{7}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))^2\right.\right.\right.$$

$$\frac{2}{11}bcd^3\left(\frac{\left(c^2x^2+1\right)^{11/2}(a+b\text{arcsinh}(cx))}{11c^6}-\frac{2\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^6}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx\right.}{7c^6}\right.$$

input
```
Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

3.216. $\int x^4\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(d^3*x^5*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/11 - (2*b*c*d^3*(-1/693*(
b*(8*x - (4*c^2*x^3)/3 + (3*c^4*x^5)/5 + (113*c^6*x^7)/7 + (161*c^8*x^9)/9
 + (63*c^10*x^11)/11))/c^5 + ((1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7
*c^6) - (2*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^6) + ((1 + c^2*x
^2)^(11/2)*(a + b*ArcSinh[c*x]))/(11*c^6)))/11 + (6*d^3*((x^5*(1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x])^2)/9 - (2*b*c*(-1/315*(b*(8*x - (4*c^2*x^3)/3 + (
3*c^4*x^5)/5 + (50*c^6*x^7)/7 + (35*c^8*x^9)/9))/c^5 + ((1 + c^2*x^2)^(5/2
)*(a + b*ArcSinh[c*x]))/(5*c^6) - (2*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*
x]))/(7*c^6) + ((1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^6)))/9 + (4
*((x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/7 - (2*b*c*(-1/105*(b*(8*x -
(4*c^2*x^3)/3 + (3*c^4*x^5)/5 + (15*c^6*x^7)/7))/c^5 + ((1 + c^2*x^2)^(3/2
)*(a + b*ArcSinh[c*x]))/(3*c^6) - (2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x]))/(5*c^6) + ((1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^6)))/7 + (2
*((x^5*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*(-1/25*(b*x^5)/c + (x^4*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (4*(-1/9*(b*x^3)/c + (x^2*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) - (2*(-((b*x)/c) + (Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/c^2))/(3*c^2)))/(5*c^2)))/5))/7))/9))/11
```

### 3.216.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.216. $\int x^4 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx$

### 3.216.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| parts | $d^3a^2\left(\frac{1}{11}c^6x^{11}+\frac{1}{3}c^4x^9+\frac{3}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3(c^2x^2+1)^4}{11}-\frac{\operatorname{arcsinh}(cx)^2cx(c^2x^2+1)^4}{33}+\frac{16}{}\right.}{}$ |
| derivativedivides | $\frac{d^3a^2\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3(c^2x^2+1)^4}{11}-\frac{\operatorname{arcsinh}(cx)^2cx(c^2x^2+1)^4}{33}+\frac{16\operatorname{arcsinh}(cx)^2x}{1155}\right.}{}$ |
| default | $\frac{d^3a^2\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3(c^2x^2+1)^4}{11}-\frac{\operatorname{arcsinh}(cx)^2cx(c^2x^2+1)^4}{33}+\frac{16\operatorname{arcsinh}(cx)^2x}{1155}\right.}{}$ |

input
```
int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/11*c^6*x^11+1/3*c^4*x^9+3/7*c^2*x^7+1/5*x^5)+d^3*b^2/c^5*(1/11*
arcsinh(c*x)^2*c^3*x^3*(c^2*x^2+1)^4-1/33*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^4
+16/1155*arcsinh(c*x)^2*x*c+1/231*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+2/385*a
rcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+8/1155*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/16
17*arcsinh(c*x)*(c^2*x^2+1)^(7/2)-428/323433*c*x*(c^2*x^2+1)^4-16/3465*arc
sinh(c*x)*(c^2*x^2+1)^(3/2)-606416/13867189875*c*x*(c^2*x^2+1)-5487704/462
2396625*c*x*(c^2*x^2+1)^2-148174/110937519*c*x*(c^2*x^2+1)^3-4/1925*arcsin
h(c*x)*(c^2*x^2+1)^(5/2)-2/121*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(9/2)+3829
86368/13867189875*c*x-32/1155*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+34/3267*arcsi
nh(c*x)*(c^2*x^2+1)^(9/2)+2/1331*c*x*(c^2*x^2+1)^5)+2*d^3*a*b/c^5*(1/11*ar
csinh(c*x)*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+1/5
*arcsinh(c*x)*c^5*x^5-91/3267*c^8*x^8*(c^2*x^2+1)^(1/2)-4705/160083*c^6*x^
6*(c^2*x^2+1)^(1/2)-6311/1334025*c^4*x^4*(c^2*x^2+1)^(1/2)+25244/4002075*c
^2*x^2*(c^2*x^2+1)^(1/2)-50488/4002075*(c^2*x^2+1)^(1/2)-1/121*c^10*x^10*(
c^2*x^2+1)^(1/2))
```

### 3.216.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.95

$$\int x^4(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$=\frac{10418625\,(121\,a^2+2\,b^2)c^{11}d^3x^{11}+471625\,(9801\,a^2+182\,b^2)c^9d^3x^9+12375\,(480249\,a^2+9410\,b^2)c^7d^3x^7}{}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/13867189875*(10418625*(121*a^2 + 2*b^2)*c^11*d^3*x^11 + 471625*(9801*a^2
 + 182*b^2)*c^9*d^3*x^9 + 12375*(480249*a^2 + 9410*b^2)*c^7*d^3*x^7 + 2079
*(1334025*a^2 + 12622*b^2)*c^5*d^3*x^5 - 58313640*b^2*c^3*d^3*x^3 + 349881
840*b^2*c*d^3*x + 12006225*(105*b^2*c^11*d^3*x^11 + 385*b^2*c^9*d^3*x^9 +
495*b^2*c^7*d^3*x^7 + 231*b^2*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2
+ 6930*(363825*a*b*c^11*d^3*x^11 + 1334025*a*b*c^9*d^3*x^9 + 1715175*a*b*c
^7*d^3*x^7 + 800415*a*b*c^5*d^3*x^5 - (33075*b^2*c^10*d^3*x^10 + 111475*b^
2*c^8*d^3*x^8 + 117625*b^2*c^6*d^3*x^6 + 18933*b^2*c^4*d^3*x^4 - 25244*b^2
*c^2*d^3*x^2 + 50488*b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
1)) - 6930*(33075*a*b*c^10*d^3*x^10 + 111475*a*b*c^8*d^3*x^8 + 117625*a*b*
c^6*d^3*x^6 + 18933*a*b*c^4*d^3*x^4 - 25244*a*b*c^2*d^3*x^2 + 50488*a*b*d^
3)*sqrt(c^2*x^2 + 1))/c^5
```

### 3.216.6 Sympy [A] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.51

$$\int x^4 \left(d + c^2 d x^2\right)^3 (a + b\text{arcsinh}(cx))^2 \, dx$$

$$= \begin{cases} \frac{a^2c^6d^3x^{11}}{11} + \frac{a^2c^4d^3x^9}{3} + \frac{3a^2c^2d^3x^7}{7} + \frac{a^2d^3x^5}{5} + \frac{2abc^6d^3x^{11}\operatorname{asinh}(cx)}{11} - \frac{2abc^5d^3x^{10}\sqrt{c^2x^2+1}}{121} + \frac{2abc^4d^3x^9\operatorname{asinh}(cx)}{3} - \frac{182abc^3 \\ \frac{a^2d^3x^5}{5} \end{cases}$$

input
```
integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**6*d**3*x**11/11 + a**2*c**4*d**3*x**9/3 + 3*a**2*c**2*d
**3*x**7/7 + a**2*d**3*x**5/5 + 2*a*b*c**6*d**3*x**11*asinh(c*x)/11 - 2*a*
b*c**5*d**3*x**10*sqrt(c**2*x**2 + 1)/121 + 2*a*b*c**4*d**3*x**9*asinh(c*x
)/3 - 182*a*b*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)/3267 + 6*a*b*c**2*d**3*x*
*7*asinh(c*x)/7 - 9410*a*b*c*d**3*x**6*sqrt(c**2*x**2 + 1)/160083 + 2*a*b*
d**3*x**5*asinh(c*x)/5 - 12622*a*b*d**3*x**4*sqrt(c**2*x**2 + 1)/(1334025*
c) + 50488*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(4002075*c**3) - 100976*a*b*d
**3*sqrt(c**2*x**2 + 1)/(4002075*c**5) + b**2*c**6*d**3*x**11*asinh(c*x)**
2/11 + 2*b**2*c**6*d**3*x**11/1331 - 2*b**2*c**5*d**3*x**10*sqrt(c**2*x**2
 + 1)*asinh(c*x)/121 + b**2*c**4*d**3*x**9*asinh(c*x)**2/3 + 182*b**2*c**4
*d**3*x**9/29403 - 182*b**2*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)*asinh(c*x)/
3267 + 3*b**2*c**2*d**3*x**7*asinh(c*x)**2/7 + 9410*b**2*c**2*d**3*x**7/11
20581 - 9410*b**2*c*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/160083 + b**2
*d**3*x**5*asinh(c*x)**2/5 + 12622*b**2*d**3*x**5/6670125 - 12622*b**2*d**
3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1334025*c) - 50488*b**2*d**3*x**3/(
12006225*c**2) + 50488*b**2*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(4002
075*c**3) + 100976*b**2*d**3*x/(4002075*c**4) - 100976*b**2*d**3*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(4002075*c**5), Ne(c, 0)), (a**2*d**3*x**5/5, True))
```

### 3.216.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. 2(413) = 826.

Time = 0.23 (sec) , antiderivative size = 1109, normalized size of antiderivative = 2.38

$$\int x^4 \left(d + c^2 d x^2\right)^3 (a + b\operatorname{arcsinh}(cx))^2\, dx = \text{Too large to display}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/11*b^2*c^6*d^3*x^11*arcsinh(c*x)^2 + 1/11*a^2*c^6*d^3*x^11 + 1/3*b^2*c^4
*d^3*x^9*arcsinh(c*x)^2 + 1/3*a^2*c^4*d^3*x^9 + 3/7*b^2*c^2*d^3*x^7*arcsin
h(c*x)^2 + 3/7*a^2*c^2*d^3*x^7 + 2/7623*(693*x^11*arcsinh(c*x) - (63*sqrt(
c^2*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1
)*x^6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10
- 256*sqrt(c^2*x^2 + 1)/c^12)*c)*a*b*c^6*d^3 - 2/26413695*(3465*(63*sqrt(c
^2*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)
*x^6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 -
 256*sqrt(c^2*x^2 + 1)/c^12)*c*arcsinh(c*x) - (19845*c^10*x^11 - 26950*c^8
*x^9 + 39600*c^6*x^7 - 66528*c^4*x^5 + 147840*c^2*x^3 - 887040*x)/c^10)*b^
2*c^6*d^3 + 1/5*b^2*d^3*x^5*arcsinh(c*x)^2 + 2/945*(315*x^9*arcsinh(c*x) -
 (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^
2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/
c^10)*c)*a*b*c^4*d^3 - 2/297675*(315*(35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sq
rt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 +
 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*arcsinh(c*x) - (1225*c^8*x^9 -
 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 + 40320*x)/c^8)*b^2*c^4*d^3 +
1/5*a^2*d^3*x^5 + 6/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^
2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^
2*x^2 + 1)/c^8)*c)*a*b*c^2*d^3 - 2/8575*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c...
```

### 3.216.8 Giac [F(-2)]

Exception generated.

$$\int x^4 (d + c^2 d x^2)^3 (a + b \operatorname{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.216.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(d+c^2 d x^2\right)^3 (a+b \text{arcsinh}(c x))^2 \, dx=\int x^4 \, (a+b \, \text{asinh}(c \, x))^2 \, \left(d \, c^2 \, x^2+d\right)^3 \, dx$$

input
```
int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

output
```
int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

# 3.217 $\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$



## 3.217.1 Optimal result

Integrand size = 26, antiderivative size = 376

$$\begin{aligned}\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = &-\frac{79b^2d^3x^2}{5120c^2}+\frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}\\ &+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}\\ &+\frac{79bd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2560c^3}\\ &-\frac{79bd^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3840c}\\ &-\frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\\ &-\frac{1}{32}bcd^3x^5(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\\ &-\frac{1}{50}bcd^3x^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\\ &-\frac{79d^3(a+b\operatorname{arcsinh}(cx))^2}{5120c^4}\\ &+\frac{1}{40}d^3x^4(a+b\operatorname{arcsinh}(cx))^2\\ &+\frac{1}{20}d^3x^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2\\ &+\frac{3}{40}d^3x^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2\\ &+\frac{1}{10}d^3x^4(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2\end{aligned}$$

output
```
-79/5120*b^2*d^3*x^2/c^2+79/15360*b^2*d^3*x^4+401/28800*b^2*c^2*d^3*x^6+57
/6400*b^2*c^4*d^3*x^8+1/500*b^2*c^6*d^3*x^10-1/32*b*c*d^3*x^5*(c^2*x^2+1)^
(3/2)*(a+b*arcsinh(c*x))-1/50*b*c*d^3*x^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c
*x))-79/5120*d^3*(a+b*arcsinh(c*x))^2/c^4+1/40*d^3*x^4*(a+b*arcsinh(c*x))^
2+1/20*d^3*x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+3/40*d^3*x^4*(c^2*x^2+1)^2
*(a+b*arcsinh(c*x))^2+1/10*d^3*x^4*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+79/2
560*b*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-79/3840*b*d^3*x^3*(a+
b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-31/960*b*c*d^3*x^5*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)
```

### 3.217.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.76

$$\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{d^3\left(cx\left(28800a^2c^3x^3(10+20c^2x^2+15c^4x^4+4c^6x^6)-30ab\sqrt{1+c^2x^2}(-1185+790c^2x^2+3208c^4x^4+273\right.\right.}{}$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*(c*x*(28800*a^2*c^3*x^3*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) -
30*a*b*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^
6 + 768*c^8*x^8) + b^2*c*x*(-17775 + 5925*c^2*x^2 + 16040*c^4*x^4 + 10260*
c^6*x^6 + 2304*c^8*x^8)) + 30*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^
2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8*x^8)) + 15*a*(-79 + 1280*c^4
*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10))*ArcSinh[c*x] + 225*b^
2*(-79 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10)*ArcSi
nh[c*x]^2))/(1152000*c^4)
```

### 3.217.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 899 vs. $2(376) = 752$.

Time = 3.90 (sec) , antiderivative size = 899, normalized size of antiderivative = 2.39, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {6223, 27, 6223, 243, 49, 2009, 6223, 244, 2009, 6191, 6221, 15, 6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.217. $\int x^3(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$

$$\int x^3 \left(c^2 d x^2+d\right)^3 (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{1}{5} b c d^3 \int x^4 \left(c^2 x^2+1\right)^{5/2} (a+b\text{arcsinh}(cx))dx+\frac{3}{5} d \int d^2 x^3 \left(c^2 x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2dx+\frac{1}{10} d^3 x^4 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5} b c d^3 \int x^4 \left(c^2 x^2+1\right)^{5/2} (a+b\text{arcsinh}(cx))dx+\frac{3}{5} d^3 \int x^3 \left(c^2 x^2+1\right)^2 (a+b\text{arcsinh}(cx))^2dx+\frac{1}{10} d^3 x^4 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6223}$$

$$-\frac{1}{5} b c d^3 \left(\frac{1}{2} \int x^4 \left(c^2 x^2+1\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{1}{10} b c \int x^5 \left(c^2 x^2+1\right)^2 dx+\frac{1}{10} x^5 \left(c^2 x^2+1\right)^{5/2} (a+b\text{arcsin}\right.$$
$$\frac{3}{5} d^3 \left(-\frac{1}{4} b c \int x^4 \left(c^2 x^2+1\right)^{3/2} (a+b\text{arcsinh}(cx))dx+\frac{1}{2} \int x^3 \left(c^2 x^2+1\right) (a+b\text{arcsinh}(cx))^2dx+\frac{1}{8} x^4 \left(c^2 x^2+1\right)^2\right.$$
$$\frac{1}{10} d^3 x^4 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{5} b c d^3 \left(\frac{1}{2} \int x^4 \left(c^2 x^2+1\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{1}{20} b c \int x^4 \left(c^2 x^2+1\right)^2 dx^2+\frac{1}{10} x^5 \left(c^2 x^2+1\right)^{5/2} (a+b\text{arcsi}\right.$$
$$\frac{3}{5} d^3 \left(-\frac{1}{4} b c \int x^4 \left(c^2 x^2+1\right)^{3/2} (a+b\text{arcsinh}(cx))dx+\frac{1}{2} \int x^3 \left(c^2 x^2+1\right) (a+b\text{arcsinh}(cx))^2dx+\frac{1}{8} x^4 \left(c^2 x^2+1\right)^2\right.$$
$$\frac{1}{10} d^3 x^4 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{49}$$

$$\frac{3}{5} d^3 \left(-\frac{1}{4} b c \int x^4 \left(c^2 x^2+1\right)^{3/2} (a+b\text{arcsinh}(cx))dx+\frac{1}{2} \int x^3 \left(c^2 x^2+1\right) (a+b\text{arcsinh}(cx))^2dx+\frac{1}{8} x^4 \left(c^2 x^2+1\right)^2\right.$$
$$\frac{1}{5} b c d^3 \left(\frac{1}{2} \int x^4 \left(c^2 x^2+1\right)^{3/2} (a+b\text{arcsinh}(cx))dx-\frac{1}{20} b c \int \left(c^4 x^8+2 c^2 x^6+x^4\right) dx^2+\frac{1}{10} x^5 \left(c^2 x^2+1\right)^{5/2} (a+b\text{a}\right.$$
$$\frac{1}{10} d^3 x^4 \left(c^2 x^2+1\right)^3 (a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

3.217. $\int x^3 (d+c^2 d x^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$

$$\frac{3}{5}d^3\left(-\frac{1}{4}bc\int x^4\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{2}\int x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{8}x^4\left(c^2x^2+1\right)^2$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\int x^4\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{10}x^5\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}+\frac{c^2x^8}{2}\right.\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 6223

$$\frac{3}{5}d^3\left(-\frac{1}{4}bc\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{8}bc\int x^5\left(c^2x^2+1\right)dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(c\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{8}bc\int x^5\left(c^2x^2+1\right)dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx\right.\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 244

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(-\frac{1}{3}bc\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}\int x^3(a+b\text{arcsinh}(cx))^2dx+\frac{1}{6}x^4\left(c^2x^2+1\right)(a+b\text{arcsin}\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{8}bc\int\left(c^2x^7+x^5\right)dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right.\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 2009

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(-\frac{1}{3}bc\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}\int x^3(a+b\text{arcsinh}(cx))^2dx+\frac{1}{6}x^4\left(c^2x^2+1\right)(a+b\text{arcsin}\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{8}bc\left(\frac{c^2x^8}{8}+\frac{x^6}{6}\right)\right)+\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 6191

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{3}bc\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))d\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{8}bc\left(\frac{c^2x^8}{8}+\frac{x^6}{6}\right)\right)+\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\big\downarrow$ 6221

3.217. $\int x^3\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{6}b\right.\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{1}{6}bc\int x^5dx+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{8}x^5\left(c^2x^2+1\right)\right.\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \; 15$$

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}x\right.\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{36}bcx^6\right)+\frac{1}{8}x^5\left(c^2x^2+1\right)^{3/2}(a\right.\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \; 6227$$

$$\frac{3}{5}d^3\left(-\frac{1}{4}bc\left(\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}\right)+\frac{1}{6}x^5\sqrt{c^2x^2+1}(\right.\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}-\frac{b\int x^3dx}{4c}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}\right)+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a\right.\right.\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \; 15$$

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx^4}{16c}\right.\right.\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}x^5\sqrt{c^2x^2+1}(a+ba\right.\right.\right.$$
$$\frac{1}{10}d^3x^4\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \; 6227$$

3.217. $\int x^3\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{10}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2x^4-$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))x^5-\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}+\frac{c^2x^8}{2}+\frac{x^6}{3}\right)+\frac{1}{2}\left(\frac{1}{8}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(c\right.\right.$$

$$\frac{3}{5}d^3\left(\frac{1}{8}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2x^4-\frac{1}{4}bc\left(\frac{1}{8}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))x^5-\frac{1}{8}bc\left(\frac{c^2x^8}{8}+\frac{x^6}{6}\right)+\frac{3}{8}\left(\right.\right.\right.$$

$\downarrow$ 15

$$\frac{1}{10}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2x^4-$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))x^5-\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}+\frac{c^2x^8}{2}+\frac{x^6}{3}\right)+\frac{1}{2}\left(\frac{1}{8}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(c\right.\right.$$

$$\frac{3}{5}d^3\left(\frac{1}{8}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2x^4-\frac{1}{4}bc\left(\frac{1}{8}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))x^5-\frac{1}{8}bc\left(\frac{c^2x^8}{8}+\frac{x^6}{6}\right)+\frac{3}{8}\left(\right.\right.\right.$$

$\downarrow$ 6198

$$\frac{1}{10}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2x^4-$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))x^5-\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}+\frac{c^2x^8}{2}+\frac{x^6}{3}\right)+\frac{1}{2}\left(\frac{1}{8}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(c\right.\right.$$

$$\frac{3}{5}d^3\left(\frac{1}{8}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2x^4-\frac{1}{4}bc\left(\frac{1}{8}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))x^5-\frac{1}{8}bc\left(\frac{c^2x^8}{8}+\frac{x^6}{6}\right)+\frac{3}{8}\left(\right.\right.\right.$$

input
```
Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

3.217. $\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(d^3*x^4*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/10 - (b*c*d^3*(-1/20*(b*c
*(x^6/3 + (c^2*x^8)/2 + (c^4*x^10)/5)) + (x^5*(1 + c^2*x^2)^(5/2)*(a + b*A
rcSinh[c*x]))/10 + (-1/8*(b*c*(x^6/6 + (c^2*x^8)/8)) + (x^5*(1 + c^2*x^2)^
(3/2)*(a + b*ArcSinh[c*x]))/8 + (3*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/6 + (-1/16*(b*x^4)/c + (x^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(4*c^2) - (3*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3)))/(4*c^2))/6
))/8)/2))/5 + (3*d^3*((x^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/8 - (b*
c*(-1/8*(b*c*(x^6/6 + (c^2*x^8)/8)) + (x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/8 + (3*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x]))/6 + (-1/16*(b*x^4)/c + (x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
)/(4*c^2) - (3*(-1/4*(b*x^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
)/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/8))/4 + ((x^4*
(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/6 - (b*c*(-1/36*(b*c*x^6) + (x^5*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/6 + (-1/16*(b*x^4)/c + (x^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*c^2) - (3*(-1/4*(b*x^2)/c + (x*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3))
)/(4*c^2))/6))/3 + ((x^4*(a + b*ArcSinh[c*x])^2)/4 - (b*c*(-1/16*(b*x^4)/c
 + (x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*c^2) - (3*(-1/4*(b*x^2)
/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSin...
```

### 3.217.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

## 3.217. $\int x^3(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.217.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| parts | $d^3a^2\left(\frac{1}{10}c^6x^{10}+\frac{3}{8}c^4x^8+\frac{1}{2}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^4}{10}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{40}-\operatorname{arcs}\right.}{}$ |
| derivativedivides | $\frac{d^3a^2\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^4}{10}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{40}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x\right.}{50}\right.}{}$ |
| default | $\frac{d^3a^2\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2\left(c^2x^2+1\right)^4}{10}-\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{40}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x\right.}{50}\right.}{}$ |

input
```
int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/10*c^6*x^10+3/8*c^4*x^8+1/2*c^2*x^6+1/4*x^4)+d^3*b^2/c^4*(1/10*
arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)^4-1/40*arcsinh(c*x)^2*(c^2*x^2+1)^4-1/5
0*arcsinh(c*x)*c*x*(c^2*x^2+1)^(9/2)+7/800*arcsinh(c*x)*c*x*(c^2*x^2+1)^(7
/2)+49/4800*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+49/3840*arcsinh(c*x)*c*x*(c
^2*x^2+1)^(3/2)+49/2560*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+49/5120*arcsinh
(c*x)^2+1/500*(c^2*x^2+1)^5-7/6400*(c^2*x^2+1)^4-49/28800*(c^2*x^2+1)^3-49
/15360*(c^2*x^2+1)^2-49/5120*c^2*x^2-49/5120)+2*d^3*a*b/c^4*(1/10*arcsinh(
c*x)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+1/4*arcsi
nh(c*x)*c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(c^2*x^2+1
)^(1/2)-401/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^2+1)^(1/
2)+79/5120*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x))
```

### 3.217.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.13

$$\int x^3 \left(d + c^2 d x^2\right)^3 (a + b \operatorname{arcsinh}(cx))^2 \, dx$$
$$= \frac{2304 \left(50 a^2 + b^2\right) c^{10} d^3 x^{10} + 540 \left(800 a^2 + 19 b^2\right) c^8 d^3 x^8 + 40 \left(14400 a^2 + 401 b^2\right) c^6 d^3 x^6 + 75 \left(3840 a^2 + 79 b\right.}{}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/1152000*(2304*(50*a^2 + b^2)*c^10*d^3*x^10 + 540*(800*a^2 + 19*b^2)*c^8*
d^3*x^8 + 40*(14400*a^2 + 401*b^2)*c^6*d^3*x^6 + 75*(3840*a^2 + 79*b^2)*c^
4*d^3*x^4 - 17775*b^2*c^2*d^3*x^2 + 225*(512*b^2*c^10*d^3*x^10 + 1920*b^2*
c^8*d^3*x^8 + 2560*b^2*c^6*d^3*x^6 + 1280*b^2*c^4*d^3*x^4 - 79*b^2*d^3)*lo
g(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(7680*a*b*c^10*d^3*x^10 + 28800*a*b*c^8*
d^3*x^8 + 38400*a*b*c^6*d^3*x^6 + 19200*a*b*c^4*d^3*x^4 - 1185*a*b*d^3 - (
768*b^2*c^9*d^3*x^9 + 2736*b^2*c^7*d^3*x^7 + 3208*b^2*c^5*d^3*x^5 + 790*b^
2*c^3*d^3*x^3 - 1185*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^
2 + 1)) - 30*(768*a*b*c^9*d^3*x^9 + 2736*a*b*c^7*d^3*x^7 + 3208*a*b*c^5*d^
3*x^5 + 790*a*b*c^3*d^3*x^3 - 1185*a*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

### 3.217.6 Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.74

$$\int x^3 \left(d + c^2 d x^2\right)^3 (a + b \operatorname{arcsinh}(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2 c^6 d^3 x^{10}}{10} + \frac{3a^2 c^4 d^3 x^8}{8} + \frac{a^2 c^2 d^3 x^6}{2} + \frac{a^2 d^3 x^4}{4} + \frac{abc^6 d^3 x^{10} \operatorname{asinh}(cx)}{5} - \frac{abc^5 d^3 x^9 \sqrt{c^2 x^2 + 1}}{50} + \frac{3abc^4 d^3 x^8 \operatorname{asinh}(cx)}{4} - \frac{57abc^3 d^3 x^7}{80} \\ \frac{a^2 d^3 x^4}{4} \end{cases}$$

input
```
integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**6*d**3*x**10/10 + 3*a**2*c**4*d**3*x**8/8 + a**2*c**2*d
**3*x**6/2 + a**2*d**3*x**4/4 + a*b*c**6*d**3*x**10*asinh(c*x)/5 - a*b*c**
5*d**3*x**9*sqrt(c**2*x**2 + 1)/50 + 3*a*b*c**4*d**3*x**8*asinh(c*x)/4 - 5
7*a*b*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)/800 + a*b*c**2*d**3*x**6*asinh(c*
x) - 401*a*b*c*d**3*x**5*sqrt(c**2*x**2 + 1)/4800 + a*b*d**3*x**4*asinh(c*
x)/2 - 79*a*b*d**3*x**3*sqrt(c**2*x**2 + 1)/(3840*c) + 79*a*b*d**3*x*sqrt(
c**2*x**2 + 1)/(2560*c**3) - 79*a*b*d**3*asinh(c*x)/(2560*c**4) + b**2*c**
6*d**3*x**10*asinh(c*x)**2/10 + b**2*c**6*d**3*x**10/500 - b**2*c**5*d**3*
x**9*sqrt(c**2*x**2 + 1)*asinh(c*x)/50 + 3*b**2*c**4*d**3*x**8*asinh(c*x)*
*2/8 + 57*b**2*c**4*d**3*x**8/6400 - 57*b**2*c**3*d**3*x**7*sqrt(c**2*x**2
 + 1)*asinh(c*x)/800 + b**2*c**2*d**3*x**6*asinh(c*x)**2/2 + 401*b**2*c**2
*d**3*x**6/28800 - 401*b**2*c*d**3*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/480
0 + b**2*d**3*x**4*asinh(c*x)**2/4 + 79*b**2*d**3*x**4/15360 - 79*b**2*d**
3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3840*c) - 79*b**2*d**3*x**2/(5120*c
**2) + 79*b**2*d**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2560*c**3) - 79*b**2
*d**3*asinh(c*x)**2/(5120*c**4), Ne(c, 0)), (a**2*d**3*x**4/4, True))
```

### 3.217.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(336) = 672.

Time = 0.28 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.96

$$\int x^3 \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/10*b^2*c^6*d^3*x^10*arcsinh(c*x)^2 + 1/10*a^2*c^6*d^3*x^10 + 3/8*b^2*c^4
*d^3*x^8*arcsinh(c*x)^2 + 3/8*a^2*c^4*d^3*x^8 + 1/2*b^2*c^2*d^3*x^6*arcsin
h(c*x)^2 + 1/2*a^2*c^2*d^3*x^6 + 1/6400*(1280*x^10*arcsinh(c*x) - (128*sqr
t(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(c^2*x^2
+ 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(c^2*x^2 + 1)*x/c^1
0 - 315*arcsinh(c*x)/c^11)*c)*a*b*c^6*d^3 + 1/64000*((128*x^10/c^2 - 180*x
^8/c^4 + 280*x^6/c^6 - 525*x^4/c^8 + 1575*x^2/c^10 - 1575*log(c*x + sqrt(c
^2*x^2 + 1))^2/c^12)*c^2 - 10*(128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^
2*x^2 + 1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)
*x^3/c^8 + 315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c*arcsinh
(c*x))*b^2*c^6*d^3 + 1/4*b^2*d^3*x^4*arcsinh(c*x)^2 + 1/512*(384*x^8*arcsi
nh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 7
0*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*
x)/c^9)*c)*a*b*c^4*d^3 + 1/3072*((36*x^8/c^2 - 56*x^6/c^4 + 105*x^4/c^6 -
315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^10)*c^2 - 6*(48*sqrt(c^
2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x
^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c*arcsinh(c*x
))*b^2*c^4*d^3 + 1/4*a^2*d^3*x^4 + 1/48*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2
*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/
c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^2*d^3 + 1/288*((8*x^6/c^2 - 15*x^4/...
```

### 3.217.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.217.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \int x^3\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

output
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

# 3.218 $\int x^2(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$



## 3.218.1 Optimal result

Integrand size = 26, antiderivative size = 382

$$\begin{aligned}&\int x^2\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx\\&=-\frac{10516b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}+\frac{4198b^2c^2d^3x^5}{165375}+\frac{374b^2c^4d^3x^7}{27783}+\frac{2}{729}b^2c^6d^3x^9\\&+\frac{64bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{945c^3}-\frac{32bd^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{945c}\\&+\frac{16bd^3(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{315c^3}+\frac{4bd^3(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{525c^3}\\&+\frac{2bd^3(1+c^2x^2)^{7/2}(a+b\operatorname{arcsinh}(cx))}{441c^3}-\frac{2bd^3(1+c^2x^2)^{9/2}(a+b\operatorname{arcsinh}(cx))}{81c^3}\\&+\frac{16}{315}d^3x^3(a+b\operatorname{arcsinh}(cx))^2+\frac{8}{105}d^3x^3\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))^2+\frac{2}{21}d^3x^3\left(1+c^2x^2\right)^2(a+b\operatorname{arcsinh}(cx\end{aligned}$$

output
```
-10516/99225*b^2*d^3*x/c^2+5258/297675*b^2*d^3*x^3+4198/165375*b^2*c^2*d^3
*x^5+374/27783*b^2*c^4*d^3*x^7+2/729*b^2*c^6*d^3*x^9+16/315*b*d^3*(c^2*x^2
+1)^(3/2)*(a+b*arcsinh(c*x))/c^3+4/525*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsin
h(c*x))/c^3+2/441*b*d^3*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^3-2/81*b*d^
3*(c^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^3+16/315*d^3*x^3*(a+b*arcsinh(c*x
))^2+8/105*d^3*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+2/21*d^3*x^3*(c^2*x^2+
1)^2*(a+b*arcsinh(c*x))^2+1/9*d^3*x^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+6
4/945*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/945*b*d^3*x^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.218.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.72

$$\int x^2 \left(d+c^2 d x^2\right)^3 (a+b \operatorname{arcsinh}(c x))^2 \, dx$$
$$= \frac{d^3 \left(99225 a^2 c^3 x^3 (105+189 c^2 x^2+135 c^4 x^4+35 c^6 x^6)-630 a b \sqrt{1+c^2 x^2} (-5258+2629 c^2 x^2+6297 c^4 x^4+\right.}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*(99225*a^2*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - 6
30*a*b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x
^6 + 1225*c^8*x^8) + b^2*(-3312540*c*x + 552090*c^3*x^3 + 793422*c^5*x^5 +
 420750*c^7*x^7 + 85750*c^9*x^9) - 630*b*(-315*a*c^3*x^3*(105 + 189*c^2*x^
2 + 135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2
+ 6297*c^4*x^4 + 4675*c^6*x^6 + 1225*c^8*x^8))*ArcSinh[c*x] + 99225*b^2*c^
3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSinh[c*x]^2))/(312
55875*c^3)
```

### 3.218.3 Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.38, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {6223, 27, 6219, 27, 290, 2009, 6223, 6219, 27, 290, 2009, 6223, 6191, 6219, 27, 2009, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2 d x^2+d\right)^3 (a+b \operatorname{arcsinh}(c x))^2 \, dx$$

$$\Big\downarrow \ 6223$$

$$-\frac{2}{9} b c d^3 \int x^3 \left(c^2 x^2+1\right)^{5/2} (a+b \operatorname{arcsinh}(c x))dx+\frac{2}{3} d \int d^2 x^2 \left(c^2 x^2+1\right)^2 (a+b \operatorname{arcsinh}(c x))^2dx+\frac{1}{9} d^3 x^3 \left(c^2 x^2+1\right)^3 (a+b \operatorname{arcsinh}(c x))^2$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}d^3\int x^2\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2dx-\frac{2}{9}bcd^3\int x^3\left(c^2x^2+1\right)^{5/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2$$

$\Big\downarrow$ 6219

$$\frac{2}{3}d^3\int x^2\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2dx-\frac{2}{9}bcd^3\left(-bc\int -\frac{\left(2-7c^2x^2\right)\left(c^2x^2+1\right)^3}{63c^4}dx+\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx)}{7c^4}\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2$$

$\Big\downarrow$ 27

$$\frac{2}{3}d^3\int x^2\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2dx-\frac{2}{9}bcd^3\left(\frac{b\int\left(2-7c^2x^2\right)\left(c^2x^2+1\right)^3dx}{63c^3}+\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4}\right)+\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2$$

$\Big\downarrow$ 290

$$\frac{2}{3}d^3\int x^2\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2dx-\frac{2}{9}bcd^3\left(\frac{b\int\left(-7c^8x^8-19c^6x^6-15c^4x^4-c^2x^2+2\right)dx}{63c^3}+\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b}{7c^4}\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{2}{3}d^3\int x^2\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2-\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\Big\downarrow$ 6223

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))^2dx-\frac{2}{7}bc\int x^3\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{7}x^3\left(c^2x^2+1\right)^2(a\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

3.218. $\int x^2\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$

$\downarrow$ 6219

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{7}bc\left(-bc\int-\frac{\left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2}{35c^4}dx+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 27

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{7}bc\left(\frac{b\int\left(2-5c^2x^2\right)\left(c^2x^2+1\right)^2dx}{35c^3}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsin}}{7c^4}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 290

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx-\frac{2}{7}bc\left(\frac{b\int\left(-5c^6x^6-8c^4x^4-c^2x^2+2\right)dx}{35c^3}+\frac{\left(c^2x^2+1\right)^{7/2}(a+b}{7c^4}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 2009

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2-\frac{2}{7}bc\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b}{7c^4}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 6223

3.218. $\int x^2\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(-\frac{2}{5}bc\int x^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx+\frac{2}{5}\int x^2(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{5}x^3\left(c^2x^2+1\right)(a+b\operatorname{arcsi}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 6191

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bc\int x^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 6219

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bc\left(-bc\int-\frac{-3c^4x^4-c^2x^2+2}{15c^4}dx+\right.\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 27

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{5}bc\left(\frac{b\int\left(-3c^4x^4-c^2x^2+2\right)dx}{15c^3}+\right.\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\operatorname{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 2009

3.218. $\int x^2(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{5}x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2-\frac{2}{5}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 6227

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(c}{3c^2}\right.\right.\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 15

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)\right.\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 6213

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2-\frac{2}{3}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}}{3c^2}\right.\right.\right.\right.$$
$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 24

---

3.218. $\int x^2(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2+$$
$$\frac{2}{3}d^3\left(\frac{1}{7}x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{7}\left(\frac{1}{5}x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{5}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^2\right.\right.\right.$$
$$\frac{2}{9}bcd^3\left(\frac{\left(c^2x^2+1\right)^{9/2}(a+b\text{arcsinh}(cx))}{9c^4}-\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^4}+\frac{b\left(-\frac{7}{9}c^8x^9-\frac{19c^6x^7}{7}-3c^4x^5-\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

input
```
Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*x^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/9 - (2*b*c*d^3*((b*(2*x -
 (c^2*x^3)/3 - 3*c^4*x^5 - (19*c^6*x^7)/7 - (7*c^8*x^9)/9))/(63*c^3) - ((1
 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4) + ((1 + c^2*x^2)^(9/2)*(a
+ b*ArcSinh[c*x]))/(9*c^4)))/9 + (2*d^3*((x^3*(1 + c^2*x^2)^2*(a + b*ArcSi
nh[c*x])^2)/7 - (2*b*c*((b*(2*x - (c^2*x^3)/3 - (8*c^4*x^5)/5 - (5*c^6*x^7
)/7))/(35*c^3) - ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4) + ((1
+ c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4)))/7 + (4*((x^3*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*((b*(2*x - (c^2*x^3)/3 - (3*c^4*x^5)/
5))/(15*c^3) - ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4) + ((1 +
c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4)))/5 + (2*((x^3*(a + b*ArcSinh
[c*x])^2)/3 - (2*b*c*(-1/9*(b*x^3)/c + (x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]))/(3*c^2) - (2*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/c^2))/(3*c^2)))/3))/5))/7))/3
```

### 3.218.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.218. $\int x^2 (d + c^2 d x^2)^3 (a + b\operatorname{arcsinh}(cx))^2 \, dx$

### 3.218.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| parts | $d^3a^2\left(\frac{1}{9}c^6x^9+\frac{3}{7}c^4x^7+\frac{3}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^4}{9}-\frac{16\operatorname{arcsinh}(cx)^2xc}{315}-\frac{\operatorname{arcsinh}(cx)^2cx\left(\right.}{63}\right.}{}$ |
| derivativedivides | $\frac{d^3a^2\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^4}{9}-\frac{16\operatorname{arcsinh}(cx)^2xc}{315}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{63}-\frac{2}{}\right.}{}$ |
| default | $\frac{d^3a^2\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^4}{9}-\frac{16\operatorname{arcsinh}(cx)^2xc}{315}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{63}-\frac{2}{}\right.}{}$ |

input
```
int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/9*c^6*x^9+3/7*c^4*x^7+3/5*c^2*x^5+1/3*x^3)+d^3*b^2/c^3*(1/9*arc
sinh(c*x)^2*c*x*(c^2*x^2+1)^4-16/315*arcsinh(c*x)^2*x*c-1/63*arcsinh(c*x)^
2*c*x*(c^2*x^2+1)^3-2/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2-8/315*arcsinh(c
*x)^2*c*x*(c^2*x^2+1)-2/81*arcsinh(c*x)*(c^2*x^2+1)^(9/2)-3406208/31255875
*c*x+2/729*c*x*(c^2*x^2+1)^4+622/250047*c*x*(c^2*x^2+1)^3+15224/10418625*c
*x*(c^2*x^2+1)^2-115504/31255875*c*x*(c^2*x^2+1)+32/315*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)+2/441*arcsinh(c*x)*(c^2*x^2+1)^(7/2)+4/525*arcsinh(c*x)*(c^2*
x^2+1)^(5/2)+16/945*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^3*a*b/c^3*(1/9*arc
sinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*ar
csinh(c*x)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^
2+1)^(1/2)-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^
2+1)^(1/2)+5258/99225*(c^2*x^2+1)^(1/2))
```

### 3.218.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.05

$$\int x^2\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{42875\,(81\,a^2+2\,b^2)c^9d^3x^9+1125\,(11907\,a^2+374\,b^2)c^7d^3x^7+189\,(99225\,a^2+4198\,b^2)c^5d^3x^5+105\,(9922}{}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^3*x^9 + 1125*(11907*a^2 + 374*b^2
)*c^7*d^3*x^7 + 189*(99225*a^2 + 4198*b^2)*c^5*d^3*x^5 + 105*(99225*a^2 +
5258*b^2)*c^3*d^3*x^3 - 3312540*b^2*c*d^3*x + 99225*(35*b^2*c^9*d^3*x^9 +
135*b^2*c^7*d^3*x^7 + 189*b^2*c^5*d^3*x^5 + 105*b^2*c^3*d^3*x^3)*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^3*x^9 + 42525*a*b*c^7*d^3*x^7
 + 59535*a*b*c^5*d^3*x^5 + 33075*a*b*c^3*d^3*x^3 - (1225*b^2*c^8*d^3*x^8 +
 4675*b^2*c^6*d^3*x^6 + 6297*b^2*c^4*d^3*x^4 + 2629*b^2*c^2*d^3*x^2 - 5258
*b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*
c^8*d^3*x^8 + 4675*a*b*c^6*d^3*x^6 + 6297*a*b*c^4*d^3*x^4 + 2629*a*b*c^2*d
^3*x^2 - 5258*a*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

### 3.218.6 Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.64

$$\int x^2 \left(d+c^2 d x^2\right)^3 (a+b\text{arcsinh}(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2c^6d^3x^9}{9} + \frac{3a^2c^4d^3x^7}{7} + \frac{3a^2c^2d^3x^5}{5} + \frac{a^2d^3x^3}{3} + \frac{2abc^6d^3x^9 \operatorname{asinh}(cx)}{9} - \frac{2abc^5d^3x^8\sqrt{c^2x^2+1}}{81} + \frac{6abc^4d^3x^7 \operatorname{asinh}(cx)}{7} - \frac{374abc^3d^3}{3} \\ \frac{a^2d^3x^3}{3} \end{cases}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**6*d**3*x**9/9 + 3*a**2*c**4*d**3*x**7/7 + 3*a**2*c**2*d
**3*x**5/5 + a**2*d**3*x**3/3 + 2*a*b*c**6*d**3*x**9*asinh(c*x)/9 - 2*a*b*
c**5*d**3*x**8*sqrt(c**2*x**2 + 1)/81 + 6*a*b*c**4*d**3*x**7*asinh(c*x)/7
- 374*a*b*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)/3969 + 6*a*b*c**2*d**3*x**5*a
sinh(c*x)/5 - 4198*a*b*c*d**3*x**4*sqrt(c**2*x**2 + 1)/33075 + 2*a*b*d**3*
x**3*asinh(c*x)/3 - 5258*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(99225*c) + 105
16*a*b*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3) + b**2*c**6*d**3*x**9*asinh(c
*x)**2/9 + 2*b**2*c**6*d**3*x**9/729 - 2*b**2*c**5*d**3*x**8*sqrt(c**2*x**
2 + 1)*asinh(c*x)/81 + 3*b**2*c**4*d**3*x**7*asinh(c*x)**2/7 + 374*b**2*c*
*4*d**3*x**7/27783 - 374*b**2*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x
)/3969 + 3*b**2*c**2*d**3*x**5*asinh(c*x)**2/5 + 4198*b**2*c**2*d**3*x**5/
165375 - 4198*b**2*c*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/33075 + b**2
*d**3*x**3*asinh(c*x)**2/3 + 5258*b**2*d**3*x**3/297675 - 5258*b**2*d**3*x
**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225*c) - 10516*b**2*d**3*x/(99225*c*
*2) + 10516*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225*c**3), Ne(c, 0
)), (a**2*d**3*x**3/3, True))
```

### 3.218.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(340) = 680.

Time = 0.25 (sec) , antiderivative size = 922, normalized size of antiderivative = 2.41

$$\int x^2 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/9*b^2*c^6*d^3*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^6*d^3*x^9 + 3/7*b^2*c^4*d^3
*x^7*arcsinh(c*x)^2 + 3/7*a^2*c^4*d^3*x^7 + 3/5*b^2*c^2*d^3*x^5*arcsinh(c*
x)^2 + 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*s
qrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2
+ 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^6*d^3 - 2/893025*(315*
(35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2
*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c
^10)*c*arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c
^2*x^3 + 40320*x)/c^8)*b^2*c^6*d^3 + 3/5*a^2*c^2*d^3*x^5 + 6/245*(35*x^7*a
rcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 +
 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^3 -
2/8575*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8
*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (7
5*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^3 + 1/3*b^2
*d^3*x^3*arcsinh(c*x)^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)
*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c
^2*d^3 - 2/375*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 +
120*x)/c^4)*b^2*c^2*d^3 + 1/3*a^2*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(s
qrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^3 - 2/27*(3*...
```

### 3.218.8 Giac [F(-2)]

Exception generated.

$$\int x^2 (d + c^2 d x^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.218.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))^2\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

output
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

# 3.219 $\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx$



## 3.219.1 Optimal result

Integrand size = 24, antiderivative size = 261

$$\begin{aligned}\int x(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2\,dx &= \frac{175b^2d^3x^2}{3072}+\frac{35b^2c^2d^3x^4}{3072}\\ &+\frac{7b^2d^3(1+c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1+c^2x^2)^4}{256c^2}\\ &-\frac{35bd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{512c}\\ &-\frac{35bd^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{768c}\\ &-\frac{7bd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{192c}\\ &-\frac{bd^3x(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{32c}\\ &-\frac{35d^3(a+b\text{arcsinh}(cx))^2}{1024c^2}\\ &+\frac{d^3(1+c^2x^2)^4(a+b\text{arcsinh}(cx))^2}{8c^2}\end{aligned}$$

output
```
175/3072*b^2*d^3*x^2+35/3072*b^2*c^2*d^3*x^4+7/1152*b^2*d^3*(c^2*x^2+1)^3/
c^2+1/256*b^2*d^3*(c^2*x^2+1)^4/c^2-35/768*b*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))/c-7/192*b*d^3*x*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-1/32*
b*d^3*x*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c-35/1024*d^3*(a+b*arcsinh(c*
x))^2/c^2+1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))^2/c^2-35/512*b*d^3*x*(a
+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.219.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98

$$\int x\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d^3\left(cx\left(1152a^2cx(4+6c^2x^2+4c^4x^4+c^6x^6)+b^2cx(837+489c^2x^2+200c^4x^4+36c^6x^6)-6ab\sqrt{1+c^2x^2}(2\right.\right.}{}$$

input
```
Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*(c*x*(1152*a^2*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) + b^2*c*x*(8
37 + 489*c^2*x^2 + 200*c^4*x^4 + 36*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(27
9 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x
^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*a*(93 + 512*c^2*x^
2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(93 + 5
12*c^2*x^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]^2))/(92
16*c^2)
```

### 3.219.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6213, 6201, 241, 6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^3(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\int\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))dx}{4c}$$

$$\Big\downarrow \text{6201}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\left(\frac{7}{8}\int\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{8}bc\int x\left(c^2x^2+1\right)^3dx+\frac{1}{8}x\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))\right)}{4c}$$

$$\Big\downarrow \text{241}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\left(\frac{7}{8}\int\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{8}x\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))-\frac{b\left(c^2x^2+1\right)^4}{64c}\right)}{4c}$$

$$\Big\downarrow \text{6201}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\int\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}bc\int x\left(c^2x^2+1\right)^2dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))\right)+\right.}{4c}$$

$$\Big\downarrow \text{241}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\int\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{b\left(c^2x^2+1\right)^3}{36c}\right)+\frac{1}{8}x\left(c^2x^2+\right.\right.}{4c}$$

$$\Big\downarrow \text{6201}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}\right.\right.}{4c}$$

$$\Big\downarrow \text{244}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int\left(c^2x^3+x\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)+\frac{1}{6}x\right.\right.}{4c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\left(c^2x^2+1\right)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)+\frac{1}{6}x\left(c^2\right.\right.\right.}{4c}$$

$$\Big\downarrow \text{6200}$$

3.219. $\int x\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2dx$

$$\frac{d^3(c^2x^2+1)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-$$
$$bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+\text{barcsi}\right.\right.\right.$$

$\downarrow$ 15

$$\frac{d^3(c^2x^2+1)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-$$
$$bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(\right.\right.\right.$$

$\downarrow$ 6198

$$\frac{d^3(c^2x^2+1)^4(a+b\text{arcsinh}(cx))^2}{8c^2}-$$
$$bd^3\left(\frac{1}{8}x(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))+\frac{7}{8}\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+\text{barcs}\right.\right.\right.$$

input
```
Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x])^2)/(8*c^2) - (b*d^3*(-1/64*(b*(1
 + c^2*x^2)^4)/c + (x*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/8 + (7*(-1
/36*(b*(1 + c^2*x^2)^3)/c + (x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6
 + (5*(-1/4*(b*c*(x^2/2 + (c^2*x^4)/4)) + (x*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*c)))/4))/6))/8))/(4*c)
```

### 3.219.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.219. $\int x(d + c^2dx^2)^3 (a + b\operatorname{arcsinh}(cx))^2 \, dx$

### 3.219.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d^3a^2(c^2x^2+1)^4}{8}+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^4}{8}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{7}{2}}}{32}-\frac{7\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{192}-\frac{35\operatorname{arcsinh}(cx)cx(c}{768}\right.}{}$ |
| default | $\frac{\frac{d^3a^2(c^2x^2+1)^4}{8}+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^4}{8}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{7}{2}}}{32}-\frac{7\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{192}-\frac{35\operatorname{arcsinh}(cx)cx(c}{768}\right.}{}$ |
| parts | $\frac{d^3a^2(c^2x^2+1)^4}{8c^2}+\frac{d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^4}{8}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{7}{2}}}{32}-\frac{7\operatorname{arcsinh}(cx)cx(c^2x^2+1)^{\frac{5}{2}}}{192}-\frac{35\operatorname{arcsinh}(cx)cx}{768}\right.}{}$ |

input
```
int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/8*d^3*a^2*(c^2*x^2+1)^4+d^3*b^2*(1/8*arcsinh(c*x)^2*(c^2*x^2+1)^4
-1/32*arcsinh(c*x)*c*x*(c^2*x^2+1)^(7/2)-7/192*arcsinh(c*x)*c*x*(c^2*x^2+1
)^(5/2)-35/768*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)-35/512*arcsinh(c*x)*c*x*
(c^2*x^2+1)^(1/2)-35/1024*arcsinh(c*x)^2+1/256*(c^2*x^2+1)^4+7/1152*(c^2*x
^2+1)^3+35/3072*(c^2*x^2+1)^2+35/1024*c^2*x^2+35/1024)+2*d^3*a*b*(1/8*arcs
inh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^4*x^4+1/2*arc
sinh(c*x)*c^2*x^2+93/1024*arcsinh(c*x)-1/64*c*x*(c^2*x^2+1)^(7/2)-7/384*c*
x*(c^2*x^2+1)^(5/2)-35/1536*c*x*(c^2*x^2+1)^(3/2)-35/1024*c*x*(c^2*x^2+1)^
(1/2)))
```

### 3.219.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.47

$$\int x\left(d+c^2dx^2\right)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{36\,(32\,a^2+b^2)c^8d^3x^8+8\,(576\,a^2+25\,b^2)c^6d^3x^6+3\,(2304\,a^2+163\,b^2)c^4d^3x^4+9\,(512\,a^2+93\,b^2)c^2d^3x^2+}{}$$

input
```
integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/9216*(36*(32*a^2 + b^2)*c^8*d^3*x^8 + 8*(576*a^2 + 25*b^2)*c^6*d^3*x^6 +
 3*(2304*a^2 + 163*b^2)*c^4*d^3*x^4 + 9*(512*a^2 + 93*b^2)*c^2*d^3*x^2 + 9
*(128*b^2*c^8*d^3*x^8 + 512*b^2*c^6*d^3*x^6 + 768*b^2*c^4*d^3*x^4 + 512*b^
2*c^2*d^3*x^2 + 93*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(384*a*b*c^
8*d^3*x^8 + 1536*a*b*c^6*d^3*x^6 + 2304*a*b*c^4*d^3*x^4 + 1536*a*b*c^2*d^3
*x^2 + 279*a*b*d^3 - (48*b^2*c^7*d^3*x^7 + 200*b^2*c^5*d^3*x^5 + 326*b^2*c
^3*d^3*x^3 + 279*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
1)) - 6*(48*a*b*c^7*d^3*x^7 + 200*a*b*c^5*d^3*x^5 + 326*a*b*c^3*d^3*x^3 +
279*a*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^2
```

### 3.219.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(252) = 504$.

Time = 1.39 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.20

$$\int x(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2c^6d^3x^8}{8} + \frac{a^2c^4d^3x^6}{2} + \frac{3a^2c^2d^3x^4}{4} + \frac{a^2d^3x^2}{2} + \frac{abc^6d^3x^8 \operatorname{asinh}(cx)}{4} - \frac{abc^5d^3x^7\sqrt{c^2x^2+1}}{32} + abc^4d^3x^6 \operatorname{asinh}(cx) - \frac{25abc^3d}{} \\ \frac{a^2d^3x^2}{2} \end{cases}$$

input
```
integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**6*d**3*x**8/8 + a**2*c**4*d**3*x**6/2 + 3*a**2*c**2*d**
3*x**4/4 + a**2*d**3*x**2/2 + a*b*c**6*d**3*x**8*asinh(c*x)/4 - a*b*c**5*d
**3*x**7*sqrt(c**2*x**2 + 1)/32 + a*b*c**4*d**3*x**6*asinh(c*x) - 25*a*b*c
**3*d**3*x**5*sqrt(c**2*x**2 + 1)/192 + 3*a*b*c**2*d**3*x**4*asinh(c*x)/2
- 163*a*b*c*d**3*x**3*sqrt(c**2*x**2 + 1)/768 + a*b*d**3*x**2*asinh(c*x) -
 93*a*b*d**3*x*sqrt(c**2*x**2 + 1)/(512*c) + 93*a*b*d**3*asinh(c*x)/(512*c
**2) + b**2*c**6*d**3*x**8*asinh(c*x)**2/8 + b**2*c**6*d**3*x**8/256 - b**
2*c**5*d**3*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c**4*d**3*x**6*a
sinh(c*x)**2/2 + 25*b**2*c**4*d**3*x**6/1152 - 25*b**2*c**3*d**3*x**5*sqrt
(c**2*x**2 + 1)*asinh(c*x)/192 + 3*b**2*c**2*d**3*x**4*asinh(c*x)**2/4 + 1
63*b**2*c**2*d**3*x**4/3072 - 163*b**2*c*d**3*x**3*sqrt(c**2*x**2 + 1)*asi
nh(c*x)/768 + b**2*d**3*x**2*asinh(c*x)**2/2 + 93*b**2*d**3*x**2/1024 - 93
*b**2*d**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(512*c) + 93*b**2*d**3*asinh(c
*x)**2/(1024*c**2), Ne(c, 0)), (a**2*d**3*x**2/2, True))
```

### 3.219.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(233) = 466.

Time = 0.25 (sec) , antiderivative size = 925, normalized size of antiderivative = 3.54

$$\int x(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*c^6*d^3*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^6*d^3*x^8 + 1/2*b^2*c^4*d^3
*x^6*arcsinh(c*x)^2 + 1/2*a^2*c^4*d^3*x^6 + 3/4*b^2*c^2*d^3*x^4*arcsinh(c*
x)^2 + 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*s
qrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2
 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^6*d^3 + 1/9216*((36*x^8/c^2 -
 56*x^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))
^2/c^10)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/
c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arc
sinh(c*x)/c^9)*c*arcsinh(c*x))*b^2*c^6*d^3 + 3/4*a^2*c^2*d^3*x^4 + 1/48*(4
8*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x
^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^3
+ 1/288*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2
+ 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x
^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))
*b^2*c^4*d^3 + 1/2*b^2*d^3*x^2*arcsinh(c*x)^2 + 3/16*(8*x^4*arcsinh(c*x) -
 (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)
/c^5)*c)*a*b*c^2*d^3 + 3/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x
^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)
*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/2*a^2*d^3*x^2
 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/...
```

### 3.219.8 Giac [F(-2)]

Exception generated.

$$\int x(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.219.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2\,dx = \int x\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

output
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

# 3.220 $\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$



## 3.220.1 Optimal result

Integrand size = 23, antiderivative size = 291

$$\begin{aligned}
&\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx \\
&= \frac{4322b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 \\
&\quad - \frac{32bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{35c} - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{105c} \\
&\quad - \frac{12bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c} \\
&\quad + \frac{16}{35}d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{35}d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{35}d^3x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 +
\end{aligned}$$

output
```
4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3+234/6125*b^2*c^4*d^3*x^5+2/
343*b^2*c^6*d^3*x^7-16/105*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-12
/175*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-2/49*b*d^3*(c^2*x^2+1)^(
7/2)*(a+b*arcsinh(c*x))/c+16/35*d^3*x*(a+b*arcsinh(c*x))^2+8/35*d^3*x*(c^2
*x^2+1)*(a+b*arcsinh(c*x))^2+6/35*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2
+1/7*d^3*x*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2-32/35*b*d^3*(a+b*arcsinh(c*x
))*(c^2*x^2+1)^(1/2)/c
```

### 3.220.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.82

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d^3\left(11025a^2cx(35+35c^2x^2+21c^4x^4+5c^6x^6)-210ab\sqrt{1+c^2x^2}(2161+757c^2x^2+351c^4x^4+75c^6x^6)+\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*(11025*a^2*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - 210*a*b*S
qrt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 2*b^2*c
*x*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6) - 210*b*(-105*a*
c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(2161
 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c*x*(
35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]^2))/(385875*c)
```

### 3.220.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6201, 27, 6201, 6201, 6187, 6213, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{6}{7}d\int d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{6}{7}d^3\int \left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{6}{7}d^3\left(-\frac{2}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}\int\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}x\left(c^2x^2+1\right)^2(a+\right.$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{6}{7}d^3\left(-\frac{2}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}\left(-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}\int(a+ba\right.\right.$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\right.\right.$$
$$\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\right.}{3c^2}\right.\right.\right.$$
$$\frac{2}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\int\left(c^2x^2+1\right)^3dx}{7c}\right)+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+$$
$$b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 24}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int\left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\int\left(c^2x^2+1\right)^3dx}{7c}\right)+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+$$
$$b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 210}$$

3.220. $\int\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-\frac{2}{3}bc\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int(c^2x^2+1)\,dx}{3c}\right)+\frac{1}{3}x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(\right.\right.\right.$$

$$\frac{2}{7}bcd^3\left(\frac{(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\int(c^6x^6+3c^4x^4+3c^2x^2+1)\,dx}{7c}\right)+$$

$$\frac{1}{7}d^3x(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}d^3x(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x(c^2x^2+1)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\right.\right.\right.$$

$$\frac{2}{7}bcd^3\left(\frac{(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\left(\frac{c^6x^7}{7}+\frac{3c^4x^5}{5}+c^2x^3+x\right)}{7c}\right)$$

input
```
Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*x*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/7 - (2*b*c*d^3*(-1/7*(b*(x
+ c^2*x^3 + (3*c^4*x^5)/5 + (c^6*x^7)/7))/c + ((1 + c^2*x^2)^(7/2)*(a + b*
ArcSinh[c*x]))/(7*c^2)))/7 + (6*d^3*((x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x
])^2)/5 - (2*b*c*(-1/5*(b*(x + (2*c^2*x^3)/3 + (c^4*x^5)/5))/c + ((1 + c^2
*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2)))/5 + (4*((x*(1 + c^2*x^2)*(a +
b*ArcSinh[c*x])^2)/3 - (2*b*c*(-1/3*(b*(x + (c^2*x^3)/3))/c + ((1 + c^2*x^
2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSinh[c*x])^2
- 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2)))/3))/
5))/7
```

### 3.220.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.220.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| derivativedivides | $\frac{d^3a^2\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b^2\left(\frac{16\,\mathrm{arcsinh}(cx)^2xc}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{8\,\mathrm{arcsin}}{}\right.}{}$ |
| default | $\frac{d^3a^2\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b^2\left(\frac{16\,\mathrm{arcsinh}(cx)^2xc}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{8\,\mathrm{arcsin}}{}\right.}{}$ |
| parts | $d^3a^2\left(\frac{1}{7}c^6x^7+\frac{3}{5}c^4x^5+x^3c^2+x\right)+\frac{d^3b^2\left(\frac{16\,\mathrm{arcsinh}(cx)^2xc}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,\mathrm{arcsinh}(cx)^2cx\left(c^2\right.}{35}\right.}{}$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^3*a^2*(1/7*c^7*x^7+3/5*c^5*x^5+c^3*x^3+c*x)+d^3*b^2*(16/35*arcsinh(
c*x)^2*x*c+1/7*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+6/35*arcsinh(c*x)^2*c*x*(c
^2*x^2+1)^2+8/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-32/35*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)+413312/385875*c*x-2/49*arcsinh(c*x)*(c^2*x^2+1)^(7/2)+2/343*c*x
*(c^2*x^2+1)^3+888/42875*c*x*(c^2*x^2+1)^2+30256/385875*c*x*(c^2*x^2+1)-12
/175*arcsinh(c*x)*(c^2*x^2+1)^(5/2)-16/105*arcsinh(c*x)*(c^2*x^2+1)^(3/2))
+2*d^3*a*b*(1/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)
*c^3*x^3+arcsinh(c*x)*c*x-2161/3675*(c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(c^2*x^
2+1)^(1/2)-117/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(c^2*x^2+1)
^(1/2)))
```

## 3.220.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.22

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1125\,(49\,a^2+2\,b^2)c^7d^3x^7+189\,(1225\,a^2+78\,b^2)c^5d^3x^5+35\,(11025\,a^2+1514\,b^2)c^3d^3x^3+105\,(3675\,a^2+}{}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d^3*x^7 + 189*(1225*a^2 + 78*b^2)*c^5*
d^3*x^5 + 35*(11025*a^2 + 1514*b^2)*c^3*d^3*x^3 + 105*(3675*a^2 + 4322*b^2
)*c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 + 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^3
*x^3 + 35*b^2*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d
^3*x^7 + 2205*a*b*c^5*d^3*x^5 + 3675*a*b*c^3*d^3*x^3 + 3675*a*b*c*d^3*x -
(75*b^2*c^6*d^3*x^6 + 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 + 2161*b^2
*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d^
3*x^6 + 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 + 2161*a*b*d^3)*sqrt(c^2
*x^2 + 1))/c
```

### 3.220.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.80

$$\int \left(d+c^2dx^2\right)^3 (a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2c^6d^3x^7}{7}+\frac{3a^2c^4d^3x^5}{5}+a^2c^2d^3x^3+a^2d^3x+\frac{2abc^6d^3x^7\operatorname{asinh}(cx)}{7}-\frac{2abc^5d^3x^6\sqrt{c^2x^2+1}}{49}+\frac{6abc^4d^3x^5\operatorname{asinh}(cx)}{5}-\frac{234abc^3d}{} \\ a^2d^3x\end{cases}$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 + a**2*c**2*d**
3*x**3 + a**2*d**3*x + 2*a*b*c**6*d**3*x**7*asinh(c*x)/7 - 2*a*b*c**5*d**3
*x**6*sqrt(c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*asinh(c*x)/5 - 234*a*b
*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)/1225 + 2*a*b*c**2*d**3*x**3*asinh(c*x)
 - 1514*a*b*c*d**3*x**2*sqrt(c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*asinh(c*x)
 - 4322*a*b*d**3*sqrt(c**2*x**2 + 1)/(3675*c) + b**2*c**6*d**3*x**7*asinh(
c*x)**2/7 + 2*b**2*c**6*d**3*x**7/343 - 2*b**2*c**5*d**3*x**6*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/49 + 3*b**2*c**4*d**3*x**5*asinh(c*x)**2/5 + 234*b**2*c
**4*d**3*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x
)/1225 + b**2*c**2*d**3*x**3*asinh(c*x)**2 + 1514*b**2*c**2*d**3*x**3/1102
5 - 1514*b**2*c*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/3675 + b**2*d**3*
x*asinh(c*x)**2 + 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(c**2*x**2 +
1)*asinh(c*x)/(3675*c), Ne(c, 0)), (a**2*d**3*x, True))
```

### 3.220.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(259) = 518.

Time = 0.22 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.45

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{7}\,b^2c^6d^3x^7\,\text{arsinh}\,(cx)^2 + \frac{1}{7}\,a^2c^6d^3x^7 + \frac{3}{5}\,b^2c^4d^3x^5\,\text{arsinh}\,(cx)^2 + \frac{3}{5}\,a^2c^4d^3x^5$$
$$+ \frac{2}{245}\left(35\,x^7\,\text{arsinh}\,(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abc^6$$
$$- \frac{2}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\,\text{arsinh}\,(cx) - \frac{75\,c^6}{}\right.$$
$$+ b^2c^2d^3x^3\,\text{arsinh}\,(cx)^2$$
$$+ \frac{2}{25}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^4d^3$$
$$- \frac{2}{375}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right)b$$
$$+ a^2c^2d^3x^3 + \frac{2}{3}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d^3$$
$$- \frac{2}{9}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3 - 6\,x}{c^2}\right)b^2c^2d^3$$
$$+ b^2d^3x\,\text{arsinh}\,(cx)^2 + 2\,b^2d^3\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right)$$
$$+ a^2d^3x + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd^3}{c}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

3.220. $\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$

output
```
1/7*b^2*c^6*d^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3
*x^5*arcsinh(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (
5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2
 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 - 2/25725*(105*(5
*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2
+ 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 12
6*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^6*d^3 + b^2*c^2*d^3*x^3*arcsi
nh(c*x)^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*s
qrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 - 2/375
*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c
^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^
2*c^4*d^3 + a^2*c^2*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 - 2/9*(3*c*(sqrt(c^2*x^
2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c
^2)*b^2*c^2*d^3 + b^2*d^3*x*arcsinh(c*x)^2 + 2*b^2*d^3*(x - sqrt(c^2*x^2 +
 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))
*a*b*d^3/c
```

### 3.220.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.220.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx=\int \left(a+b\,\text{asinh}(c\,x)\right)^2\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

3.220. $\int (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2 \, dx$

# 3.221 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$



## 3.221.1 Optimal result

Integrand size = 26, antiderivative size = 337

$$
\begin{aligned}
\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx &= \frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3(1+c^2x^2)^3 \\
&\quad -\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\
&\quad -\frac{7}{36}bcd^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx)) \\
&\quad -\frac{1}{18}bcd^3x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) \\
&\quad -\frac{19}{48}d^3(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{2}d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{4}d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{1}{6}d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2 \\
&\quad +\frac{d^3(a+b\text{arcsinh}(cx))^3}{3b} \\
&\quad +d^3(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right) \\
&\quad -bd^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right) \\
&\quad -\frac{1}{2}b^2d^3\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)
\end{aligned}
$$

output
```
71/144*b^2*c^2*d^3*x^2+7/144*b^2*c^4*d^3*x^4+1/108*b^2*d^3*(c^2*x^2+1)^3-7
/36*b*c*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/18*b*c*d^3*x*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))-19/48*d^3*(a+b*arcsinh(c*x))^2+1/2*d^3*(c^2*x
^2+1)*(a+b*arcsinh(c*x))^2+1/4*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/6*
d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+1/3*d^3*(a+b*arcsinh(c*x))^3/b+d^3*
(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-b*d^3*(a+b*arcsinh(
c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*d^3*polylog(3,1/(c*x+
(c^2*x^2+1)^(1/2))^2)-19/24*b*c*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.221.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.24

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$$
$$= \frac{d^3\left(5184a^2c^2x^2+2592a^2c^4x^4+576a^2c^6x^6-3600abcx\sqrt{1+c^2x^2}-1056abc^3x^3\sqrt{1+c^2x^2}-192abc^5x^5\sqrt{1\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

output
```
(d^3*(5184*a^2*c^2*x^2 + 2592*a^2*c^4*x^4 + 576*a^2*c^6*x^6 - 3600*a*b*c*x
*Sqrt[1 + c^2*x^2] - 1056*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 192*a*b*c^5*x^5*
Sqrt[1 + c^2*x^2] + 10368*a*b*c^2*x^2*ArcSinh[c*x] + 5184*a*b*c^4*x^4*ArcS
inh[c*x] + 1152*a*b*c^6*x^6*ArcSinh[c*x] - 3456*a*b*ArcSinh[c*x]^2 - 1152*
b^2*ArcSinh[c*x]^3 + 783*b^2*Cosh[2*ArcSinh[c*x]] + 1566*b^2*ArcSinh[c*x]^
2*Cosh[2*ArcSinh[c*x]] + 27*b^2*Cosh[4*ArcSinh[c*x]] + 216*b^2*ArcSinh[c*x
]^2*Cosh[4*ArcSinh[c*x]] + b^2*Cosh[6*ArcSinh[c*x]] + 18*b^2*ArcSinh[c*x]^
2*Cosh[6*ArcSinh[c*x]] + 6912*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])]
 + 3456*b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3456*a^2*Log[c*x]
 - 3600*a*b*Log[-(c*x) + Sqrt[1 + c^2*x^2]] + 3456*b*(a + b*ArcSinh[c*x])*
PolyLog[2, E^(2*ArcSinh[c*x])] - 1728*b^2*PolyLog[3, E^(2*ArcSinh[c*x])] -
 1566*b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 108*b^2*ArcSinh[c*x]*Sinh[4*
ArcSinh[c*x]] - 6*b^2*ArcSinh[c*x]*Sinh[6*ArcSinh[c*x]]))/3456
```

### 3.221.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.69, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {6223, 27, 6201, 241, 6201, 244, 2009, 6200, 15, 6198, 6223, 6201, 244, 2009, 6200, 15, 6198, 6223, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 6200, 15, 6198, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3 (a+b\text{arcsinh}(cx))^2}{x} \, dx$$

$$\Big\downarrow \text{ 6223}$$

$$-\frac{1}{3}bcd^3\int (c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))dx+d\int \frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}bcd^3\int (c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))dx+d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{1}{3}bcd^3\left(\frac{5}{6}\int (c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}bc\int x(c^2x^2+1)^2dx+\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))\right)+d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{1}{3}bcd^3\left(\frac{5}{6}\int (c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))-\frac{b(c^2x^2+1)^3}{36c}\right)+d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)\right.$$
$$d^3\int\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 244

$$d^3\int\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx-$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int\left(c^2x^3+x\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)+\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 2009

$$d^3\int\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx-$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)+\frac{1}{6}\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6200

$$d^3\int\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx-$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 15

$$d^3\int\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx-$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arc}\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6198

$$d^3\int\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

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3.221. $\int\frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$\Big\downarrow \text{6223}$$

$$d^3\left(-\frac{1}{2}bc\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcs}\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \text{6201}$$

$$d^3\left(-\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \text{244}$$

$$d^3\left(\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int \left(c^2x^3+x\right)dx+\frac{1}{4}\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$d^3\left(\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arc}\right.\right.$$
$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \text{6200}$$

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3.221. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d^3\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx - \frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx - \frac{1}{2}bc\int xdx + \frac{1}{2}x\sqrt{c^2x^2+1}(a+ba\right.\right.\right.$$
$$\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2 -$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barc\right.\right.\right.$$

$\downarrow$ 15

$$d^3\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx - \frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx + \frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)) -\right.\right.\right.$$
$$\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2 -$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barc\right.\right.\right.$$

$\downarrow$ 6198

$$d^3\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{x}dx + \frac{1}{4}(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2 - \frac{1}{2}bc\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+barcsin\right.\right.$$
$$\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2 -$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barc\right.\right.\right.$$

$\downarrow$ 6223

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx + \int \frac{(a+b\text{arcsinh}(cx))^2}{x}dx + \frac{1}{4}(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2 + \frac{1}{2}(c\right.$$
$$\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2 -$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barc\right.\right.\right.$$

$\downarrow$ 6190

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx + \frac{\int -(a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx}{b}\right.$$
$$\frac{1}{6}d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2 -$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx)) + \frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barc\right.\right.\right.$$

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3.221. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$\Big\downarrow \; 25$$

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int (a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int -i(a+b\text{arcsinh}(cx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arc}}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \; 26$$

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\int (a+b\text{arcsinh}(cx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \; 4201$$

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \; 2620$$

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3.221. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\int(a+b\text{arcsinh}(cx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+\right.\right.\right.}{}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barc}\right.\right.\right.$$

$\downarrow$ 3011

$$d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)\right.\right.\right.}{}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barc}\right.\right.\right.$$

$\downarrow$ 2720

$$d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+\text{barcsi}\right.\right.\right.}{}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barc}\right.\right.\right.$$

$\downarrow$ 6200

$$d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+\text{barcsi}\right.\right.\right.}{}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barc}\right.\right.\right.$$

$\downarrow$ 15

---

3.221. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsin}\right.\right.\right.}{}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \text{ 6198}$$

$$d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arcsin}\right.\right.\right.}{}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

$$\Big\downarrow \text{ 7143}$$

$$d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\text{PolyLog}(3,-a-b\text{arcsinh}(cx))+\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)\right)\right.\right.}{b}\right.$$

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2-$$

$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arc}\right.\right.\right.$$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

## 3.221. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

output
```
(d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/6 - (b*c*d^3*(-1/36*(b*(1 + c
^2*x^2)^3)/c + (x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*(-1/4*(
b*c*(x^2/2 + (c^2*x^4)/4)) + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/
4 + (3*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a
 + b*ArcSinh[c*x])^2/(4*b*c)))/4))/6))/3 + d^3*(((1 + c^2*x^2)*(a + b*ArcS
inh[c*x])^2)/2 + ((1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - b*c*(-1/4*(b
*c*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*
x])^2/(4*b*c)) - (b*c*(-1/4*(b*c*(x^2/2 + (c^2*x^4)/4)) + (x*(1 + c^2*x^2)
^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*c)))/4))/2 + (I*((-
1/3*I)*(a + b*ArcSinh[c*x])^3 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])^2*Log[
1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)]) + b*((b*(a + b*ArcSi
nh[c*x])*PolyLog[2, -E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)])/2 +
 (b^2*PolyLog[3, -a - b*ArcSinh[c*x]])/4))))/b)
```

### 3.221.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

## 3.221. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.221. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x}\,dx$

### 3.221.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(334) = 668.

Time = 0.33 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| parts | $-\frac{d^3ab\,c^5x^5\sqrt{c^2x^2+1}}{18}-\frac{11d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{36}-\frac{25d^3abcx\sqrt{c^2x^2+1}}{24}-2d^3b^2\operatorname{polylog}\left(3,-cx-\sqrt{c^2x^2\right.$ |
| derivativedivides | $d^3a^2\left(\frac{c^6x^6}{6}+\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}+\ln(cx)\right)-\frac{d^3ab\,c^5x^5\sqrt{c^2x^2+1}}{18}-\frac{11d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{36}-\frac{25d^3abcx\sqrt{c^2x^2}}{24}$ |
| default | $d^3a^2\left(\frac{c^6x^6}{6}+\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}+\ln(cx)\right)-\frac{d^3ab\,c^5x^5\sqrt{c^2x^2+1}}{18}-\frac{11d^3ab\,c^3x^3\sqrt{c^2x^2+1}}{36}-\frac{25d^3abcx\sqrt{c^2x^2}}{24}$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/18*d^3*a*b*c^5*x^5*(c^2*x^2+1)^(1/2)-11/36*d^3*a*b*c^3*x^3*(c^2*x^2+1)^
(1/2)-25/24*d^3*a*b*c*x*(c^2*x^2+1)^(1/2)-2*d^3*b^2*polylog(3,-c*x-(c^2*x^
2+1)^(1/2))-1/3*d^3*b^2*arcsinh(c*x)^3+25/48*d^3*b^2*arcsinh(c*x)^2-2*d^3*
b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*
x^2+1)^(1/2))+2*d^3*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/6*d^3*b
^2*arcsinh(c*x)^2*c^6*x^6+3/4*d^3*b^2*arcsinh(c*x)^2*c^4*x^4+3/2*d^3*b^2*a
rcsinh(c*x)^2*c^2*x^2+1/3*d^3*a*b*arcsinh(c*x)*c^6*x^6+3/2*d^3*a*b*arcsinh
(c*x)*c^4*x^4+3*d^3*a*b*arcsinh(c*x)*c^2*x^2-1/18*d^3*b^2*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)*c^5*x^5-11/36*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^
3-25/24*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+811/3456*d^3*b^2+d^3*a^
2*(1/6*c^6*x^6+3/4*c^4*x^4+3/2*c^2*x^2+ln(x))+25/48*b^2*c^2*d^3*x^2+11/144
*b^2*c^4*d^3*x^4+d^3*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d^3*
b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+25/24*d^3*a*b*arcsinh(c
*x)-d^3*a*b*arcsinh(c*x)^2+2*d^3*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*d^
3*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+1/108*d^3*b^2*c^6*x^6+d^3*b^2*arcs
inh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d^3*b^2*arcsinh(c*x)*polylog(2,c*
x+(c^2*x^2+1)^(1/2))
```

### 3.221.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^
3 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*ar
csinh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
+ a*b*d^3)*arcsinh(c*x))/x, x)
```

### 3.221.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))^2}{x}\,dx = d^3\left(\int \frac{a^2}{x}\,dx + \int 3a^2c^2x\,dx + \int 3a^2c^4x^3\,dx + \int a^2c^6x^5\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x}\,dx + \int 3b^2c^2x\operatorname{asinh}^2(cx)\,dx + \int 3b^2c^4x^3\operatorname{asinh}^2(cx)\,dx + \int b^2c^6x^5\operatorname{asinh}^2(cx)\,dx + \int 6abc^2x\operatorname{asinh}(cx)\,dx + \int 6abc^4x^3\operatorname{asinh}(cx)\,dx + \int 2abc^6x^5\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x,x)
```

output
```
d**3*(Integral(a**2/x, x) + Integral(3*a**2*c**2*x, x) + Integral(3*a**2*c
**4*x**3, x) + Integral(a**2*c**6*x**5, x) + Integral(b**2*asinh(c*x)**2/x
, x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(3*b**2*c**2*x*asinh(c*x)
**2, x) + Integral(3*b**2*c**4*x**3*asinh(c*x)**2, x) + Integral(b**2*c**6
*x**5*asinh(c*x)**2, x) + Integral(6*a*b*c**2*x*asinh(c*x), x) + Integral(
6*a*b*c**4*x**3*asinh(c*x), x) + Integral(2*a*b*c**6*x**5*asinh(c*x), x))
```

### 3.221.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x} dx = \int \frac{(c^2dx^2+d)^3 (b\,\text{arsinh}\,(cx)+a)^2}{x} dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/6*a^2*c^6*d^3*x^6 + 3/4*a^2*c^4*d^3*x^4 + 3/2*a^2*c^2*d^3*x^2 + a^2*d^3*
log(x) + integrate(b^2*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*
c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x^3*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 6*a*b*c^4*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2
*c^2*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b*c^2*d^3*x*log(c*x + sqrt
(c^2*x^2 + 1)) + b^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d^3*log(
c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.221.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x} dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2 (d\,c^2\,x^2+d)^3}{x} dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x, x)
```

# 3.222 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$



## 3.222.1 Optimal result

Integrand size = 26, antiderivative size = 307

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx &= \frac{122}{25}b^2c^2d^3x+\frac{14}{75}b^2c^4d^3x^3+\frac{2}{125}b^2c^6d^3x^5 \\ &\quad -\frac{22}{5}bcd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{2}{5}bcd^3\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx)) \\ &\quad -\frac{2}{25}bcd^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx)) \\ &\quad +\frac{16}{5}c^2d^3x(a+b\text{arcsinh}(cx))^2 \\ &\quad +\frac{8}{5}c^2d^3x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^2 \\ &\quad +\frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^2 \\ &\quad -\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{x} \\ &\quad -4bcd^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right) \\ &\quad -2b^2cd^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right) \\ &\quad +2b^2cd^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
122/25*b^2*c^2*d^3*x+14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-2/5*b*c*d
^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-2/25*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+
b*arcsinh(c*x))+16/5*c^2*d^3*x*(a+b*arcsinh(c*x))^2+8/5*c^2*d^3*x*(c^2*x^2
+1)*(a+b*arcsinh(c*x))^2+6/5*c^2*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-
d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x-4*b*c*d^3*(a+b*arcsinh(c*x))*arct
anh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2
*b^2*c*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-22/5*b*c*d^3*(a+b*arcsinh(c*x)
)*(c^2*x^2+1)^(1/2)
```

### 3.222.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.52

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \frac{1}{720}d^3\Bigg(-\frac{720a^2}{x}+2160a^2c^2x+3460b^2c^2x+720a^2c^4x^3+144a^2c^6x^5-\frac{17568}{5}abc\sqrt{1+c^2x^2}-\frac{2016}{5}abc^3x^2\sqrt{1+c^2x^2}-\frac{288}{5}abc^5x^4\sqrt{1+c^2x^2}-\frac{1440ab\text{arcsinh}(cx)}{x}+4320abc^2x\text{arcsinh}(cx)+1440abc^4x^3\text{arcsinh}(cx)+288abc^6x^5\text{arcsinh}(cx)-3420b^2c\sqrt{1+c^2x^2}\text{arcsinh}(cx)-\frac{720b^2\text{arcsinh}(cx)^2}{x}+1890b^2c^2x\text{arcsinh}(cx)^2-1440abc\text{arctanh}\left(\sqrt{1+c^2x^2}\right)+80b^2c^2x\cosh(2\text{arcsinh}(cx))+360b^2c^2x\text{arcsinh}(cx)^2\cosh(2\text{arcsinh}(cx))-90b^2c\text{arcsinh}(cx)\cosh(3\text{arcsinh}(cx))-\frac{18}{5}b^2c\text{arcsinh}(cx)\cosh(5\text{arcsinh}(cx))+1440b^2c\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)-1440b^2c\text{arcsinh}(cx)\log\left(1+e^{-\text{arcsinh}(cx)}\right)+1440b^2c\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(cx)}\right)-1440b^2c\,\text{PolyLog}\left(2,e^{-\text{arcsinh}(cx)}\right)-10b^2c\sinh(3\text{arcsinh}(cx))-45b^2c\text{arcsinh}(cx)^2\sinh(3\text{arcsinh}(cx))+\frac{18}{25}b^2c\sinh(5\text{arcsinh}(cx))+9b^2c\text{arcsinh}(cx)^2\sinh(5\text{arcsinh}(cx))\Bigg)$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
(d^3*((-720*a^2)/x + 2160*a^2*c^2*x + 3460*b^2*c^2*x + 720*a^2*c^4*x^3 + 1
44*a^2*c^6*x^5 - (17568*a*b*c*Sqrt[1 + c^2*x^2])/5 - (2016*a*b*c^3*x^2*Sqr
t[1 + c^2*x^2])/5 - (288*a*b*c^5*x^4*Sqrt[1 + c^2*x^2])/5 - (1440*a*b*ArcS
inh[c*x])/x + 4320*a*b*c^2*x*ArcSinh[c*x] + 1440*a*b*c^4*x^3*ArcSinh[c*x]
+ 288*a*b*c^6*x^5*ArcSinh[c*x] - 3420*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
 - (720*b^2*ArcSinh[c*x]^2)/x + 1890*b^2*c^2*x*ArcSinh[c*x]^2 - 1440*a*b*c
*ArcTanh[Sqrt[1 + c^2*x^2]] + 80*b^2*c^2*x*Cosh[2*ArcSinh[c*x]] + 360*b^2*
c^2*x*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] - 90*b^2*c*ArcSinh[c*x]*Cosh[3*A
rcSinh[c*x]] - (18*b^2*c*ArcSinh[c*x]*Cosh[5*ArcSinh[c*x]])/5 + 1440*b^2*c
*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 1440*b^2*c*ArcSinh[c*x]*Log[1 +
 E^(-ArcSinh[c*x])] + 1440*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 1440*b^2
*c*PolyLog[2, E^(-ArcSinh[c*x])] - 10*b^2*c*Sinh[3*ArcSinh[c*x]] - 45*b^2*
c*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]] + (18*b^2*c*Sinh[5*ArcSinh[c*x]])/25
 + 9*b^2*c*ArcSinh[c*x]^2*Sinh[5*ArcSinh[c*x]]))/720
```

### 3.222.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.17 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.47, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {6222, 27, 6201, 6201, 6187, 6213, 24, 210, 2009, 6223, 210, 2009, 6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^3(a+b\text{arcsinh}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{6222}$$

$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx+6c^2d\int d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx-\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{27}$$

$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx+6c^2d^3\int \left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx-\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6201}$$

$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx+6c^2d^3\left(-\frac{2}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}x\left(c^2x^2+1\right)^2(a\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6201}$$

$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx+6c^2d^3\left(-\frac{2}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}\left(-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}\int (a+b\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6187}$$

$$6c^2d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\right.\right.$$
$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6213}$$

$$6c^2d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+}{3c^2}\right.\right.\right.$$
$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{24}$$

$$6c^2d^3\left(\frac{4}{5}\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\right.\right.$$
$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{210}$$

---

3.222. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx+$$
$$6c^2d^3\left(\frac{4}{5}\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}$$

$\downarrow$ 2009

$$2bcd^3\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$\downarrow$ 6223

$$2bcd^3\left(\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{5}bc\int \left(c^2x^2+1\right)^2dx+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))\right)-$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$\downarrow$ 210

$$2bcd^3\left(\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{5}bc\int \left(c^4x^4+2c^2x^2+1\right)dx+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx)\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$\downarrow$ 2009

$$2bcd^3\left(\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{1}{5}bc\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)\right)-$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

---

3.222. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$$\Big\downarrow \text{6223}$$

$$2bcd^3\left(\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{3}bc\int \left(c^2x^2+1\right)dx+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(c^2x^2+\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2bcd^3\left(\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{6221}$$

$$2bcd^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int 1dx+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{24}$$

$$2bcd^3\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{6231}$$

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## 3.222. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$2bcd^3\left(\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$

$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$2bcd^3\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(c^2x^2+1\right)^{3/}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$

$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$2bcd^3\left(i\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{5}\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{1}{3}\left(c^2x^2+1\right)^{3/}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$

$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{4670}$$

$$2bcd^3\left(i\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$

$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \text{2715}$$

---

3.222. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$2bcd^3\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

$$\Big\downarrow \; 2838$$

$$2bcd^3\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{x}+$$
$$6c^2d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
-((d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/x) + 6*c^2*d^3*((x*(1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*(-1/5*(b*(x + (2*c^2*x^3)/3 + (
c^4*x^5)/5))/c + ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2)))/5 +
(4*((x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*(-1/3*(b*(x + (c^2
*x^3)/3))/c + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2)))/3 + (2*
(x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x]))/c^2)))/3))/5) + 2*b*c*d^3*(-(b*c*x) - (b*c*(x + (c^2*x^3)/3
))/3 - (b*c*(x + (2*c^2*x^3)/3 + (c^4*x^5)/5))/5 + Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((1 + c^2
*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/5 + I*((2*I)*(a + b*ArcSinh[c*x])*ArcTan
h[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^Arc
Sinh[c*x]]))
```

## 3.222. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

### 3.222.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

---

3.222. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

## 3.222. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

### 3.222.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.51

| method | result |
|---|---|
| derivativedivides | $c\left(d^3a^2\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+\frac{14d^3b^2c^3x^3}{75}+\frac{122d^3b^2cx}{25}+2d^3b^2\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2}\right.\right.$ |
| default | $c\left(d^3a^2\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+\frac{14d^3b^2c^3x^3}{75}+\frac{122d^3b^2cx}{25}+2d^3b^2\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2}\right.\right.$ |
| parts | $d^3a^2\left(\frac{c^6x^5}{5}+c^4x^3+3c^2x-\frac{1}{x}\right)-2d^3b^2c\operatorname{arcsinh}(cx)\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2d^3b^2c$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(d^3*a^2*(1/5*c^5*x^5+c^3*x^3+3*c*x-1/c/x)+14/75*d^3*b^2*c^3*x^3+122/25*
d^3*b^2*c*x+2*d^3*b^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*d^3*b^2*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))-2/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4*x
^4-14/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+2*d^3*b^2*arcsinh(
c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2/125*d^3*b^2*c^5*x^5-2*d^3*b^2*arcsinh(c
*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-d^3*b^2*arcsinh(c*x)^2/c/x+1/5*d^3*b^2*arc
sinh(c*x)^2*c^5*x^5+d^3*b^2*arcsinh(c*x)^2*c^3*x^3+3*d^3*b^2*arcsinh(c*x)^
2*c*x-122/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*d^3*a*b*(1/5*arcsinh
(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/2
5*c^4*x^4*(c^2*x^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+
1)^(1/2)-arctanh(1/(c^2*x^2+1)^(1/2))))
```

### 3.222.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^
3 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*ar
csinh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
+ a*b*d^3)*arcsinh(c*x))/x^2, x)
```

### 3.222.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = d^3\left(\int 3a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int 3a^2c^4x^2\,dx + \int a^2c^6x^4\,dx + \int 3b^2c^2\operatorname{asinh}^2(cx)\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^2}\,dx + \int 6abc^2\operatorname{asinh}(cx)\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^2}\,dx + \int 3b^2c^4x^2\operatorname{asinh}^2(cx)\,dx + \int b^2c^6x^4\operatorname{asinh}^2(cx)\,dx + \int 6abc^4x^2\operatorname{asinh}(cx)\,dx + \int 2abc^6x^4\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**2,x)
```

output
```
d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*
c**4*x**2, x) + Integral(a**2*c**6*x**4, x) + Integral(3*b**2*c**2*asinh(c
*x)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*as
inh(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x) + Integral(3*b**2*c**4*x
**2*asinh(c*x)**2, x) + Integral(b**2*c**6*x**4*asinh(c*x)**2, x) + Integr
al(6*a*b*c**4*x**2*asinh(c*x), x) + Integral(2*a*b*c**6*x**4*asinh(c*x), x
))
```

### 3.222.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2} dx = \int \frac{(c^2dx^2+d)^3 (b\,\text{arsinh}\,(cx)+a)^2}{x^2} dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
1/5*a^2*c^6*d^3*x^5 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4
/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^6*d
^3 + a^2*c^4*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/
c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^3 + 3*b^2*c^2*d^3*x*arcsinh(c*x)
^2 + 6*b^2*c^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^2*d^3*
x + 6*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^3 - 2*(c*arcsinh(1/(c
*abs(x))) + arcsinh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/5*(b^2*c^6*d^3*x^6 + 5
*b^2*c^4*d^3*x^4 - 5*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate
(2/5*(b^2*c^9*d^3*x^8 + 6*b^2*c^7*d^3*x^6 + 5*b^2*c^5*d^3*x^4 - 5*b^2*c^3*
d^3*x^2 - 5*b^2*c*d^3 + (b^2*c^8*d^3*x^7 + 5*b^2*c^6*d^3*x^5 - 5*b^2*c^2*d
^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (
c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.222.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^3}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2, x)
```

# 3.223 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$



## 3.223.1 Optimal result

Integrand size = 26, antiderivative size = 354

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx =& \frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4\\&-\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\&+\frac{7}{8}bc^3d^3x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\\&-\frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{x}\\&-\frac{3}{32}c^2d^3(a+b\text{arcsinh}(cx))^2\\&+\frac{3}{2}c^2d^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\\&+\frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2\\&-\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{2x^2}\\&+\frac{c^2d^3(a+b\text{arcsinh}(cx))^3}{b}\\&+3c^2d^3(a+b\text{arcsinh}(cx))^2\log\left(1-e^{-2\text{arcsinh}(cx)}\right)\\&+b^2c^2d^3\log(x)\\&-3bc^2d^3(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)\\&-\frac{3}{2}b^2c^2d^3\text{PolyLog}\left(3,e^{-2\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
21/32*b^2*c^4*d^3*x^2+1/32*b^2*c^6*d^3*x^4+7/8*b*c^3*d^3*x*(c^2*x^2+1)^(3/
2)*(a+b*arcsinh(c*x))-b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/x-3/32*
c^2*d^3*(a+b*arcsinh(c*x))^2+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+
3/4*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-1/2*d^3*(c^2*x^2+1)^3*(a+b*
arcsinh(c*x))^2/x^2+c^2*d^3*(a+b*arcsinh(c*x))^3/b+3*c^2*d^3*(a+b*arcsinh(
c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^3*ln(x)-3*b*c^2*d^3*(a
+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b^2*c^2*d^3*po
lylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/16*b*c^3*d^3*x*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)
```

## 3.223.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.40

$$\int \frac{(d+c^2dx^2)^3\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$$
$$= \frac{d^3\left(-128a^2+384a^2c^4x^4+64a^2c^6x^6-256abcx\sqrt{1+c^2x^2}-336abc^3x^3\sqrt{1+c^2x^2}-32abc^5x^5\sqrt{1+c^2x^2}-\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
(d^3*(-128*a^2 + 384*a^2*c^4*x^4 + 64*a^2*c^6*x^6 - 256*a*b*c*x*Sqrt[1 + c
^2*x^2] - 336*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 32*a*b*c^5*x^5*Sqrt[1 + c^2*
x^2] - 256*a*b*ArcSinh[c*x] + 768*a*b*c^4*x^4*ArcSinh[c*x] + 128*a*b*c^6*x
^6*ArcSinh[c*x] - 256*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 128*b^2*Arc
Sinh[c*x]^2 - 768*a*b*c^2*x^2*ArcSinh[c*x]^2 - 256*b^2*c^2*x^2*ArcSinh[c*x
]^3 + 80*b^2*c^2*x^2*Cosh[2*ArcSinh[c*x]] + 160*b^2*c^2*x^2*ArcSinh[c*x]^2
*Cosh[2*ArcSinh[c*x]] + b^2*c^2*x^2*Cosh[4*ArcSinh[c*x]] + 8*b^2*c^2*x^2*A
rcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + 1536*a*b*c^2*x^2*ArcSinh[c*x]*Log[1 -
 E^(2*ArcSinh[c*x])] + 768*b^2*c^2*x^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh
[c*x])] + 768*a^2*c^2*x^2*Log[x] + 256*b^2*c^2*x^2*Log[c*x] - 336*a*b*c^2*
x^2*Log[-(c*x) + Sqrt[1 + c^2*x^2]] + 768*b*c^2*x^2*(a + b*ArcSinh[c*x])*P
olyLog[2, E^(2*ArcSinh[c*x])] - 384*b^2*c^2*x^2*PolyLog[3, E^(2*ArcSinh[c*
x])] - 160*b^2*c^2*x^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 4*b^2*c^2*x^2*A
rcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(256*x^2)
```

### 3.223.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3 (a+b\text{arcsinh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{6222}$$

$$bcd^3\int \frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}dx+3c^2d\int \frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$bcd^3\int \frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x^2}dx+3c^2d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6222}$$

$$bcd^3\left(5c^2\int (c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx+bc\int \frac{(c^2x^2+1)^2}{x}dx-\frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x}\right)+3c^2d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$bcd^3\left(5c^2\int (c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{2}bc\int \frac{(c^2x^2+1)^2}{x^2}dx^2-\frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x}\right)+3c^2d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{49}$$

$$3c^2d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x}dx+bcd^3\left(5c^2\int (c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{2}bc\int \left(x^2c^4+2c^2+\frac{1}{x^2}\right)dx^2-\frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(c$$

$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$3c^2d^3\int \frac{(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^3\left(5c^2\int (c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x}+\frac{1}{2}bc\left(\frac{c^4x^4}{2}+2c^2x^2+\log (x\right.\right.$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 6201

$$3c^2d^3\int \frac{(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^3\left(5c^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x(c^2x^2+1)\,dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\right)-\right.$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 244

$$3c^2d^3\int \frac{(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^3\left(5c^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int (c^2x^3+x)\,dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))\right)-\right.$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 2009

$$3c^2d^3\int \frac{(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^3\left(5c^2\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)-\frac{(c\right.$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 6200

$$3c^2d^3\int \frac{(c^2x^2+1)^2 (a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}(a+\right.\right.$$
$$\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 15

---

3.223. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$3c^2d^3\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$bcd^3\left(5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+\text{barc}\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 6198

$$3c^2d^3\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barcs}\right.\right.\right.$$

$\downarrow$ 6223

$$3c^2d^3\left(-\frac{1}{2}bc\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+\text{ba}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barcs}\right.\right.\right.$$

$\downarrow$ 6201

$$3c^2d^3\left(-\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx)\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barcs}\right.\right.\right.$$

$\downarrow$ 244

$$3c^2d^3\left(\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int \left(c^2x^3+x\right)dx+\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+\text{barcs}\right.\right.\right.$$

$\downarrow$ 2009

---

3.223. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$3c^2d^3\left(\int \frac{(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\,(a+$$
$$\frac{d^3(c^2x^2+1)^3\,(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{(c^2x^2+1)^{5/2}\,(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\big\downarrow$ 6200

$$3c^2d^3\left(\int \frac{(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a-\right.\right.\right.$$
$$\frac{d^3(c^2x^2+1)^3\,(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{(c^2x^2+1)^{5/2}\,(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\big\downarrow$ 15

$$3c^2d^3\left(\int \frac{(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx\right.\right.\right.$$
$$\frac{d^3(c^2x^2+1)^3\,(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{(c^2x^2+1)^{5/2}\,(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\big\downarrow$ 6198

$$3c^2d^3\left(\int \frac{(c^2x^2+1)\,(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{4}(c^2x^2+1)^2\,(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(\frac{1}{4}x(c^2x^2+1)^{3/2}\,(a+b\text{arc}\right.\right.$$
$$\frac{d^3(c^2x^2+1)^3\,(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{(c^2x^2+1)^{5/2}\,(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x(c^2x^2+1)^{3/2}\,(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\big\downarrow$ 6223

3.223. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$3c^2d^3\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\int\frac{(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\downarrow$ 6190

$$3c^2d^3\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{\int-(a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}}{b}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\downarrow$ 25

$$3c^2d^3\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int(a+b\text{arcsinh}(cx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(}{b}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\downarrow$ 3042

$$3c^2d^3\left(-bc\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{\int-i(a+b\text{arcsinh}(cx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b}{b}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$
$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$\downarrow$ 26

3.223. $\int\frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$3c^2d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\int (a+b\text{arcsinh}(cx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+\right.}{b}$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$$\Big\downarrow \; 4201$$

$$3c^2d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))\right.}{b}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$$\Big\downarrow \; 2620$$

$$3c^2d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\int (a+b\text{arcsinh}(cx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a\right.\right.}{}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$$\Big\downarrow \; 3011$$

$$3c^2d^3\left(-bc\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}-\right.\right.\right.\right.}{}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcs}\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

---

3.223. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$3c^2d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+bar\right.\right.\right.}{}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barcs\right.\right.\right.$$

$$\Big\downarrow \text{ 6200}$$

$$3c^2d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+bar\right.\right.\right.}{}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barcs\right.\right.\right.$$

$$\Big\downarrow \text{ 15}$$

$$3c^2d^3\left(\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\text{PolyLog}(2,-a-b\text{arcsinh}(cx))de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}+\frac{1}{2}b(a+bar\right.\right.\right.}{}\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{2x^2}+$$

$$bcd^3\left(-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{x}+5c^2\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+barcs\right.\right.\right.$$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
$Aborted
```

3.223. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

### 3.223.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

## 3.223. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

---

## 3.223. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^3} dx$

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

### 3.223.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(357) = 714$.

Time = 0.34 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.18

| method | result |
|---|---|
| derivativedivides | $c^{2}\left(d^{3}a^{2}\left(\frac{c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}+3\ln\left(cx\right)-\frac{1}{2c^{2}x^{2}}\right)+d^{3}b^{2}\operatorname{arcsinh}\left(cx\right)-\frac{d^{3}ab\,c^{3}x^{3}\sqrt{c^{2}x^{2}+1}}{8}-\frac{21d^{3}abc$ |
| default | $c^{2}\left(d^{3}a^{2}\left(\frac{c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}+3\ln\left(cx\right)-\frac{1}{2c^{2}x^{2}}\right)+d^{3}b^{2}\operatorname{arcsinh}\left(cx\right)-\frac{d^{3}ab\,c^{3}x^{3}\sqrt{c^{2}x^{2}+1}}{8}-\frac{21d^{3}abc$ |
| parts | $-\frac{d^{3}ab\operatorname{arcsinh}(cx)}{x^{2}}+\frac{21d^{3}ab\,c^{2}\operatorname{arcsinh}(cx)}{16}-3d^{3}ab\,c^{2}\operatorname{arcsinh}\left(cx\right)^{2}+6d^{3}ab\,c^{2}\operatorname{polylog}\left(2,-cx-$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^3*a^2*(1/4*c^4*x^4+3/2*c^2*x^2+3*ln(c*x)-1/2/c^2/x^2)+d^3*b^2*arcsi
nh(c*x)-1/8*d^3*a*b*c^3*x^3*(c^2*x^2+1)^(1/2)-21/16*d^3*a*b*c*x*(c^2*x^2+1
)^(1/2)-6*d^3*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-d^3*b^2*arcsinh(c*x)^3
+21/32*d^3*b^2*arcsinh(c*x)^2-6*d^3*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))+6
*d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+6*d^3*a*b*arcsinh(c*x)*l
n(1-c*x-(c^2*x^2+1)^(1/2))+1/4*d^3*b^2*arcsinh(c*x)^2*c^4*x^4+3/2*d^3*b^2*
arcsinh(c*x)^2*c^2*x^2+1/2*d^3*a*b*arcsinh(c*x)*c^4*x^4+3*d^3*a*b*arcsinh(
c*x)*c^2*x^2-1/8*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-21/16*d^3*
b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+d^3*a*b+81/256*d^3*b^2-d^3*b^2*arcs
inh(c*x)/c/x*(c^2*x^2+1)^(1/2)-d^3*a*b*arcsinh(c*x)/c^2/x^2+d^3*b^2*ln(c*x
+(c^2*x^2+1)^(1/2)-1)+d^3*b^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*d^3*b^2*ln(c*x
+(c^2*x^2+1)^(1/2))+21/32*b^2*c^2*d^3*x^2+1/32*b^2*c^4*d^3*x^4+3*d^3*b^2*a
rcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+6*d^3*b^2*arcsinh(c*x)*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))+21/16*d^3*a*b*arcsinh(c*x)-3*d^3*a*b*arcsinh(c*x)
^2+6*d^3*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+6*d^3*a*b*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))-d^3*a*b/c/x*(c^2*x^2+1)^(1/2)+3*d^3*b^2*arcsinh(c*x)^2*ln(
1+c*x+(c^2*x^2+1)^(1/2))+6*d^3*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^
(1/2))-1/2*d^3*b^2*arcsinh(c*x)^2/c^2/x^2)
```

### 3.223.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^3(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^
3 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*ar
csinh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
+ a*b*d^3)*arcsinh(c*x))/x^3, x)
```

### 3.223.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = d^3\left(\int \frac{a^2}{x^3}\,dx + \int \frac{3a^2c^2}{x}\,dx + \int 3a^2c^4x\,dx + \int a^2c^6x^3\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{3b^2c^2\operatorname{asinh}^2(cx)}{x}\,dx + \int 3b^2c^4x\operatorname{asinh}^2(cx)\,dx + \int b^2c^6x^3\operatorname{asinh}^2(cx)\,dx + \int \frac{6abc^2\operatorname{asinh}(cx)}{x}\,dx + \int 6abc^4x\operatorname{asinh}(cx)\,dx + \int 2abc^6x^3\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**3,x)
```

output
```
d**3*(Integral(a**2/x**3, x) + Integral(3*a**2*c**2/x, x) + Integral(3*a**
2*c**4*x, x) + Integral(a**2*c**6*x**3, x) + Integral(b**2*asinh(c*x)**2/x
**3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(3*b**2*c**2*asinh(
c*x)**2/x, x) + Integral(3*b**2*c**4*x*asinh(c*x)**2, x) + Integral(b**2*c
**6*x**3*asinh(c*x)**2, x) + Integral(6*a*b*c**2*asinh(c*x)/x, x) + Integr
al(6*a*b*c**4*x*asinh(c*x), x) + Integral(2*a*b*c**6*x**3*asinh(c*x), x))
```

### 3.223.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/4*a^2*c^6*d^3*x^4 + 3/2*a^2*c^4*d^3*x^2 + 3*a^2*c^2*d^3*log(x) - a*b*d^3
*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a^2*d^3/x^2 + integrate(
b^2*c^6*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^6*d^3*x^3*log(c*x
 + sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6
*a*b*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2*d^3*log(c*x + sqrt
(c^2*x^2 + 1))^2/x + 6*a*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d^
3*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

### 3.223.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^3}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3, x)
```

# 3.224 $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$



## 3.224.1 Optimal result

Integrand size = 26, antiderivative size = 326

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx &= -\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3\\
&\quad-5bc^3d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\\
&\quad+\frac{1}{9}bc^3d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\\
&\quad-\frac{bcd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{3x^2}\\
&\quad+\frac{16}{3}c^4d^3x(a+b\text{arcsinh}(cx))^2\\
&\quad+\frac{8}{3}c^4d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2\\
&\quad-\frac{2c^2d^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{x}\\
&\quad-\frac{d^3(1+c^2x^2)^3(a+b\text{arcsinh}(cx))^2}{3x^3}\\
&\quad-\frac{34}{3}bc^3d^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)\\
&\quad-\frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)\\
&\quad+\frac{17}{3}b^2c^3d^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d^3/x+50/9*b^2*c^4*d^3*x+2/27*b^2*c^6*d^3*x^3+1/9*b*c^3*d^3*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/3*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))/x^2+16/3*c^4*d^3*x*(a+b*arcsinh(c*x))^2+8/3*c^4*d^3*x*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^2-2*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-1/3
*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x^3-34/3*b*c^3*d^3*(a+b*arcsinh(c*
x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-17/3*b^2*c^3*d^3*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))+17/3*b^2*c^3*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-5*b*c^3*d^3*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

### 3.224.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.41

$$\int \frac{(d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^2}{x^4} dx$$
$$= \frac{d^3(-9a^2 - 81a^2c^2x^2 - 9b^2c^2x^2 + 81a^2c^4x^4 + 150b^2c^4x^4 + 9a^2c^6x^6 + 2b^2c^6x^6 - 9abcx\sqrt{1 + c^2x^2} - 150abc}{}$$

input
```
Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
(d^3*(-9*a^2 - 81*a^2*c^2*x^2 - 9*b^2*c^2*x^2 + 81*a^2*c^4*x^4 + 150*b^2*c
^4*x^4 + 9*a^2*c^6*x^6 + 2*b^2*c^6*x^6 - 9*a*b*c*x*Sqrt[1 + c^2*x^2] - 150
*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 6*a*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 18*a*b*
ArcSinh[c*x] - 162*a*b*c^2*x^2*ArcSinh[c*x] + 162*a*b*c^4*x^4*ArcSinh[c*x]
 + 18*a*b*c^6*x^6*ArcSinh[c*x] - 9*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
- 150*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^5*x^5*Sqrt[1 +
c^2*x^2]*ArcSinh[c*x] - 9*b^2*ArcSinh[c*x]^2 - 81*b^2*c^2*x^2*ArcSinh[c*x]
^2 + 81*b^2*c^4*x^4*ArcSinh[c*x]^2 + 9*b^2*c^6*x^6*ArcSinh[c*x]^2 - 153*a*
b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] + 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1
- E^(-ArcSinh[c*x])] - 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*
x])] + 153*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 153*b^2*c^3*x^3*Po
lyLog[2, E^(-ArcSinh[c*x])]))/(27*x^3)
```

### 3.224.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.62, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {6222, 27, 6222, 244, 2009, 6201, 6187, 6213, 24, 2009, 6223, 2009, 6221, 24, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3 (a+b\text{arcsinh}(cx))^2}{x^4} \, dx$$

$$\Big\downarrow \text{6222}$$

$$\frac{2}{3}bcd^3\int \frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}dx+2c^2d\int \frac{d^2(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$2c^2d^3\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^2}{x^2}dx+\frac{2}{3}bcd^3\int \frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{x^3}dx-\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6222}$$

$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int \frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bc\int \frac{(c^2x^2+1)^2}{x^2}dx-\frac{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}\right)+$$
$$2c^2d^3\left(4c^2\int (c^2x^2+1)(a+b\text{arcsinh}(cx))^2dx+2bc\int \frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{(c^2x^2+1)^2(a+ba}{x}\right.$$
$$-\frac{d^3(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{244}$$

$$2c^2d^3\left(4c^2\int\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+2bc\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^2(a+ba}{x}\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bc\int\left(x^2c^4+2c^2+\frac{1}{x^2}\right)dx-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(}{2x^2}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2009$$

$$2c^2d^3\left(4c^2\int\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+2bc\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^2(a+ba}{x}\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\right)$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 6201$$

$$2c^2d^3\left(4c^2\left(-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}\int(a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\right)$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 6187$$

$$2c^2d^3\left(4c^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\right)$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 6213$$

---

3.224. $\int\frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$2c^2d^3\left(4c^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a}{3c}\right.\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 24$$

$$2c^2d^3\left(4c^2\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int\left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2009$$

$$2c^2d^3\left(2bc\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2}{x}+4c^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{a}\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\int\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{x}dx-\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(\frac{c^4x^3}{3}+2c^2x-\frac{1}{x}\right)\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 6223$$

$$2c^2d^3\left(2bc\left(\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{3}bc\int\left(c^2x^2+1\right)dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)-\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx-\frac{1}{3}bc\int\left(c^2x^2+1\right)dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)-\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2009$$

3.224. $\int\frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$2c^2d^3\left(2bc\left(\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\left(\frac{c^2x^3}{3}+x\right)\right)-\frac{(c^2x^2}{}\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\left(\frac{c^2x^3}{3}+x\right)\right)-\frac{(c^2x}{}\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6221

$$2c^2d^3\left(2bc\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int 1dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int 1dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(c\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 24

$$2c^2d^3\left(2bc\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\left(\frac{c}{}\right.\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\left(\right.\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6231

$$2c^2d^3\left(2bc\left(\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 3042

---

3.224. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$2c^2d^3\left(2bc\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 26

$$2c^2d^3\left(2bc\left(i\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(i\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{1}{3}\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 4670

$$2c^2d^3\left(2bc\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsi}}\right.\right.\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arc}}\right.\right.\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2715

$$2c^2d^3\left(2bc\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(}\right.\right.\right.$$
$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}}\right.\right.\right.$$
$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2838

3.224. $\int \frac{\left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$2c^2d^3\left(2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx}\right.\right.\right.\right.$$

$$\frac{2}{3}bcd^3\left(\frac{5}{2}c^2\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c}\right.\right.\right.\right.$$

$$\frac{d^3\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2}{3x^3}$$

input
```
Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3 + 2*c^2*d^3*(-(((1 +
 c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/x) + 4*c^2*((x*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/3 - (2*b*c*(-1/3*(b*(x + (c^2*x^3)/3))/c + ((1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSinh[c*x])^2 - 2*
b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2)))/3) + 2*b
*c*(-(b*c*x) - (b*c*(x + (c^2*x^3)/3))/3 + Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + I*((2*I)*(a + b*A
rcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I
*b*PolyLog[2, E^ArcSinh[c*x]]))) + (2*b*c*d^3*((b*c*(-x^(-1) + 2*c^2*x + (
c^4*x^3)/3))/2 - ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (5*c
^2*(-(b*c*x) - (b*c*(x + (c^2*x^3)/3))/3 + Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + I*((2*I)*(a + b*A
rcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I
*b*PolyLog[2, E^ArcSinh[c*x]])))/2))/3
```

### 3.224.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

## 3.224. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

---

## 3.224. $\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

### 3.224.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| derivativedivides | $c^3 \left( d^3 a^2 \left( \frac{c^3 x^3}{3} + 3cx - \frac{1}{3c^3 x^3} - \frac{3}{cx} \right) - \frac{d^3 b^2 \operatorname{arcsinh}(cx) \sqrt{c^2 x^2+1}}{3c^2 x^2} - \frac{2 d^3 b^2 \operatorname{arcsinh}(cx) \sqrt{c^2 x^2+1}\, c^2 x^2}{9} + \right.$ |
| default | $c^3 \left( d^3 a^2 \left( \frac{c^3 x^3}{3} + 3cx - \frac{1}{3c^3 x^3} - \frac{3}{cx} \right) - \frac{d^3 b^2 \operatorname{arcsinh}(cx) \sqrt{c^2 x^2+1}}{3c^2 x^2} - \frac{2 d^3 b^2 \operatorname{arcsinh}(cx) \sqrt{c^2 x^2+1}\, c^2 x^2}{9} + \right.$ |
| parts | $d^3 a^2 \left( \frac{c^6 x^3}{3} + 3c^4 x - \frac{3c^2}{x} - \frac{1}{3x^3} \right) - \frac{d^3 b^2 c \operatorname{arcsinh}(cx) \sqrt{c^2 x^2+1}}{3x^2} - \frac{2 d^3 b^2 c^5 \operatorname{arcsinh}(cx) \sqrt{c^2 x^2+1}\, x^2}{9} - \frac{17b^3}{}$ |

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(d^3*a^2*(1/3*c^3*x^3+3*c*x-1/3/c^3/x^3-3/c/x)-1/3*d^3*b^2/c^2/x^2*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)-2/9*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2
*x^2+2/27*d^3*b^2*c^3*x^3-1/3*d^3*b^2/c/x+50/9*d^3*b^2*c*x-3*d^3*b^2*arcsi
nh(c*x)^2/c/x-1/3*d^3*b^2/c^3/x^3*arcsinh(c*x)^2+1/3*d^3*b^2*arcsinh(c*x)^
2*c^3*x^3+3*d^3*b^2*arcsinh(c*x)^2*c*x-50/9*d^3*b^2*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)+17/3*d^3*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-17/3*d^3*b^
2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+17/3*d^3*b^2*polylog(2,c*x+(c^2
*x^2+1)^(1/2))-17/3*d^3*b^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d^3*a*b*(1
/3*arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-1/3*arcsinh(c*x)/c^3/x^3-3*arcs
inh(c*x)/c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-1/6/c^2/
x^2*(c^2*x^2+1)^(1/2)-17/6*arctanh(1/(c^2*x^2+1)^(1/2))))
```

### 3.224.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(c^2dx^2+d)^3 (b\operatorname{arsinh}(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^
3 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*ar
csinh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
+ a*b*d^3)*arcsinh(c*x))/x^4, x)
```

### 3.224.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^3(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = d^3\left(\int 3a^2c^4\,dx + \int \frac{a^2}{x^4}\,dx + \int \frac{3a^2c^2}{x^2}\,dx + \int a^2c^6x^2\,dx + \int 3b^2c^4\operatorname{asinh}^2(cx)\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^4}\,dx + \int 6abc^4\operatorname{asinh}(cx)\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{3b^2c^2\operatorname{asinh}^2(cx)}{x^2}\,dx + \int b^2c^6x^2\operatorname{asinh}^2(cx)\,dx + \int \frac{6abc^2\operatorname{asinh}(cx)}{x^2}\,dx + \int 2abc^6x^2\operatorname{asinh}(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**4,x)
```

output
```
d**3*(Integral(3*a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(3*a**2*
c**2/x**2, x) + Integral(a**2*c**6*x**2, x) + Integral(3*b**2*c**4*asinh(c
*x)**2, x) + Integral(b**2*asinh(c*x)**2/x**4, x) + Integral(6*a*b*c**4*as
inh(c*x), x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(3*b**2*c**2*a
sinh(c*x)**2/x**2, x) + Integral(b**2*c**6*x**2*asinh(c*x)**2, x) + Integr
al(6*a*b*c**2*asinh(c*x)/x**2, x) + Integral(2*a*b*c**6*x**2*asinh(c*x), x
))
```

### 3.224.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(c^2dx^2+d)^3(b\operatorname{arsinh}(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
1/3*a^2*c^6*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c
^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^6*d^3 + 3*b^2*c^4*d^3*x*arcsinh(c*x)^
2 + 6*b^2*c^4*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^4*d^3*x
 + 6*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3*d^3 - 6*(c*arcsinh(1/(
c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d^3 + 1/3*((c^2*arcsinh(1/(c*abs(x)))
 - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/
x - 1/3*a^2*d^3/x^3 + 1/3*(b^2*c^6*d^3*x^6 - 9*b^2*c^2*d^3*x^2 - b^2*d^3)*
log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 - integrate(2/3*(b^2*c^9*d^3*x^8 + b^2*
c^7*d^3*x^6 - 9*b^2*c^5*d^3*x^4 - 10*b^2*c^3*d^3*x^2 - b^2*c*d^3 + (b^2*c^
8*d^3*x^7 - 9*b^2*c^4*d^3*x^3 - b^2*c^2*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))
, x)
```

### 3.224.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3 (a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2}{x^4} \, dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^3}{x^4} \, dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4, x)
```

# 3.225 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2} dx$



## 3.225.1 Optimal result

Integrand size = 26, antiderivative size = 277

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2} dx = &-\frac{22b^2x}{9c^4d}+\frac{2b^2x^3}{27c^2d}+\frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^5d} \\ &-\frac{2bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d} \\ &-\frac{x(a+b\text{arcsinh}(cx))^2}{c^4d}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d} \\ &+\frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^5d} \\ &-\frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} \\ &+\frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^5d} \\ &+\frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^5d} \\ &-\frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^5d}\end{aligned}$$

output
```
-22/9*b^2*x/c^4/d+2/27*b^2*x^3/c^2/d-x*(a+b*arcsinh(c*x))^2/c^4/d+1/3*x^3*
(a+b*arcsinh(c*x))^2/c^2/d+2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(
1/2))/c^5/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))
/c^5/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d
+2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d-2*I*b^2*polylog(3,I*(
c*x+(c^2*x^2+1)^(1/2)))/c^5/d+22/9*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/
c^5/d-2/9*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

### 3.225.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.32

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$$
$$= \frac{-3a^2cx+a^2c^3x^3+3a^2\arctan(cx)-\frac{2}{3}ab\left(-11\sqrt{1+c^2x^2}+c^2x^2\sqrt{1+c^2x^2}+9cx\text{arcsinh}(cx)-3c^3x^3\text{arcsi}\right.}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

output
```
(-3*a^2*c*x + a^2*c^3*x^3 + 3*a^2*ArcTan[c*x] - (2*a*b*(-11*Sqrt[1 + c^2*x
^2] + c^2*x^2*Sqrt[1 + c^2*x^2] + 9*c*x*ArcSinh[c*x] - 3*c^3*x^3*ArcSinh[c
*x] - (9*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (9*I)*ArcSinh[c*x]*Lo
g[1 + I*E^ArcSinh[c*x]] + (9*I)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9*I)*Po
lyLog[2, I*E^ArcSinh[c*x]]))/3 + 3*b^2*((5*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])
/2 - (5*c*x*(2 + ArcSinh[c*x]^2))/4 - (ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]])/
18 + I*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^ArcSinh[
c*x]])) - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E
^ArcSinh[c*x]]) - 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] + 2*PolyLog[3, I/E^Arc
Sinh[c*x]]) + ((2 + 9*ArcSinh[c*x]^2)*Sinh[3*ArcSinh[c*x]])/108))/(3*c^5*d
)
```

### 3.225.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6227, 27, 6227, 15, 6204, 3042, 4668, 3011, 2720, 6213, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2dx^2+d}\,dx$$
$$\Big\downarrow \; 6227$$
$$-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{c^2}-\frac{2b\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3cd}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$
$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{c^2d}-\frac{2b\int \frac{x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3cd}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{6227}$$

$$-\frac{2b\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}\right)}{3cd}-\frac{-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{c}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2}}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{15}$$

$$-\frac{-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{c}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2}}{c^2d}-\frac{2b\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{6204}$$

$$-\frac{2b\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}-\frac{-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{c}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2}}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}-\frac{-\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2}}{c^2d}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{4668}$$

---

3.225. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}dx$

$$-\frac{-\frac{-2ib\int(a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{ar}}\right.}{c^3}}{c^2d}$$

$$\frac{2b\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$\downarrow$ 3011

$$-\frac{-\frac{2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)d\text{ar}\right.}{c^3}}{}$$

$$\frac{2b\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$\downarrow$ 2720

$$-\frac{-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Pol}\right.}{c^3}}{}$$

$$\frac{2b\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$\downarrow$ 6213

$$-\frac{-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Pol}\right.}{c^3}}{}$$

$$\frac{2b\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}+\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$\downarrow$ 24

$$-\frac{-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Pol}\right.}{c^3}}{}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\left(\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}$$

3.225. $\int\frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}dx$

$\downarrow$ 7143

$$-\frac{-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\,\text{Pol}\right.}{c^3}}{c^2d}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\left(\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

output
```
(x^3*(a + b*ArcSinh[c*x])^2)/(3*c^2*d) - (2*b*(-1/9*(b*x^3)/c + (x^2*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) - (2*(-((b*x)/c) + (Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/c^2))/(3*c^2)))/(3*c*d) - ((x*(a + b*ArcSinh[
c*x])^2)/c^2 - (2*b*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/c^2))/c - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((
a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E
^ArcSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c
*x]]) + b*PolyLog[3, I*E^ArcSinh[c*x]]))/c^3)/(c^2*d)
```

### 3.225.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.225. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.225.4 Maple [F]

$$\int \frac{x^4(a+b\,\mathrm{arcsinh}\,(cx))^2}{c^2d\,x^2+d}dx$$

input
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

output
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

### 3.225.5 Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^4}{c^2dx^2+d}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^2*
d*x^2 + d), x)
```

### 3.225.6 Sympy [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{\int \frac{a^2x^4}{c^2x^2+1}\,dx + \int \frac{b^2x^4\,\mathrm{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^4\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2*x**4/(c**2*x**2 + 1), x) + Integral(b**2*x**4*asinh(c*x)**2
/(c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh(c*x)/(c**2*x**2 + 1), x))
/d
```

### 3.225.7 Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{c^2dx^2+d}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/3*a^2*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + integrate(b^2*
x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + s
qrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

### 3.225.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{x^4\,\left(a+b\,\text{asinh}(c\,x)\right)^2}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

output
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

# 3.226 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$



## 3.226.1 Optimal result

Integrand size = 26, antiderivative size = 199

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d} + \frac{(a+b\text{arcsinh}(cx))^2}{4c^4d} + \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d} + \frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d}$$

output
```
1/4*b^2*x^2/c^2/d+1/4*(a+b*arcsinh(c*x))^2/c^4/d+1/2*x^2*(a+b*arcsinh(c*x)
)^2/c^2/d+1/3*(a+b*arcsinh(c*x))^3/b/c^4/d-(a+b*arcsinh(c*x))^2*ln(1+(c*x+
(c^2*x^2+1)^(1/2))^2)/c^4/d-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+
1)^(1/2))^2)/c^4/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/2
*b*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

### 3.226.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.48

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx =$$
$$-\frac{-12a^2c^2x^2+12abcx\sqrt{1+c^2x^2}-24abc^2x^2\text{arcsinh}(cx)-24ab\text{arcsinh}(cx)^2+8b^2\text{arcsinh}(cx)^3-3b^2\cos}{}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

output
```
-1/24*(-12*a^2*c^2*x^2 + 12*a*b*c*x*Sqrt[1 + c^2*x^2] - 24*a*b*c^2*x^2*Arc
Sinh[c*x] - 24*a*b*ArcSinh[c*x]^2 + 8*b^2*ArcSinh[c*x]^3 - 3*b^2*Cosh[2*Ar
cSinh[c*x]] - 6*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + 24*b^2*ArcSinh[c
*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + 48*a*b*ArcSinh[c*x]*Log[1 - I*E^ArcSi
nh[c*x]] + 48*a*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + 12*a^2*Log[1 +
c^2*x^2] + 12*a*b*Log[-(c*x) + Sqrt[1 + c^2*x^2]] - 24*b^2*ArcSinh[c*x]*Po
lyLog[2, -E^(-2*ArcSinh[c*x])] + 48*a*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] +
48*a*b*PolyLog[2, I*E^ArcSinh[c*x]] - 12*b^2*PolyLog[3, -E^(-2*ArcSinh[c*x
])] + 6*b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]])/(c^4*d)
```

### 3.226.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6227, 27, 6212, 3042, 26, 4201, 2620, 3011, 2720, 6227, 15, 6198, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}-\frac{\int \frac{x(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{c^2}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}-\frac{\int \frac{x(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{c^2d}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \; 6212$$

$$-\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}-\frac{\int \frac{cx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^4d}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int -i(a+b\text{arcsinh}(cx))^2\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d}-\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \; 26$$

$$\frac{i\int (a+b\text{arcsinh}(cx))^2\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d}-\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \; 4201$$

$$\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^3}{3b}\right)}{c^4d}-\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \; 2620$$

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\int (a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a-}{}\right.}{c^4d}$$

$$\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \; 3011$$

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(2,\right.\right.\right.\right.}{c^4d}$$

$$\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \; 2720$$

---

3.226. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}dx$

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\right.\right.\right.}{c^4d}$$

$$\frac{b\int\frac{x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$\downarrow$ 6227

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\right.\right.\right.}{c^4d}$$

$$\frac{b\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int xdx}{2c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}\right)}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$\downarrow$ 15

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\right.\right.\right.}{c^4d}$$

$$\frac{b\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{cd}+\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$\downarrow$ 6198

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\right.\right.\right.}{c^4d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{b\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{cd}$$

$\downarrow$ 7143

$$\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(\right.\right.\right.}{c^4d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{b\left(-\frac{(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{cd}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

3.226. $\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

output
```
(x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*d) - (b*(-1/4*(b*x^2)/c + (x*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) - (a + b*ArcSinh[c*x])^2/(4*b*c^3))
)/(c*d) + (I*(((-1/3*I)*(a + b*ArcSinh[c*x])^3)/b + (2*I)*(((a + b*ArcSinh
[c*x])^2*Log[1 + E^(2*ArcSinh[c*x])])/2 - b*(-1/2*((a + b*ArcSinh[c*x])*Po
lyLog[2, -E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(2*ArcSinh[c*x])])/4)))
/(c^4*d)
```

### 3.226.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

---

3.226. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.226. $\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

### 3.226.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b^2\operatorname{arcsinh}(cx)^3}{3d}+\frac{b^2\operatorname{arcsinh}(cx)^2c^2x^2}{2d}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2d}+\frac{b^2\operatorname{arcsinh}(cx)^2}{4d}+\frac{b^2c^2x^2}{4d}+\frac{b^2}{8d}-$ |
| default | $\frac{a^2\left(\frac{c^2x^2}{2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d}+\frac{b^2\operatorname{arcsinh}(cx)^3}{3d}+\frac{b^2\operatorname{arcsinh}(cx)^2c^2x^2}{2d}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{2d}+\frac{b^2\operatorname{arcsinh}(cx)^2}{4d}+\frac{b^2c^2x^2}{4d}+\frac{b^2}{8d}-$ |
| parts | $\frac{a^2\left(\frac{x^2}{2c^2}-\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d}+\frac{b^2\operatorname{arcsinh}(cx)^3}{3d\,c^4}+\frac{b^2\operatorname{arcsinh}(cx)^2x^2}{2d\,c^2}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x}{2d\,c^3}+\frac{b^2x^2}{4c^2d}+\frac{b^2\operatorname{arcsin}}{4d\,c}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2/d*(1/2*c^2*x^2-1/2*ln(c^2*x^2+1))+1/3*b^2/d*arcsinh(c*x)^3+1/2*
b^2/d*arcsinh(c*x)^2*c^2*x^2-1/2*b^2/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+
1/4*b^2/d*arcsinh(c*x)^2+1/4*b^2/d*c^2*x^2+1/8*b^2/d-b^2/d*arcsinh(c*x)^2*
ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))^2)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+a*b/d*arcsi
nh(c*x)^2+a*b/d*arcsinh(c*x)*c^2*x^2-1/2*a*b/d*c*x*(c^2*x^2+1)^(1/2)+1/2*a
*b/d*arcsinh(c*x)-2*a*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-a*b
/d*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2))
```

### 3.226.5 Fricas [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^3}{c^2dx^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^2*
d*x^2 + d), x)
```

### 3.226.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{\int \frac{a^2x^3}{c^2x^2+1}\,dx + \int \frac{b^2x^3\,\text{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^3\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2*x**3/(c**2*x**2 + 1), x) + Integral(b**2*x**3*asinh(c*x)**2
/(c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh(c*x)/(c**2*x**2 + 1), x))
/d
```

### 3.226.7 Maxima [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{c^2dx^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) + 1/2*(b^2*c^2*x^2 - b^2*
log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d) + integrate(-(b^2
*c^2*x^2 - (2*a*b*c^4 - b^2*c^4)*x^4 - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1
) - (b^2*c*x*log(c^2*x^2 + 1) + (2*a*b*c^3 - b^2*c^3)*x^3)*sqrt(c^2*x^2 +
1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d
)*sqrt(c^2*x^2 + 1)), x)
```

### 3.226.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

output
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

# 3.227 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$



## 3.227.1 Optimal result

Integrand size = 26, antiderivative size = 198

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{2b^2x}{c^2d}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d} -\frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d} +\frac{2ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} -\frac{2ib(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d} -\frac{2ib^2\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^3d} +\frac{2ib^2\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^3d}$$

output
```
2*b^2*x/c^2/d+x*(a+b*arcsinh(c*x))^2/c^2/d-2*(a+b*arcsinh(c*x))^2*arctan(c
*x+(c^2*x^2+1)^(1/2))/c^3/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))/c^3/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+
1)^(1/2)))/c^3/d-2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d+2*I*b
^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-2*b*(a+b*arcsinh(c*x))*(c^2*
x^2+1)^(1/2)/c^3/d
```

### 3.227.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.60

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{a^2x}{c^2d} - \frac{a^2\arctan(cx)}{c^3d}$$
$$+ \frac{2ab\left(-\sqrt{1+c^2x^2} + cx\text{arcsinh}(cx) + \frac{1}{2}i\left(-\frac{1}{2}\text{arcsinh}(cx)^2 + 2\text{arcsinh}(cx)\log\left(1+ie^{\text{arcsinh}(cx)}\right) + 2\,\text{PolyLo}\right.\right.}{}$$
$$+ \frac{b^2\left(-2\sqrt{1+c^2x^2}\text{arcsinh}(cx) + cx(2+\text{arcsinh}(cx)^2) - i\left(-\text{arcsinh}(cx)^2\left(\log\left(1-ie^{-\text{arcsinh}(cx)}\right) - \log(1\right.\right.\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

output
```
(a^2*x)/(c^2*d) - (a^2*ArcTan[c*x])/(c^3*d) + (2*a*b*(-Sqrt[1 + c^2*x^2] +
 c*x*ArcSinh[c*x] + (I/2)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 + I*
E^ArcSinh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - (I/2)*(-1/2*ArcSinh
[c*x]^2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2, I*E^ArcS
inh[c*x]])))/(c^3*d) + (b^2*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*(2 +
ArcSinh[c*x]^2) - I*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 +
 I/E^ArcSinh[c*x]])) - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - P
olyLog[2, I/E^ArcSinh[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSinh[c*x]] - PolyLo
g[3, I/E^ArcSinh[c*x]]))))/(c^3*d)
```

### 3.227.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6227, 27, 6204, 3042, 4668, 3011, 2720, 6213, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \; 6227$$

$$-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd} - \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{c^2} + \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{c^2d}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \ 6204$$

$$-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3d}-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \ 4668$$

$$-\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arc}}{c^3d}$$
$$\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \ 3011$$

$$-\frac{2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyL}\right.}{}$$
$$\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \ 2720$$

$$-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib}{}$$
$$\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{cd}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \ 6213$$

$$-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib}{}$$
$$\frac{2b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)}{cd}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$$\Big\downarrow \ 24$$

3.227. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

$$-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib}{}$$

$$\frac{2b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)}{cd}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

$\big\downarrow$ 7143

$$-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+ba\right.}{c^3d}$$

$$\frac{2b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)}{cd}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*b*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x]))/c^2))/(c*d) - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^Ar
cSinh[c*x]] + (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*
x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x]
)*PolyLog[2, I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*E^ArcSinh[c*x]]))/(c^3*d)
```

### 3.227.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.227. $\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.227.4 Maple [F]

$$\int \frac{x^2(a + b\,\mathrm{arcsinh}\,(cx))^2}{c^2 d\,x^2 + d}dx$$

input
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

output
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

### 3.227.5 Fricas [F]

$$\int \frac{x^2(a + b\mathrm{arcsinh}(cx))^2}{d + c^2dx^2}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx) + a)^2x^2}{c^2dx^2 + d}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^2*
d*x^2 + d), x)
```

### 3.227.6 Sympy [F]

$$\int \frac{x^2(a + b\mathrm{arcsinh}(cx))^2}{d + c^2dx^2}\,dx = \frac{\int \frac{a^2x^2}{c^2x^2+1}\,dx + \int \frac{b^2x^2\,\mathrm{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^2\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2*x**2/(c**2*x**2 + 1), x) + Integral(b**2*x**2*asinh(c*x)**2
/(c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh(c*x)/(c**2*x**2 + 1), x))
/d
```

### 3.227.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{c^2dx^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
a^2*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + integrate(b^2*x^2*log(c*x + sqrt(c
^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c
^2*d*x^2 + d), x)
```

### 3.227.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{c^2dx^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d), x)
```

### 3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{x^2\,\left(a+b\,\text{asinh}(c\,x)\right)^2}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

output
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

# 3.228 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$



## 3.228.1 Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=-\frac{(a+b\text{arcsinh}(cx))^3}{3bc^2d}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^2d}+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^2d}-\frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^2d}$$

output
```
-1/3*(a+b*arcsinh(c*x))^3/b/c^2/d+(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+
1)^(1/2))^2)/c^2/d+b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))
^2)/c^2/d-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

## 3.228.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(105) = 210.

Time = 0.21 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.68

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{-6ab\text{arcsinh}(cx)^2-2b^2\text{arcsinh}(cx)^3+12ab\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)+6b^2\text{arcsinh}(cx)^2\log\left(1+\right.}{}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

output
```
(-6*a*b*ArcSinh[c*x]^2 - 2*b^2*ArcSinh[c*x]^3 + 12*a*b*ArcSinh[c*x]*Log[1
+ (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSi
nh[c*x])/Sqrt[-c^2]] + 12*a*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c
*x])/c] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*
a^2*Log[1 + c^2*x^2] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c
*x])/Sqrt[-c^2]] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcS
inh[c*x])/c] - 12*b^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^2*P
olyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*c^2*d)
```

### 3.228.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6212, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \text{6212}$$

$$\frac{\int \frac{cx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i(a+b\text{arcsinh}(cx))^2\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^2d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (a+b\text{arcsinh}(cx))^2\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^2d}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^3}{3b}\right)}{c^2d}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\int(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(}{}\right.}{c^2d}$$

$\Big\downarrow$ 3011

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{2}b\int\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(\right.\right.\right.}{c^2d}$$

$\Big\downarrow$ 2720

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\right.\right.\right.}{c^2d}$$

$\Big\downarrow$ 7143

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right.\right.\right.}{c^2d}$$

input
```
Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

output
```
((-I)*(((-1/3*I)*(a + b*ArcSinh[c*x])^3)/b + (2*I)*(((a + b*ArcSinh[c*x])^
2*Log[1 + E^(2*ArcSinh[c*x])])/2 - b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog[2
, -E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(2*ArcSinh[c*x])])/4))))/(c^2*d
)
```

### 3.228.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.228. $\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

### 3.228.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^2\ln\left(c^2x^2+1\right)}{2d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)-\frac{\operatorname{polylog}}{}\right.}{d}}{c^2}$ |
| default | $\frac{\frac{a^2\ln\left(c^2x^2+1\right)}{2d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)-\frac{\operatorname{polylog}}{}\right.}{d}}{c^2}$ |
| parts | $\frac{a^2\ln(c^2x^2+1)}{2d\,c^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\operatorname{arcsinh}(cx)^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+\right.\right.\right.}{d\,c^2}$ |

input
```
int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*a^2/d*ln(c^2*x^2+1)+b^2/d*(-1/3*arcsinh(c*x)^3+arcsinh(c*x)^2*l
n(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))^2)-1/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b/d*(-1/2*arcsinh(c
*x)^2+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)))
```

### 3.228.5 Fricas [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{c^2dx^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a^2*x)/(c^2*d*x^2
+ d), x)
```

### 3.228.6 Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{\int \frac{a^2x}{c^2x^2+1}\,dx + \int \frac{b^2x\,\text{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2abx\,\text{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2*x/(c**2*x**2 + 1), x) + Integral(b**2*x*asinh(c*x)**2/(c**2
*x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

### 3.228.7 Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{c^2dx^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*b^2*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d) + 1/2*a^2*
log(c^2*d*x^2 + d)/(c^2*d) - integrate(-(2*a*b*c^2*x^2 - (b^2*c^2*x^2 + b^
2)*log(c^2*x^2 + 1) - (b^2*c*x*log(c^2*x^2 + 1) - 2*a*b*c*x)*sqrt(c^2*x^2
+ 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d*x^3 + c^2*d*x + (c^3*d*x^2 + c*d
)*sqrt(c^2*x^2 + 1)), x)
```

### 3.228.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{c^2dx^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d), x)
```

### 3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{x\,(a+b\,\text{asinh}(c\,x))^2}{d\,c^2\,x^2+d}\,dx$$

input
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

output
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

# 3.229 $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$



## 3.229.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

output
```
2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-2*I*b*(a+b*arcsin
h(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b*(a+b*arcsinh(c*x))
*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b^2*polylog(3,-I*(c*x+(c^2*x
^2+1)^(1/2)))/c/d-2*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

### 3.229.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.99

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx =$$
$$-\frac{c\left(a^2\sqrt{-c^2}\arctan(cx)-2abc\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)-b^2c\text{arcsinh}(cx)^2\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)+\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

output
```
-((c*(a^2*Sqrt[-c^2]*ArcTan[c*x] - 2*a*b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSi
nh[c*x])/Sqrt[-c^2]] - b^2*c*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sqr
t[-c^2]] + 2*a*b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b
^2*c*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b*c*(a + b*
ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 2*b*c*(a + b*Arc
Sinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b^2*c*PolyLog[3,
(c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 2*b^2*c*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[
c*x])/c]))/((-c^2)^(3/2)*d))
```

### 3.229.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6204, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \text{ 6204}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{cd}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsi}}{cd}$$

$\big\downarrow$ 3011

$$2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyLog}\right.$$

$\big\downarrow$ 2720

$$2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib(b$$

$\big\downarrow$ 7143

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arc}\right.}{cd}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

output
```
(2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSi
nh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*
x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*P
olyLog[3, I*E^ArcSinh[c*x]]))/(c*d)
```

### 3.229.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.229. $\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.229.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{c^2d\,x^2+d}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

### 3.229.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^2 + d),
x)
```

### 3.229.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \frac{\int \frac{a^2}{c^2x^2+1}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2/(c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**
2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

### 3.229.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
a^2*arctan(c*x)/(c*d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*
d*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

### 3.229.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d), x)
```

### 3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2}\,dx = \int \frac{\left(a + b\,\text{asinh}(c\,x)\right)^2}{d\,c^2\,x^2 + d}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2), x)
```

# 3.230 $\int \frac{(a+b\,\mathrm{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx$



## 3.230.1 Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{(a+b\,\mathrm{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx = -\frac{2(a+b\,\mathrm{arcsinh}(cx))^2\mathrm{arctanh}\left(e^{2\mathrm{arcsinh}(cx)}\right)}{d} - \frac{b(a+b\,\mathrm{arcsinh}(cx))\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{arcsinh}(cx)}\right)}{d} + \frac{b(a+b\,\mathrm{arcsinh}(cx))\,\mathrm{PolyLog}\left(2,e^{2\mathrm{arcsinh}(cx)}\right)}{d} + \frac{b^2\,\mathrm{PolyLog}\left(3,-e^{2\mathrm{arcsinh}(cx)}\right)}{2d} - \frac{b^2\,\mathrm{PolyLog}\left(3,e^{2\mathrm{arcsinh}(cx)}\right)}{2d}$$

output
```
-2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-b*(a+b*arcsin
h(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+b*(a+b*arcsinh(c*x))*polyl
og(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2
))^2)/d-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d
```

### 3.230.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 400 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.45

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx = -\frac{2a^3+6a^2b\text{arcsinh}(cx)+12ab^2\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)+6b^3\text{arcsinh}(cx)^2\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

output
```
-1/6*(2*a^3 + 6*a^2*b*ArcSinh[c*x] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (c*E^Ar
cSinh[c*x])/Sqrt[-c^2]] + 6*b^3*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/
Sqrt[-c^2]] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
 + 6*b^3*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6*a^2*b*L
og[1 - E^(2*ArcSinh[c*x])] - 12*a*b^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*
x])] - 6*b^3*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3*a^2*b*Log[1 +
c^2*x^2] + 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] - 6*a*b^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 6*b^3*ArcSinh[c*x]*PolyLog
[2, E^(2*ArcSinh[c*x])] - 12*b^3*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
 - 12*b^3*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*b^3*PolyLog[3, E^(
2*ArcSinh[c*x])])/(b*d)
```

### 3.230.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6214, 5984, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)}\,dx$$

$$\Big\downarrow \text{6214}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{ 5984}$$

$$\frac{2\int (a+b\text{arcsinh}(cx))^2\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int i(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2i\int (a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{2i\left(ib\int (a+b\text{arcsinh}(cx))\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int (a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsin}\right.}{d}$$

$$\Big\downarrow \text{ 3011}$$

$$2i\left(-ib\left(\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+ib\left(\frac{1}{2}b\int\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$2i\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\right.\right.$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-ib\left(\frac{1}{4}b\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a\right.\right.}{d}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

output
```
((2*I)*(I*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])] - I*b*(-1/2*(
(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(
2*ArcSinh[c*x])])/4) + I*b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog[2, E^(2*Arc
Sinh[c*x])]) + (b*PolyLog[3, E^(2*ArcSinh[c*x])])/4)))/d
```

3.230. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx$

### 3.230.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.230. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.230.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(157) = 314$.

Time = 0.24 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.38

| method | result |
|---|---|
| parts | $\frac{a^2\left(-\frac{\ln(c^2x^2+1)}{2}+\ln(x)\right)}{d}+\frac{b^2\left(\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-2\,\text{po}\right.}{}$ |
| derivativedivides | $\frac{a^2\left(\ln(cx)-\frac{\ln(c^2x^2+1)}{2}\right)}{d}+\frac{b^2\left(\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-2\,\text{po}\right.}{}$ |
| default | $\frac{a^2\left(\ln(cx)-\frac{\ln(c^2x^2+1)}{2}\right)}{d}+\frac{b^2\left(\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)-2\,\text{po}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(-1/2*ln(c^2*x^2+1)+ln(x))+b^2/d*(arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1
)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*polylog(3,-c*x
-(c^2*x^2+1)^(1/2))-arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh
(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(c*x+(c^2*x^2+1
)^(1/2))^2)+arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*poly
log(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b/d*(
arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(1/2))
-arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+(c^2*
x^2+1)^(1/2)))
```

### 3.230.5 Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x(d + c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^3 + d*x)
, x)
```

### 3.230.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x(d + c^2dx^2)}\,dx = \frac{\int \frac{a^2}{c^2x^3+x}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^2x^3+x}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^2x^3+x}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2/(c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**
3 + x), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**3 + x), x))/d
```

### 3.230.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x(d + c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + integrate(b^2*log(c*x + sqrt(
c^2*x^2 + 1))^2/(c^2*d*x^3 + d*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^
2*d*x^3 + d*x), x)
```

### 3.230.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x\,(d + c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x), x)
```

### 3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x\,(d + c^2dx^2)}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{x\,(d\,c^2\,x^2 + d)}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)), x)
```

# 3.231 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx$



## 3.231.1 Optimal result

Integrand size = 26, antiderivative size = 204

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx &= -\frac{(a+b\text{arcsinh}(cx))^2}{dx} \\ &- \frac{2c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d} \\ &- \frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d} \\ &- \frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d} \\ &+ \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d} \\ &- \frac{2ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d} \\ &+ \frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d} - \frac{2ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d} \\ &+ \frac{2ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d}\end{aligned}$$

output
```
-(a+b*arcsinh(c*x))^2/d/x-2*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^
(1/2))/d-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d-2*b^2*c
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d+2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,
-I*(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+
(c^2*x^2+1)^(1/2)))/d+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d-2*I*b^2*c
*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+2*I*b^2*c*polylog(3,I*(c*x+(c^2*x
^2+1)^(1/2)))/d
```

## 3.231.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx =$$
$$-\frac{\frac{a^2}{x}+\frac{2ab\text{arcsinh}(cx)}{x}+a^2c\arctan(cx)+2abc\text{arctanh}\left(\sqrt{1+c^2x^2}\right)+\frac{1}{2}iabc\left(\text{arcsinh}(cx)\left(\text{arcsinh}(cx)-4\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

output
```
-((a^2/x + (2*a*b*ArcSinh[c*x])/x + a^2*c*ArcTan[c*x] + 2*a*b*c*ArcTanh[Sq
rt[1 + c^2*x^2]] + (I/2)*a*b*c*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E
^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - (I/2)*a*b*c*(ArcSin
h[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^Arc
Sinh[c*x]]) - b^2*c*(-(ArcSinh[c*x]^2/(c*x)) + 2*ArcSinh[c*x]*Log[1 - E^(-
ArcSinh[c*x])] + I*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - I*ArcSinh[c*
x]^2*Log[1 + I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])]
 + 2*PolyLog[2, -E^(-ArcSinh[c*x])] + (2*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E
^ArcSinh[c*x]] - (2*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*PolyL
og[2, E^(-ArcSinh[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (2*I)*P
olyLog[3, I/E^ArcSinh[c*x]]))/d)
```

### 3.231.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {6224, 27, 6204, 3042, 4668, 3011, 2720, 6231, 3042, 26, 4670, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(c^2dx^2+d\right)}dx$$

$$\Big\downarrow \text{ 6224}$$

$$c^2\left(-\int \frac{(a+b\text{arcsinh}(cx))^2}{d\left(c^2x^2+1\right)}dx\right)+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{d}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \text{ 6204}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{c\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{c\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \text{ 4668}$$

$$-\frac{c\left(-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)da\right.}{d} \frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{c\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{Pol}\right.\right.}{} \frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \; 2720$$

$$-\frac{c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2\right.}{}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \; 6231$$

$$-\frac{c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2\right.}{}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \; 3042$$

$$-\frac{c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2\right.}{}$$

$$\frac{2bc\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \; 26$$

$$-\frac{c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2\right.}{}$$

$$\frac{2ibc\int (a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \; 4670$$

$$-\frac{c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2\right.}{}$$

$$\frac{2ibc\left(ib\int \log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int \log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\right.}{d}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$$\Big\downarrow \; 2715$$

$$-\frac{c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2\right.}{}$$

$$\frac{2ibc\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arcta}\right.}{d}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

---

3.231. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx$

$\Big\downarrow$ 2838

$$-\frac{c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2\right.}{}$$
$$\frac{2ibc\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

$\Big\downarrow$ 7143

$$-\frac{c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+\right.\right.}{d}$$
$$\frac{2ibc\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{dx}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(d*x)) + ((2*I)*b*c*((2*I)*(a + b*ArcSinh[c*x])*A
rcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2,
E^ArcSinh[c*x]]))/d - (c*(2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]]
+ (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*Pol
yLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2,
 I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*E^ArcSinh[c*x]])))/d
```

### 3.231.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.231. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

---

## 3.231. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.231.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^2\,(c^2d\,x^2+d)}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

### 3.231.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^4 + d*x^
2), x)
```

### 3.231.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)}\,dx = \frac{\int \frac{a^2}{c^2x^4+x^2}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^2x^4+x^2}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^2x^4+x^2}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2/(c**2*x**4 + x**2), x) + Integral(b**2*asinh(c*x)**2/(c**2*
x**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**4 + x**2), x))/d
```

### 3.231.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-a^2*(c*arctan(c*x)/d + 1/(d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
1))^2/(c^2*d*x^4 + d*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^4
+ d*x^2), x)
```

### 3.231.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^2), x)
```

### 3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^2\,(d\,c^2\,x^2+d)}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)), x)
```

# 3.232 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx$



## 3.232.1 Optimal result

Integrand size = 26, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx &= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}\\ &\quad+\frac{2c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d}+\frac{b^2c^2\log(x)}{d}\\ &\quad+\frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d}\\ &\quad-\frac{bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d}\\ &\quad-\frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d}+\frac{b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d}\end{aligned}$$

output
```
-1/2*(a+b*arcsinh(c*x))^2/d/x^2+2*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)/d+b^2*c^2*ln(x)/d+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)/d-b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*
x^2+1)^(1/2))^2)/d-1/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2
*b^2*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d-b*c*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/d/x
```

### 3.232.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx =$$
$$-\frac{\frac{a^2}{x^2}+\frac{2ab\left(cx\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{x^2}+2a^2c^2\log(x)-a^2c^2\log(1+c^2x^2)+abc^2\left(\text{arcsinh}(cx)\left(\text{arcsinh}(cx)-\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

output
```
-1/2*(a^2/x^2 + (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + 2*a^2
*c^2*Log[x] - a^2*c^2*Log[1 + c^2*x^2] + a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*
x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + a
*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*Poly
Log[2, I*E^ArcSinh[c*x]]) - 2*a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] - 2*Log[
1 - E^(2*ArcSinh[c*x])]) - PolyLog[2, E^(2*ArcSinh[c*x])]) - 2*b^2*c^2*((-
1/24*I)*Pi^3 - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - ArcSinh[c*x]^2/(2*
c^2*x^2) + (2*ArcSinh[c*x]^3)/3 + ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x
])] - ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + Log[(c*x)/Sqrt[1 + c^2*
x^2]] + Log[1 + c^2*x^2]/2 - ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])]
 - ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - PolyLog[3, -E^(-2*ArcSinh
[c*x])]/2 + PolyLog[3, E^(2*ArcSinh[c*x])]/2))/d
```

### 3.232.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6224, 27, 6214, 5984, 3042, 26, 4670, 3011, 2720, 6215, 14, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(c^2dx^2+d)}\,dx$$

$$\Big\downarrow \; 6224$$

$$c^2\left(-\int \frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2x^2+1)}dx\right)+\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\downarrow$ 27

$$-\frac{c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx}{d}+\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\downarrow$ 6214

$$-\frac{c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d}+\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\downarrow$ 5984

$$\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{2c^2\int (a+b\text{arcsinh}(cx))^2\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\downarrow$ 3042

$$\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{2c^2\int i(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\downarrow$ 26

$$\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{2ic^2\int (a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\downarrow$ 4670

$$-\frac{2ic^2\left(ib\int (a+b\text{arcsinh}(cx))\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int (a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{ar}\right.}{d}$$
$$\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\downarrow$ 3011

$$-\frac{2ic^2\left(-ib\left(\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+ib\left(\frac{1}{2}\right.\right.}{}$$
$$\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

---

3.232. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}dx$

$\Big\downarrow$ 2720

$$-\frac{2ic^2\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(\right.\right.}{}$$

$$\frac{bc\int\frac{a+b\text{arcsinh}(cx)}{x^2\sqrt{c^2x^2+1}}dx}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 6215

$$-\frac{2ic^2\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(\right.\right.}{}$$

$$\frac{bc\left(bc\int\frac{1}{x}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}\right)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 14

$$-\frac{2ic^2\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(\right.\right.}{}$$

$$\frac{bc\left(bc\log(x)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}\right)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 7143

$$-\frac{2ic^2\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-ib\left(\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right.\right.\right.}{d}$$

$$\frac{bc\left(bc\log(x)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{x}\right)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

input

```
Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

output

```
-1/2*(a + b*ArcSinh[c*x])^2/(d*x^2) + (b*c*(-((Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/x) + b*c*Log[x]))/d - ((2*I)*c^2*(I*(a + b*ArcSinh[c*x])^2*Ar
cTanh[E^(2*ArcSinh[c*x])] - I*b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog[2, -E^
(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(2*ArcSinh[c*x])])/4) + I*b*(-1/2*((
a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, E^(2*A
rcSinh[c*x])])/4)))/d
```

## 3.232. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx$

### 3.232.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

---

## 3.232. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx$

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.232.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(231) = 462$.

Time = 0.27 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.78

| method | result |
|---|---|
| derivativedivides | $c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(c^2x^2+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\ln\left(1+cx+\sqrt{c^2x^2+1}\right.\right.}{}\right.$ |
| default | $c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(c^2x^2+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\ln\left(1+cx+\sqrt{c^2x^2+1}\right.\right.}{}\right.$ |
| parts | $\frac{a^2\left(\frac{c^2\ln(c^2x^2+1)}{2}-\frac{1}{2x^2}-c^2\ln(x)\right)}{d}+\frac{b^2c^2\left(-\frac{\operatorname{arcsinh}(cx)\left(-2c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{2c^2x^2}+\ln\left(1+cx+\sqrt{c^2x^2+1}\right)-\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
c^2*(a^2/d*(-1/2/c^2/x^2-ln(c*x)+1/2*ln(c^2*x^2+1))+b^2/d*(-1/2*arcsinh(c*
x)*(-2*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/c^2/x^2+ln(1+c*x+(c^2
*x^2+1)^(1/2))-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln(c*x+(c^2*x^2+1)^(1/2)-1)-arc
sinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*arcsinh(c*x)*polylog(2,-c*x-(c^2
*x^2+1)^(1/2))+2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+arcsinh(c*x)^2*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)
-1/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh(c*x)^2*ln(1-c*x-(c^2*x^
2+1)^(1/2))-2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*polylog(3,c*
x+(c^2*x^2+1)^(1/2)))+2*a*b/d*(-1/2*(c*x*(c^2*x^2+1)^(1/2)-c^2*x^2+arcsinh
(c*x))/c^2/x^2-arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-polylog(2,-c*x-(c^
2*x^2+1)^(1/2))+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2
,-(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-poly
log(2,c*x+(c^2*x^2+1)^(1/2))))
```

### 3.232.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^5 + d*x^
3), x)
```

### 3.232.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx = \frac{\int \frac{a^2}{c^2x^5+x^3}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^5+x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^5+x^3}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2/(c**2*x**5 + x**3), x) + Integral(b**2*asinh(c*x)**2/(c**2*
x**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**5 + x**3), x))/d
```

### 3.232.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a^2 + integrate(
b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^5 + d*x^3) + 2*a*b*log(c*x + s
qrt(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

### 3.232.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^3), x)
```

### 3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)}\,dx = \int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2}{x^3\,\left(d\,c^2\,x^2+d\right)}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)), x)
```

# 3.233 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx$



## 3.233.1 Optimal result

Integrand size = 26, antiderivative size = 297

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx =& -\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3dx^2}\\ &-\frac{(a+b\operatorname{arcsinh}(cx))^2}{3dx^3}+\frac{c^2(a+b\operatorname{arcsinh}(cx))^2}{dx}\\ &+\frac{2c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{d}\\ &+\frac{14bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d}\\ &+\frac{7b^2c^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{3d}\\ &-\frac{2ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{d}\\ &+\frac{2ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d}\\ &-\frac{7b^2c^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{3d}\\ &+\frac{2ib^2c^3\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{d}\\ &-\frac{2ib^2c^3\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{d}\end{aligned}$$

output
```
-1/3*b^2*c^2/d/x-1/3*(a+b*arcsinh(c*x))^2/d/x^3+c^2*(a+b*arcsinh(c*x))^2/d
/x+2*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d+14/3*b*c^3*(
a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d+7/3*b^2*c^3*polylog(2,-
c*x-(c^2*x^2+1)^(1/2))/d-2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c
^2*x^2+1)^(1/2)))/d+2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/d-7/3*b^2*c^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d+2*I*b^2*c^3*p
olylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x
^2+1)^(1/2)))/d-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x^2
```

### 3.233.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 602 vs. 2(297) = 594.

Time = 7.33 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.03

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx = -\frac{a^2}{3dx^3}+\frac{a^2c^2}{dx}+\frac{a^2c^3\arctan(cx)}{d}$$

$$+\frac{2ab\left(-\frac{c\sqrt{1+c^2x^2}}{6x^2}-\frac{\text{arcsinh}(cx)}{3x^3}+\frac{1}{6}c^3\text{arctanh}\left(\sqrt{1+c^2x^2}\right)-c^2\left(-\frac{\text{arcsinh}(cx)}{x}-c\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)\right)-\right.}{}$$

$$+\frac{b^2c^3\left(-4\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)+14\text{arcsinh}(cx)^2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-2\text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

output
```
-1/3*a^2/(d*x^3) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcTan[c*x])/d + (2*a*b*(-1/
6*(c*Sqrt[1 + c^2*x^2])/x^2 - ArcSinh[c*x]/(3*x^3) + (c^3*ArcTanh[Sqrt[1 +
 c^2*x^2]])/6 - c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) - (
I/2)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]
])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + (I/2)*c^4*(-1/2*ArcSinh[c*
x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^
ArcSinh[c*x]])/c)))/d + (b^2*c^3*(-4*Coth[ArcSinh[c*x]/2] + 14*ArcSinh[c*x
]^2*Coth[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*Ar
cSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 56*ArcSinh[c*x]*Log[1 - E^(-ArcSi
nh[c*x])] - (24*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSi
nh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 56*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] - 56*PolyLog[2, -E^(-ArcSinh[c*x])] - (48*I)*ArcSinh[c*x]*PolyLog[2
, (-I)/E^ArcSinh[c*x]] + (48*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]]
+ 56*PolyLog[2, E^(-ArcSinh[c*x])] - (48*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]
] + (48*I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]
/2]^2 - (8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSi
nh[c*x]/2] - 14*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d)
```

### 3.233.3 Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6224, 27, 6224, 15, 6204, 3042, 4668, 3011, 2720, 6231, 3042, 26, 4670, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(c^2dx^2+d)}dx$$

$$\Big\downarrow \textbf{6224}$$

$$c^2\left(-\int \frac{(a+b\text{arcsinh}(cx))^2}{dx^2(c^2x^2+1)}dx\right)+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{c^2x^2+1}}dx}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(c^2x^2+1)}dx}{d}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x^3\sqrt{c^2x^2+1}}dx}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \textbf{6224}$$

$$-\frac{c^2\left(c^2\left(-\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)}{d}+\frac{2bc\left(-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{1}{2}bc\int \frac{1}{x^2}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 15

$$-\frac{c^2\left(c^2\left(-\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)}{d}+\frac{2bc\left(-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 6204

$$-\frac{c^2\left(2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-c\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)}{d}+\frac{2bc\left(-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 3042

$$\frac{2bc\left(-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{c^2\left(2bc\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-c\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^2}{x}\right)}{d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 4668

$$-\frac{c^2\left(-c\left(-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(c\right.\right.\right.}{d}$$
$$\frac{2bc\left(-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 3011

$$-\frac{c^2\left(-c\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\right.\right.\right.}{}$$
$$\frac{2bc\left(-\frac{1}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\Big\downarrow$ 2720

---

3.233. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}dx$

$$-\frac{c^2\Big(-c\big(2ib\big(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\big)}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\downarrow$ 6231

$$-\frac{c^2\Big(-c\big(2ib\big(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\big)}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\downarrow$ 3042

$$-\frac{c^2\Big(-c\big(2ib\big(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\big)}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\downarrow$ 26

$$-\frac{c^2\Big(-c\big(2ib\big(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\big)}{}$$

$$\frac{2bc\left(-\frac{1}{2}ic^2\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2x^2}-\frac{bc}{2x}\right)}{3d}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\downarrow$ 4670

$$-\frac{c^2\Big(-c\big(2ib\big(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\big)}{}$$

$$\frac{2bc\Big(-\frac{1}{2}ic^2\big(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\big(e^{\text{arcsinh}(cx)}\big)}{3d}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$\downarrow$ 2715

3.233. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx$

$$-\frac{c^2\left(-c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\right.\right.\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}ic^2\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\right.\right.}{3d}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{c^2\left(-c\left(2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\right.\right.\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}ic^2\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.}{3d}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{c^2\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)\right.\right.\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}ic^2\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.}{3d}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])^2/(d*x^3) + (2*b*c*(-1/2*(b*c)/x - (Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(2*x^2) - (I/2)*c^2*((2*I)*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[
2, E^ArcSinh[c*x]])))/(3*d) - (c^2*(-((a + b*ArcSinh[c*x])^2/x) + (2*I)*b*
c*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^
ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]]) - c*(2*(a + b*ArcSinh[c*x]
)^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, (-
I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)*b*(-((a +
 b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*E^ArcSinh[
c*x]]))))/d
```

## 3.233. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx$

### 3.233.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

## 3.233. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.233. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx$

### 3.233.4 Maple [F]

$$\int \frac{(a+b \operatorname{arcsinh}(cx))^2}{x^4 (c^2 d x^2+d)} dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

### 3.233.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4 (d+c^2dx^2)} dx = \int \frac{(b \operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)x^4} dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^6 + d*x^
4), x)
```

### 3.233.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4 (d+c^2dx^2)} dx = \frac{\int \frac{a^2}{c^2x^6+x^4} dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^2x^6+x^4} dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^2x^6+x^4} dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2/(c**2*x**6 + x**4), x) + Integral(b**2*asinh(c*x)**2/(c**2*
x**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**6 + x**4), x))/d
```

### 3.233.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a^2 + integrate(b^2*lo
g(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^6 + d*x^4) + 2*a*b*log(c*x + sqrt(c^
2*x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

### 3.233.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^4), x)
```

### 3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^4\,(d\,c^2\,x^2+d)}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)), x)
```

# 3.234 $\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$



## 3.234.1 Optimal result

Integrand size = 26, antiderivative size = 279

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{2b^2x}{c^4d^2}+\frac{b(a+b\operatorname{arcsinh}(cx))}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c^5d^2}+\frac{3x(a+b\operatorname{arcsinh}(cx))^2}{2c^4d^2}-\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}-\frac{3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2}-\frac{b^2\arctan(cx)}{c^5d^2}+\frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2}-\frac{3ib(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2}-\frac{3ib^2\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2}+\frac{3ib^2\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{c^5d^2}$$

output
```
2*b^2*x/c^4/d^2+3/2*x*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*
x))^2/c^2/d^2/(c^2*x^2+1)-3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1
/2))/c^5/d^2-b^2*arctan(c*x)/c^5/d^2+3*I*b*(a+b*arcsinh(c*x))*polylog(2,-I
*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*
x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2))
)/c^5/d^2+3*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2+b*(a+b*arcs
inh(c*x))/c^5/d^2/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/
2)/c^5/d^2
```

### 3.234.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.73

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$$

$$= \frac{\frac{2a^2x}{c^4}+\frac{a^2x}{c^4+c^6x^2}-\frac{3a^2\arctan(cx)}{c^5}-\frac{2ab\left(\sqrt{1+c^2x^2}+2c^2x^2\sqrt{1+c^2x^2}-3cx\operatorname{arcsinh}(cx)-2c^3x^3\operatorname{arcsinh}(cx)+3i\operatorname{arcsinh}(cx)\log\left(1-ie^{\ldots}\right.\right.}{\ldots}}{\ldots}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

output
```
((2*a^2*x)/c^4 + (a^2*x)/(c^4 + c^6*x^2) - (3*a^2*ArcTan[c*x])/c^5 - (2*a*
b*(Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^2] - 3*c*x*ArcSinh[c*x] -
2*c^3*x^3*ArcSinh[c*x] + (3*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3
*I)*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]*Lo
g[1 + I*E^ArcSinh[c*x]] - (3*I)*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c
*x]] - (3*I)*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*(1 + c^
2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]]))/(c^5 + c^7*x^2) + (2*b^2*(ArcSinh[c*
x]/Sqrt[1 + c^2*x^2] - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + (c*x*ArcSinh[c*x
]^2)/(2 + 2*c^2*x^2) + c*x*(2 + ArcSinh[c*x]^2) + (I/2)*((4*I)*ArcTan[Tanh
[ArcSinh[c*x]/2]] + 3*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - 3*ArcSinh
[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 6*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSi
nh[c*x]] - 6*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 6*PolyLog[3, (-I)
/E^ArcSinh[c*x]] - 6*PolyLog[3, I/E^ArcSinh[c*x]])))/c^5)/(2*d^2)
```

### 3.234.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {6225, 27, 6219, 27, 299, 216, 6227, 6204, 3042, 4668, 3011, 2720, 6213, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(c^2dx^2+d)^2}\,dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{b\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}+\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{2c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{6219}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\left(-bc\int \frac{c^2x^2+2}{c^4(c^2x^2+1)}dx+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\left(-\frac{b\int \frac{c^2x^2+2}{c^2x^2+1}dx}{c^3}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{299}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\left(-\frac{b\left(\int \frac{1}{c^2x^2+1}dx+x\right)}{c^3}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2c^2d^2}+\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{6227}$$

$$\frac{3\left(-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{c}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2}\right)}{2c^2d^2}+\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

3.234. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

$\downarrow$ 6204

$$\frac{3\left(-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{c}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2}\right)}{2c^2d^2}+\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$\downarrow$ 3042

$$\frac{3\left(-\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}-\frac{2b\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))^2}{c^2}\right)}{2c^2d^2}+\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$\downarrow$ 4668

$$\frac{3\left(-\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e\right.}{c^3}\right.}{2c^2d^2}$$

$$\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$\downarrow$ 3011

$$3\left(-\frac{2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)d\right.}{c^3}\right.$$

$$\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$\downarrow$ 2720

---

3.234. $\displaystyle\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx$

$$3\left(-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{P}\right.}{c^3}\right.$$

$$\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$\downarrow$ 6213

$$3\left(-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{P}\right.}{c^3}\right.$$

$$\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$\downarrow$ 24

$$3\left(-\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{P}\right.}{c^3}\right.$$

$$\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$\downarrow$ 7143

$$3\left(-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\,\text{P}\right.}{c^3}\right.$$

$$2c^2$$

$$\frac{b\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^4}+\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}-\frac{b\left(\frac{\arctan(cx)}{c}+x\right)}{c^3}\right)}{cd^2}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

3.234. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

output
```
-1/2*(x^3*(a + b*ArcSinh[c*x])^2)/(c^2*d^2*(1 + c^2*x^2)) + (b*((a + b*Arc
Sinh[c*x])/(c^4*Sqrt[1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/c^4 - (b*(x + ArcTan[c*x]/c))/c^3))/(c*d^2) + (3*((x*(a + b*ArcSinh[c*
x])^2)/c^2 - (2*b*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c
^2))/c - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a
+ b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^A
rcSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x
]]) + b*PolyLog[3, I*E^ArcSinh[c*x]]))/c^3))/(2*c^2*d^2)
```

### 3.234.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.234. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.234. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.234.4 Maple [F]

$$\int \frac{x^4 (a + b \operatorname{arcsinh}(cx))^2}{(c^2 d x^2 + d)^2} dx$$

input
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

output
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

### 3.234.5 Fricas [F]

$$\int \frac{x^4 (a + b\text{arcsinh}(cx))^2}{(d + c^2 dx^2)^2} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^4}{(c^2 dx^2 + d)^2} dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^4*
d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.234.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x^4}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x^4\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx^4\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**4
*asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asi
nh(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.234.7 Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^
2)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^
2*d^2*x^2 + d^2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2
*c^2*d^2*x^2 + d^2), x)
```

### 3.234.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

output
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

# 3.235 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$



## 3.235.1 Optimal result

Integrand size = 26, antiderivative size = 213

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = -\frac{bx(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}-\frac{(a+b\text{arcsinh}(cx))^3}{3bc^4d^2} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^4d^2}+\frac{b^2\log(1+c^2x^2)}{2c^4d^2} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^4d^2} - \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2c^4d^2}$$

output
```
1/2*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2
*x^2+1)-1/3*(a+b*arcsinh(c*x))^3/b/c^4/d^2+(a+b*arcsinh(c*x))^2*ln(1+(c*x+
(c^2*x^2+1)^(1/2))^2)/c^4/d^2+1/2*b^2*ln(c^2*x^2+1)/c^4/d^2+b*(a+b*arcsinh
(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b^2*polylog(3,-(c
*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-b*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1
)^(1/2)
```

### 3.235.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$$
$$= \frac{\frac{a^2}{1+c^2x^2} - \frac{ab\left(\sqrt{1+c^2x^2}-i\text{arcsinh}(cx)\right)}{i+cx} - \frac{ab\left(\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)\right)}{-i+cx} - ab\text{arcsinh}(cx)\left(\text{arcsinh}(cx) - 4\log\left(1 - ie^{\text{ar}}\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

output
```
(a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x)
- (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - a*b*ArcSinh[c*x]
*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - a*b*ArcSinh[c*x]*(ArcSinh[
c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) + a^2*Log[1 + c^2*x^2] + 4*a*b*PolyLog
[2, (-I)*E^ArcSinh[c*x]] + 4*a*b*PolyLog[2, I*E^ArcSinh[c*x]] + 2*b^2*(-((
c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2]) + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) + Ar
cSinh[c*x]^3/3 + ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + Log[1 + c^2
*x^2]/2 - ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - PolyLog[3, -E^(-
2*ArcSinh[c*x])]/2))/(2*c^4*d^2)
```

### 3.235.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6225, 27, 6212, 3042, 26, 4201, 2620, 3011, 2720, 6225, 240, 6198, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^2}\,dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2} + \frac{\int \frac{x(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{c^2d} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}+\frac{\int \frac{x(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{c^2d^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \; 6212$$

$$\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}+\frac{\int \frac{cx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^4d^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -i(a+b\text{arcsinh}(cx))^2\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d^2}+\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\int(a+b\text{arcsinh}(cx))^2\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4d^2}+\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \; 4201$$

$$-\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^3}{3b}\right)}{c^4d^2}+\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \; 2620$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\int(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(}{}\right.}{c^4d^2}$$
$$\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \; 3011$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\,(\right.\right.\right.}{c^4d^2}$$
$$\frac{b\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

---

3.235. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

$$\Big\downarrow \text{2720}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\right.\right.\right.}{c^4d^2}$$

$$\frac{b\int\frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{6225}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\right.\right.\right.}{c^4d^2}$$

$$\frac{b\left(\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{c^2}+\frac{b\int\frac{x}{c^2x^2+1}dx}{c}-\frac{x(a+b\text{arcsinh}(cx))}{c^2\sqrt{c^2x^2+1}}\right)}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{240}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\right.\right.\right.}{c^4d^2}$$

$$\frac{b\left(\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{c^2}-\frac{x(a+b\text{arcsinh}(cx))}{c^2\sqrt{c^2x^2+1}}+\frac{b\log(c^2x^2+1)}{2c^3}\right)}{cd^2}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\right.\right.\right.}{c^4d^2}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}+\frac{b\left(\frac{(a+b\text{arcsinh}(cx))^2}{2bc^3}-\frac{x(a+b\text{arcsinh}(cx))}{c^2\sqrt{c^2x^2+1}}+\frac{b\log(c^2x^2+1)}{2c^3}\right)}{cd^2}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))^2-b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right.\right.\right.}{c^4d^2}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}+\frac{b\left(\frac{(a+b\text{arcsinh}(cx))^2}{2bc^3}-\frac{x(a+b\text{arcsinh}(cx))}{c^2\sqrt{c^2x^2+1}}+\frac{b\log(c^2x^2+1)}{2c^3}\right)}{cd^2}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

3.235. $\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}dx$

output
```
-1/2*(x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d^2*(1 + c^2*x^2)) + (b*(-((x*(a +
b*ArcSinh[c*x]))/(c^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c*x])^2/(2*b*c^
3) + (b*Log[1 + c^2*x^2])/(2*c^3)))/(c*d^2) - (I*(((-1/3*I)*(a + b*ArcSinh
[c*x])^3)/b + (2*I)*(((a + b*ArcSinh[c*x])^2*Log[1 + E^(2*ArcSinh[c*x])])/
2 - b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])]) + (b*Po
lyLog[3, -E^(2*ArcSinh[c*x])])/4))))/(c^4*d^2)
```

### 3.235.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

## 3.235. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.235. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

### 3.235.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(\right.\right.}{}$ |
| default | $\frac{a^2\left(\frac{1}{2c^2x^2+2}+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(\right.\right.}{}$ |
| parts | $\frac{a^2\left(\frac{1}{2c^4\left(c^2x^2+1\right)}+\frac{\ln\left(c^2x^2+1\right)}{2c^4}\right)}{d^2}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(cx)^3}{3}+\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)+2\right)\operatorname{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2\right.\right.}{}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2/d^2*(1/2/(c^2*x^2+1)+1/2*ln(c^2*x^2+1))+b^2/d^2*(-1/3*arcsinh(c
*x)^3+1/2*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)+2)*arcsinh(c*x)/
(c^2*x^2+1)-2*ln(c*x+(c^2*x^2+1)^(1/2))+ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+ar
csinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))^2)-1/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b/d^2
*(-1/2*arcsinh(c*x)^2+1/2*(-c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/
(c^2*x^2+1)+arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)))
```

### 3.235.5 Fricas [F]

$$\int\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx=\int\frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^3}{\left(c^2dx^2+d\right)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^4*
d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.235.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x^3}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x^3\,\text{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx^3\,\text{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**3
*asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asi
nh(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.235.7 Maxima [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}(cx)+a)^2x^3}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2)) + 1/2*(b^
2 + (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^2*x^2 + c^4*d^2) - integrate(-(2*a*b*c^4*x^4 - b^2*c^2*x^2 - b^2 - (
b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + (2*a*b*c^3*x^3 - b^2
*c*x - (b^2*c^3*x^3 + b^2*c*x)*log(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1))/(c^8*d^2*x^5 + 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*d^2
*x^4 + 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x)
```

### 3.235.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{x^3\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

output
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

# 3.236 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx$



## 3.236.1 Optimal result

Integrand size = 26, antiderivative size = 213

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^2}\,dx = & -\frac{b(a+b\text{arcsinh}(cx))}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(1+c^2x^2\right)}\\ & +\frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^3d^2}+\frac{b^2\arctan(cx)}{c^3d^2}\\ & -\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^3d^2}\\ & +\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3d^2}\\ & +\frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{c^3d^2}-\frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{c^3d^2}\end{aligned}$$

output
```
-1/2*x*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arcta
n(c*x+(c^2*x^2+1)^(1/2))/c^3/d^2+b^2*arctan(c*x)/c^3/d^2-I*b*(a+b*arcsinh(
c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b*(a+b*arcsinh(c*x))
*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b^2*polylog(3,-I*(c*x+(c^2
*x^2+1)^(1/2)))/c^3/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2
-b*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)
```

### 3.236.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.81

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx =$$

$$-\frac{\frac{a^2cx}{1+c^2x^2}+\frac{2b^2\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}+\frac{b^2cx\text{arcsinh}(cx)^2}{1+c^2x^2}+\frac{ab\left(-i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{-i+cx}+\frac{ab\left(i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{i+cx}-a^2\,\text{arc}}{}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*((a^2*c*x)/(1 + c^2*x^2) + (2*b^2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
b^2*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + (a*b*((-I)*Sqrt[1 + c^2*x^2] + Arc
Sinh[c*x]))/(-I + c*x) + (a*b*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c
*x) - a^2*ArcTan[c*x] - (I/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 +
I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + (I/2)*a*b*(ArcSi
nh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^Ar
cSinh[c*x]]) + I*b^2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSinh[c*x]^2*
Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 2*A
rcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyLog[2, I/
E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3, I/E^Arc
Sinh[c*x]]))/(c^3*d^2)
```

### 3.236.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6225, 27, 6204, 3042, 4668, 3011, 2720, 6213, 216, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^2}\,dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{2c^2d}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\,(c^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2c^2d^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 6204}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c^3d^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c^3d^2}+\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsi}}{2c^3d^2}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 3011}$$

$$\underline{2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyLog}\right.}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 2720}$$

$$\underline{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib(}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 6213}$$

$$\underline{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib(}$$

$$\frac{b\left(\frac{b\int \frac{1}{c^2x^2+1}dx}{c}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{cd^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2\left(c^2x^2+1\right)}$$

---

3.236. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

$$\Big\downarrow \text{216}$$

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib(b}{}$$

$$\frac{b\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{cd^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\operatorname{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arc}\right.}{2c^3d^2}$$

$$\frac{b\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{cd^2}-\frac{x(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*(x*(a + b*ArcSinh[c*x])^2)/(c^2*d^2*(1 + c^2*x^2)) + (b*(-((a + b*Arc
Sinh[c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x])/c^2))/(c*d^2) + (2*(
a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSinh[c
*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*x]])
 - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*PolyL
og[3, I*E^ArcSinh[c*x]]))/(2*c^3*d^2)
```

### 3.236.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

---

3.236. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.236. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.236.4 Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arcsinh}\,(cx))^2}{(c^2d\,x^2+d)^2}dx$$

input
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

output
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

### 3.236.5 Fricas [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^4*
d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.236.6 Sympy [F]

$$\int \frac{x^2(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\frac{\int \frac{a^2x^2}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{b^2x^2\,\mathrm{asinh}^2\,(cx)}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{2abx^2\,\mathrm{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**2
*asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asi
nh(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.236.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + integrate(b
^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)
+ 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^
2), x)
```

### 3.236.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^2, x)
```

### 3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{x^2\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

output
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

# 3.237 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$



## 3.237.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\frac{bx(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2(1+c^2x^2)}-\frac{b^2\log(1+c^2x^2)}{2c^2d^2}$$

output
```
-1/2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)-1/2*b^2*ln(c^2*x^2+1)/c^2/d^
2+b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)
```

## 3.237.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx&=-\frac{a^2}{2c^2d^2(1+c^2x^2)}+\frac{abx}{cd^2\sqrt{1+c^2x^2}}\\&\quad+\frac{b\left(-a+bcx\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)}{c^2d^2(1+c^2x^2)}\\&\quad-\frac{b^2\text{arcsinh}(cx)^2}{2c^2d^2(1+c^2x^2)}-\frac{b^2\log(1+c^2x^2)}{2c^2d^2}\end{aligned}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*a^2/(c^2*d^2*(1 + c^2*x^2)) + (a*b*x)/(c*d^2*Sqrt[1 + c^2*x^2]) + (b*
(-a + b*c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x])/(c^2*d^2*(1 + c^2*x^2)) - (b^
2*ArcSinh[c*x]^2)/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^
2*d^2)
```

### 3.237.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6213, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^2}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx}{cd^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{6202}$$

$$\frac{b\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)}{cd^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{240}$$

$$\frac{b\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2c^2d^2(c^2x^2+1)}$$

input
```
Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(c^2*d^2*(1 + c^2*x^2)) + (b*((x*(a + b*ArcSin
h[c*x]))/Sqrt[1 + c^2*x^2] - (b*Log[1 + c^2*x^2])/(2*c)))/(c*d^2)
```

### 3.237.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.237.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.73

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a^2}{2d^2\left(c^2x^2+1\right)}+\frac{b^2\left(2\,\mathrm{arcsinh}(cx)-\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)+2\right)\mathrm{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)\right)}{d^2}+\frac{2ab\left(-\frac{\mathrm{arc}}{2(}\right.}{}}{c^2}$ |
| default | $\frac{-\frac{a^2}{2d^2\left(c^2x^2+1\right)}+\frac{b^2\left(2\,\mathrm{arcsinh}(cx)-\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)+2\right)\mathrm{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)\right)}{d^2}+\frac{2ab\left(-\frac{\mathrm{arc}}{2(}\right.}{}}{c^2}$ |
| parts | $-\frac{a^2}{2d^2c^2(c^2x^2+1)}+\frac{b^2\left(2\,\mathrm{arcsinh}(cx)-\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)+2\right)\mathrm{arcsinh}(cx)}{2\left(c^2x^2+1\right)}-\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)\right)}{d^2c^2}$ |

input
```
int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*a^2/d^2/(c^2*x^2+1)+b^2/d^2*(2*arcsinh(c*x)-1/2*(2*c^2*x^2-2*c
*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)+2)*arcsinh(c*x)/(c^2*x^2+1)-ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2))+2*a*b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x/(c^2
*x^2+1)^(1/2)))
```

### 3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.18

$$\int \frac{x(a + b\operatorname{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx = \frac{2\,abc^2x^2 + 2\,\sqrt{c^2x^2+1}abcx - b^2\log\left(cx + \sqrt{c^2x^2+1}\right)^2 - a^2 + 2\,ab - (b^2c^2x^2 + b^2)\log\left(c^2x^2+1\right) + 2\left(abc\right.}{2\left(c^4d^2x^2 + c^2d^2\right)}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*c^2*x^2 + 2*sqrt(c^2*x^2 + 1)*a*b*c*x - b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2 - a^2 + 2*a*b - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(a*b*
c^2*x^2 + sqrt(c^2*x^2 + 1)*b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b
*c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^2*x^2 + c^2*d^2)
```

### 3.237.6 Sympy [F]

$$\int \frac{x(a + b\operatorname{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x*asinh
(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(
c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.237.7 Maxima [F]

$$\int \frac{x(a + b\operatorname{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2x}{(c^2dx^2 + d)^2}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^2 + c^2*d^2) - 1/2*a^2/
(c^4*d^2*x^2 + c^2*d^2) + integrate(((2*a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*
x^2 + 1)*(2*a*b*c + b^2*c)*x + b^2)*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^2*
x^5 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2)*sq
rt(c^2*x^2 + 1)), x)
```

### 3.237.8 Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^2} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2 x}{(c^2dx^2 + d)^2} dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^2, x)
```

### 3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^2} dx = \int \frac{x(a + b \operatorname{asinh}(c x))^2}{(d c^2 x^2 + d)^2} dx$$

input
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

output
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

# 3.238 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$



## 3.238.1 Optimal result

Integrand size = 23, antiderivative size = 210

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx &= \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)}\\ &+\frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2}-\frac{b^2\arctan(cx)}{cd^2}\\ &-\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\ &+\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\ &+\frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}-\frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\end{aligned}$$

output
```
1/2*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan(c*x
+(c^2*x^2+1)^(1/2))/c/d^2-b^2*arctan(c*x)/c/d^2-I*b*(a+b*arcsinh(c*x))*pol
ylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b*(a+b*arcsinh(c*x))*polylog(2,
I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)
))/c/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+b*(a+b*arcsinh(c
*x))/c/d^2/(c^2*x^2+1)^(1/2)
```

### 3.238.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.92

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^2}\,dx$$

$$= \frac{\frac{a^2x}{1+c^2x^2} + \frac{a^2\arctan(cx)}{c} + \frac{2ab\left(\sqrt{1+c^2x^2}+cx\text{arcsinh}(cx)+i\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx)}\right)+ic^2x^2\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx}\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

output
```
((a^2*x)/(1 + c^2*x^2) + (a^2*ArcTan[c*x])/c + (2*a*b*(Sqrt[1 + c^2*x^2] +
 c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*c^2*x^2*A
rcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*ArcSinh[c*x]*Log[1 + I*E^ArcSinh
[c*x]] - I*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*(1 + c^2*x^2
)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh
[c*x]]))/(c + c^3*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt[1 + c^2*x^2] + (c*x*Arc
Sinh[c*x]^2)/(2 + 2*c^2*x^2) - (I/2)*((-4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
+ ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^Ar
cSinh[c*x]] + 2*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c
*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*P
olyLog[3, I/E^ArcSinh[c*x]])))/c)/(2*d^2)
```

### 3.238.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6203, 27, 6204, 3042, 4668, 3011, 2720, 6213, 216, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(c^2dx^2 + d)^2}\,dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{2d} + \frac{x(a + b\text{arcsinh}(cx))^2}{2d^2\,(c^2x^2 + 1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \; 6204$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \; 4668$$

$$\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsi}}{2cd^2}$$
$$\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \; 3011$$

$$2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyLog}\right.$$
$$\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \; 2720$$

$$2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib($$
$$\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \; 6213$$

$$2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib($$
$$\frac{bc\left(\frac{b\int \frac{1}{c^2x^2+1}dx}{c}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

---

3.238. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}dx$

$$\Big\downarrow \ 216$$

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib(b}{}$$

$$\frac{bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \ 7143$$

$$-\frac{bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{d^2}+$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arc}\right.}{2cd^2}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*x^2)) - (b*c*(-((a + b*ArcSinh[
c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x])/c^2))/d^2 + (2*(a + b*Arc
Sinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSinh[c*x])*Poly
Log[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)*
b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*E
^ArcSinh[c*x]]))/(2*c*d^2)
```

### 3.238.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.238. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

---

## 3.238. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.238.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(c^2 d\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

## 3.238.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

### 3.238.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x
)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**4*x
**4 + 2*c**2*x**2 + 1), x))/d**2
```

### 3.238.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + integrate(b^2*log(
c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 2*a*b*log
(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.238.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^2, x)
```

### 3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2, x)
```

# 3.239 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx$



## 3.239.1 Optimal result

Integrand size = 26, antiderivative size = 193

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx = &-\frac{bcx(a+b\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)}\\ &-\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2}+\frac{b^2\log(1+c^2x^2)}{2d^2}\\ &-\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2}\\ &+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2}\\ &+\frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^2}-\frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^2}\end{aligned}$$

output
```
1/2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c
*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*ln(c^2*x^2+1)/d^2-b*(a+b*arcsinh(c*x)
)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b*(a+b*arcsinh(c*x))*polylog(2
,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))
^2)/d^2-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*x*(a+b*arcsin
h(c*x))/d^2/(c^2*x^2+1)^(1/2)
```

### 3.239.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx$$
$$= \frac{\frac{a^2}{1+c^2x^2} - \frac{ab\left(\sqrt{1+c^2x^2}-i\text{arcsinh}(cx)\right)}{i+cx} - \frac{ab\left(\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)\right)}{-i+cx} - 2ab\text{arcsinh}(cx)^2 + 4ab\text{arcsinh}(cx)\log\left(1-e\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

output
```
(a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x)
- (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 2*a*b*ArcSinh[c*
x]^2 + 4*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a^2*Log[c*x] - a
^2*Log[1 + c^2*x^2] + a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcS
inh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + a*b*(ArcSinh[c*x]*(ArcSi
nh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) +
 2*a*b*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*((I/24)*Pi^3 - (c*x*ArcSinh[
c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) - (2*ArcSinh[c*x]
^3)/3 - ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + ArcSinh[c*x]^2*Log[1
 - E^(2*ArcSinh[c*x])] + Log[1 + c^2*x^2]/2 + ArcSinh[c*x]*PolyLog[2, -E^(
-2*ArcSinh[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + PolyLog[
3, -E^(-2*ArcSinh[c*x])]/2 - PolyLog[3, E^(2*ArcSinh[c*x])]/2))/(2*d^2)
```

### 3.239.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6226, 27, 6202, 240, 6214, 5984, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^2}\,dx$$
$$\Big\downarrow \text{ 6226}$$

$$-\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{dx(c^2x^2+1)}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx}{d^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 6202}$$

$$-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx}{d^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx}{d^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{ 6214}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{ 5984}$$

$$\frac{2\int (a+b\text{arcsinh}(cx))^2\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int i(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2i\int (a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{ 4670}$$

3.239. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}dx$

$$\frac{2i\left(ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsin}\right.}{d^2}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \ 3011$$

$$2i\left(-ib\left(\frac{1}{2}b\int\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+ib\left(\frac{1}{2}b\int\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \ 2720$$

$$2i\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

$$\Big\downarrow \ 7143$$

$$\frac{2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-ib\left(\frac{1}{4}b\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a\right.\right.}{d^2}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^2}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

output
```
(a + b*ArcSinh[c*x])^2/(2*d^2*(1 + c^2*x^2)) - (b*c*((x*(a + b*ArcSinh[c*x
]))/Sqrt[1 + c^2*x^2] - (b*Log[1 + c^2*x^2])/(2*c)))/d^2 + ((2*I)*(I*(a +
b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])] - I*b*(-1/2*((a + b*ArcSinh[
c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(2*ArcSinh[c*x])
])/4) + I*b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])]) +
(b*PolyLog[3, E^(2*ArcSinh[c*x])])/4)))/d^2
```

## 3.239. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx$

### 3.239.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

## 3.239. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)^2}\,dx$

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.239.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(228) = 456$.

Time = 0.26 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.76

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)+2\right)\text{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(\right.\right.}{}$ |
| default | $\frac{a^2\left(\ln(cx)+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^2}+\frac{b^2\left(\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)+2\right)\text{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+1}\right)+\ln\left(\right.\right.}{}$ |
| parts | $\frac{a^2}{2d^2(c^2x^2+1)}-\frac{a^2\ln(c^2x^2+1)}{2d^2}+\frac{a^2\ln(x)}{d^2}+\frac{b^2\left(\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+\text{arcsinh}(cx)+2\right)\text{arcsinh}(cx)}{2c^2x^2+2}-2\ln\left(cx+\sqrt{c^2x^2+\right.\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^2*(ln(c*x)+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b^2/d^2*(1/2*(2*c^2*x^
2-2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)+2)*arcsinh(c*x)/(c^2*x^2+1)-2*ln(c*
x+(c^2*x^2+1)^(1/2))+ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)^2*ln(1+c
*x+(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*p
olylog(3,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2
))^2)-arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(c
*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*arcs
inh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(3,c*x+(c^2*x^2+1)^(1/2
)))+2*a*b/d^2*(1/2*(-c*x*(c^2*x^2+1)^(1/2)+c^2*x^2+arcsinh(c*x)+1)/(c^2*x^
2+1)+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+polylog(2,-c*x-(c^2*x^2+1)^(
1/2))-arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+polylog(2,c*x+
(c^2*x^2+1)^(1/2)))
```

### 3.239.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^5 + 2*
c^2*d^2*x^3 + d^2*x), x)
```

### 3.239.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\frac{\int \frac{a^2}{c^4x^5+2c^2x^3+x}\,dx+\int \frac{b^2\operatorname{asinh}^2\left(cx\right)}{c^4x^5+2c^2x^3+x}\,dx+\int \frac{2ab\operatorname{asinh}\left(cx\right)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b**2*asinh(c*x
)**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(2*a*b*asinh(c*x)/(c**4*x
**5 + 2*c**2*x**3 + x), x))/d**2
```

### 3.239.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + in
tegrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 +
d^2*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 +
 d^2*x), x)
```

### 3.239.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x), x)
```

### 3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{x\,\left(d\,c^2\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2), x)
```

# 3.240 $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx$



## 3.240.1 Optimal result

Integrand size = 26, antiderivative size = 287

$$\begin{aligned}\int \frac{(a+b\,\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx =& -\frac{bc(a+b\,\text{arcsinh}(cx))}{d^2\sqrt{1+c^2x^2}}-\frac{(a+b\,\text{arcsinh}(cx))^2}{d^2x(1+c^2x^2)}\\&-\frac{3c^2x(a+b\,\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)}\\&-\frac{3c(a+b\,\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}\\&+\frac{b^2c\arctan(cx)}{d^2}-\frac{4bc(a+b\,\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2}\\&-\frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2}\\&+\frac{3ibc(a+b\,\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d^2}\\&-\frac{3ibc(a+b\,\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d^2}\\&+\frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2}-\frac{3ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d^2}\\&+\frac{3ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d^2}\end{aligned}$$

output
```
-(a+b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))^2/d^2
/(c^2*x^2+1)-3*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+b^
2*c*arctan(c*x)/d^2-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2)
)/d^2-2*b^2*c*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2+3*I*b*c*(a+b*arcsinh(c
*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-3*I*b*c*(a+b*arcsinh(c*x))*
polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)
^(1/2))/d^2-3*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+3*I*b^2*c*
polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-b*c*(a+b*arcsinh(c*x))/d^2/(c^2*x
^2+1)^(1/2)
```

### 3.240.2 Mathematica [A] (verified)

Time = 7.06 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.91

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{a^2}{d^2x}-\frac{a^2c^2x}{2d^2\left(1+c^2x^2\right)}-\frac{3a^2c\arctan(cx)}{2d^2}$$
$$+\frac{2abc\left(\frac{\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)}{4(-1-icx)}-\frac{\text{arcsinh}(cx)}{cx}-\frac{i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)}{4(i+cx)}-\text{arctanh}\left(\sqrt{1+c^2x^2}\right)+\frac{3}{4}i\left(-\frac{1}{2}\text{arcsinh}(\right.\right.}{}$$
$$+\frac{b^2c\left(-\frac{2\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}-\frac{cx\text{arcsinh}(cx)^2}{1+c^2x^2}+4\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)-\text{arcsinh}(cx)^2\coth\left(\frac{1}{2}\text{arcsinh}(cx\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

output
```
-(a^2/(d^2*x)) - (a^2*c^2*x)/(2*d^2*(1 + c^2*x^2)) - (3*a^2*c*ArcTan[c*x])
/(2*d^2) + (2*a*b*c*((Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x])/(4*(-1 - I*c*x))
 - ArcSinh[c*x]/(c*x) - (I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x])/(4*(I + c*x))
 - ArcTanh[Sqrt[1 + c^2*x^2]] + ((3*I)/4)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh
[c*x]*Log[1 + I*E^ArcSinh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((3
*I)/4)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2
*PolyLog[2, I*E^ArcSinh[c*x]])))/d^2 + (b^2*c*((-2*ArcSinh[c*x])/Sqrt[1 +
c^2*x^2] - (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 4*ArcTan[Tanh[ArcSinh[c*x]
/2]] - ArcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 4*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] + (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (3*I)*ArcS
inh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] + (6*I)*ArcSinh[c*x]*PolyLog[2,
(-I)/E^ArcSinh[c*x]] - (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 4
*PolyLog[2, E^(-ArcSinh[c*x])] + (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (
6*I)*PolyLog[3, I/E^ArcSinh[c*x]] + ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/
(2*d^2)
```

### 3.240.3 Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.06, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {6224, 27, 6203, 6204, 3042, 4668, 3011, 2720, 6213, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(c^2dx^2+d\right)^2}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{d^2\left(c^2x^2+1\right)^2}dx+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{3c^2\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{2\left(c^2x^2+1\right)}\right)}{d^2}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 6204}$$

$$-\frac{3c^2\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))^2}{2\left(c^2x^2+1\right)}\right)}{d^2}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx}{d^2} - \frac{3c^2\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx + \frac{\int (a+b\text{arcsinh}(cx))^2 \csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c} + \frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)}{d^2} - \frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 4668

$$-\frac{3c^2\left(\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan}{2c}\right.}{d^2}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx}{d^2} - \frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 3011

$$-\frac{3c^2\left(\frac{2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right.}{2c}\right.}{}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx}{d^2} - \frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 2720

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx}{d^2} - \frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 6213

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{}$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx}{d^2} - \frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 216

3.240. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx$

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{}$$

$$\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 6226

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{}$$

$$\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int\frac{1}{c^2x^2+1}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 216

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{}$$

$$\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 6231

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{}$$

$$\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 3042

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{}$$

$$\frac{2bc\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\,(c^2x^2+1)}$$

$\downarrow$ 26

3.240. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}dx$

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{\frac{2bc\left(i\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{d^2x(c^2x^2+1)}}$$

$\downarrow$ 4670

$$\frac{2bc\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+\right.\right.}{d^2}$$
$$\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{P}\right.}{2c}\right.}{\frac{(a+b\text{arcsinh}(cx))^2}{d^2x(c^2x^2+1)}}$$

$\downarrow$ 2715

$$\frac{2bc\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arct}\right.\right.}{d^2}$$
$$\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{P}\right.}{2c}\right.}{\frac{(a+b\text{arcsinh}(cx))^2}{d^2x(c^2x^2+1)}}$$

$\downarrow$ 2838

$$-\frac{3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx}\right.}{2c}\right.}{\frac{2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)+\frac{a+b}{}\right.}{d^2}\frac{(a+b\text{arcsinh}(cx))^2}{d^2x(c^2x^2+1)}}$$

$\downarrow$ 7143

---

3.240. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx$

$$\frac{2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)+\frac{a+b}{}\right.}{d^2}$$

$$\frac{3c^2\left(-bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)+\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2\right.\right.}{}\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d^2x\left(c^2x^2+1\right)}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(d^2*x*(1 + c^2*x^2))) + (2*b*c*((a + b*ArcSinh[c
*x])/Sqrt[1 + c^2*x^2] - b*ArcTan[c*x] + I*((2*I)*(a + b*ArcSinh[c*x])*Arc
Tanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^
ArcSinh[c*x]])))/d^2 - (3*c^2*((x*(a + b*ArcSinh[c*x])^2)/(2*(1 + c^2*x^2)
) - b*c*(-((a + b*ArcSinh[c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x])
/c^2) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a +
 b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^Ar
cSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]
]) + b*PolyLog[3, I*E^ArcSinh[c*x]]))/(2*c)))/d^2
```

### 3.240.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.240. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.240. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

## 3.240. $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.240.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^2\,(c^2d\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

### 3.240.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^2\,(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^6 + 2*
c^2*d^2*x^4 + d^2*x^2), x)
```

### 3.240.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a^2}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{b^2\,\text{asinh}^2(cx)}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{2ab\,\text{asinh}(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b**2*asinh(
c*x)**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(
c**4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

### 3.240.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + i
ntegrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 +
 d^2*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^
4 + d^2*x^2), x)
```

### 3.240.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^2), x)
```

### 3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{x^2\,\left(d\,c^2\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2), x)
```

# 3.241 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$



## 3.241.1 Optimal result

Integrand size = 26, antiderivative size = 253

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx = &-\frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}} \\ &-\frac{c^2(a+b\text{arcsinh}(cx))^2}{d^2(1+c^2x^2)}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(1+c^2x^2)} \\ &+\frac{4c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2} \\ &+\frac{b^2c^2\log(x)}{d^2}-\frac{b^2c^2\log(1+c^2x^2)}{2d^2} \\ &+\frac{2bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^2} \\ &-\frac{2bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2} \\ &-\frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{d^2}+\frac{b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{d^2}\end{aligned}$$

output
```
-c^2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-1/2*(a+b*arcsinh(c*x))^2/d^2/x^2
/(c^2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)
/d^2+b^2*c^2*ln(x)/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2+2*b*c^2*(a+b*arcsinh(
c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-2*b*c^2*(a+b*arcsinh(c*x))
*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b^2*c^2*polylog(3,-(c*x+(c^2*x^2
+1)^(1/2))^2)/d^2+b^2*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*(a+
b*arcsinh(c*x))/d^2/x/(c^2*x^2+1)^(1/2)
```

### 3.241.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx = \frac{-\frac{a^2}{x^2}-\frac{a^2c^2}{1+c^2x^2}-4a^2c^2\log(x)+2a^2c^2\log(1+c^2x^2)+ab\left(-\frac{2c\sqrt{1+c^2x^2}}{x}+\frac{c^2\sqrt{1+c^2x^2}}{-i+cx}+\frac{c^2\sqrt{1+c^2x^2}}{i+cx}-\frac{2\text{arcsinh}(cx)}{x^2}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

output
```
(-(a^2/x^2) - (a^2*c^2)/(1 + c^2*x^2) - 4*a^2*c^2*Log[x] + 2*a^2*c^2*Log[1
 + c^2*x^2] + a*b*((-2*c*Sqrt[1 + c^2*x^2])/x + (c^2*Sqrt[1 + c^2*x^2])/(-
I + c*x) + (c^2*Sqrt[1 + c^2*x^2])/(I + c*x) - (2*ArcSinh[c*x])/x^2 + (c^2
*ArcSinh[c*x])/(-1 - I*c*x) - (I*c^2*ArcSinh[c*x])/(I + c*x) + 8*c^2*ArcSi
nh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 8*c^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh
[c*x]] - 8*c^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 8*c^2*PolyLog[2,
 (-I)*E^ArcSinh[c*x]] + 8*c^2*PolyLog[2, I*E^ArcSinh[c*x]] - 4*c^2*PolyLog
[2, E^(2*ArcSinh[c*x])]) + b^2*c^2*((2*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2]
 - (2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - ArcSinh[c*x]^2/(c^2*x^2) - A
rcSinh[c*x]^2/(1 + c^2*x^2) - 4*ArcSinh[c*x]^2*Log[1 - E^(-2*ArcSinh[c*x])
] + 4*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + 2*Log[(c*x)/Sqrt[1 + c
^2*x^2]] - 4*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 4*ArcSinh[c*x
]*PolyLog[2, E^(-2*ArcSinh[c*x])] - 2*PolyLog[3, -E^(-2*ArcSinh[c*x])] + 2
*PolyLog[3, E^(-2*ArcSinh[c*x])]))/(2*d^2)
```

### 3.241.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.23, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {6224, 27, 6219, 25, 354, 86, 2009, 6226, 6202, 240, 6214, 5984, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(c^2dx^2+d\right)^2}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{d^2x\left(c^2x^2+1\right)^2}dx+\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(c^2x^2+1\right)^{3/2}}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 6219}$$

$$-\frac{2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\left(-bc\int -\frac{2c^2x^2+1}{x\left(c^2x^2+1\right)}dx-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\left(bc\int \frac{2c^2x^2+1}{x\left(c^2x^2+1\right)}dx-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\left(\frac{1}{2}bc\int \frac{2c^2x^2+1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^2}+\frac{bc\left(\frac{1}{2}bc\int \left(\frac{c^2}{c^2x^2+1}+\frac{1}{x^2}\right)dx^2-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

---

3.241. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^2}dx$

$$\Big\downarrow \; 6226$$

$$-\frac{2c^2\left(-bc\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 6202$$

$$-\frac{2c^2\left(-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 240$$

$$-\frac{2c^2\left(\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 6214$$

$$-\frac{2c^2\left(\int \frac{(a+b\text{arcsinh}(cx))^2}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)\right)}{d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 5984$$

$$-\frac{2c^2\left(2\int (a+b\text{arcsinh}(cx))^2\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x\ldots}{2c}\right.\right.}{d^2}\ldots$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2c^2\left(2\int i(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2\ldots}{2c\ldots}\right.\right.}{d^2}\ldots$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2(c^2x^2+1)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

---

3.241. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}dx$

$$\Big\downarrow \; 26$$

$$-\frac{2c^2\left(2i\int(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2}{2c}\right.\right.}{d^2}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 4670$$

$$-\frac{2c^2\left(2i\left(ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 3011$$

$$-\frac{2c^2\left(2i\left(-ib\left(\frac{1}{2}b\int\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+ib\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 2720$$

$$-\frac{2c^2\left(2i\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsin}\right.\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$$\Big\downarrow \; 7143$$

$$-\frac{2c^2\left(2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-ib\left(\frac{1}{4}b\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(\right.\right.\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^2x^2\left(c^2x^2+1\right)}+\frac{bc\left(-\frac{2c^2x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(\log\left(c^2x^2+1\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

3.241. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(d^2*x^2*(1 + c^2*x^2)) + (b*c*(-((a + b*ArcSi
nh[c*x])/(x*Sqrt[1 + c^2*x^2])) - (2*c^2*x*(a + b*ArcSinh[c*x]))/Sqrt[1 +
c^2*x^2] + (b*c*(Log[x^2] + Log[1 + c^2*x^2]))/2))/d^2 - (2*c^2*((a + b*Ar
cSinh[c*x])^2/(2*(1 + c^2*x^2)) - b*c*((x*(a + b*ArcSinh[c*x]))/Sqrt[1 + c
^2*x^2] - (b*Log[1 + c^2*x^2])/(2*c)) + (2*I)*(I*(a + b*ArcSinh[c*x])^2*Ar
cTanh[E^(2*ArcSinh[c*x])] - I*b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog[2, -E^
(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(2*ArcSinh[c*x])])/4) + I*b*(-1/2*((
a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, E^(2*A
rcSinh[c*x])])/4))))/d^2
```

### 3.241.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

---

3.241. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

---

## 3.241. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

---

3.241. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$

### 3.241.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(292) = 584$.

Time = 0.30 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.32

| method | result |
|---|---|
| derivativedivides | $c^2 \left( \frac{a^2\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2(c^2x^2+1)}+\ln(c^2x^2+1)\right)}{d^2} + \frac{b^2\left(-\frac{(2\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx))\operatorname{arcsinh}(c\cdots}{2c^2x^2(c^2x^2+1)}\right.}{\cdots}\right.$ |
| default | $c^2 \left( \frac{a^2\left(-\frac{1}{2c^2x^2}-2\ln(cx)-\frac{1}{2(c^2x^2+1)}+\ln(c^2x^2+1)\right)}{d^2} + \frac{b^2\left(-\frac{(2\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx))\operatorname{arcsinh}(c\cdots}{2c^2x^2(c^2x^2+1)}\right.}{\cdots}\right.$ |
| parts | $\frac{a^2\left(\frac{c^4\left(-\frac{1}{c^2(c^2x^2+1)}+\frac{2\ln(c^2x^2+1)}{c^2}\right)}{2}-\frac{1}{2x^2}-2c^2\ln(x)\right)}{d^2} + \frac{b^2c^2\left(-\frac{(2\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx))\operatorname{arcsi}\cdots}{2c^2x^2(c^2x^2+1)}\right.}{\cdots}$ |

input
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a^2/d^2*(-1/2/c^2/x^2-2*ln(c*x)-1/2/(c^2*x^2+1)+ln(c^2*x^2+1))+b^2/d^
2*(-1/2*(2*arcsinh(c*x)*c^2*x^2+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))*arcs
inh(c*x)/c^2/x^2/(c^2*x^2+1)+ln(1+c*x+(c^2*x^2+1)^(1/2))-ln(1+(c*x+(c^2*x^
2+1)^(1/2))^2)+ln(c*x+(c^2*x^2+1)^(1/2)-1)-2*arcsinh(c*x)^2*ln(1+c*x+(c^2*
x^2+1)^(1/2))-4*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+4*polylog(3
,-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+
2*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-polylog(3,-(c*x+(c^2*
x^2+1)^(1/2))^2)-2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-4*arcsinh(c*
x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+4*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*
a*b/d^2*(-1/2*(2*arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/
c^2/x^2/(c^2*x^2+1)-2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+po
lylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1
/2))-2*polylog(2,c*x+(c^2*x^2+1)^(1/2))))
```

### 3.241.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3 (d+c^2dx^2)^2} \, dx = \int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^2x^3} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^7 + 2*
c^2*d^2*x^5 + d^2*x^3), x)
```

### 3.241.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3 (d+c^2dx^2)^2} \, dx = \frac{\int \frac{a^2}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b**2*asinh(
c*x)**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(
c**4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

### 3.241.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3 (d+c^2dx^2)^2} \, dx = \int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^2x^3} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(
c^2*d^2*x^4 + d^2*x^2)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^
4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/
(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

### 3.241.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^2x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^3), x)
```

### 3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^3\,(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2), x)
```

# 3.242 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx$



## 3.242.1 Optimal result

Integrand size = 26, antiderivative size = 401

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx &= -\frac{b^2c^2}{3d^2x}+\frac{2bc^3(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}}\\
&\quad-\frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(1+c^2x^2)}\\
&\quad+\frac{5c^2(a+b\text{arcsinh}(cx))^2}{3d^2x(1+c^2x^2)}+\frac{5c^4x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)}\\
&\quad+\frac{5c^3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d^2}-\frac{b^2c^3\arctan(cx)}{d^2}\\
&\quad+\frac{26bc^3(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{3d^2}\\
&\quad+\frac{13b^2c^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{3d^2}\\
&\quad-\frac{5ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d^2}\\
&\quad+\frac{5ibc^3(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d^2}\\
&\quad-\frac{13b^2c^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{3d^2}\\
&\quad+\frac{5ib^2c^3\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{d^2}\\
&\quad-\frac{5ib^2c^3\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{d^2}\end{aligned}$$

output
```
-1/3*b^2*c^2/d^2/x-1/3*(a+b*arcsinh(c*x))^2/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a
+b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c
^2*x^2+1)+5*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b^2
*c^3*arctan(c*x)/d^2+26/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)
^(1/2))/d^2+13/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2-5*I*b*c^3*(
a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5*I*b*c^3*(a+b
*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-13/3*b^2*c^3*polyl
og(2,c*x+(c^2*x^2+1)^(1/2))/d^2+5*I*b^2*c^3*polylog(3,-I*(c*x+(c^2*x^2+1)^
(1/2)))/d^2-5*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2/3*b*c^3
*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2/x
^2/(c^2*x^2+1)^(1/2)
```

## 3.242.2 Mathematica [A] (verified)

Time = 8.04 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.91

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx = -\frac{a^2}{3d^2x^3}+\frac{2a^2c^2}{d^2x}+\frac{a^2c^4x}{2d^2(1+c^2x^2)}+\frac{5a^2c^3\arctan(cx)}{2d^2}$$
$$+\frac{2ab\left(-\frac{c\sqrt{1+c^2x^2}}{6x^2}-\frac{c^3\left(\sqrt{1+c^2x^2}+i\text{arcsinh}(cx)\right)}{4(-1-icx)}-\frac{\text{arcsinh}(cx)}{3x^3}+\frac{c^4\left(i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{4(ic+c^2x)}+\frac{1}{6}c^3\text{arctanh}\left(\sqrt{1+\right.}{}$$
$$+\frac{b^2c^3\left(\frac{24\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}+\frac{12cx\text{arcsinh}(cx)^2}{1+c^2x^2}-48\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)-4\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)+26\text{a}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

output
```
-1/3*a^2/(d^2*x^3) + (2*a^2*c^2)/(d^2*x) + (a^2*c^4*x)/(2*d^2*(1 + c^2*x^2
)) + (5*a^2*c^3*ArcTan[c*x])/(2*d^2) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/
x^2 - (c^3*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(4*(-1 - I*c*x)) - ArcSin
h[c*x]/(3*x^3) + (c^4*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(4*(I*c + c^2*
x)) + (c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/6 - 2*c^2*(-(ArcSinh[c*x]/x) - c*Ar
cTanh[Sqrt[1 + c^2*x^2]]) - ((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcS
inh[c*x]*Log[1 + I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]]
)/c) + ((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^
ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c)))/d^2 + (b^2*c^3*((
24*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (12*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)
 - 48*ArcTan[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 26*ArcSinh[c
*x]^2*Coth[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*
ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 104*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] - (60*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (60*I)*Ar
cSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 104*ArcSinh[c*x]*Log[1 + E^(-ArcS
inh[c*x])] - 104*PolyLog[2, -E^(-ArcSinh[c*x])] - (120*I)*ArcSinh[c*x]*Pol
yLog[2, (-I)/E^ArcSinh[c*x]] + (120*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh
[c*x]] + 104*PolyLog[2, E^(-ArcSinh[c*x])] - (120*I)*PolyLog[3, (-I)/E^Arc
Sinh[c*x]] + (120*I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[Ar
cSinh[c*x]/2]^2 - (8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) +...
```

## 3.242.3 Rubi [A] (verified)

Time = 4.39 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.21, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {6224, 27, 6224, 264, 216, 6203, 6204, 3042, 4668, 3011, 2720, 6213, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(c^2dx^2+d\right)^2}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{5}{3}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{d^2x^2\left(c^2x^2+1\right)^2}dx+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(c^2x^2+1\right)^{3/2}}dx}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(c^2x^2+1)^2}dx}{3d^2}+\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x^3(c^2x^2+1)^{3/2}}dx}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$\downarrow$ 6224

$$-\frac{5c^2\left(-3c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx+2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}\right)}{3d^2}+\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$\downarrow$ 264

$$-\frac{5c^2\left(-3c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx+2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}\right)}{3d^2}+\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx+\frac{1}{2}bc\left(c^2\left(-\int\frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$\downarrow$ 216

$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{5c^2\left(-3c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx+2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$\downarrow$ 6203

$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{5c^2\left(-3c^2\left(-bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)+2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-(\right.}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$\downarrow$ 6204

3.242. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}dx$

$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-$$
$$\frac{5c^2\left(-3c^2\left(-bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)+2bc\int\frac{a+b\text{arcsinh}(c}{x(c^2x^2+1)^{3/2}}\right.}{3d^2}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-$$
$$\frac{5c^2\left(2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-3c^2\left(-bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int(a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}\right.\right.}{3d^2}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-$$
$$\frac{5c^2\left(-3c^2\left(\frac{-2ib\int(a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2}{2c}\right.\right.}{}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\text{PolyLog}\left(2,ie^{\text{arcs}}\right.\right.}{2c}\right.\right.}{}$$
$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{}$$
$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3(c^2x^2+1)}$$

---

3.242. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}dx$

$\downarrow$ 6213

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 216

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(-\frac{3}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 6226

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(-\frac{3}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int\frac{1}{c^2x^2+1}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 216

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(-\frac{3}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 6231

---

3.242. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx$

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{2bc\left(-\frac{3}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}+\frac{1}{2}bc(-c\arctan(cx)\right.}$$

$$3d^2$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 3042

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{2bc\left(-\frac{3}{2}c^2\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}\right.}$$

$$3d^2$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 26

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{2bc\left(-\frac{3}{2}c^2\left(i\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-b\arctan(cx)\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2\sqrt{c^2x^2+1}}\right.}$$

$$3d^2$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 4670

$$-\frac{5c^2\left(2bc\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.\right.\right.}{2bc\left(-\frac{3}{2}c^2\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.\right.\right.}$$

$$3d^2$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$\downarrow$ 2715

3.242. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^2}dx$

$$-\frac{5c^2\left(2bc\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.\right.\right.}{2bc\left(-\frac{3}{2}c^2\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+\right.\right.\right.}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{5c^2\left(-3c^2\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{ar}}\right.}{}\right.\right.}{2bc\left(-\frac{3}{2}c^2\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.\right.}\ \frac{}{3d^2}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{5c^2\left(2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.\right.}{2bc\left(-\frac{3}{2}c^2\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.\right.}\ \frac{}{3d^2}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^2x^3\left(c^2x^2+1\right)}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

## 3.242. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx$

output
```
-1/3*(a + b*ArcSinh[c*x])^2/(d^2*x^3*(1 + c^2*x^2)) + (2*b*c*(-1/2*(a + b*
ArcSinh[c*x])/(x^2*Sqrt[1 + c^2*x^2]) + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2
- (3*c^2*((a + b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - b*ArcTan[c*x] + I*((2*I
)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh
[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]])))/2))/(3*d^2) - (5*c^2*(-((a + b*
ArcSinh[c*x])^2/(x*(1 + c^2*x^2))) + 2*b*c*((a + b*ArcSinh[c*x])/Sqrt[1 +
c^2*x^2] - b*ArcTan[c*x] + I*((2*I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh
[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]])
) - 3*c^2*((x*(a + b*ArcSinh[c*x])^2)/(2*(1 + c^2*x^2)) - b*c*(-((a + b*Ar
cSinh[c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x])/c^2) + (2*(a + b*Ar
cSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSinh[c*x])*Pol
yLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)
*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*
E^ArcSinh[c*x]]))/(2*c))))/(3*d^2)
```

### 3.242.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

---

3.242. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.242. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

## 3.242. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^2}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.242.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^4\,(c^2d\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

## 3.242.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^4\,(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^2x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^8 + 2*
c^2*d^2*x^6 + d^2*x^4), x)
```

### 3.242.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = \frac{\int \frac{a^2}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b**2*asinh(
c*x)**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(
c**4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

### 3.242.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = \int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^2x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
 d^2*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^8
 + 2*c^2*d^2*x^6 + d^2*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*
x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

### 3.242.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = \int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^2x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^4), x)
```

### 3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^2}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{x^4\,\left(d\,c^2\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2), x)
```

# 3.243 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$



## 3.243.1 Optimal result

Integrand size = 26, antiderivative size = 320

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx =& -\frac{b^2x}{12c^4d^3(1+c^2x^2)}+\frac{b(a+b\text{arcsinh}(cx))}{6c^5d^3(1+c^2x^2)^{3/2}}\\&-\frac{5b(a+b\text{arcsinh}(cx))}{4c^5d^3\sqrt{1+c^2x^2}}\\&-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}-\frac{3x(a+b\text{arcsinh}(cx))^2}{8c^4d^3(1+c^2x^2)}\\&+\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^5d^3}+\frac{7b^2\arctan(cx)}{6c^5d^3}\\&-\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\\&+\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\\&+\frac{3ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\\&-\frac{3ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4c^5d^3}\end{aligned}$$

output
```
-1/12*b^2*x/c^4/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+
1)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-3/8*x*(a+b*arc
sinh(c*x))^2/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*
x^2+1)^(1/2))/c^5/d^3+7/6*b^2*arctan(c*x)/c^5/d^3-3/4*I*b*(a+b*arcsinh(c*x
))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b*(a+b*arcsinh(c*x)
)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b^2*polylog(3,-I*(c*x
+(c^2*x^2+1)^(1/2)))/c^5/d^3-3/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2))
)/c^5/d^3-5/4*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+1)^(1/2)
```

### 3.243.2 Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.72

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{\frac{6a^2cx}{(1+c^2x^2)^2} - \frac{15a^2cx}{1+c^2x^2} + \frac{15ab\left(\sqrt{1+c^2x^2}-i\operatorname{arcsinh}(cx)\right)}{-1+icx} + \frac{15ab\left(\sqrt{1+c^2x^2}+i\operatorname{arcsinh}(cx)\right)}{-1-icx} - \frac{iab\left((-2i+cx)\sqrt{1+c^2x^2}+3\operatorname{arcsinh}(c\right.}{(-i+cx)^2}}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

output
```
((6*a^2*c*x)/(1 + c^2*x^2)^2 - (15*a^2*c*x)/(1 + c^2*x^2) + (15*a*b*(Sqrt[
1 + c^2*x^2] - I*ArcSinh[c*x]))/(-1 + I*c*x) + (15*a*b*(Sqrt[1 + c^2*x^2]
+ I*ArcSinh[c*x]))/(-1 - I*c*x) - (I*a*b*((-2*I + c*x)*Sqrt[1 + c^2*x^2] +
 3*ArcSinh[c*x]))/(-I + c*x)^2 + (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3
*ArcSinh[c*x]))/(I + c*x)^2 + 9*a^2*ArcTan[c*x] + ((9*I)/2)*a*b*(ArcSinh[c
*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^Arc
Sinh[c*x]]) - ((9*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^Ar
cSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((-2*c*x)/(1 + c^2*x^
2) + (4*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (30*ArcSinh[c*x])/Sqrt[1 + c^2
*x^2] + (6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 - (15*c*x*ArcSinh[c*x]^2)/(
1 + c^2*x^2) + 56*ArcTan[Tanh[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[
1 - I/E^ArcSinh[c*x]] + (9*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (
18*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*P
olyLog[2, I/E^ArcSinh[c*x]] - (18*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18
*I)*PolyLog[3, I/E^ArcSinh[c*x]]))/(24*c^5*d^3)
```

### 3.243.3 Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {6225, 27, 6219, 27, 298, 216, 6225, 6204, 3042, 4668, 3011, 2720, 6213, 216, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^3}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{b\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}+\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{d^2(c^2x^2+1)^2}dx}{4c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{6219}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\left(-bc\int -\frac{3c^2x^2+2}{3c^4(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\left(\frac{b\int \frac{3c^2x^2+2}{(c^2x^2+1)^2}dx}{3c^3}-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\left(\frac{b\left(\frac{5}{2}\int \frac{1}{c^2x^2+1}dx-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx}{4c^2d^3}+\frac{b\left(-\frac{a+b\operatorname{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\operatorname{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{6225}$$

$$\frac{3\left(\frac{b\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{c}+\frac{\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{c^2x^2+1}dx}{2c^2}-\frac{x(a+b\operatorname{arcsinh}(cx))^2}{2c^2(c^2x^2+1)}\right)}{4c^2d^3}+\frac{b\left(-\frac{a+b\operatorname{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\operatorname{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{6204}$$

$$\frac{3\left(\frac{b\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{c}+\frac{\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\operatorname{arcsinh}(cx)}{2c^3}-\frac{x(a+b\operatorname{arcsinh}(cx))^2}{2c^2(c^2x^2+1)}\right)}{4c^2d^3}+\frac{b\left(-\frac{a+b\operatorname{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\operatorname{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int (a+b\operatorname{arcsinh}(cx))^2\csc\left(i\operatorname{arcsinh}(cx)+\frac{\pi}{2}\right)d\operatorname{arcsinh}(cx)}{2c^3}+\frac{b\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{c}-\frac{x(a+b\operatorname{arcsinh}(cx))^2}{2c^2(c^2x^2+1)}\right)}{4c^2d^3}+\frac{b\left(-\frac{a+b\operatorname{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\operatorname{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{4668}$$

3.243. $\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

$$3\left(\frac{-2ib\int(a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{ar}}\right.}{2c^3}\right.$$

$$4c^2d^3$$

$$\frac{b\left(-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$\downarrow$ 3011

$$3\left(\frac{2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)d\text{arc}\right.}{2c^3}\right.$$

$$\frac{b\left(-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$\downarrow$ 2720

$$3\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Pol}\right.}{2c^3}\right.$$

$$\frac{b\left(-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$\downarrow$ 6213

$$3\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Pol}\right.}{2c^3}\right.$$

$$\frac{b\left(-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

---

3.243. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

$\Big\downarrow$ 216

$$3\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{Pol}\right.}{2c^3}\right.$$

$$\frac{b\left(-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$\Big\downarrow$ 7143

$$3\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\,\text{Pol}\right.}{2c^3}\right.$$

$$4c^2d^3$$

$$\frac{b\left(-\frac{a+b\text{arcsinh}(cx)}{c^4\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3c^4(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{5\arctan(cx)}{2c}-\frac{x}{2(c^2x^2+1)}\right)}{3c^3}\right)}{2cd^3}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

output
```
-1/4*(x^3*(a + b*ArcSinh[c*x])^2)/(c^2*d^3*(1 + c^2*x^2)^2) + (b*((a + b*A
rcSinh[c*x])/(3*c^4*(1 + c^2*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*Sqrt[
1 + c^2*x^2]) + (b*(-1/2*x/(1 + c^2*x^2) + (5*ArcTan[c*x])/(2*c)))/(3*c^3)
))/(2*c*d^3) + (3*(-1/2*(x*(a + b*ArcSinh[c*x])^2)/(c^2*(1 + c^2*x^2)) + (
b*(-((a + b*ArcSinh[c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x])/c^2))
/c + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSi
nh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])
+ b*PolyLog[3, I*E^ArcSinh[c*x]]))/(2*c^3)))/(4*c^2*d^3)
```

3.243. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

### 3.243.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.243. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.243. $\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

### 3.243.4 Maple [F]

$$\int \frac{x^4(a+b \operatorname{arcsinh}(cx))^2}{(c^2 d x^2+d)^3}dx$$

input
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

output
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

### 3.243.5 Fricas [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b \operatorname{arsinh}(cx)+a)^2 x^4}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^6*
d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.243.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2x^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^4\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^4\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Inte
gral(b**2*x**4*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1),
x) + Integral(2*a*b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2
 + 1), x))/d**3
```

### 3.243.7 Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/8*a^2*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*ar
ctan(c*x)/(c^5*d^3)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c
^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^4*log(c*x + sq
rt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.243.8 Giac [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^3, x)
```

### 3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{x^4\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

output
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

# 3.244 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$



## 3.244.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = -\frac{b^2}{12c^4d^3(1+c^2x^2)}+\frac{bx^3(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{bx(a+b\text{arcsinh}(cx))}{2c^3d^3\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^4d^3}+\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}-\frac{b^2\log(1+c^2x^2)}{3c^4d^3}$$

output
```
-1/12*b^2/c^4/d^3/(c^2*x^2+1)+1/6*b*x^3*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+
1)^(3/2)-1/4*(a+b*arcsinh(c*x))^2/c^4/d^3+1/4*x^4*(a+b*arcsinh(c*x))^2/d^3
/(c^2*x^2+1)^2-1/3*b^2*ln(c^2*x^2+1)/c^4/d^3+1/2*b*x*(a+b*arcsinh(c*x))/c^
3/d^3/(c^2*x^2+1)^(1/2)
```

## 3.244.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = -\frac{3a^2+b^2+6a^2c^2x^2+b^2c^2x^2-6abcx\sqrt{1+c^2x^2}-8abc^3x^3\sqrt{1+c^2x^2}+2b\left(-bcx\sqrt{1+c^2x^2}(3+4c^2x^2)-\right.}{12c^4d^3(1+c^2x^2)}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

output
```
-1/12*(3*a^2 + b^2 + 6*a^2*c^2*x^2 + b^2*c^2*x^2 - 6*a*b*c*x*Sqrt[1 + c^2*
x^2] - 8*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(3
 + 4*c^2*x^2)) + a*(3 + 6*c^2*x^2))*ArcSinh[c*x] + 3*b^2*(1 + 2*c^2*x^2)*A
rcSinh[c*x]^2 + 4*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(c^4*d^3*(1 + c^2*x^
2)^2)
```

## 3.244.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6215, 6225, 243, 49, 2009, 6225, 240, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^3}dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}-\frac{bc\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}$$

$$\Big\downarrow \text{6225}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}-\frac{bc\left(\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{c^2}+\frac{b\int \frac{x^3}{(c^2x^2+1)^2}dx}{3c}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}-\frac{bc\left(\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{c^2}+\frac{b\int \frac{x^2}{(c^2x^2+1)^2}dx^2}{6c}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{49}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{c^2}+\frac{b\int \left(\frac{1}{c^2\left(c^2x^2+1\right)}-\frac{1}{c^2\left(c^2x^2+1\right)^2}\right)dx^2}{6c}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2(c^2x^2+1)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{1}{c^4\left(c^2x^2+1\right)}+\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{6c}\right)}{2d^3}$$

$$\Big\downarrow \text{6225}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{c^2}+\frac{b\int \frac{x}{c^2x^2+1}dx}{c}-\frac{x(a+b\text{arcsinh}(cx))}{c^2\sqrt{c^2x^2+1}}}{c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{1}{c^4\left(c^2x^2+1\right)}+\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{6c}\right)}{2d^3}$$

$$\Big\downarrow \text{240}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{c^2}-\frac{x(a+b\text{arcsinh}(cx))}{c^2\sqrt{c^2x^2+1}}+\frac{b\log\left(c^2x^2+1\right)}{2c^3}}{c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2(c^2x^2+1)^{3/2}}+\frac{b\left(\frac{1}{c^4\left(c^2x^2+1\right)}+\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{6c}\right)}{2d^3}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(-\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2(c^2x^2+1)^{3/2}}+\frac{\frac{(a+b\text{arcsinh}(cx))^2}{2bc^3}-\frac{x(a+b\text{arcsinh}(cx))}{c^2\sqrt{c^2x^2+1}}+\frac{b\log\left(c^2x^2+1\right)}{2c^3}}{c^2}+\frac{b\left(\frac{1}{c^4\left(c^2x^2+1\right)}+\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{6c}\right)}{2d^3}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

---

3.244. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^3}\,dx$

output
```
(x^4*(a + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (b*c*(-1/3*(x^3*(a
+ b*ArcSinh[c*x]))/(c^2*(1 + c^2*x^2)^(3/2)) + (b*(1/(c^4*(1 + c^2*x^2)) +
 Log[1 + c^2*x^2]/c^4))/(6*c) + (-((x*(a + b*ArcSinh[c*x]))/(c^2*Sqrt[1 +
c^2*x^2])) + (a + b*ArcSinh[c*x])^2/(2*b*c^3) + (b*Log[1 + c^2*x^2])/(2*c^
3))/c^2))/(2*d^3)
```

### 3.244.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.244. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

### 3.244.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(\frac{1}{4\left(c^2x^2+1\right)^2}-\frac{1}{2\left(c^2x^2+1\right)}\right)}{d^3}+\frac{b^2\left(\frac{4\,\mathrm{arcsinh}(cx)}{3}-\frac{-8\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+8\,\mathrm{arcsinh}(cx)c^4x^4+6\,\mathrm{arcsinh}(cx)^2x^2c^2-6\ \mathrm{a}}{12\left(c^4\right.}\right.}{}$ |
| default | $\frac{a^2\left(\frac{1}{4\left(c^2x^2+1\right)^2}-\frac{1}{2\left(c^2x^2+1\right)}\right)}{d^3}+\frac{b^2\left(\frac{4\,\mathrm{arcsinh}(cx)}{3}-\frac{-8\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+8\,\mathrm{arcsinh}(cx)c^4x^4+6\,\mathrm{arcsinh}(cx)^2x^2c^2-6\ \mathrm{a}}{12\left(c^4\right.}\right.}{}$ |
| parts | $\frac{a^2\left(-\frac{1}{2c^4\left(c^2x^2+1\right)}+\frac{1}{4c^4\left(c^2x^2+1\right)^2}\right)}{d^3}+\frac{b^2\left(\frac{4\,\mathrm{arcsinh}(cx)}{3}-\frac{-8\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+8\,\mathrm{arcsinh}(cx)c^4x^4+6\,\mathrm{arcsinh}(cx)^2}{}\right.}{}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2/d^3*(1/4/(c^2*x^2+1)^2-1/2/(c^2*x^2+1))+b^2/d^3*(4/3*arcsinh(c*
x)-1/12*(-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+8*arcsinh(c*x)*c^4*x^4+
6*arcsinh(c*x)^2*x^2*c^2-6*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+16*arcsinh(c
*x)*c^2*x^2+3*arcsinh(c*x)^2+c^2*x^2+8*arcsinh(c*x)+1)/(c^4*x^4+2*c^2*x^2+
1)-2/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b/d^3*(1/4/(c^2*x^2+1)^2*arcsi
nh(c*x)-1/2/(c^2*x^2+1)*arcsinh(c*x)-1/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c*x/(c
^2*x^2+1)^(1/2)))
```

### 3.244.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.69

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{8\,abc^4x^4-(6\,a^2-16\,ab+b^2)c^2x^2-3\,(2\,b^2c^2x^2+b^2)\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+8\,ab-b^2-4\,(b^2c^4x^4}{}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/12*(8*a*b*c^4*x^4 - (6*a^2 - 16*a*b + b^2)*c^2*x^2 - 3*(2*b^2*c^2*x^2 +
b^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 3*a^2 + 8*a*b - b^2 - 4*(b^2*c^4*x^4
 + 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + (4*b^2*c^3*x
^3 + 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c
^4*x^4 + 2*a*b*c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(4*a*b*c^3
*x^3 + 3*a*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^
3)
```

### 3.244.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{\int \frac{a^2x^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{2abx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Inte
gral(b**2*x**3*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1),
x) + Integral(2*a*b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2
 + 1), x))/d**3
```

### 3.244.7 Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^3}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*c^2*x^2 + 1)*a^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 1/4*(2*
b^2*c^2*x^2 + b^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^8*d^3*x^4 + 2*c^6*d^3
*x^2 + c^4*d^3) + integrate(1/2*(3*b^2*c^2*x^2 + 2*(2*a*b*c^4 + b^2*c^4)*x
^4 + b^2 + (b^2*c*x + 2*(2*a*b*c^3 + b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*log(
c*x + sqrt(c^2*x^2 + 1))/(c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c
^4*d^3*x + (c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^
2*x^2 + 1)), x)
```

### 3.244.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{x^3\,(a+b\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

output
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

# 3.245 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$



## 3.245.1 Optimal result

Integrand size = 26, antiderivative size = 318

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx =& \frac{b^2x}{12c^2d^3(1+c^2x^2)}-\frac{b(a+b\text{arcsinh}(cx))}{6c^3d^3(1+c^2x^2)^{3/2}}+\frac{b(a+b\text{arcsinh}(cx))}{4c^3d^3\sqrt{1+c^2x^2}}\\&-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{8c^2d^3(1+c^2x^2)}\\&+\frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4c^3d^3}-\frac{b^2\arctan(cx)}{6c^3d^3}\\&-\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3}\\&+\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3}\\&+\frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3}-\frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4c^3d^3}\end{aligned}$$

output
```
1/12*b^2*x/c^2/d^3/(c^2*x^2+1)-1/6*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1
)^(3/2)-1/4*x*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2+1/8*x*(a+b*arcsin
h(c*x))^2/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2
+1)^(1/2))/c^3/d^3-1/6*b^2*arctan(c*x)/c^3/d^3-1/4*I*b*(a+b*arcsinh(c*x))*
polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b*(a+b*arcsinh(c*x))*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b^2*polylog(3,-I*(c*x+(c
^2*x^2+1)^(1/2)))/c^3/d^3-1/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c
^3/d^3+1/4*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1)^(1/2)
```

### 3.245.2 Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.73

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{-\frac{6a^2cx}{(1+c^2x^2)^2}+\frac{3a^2cx}{1+c^2x^2}+\frac{ab\left((2+icx)\sqrt{1+c^2x^2}+3i\text{arcsinh}(cx)\right)}{(-i+cx)^2}+\frac{3ab\left(-i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{-i+cx}+\frac{3ab\left(i\sqrt{1+c^2x^2}+\text{arcsinh}(c\right.}{i+cx}}{}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

output
```
((-6*a^2*c*x)/(1 + c^2*x^2)^2 + (3*a^2*c*x)/(1 + c^2*x^2) + (a*b*((2 + I*c
*x)*Sqrt[1 + c^2*x^2] + (3*I)*ArcSinh[c*x]))/(-I + c*x)^2 + (3*a*b*((-I)*S
qrt[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (3*a*b*(I*Sqrt[1 + c^2*x^2]
 + ArcSinh[c*x]))/(I + c*x) - (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*Ar
cSinh[c*x]))/(I + c*x)^2 + 3*a^2*ArcTan[c*x] + ((3*I)/2)*a*b*(ArcSinh[c*x]
*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSin
h[c*x]]) - ((3*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSi
nh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((2*c*x)/(1 + c^2*x^2) -
 (4*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2]
 - (6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (3*c*x*ArcSinh[c*x]^2)/(1 + c^
2*x^2) - 8*ArcTan[Tanh[ArcSinh[c*x]/2]] - (3*I)*ArcSinh[c*x]^2*Log[1 - I/E
^ArcSinh[c*x]] + (3*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (6*I)*Ar
cSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (6*I)*ArcSinh[c*x]*PolyLog[2,
 I/E^ArcSinh[c*x]] - (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (6*I)*PolyLog
[3, I/E^ArcSinh[c*x]]))/(24*c^3*d^3)
```

### 3.245.3 Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6225, 27, 6203, 6204, 3042, 4668, 3011, 2720, 6213, 215, 216, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^3}\,dx$$
$$\Big\downarrow \; 6225$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d^2(c^2x^2+1)^2}dx}{4c^2d}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 27

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 6203

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}+\frac{-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 6204

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}+\frac{-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 3042

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}+\frac{-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}}{4c^2d^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 4668

$$\frac{\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}}\right.}{2c}}{4c^2d^3}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 3011

---

3.245. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

$$\frac{2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)d\text{arcsin}\right.}{2c}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 2720

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\right.}{2c}$$

$$\frac{b\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2cd^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 6213

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\right.}{2c}$$

$$\frac{b\left(\frac{b\int \frac{1}{\left(c^2x^2+1\right)^2}dx}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 215

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\right.}{2c}$$

$$\frac{b\left(\frac{b\left(\frac{1}{2}\int \frac{1}{c^2x^2+1}dx+\frac{x}{2\left(c^2x^2+1\right)}\right)}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 216

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\right.}{2c}$$

$$\frac{b\left(\frac{b\left(\frac{\arctan(cx)}{2c}+\frac{x}{2\left(c^2x^2+1\right)}\right)}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

---

3.245. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}dx$

$\Big\downarrow$ 7143

$$\frac{-bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)+\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{ar}}\right.\right.}{}}{4c^2d^3}$$

$$\frac{b\left(\frac{b\left(\frac{\arctan(cx)}{2c}+\frac{x}{2\left(c^2x^2+1\right)}\right)}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{x(a+b\text{arcsinh}(cx))^2}{4c^2d^3\left(c^2x^2+1\right)^2}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

output
```
-1/4*(x*(a + b*ArcSinh[c*x])^2)/(c^2*d^3*(1 + c^2*x^2)^2) + (b*(-1/3*(a +
b*ArcSinh[c*x])/(c^2*(1 + c^2*x^2)^(3/2)) + (b*(x/(2*(1 + c^2*x^2)) + ArcT
an[c*x]/(2*c)))/(3*c)))/(2*c*d^3) + ((x*(a + b*ArcSinh[c*x])^2)/(2*(1 + c^
2*x^2)) - b*c*(-((a + b*ArcSinh[c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan
[c*x])/c^2) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(
-((a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I
)*E^ArcSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSin
h[c*x]]) + b*PolyLog[3, I*E^ArcSinh[c*x]]))/(2*c))/(4*c^2*d^3)
```

### 3.245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.245. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

---

## 3.245. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3} dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.245.4 Maple [F]

$$\int \frac{x^2(a + b\,\mathrm{arcsinh}\,(cx))^2}{(c^2 d\,x^2 + d)^3} dx$$

input
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

output
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

### 3.245.5 Fricas [F]

$$\int \frac{x^2(a + b\mathrm{arcsinh}(cx))^2}{(d + c^2dx^2)^3}\, dx = \int \frac{(b\,\mathrm{arsinh}\,(cx) + a)^2 x^2}{(c^2dx^2 + d)^3}\, dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^6*
d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.245.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2x^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^2\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^2\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Inte
gral(b**2*x**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1),
x) + Integral(2*a*b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2
 + 1), x))/d**3
```

### 3.245.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^2}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/8*a^2*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*
x)/(c^3*d^3)) + integrate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*
x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^2*log(c*x + sqrt(c^2*
x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.245.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^3}\,dx=\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2x^2}{\left(c^2dx^2+d\right)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^3, x)
```

### 3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\left(d+c^2dx^2\right)^3}\,dx=\int \frac{x^2\,\left(a+b\,\text{asinh}(c\,x)\right)^2}{\left(d\,c^2\,x^2+d\right)^3}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

output
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

# 3.246 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$



## 3.246.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{b^2}{12c^2d^3(1+c^2x^2)} + \frac{bx(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}} + \frac{bx(a+b\text{arcsinh}(cx))}{3cd^3\sqrt{1+c^2x^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(1+c^2x^2)^2} - \frac{b^2\log(1+c^2x^2)}{6c^2d^3}$$

output
```
1/12*b^2/c^2/d^3/(c^2*x^2+1)+1/6*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^
(3/2)-1/4*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-1/6*b^2*ln(c^2*x^2+1)
/c^2/d^3+1/3*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(1/2)
```

## 3.246.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{-3a^2+b^2+b^2c^2x^2+6abcx\sqrt{1+c^2x^2}+4abc^3x^3\sqrt{1+c^2x^2}+2b\left(-3a+bcx\sqrt{1+c^2x^2}(3+2c^2x^2)\right)\text{arcsin}}{12d^3(c+c^3x^2)^2}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

output
```
(-3*a^2 + b^2 + b^2*c^2*x^2 + 6*a*b*c*x*Sqrt[1 + c^2*x^2] + 4*a*b*c^3*x^3*
Sqrt[1 + c^2*x^2] + 2*b*(-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2))*A
rcSinh[c*x] - 3*b^2*ArcSinh[c*x]^2 - 2*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])
/(12*d^3*(c + c^3*x^2)^2)
```

### 3.246.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6213, 6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^3}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{b\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{5/2}}dx}{2cd^3}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{6203}$$

$$\frac{b\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\int \frac{x}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}\right)}{2cd^3}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{b\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)}{2cd^3}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{6202}$$

$$\frac{b\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)}{2cd^3}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{b\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2cd^3}-\frac{(a+b\text{arcsinh}(cx))^2}{4c^2d^3(c^2x^2+1)^2}$$

input
```
Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcSinh[c*x])^2/(c^2*d^3*(1 + c^2*x^2)^2) + (b*(b/(6*c*(1 + c^
2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*(1 + c^2*x^2)^(3/2)) + (2*((x*(a + b
*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*Log[1 + c^2*x^2])/(2*c)))/3))/(2*c*
d^3)
```

### 3.246.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.246.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.57

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a^2}{4d^3(c^2x^2+1)^2}+\frac{b^2\left(\frac{2\,\mathrm{arcsinh}(cx)}{3}-\frac{-4\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+4\,\mathrm{arcsinh}(cx)c^4x^4-6\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}+8\,\mathrm{arcsinh}(cx)}{12(c^4x^4+2c^2x^2+1)}\right.}{d^3}}{c^2}$ |
| default | $\frac{-\frac{a^2}{4d^3(c^2x^2+1)^2}+\frac{b^2\left(\frac{2\,\mathrm{arcsinh}(cx)}{3}-\frac{-4\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+4\,\mathrm{arcsinh}(cx)c^4x^4-6\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}+8\,\mathrm{arcsinh}(cx)}{12(c^4x^4+2c^2x^2+1)}\right.}{d^3}}{c^2}$ |
| parts | $-\frac{a^2}{4d^3c^2(c^2x^2+1)^2}+\frac{b^2\left(\frac{2\,\mathrm{arcsinh}(cx)}{3}-\frac{-4\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+4\,\mathrm{arcsinh}(cx)c^4x^4-6\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}+8\,\mathrm{arcsi}}{12(c^4x^4+2c^2x^2+1)}\right.}{d^3c^2}$ |

input
```
int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/4*a^2/d^3/(c^2*x^2+1)^2+b^2/d^3*(2/3*arcsinh(c*x)-1/12*(-4*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+4*arcsinh(c*x)*c^4*x^4-6*arcsinh(c*x)*c*
x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)*c^2*x^2+3*arcsinh(c*x)^2-c^2*x^2+4*arcs
inh(c*x)-1)/(c^4*x^4+2*c^2*x^2+1)-1/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2))+2*a
*b/d^3*(-1/4/(c^2*x^2+1)^2*arcsinh(c*x)+1/12/(c^2*x^2+1)^(3/2)*c*x+1/6*c*x
/(c^2*x^2+1)^(1/2)))
```

### 3.246.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(131) = 262.

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.88

$$\int \frac{x(a+b\,\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$=\frac{4\,abc^4x^4+(8\,ab+b^2)c^2x^2-3\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+4\,ab+b^2-2\,(b^2c^4x^4+2\,b^2c^2x^2+b^2)\log}{}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/12*(4*a*b*c^4*x^4 + (8*a*b + b^2)*c^2*x^2 - 3*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 - 3*a^2 + 4*a*b + b^2 - 2*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log
(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + (2*b^2*c^3*x^3 + 3*b^2*
c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x^4 + 2*
a*b*c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(2*a*b*c^3*x^3 + 3*a*
b*c*x)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

### 3.246.6 Sympy [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \frac{\int \frac{a^2x}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(b**2*x*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + I
ntegral(2*a*b*x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
)/d**3
```

### 3.246.7 Maxima [F]

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2
*d^3) - 1/4*a^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + integrate(1/2*((
4*a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^2 + 1)*(4*a*b*c + b^2*c)*x + b^2)*lo
g(c*x + sqrt(c^2*x^2 + 1))/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3 +
c^2*d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^2
*x^2 + 1)), x)
```

### 3.246.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^3, x)
```

### 3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{x\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

output
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

# 3.247 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$



## 3.247.1 Optimal result

Integrand size = 23, antiderivative size = 309

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx &= -\frac{b^2x}{12d^3(1+c^2x^2)}+\frac{b(a+b\text{arcsinh}(cx))}{6cd^3(1+c^2x^2)^{3/2}}+\frac{3b(a+b\text{arcsinh}(cx))}{4cd^3\sqrt{1+c^2x^2}}\\ &\quad+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}+\frac{3x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)}\\ &\quad+\frac{3(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{5b^2\arctan(cx)}{6cd^3}\\ &\quad-\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}\\ &\quad+\frac{3ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}\\ &\quad+\frac{3ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{3ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4cd^3}\end{aligned}$$

output
```
-1/12*b^2*x/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(3/
2)+1/4*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+3/8*x*(a+b*arcsinh(c*x))^2
/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/
d^3-5/6*b^2*arctan(c*x)/c/d^3-3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x
+(c^2*x^2+1)^(1/2)))/c/d^3+3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^
2*x^2+1)^(1/2)))/c/d^3+3/4*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
^3-3/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/4*b*(a+b*arcsinh
(c*x))/c/d^3/(c^2*x^2+1)^(1/2)
```

### 3.247.2 Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{\frac{6a^2x}{(1+c^2x^2)^2}+\frac{9a^2x}{1+c^2x^2}+\frac{9a^2\arctan(cx)}{c}+\frac{ab\left(\frac{9\left(-i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{-i+cx}+\frac{9\left(i\sqrt{1+c^2x^2}+\text{arcsinh}(cx)\right)}{i+cx}-\frac{i\left((-2i+cx)\sqrt{1+c^2x^2}+3\text{arcsi}\right.}{(-i+cx)^2}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

output
```
((6*a^2*x)/(1 + c^2*x^2)^2 + (9*a^2*x)/(1 + c^2*x^2) + (9*a^2*ArcTan[c*x])
/c + (a*b*((9*((-I)*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (9*(I*
Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x) - (I*((-2*I + c*x)*Sqrt[1 + c
^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (I*((2*I + c*x)*Sqrt[1 + c^2*x^2
] + 3*ArcSinh[c*x]))/(I + c*x)^2 + ((9*I)/2)*(ArcSinh[c*x]*(ArcSinh[c*x] -
 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((9*I
)/2)*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLo
g[2, I*E^ArcSinh[c*x]])))/c + (b^2*((-2*c*x)/(1 + c^2*x^2) + (4*ArcSinh[c*
x])/(1 + c^2*x^2)^(3/2) + (18*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (6*c*x*Arc
Sinh[c*x]^2)/(1 + c^2*x^2)^2 + (9*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 40*A
rcTan[Tanh[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x
]] + (9*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSinh[c*x]*
PolyLog[2, (-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSi
nh[c*x]] - (18*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3, I/E^
ArcSinh[c*x]]))/c)/(24*d^3)
```

### 3.247.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6203, 27, 6203, 6204, 3042, 4668, 3011, 2720, 6213, 215, 216, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^3}\,dx$$

$$\Big\downarrow \; 6203$$

$$-\frac{bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{3\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d^2(c^2x^2+1)^2}dx}{4d}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{3\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx}{4d^3}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 6203$$

$$-\frac{bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{3\left(-bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{c^2x^2+1}dx+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)}{4d^3}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 6204$$

$$-\frac{bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{3\left(-bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\operatorname{arcsinh}(cx)}{2c}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)}{4d^3}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{3\left(-bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int (a+b\operatorname{arcsinh}(cx))^2\csc\left(i\operatorname{arcsinh}(cx)+\frac{\pi}{2}\right)d\operatorname{arcsinh}(cx)}{2c}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)}{4d^3}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 4668$$

---

3.247. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

$$3\left(\frac{-2ib\int(a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arc}\cdots}\right.}{2c}\right.$$
$$\overline{\qquad 4d^3 \qquad}$$
$$\frac{bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}$$

$\downarrow$ 3011

$$3\left(\frac{2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)d\text{arc}\cdots\right.}{2c}\right.$$
$$\frac{bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}$$

$\downarrow$ 2720

$$3\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Poly}\cdots\right.}{2c}\right.$$
$$\frac{bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{2d^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}$$

$\downarrow$ 6213

$$3\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Poly}\cdots\right.}{2c}\right.$$
$$\frac{bc\left(\frac{b\int\frac{1}{(c^2x^2+1)^2}dx}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2d^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}$$

$\downarrow$ 215

$$3\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{Poly}\cdots\right.}{2c}\right.$$
$$\frac{bc\left(\frac{b\left(\frac{1}{2}\int\frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2d^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3(c^2x^2+1)^2}$$

---

3.247. $\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}dx$

$\downarrow$ 216

$$3\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{Poly}\right.}{2c}\right.$$

$$\frac{bc\left(\frac{b\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2d^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 7143

$$3\left(-bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)+\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-\right.\right.}{4}\right.$$

$$\frac{bc\left(\frac{b\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)}{2d^3}+\frac{x(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (b*c*(-1/3*(a + b*Arc
Sinh[c*x])/(c^2*(1 + c^2*x^2)^(3/2)) + (b*(x/(2*(1 + c^2*x^2)) + ArcTan[c*
x]/(2*c)))/(3*c)))/(2*d^3) + (3*((x*(a + b*ArcSinh[c*x])^2)/(2*(1 + c^2*x^
2)) - b*c*(-((a + b*ArcSinh[c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x
])/c^2) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a
 + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^
ArcSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]]) + b*PolyLog[3, I*E^ArcSinh[c*x]]))/(2*c)))/(4*d^3)
```

## 3.247. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

### 3.247.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.247. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.247.4 Maple [F]

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{3}}d x$$

input
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

output
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

### 3.247.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.247.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx$$
$$=\frac{\int \frac{a^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{b^2\,\text{asinh}^2\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{2ab\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(
b**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(2*a*b*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

### 3.247.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/8*a^2*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(
c*x)/(c*d^3)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^6
+ 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1)
)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.247.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^3, x)
```

### 3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3, x)
```

# 3.248 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx$



## 3.248.1 Optimal result

Integrand size = 26, antiderivative size = 275

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx = &-\frac{b^2}{12d^3(1+c^2x^2)}-\frac{bcx(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}}\\ &-\frac{4bcx(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}}\\ &+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}+\frac{(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)}\\ &-\frac{2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3}+\frac{2b^2\log(1+c^2x^2)}{3d^3}\\ &-\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^3}\\ &+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^3}\\ &+\frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^3}-\frac{b^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^3}\end{aligned}$$

output
```
-1/12*b^2/d^3/(c^2*x^2+1)-1/6*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/
2)+1/4*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+1/2*(a+b*arcsinh(c*x))^2/d^3
/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3
+2/3*b^2*ln(c^2*x^2+1)/d^3-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1
)^(1/2))^2)/d^3+b*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/
d^3+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-1/2*b^2*polylog(3,(c
*x+(c^2*x^2+1)^(1/2))^2)/d^3-4/3*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^
(1/2)
```

### 3.248.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx$$

$$= \frac{\frac{6a^2}{(1+c^2x^2)^2}+\frac{12a^2}{1+c^2x^2}+24a^2\log(cx)-12a^2\log(1+c^2x^2)+ab\left(-\frac{15\left(\sqrt{1+c^2x^2}-i\text{arcsinh}(cx)\right)}{i+cx}-\frac{15\left(\sqrt{1+c^2x^2}+i\text{arcs}\right.}{-i+cx}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

output
```
((6*a^2)/(1 + c^2*x^2)^2 + (12*a^2)/(1 + c^2*x^2) + 24*a^2*Log[c*x] - 12*a
^2*Log[1 + c^2*x^2] + a*b*((-15*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I +
 c*x) - (15*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 24*ArcSinh[
c*x]^2 - ((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(-I + c*x)^2 -
((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(I + c*x)^2 + 48*ArcSinh[
c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 12*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[
1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + 12*(ArcSinh[
c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSi
nh[c*x]]) + 24*PolyLog[2, E^(2*ArcSinh[c*x])]) + b^2*(I*Pi^3 - 2/(1 + c^2*
x^2) - (4*c*x*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (32*c*x*ArcSinh[c*x])/Sq
rt[1 + c^2*x^2] + (6*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (12*ArcSinh[c*x]^2)
/(1 + c^2*x^2) - 16*ArcSinh[c*x]^3 - 24*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSi
nh[c*x])] + 24*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 16*Log[1 + c^2
*x^2] + 24*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 24*ArcSinh[c*x]
*PolyLog[2, E^(2*ArcSinh[c*x])] + 12*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 12
*PolyLog[3, E^(2*ArcSinh[c*x])]))/(24*d^3)
```

### 3.248.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.18, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.654, Rules used = {6226, 27, 6203, 241, 6202, 240, 6226, 6202, 240, 6214, 5984, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.248. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^3}dx$$

$$\Big\downarrow \text{ 6226}$$

$$-\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^{5/2}}dx}{2d^3}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d^2x\left(c^2x^2+1\right)^2}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^{5/2}}dx}{2d^3}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{bc\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^{3/2}}dx-\frac{1}{3}bc\int \frac{x}{\left(c^2x^2+1\right)^2}dx+\frac{x(a+b\text{arcsinh}(cx))}{3\left(c^2x^2+1\right)^{3/2}}\right)}{2d^3}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{bc\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{3\left(c^2x^2+1\right)^{3/2}}+\frac{b}{6c\left(c^2x^2+1\right)}\right)}{2d^3}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 6202}$$

$$-\frac{bc\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)+\frac{x(a+b\text{arcsinh}(cx))}{3\left(c^2x^2+1\right)^{3/2}}+\frac{b}{6c\left(c^2x^2+1\right)}\right)}{2d^3}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2x^2+1\right)^2}dx}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3\left(c^2x^2+1\right)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)+\frac{b}{6c\left(c^2x^2+1\right)}\right)}{2d^3}$$

$$\Big\downarrow \text{ 6226}$$

---

3.248. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^3}dx$

$$\frac{-bc\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$$\Big\downarrow \text{ 6214}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$$\Big\downarrow \text{ 5984}$$

$$\frac{2\int (a+b\text{arcsinh}(cx))^2\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int i(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^3}+\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$$\Big\downarrow \text{ 26}$$

---

3.248. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx$

$$\frac{2i\int (a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$\downarrow$ 4670

$$\frac{2i\left(ib\int (a+b\text{arcsinh}(cx))\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int (a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsin}}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$\downarrow$ 3011

$$2i\left(-ib\left(\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+ib\left(\frac{1}{2}b\int\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$\downarrow$ 2720

$$2i\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

$\downarrow$ 7143

$$2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-ib\left(\frac{1}{4}b\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{2d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

---

3.248. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx$

output
```
(a + b*ArcSinh[c*x])^2/(4*d^3*(1 + c^2*x^2)^2) - (b*c*(b/(6*c*(1 + c^2*x^2
)) + (x*(a + b*ArcSinh[c*x]))/(3*(1 + c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcS
inh[c*x]))/Sqrt[1 + c^2*x^2] - (b*Log[1 + c^2*x^2])/(2*c)))/3))/(2*d^3) +
((a + b*ArcSinh[c*x])^2/(2*(1 + c^2*x^2)) - b*c*((x*(a + b*ArcSinh[c*x]))/
Sqrt[1 + c^2*x^2] - (b*Log[1 + c^2*x^2])/(2*c)) + (2*I)*(I*(a + b*ArcSinh[
c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])] - I*b*(-1/2*((a + b*ArcSinh[c*x])*Poly
Log[2, -E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(2*ArcSinh[c*x])])/4) + I*
b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])]) + (b*PolyLog
[3, E^(2*ArcSinh[c*x])])/4)))/d^3
```

### 3.248.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.248. $\int \frac{(a+b\,\mathrm{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.248. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^3}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.248.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(300) = 600$.

Time = 0.31 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.46

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(\ln(cx)+\frac{1}{4\left(c^2x^2+1\right)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^3}+\frac{b^2\left(\frac{-16\ \mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+16\ \mathrm{arcsinh}(cx)c^4x^4+6\ \mathrm{arcsinh}(cx)}{}\right.}{}$ |
| default | $\frac{a^2\left(\ln(cx)+\frac{1}{4\left(c^2x^2+1\right)^2}+\frac{1}{2c^2x^2+2}-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{d^3}+\frac{b^2\left(\frac{-16\ \mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+16\ \mathrm{arcsinh}(cx)c^4x^4+6\ \mathrm{arcsinh}(cx)}{}\right.}{}$ |
| parts | $\frac{a^2\left(-\frac{c^2\left(-\frac{1}{c^2\left(c^2x^2+1\right)}-\frac{1}{2c^2\left(c^2x^2+1\right)^2}+\frac{\ln\left(c^2x^2+1\right)}{c^2}\right)}{2}+\ln(x)\right)}{d^3}+\frac{b^2\left(\frac{-16\ \mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+16\ \mathrm{arcsinh}(cx)c^4x}{}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^3*(ln(c*x)+1/4/(c^2*x^2+1)^2+1/2/(c^2*x^2+1)-1/2*ln(c^2*x^2+1))+b^2/
d^3*(1/12*(-16*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+16*arcsinh(c*x)*c^4*
x^4+6*arcsinh(c*x)^2*x^2*c^2-18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+32*arcs
inh(c*x)*c^2*x^2+9*arcsinh(c*x)^2-c^2*x^2+16*arcsinh(c*x)-1)/(c^4*x^4+2*c^
2*x^2+1)-8/3*ln(c*x+(c^2*x^2+1)^(1/2))+4/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
+arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,-c*x-
(c^2*x^2+1)^(1/2))-2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)^2*ln(1
+(c*x+(c^2*x^2+1)^(1/2))^2)-arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)+1/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)^2*ln(1-c*x-(c^
2*x^2+1)^(1/2))+2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*polylog(
3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b/d^3*(1/12*(-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*
c^4*x^4+6*arcsinh(c*x)*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+16*c^2*x^2+9*arcsin
h(c*x)+8)/(c^4*x^4+2*c^2*x^2+1)+arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))-arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)-1/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)*ln(1-c*x-(c^2*x
^2+1)^(1/2))+polylog(2,c*x+(c^2*x^2+1)^(1/2)))
```

### 3.248.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^3}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^7 + 3*
c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

### 3.248.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^3}\,dx=\frac{\int \frac{a^2}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int \frac{b^2\,\text{asinh}^2\left(cx\right)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int \frac{2ab\,\text{asinh}\left(cx\right)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(
b**2*asinh(c*x)**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integ
ral(2*a*b*asinh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

### 3.248.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a^2*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x
^2 + 1)/d^3 + 4*log(x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2
/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x) + 2*a*b*log(c*x + s
qrt(c^2*x^2 + 1))/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x
)
```

### 3.248.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x), x)
```

### 3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^3}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x\,(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3), x)
```

# 3.249 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$



## 3.249.1 Optimal result

Integrand size = 26, antiderivative size = 389

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx &= \frac{b^2c^2x}{12d^3(1+c^2x^2)}-\frac{bc(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}}\\&-\frac{7bc(a+b\text{arcsinh}(cx))}{4d^3\sqrt{1+c^2x^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x(1+c^2x^2)^2}\\&-\frac{5c^2x(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2}-\frac{15c^2x(a+b\text{arcsinh}(cx))^2}{8d^3(1+c^2x^2)}\\&-\frac{15c(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4d^3}\\&+\frac{11b^2c\arctan(cx)}{6d^3}-\frac{4bc(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^3}\\&-\frac{2b^2c\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^3}\\&+\frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{4d^3}\\&-\frac{15ibc(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{4d^3}\\&+\frac{2b^2c\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^3}\\&-\frac{15ib^2c\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{4d^3}\\&+\frac{15ib^2c\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{4d^3}\end{aligned}$$

output
```
1/12*b^2*c^2*x/d^3/(c^2*x^2+1)-1/6*b*c*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^
(3/2)-(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2-5/4*c^2*x*(a+b*arcsinh(c*x)
)^2/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)-15/4
*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3+11/6*b^2*c*arcta
n(c*x)/d^3-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d^3-2*b
^2*c*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3+15/4*I*b*c*(a+b*arcsinh(c*x))*p
olylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-15/4*I*b*c*(a+b*arcsinh(c*x))*pol
ylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1
/2))/d^3-15/4*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+15/4*I*b^2
*c*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/4*b*c*(a+b*arcsinh(c*x))/d^3
/(c^2*x^2+1)^(1/2)
```

## 3.249.2 Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.84

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx = -\frac{a^2}{d^3x}-\frac{a^2c^2x}{4d^3(1+c^2x^2)^2}-\frac{7a^2c^2x}{8d^3(1+c^2x^2)}-\frac{15a^2c\arctan(cx)}{8d^3}$$
$$+\frac{2abc\left(\frac{7\left(\sqrt{1+c^2x^2}+i\operatorname{arcsinh}(cx)\right)}{16(-1-icx)}-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{7\left(i\sqrt{1+c^2x^2}+\operatorname{arcsinh}(cx)\right)}{16(i+cx)}+\frac{i\left((-2i+cx)\sqrt{1+c^2x^2}+3\operatorname{arcsinh}(cx)\right)}{48(-i+cx)^2}-\right.}{}$$
$$+\frac{b^2c\left(\frac{2cx}{1+c^2x^2}-\frac{4\operatorname{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}-\frac{42\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}-\frac{6cx\operatorname{arcsinh}(cx)^2}{(1+c^2x^2)^2}-\frac{21cx\operatorname{arcsinh}(cx)^2}{1+c^2x^2}+88\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arcsin}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

output
```
-(a^2/(d^3*x)) - (a^2*c^2*x)/(4*d^3*(1 + c^2*x^2)^2) - (7*a^2*c^2*x)/(8*d^
3*(1 + c^2*x^2)) - (15*a^2*c*ArcTan[c*x])/(8*d^3) + (2*a*b*c*((7*(Sqrt[1 +
 c^2*x^2] + I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(c*x) - (7*(
I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I + c*x)) + ((I/48)*((-2*I + c*x
)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 - ((I/48)*((2*I + c*x)
*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - ArcTanh[Sqrt[1 + c^2*x
^2]] + ((15*I)/16)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 + I*E^ArcSi
nh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((15*I)/16)*(-1/2*ArcSinh[
c*x]^2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2, I*E^ArcSi
nh[c*x]])))/d^3 + (b^2*c*((2*c*x)/(1 + c^2*x^2) - (4*ArcSinh[c*x])/(1 + c^
2*x^2)^(3/2) - (42*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x*ArcSinh[c*x]^2
)/(1 + c^2*x^2)^2 - (21*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 88*ArcTan[Tanh
[ArcSinh[c*x]/2]] - 12*ArcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 48*ArcSinh[c*
x]*Log[1 - E^(-ArcSinh[c*x])] + (45*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[
c*x]] - (45*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - 48*ArcSinh[c*x]*
Log[1 + E^(-ArcSinh[c*x])] + 48*PolyLog[2, -E^(-ArcSinh[c*x])] + (90*I)*Ar
cSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (90*I)*ArcSinh[c*x]*PolyLog[2
, I/E^ArcSinh[c*x]] - 48*PolyLog[2, E^(-ArcSinh[c*x])] + (90*I)*PolyLog[3,
 (-I)/E^ArcSinh[c*x]] - (90*I)*PolyLog[3, I/E^ArcSinh[c*x]] + 12*ArcSinh[c
*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^3)
```

### 3.249.3 Rubi [A] (verified)

Time = 4.19 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.20, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {6224, 27, 6203, 6203, 6204, 3042, 4668, 3011, 2720, 6213, 215, 216, 6226, 215, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(c^2dx^2+d\right)^3}dx$$

$$\Big\downarrow \ 6224$$

$$-5c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{d^3\left(c^2x^2+1\right)^3}dx+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{5c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^3}dx}{d^3}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 6203

$$-\frac{5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}\right)}{d^3}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 6203

$$-\frac{5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}\right)\right.}{d^3}$$
$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 6204

$$-\frac{5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}}{2(c^2x^2+1)}\right.\right.}{d^3}$$
$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-$$
$$\frac{5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx}{2c}\right.\right.}{d^3}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 4668

3.249. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{-2ib\int(a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\,\text{ar}}{2c}\right.\right.}{\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\downarrow$ 3011

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\text{PolyLog}\left(2,ie^{\text{arcsinh}(}\right.\right.}{2c}\right.\right.}{\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\downarrow$ 2720

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\downarrow$ 6213

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\downarrow$ 215

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

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3.249. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

$\Big\downarrow$ 216

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{\frac{2bc\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\Big\downarrow$ 6226

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\int\frac{1}{(c^2x^2+1)^2}dx+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\Big\downarrow$ 215

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\left(\frac{1}{2}\int\frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\Big\downarrow$ 216

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\,(c^2x^2+1)^2}}$$

$\Big\downarrow$ 6226

3.249. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int\frac{1}{c^2x^2+1}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x(c^2x^2+1)^2}$$

$\downarrow$ 216

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)-b\arctan(cx)\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x(c^2x^2+1)^2}$$

$\downarrow$ 6231

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)-b\arctan(cx)\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x(c^2x^2+1)^2}$$

$\downarrow$ 3042

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2\right.\right.}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{d^3x(c^2x^2+1)^2}$$

---

3.249. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

$$\Big\downarrow \; 26$$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}}\right.}{}\right.\right.}{}$$

$$\frac{2bc\left(i\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{}{2(c^2}\right.\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 4670$$

$$\frac{2bc\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+\right.\right.}{d^3}$$

$$\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(}\right.}{}\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 2715$$

$$\frac{2bc\left(i\left(ib\int e^{-\text{arcsinh}(cx)}\log\left(1-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-ib\int e^{-\text{arcsinh}(cx)}\log\left(1+e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2i\text{arct}\right.\right.}{}$$

$$\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsinh}(}\right.}{}\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \; 2838$$

3.249. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^3}\,dx$

$$-\frac{5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int e^{-\text{arcsin}\cdots}\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)+\frac{a+b\cdots}{\cdots}\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

$\big\downarrow$ 7143

$$\frac{2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)+\frac{a+b\cdots}{\cdots}\right.}{d^3}$$

$$\frac{5c^2\left(\frac{3}{4}\left(-bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)+\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLo}\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^2}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(d^3*x*(1 + c^2*x^2)^2)) + (2*b*c*((a + b*ArcSinh
[c*x])/(3*(1 + c^2*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] -
b*ArcTan[c*x] - (b*c*(x/(2*(1 + c^2*x^2)) + ArcTan[c*x]/(2*c)))/3 + I*((2*
I)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSin
h[c*x]] - I*b*PolyLog[2, E^ArcSinh[c*x]])))/d^3 - (5*c^2*((x*(a + b*ArcSin
h[c*x])^2)/(4*(1 + c^2*x^2)^2) - (b*c*(-1/3*(a + b*ArcSinh[c*x])/(c^2*(1 +
 c^2*x^2)^(3/2)) + (b*(x/(2*(1 + c^2*x^2)) + ArcTan[c*x]/(2*c)))/(3*c)))/2
 + (3*((x*(a + b*ArcSinh[c*x])^2)/(2*(1 + c^2*x^2)) - b*c*(-((a + b*ArcSin
h[c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x])/c^2) + (2*(a + b*ArcSin
h[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog
[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)*b*(
-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*E^Ar
cSinh[c*x]]))/(2*c)))/4))/d^3
```

3.249. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

### 3.249.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

## 3.249. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.249. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^3}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.249.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^2\,(c^2d\,x^2+d)^3}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

### 3.249.5 Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)^3} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^3 x^2} dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^8 + 3*
c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

### 3.249.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)^3} dx = \frac{\int \frac{a^2}{c^6x^8+3c^4x^6+3c^2x^4+x^2} dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2} dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2} dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integr
al(b**2*asinh(c*x)**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) +
 Integral(2*a*b*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2),
 x))/d**3
```

### 3.249.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2 (d + c^2dx^2)^3} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^3 x^2} dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/8*a^2*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3
*x) + 15*c*arctan(c*x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2
/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2) + 2*a*b*log(c*x +
 sqrt(c^2*x^2 + 1))/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2
), x)
```

### 3.249.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^2), x)
```

### 3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^3}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^2\,(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3), x)
```

# 3.250 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$



## 3.250.1 Optimal result

Integrand size = 26, antiderivative size = 381

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx = {} & \frac{b^2c^2}{12d^3(1+c^2x^2)} - \frac{bc(a+b\text{arcsinh}(cx))}{d^3x(1+c^2x^2)^{3/2}} \\ & - \frac{5bc^3x(a+b\text{arcsinh}(cx))}{6d^3(1+c^2x^2)^{3/2}} \\ & + \frac{4bc^3x(a+b\text{arcsinh}(cx))}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{4d^3(1+c^2x^2)^2} \\ & - \frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(1+c^2x^2)^2} - \frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d^3(1+c^2x^2)} \\ & + \frac{6c^2(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & + \frac{b^2c^2\log(x)}{d^3} - \frac{7b^2c^2\log(1+c^2x^2)}{6d^3} \\ & + \frac{3bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & - \frac{3bc^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^3} \\ & - \frac{3b^2c^2\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)}{2d^3} \\ & + \frac{3b^2c^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(cx)}\right)}{2d^3}\end{aligned}$$

output
```
1/12*b^2*c^2/d^3/(c^2*x^2+1)-b*c*(a+b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^(3/2
)-5/6*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-3/4*c^2*(a+b*arcsin
h(c*x))^2/d^3/(c^2*x^2+1)^2-1/2*(a+b*arcsinh(c*x))^2/d^3/x^2/(c^2*x^2+1)^2
-3/2*c^2*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c*x))^2*a
rctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+b^2*c^2*ln(x)/d^3-7/6*b^2*c^2*ln(c^2
*x^2+1)/d^3+3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^
2)/d^3-3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3
-3/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b^2*c^2*polylog
(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+4/3*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*
x^2+1)^(1/2)
```

### 3.250.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.93 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.84

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx =$$

$$-\frac{\frac{2a^2}{x^2}+\frac{a^2c^2}{(1+c^2x^2)^2}+\frac{4a^2c^2}{1+c^2x^2}+12a^2c^2\log(x)-6a^2c^2\log(1+c^2x^2)-\frac{1}{6}ab\left(\frac{27c^2\left(\sqrt{1+c^2x^2}-i\text{arcsinh}(cx)\right)}{i+cx}+\frac{27c^2(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

output
```
-1/4*((2*a^2)/x^2 + (a^2*c^2)/(1 + c^2*x^2)^2 + (4*a^2*c^2)/(1 + c^2*x^2)
+ 12*a^2*c^2*Log[x] - 6*a^2*c^2*Log[1 + c^2*x^2] - (a*b*((27*c^2*(Sqrt[1 +
 c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) + (27*c^2*(Sqrt[1 + c^2*x^2] + I*Ar
cSinh[c*x]))/(-I + c*x) - (24*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2
+ (c^2*((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (
c^2*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - 36*c^2
*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2,
 (-I)*E^ArcSinh[c*x]]) - 36*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*
E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + 72*c^2*(ArcSinh[c*x]*
(ArcSinh[c*x] - 2*Log[1 - E^(2*ArcSinh[c*x])]) - PolyLog[2, E^(2*ArcSinh[c
*x])])))/6 - 4*b^2*c^2*((-1/8*I)*Pi^3 + (12 + 12*c^2*x^2)^(-1) + (c*x*ArcS
inh[c*x])/(6*(1 + c^2*x^2)^(3/2)) + (7*c*x*ArcSinh[c*x])/(3*Sqrt[1 + c^2*x
^2]) - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - ArcSinh[c*x]^2/(2*c^2*x^2)
 - ArcSinh[c*x]^2/(4*(1 + c^2*x^2)^2) - ArcSinh[c*x]^2/(1 + c^2*x^2) + 2*A
rcSinh[c*x]^3 + 3*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] - 3*ArcSinh[
c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - (2*Log
[1 + c^2*x^2])/3 - 3*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - 3*Arc
Sinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - (3*PolyLog[3, -E^(-2*ArcSinh[c*
x])])/2 + (3*PolyLog[3, E^(2*ArcSinh[c*x])])/2))/d^3
```

### 3.250.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.27, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.885, Rules used = {6224, 27, 6219, 27, 1578, 1195, 2009, 6226, 6203, 241, 6202, 240, 6226, 6202, 240, 6214, 5984, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(c^2dx^2+d\right)^3}dx$$

$$\Big\downarrow \text{6224}$$

$$-3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{d^3x\left(c^2x^2+1\right)^3}dx+\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(c^2x^2+1\right)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)^{5/2}}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 6219}$$

$$-\frac{3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(-bc\int -\frac{8c^4x^4+12c^2x^2+3}{3x(c^2x^2+1)^2}dx-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(\frac{1}{3}bc\int \frac{8c^4x^4+12c^2x^2+3}{x(c^2x^2+1)^2}dx-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 1578}$$

$$-\frac{3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(\frac{1}{6}bc\int \frac{8c^4x^4+12c^2x^2+3}{x^2(c^2x^2+1)^2}dx^2-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 1195}$$

$$-\frac{3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^3}dx}{d^3}+\frac{bc\left(\frac{1}{6}bc\int \left(\frac{5c^2}{c^2x^2+1}+\frac{c^2}{(c^2x^2+1)^2}+\frac{3}{x^2}\right)dx^2-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

---

3.250. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

$$-\frac{3c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^3}dx}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 6226

$$-\frac{3c^2\left(-\frac{1}{2}bc\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{5/2}}dx+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}dx+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 6203

$$-\frac{3c^2\left(-\frac{1}{2}bc\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\int \frac{x}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}\right)+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}dx+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 241

$$-\frac{3c^2\left(-\frac{1}{2}bc\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}dx+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 6202

$$-\frac{3c^2\left(-\frac{1}{2}bc\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)+\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}dx+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}\right)}{d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 240

---

3.250. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

$$-\frac{3c^2\left(\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}dx+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-\frac{1}{2}bc\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{}{6}\right.\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \ 6226$$

$$-\frac{3c^2\left(-bc\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx+\int\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-\frac{1}{2}bc\left(\frac{x(a+b\text{arcsinh}(\right.\right.}{3(c^2x^2+1)^{3/2}}}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \ 6202$$

$$-\frac{3c^2\left(-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int\frac{x}{c^2x^2+1}dx\right)+\int\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-\frac{1}{2}\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \ 240$$

$$-\frac{3c^2\left(\int\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)}dx+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)-\frac{1}{2}bc\left(\right.\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \ 6214$$

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3.250. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

$$-\frac{3c^2\left(\int \frac{(a+b\text{arcsinh}(cx))^2}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-bc\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right.\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$\downarrow$ 5984

$$-\frac{3c^2\left(2\int(a+b\text{arcsinh}(cx))^2\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-bc\left(\frac{x(a+b\text{arc}}{\sqrt{c^2}}\right.\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$\downarrow$ 3042

$$-\frac{3c^2\left(2\int i(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-bc\left(\frac{x(a+ba}{\sqrt{}}\right.\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$\downarrow$ 26

$$-\frac{3c^2\left(2i\int(a+b\text{arcsinh}(cx))^2\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}+\frac{(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-bc\left(\frac{x(a+ba}{\sqrt{c}}\right.\right.}{d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2(c^2x^2+1)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log(c^2x^2+1)+3\log(x^2)\right)\right)}{d^3}$$

$\downarrow$ 4670

3.250. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

$$-\frac{3c^2\left(2i\left(ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\downarrow$ 3011

$$-\frac{3c^2\left(2i\left(-ib\left(\frac{1}{2}b\int\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+ib\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\downarrow$ 2720

$$-\frac{3c^2\left(2i\left(-ib\left(\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsi}\right.\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\downarrow$ 7143

$$-\frac{3c^2\left(2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-ib\left(\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(\right.\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2d^3x^2\left(c^2x^2+1\right)^2}+$$

$$\frac{bc\left(-\frac{8c^2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{4c^2x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}+\frac{1}{6}bc\left(-\frac{1}{c^2x^2+1}+5\log\left(c^2x^2+1\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

---

3.250. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(d^3*x^2*(1 + c^2*x^2)^2) + (b*c*(-((a + b*Arc
Sinh[c*x])/(x*(1 + c^2*x^2)^(3/2))) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*(1
 + c^2*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2])
+ (b*c*(-(1 + c^2*x^2)^(-1) + 3*Log[x^2] + 5*Log[1 + c^2*x^2]))/6))/d^3 -
(3*c^2*((a + b*ArcSinh[c*x])^2/(4*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])^
2/(2*(1 + c^2*x^2)) - b*c*((x*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b
*Log[1 + c^2*x^2])/(2*c)) - (b*c*(b/(6*c*(1 + c^2*x^2)) + (x*(a + b*ArcSin
h[c*x]))/(3*(1 + c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/Sqrt[1 + c
^2*x^2] - (b*Log[1 + c^2*x^2])/(2*c)))/3))/2 + (2*I)*(I*(a + b*ArcSinh[c*x
])^2*ArcTanh[E^(2*ArcSinh[c*x])] - I*b*(-1/2*((a + b*ArcSinh[c*x])*PolyLog
[2, -E^(2*ArcSinh[c*x])]) + (b*PolyLog[3, -E^(2*ArcSinh[c*x])])/4) + I*b*(
-1/2*((a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])]) + (b*PolyLog[3,
 E^(2*ArcSinh[c*x])])/4))))/d^3
```

### 3.250.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

## 3.250. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

## 3.250. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3} \, dx$

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6219
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSi
nh[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[S
implifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)
/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.250. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.250.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(402) = 804$.

Time = 0.36 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.18

| method | result |
|---|---|
| derivativedivides | $c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b^2\left(-\frac{-16\ \mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5c^5+16\ \mathrm{arcsinh}(c\right.}{}\right.\right.$ |
| default | $c^2\left(\frac{a^2\left(-\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{4(c^2x^2+1)^2}-\frac{1}{c^2x^2+1}+\frac{3\ln(c^2x^2+1)}{2}\right)}{d^3}+\frac{b^2\left(-\frac{-16\ \mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5c^5+16\ \mathrm{arcsinh}(c\right.}{}\right.\right.$ |
| parts | $\frac{a^2\left(\frac{c^4\left(-\frac{2}{c^2(c^2x^2+1)}-\frac{1}{2c^2(c^2x^2+1)^2}+\frac{3\ln(c^2x^2+1)}{c^2}\right)}{2}-\frac{1}{2x^2}-3c^2\ln(x)\right)}{d^3}+\frac{b^2c^2\left(-\frac{-16\ \mathrm{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^5c^5+16\right.}{}\right.}{}$ |

input
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a^2/d^3*(-1/2/c^2/x^2-3*ln(c*x)-1/4/(c^2*x^2+1)^2-1/(c^2*x^2+1)+3/2*l
n(c^2*x^2+1))+b^2/d^3*(-1/12/(c^4*x^4+2*c^2*x^2+1)/c^2/x^2*(-16*arcsinh(c*
x)*(c^2*x^2+1)^(1/2)*x^5*c^5+16*arcsinh(c*x)*c^6*x^6+18*arcsinh(c*x)^2*x^4
*c^4-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+32*arcsinh(c*x)*c^4*x^4+27*a
rcsinh(c*x)^2*x^2*c^2-c^4*x^4+12*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+16*arc
sinh(c*x)*c^2*x^2+6*arcsinh(c*x)^2-c^2*x^2)+ln(1+c*x+(c^2*x^2+1)^(1/2))+8/
3*ln(c*x+(c^2*x^2+1)^(1/2))-7/3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+ln(c*x+(c^
2*x^2+1)^(1/2)-1)-3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-6*arcsinh(c
*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+6*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+
3*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+3*arcsinh(c*x)*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)-3*ar
csinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-6*arcsinh(c*x)*polylog(2,c*x+(c^2
*x^2+1)^(1/2))+6*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b/d^3*(-1/12/(c^4*x
^4+2*c^2*x^2+1)/c^2/x^2*(-8*c^5*x^5*(c^2*x^2+1)^(1/2)+8*c^6*x^6+18*arcsinh
(c*x)*c^4*x^4-3*c^3*x^3*(c^2*x^2+1)^(1/2)+16*c^4*x^4+27*arcsinh(c*x)*c^2*x
^2+6*c*x*(c^2*x^2+1)^(1/2)+8*c^2*x^2+6*arcsinh(c*x))-3*arcsinh(c*x)*ln(1+c
*x+(c^2*x^2+1)^(1/2))-3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3*arcsinh(c*x)*l
n(1+(c*x+(c^2*x^2+1)^(1/2))^2)+3/2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-3
*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-3*polylog(2,c*x+(c^2*x^2+1)^(1/2
))))
```

### 3.250.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^3}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^9 + 3*
c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

### 3.250.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^3}\,dx = \frac{\int \frac{a^2}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integr
al(b**2*asinh(c*x)**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) +
 Integral(2*a*b*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3),
 x))/d**3
```

### 3.250.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^3}\,dx = \int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x
^2) - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + integrate(b^2*log(
c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 +
d^3*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7
 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

### 3.250.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^3}\,dx = \int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^3), x)
```

### 3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx=\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2}{x^3\,\left(d\,c^2\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3), x)
```

# 3.251 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

### 3.251.1 Optimal result

Integrand size = 26, antiderivative size = 529

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^3}\,dx = & -\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)}+\frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} \\
& -\frac{bc^3(a+b\operatorname{arcsinh}(cx))}{6d^3\left(1+c^2x^2\right)^{3/2}}-\frac{bc(a+b\operatorname{arcsinh}(cx))}{3d^3x^2\left(1+c^2x^2\right)^{3/2}} \\
& +\frac{29bc^3(a+b\operatorname{arcsinh}(cx))}{12d^3\sqrt{1+c^2x^2}}-\frac{(a+b\operatorname{arcsinh}(cx))^2}{3d^3x^3\left(1+c^2x^2\right)^2} \\
& +\frac{7c^2(a+b\operatorname{arcsinh}(cx))^2}{3d^3x\left(1+c^2x^2\right)^2}+\frac{35c^4x(a+b\operatorname{arcsinh}(cx))^2}{12d^3\left(1+c^2x^2\right)^2} \\
& +\frac{35c^4x(a+b\operatorname{arcsinh}(cx))^2}{8d^3\left(1+c^2x^2\right)} \\
& +\frac{35c^3(a+b\operatorname{arcsinh}(cx))^2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{4d^3} \\
& -\frac{17b^2c^3\arctan(cx)}{6d^3} \\
& +\frac{38bc^3(a+b\operatorname{arcsinh}(cx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{3d^3} \\
& +\frac{19b^2c^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{3d^3} \\
& -\frac{35ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3} \\
& +\frac{35ibc^3(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3} \\
& -\frac{19b^2c^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{3d^3} \\
& +\frac{35ib^2c^3\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3} \\
& -\frac{35ib^2c^3\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(cx)}\right)}{4d^3}
\end{aligned}
$$

output
```
-1/2*b^2*c^2/d^3/x+1/6*b^2*c^2/d^3/x/(c^2*x^2+1)+1/12*b^2*c^4*x/d^3/(c^2*x
^2+1)-1/6*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-1/3*b*c*(a+b*arcs
inh(c*x))/d^3/x^2/(c^2*x^2+1)^(3/2)-1/3*(a+b*arcsinh(c*x))^2/d^3/x^3/(c^2*
x^2+1)^2+7/3*c^2*(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2+35/12*c^4*x*(a+b
*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(a+b*arcsinh(c*x))^2/d^3/(c^
2*x^2+1)+35/4*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3-1
7/6*b^2*c^3*arctan(c*x)/d^3+38/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2
*x^2+1)^(1/2))/d^3+19/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3-35/4
*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+35/4
*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-19/3*
b^2*c^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d^3+35/4*I*b^2*c^3*polylog(3,-I*(
c*x+(c^2*x^2+1)^(1/2)))/d^3-35/4*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(1
/2)))/d^3+29/12*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1/2)
```

### 3.251.2 Mathematica [A] (verified)

Time = 9.00 (sec) , antiderivative size = 937, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$$

$$= -\frac{a^2}{3d^3x^3}+\frac{3a^2c^2}{d^3x}+\frac{a^2c^4x}{4d^3(1+c^2x^2)^2}+\frac{11a^2c^4x}{8d^3(1+c^2x^2)}+\frac{35a^2c^3\arctan(cx)}{8d^3}$$

$$+\frac{2ab\left(-\frac{c\sqrt{1+c^2x^2}}{6x^2}+\frac{ic^3\left((2i-cx)\sqrt{1+c^2x^2}-3\operatorname{arcsinh}(cx)\right)}{48(-i+cx)^2}-\frac{11c^3\left(\sqrt{1+c^2x^2}+i\operatorname{arcsinh}(cx)\right)}{16(-1-icx)}-\frac{\operatorname{arcsinh}(cx)}{3x^3}+\frac{11c^4\left(i\sqrt{1+c\right.}{16\cdots}}{}$$

$$+\frac{b^2c^3\left(-\frac{2cx}{1+c^2x^2}+\frac{4\operatorname{arcsinh}(cx)}{(1+c^2x^2)^{3/2}}+\frac{66\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}+\frac{6cx\operatorname{arcsinh}(cx)^2}{(1+c^2x^2)^2}+\frac{33cx\operatorname{arcsinh}(cx)^2}{1+c^2x^2}-136\arctan\left(\tanh\left(\frac{1}{2}a\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

output
```
-1/3*a^2/(d^3*x^3) + (3*a^2*c^2)/(d^3*x) + (a^2*c^4*x)/(4*d^3*(1 + c^2*x^2
)^2) + (11*a^2*c^4*x)/(8*d^3*(1 + c^2*x^2)) + (35*a^2*c^3*ArcTan[c*x])/(8*
d^3) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/x^2 + ((I/48)*c^3*((2*I - c*x)*S
qrt[1 + c^2*x^2] - 3*ArcSinh[c*x]))/(-I + c*x)^2 - (11*c^3*(Sqrt[1 + c^2*x
^2] + I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(3*x^3) + (11*c^4*
(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I*c + c^2*x)) + ((I/48)*c^3*((2
*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 + (c^3*ArcTanh[
Sqrt[1 + c^2*x^2]])/6 - 3*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*
x^2]]) - ((35*I)/16)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 +
I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + ((35*I)/16
)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/
c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c)))/d^3 + (b^2*c^3*((-2*c*x)/(1 + c^
2*x^2) + (4*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (66*ArcSinh[c*x])/Sqrt[1 +
 c^2*x^2] + (6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (33*c*x*ArcSinh[c*x]^
2)/(1 + c^2*x^2) - 136*ArcTan[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/
2] + 38*ArcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[
c*x]/2]^2 - (c*x*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 152*ArcSinh[c*
x]*Log[1 - E^(-ArcSinh[c*x])] - (105*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh
[c*x]] + (105*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 152*ArcSinh[c*
x]*Log[1 + E^(-ArcSinh[c*x])] - 152*PolyLog[2, -E^(-ArcSinh[c*x])] - (2...
```

### 3.251.3 Rubi [A] (verified)

Time = 6.15 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.37, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {6224, 27, 6224, 253, 264, 216, 6203, 6203, 6204, 3042, 4668, 3011, 2720, 6213, 215, 216, 6226, 215, 216, 6226, 216, 6231, 3042, 26, 4670, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(c^2dx^2+d\right)^3}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{7}{3}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{d^3x^2\left(c^2x^2+1\right)^3}dx+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x^3\left(c^2x^2+1\right)^{5/2}}dx}{3d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{7c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(c^2x^2+1)^3}dx}{3d^3}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{x^3(c^2x^2+1)^{5/2}}dx}{3d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{6224}$$

$$-\frac{7c^2\left(-5c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^3}dx+2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}\right)}{3d^3}+\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}\right)}{3d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{253}$$

$$-\frac{7c^2\left(-5c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^3}dx+2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}\right)}{3d^3}+\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx+\frac{1}{2}bc\left(\frac{3}{2}\int \frac{1}{x^2(c^2x^2+1)}dx+\frac{1}{2x(c^2x^2+1)}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}\right)}{3d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{264}$$

$$-\frac{7c^2\left(-5c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^3}dx+2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}\right)}{3d^3}+\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx+\frac{1}{2}bc\left(\frac{3}{2}\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}\right)}{3d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-\frac{7c^2\left(-5c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^3}dx+2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2x^2+1)^2}\right)}{3d^3}-\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{6203}$$

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3.251. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

$$\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-\frac{7c^2\left(-5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}\right)+2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\right.}{3d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 6203

$$\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-\frac{7c^2\left(-5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}\right.\right.\right.}{3d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 6204

$$\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-\frac{7c^2\left(-5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{ar}}{2(c}\right.\right.\right.}{3d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{2bc\left(-\frac{5}{2}c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-\frac{7c^2\left(2bc\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-5c^2\left(-\frac{1}{2}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx+\frac{3}{4}\left(-bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx+\frac{\int(a+b\text{arcsinh}(cx))}{}\right.\right.\right.}{3d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\Big\downarrow$ 4668

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3.251. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

$$\frac{2bc\left(-\frac{5}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-$$

$$7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{-2ib\int(a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx}{2c}\right.\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 3011

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\text{PolyLog}\left(2,i\right.\right.}{2c}\right.\right.\right.}{}$$

$$\frac{2bc\left(-\frac{5}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 2720

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{}$$

$$\frac{2bc\left(-\frac{5}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 6213

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{}$$

$$\frac{2bc\left(-\frac{5}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}-$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 215

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3.251. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}dx$

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{\frac{2bc\left(-\frac{5}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\big\downarrow$ 216

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{\frac{2bc\left(-\frac{5}{2}c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{5/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\frac{1}{x}\right)+\frac{1}{2x(c^2x^2+1)}\right)\right)}{3d^3}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\big\downarrow$ 6226

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{\frac{2bc\left(-\frac{5}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\int\frac{1}{(c^2x^2+1)^2}dx+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\arctan(cx)-\right.\right.\right.}{3d^3}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\big\downarrow$ 215

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{\frac{2bc\left(-\frac{5}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\left(\frac{1}{2}\int\frac{1}{c^2x^2+1}dx+\frac{x}{2(c^2x^2+1)}\right)+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\right.\right.\right.}{3d^3}} - \frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

3.251. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}dx$

$\downarrow$ 216

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{\;}$$

$$\frac{2bc\left(-\frac{5}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)^{3/2}}+\frac{1}{2}bc\left(\frac{3}{2}\left(-c\,\text{arcta}\right.\right.\right.}{3d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 6226

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{\;}$$

$$\frac{2bc\left(-\frac{5}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx-bc\int\frac{1}{c^2x^2+1}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)\right)-\right.}{3d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 216

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{\;}$$

$$\frac{2bc\left(-\frac{5}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x\sqrt{c^2x^2+1}}dx+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)-b\arctan(cx)\right)-\right.}{3d^3}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 6231

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3.251. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{2bc\left(-\frac{5}{2}c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{cx}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}+\frac{x}{2(c^2x^2+1)}\right)-b\,\text{arcta}\right.\right.}$$

$$3d^3$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 3042

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{2bc\left(-\frac{5}{2}c^2\left(\int i(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}\right.\right.\right.}$$

$$3d^3$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 26

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.}{}\right.\right.\right.}{2bc\left(-\frac{5}{2}c^2\left(i\int(a+b\text{arcsinh}(cx))\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-\frac{1}{3}bc\left(\frac{\arctan(cx)}{2c}\right.\right.\right.}$$

$$3d^3$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 4670

$$-\frac{7c^2\left(2bc\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.\right.\right.}{2bc\left(-\frac{5}{2}c^2\left(i\left(ib\int\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-ib\int\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right.\right.\right.\right.}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

---

3.251. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

$\downarrow$ 2715

$$-\frac{7\left(-5\left(\frac{x(a+b\text{arcsinh}(cx))^2}{4(c^2x^2+1)^2}-\frac{1}{2}bc\left(\frac{b\left(\frac{x}{2(c^2x^2+1)}+\frac{\arctan(cx)}{2c}\right)}{3c}-\frac{a+b\text{arcsinh}(cx)}{3c^2(c^2x^2+1)^{3/2}}\right)+\frac{3}{4}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{2(c^2x^2+1)}-bc\left(\frac{b\arctan}{c^2}\right.\right.\right.\right.}{2b\left(-\frac{5}{2}\left(\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}-b\arctan(cx)-\frac{1}{3}bc\left(\frac{x}{2(c^2x^2+1)}+\frac{\arctan(cx)}{2c}\right)+i(2i(a+b\text{arcsinh}(cx))\text{arct}\right.\right.}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 2838

$$-\frac{7c^2\left(-5c^2\left(\frac{3}{4}\left(\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int\right.\right.}{}\right.\right.\right.}{2bc\left(-\frac{5}{2}c^2\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.\right.}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

$\downarrow$ 7143

$$-\frac{7c^2\left(2bc\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right.\right.\right.}{2bc\left(-\frac{5}{2}c^2\left(i\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+ib\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)\right)\right.\right.}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d^3x^3\left(c^2x^2+1\right)^2}$$

input

```
Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

3.251. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

output
```
-1/3*(a + b*ArcSinh[c*x])^2/(d^3*x^3*(1 + c^2*x^2)^2) + (2*b*c*(-1/2*(a +
b*ArcSinh[c*x])/(x^2*(1 + c^2*x^2)^(3/2)) + (b*c*(1/(2*x*(1 + c^2*x^2)) +
(3*(-x^(-1) - c*ArcTan[c*x]))/2))/2 - (5*c^2*((a + b*ArcSinh[c*x])/(3*(1 +
 c^2*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - b*ArcTan[c*x]
- (b*c*(x/(2*(1 + c^2*x^2)) + ArcTan[c*x]/(2*c)))/3 + I*((2*I)*(a + b*ArcS
inh[c*x])*ArcTanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*
PolyLog[2, E^ArcSinh[c*x]])))/2))/(3*d^3) - (7*c^2*(-((a + b*ArcSinh[c*x])
^2/(x*(1 + c^2*x^2)^2)) + 2*b*c*((a + b*ArcSinh[c*x])/(3*(1 + c^2*x^2)^(3/
2)) + (a + b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - b*ArcTan[c*x] - (b*c*(x/(2*
(1 + c^2*x^2)) + ArcTan[c*x]/(2*c)))/3 + I*((2*I)*(a + b*ArcSinh[c*x])*Arc
Tanh[E^ArcSinh[c*x]] + I*b*PolyLog[2, -E^ArcSinh[c*x]] - I*b*PolyLog[2, E^
ArcSinh[c*x]])) - 5*c^2*((x*(a + b*ArcSinh[c*x])^2)/(4*(1 + c^2*x^2)^2) -
(b*c*(-1/3*(a + b*ArcSinh[c*x])/(c^2*(1 + c^2*x^2)^(3/2)) + (b*(x/(2*(1 +
c^2*x^2)) + ArcTan[c*x]/(2*c)))/(3*c)))/2 + (3*((x*(a + b*ArcSinh[c*x])^2)
/(2*(1 + c^2*x^2)) - b*c*(-((a + b*ArcSinh[c*x])/(c^2*Sqrt[1 + c^2*x^2]))
+ (b*ArcTan[c*x])/c^2) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]]
+ (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*Pol
yLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2,
 I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*E^ArcSinh[c*x]]))/(2*c)))/4)))/(3*d^3
)
```

### 3.251.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.251. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

## 3.251. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

---

3.251. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.251.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^4\,(c^2d\,x^2+d)^3}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

### 3.251.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^3}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^3x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^10 + 3
*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

### 3.251.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^3}\,dx$$
$$=\frac{\int \frac{a^2}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{b^2\,\text{asinh}^2\left(cx\right)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{2ab\,\text{asinh}\left(cx\right)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integ
ral(b**2*asinh(c*x)**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x)
 + Integral(2*a*b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**
4), x))/d**3
```

### 3.251.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^3}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^3x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/24*a^2*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^
2 - 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + integrate(b^2*log(c*x +
sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x
^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3
*c^2*d^3*x^6 + d^3*x^4), x)
```

### 3.251.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^3x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^4), x)
```

### 3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^3}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^4\,(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3), x)
```

# 3.252 $\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx$



## 3.252.1 Optimal result

Integrand size = 25, antiderivative size = 300

$$\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{245b^2\pi^{5/2}x\sqrt{1+c^2x^2}}{1152} + \frac{65b^2\pi^{5/2}x(1+c^2x^2)^{3/2}}{1728}$$
$$+\frac{1}{108}b^2\pi^{5/2}x(1+c^2x^2)^{5/2} - \frac{115b^2\pi^{5/2}\text{arcsinh}(cx)}{1152c} - \frac{5}{16}bc\pi^{5/2}x^2(a+b\text{arcsinh}(cx)) - \frac{5b\pi^{5/2}(1+c^2x^2)^2(a+ba}{48c}$$

output
```
65/1728*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(3/2)+1/108*b^2*Pi^(5/2)*x*(c^2*x^2+1)^
(5/2)-115/1152*b^2*Pi^(5/2)*arcsinh(c*x)/c-5/16*b*c*Pi^(5/2)*x^2*(a+b*arcs
inh(c*x))-5/48*b*Pi^(5/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c-1/18*b*Pi^(5/
2)*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b
*arcsinh(c*x))^2+1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2+5/48*Pi^
(5/2)*(a+b*arcsinh(c*x))^3/b/c+245/1152*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(1/2)+5
/16*Pi^2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.252.2 Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95

$$\int (\pi + c^2\pi x^2)^{5/2} (a$$
$$+b\text{arcsinh}(cx))^2 \, dx = \frac{\pi^{5/2}\left(9504a^2cx\sqrt{1+c^2x^2} + 7488a^2c^3x^3\sqrt{1+c^2x^2} + 2304a^2c^5x^5\sqrt{1+c^2x^2} + 1440b^2\text{arc}\right.}{}$$

input
```
Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Pi^(5/2)*(9504*a^2*c*x*Sqrt[1 + c^2*x^2] + 7488*a^2*c^3*x^3*Sqrt[1 + c^2*
x^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2] + 1440*b^2*ArcSinh[c*x]^3 - 3240
*a*b*Cosh[2*ArcSinh[c*x]] - 324*a*b*Cosh[4*ArcSinh[c*x]] - 24*a*b*Cosh[6*A
rcSinh[c*x]] + 1620*b^2*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sinh[4*ArcSinh[c*x]]
 + 4*b^2*Sinh[6*ArcSinh[c*x]] + 72*b*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*Ar
cSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]) + 12*ArcS
inh[c*x]*(360*a^2 - 270*b^2*Cosh[2*ArcSinh[c*x]] - 27*b^2*Cosh[4*ArcSinh[c
*x]] - 2*b^2*Cosh[6*ArcSinh[c*x]] + 540*a*b*Sinh[2*ArcSinh[c*x]] + 108*a*b
*Sinh[4*ArcSinh[c*x]] + 12*a*b*Sinh[6*ArcSinh[c*x]])))/(13824*c)
```

### 3.252.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {6201, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{1}{3}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\pi\int \left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6201}$$

$$-\frac{1}{3}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^3\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6200}$$

$$-\frac{1}{3}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int x(a+b\text{arcsi}\right.\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6191

$$-\frac{1}{3}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\left(-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right.\right.\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 262

$$-\frac{1}{3}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\left(-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\right.\right.\right.\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 222

$$-\frac{1}{3}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+\right.\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6198

$$-\frac{1}{3}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+}\right.\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6213

---

3.252. $\int\left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$-\frac{1}{3}\pi^{5/2}bc\left(\frac{\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\int \left(c^2x^2+1\right)^{5/2}dx}{6c}\right)+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int \left(c^2x^2+1\right)^{3/2}dx}{4c}\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow 211$$

$$-\frac{1}{3}\pi^{5/2}bc\left(\frac{\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\int \left(c^2x^2+1\right)^{3/2}dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)}{6c}\right)+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow 211$$

$$-\frac{1}{3}\pi^{5/2}bc\left(\frac{\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}\right)}{6c}\right)+$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}\right)+$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow 211$$

$$-\frac{1}{3}\pi^{5/2}bc\left(\frac{\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x(c\right.}{6c}\right.$$
$$\frac{5}{6}\pi\left(-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}\right)+$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow 222$$

---

3.252. $\int \left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2-$$
$$\frac{1}{3}\pi^{5/2}bc\left(\frac{\left(c^2x^2+1\right)^3(a+b\operatorname{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\operatorname{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)+\frac{1}{6}x\left(c^2x^2\right.\right.}{6c}\right.$$
$$\frac{5}{6}\pi\left(\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\operatorname{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{\right.\right.}}{}\right.\right.$$

input
```
Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 - (b*c*Pi^(5/2)*(((1
+ c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2) - (b*((x*(1 + c^2*x^2)^(5/2))/6
 + (5*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c
*x]/(2*c)))/4))/6))/(6*c)))/3 + (5*Pi*((x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*A
rcSinh[c*x])^2)/4 + (3*Pi*((x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2
)/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^3)/(6*b*c) - b*c*Sqrt[Pi]*((x^2*(a +
b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2
*c^3)))/2)))/4 - (b*c*Pi^(3/2)*(((1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*
c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSi
nh[c*x]/(2*c)))/4))/(4*c)))/2))/6
```

### 3.252.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.252. $\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx$

### 3.252.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| default | $\frac{a^2x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6}+\frac{5a^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a^2\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a^2\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b^2\pi^{\frac{5}{2}}\left(576\,\mathrm{arcsinh}(cx)^2\sqrt{}\right.}{}$ |
| parts | $\frac{a^2x(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6}+\frac{5a^2\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a^2\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a^2\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b^2\pi^{\frac{5}{2}}\left(576\,\mathrm{arcsinh}(cx)^2\sqrt{}\right.}{}$ |

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/24*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a
^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a^2*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(
Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3456*b^2*Pi^(5/2)*(576*arcsinh(c*x)
^2*(c^2*x^2+1)^(1/2)*x^5*c^5-192*arcsinh(c*x)*c^6*x^6+32*c^5*x^5*(c^2*x^2+
1)^(1/2)+1872*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^3*c^3-936*arcsinh(c*x)*c^
4*x^4+194*c^3*x^3*(c^2*x^2+1)^(1/2)+2376*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*
c*x-2376*arcsinh(c*x)*c^2*x^2+897*c*x*(c^2*x^2+1)^(1/2)+360*arcsinh(c*x)^3
-897*arcsinh(c*x))/c+1/144*a*b*Pi^(5/2)*(48*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
*x^5*c^5-8*c^6*x^6+156*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-39*c^4*x^4+1
98*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-99*c^2*x^2+45*arcsinh(c*x)^2-68)/c
```

### 3.252.5 Fricas [F]

$$\int \left(\pi+c^2\pi x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a^2*c^4*x^4 + 2*pi^2*a^2*c^2*x^2 + pi
^2*a^2 + (pi^2*b^2*c^4*x^4 + 2*pi^2*b^2*c^2*x^2 + pi^2*b^2)*arcsinh(c*x)^2
 + 2*(pi^2*a*b*c^4*x^4 + 2*pi^2*a*b*c^2*x^2 + pi^2*a*b)*arcsinh(c*x)), x)
```

### 3.252.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(287) = 574$.

Time = 28.50 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.94

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))^2 \, dx = \begin{cases} \frac{\pi^{\frac{5}{2}} a^2 c^4 x^5 \sqrt{c^2x^2+1}}{6} + \frac{13\pi^{\frac{5}{2}} a^2 c^2 x^3 \sqrt{c^2x^2+1}}{24} + \frac{11\pi^{\frac{5}{2}} a^2 x \sqrt{c^2x^2+1}}{16} + \frac{5\pi^{\frac{5}{2}} a^2 \operatorname{asinh}(cx)}{16c} - \frac{\pi^{\frac{5}{2}} abc^5 x^6}{18} + \frac{\pi^{\frac{5}{2}}}{} \\ \pi^{\frac{5}{2}} a^2 x \end{cases}$$

input

```
integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output

```
Piecewise((pi**(5/2)*a**2*c**4*x**5*sqrt(c**2*x**2 + 1)/6 + 13*pi**(5/2)*a
**2*c**2*x**3*sqrt(c**2*x**2 + 1)/24 + 11*pi**(5/2)*a**2*x*sqrt(c**2*x**2
+ 1)/16 + 5*pi**(5/2)*a**2*asinh(c*x)/(16*c) - pi**(5/2)*a*b*c**5*x**6/18
+ pi**(5/2)*a*b*c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/3 - 13*pi**(5/2)*
a*b*c**3*x**4/48 + 13*pi**(5/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*
x)/12 - 11*pi**(5/2)*a*b*c*x**2/16 + 11*pi**(5/2)*a*b*x*sqrt(c**2*x**2 + 1
)*asinh(c*x)/8 + 5*pi**(5/2)*a*b*asinh(c*x)**2/(16*c) - pi**(5/2)*b**2*c**
5*x**6*asinh(c*x)/18 + pi**(5/2)*b**2*c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(
c*x)**2/6 + pi**(5/2)*b**2*c**4*x**5*sqrt(c**2*x**2 + 1)/108 - 13*pi**(5/2
)*b**2*c**3*x**4*asinh(c*x)/48 + 13*pi**(5/2)*b**2*c**2*x**3*sqrt(c**2*x**
2 + 1)*asinh(c*x)**2/24 + 97*pi**(5/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/
1728 - 11*pi**(5/2)*b**2*c*x**2*asinh(c*x)/16 + 11*pi**(5/2)*b**2*x*sqrt(c
**2*x**2 + 1)*asinh(c*x)**2/16 + 299*pi**(5/2)*b**2*x*sqrt(c**2*x**2 + 1)/
1152 + 5*pi**(5/2)*b**2*asinh(c*x)**3/(48*c) - 299*pi**(5/2)*b**2*asinh(c*
x)/(1152*c), Ne(c, 0)), (pi**(5/2)*a**2*x, True))
```

### 3.252.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.252.8 Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.252.9 Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b\,\text{asinh}(c\,x))^2 \left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2} \, dx$$

input
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.253 $\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx$



## 3.253.1 Optimal result

Integrand size = 25, antiderivative size = 210

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{15}{64} b^2 \pi^{3/2} x \sqrt{1 + c^2x^2} + \frac{1}{32} b^2 \pi^{3/2} x \left(1 + c^2x^2\right)^{3/2} - \frac{9b^2\pi^{3/2}\text{arcsinh}(cx)}{64c} - \frac{3}{8} bc\pi^{3/2} x^2 (a + b\text{arcsinh}(cx)) - \frac{b\pi^{3/2}\left(1 + c^2x^2\right)^2 (a + b\text{arcsinh}$$

output
```
1/32*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(3/2)-9/64*b^2*Pi^(3/2)*arcsinh(c*x)/c-3/8
*b*c*Pi^(3/2)*x^2*(a+b*arcsinh(c*x))-1/8*b*Pi^(3/2)*(c^2*x^2+1)^2*(a+b*arc
sinh(c*x))/c+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*Pi^(3/2)
*(a+b*arcsinh(c*x))^3/b/c+15/64*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(1/2)+3/8*Pi*x*
(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.253.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.96

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{\pi^{3/2}\left(160a^2cx\sqrt{1 + c^2x^2} + 64a^2c^3x^3\sqrt{1 + c^2x^2} + 32b^2\text{arcsinh}(cx)^3 - 64ab\cosh(2\text{arcsin}$$

input
```
Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Pi^(3/2)*(160*a^2*c*x*Sqrt[1 + c^2*x^2] + 64*a^2*c^3*x^3*Sqrt[1 + c^2*x^2
] + 32*b^2*ArcSinh[c*x]^3 - 64*a*b*Cosh[2*ArcSinh[c*x]] - 4*a*b*Cosh[4*Arc
Sinh[c*x]] + 32*b^2*Sinh[2*ArcSinh[c*x]] + b^2*Sinh[4*ArcSinh[c*x]] + 8*b*
ArcSinh[c*x]^2*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]])
+ 4*ArcSinh[c*x]*(-16*b^2*Cosh[2*ArcSinh[c*x]] - b^2*Cosh[4*ArcSinh[c*x]]
+ 4*a*(6*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))))/(256*c)
```

### 3.253.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6200}$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int x(a+b\text{arcsinh}(cx))dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 6191}$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\pi\left(-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\right.$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+$$
$$\frac{3}{4}\pi\left(-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)+\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\right.$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\downarrow 222$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\right.\right.\right.$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\downarrow 6198$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)+\right.$$

$$\downarrow 6213$$

$$-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int\left(c^2x^2+1\right)^{3/2}dx}{4c}\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+$$
$$b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)+\right.$$

$$\downarrow 211$$

$$-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}\right)+$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)+\right.$$

$$\downarrow 211$$

---

3.253. $\int\left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx$

$$-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}\right)+$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)+\right.$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}\right)+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)+\right.$$

input
```
Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*Pi*((x*Sqrt[Pi +
 c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^3)
/(6*b*c) - b*c*Sqrt[Pi]*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 +
c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)))/4 - (b*c*Pi^(3/2)*(((1 + c
^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4 +
(3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)))/2
```

### 3.253.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.253. $\int (\pi + c^2\pi x^2)^{3/2} (a + b\operatorname{arcsinh}(cx))^2 \, dx$

### 3.253.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| default | $\frac{a^{2} x \left(\pi \,c^{2} x^{2}+\pi \right)^{\frac{3}{2}}}{4}+\frac{3 a^{2} \pi x \sqrt{\pi \,c^{2} x^{2}+\pi}}{8}+\frac{3 a^{2} \pi^{2} \ln \left(\frac{\pi \,c^{2} x}{\sqrt{\pi \,c^{2}}}+\sqrt{\pi \,c^{2} x^{2}+\pi}\right)}{8 \sqrt{\pi \,c^{2}}}+\frac{b^{2} \pi^{\frac{3}{2}} \left(16 \,\operatorname{arcsinh}\left(c x \right)^{2} \sqrt{c^{2} x^{2}+1}\, x^{3} c^{3}-8 \,\operatorname{arcsinh}(c$ |
| parts | $\frac{a^{2} x \left(\pi \,c^{2} x^{2}+\pi \right)^{\frac{3}{2}}}{4}+\frac{3 a^{2} \pi x \sqrt{\pi \,c^{2} x^{2}+\pi}}{8}+\frac{3 a^{2} \pi^{2} \ln \left(\frac{\pi \,c^{2} x}{\sqrt{\pi \,c^{2}}}+\sqrt{\pi \,c^{2} x^{2}+\pi}\right)}{8 \sqrt{\pi \,c^{2}}}+\frac{b^{2} \pi^{\frac{3}{2}} \left(16 \,\operatorname{arcsinh}\left(c x \right)^{2} \sqrt{c^{2} x^{2}+1}\, x^{3} c^{3}-8 \,\operatorname{arcsinh}(c$ |

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/8*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a^2
*Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6
4*b^2*Pi^(3/2)*(16*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^3*c^3-8*arcsinh(c*x)
*c^4*x^4+2*c^3*x^3*(c^2*x^2+1)^(1/2)+40*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c
*x-40*arcsinh(c*x)*c^2*x^2+17*c*x*(c^2*x^2+1)^(1/2)+8*arcsinh(c*x)^3-17*ar
csinh(c*x))/c+1/8*a*b*Pi^(3/2)*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3-c
^4*x^4+10*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-5*c^2*x^2+3*arcsinh(c*x)^2-4)
/c
```

### 3.253.5 Fricas [F]

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)^2 \, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a^2*c^2*x^2 + pi*a^2 + (pi*b^2*c^2*x^2
+ pi*b^2)*arcsinh(c*x)^2 + 2*(pi*a*b*c^2*x^2 + pi*a*b)*arcsinh(c*x)), x)
```

### 3.253.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(197) = 394.

Time = 2.91 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.93

$$\int \left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\begin{cases}\frac{\pi^{\frac{3}{2}}a^2c^2x^3\sqrt{c^2x^2+1}}{4}+\frac{5\pi^{\frac{3}{2}}a^2x\sqrt{c^2x^2+1}}{8}+\frac{3\pi^{\frac{3}{2}}a^2\operatorname{asinh}(cx)}{8c}-\frac{\pi^{\frac{3}{2}}abc^3x^4}{8}+\frac{\pi^{\frac{3}{2}}abc^2x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{2} \\ \pi^{\frac{3}{2}}a^2x\end{cases}$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((pi**(3/2)*a**2*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*
*2*x*sqrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a**2*asinh(c*x)/(8*c) - pi**(3/2)
*a*b*c**3*x**4/8 + pi**(3/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/
2 - 5*pi**(3/2)*a*b*c*x**2/8 + 5*pi**(3/2)*a*b*x*sqrt(c**2*x**2 + 1)*asinh
(c*x)/4 + 3*pi**(3/2)*a*b*asinh(c*x)**2/(8*c) - pi**(3/2)*b**2*c**3*x**4*a
sinh(c*x)/8 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/4
 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/32 - 5*pi**(3/2)*b**2*c*x*
*2*asinh(c*x)/8 + 5*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/8 +
 17*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)/64 + pi**(3/2)*b**2*asinh(c*x)**3
/(8*c) - 17*pi**(3/2)*b**2*asinh(c*x)/(64*c), Ne(c, 0)), (pi**(3/2)*a**2*x
, True))
```

### 3.253.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\pi+c^2\pi x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.253.8 Giac [F(-2)]

Exception generated.

$$\int \left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.253.9 Mupad [F(-1)]

Timed out.

$$\int \left(\pi+c^2\pi x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.254 $\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx$



## 3.254.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\begin{aligned}\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = {} & \frac{1}{4}b^2\sqrt{\pi}x\sqrt{1+c^2x^2} - \frac{b^2\sqrt{\pi}\text{arcsinh}(cx)}{4c} \\ & - \frac{1}{2}bc\sqrt{\pi}x^2(a + b\text{arcsinh}(cx)) \\ & + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 \\ & + \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^3}{6bc}\end{aligned}$$

output
```
-1/4*b^2*arcsinh(c*x)*Pi^(1/2)/c-1/2*b*c*x^2*(a+b*arcsinh(c*x))*Pi^(1/2)+1
/6*(a+b*arcsinh(c*x))^3*Pi^(1/2)/b/c+1/4*b^2*x*Pi^(1/2)*(c^2*x^2+1)^(1/2)+
1/2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.254.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx$$

$$= \frac{\sqrt{\pi}\left(4b^2\text{arcsinh}(cx)^3 + 6b\text{arcsinh}(cx)^2(2a + b\sinh(2\text{arcsinh}(cx))) + 3\left(4a^2cx\sqrt{1+c^2x^2} - 2ab\cosh(2\text{arcsin}\right.\right.}{2}$$

input
```
Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Sqrt[Pi]*(4*b^2*ArcSinh[c*x]^3 + 6*b*ArcSinh[c*x]^2*(2*a + b*Sinh[2*ArcSi
nh[c*x]]) + 3*(4*a^2*c*x*Sqrt[1 + c^2*x^2] - 2*a*b*Cosh[2*ArcSinh[c*x]] +
b^2*Sinh[2*ArcSinh[c*x]]) + 6*ArcSinh[c*x]*(-(b^2*Cosh[2*ArcSinh[c*x]]) +
2*a*(a + b*Sinh[2*ArcSinh[c*x]]))))/(24*c)
```

### 3.254.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6200, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6200}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int x(a+b\text{arcsinh}(cx))dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)+\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-$$
$$\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^3}{6bc}$$

input
```
Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[Pi]*(a + b*ArcS
inh[c*x])^3)/(6*b*c) - b*c*Sqrt[Pi]*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*(
(x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)
```

### 3.254.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

### 3.254.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

| method | result |
|---|---|
| default | $\frac{a^2x\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{a^2\pi\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2\sqrt{\pi\,c^2}}+\frac{b^2\sqrt{\pi}\left(6\,\mathrm{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,cx-6\,\mathrm{arcsinh}(cx)c^2x^2+3cx\sqrt{c^2x^2+1}+2\,\mathrm{arcsin}\right.}{12c}$ |
| parts | $\frac{a^2x\sqrt{\pi\,c^2x^2+\pi}}{2}+\frac{a^2\pi\ln\left(\frac{\pi\,c^2x}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{2\sqrt{\pi\,c^2}}+\frac{b^2\sqrt{\pi}\left(6\,\mathrm{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,cx-6\,\mathrm{arcsinh}(cx)c^2x^2+3cx\sqrt{c^2x^2+1}+2\,\mathrm{arcsin}\right.}{12c}$ |

input
```
int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a^2*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*
c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/12*b^2*Pi^(1/2)*(6*arcsinh(c*x)^2*(c^2
*x^2+1)^(1/2)*c*x-6*arcsinh(c*x)*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+2*arcsinh
(c*x)^3-3*arcsinh(c*x))/c+1/2*a*b*Pi^(1/2)*(2*arcsinh(c*x)*c*x*(c^2*x^2+1)
^(1/2)-c^2*x^2+arcsinh(c*x)^2-1)/c
```

### 3.254.5 Fricas [F]

$$\int\sqrt{\pi+c^2\pi x^2}(a+b\mathrm{arcsinh}(cx))^2\,dx=\int\sqrt{\pi+\pi c^2x^2}(b\,\mathrm{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2), x)
```

### 3.254.6 Sympy [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 \, dx = \sqrt{\pi}\left(\int a^2\sqrt{c^2x^2 + 1}\, dx + \int b^2\sqrt{c^2x^2 + 1}\operatorname{asinh}^2(cx)\, dx + \int 2ab\sqrt{c^2x^2 + 1}\operatorname{asinh}(cx)\, dx\right)$$

input
```
integrate((pi*c**2*x**2+pi)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
sqrt(pi)*(Integral(a**2*sqrt(c**2*x**2 + 1), x) + Integral(b**2*sqrt(c**2*
x**2 + 1)*asinh(c*x)**2, x) + Integral(2*a*b*sqrt(c**2*x**2 + 1)*asinh(c*x
), x))
```

### 3.254.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.254.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.254.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = \int (a + b\,\text{asinh}(c\,x))^2\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.255 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}} dx$



## 3.255.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}} dx = \frac{(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{\pi}}$$

output
```
1/3*(a+b*arcsinh(c*x))^3/b/c/Pi^(1/2)
```

## 3.255.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}} dx = \frac{(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{\pi}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

## 3.255.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{(a+b\text{arcsinh}(cx))^3}{3\sqrt{\pi}bc}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

### 3.255.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.255.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.88

| method | result | size |
|---|---|---|
| default | $\frac{a^2\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b^2\,\text{arcsinh}(cx)^3}{3c\sqrt{\pi}}+\frac{ab\,\text{arcsinh}(cx)^2}{c\sqrt{\pi}}$ | 72 |
| parts | $\frac{a^2\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b^2\,\text{arcsinh}(cx)^3}{3c\sqrt{\pi}}+\frac{ab\,\text{arcsinh}(cx)^2}{c\sqrt{\pi}}$ | 72 |

input
```
int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3*b
^2/c/Pi^(1/2)*arcsinh(c*x)^3+a*b*arcsinh(c*x)^2/c/Pi^(1/2)
```

### 3.255.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(pi + pi*c^2*
x^2), x)
```

### 3.255.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.

Time = 0.90 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.60

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \begin{cases} a^2\left(\begin{cases} \frac{\log\left(2\pi c^2x+2\sqrt{\pi}\sqrt{\pi c^2x^2+\pi}\sqrt{c^2}\right)}{\sqrt{\pi}\sqrt{c^2}} & \text{for } \pi c^2\neq 0 \\ \frac{x}{\sqrt{\pi}} & \text{otherwise} \end{cases}\right) & \text{for } b=0 \\ \frac{a^2x}{\sqrt{\pi}} & \text{for } c=0 \\ \frac{(a+b\,\text{asinh}\,(cx))^3}{3\sqrt{\pi}bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((a**2*Piecewise((log(2*pi*c**2*x + 2*sqrt(pi)*sqrt(pi*c**2*x**2
+ pi)*sqrt(c**2))/(sqrt(pi)*sqrt(c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi), Tru
e)), Eq(b, 0)), (a**2*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**3/(3*sqr
t(pi)*b*c), True))
```

### 3.255.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{b^2\operatorname{arsinh}(cx)^3}{3\sqrt{\pi}c} + \frac{ab\operatorname{arsinh}(cx)^2}{\sqrt{\pi}c} + \frac{a^2\operatorname{arsinh}(cx)}{\sqrt{\pi}c}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima"
)
```

output
```
1/3*b^2*arcsinh(c*x)^3/(sqrt(pi)*c) + a*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a^
2*arcsinh(c*x)/(sqrt(pi)*c)
```

### 3.255.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/sqrt(pi + pi*c^2*x^2), x)
```

### 3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2), x)
```

# 3.256 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx$



## 3.256.1 Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{(a+b\text{arcsinh}(cx))^2}{c\pi^{3/2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}} - \frac{b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c\pi^{3/2}}$$

output
```
(a+b*arcsinh(c*x))^2/c/Pi^(3/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+
1)^(1/2))^2)/c/Pi^(3/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(3/
2)+x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.256.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)^2 + 2b\text{arcsinh}(cx)\left(acx - b\sqrt{1+c^2x^2}\log\left(1\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*
c*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2
*ArcSinh[c*x])])/(c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

### 3.256.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6202, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{\pi^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{26}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{4201}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{2620}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{2715}$$

$$\frac{x(a+b\operatorname{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+$$
$$\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\operatorname{arcsinh}(cx)}+1\right)(a+b\operatorname{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\operatorname{arcsinh}(cx)}\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)de^{2\operatorname{arcsinh}(cx)}\right)-\frac{i(a+b\operatorname{ar}}{}\right.}{\pi^{3/2}c}$$

$\downarrow$ 2838

$$\frac{x(a+b\operatorname{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+$$
$$\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\operatorname{arcsinh}(cx)}+1\right)(a+b\operatorname{arcsinh}(cx))+\frac{1}{4}b\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)\right)-\frac{i(a+b\operatorname{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + ((2*I)*b*(((-1/2*I
)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*Ar
cSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c*Pi^(3/2))
```

### 3.256.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.256. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.256.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(114) = 228$.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| default | $\frac{a^2 x}{\pi\sqrt{\pi c^2x^2+\pi}} + b^2\left(-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)^2}{\pi^{\frac{3}{2}}c(c^2x^2+1)} + \frac{2\operatorname{arcsinh}(cx)^2}{c\pi^{\frac{3}{2}}} - \frac{2\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\pi^{\frac{3}{2}}} - \right.$ |
| parts | $\frac{a^2 x}{\pi\sqrt{\pi c^2x^2+\pi}} + b^2\left(-\frac{\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)^2}{\pi^{\frac{3}{2}}c(c^2x^2+1)} + \frac{2\operatorname{arcsinh}(cx)^2}{c\pi^{\frac{3}{2}}} - \frac{2\operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c\pi^{\frac{3}{2}}} - \right.$ |

input
```
int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+b^2*(-1/Pi^(3/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(
1/2)+1)*arcsinh(c*x)^2/c/(c^2*x^2+1)+2/c/Pi^(3/2)*arcsinh(c*x)^2-2/c/Pi^(3
/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c/Pi^(3/2)*polylog(2,-(
c*x+(c^2*x^2+1)^(1/2))^2))+2*a*b*(2/c/Pi^(3/2)*arcsinh(c*x)-1/Pi^(3/2)*(c^
2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/c/(c^2*x^2+1)-1/c/Pi^(3/2)*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2))
```

### 3.256.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2)/(pi^2*c^4*x^4 + 2*pi^2*c^2*x^2 + pi^2), x)
```

### 3.256.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) +
 Integral(b**2*asinh(c*x)**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x*
*2 + 1)), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) +
sqrt(c**2*x**2 + 1)), x))/pi**(3/2)
```

### 3.256.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima"
)
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(pi + pi*c^2*x^2)^(3/2), x) +
 2*a*b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a^2*x/(pi*sqrt(pi + pi*
c^2*x^2)) - a*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

### 3.256.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(3/2), x)
```

### 3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2 + \Pi)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2), x)
```

# 3.257 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx$



## 3.257.1 Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)}+\frac{2(a+b\text{arcsinh}(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{4b(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}-\frac{2b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}$$

output
```
1/3*b*(a+b*arcsinh(c*x))/c/Pi^(5/2)/(c^2*x^2+1)+2/3*(a+b*arcsinh(c*x))^2/c
/Pi^(5/2)+1/3*x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-4/3*b*(a+b*a
rcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-2/3*b^2*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-1/3*b^2*x/Pi^(5/2)/(c^2*x^2+1)^(1/2
)+2/3*x*(a+b*arcsinh(c*x))^2/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

## 3.257.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{3a^2cx-b^2cx+2a^2c^3x^3-b^2c^3x^3+ab\sqrt{1+c^2x^2}-b^2\left(-3cx-2c^3x^3+2\sqrt{1+c\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 - b^2*c^3*x^3 + a*b*Sqrt[1 + c^2*x^2]
 - b^2*(-3*c*x - 2*c^3*x^3 + 2*Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*
x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-6*a*c*x - 4*a*c^3*x^3 - b*Sqrt[1 +
 c^2*x^2] + 4*b*(1 + c^2*x^2)^(3/2)*Log[1 + E^(-2*ArcSinh[c*x])]) - 2*a*b*
Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 + c^2*x^2]*Log[1
 + c^2*x^2] + 2*b^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])])/
(3*c*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

### 3.257.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{\pi^{3/2}}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 26

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 4201

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 2620

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(c}{2b}}{\pi^{3/2}c}}{3\pi}\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 2715

---

3.257. $\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}dx$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i}{}\right.}{\pi^{3/2}c}}{3\pi}-\frac{2bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \; 6213$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\left(\frac{b\int\frac{1}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{2bc\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

3.257. $\int\frac{(a+b\text{arcsinh}(cx))^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}dx$

output
```
(x*(a + b*ArcSinh[c*x])^2)/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (2*b*c*((b*x)/
(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*(1 + c^2*x^2))))/(3*
Pi^(5/2)) + (2*((x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + ((
2*I)*b*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])
*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))
/(c*Pi^(3/2))))/(3*Pi)
```

### 3.257.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.257. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.257.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(192) = 384$.

Time = 0.26 (sec) , antiderivative size = 1729, normalized size of antiderivative = 8.48

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1729 |
| parts | Expression too large to display | 1729 |

input
```
int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
4*a*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5-4*a*b/
Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+4/3*a*b/Pi^(5/2)
/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2-3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*
arcsinh(c*x)*x^2-4/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*
x)*x^6+2*b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x
^5-22/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^2+16/3
*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4/3*b^2/Pi^(5
/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^8-2*b^2/Pi^(5/2)*c^5/(3
*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^6+16/3*b^2/Pi^(5/2)*c^5/(3*c^2*
x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6-20/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/
(c^2*x^2+1)^2*arcsinh(c*x)^2*x^4+8*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2
+1)^2*arcsinh(c*x)*x^4-4*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsin
h(c*x)*x^4+17/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c
*x)^2*x^3+6*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^4+a^2*(1/3/Pi*x
/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))+8/3*a*b/c/Pi^(5/2
)*arcsinh(c*x)-4/3*a*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-2/3*b^2*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-4/3*b^2/Pi^(5/2)/(3*c^2*x
^2+4)/(c^2*x^2+1)^(3/2)*x-4/3*b^2/c/Pi^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)+4/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2+4/3*b^2/c/P
i^(5/2)*arcsinh(c*x)^2+34/3*a*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^...
```

### 3.257.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2)/(pi^3*c^6*x^6 + 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

### 3.257.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{a^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \cdots}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2
 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b**2*asinh(c*x)**2/(c**4*x**4*
sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1
)), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2
*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(5/2)
```

### 3.257.7 Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima"
)
```

output
```
1/3*a*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5
/2)*c^2)) + 2/3*a*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi +
pi*c^2*x^2)))*arcsinh(c*x) + 1/3*a^2*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x
/(pi^2*sqrt(pi + pi*c^2*x^2))) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1)
)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

### 3.257.8 Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

### 3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2), x)
```

# 3.258 $\int x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx$



## 3.258.1 Optimal result

Integrand size = 28, antiderivative size = 358

$$\begin{aligned}\int x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx &= -\frac{52b^2\sqrt{d+c^2dx^2}}{225c^4}+\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}\\ &-\frac{26b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{675c^4}\\ &+\frac{2b^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^4}\\ &+\frac{4b^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{15c^3\sqrt{1+c^2x^2}}\\ &-\frac{2bx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{45c\sqrt{1+c^2x^2}}\\ &-\frac{2bcx^5\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}\\ &-\frac{2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{15c^4}\\ &+\frac{x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{15c^2}\\ &+\frac{1}{5}x^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\end{aligned}$$

output
```
-52/225*b^2*(c^2*d*x^2+d)^(1/2)/c^4-26/675*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(
1/2)/c^4+2/125*b^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4-2/15*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/c^2+1/5*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/15*a*b*x*(
c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/15*b^2*x*arcsinh(c*x)*(c^2*d*x^
2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/45*b*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/25*b*c*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)
```

## 3.258.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.62

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{\sqrt{d+c^2dx^2}\Big(225(-2+3c^2x^2)\left(a+ac^2x^2\right)^2-30abcx\sqrt{1+c^2x^2}(-30+5c^2x^2+9c^4x^4)+2b^2(-428-439c}{}$$

input
```
Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(225*(-2 + 3*c^2*x^2)*(a + a*c^2*x^2)^2 - 30*a*b*c*x*
Sqrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(-428 - 439*c^2*x^
2 + 16*c^4*x^4 + 27*c^6*x^6) - 30*b*(-15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2
) + b*c*x*Sqrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] +
225*(-2 + 3*c^2*x^2)*(b + b*c^2*x^2)^2*ArcSinh[c*x]^2))/(3375*c^4*(1 + c^2
*x^2))
```

## 3.258.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6221, 6191, 243, 53, 2009, 6227, 6191, 243, 53, 2009, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6221}$$

$$-\frac{2bc\sqrt{c^2dx^2+d}\int x^4(a+b\text{arcsinh}(cx))dx}{5\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{5}bc\int \frac{x^5}{\sqrt{c^2x^2+1}}dx\right)}{5\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int \frac{x^4}{\sqrt{c^2x^2+1}}dx^2\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int \left(\frac{(c^2x^2+1)^{3/2}}{c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{1}{c^4\sqrt{c^2x^2+1}}\right)dx^2\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6227}$$

3.258. $\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\int x^2(a+b\text{arcsinh}(cx))dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6191$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\int \frac{x^3}{\sqrt{c^2x^2+1}}dx\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 243$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx^2\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 53$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2009$$

---

3.258. $\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{2\sqrt{c^2 \cdots}}{c \cdots}\right.\right.}{3c}\right.}{5\sqrt{c^2x^2+1}}$$

$$\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6213}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\int(a+b\text{arcsinh}(cx))dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(\cdots\right.}{\cdots}\right.}{5\sqrt{c^2x^2+1}}$$

$$\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5\sqrt{c^2x^2+1}}+$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c}\right)}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\cdots\right.}{\cdots}\right.}{5\sqrt{c^2x^2+1}}$$

input
```
Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

3.258. $\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*Sqrt[d + c^2*d
*x^2]*(-1/10*(b*c*((2*Sqrt[1 + c^2*x^2])/c^6 - (4*(1 + c^2*x^2)^(3/2))/(3*
c^6) + (2*(1 + c^2*x^2)^(5/2))/(5*c^6))) + (x^5*(a + b*ArcSinh[c*x]))/5))/
(5*Sqrt[1 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*((x^2*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])^2)/(3*c^2) - (2*b*(-1/6*(b*c*((-2*Sqrt[1 + c^2*x^2])/c^4 +
 (2*(1 + c^2*x^2)^(3/2))/(3*c^4))) + (x^3*(a + b*ArcSinh[c*x]))/3))/(3*c)
- (2*((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c^2 - (2*b*(a*x - (b*Sqrt
[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/c))/(3*c^2)))/(5*Sqrt[1 + c^2*x^2])
```

### 3.258.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

3.258. $\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.258.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(310) = 620$.

Time = 0.38 (sec) , antiderivative size = 1162, normalized size of antiderivative = 3.25

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1162 |
| parts | Expression too large to display | 1162 |

input
```
int(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b^2
*(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28
*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1
)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+
1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+
1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/16*(d*(c
^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsi
nh(c*x)+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^
2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(
c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(
c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)+1/
4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4
*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(2
5*arcsinh(c*x)^2+10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*
x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x
^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c
*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4
/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*
(-1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c...
```

## 3.258.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.88

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{225\left(3\,b^2c^6x^6+4\,b^2c^4x^4-b^2c^2x^2-2\,b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(45\,abc^6x^6+60\,abc^4x^4-\right.}{}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
1/3375*(225*(3*b^2*c^6*x^6 + 4*b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 + 60*a*b*c
^4*x^4 - 15*a*b*c^2*x^2 - 30*a*b - (9*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 30*b^2
*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))
+ (27*(25*a^2 + 2*b^2)*c^6*x^6 + 4*(225*a^2 + 8*b^2)*c^4*x^4 - (225*a^2 +
878*b^2)*c^2*x^2 - 450*a^2 - 856*b^2 - 30*(9*a*b*c^5*x^5 + 5*a*b*c^3*x^3 -
 30*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

### 3.258.6 Sympy [F]

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^3\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}\left(cx\right))^2\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

### 3.258.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx\\&=\frac{1}{15}\,b^2\left(\frac{3\,\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,\left(c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}\left(cx\right)^2\\&\quad+\frac{2}{15}\,ab\left(\frac{3\,\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,\left(c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}\left(cx\right)\\&\quad+\frac{1}{15}\,a^2\left(\frac{3\,\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\,\left(c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\\&\quad+\frac{2}{3375}\,b^2\left(\frac{27\,\sqrt{c^2x^2+1}c^2\sqrt{d}x^4-11\,\sqrt{c^2x^2+1}\sqrt{d}x^2-\frac{428\,\sqrt{c^2x^2+1}\sqrt{d}}{c^2}}{c^2}-\frac{15\,\left(9\,c^4\sqrt{d}x^5+5\,c^2\sqrt{d}x^3-30\,\sqrt{\cdots}\right.}{c^3}\right.\\&\quad-\frac{2\,\left(9\,c^4\sqrt{d}x^5+5\,c^2\sqrt{d}x^3-30\,\sqrt{d}x\right)ab}{225\,c^3}\end{aligned}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
1/15*b^2*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c
^4*d))*arcsinh(c*x)^2 + 2/15*a*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*
(c^2*d*x^2 + d)^(3/2)/(c^4*d))*arcsinh(c*x) + 1/15*a^2*(3*(c^2*d*x^2 + d)^
(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 2/3375*b^2*((27*sqr
t(c^2*x^2 + 1)*c^2*sqrt(d)*x^4 - 11*sqrt(c^2*x^2 + 1)*sqrt(d)*x^2 - 428*sq
rt(c^2*x^2 + 1)*sqrt(d)/c^2)/c^2 - 15*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x
^3 - 30*sqrt(d)*x)*arcsinh(c*x)/c^3) - 2/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sq
rt(d)*x^3 - 30*sqrt(d)*x)*a*b/c^3
```

### 3.258.8 Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.258.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int x^3\,(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

# 3.259 $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.259.1 Optimal result

Integrand size = 28, antiderivative size = 291

$$\begin{aligned}\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d+c^2dx^2}\\&-\frac{b^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64c^3\sqrt{1+c^2x^2}}\\&-\frac{bx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{1+c^2x^2}}\\&-\frac{bcx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}}\\&+\frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^2}\\&+\frac{1}{4}x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\\&-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{24bc^3\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
1/64*b^2*x*(c^2*d*x^2+d)^(1/2)/c^2+1/32*b^2*x^3*(c^2*d*x^2+d)^(1/2)+1/8*x*
(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)-1/64*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2
+1)^(1/2)-1/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(
1/2)-1/8*b*c*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-
1/24*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

### 3.259.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx =$$

$$-\frac{-96a^2cx(1+2c^2x^2)\sqrt{d+c^2dx^2}+96a^2\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{12ab\sqrt{d+c^2dx^2}\left(8\text{arcsinh}(cx)^2+\cosh\right.}{}}{}$$

input
```
Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/768*(-96*a^2*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 96*a^2*Sqrt[d]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (12*a*b*Sqrt[d + c^2*d*x^2]*(8*A
rcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]
))/Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(32*ArcSinh[c*x]^3 + 12*Ar
cSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*ArcSinh[c*x]^2)*Sinh[4*ArcSinh[
c*x]]))/Sqrt[1 + c^2*x^2])/c^3
```

### 3.259.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6221, 6191, 262, 262, 222, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6221}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^3(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\int \frac{x^4}{\sqrt{c^2x^2+1}}dx\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int \frac{x^2}{\sqrt{c^2x^2+1}}dx}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6227}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int x(a+b\text{arcsinh}(cx))dx}{c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6191}$$

3.259. $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{c}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 262$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{c}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 222$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{c}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6198$$

---

3.259. $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcsinh}(cx))^3}{6bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{c}\right)}{4\sqrt{c^2x^2+1}}$$

$$\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2\sqrt{c^2x^2+1}}$$

input
```
Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/4 - (b*c*Sqrt[d + c^2*d*x
^2]*((x^4*(a + b*ArcSinh[c*x]))/4 - (b*c*((x^3*Sqrt[1 + c^2*x^2])/(4*c^2)
- (3*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/(4*c^2)))/4))
/(2*Sqrt[1 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*((x*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])^2)/(2*c^2) - (a + b*ArcSinh[c*x])^3/(6*b*c^3) - (b*((x^2*(a
 + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]
/(2*c^3)))/2))/c))/(4*Sqrt[1 + c^2*x^2])
```

### 3.259.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.259.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(251) = 502$.

Time = 0.27 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| default | $\frac{a^2x(c^2d\,x^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{a^2x\sqrt{c^2d\,x^2+d}}{8c^2}-\frac{a^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{24\sqrt{c^2x^2+1}\,c^3}+\frac{\sqrt{d(c^2x^2+1)}\left(8c^5\right.}{}\right.$ |
| parts | $\frac{a^2x(c^2d\,x^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{a^2x\sqrt{c^2d\,x^2+d}}{8c^2}-\frac{a^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{24\sqrt{c^2x^2+1}\,c^3}+\frac{\sqrt{d(c^2x^2+1)}\left(8c^5\right.}{}\right.$ |

input
```
int(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*
a^2/c^2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*
(-1/24*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3+1/512*(d
*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^
2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsi
nh(c*x)+1)/c^3/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^
4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+
1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)/c^3/(c^2*x^2+1))+2*a*b*(-1/1
6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/256*(d*(c^2
*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2
*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^3/(c^2*x
^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+1
2*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(1+4*arcsin
h(c*x))/c^3/(c^2*x^2+1))
```

### 3.259.5 Fricas [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(
c^2*d*x^2 + d), x)
```

### 3.259.6 Sympy [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^2\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

### 3.259.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.259.8 Giac [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

### 3.259.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int x^2\,(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

# 3.260 $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.260.1 Optimal result

Integrand size = 26, antiderivative size = 180

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{4b^2\sqrt{d+c^2dx^2}}{9c^2}+\frac{2b^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{27c^2}-\frac{2bx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

output
```
1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/c^2/d+4/9*b^2*(c^2*d*x^2+d)^(
1/2)/c^2+2/27*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*x*(a+b*arcsinh
(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/9*b*c*x^3*(a+b*arcsinh(c*
x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.260.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{\sqrt{d+c^2dx^2}\left(-6abcx\sqrt{1+c^2x^2}(3+c^2x^2)+9(a+ac^2x^2)^2+2b^2(7+8c^2x^2+c^4x^4)+6b\left(3a(1+c^2x^2)^2-\right.\right.}{27c^2\left(1+c^2x^2\right)}$$

input
```
Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-6*a*b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2) + 9*(a +
a*c^2*x^2)^2 + 2*b^2*(7 + 8*c^2*x^2 + c^4*x^4) + 6*b*(3*a*(1 + c^2*x^2)^2
- b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2))*ArcSinh[c*x] + 9*(b + b*c^2*x^2)^
2*ArcSinh[c*x]^2))/(27*c^2*(1 + c^2*x^2))
```

### 3.260.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6213, 6199, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2dx^2+d}\int\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6199}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2dx^2+d}\left(-bc\int\frac{x(c^2x^2+3)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))\right)}{3c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2dx^2+d}\left(-\frac{1}{3}bc\int\frac{x(c^2x^2+3)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\operatorname{arcsinh}(cx))+x(a+b\operatorname{arcsinh}(cx))\right)}{3c\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 353

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int\frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2x^3(a+b\operatorname{arcsinh}(cx))+x(a+b\operatorname{arcsinh}(cx))\right)}{3c\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 53

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\operatorname{arcsinh}(cx))+x(a+b\operatorname{arcsinh}(cx))\right)}{3c\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2009

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\operatorname{arcsinh}(cx))+x(a+b\operatorname{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3c\sqrt{c^2x^2+1}}$$

input
```
Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*c^2*d) - (2*b*Sqrt[d + c
^2*d*x^2]*(-1/6*(b*c*((4*Sqrt[1 + c^2*x^2])/c^2 + (2*(1 + c^2*x^2)^(3/2))/
(3*c^2))) + x*(a + b*ArcSinh[c*x]) + (c^2*x^3*(a + b*ArcSinh[c*x]))/3))/(3
*c*Sqrt[1 + c^2*x^2])
```

## 3.260. $\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx$

### 3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.260. $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.260.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(156) = 312$.

Time = 0.26 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.65

| method | result |
|---|---|
| default | $\frac{a^2(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\operatorname{arcsinh}(cx)^2-6\operatorname{arcsinh}(cx)+2\right)}{216c^2(c^2x^2+1)}+\right.$ |
| parts | $\frac{a^2(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\operatorname{arcsinh}(cx)^2-6\operatorname{arcsinh}(cx)+2\right)}{216c^2(c^2x^2+1)}+\right.$ |

input
```
int(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*(c^2*d*x^2+d)^(3/2)/c^2/d+b^2*(1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*ar
csinh(c*x)^2-6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(
c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/(c^
2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcs
inh(c*x)^2+2*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(
4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)
*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*
x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1
/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/
8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)
/c^2/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+
1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^2/(c^2*
x^2+1))
```

### 3.260.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.38

$$\int x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{9\left(b^2c^4x^4+2b^2c^2x^2+b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3abc^4x^4+6abc^2x^2+3ab-(b^2c^3x^3+3\right.}{}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/27*(9*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x +
sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + 3*a*b - (b^2*c^3
*x^3 + 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + 2*(9*a^2 + 8*b^2)*c^2*x^2 + 9*a^2
 + 14*b^2 - 6*(a*b*c^3*x^3 + 3*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2
+ d))/(c^4*x^2 + c^2)
```

### 3.260.6 Sympy [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int x\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

### 3.260.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

$$\begin{aligned}&\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx\\&=\frac{2}{27}b^2\left(\frac{\sqrt{c^2x^2+1}d^{\frac{3}{2}}x^2+\frac{7\sqrt{c^2x^2+1}d^{\frac{3}{2}}}{c^2}}{d}-\frac{3\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)\operatorname{arsinh}(cx)}{cd}\right)\\&\quad+\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b^2\operatorname{arsinh}(cx)^2}{3\,c^2d}+\frac{2\left(c^2dx^2+d\right)^{\frac{3}{2}}ab\operatorname{arsinh}(cx)}{3\,c^2d}\\&\quad-\frac{2\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)ab}{9\,cd}+\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}a^2}{3\,c^2d}\end{aligned}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
2/27*b^2*((sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 + 7*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2
)/d - 3*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*arcsinh(c*x)/(c*d)) + 1/3*(c^2*d*x
^2 + d)^(3/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*a
rcsinh(c*x)/(c^2*d) - 2/9*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*a*b/(c*d) + 1/3*
(c^2*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

### 3.260.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.260.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int x\,(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

# 3.261 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.261.1 Optimal result

Integrand size = 25, antiderivative size = 184

$$\begin{aligned}\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}\\ &\quad-\frac{bcx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\ &\quad+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\\ &\quad+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
1/4*b^2*x*(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/
2)-1/4*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^
2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsin
h(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.261.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{1}{24}\left(12a^2x\sqrt{d+c^2dx^2}+\frac{12a^2\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{c}\right.$$
$$+\frac{b^2\sqrt{d+c^2dx^2}(4\text{arcsinh}(cx)^3-6\text{arcsinh}(cx)\cosh(2\text{arcsinh}(cx))+(3+6\text{arcsinh}(cx)^2)\sinh(2\text{arcsinh}(cx)))}{c\sqrt{1+c^2x^2}}$$
$$\left.+\frac{6ab\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{c\sqrt{1+c^2x^2}}\right)$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(12*a^2*x*Sqrt[d + c^2*d*x^2] + (12*a^2*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d
 + c^2*d*x^2]])/c + (b^2*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh
[c*x]*Cosh[2*ArcSinh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))
/(c*Sqrt[1 + c^2*x^2]) + (6*a*b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]]
 + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*
x^2]))/24
```

### 3.261.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6200, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2\,dx$$
$$\Big\downarrow \text{6200}$$
$$-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \ 6191$$

$$-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \ 262$$

$$-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \ 222$$

$$\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 6198$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[d + c^2*d*x^2]
*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - Arc
Sinh[c*x]/(2*c^3)))/2))/Sqrt[1 + c^2*x^2]
```

3.261. $\int\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.261.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

### 3.261.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(158) = 316$.

Time = 0.22 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.61

| method | result |
|---|---|
| default | $\frac{a^2 x\sqrt{c^2 d\,x^2+d}}{2}+\frac{a^2 d \ln\left(\frac{c^2 d x}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{2\sqrt{c^2 d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx\right.}{16c}\right.$ |
| parts | $\frac{a^2 x\sqrt{c^2 d\,x^2+d}}{2}+\frac{a^2 d \ln\left(\frac{c^2 d x}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{2\sqrt{c^2 d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2cx\right.}{16c}\right.$ |

input
```
int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x*(c^2*d*x^2+d)^(1/2)+1/2*a^2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c*arcsinh(c*x)^3+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2
+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)/c/(
c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/
2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)/c/(c^2*x^2
+1))+2*a*b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2+1
/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^
2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/
2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*ar
csinh(c*x))/c/(c^2*x^2+1))
```

### 3.261.5 Fricas [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2), x)
```

### 3.261.6 Sympy [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\,(cx))^2\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

### 3.261.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.261.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.261.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

# 3.262 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$



## 3.262.1 Optimal result

Integrand size = 28, antiderivative size = 338

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx \\
&= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} \\
&\quad + \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{2b\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{2b^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{2b^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
2*b^2*(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c
*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*x*arcsinh(c*x)*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)
^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*polyl
og(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*(a+
b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*
x^2+1)^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)
```

### 3.262.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$$
$$= a^2\sqrt{d+c^2dx^2} + a^2\sqrt{d}\log(cx) - a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)$$
$$+ \frac{2ab\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(cx)\log\left(1-e^{-\text{arcsinh}(cx)}\right)-\text{arcsinh}(cx)\log\left(1-\right.\right.}{\sqrt{1+c^2x^2}}$$
$$+ \frac{b^2\sqrt{d+c^2dx^2}\left(2\sqrt{1+c^2x^2}-2cx\text{arcsinh}(cx)+\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2+\text{arcsinh}(cx)^2\log\left(1-e^{-\text{arcsinh}(}\right.\right.}{}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

output
```
a^2*Sqrt[d + c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[
d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^
2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*
x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2
, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(2*Sqr
t[1 + c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + A
rcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcS
inh[c*x])] + 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x
]*PolyLog[2, E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*Pol
yLog[3, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
```

### 3.262.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {6221, 2009, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}\,dx$$
$$\Big\downarrow \; 6221$$

$$-\frac{2bc\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6231$$

$$\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 26$$

$$\frac{i\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 4670$$

$$\frac{i\sqrt{c^2dx^2+d}\left(2ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int(a+b\text{arcsinh}(cx))\log\left(1+e^{\text{arcsinh}(cx)}\right)\cdots\right.}{\sqrt{c^2x^2+1}}$$
$$\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 3011$$

3.262. $\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}dx$

$$\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+2\cdots}{\cdots}$$

$$\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 2720

$$\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}\cdots\right.\right.}{\cdots}$$

$$\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 7143

$$\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(\cdots}\right.\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{c^2x^2+1}}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

output
```
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*b*c*Sqrt[d + c^2*d*x^2]*(a
*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (I*S
qrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]] -
 (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]]) + b*PolyLog[
3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSin
h[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2]
```

### 3.262.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.262. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

---

## 3.262. $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.262.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(353) = 706$.

Time = 0.32 (sec) , antiderivative size = 764, normalized size of antiderivative = 2.26

| method | result |
|---|---|
| default | $-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)a^2+a^2\sqrt{c^2d\,x^2+d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\operatorname{arcsinh}(cx)^2-2\operatorname{arcs}\right.}{2c^2x^2+2}\right.$ |
| parts | $-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)a^2+a^2\sqrt{c^2d\,x^2+d}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\operatorname{arcsinh}(cx)^2-2\operatorname{arcs}\right.}{2c^2x^2+2}\right.$ |

input
```
int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a^2+a^2*(c^2*d*x^2+d)^(
1/2)+b^2*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arc
sinh(c*x)^2-2*arcsinh(c*x)+2)/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/(c^2*x^2+1)-
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1
)^(1/2))-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,
-c*x-(c^2*x^2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(
3,-c*x-(c^2*x^2+1)^(1/2))+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)*c*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)+2*a*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(
1/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2
+1)^(1/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1
+c*x+(c^2*x^2+1)^(1/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*poly
log(2,-c*x-(c^2*x^2+1)^(1/2))
```

## 3.262.5 Fricas [F]

$$\int \frac{\sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^2}{x} \, dx = \int \frac{\sqrt{c^2 d x^2 + d} (b \operatorname{arsinh}(cx) + a)^2}{x} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/x, x)
```

## 3.262.6 Sympy [F]

$$\int \frac{\sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^2}{x} \, dx = \int \frac{\sqrt{d (c^2 x^2 + 1)} (a + b \operatorname{asinh}(cx))^2}{x} \, dx$$

input
```
integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x, x)
```

## 3.262.7 Maxima [F]

$$\int \frac{\sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^2}{x} \, dx = \int \frac{\sqrt{c^2 d x^2 + d} (b \operatorname{arsinh}(cx) + a)^2}{x} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

output
```
-(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a^2 + integrate(sqr
t(c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*sqrt(c^2*d*x^2 +
 d)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.262.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x, x)
```

# 3.263 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$



## 3.263.1 Optimal result

Integrand size = 28, antiderivative size = 209

$$\begin{aligned}\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx &= -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}+\frac{c\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\\ &\quad+\frac{c\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}}\\ &\quad+\frac{2bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\ &\quad-\frac{b^2c\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+c*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1
/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(
1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2
*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.263.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx = -\frac{a^2\sqrt{d+c^2dx^2}}{x}$$
$$+\frac{ab\sqrt{d+c^2dx^2}\left(-2\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)+cx\operatorname{arcsinh}(cx)^2+2cx\log(cx)\right)}{x\sqrt{1+c^2x^2}}$$
$$+a^2c\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)$$
$$+\frac{b^2c\sqrt{d+c^2dx^2}\left(\operatorname{arcsinh}(cx)\left(\left(3-\frac{3\sqrt{1+c^2x^2}}{cx}\right)\operatorname{arcsinh}(cx)+\operatorname{arcsinh}(cx)^2+6\log\left(1-e^{-2\operatorname{arcsinh}(cx)}\right)\right)-3\right.}{3\sqrt{1+c^2x^2}}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
-((a^2*Sqrt[d + c^2*d*x^2])/x) + (a*b*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2
*x^2]) + a^2*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b^2*c*S
qrt[d + c^2*d*x^2]*(ArcSinh[c*x]*((3 - (3*Sqrt[1 + c^2*x^2])/(c*x))*ArcSin
h[c*x] + ArcSinh[c*x]^2 + 6*Log[1 - E^(-2*ArcSinh[c*x])]) - 3*PolyLog[2, E
^(-2*ArcSinh[c*x])]))/(3*Sqrt[1 + c^2*x^2])
```

### 3.263.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6220, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \ 6220$$
$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2bc\sqrt{c^2dx^2+d}\int \frac{a+b\operatorname{arcsinh}(cx)}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6190}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2c\sqrt{c^2dx^2+d}\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{2c\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{2c\sqrt{c^2dx^2+d}\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{26}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{4201}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

---

3.263. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$\Big\downarrow \; 2620$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\right.\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \; 2715$$

$$\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \; 2838$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \; 6198$$

$$\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{c\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
-((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x) + (c*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2]) + ((2*I)*c*Sqrt[d + c^2*d
*x^2]*((-1/2*I)*(a + b*ArcSinh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x
])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)]) + (b^2*PolyLo
g[2, -a - b*ArcSinh[c*x]])/4)))/Sqrt[1 + c^2*x^2]
```

3.263. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

### 3.263.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

## 3.263. $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x^2}\,dx$

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

## 3.263.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(207) = 414$.

Time = 0.29 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.77

| method | result |
|---|---|
| default | $-\frac{a^2(c^2dx^2+d)^{\frac{3}{2}}}{dx}+a^2c^2x\sqrt{c^2d\,x^2+d}+\frac{a^2c^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3c}{3\sqrt{c^2x^2+1}}-\frac{\sqrt{d(c^2x}{}\right.$ |
| parts | $-\frac{a^2(c^2dx^2+d)^{\frac{3}{2}}}{dx}+a^2c^2x\sqrt{c^2d\,x^2+d}+\frac{a^2c^2d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3c}{3\sqrt{c^2x^2+1}}-\frac{\sqrt{d(c^2x}{}\right.$ |

input
```
int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(c^2*d*x^2+d)^(3/2)+a^2*c^2*x*(c^2*d*x^2+d)^(1/2)+a^2*c^2*d*ln(c^
2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/3*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c-(d*(c^2*x^2+1))^(1/2)*(c^2*
x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)^2/x/(c^2*x^2+1)-2*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c)+2
*a*b*(1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c-2*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c-(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/x/(c^2*x^2+1)+(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c)
```

### 3.263.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/x^2, x)
```

### 3.263.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2}{x^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**2, x)
```

### 3.263.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

output
```
(c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a^2 + integrate(sqrt(c^2*
d*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^2 + 2*sqrt(c^2*d*x^2 + d)*
a*b*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

### 3.263.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

# 3.264 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$



## 3.264.1 Optimal result

Integrand size = 28, antiderivative size = 358

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx\\
&= -\frac{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2x^2}\\
&\quad - \frac{c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\
&\quad - \frac{b^2c^2\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}}\\
&\quad - \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\
&\quad + \frac{bc^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\
&\quad + \frac{b^2c^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} - \frac{b^2c^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-1/2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2-b*c*(a+b*arcsinh(c*x))*(
c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))^2*arctanh(c*
x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*arctanh
((c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*(a+b*arcsi
nh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2)
)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.264.2 Mathematica [A] (verified)

Time = 3.79 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$$
$$= \frac{1}{8}\Bigg(-\frac{4a^2\sqrt{d+c^2dx^2}}{x^2} + 4a^2c^2\sqrt{d}\log(x) - 4a^2c^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)$$
$$+\frac{2abc^2\sqrt{d+c^2dx^2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - \text{arcsinh}(cx)\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 4\text{arcsinh}(cx)\log\left(1-e\right.\right.}{}$$
$$+\frac{b^2c^2\sqrt{d+c^2dx^2}\left(-4\text{arcsinh}(cx)\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right) - \text{arcsinh}(cx)^2\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(cx)\right) + 4\text{arcsinh}(cx\right.}{}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 + 4*a^2*c^2*Sqrt[d]*Log[x] - 4*a^2*c^2*S
qrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*Sqrt[d + c^2*d*x^
2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcS
inh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c
*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])]
 - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 +
 c^2*x^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x
]/2] - ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^
(-ArcSinh[c*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tan
h[ArcSinh[c*x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*Arc
Sinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])]
 - 8*PolyLog[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2
 + 4*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

### 3.264.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.63, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6220, 6191, 243, 73, 221, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{6220}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{x^2}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6191}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\left(bc\int \frac{1}{x\sqrt{c^2x^2+1}}dx-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\left(\frac{b\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c}-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 6231

$$\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 3042

$$\frac{c^2\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 26

$$\frac{ic^2\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 4670

$$\frac{ic^2\sqrt{c^2dx^2+d}\left(2ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int(a+b\text{arcsinh}(cx))\log\left(1+e^{\text{arcsin}}\right.\right.}{2\sqrt{c^2x^2+1}}$$
$$\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 3011

$$\frac{ic^2\sqrt{c^2dx^2+d}\left(-2ib\left(b\int\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+\right.}{}$$
$$\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-bc\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 2720

3.264. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$\frac{ic^2\sqrt{c^2dx^2+d}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsi}\right.\right.}{}$$

$$\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\big\downarrow$ 7143

$$\frac{ic^2\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\operatorname{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\operatorname{PolyLog}\left(2,-e^{\text{arcsin}}\right.\right.\right.}{2\sqrt{c^2x^2+}}$$

$$\frac{bc\sqrt{c^2dx^2+d}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
-1/2*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2 + (b*c*Sqrt[d + c^2*
d*x^2]*(-((a + b*ArcSinh[c*x])/x) - b*c*ArcTanh[Sqrt[1 + c^2*x^2]]))/Sqrt[
1 + c^2*x^2] + ((I/2)*c^2*Sqrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])^
2*ArcTanh[E^ArcSinh[c*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^
ArcSinh[c*x]]) + b*PolyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSin
h[c*x])*PolyLog[2, E^ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/Sqrt
[1 + c^2*x^2]
```

### 3.264.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

---

## 3.264. $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx$

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.264.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(371) = 742$.

Time = 0.35 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.11

| method | result |
|---|---|
| default | $a^2\left(-\frac{(c^2d\,x^2+d)^{\frac{3}{2}}}{2d\,x^2}+\frac{c^2\left(\sqrt{c^2d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)\right)}{2}\right)+b^2\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(c\right.}{2x^2(c^2x^2+1)}\right.$ |
| parts | $a^2\left(-\frac{(c^2d\,x^2+d)^{\frac{3}{2}}}{2d\,x^2}+\frac{c^2\left(\sqrt{c^2d\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)\right)}{2}\right)+b^2\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(c\right.}{2x^2(c^2x^2+1)}\right.$ |

input
```
int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

3.264. $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx$

output
```
a^2*(-1/2/d/x^2*(c^2*d*x^2+d)^(3/2)+1/2*c^2*((c^2*d*x^2+d)^(1/2)-d^(1/2)*l
n((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)))+b^2*(-1/2*(arcsinh(c*x)*c^2*x^2
+2*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)/
x^2/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2
*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2+(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2+(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1
)^(1/2))*c^2-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^
2+1)^(1/2))*c^2-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2
*x^2+1)^(1/2))*c^2)+2*a*b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2
)+arcsinh(c*x))*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2-1/2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c
^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*
x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*
x+(c^2*x^2+1)^(1/2))*c^2)
```

### 3.264.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/x^3, x)
```

### 3.264.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}\left(cx\right))^2}{x^3}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**3, x)
```

### 3.264.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}\left(cx\right)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxim
a")
```

output
```
-1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d
*x^2 + d)^(3/2)/(d*x^2))*a^2 + integrate(sqrt(c^2*d*x^2 + d)*b^2*log(c*x +
 sqrt(c^2*x^2 + 1))^2/x^3 + 2*sqrt(c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c^2*x
^2 + 1))/x^3, x)
```

### 3.264.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3, x)
```

# 3.265 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$



## 3.265.1 Optimal result

Integrand size = 28, antiderivative size = 294

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx \\
&= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}} \\
&\quad -\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} \\
&\quad +\frac{c^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}}-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3} \\
&\quad +\frac{2bc^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} \\
&\quad -\frac{b^2c^3\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/d/x^3-1/3*b^2*c^2*(c^2*d*x^2
+d)^(1/2)/x+1/3*b^2*c^3*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
+1/3*c^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*b*
c^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)-1/3*b^2*c^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)
^(1/2)*(c^2*d*x^2+d)^(1/2)/x^2
```

### 3.265.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx =$$
$$-\frac{\sqrt{d+c^2dx^2}\Big(abcx+a^2\sqrt{1+c^2x^2}+a^2c^2x^2\sqrt{1+c^2x^2}+b^2c^2x^2\sqrt{1+c^2x^2}+b^2\big(-c^3x^3+\sqrt{1+c^2x^2}+c^2}{}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
-1/3*(Sqrt[d + c^2*d*x^2]*(a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + a^2*c^2*x^2*S
qrt[1 + c^2*x^2] + b^2*c^2*x^2*Sqrt[1 + c^2*x^2] + b^2*(-(c^3*x^3) + Sqrt[
1 + c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*
(-(b*c*x) - 2*a*(1 + c^2*x^2)^(3/2) + 2*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*
x])]) - 2*a*b*c^3*x^3*Log[c*x] + b^2*c^3*x^3*PolyLog[2, E^(-2*ArcSinh[c*x]
)]))/(x^3*Sqrt[1 + c^2*x^2])
```

### 3.265.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.74, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6215, 6217, 247, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \text{6215}$$

$$\frac{2bc\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x^3}dx}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{6217}$$

$$\frac{2bc\sqrt{c^2dx^2+d}\left(c^2\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}bc\int \frac{\sqrt{c^2x^2+1}}{x^2}dx-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}\right)}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{2bc\sqrt{c^2dx^2+d}\left(c^2\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}}{x}\right)-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}\right)}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{2bc\sqrt{c^2dx^2+d}\left(c^2\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(c\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)\right)}{3\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 6190}$$

$$\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{c^2\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(c\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2bc\sqrt{c^2dx^2+d}\left(-\frac{c^2\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(car\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{ 3042}$$

3.265. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{2bc\sqrt{c^2dx^2+d}\left(-\frac{c^2\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 26

$$-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{ic^2\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 4201

$$-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}-\frac{(c^2x^2+1)(a+ba}{2x^2}\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 2620

$$-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}}\right.\right.\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 2715

$$-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3dx^3}+\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+barc\right.\right.}{b}\right.}{3}$$

$\downarrow$ 2838

---

3.265. $\int\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{3dx^3}+$$
$$\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\operatorname{arcsinh}(cx))-\frac{1}{2}b(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\operatorname{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\operatorname{arcs}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
-1/3*((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d*x^3) + (2*b*c*Sqrt[
d + c^2*d*x^2]*(-1/2*((1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/x^2 + (b*c*(-(Sq
rt[1 + c^2*x^2]/x) + c*ArcSinh[c*x]))/2 + (I*c^2*((-1/2*I)*(a + b*ArcSinh[
c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi -
(2*(a + b*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))
/b))/(3*Sqrt[1 + c^2*x^2])
```

### 3.265.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.265. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 6217
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1
 + c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

## 3.265.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(274) = 548$.

Time = 0.34 (sec) , antiderivative size = 1729, normalized size of antiderivative = 5.88

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1729 |
| parts | Expression too large to display | 1729 |

input
```
int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(2*arcsinh(c*x)*c^3*x^3-2*ln((c*x+(c^2*x^2+
1)^(1/2))^2-1)*x^3*c^3+2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)+c*x)/(c^2*x^2+1)^(1/2)/x^3-1/3*a^2/d/x^3*(c^2*d*x^2
+d)^(3/2)+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1
)^(1/2)*arcsinh(c*x)^2*c^5-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+
1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+b^2*(d*(c^2*x^2
+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^
7-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsin
h(c*x)^2*c^8-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^
2*x^2+1)*arcsinh(c*x)*c^8-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2
+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2*c^6-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^
4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-10/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-1/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c
^4-5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arc
sinh(c*x)^2*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2
*x^2+1)^(1/2)*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^
3*c^6+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polyl...
```

### 3.265.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/x^4, x)
```

### 3.265.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**4, x)
```

### 3.265.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxim
a")
```

output
```
-1/3*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^
(3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*a*b*c/d
- 1/3*b^2*((c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^
2*x^2 + 1))^2/x^3 - 3*integrate(2/3*((c^2*x^2 + 1)*c^2*sqrt(d)*x + (c^3*sq
rt(d)*x^2 + c*sqrt(d))*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c*
x^4 + sqrt(c^2*x^2 + 1)*x^3), x)) - 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcsinh(
c*x)/(d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)
```

### 3.265.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4, x)
```

# 3.266 $\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.266.1 Optimal result

Integrand size = 28, antiderivative size = 482

$$\begin{aligned}\int x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=&-\frac{304b^2d\sqrt{d+c^2dx^2}}{3675c^4}\\&+\frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{152b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{11025c^4}\\&-\frac{38b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{6125c^4}+\frac{2b^2d(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{343c^4}\\&+\frac{4b^2dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{35c^3\sqrt{1+c^2x^2}}-\frac{2bdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{105c\sqrt{1+c^2x^2}}\\&-\frac{16bcdx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{175\sqrt{1+c^2x^2}}-\frac{2bc^3dx^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}}\\&-\frac{2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^4}+\frac{dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{35c^2}\\&+\frac{3}{35}dx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{1}{7}x^4(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
1/7*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-304/3675*b^2*d*(c^2*d*x^2
+d)^(1/2)/c^4-152/11025*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-38/6125*
b^2*d*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4+2/343*b^2*d*(c^2*x^2+1)^3*(c^2
*d*x^2+d)^(1/2)/c^4-2/35*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/
35*d*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+3/35*d*x^4*(a+b*arcs
inh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/35*a*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*
x^2+1)^(1/2)+4/35*b^2*d*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)-2/105*b*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^
(1/2)-16/175*b*c*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)-2/49*b*c^3*d*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)
```

## 3.266.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.52

$$\int x^3 (d + c^2 d x^2)^{3/2} (a + b \operatorname{arcsinh}(cx))^2 \, dx = \frac{d\sqrt{d + c^2 d x^2}\Big(11025 a^2 (1 + c^2 x^2)^3 (-2 + 5c^2 x^2) - 210 abcx\sqrt{1 + c^2 x^2}(-210 + 35c^2 x^2 -$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(11025*a^2*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - 210*a
*b*c*x*Sqrt[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) +
2*b^2*(-18692 - 20371*c^2*x^2 + 499*c^4*x^4 + 3303*c^6*x^6 + 1125*c^8*x^8)
 - 210*b*(-105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2
]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2
*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^4*(1 + c^2*x^
2))
```

### 3.266.3 Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.12, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6223, 6218, 27, 354, 86, 2009, 6221, 6191, 243, 53, 2009, 6227, 6191, 243, 53, 2009, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c^2 d x^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^2 dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2bcd\sqrt{c^2dx^2+d}\int x^4 \left(c^2x^2+1\right) (a+b\text{arcsinh}(cx))dx}{7\sqrt{c^2x^2+1}}+\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6218}$$

$$\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\int \frac{x^5\left(5c^2x^2+7\right)}{35\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))\right)}{7\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{35}bc\int \frac{x^5\left(5c^2x^2+7\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))\right)}{7\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{354}$$

$$\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{70}bc\int \frac{x^4\left(5c^2x^2+7\right)}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))\right)}{7\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{86}$$

$$\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{70}bc\int\left(\frac{5(c^2x^2+1)^{5/2}}{c^4}-\frac{8(c^2x^2+1)^{3/2}}{c^4}+\frac{\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\right.}{7\sqrt{c^2x^2+1}}$$
$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 2009

$$\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 6221

$$\frac{3}{7}d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int x^4(a+b\text{arcsinh}(cx))dx}{5\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+bar\right.$$
$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 6191

$$\frac{3}{7}d\left(-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{5}bc\int\frac{x^5}{\sqrt{c^2x^2+1}}dx\right)}{5\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{}\right.$$
$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 243

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int\frac{x^4}{\sqrt{c^2x^2+1}}dx^2\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{}\right.$$
$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\right.\right.}{7\sqrt{c^2x^2+1}}$$

3.266. $\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$\downarrow$ 53

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int\left(\frac{(c^2x^2+1)^{3/2}}{c^4}-\frac{2\sqrt{c^2x^2\cdots}}{c^4}\cdots\right.\right.}{5\sqrt{c^2x^2+1}}\cdots\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\cdots}{3c\cdots}\cdots\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsi}\cdots\right.}{\cdots}\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\cdots}{3c\cdots}\cdots\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 6227

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\int x^2(a+b\text{arcsinh}(cx))dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx\cdots}\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\cdots}{3c\cdots}\cdots\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 6191

3.266. $\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\int\frac{x^3}{\sqrt{c^2x^2+1}}dx\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right.}{5\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^6}-\frac{16\left(c^2x^2+1\right)^{5/2}}{5c^6}+\frac{2(c^2x^2+}{3c}\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 243

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx^2\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right.}{5\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^6}-\frac{16\left(c^2x^2+1\right)^{5/2}}{5c^6}+\frac{2(c^2x^2+}{3c}\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 53

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int\left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+}{3c}\right.}{5\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^6}-\frac{16\left(c^2x^2+1\right)^{5/2}}{5c^6}+\frac{2(c^2x^2+}{3c}\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

3.266. $\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}\right.\right.}{3c}\right.}{5\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^6}-\frac{16\left(c^2x^2+1\right)^{5/2}}{5c^6}+\frac{2\left(c^2x^2+\right.}{3c}\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 6213

$$\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\int(a+b\text{arcsinh}(cx))dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arc}\right.}{}\right.}{5\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10\left(c^2x^2+1\right)^{7/2}}{7c^6}-\frac{16\left(c^2x^2+1\right)^{5/2}}{5c^6}+\frac{2\left(c^2x^2+\right.}{3c}\right.\right.}{7\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

3.266. $\int x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{70}bc\left(\frac{10(c^2x^2+1)^{7/2}}{7c^6}-\frac{16(c^2x^2+1)^{5/2}}{5c^6}+\frac{2(c^2x^2+\ldots}{3c^{\ldots}}\right.\right.}{7\sqrt{c^2x^2+1}}$$

$$\frac{3}{7}d\left(\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1\ldots}{3c^6}\right.\right.}{5\sqrt{c^2x^2+1}}\right.$$

input
```
Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x^4*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/7 - (2*b*c*d*Sqrt[d + c
^2*d*x^2]*(-1/70*(b*c*((4*Sqrt[1 + c^2*x^2])/c^6 + (2*(1 + c^2*x^2)^(3/2))
/(3*c^6) - (16*(1 + c^2*x^2)^(5/2))/(5*c^6) + (10*(1 + c^2*x^2)^(7/2))/(7*
c^6))) + (x^5*(a + b*ArcSinh[c*x]))/5 + (c^2*x^7*(a + b*ArcSinh[c*x]))/7))
/(7*Sqrt[1 + c^2*x^2]) + (3*d*((x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^2)/5 - (2*b*c*Sqrt[d + c^2*d*x^2]*(-1/10*(b*c*((2*Sqrt[1 + c^2*x^2])/c^
6 - (4*(1 + c^2*x^2)^(3/2))/(3*c^6) + (2*(1 + c^2*x^2)^(5/2))/(5*c^6))) +
(x^5*(a + b*ArcSinh[c*x]))/5))/(5*Sqrt[1 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2
]*((x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2) - (2*b*(-1/6*(b*
c*((-2*Sqrt[1 + c^2*x^2])/c^4 + (2*(1 + c^2*x^2)^(3/2))/(3*c^4))) + (x^3*(
a + b*ArcSinh[c*x]))/3))/(3*c) - (2*((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/c^2 - (2*b*(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/c))/(
3*c^2)))/(5*Sqrt[1 + c^2*x^2])))/7
```

### 3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.266. $\int x^3(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1765 vs. $2(420) = 840$.

Time = 0.39 (sec) , antiderivative size = 1766, normalized size of antiderivative = 3.66

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1766 |
| parts | Expression too large to display | 1766 |

input
```
int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b^2
*(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+1
44*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1
)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsin
h(c*x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c
^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^
2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*d/c^4/(
c^2*x^2+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(
1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)
+2)*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)
^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^4/(c^2*x^2+1)-3/128*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsin
h(c*x)+2)*d/c^4/(c^2*x^2+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*
x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)
^2+6*arcsinh(c*x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c
^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2
)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)
+2)*d/c^4/(c^2*x^2+1)+1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7
*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-5
6*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*a...
```

### 3.266.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.83

$$\int x^3 \left(d + c^2 d x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{11025 \left(5 b^2 c^8 d x^8 + 13 b^2 c^6 d x^6 + 9 b^2 c^4 d x^4 - b^2 c^2 d x^2 - 2 b^2 d\right) \sqrt{c^2 d x^2 + d} \log\left(cx + \sqrt{c^2\right.}{}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
1/385875*(11025*(5*b^2*c^8*d*x^8 + 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 - b^
2*c^2*d*x^2 - 2*b^2*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2
+ 210*(525*a*b*c^8*d*x^8 + 1365*a*b*c^6*d*x^6 + 945*a*b*c^4*d*x^4 - 105*a*
b*c^2*d*x^2 - 210*a*b*d - (75*b^2*c^7*d*x^7 + 168*b^2*c^5*d*x^5 + 35*b^2*c
^3*d*x^3 - 210*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 + 9*(15925*a^2 + 73
4*b^2)*c^6*d*x^6 + (99225*a^2 + 998*b^2)*c^4*d*x^4 - (11025*a^2 + 40742*b^
2)*c^2*d*x^2 - 2*(11025*a^2 + 18692*b^2)*d - 210*(75*a*b*c^7*d*x^7 + 168*a
*b*c^5*d*x^5 + 35*a*b*c^3*d*x^3 - 210*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c
^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

## 3.266.6 Sympy [F]

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx = \int x^3 \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

## 3.266.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.72

$$\int x^3 (d + c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d} - \frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)b^2\operatorname{arsinh}(cx)^2 + \frac{2}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d} - \frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)ab\operatorname{arsinh}(cx) + \frac{1}{35}\left(\frac{5\,(c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d} - \frac{2\,(c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)a^2 + \frac{2}{385875}b^2\left(\frac{1125\sqrt{c^2x^2+1}c^4d^{\frac{3}{2}}x^6 + 2178\sqrt{c^2x^2+1}c^2d^{\frac{3}{2}}x^4 - 1679\sqrt{c^2x^2+1}d^{\frac{3}{2}}x^2 - \frac{18692\sqrt{c^2x^2+1}d^{\frac{3}{2}}}{c^2}}{c^2} - \frac{10}{} \right. - \frac{2\left(75\,c^6d^{\frac{3}{2}}x^7 + 168\,c^4d^{\frac{3}{2}}x^5 + 35\,c^2d^{\frac{3}{2}}x^3 - 210\,d^{\frac{3}{2}}x\right)ab}{3675\,c^3}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d
))*b^2*arcsinh(c*x)^2 + 2/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2
*d*x^2 + d)^(5/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2
)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 + 2/385875*b^2*((1125
*sqrt(c^2*x^2 + 1)*c^4*d^(3/2)*x^6 + 2178*sqrt(c^2*x^2 + 1)*c^2*d^(3/2)*x^
4 - 1679*sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 - 18692*sqrt(c^2*x^2 + 1)*d^(3/2)/c
^2)/c^2 - 105*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x
^3 - 210*d^(3/2)*x)*arcsinh(c*x)/c^3) - 2/3675*(75*c^6*d^(3/2)*x^7 + 168*c
^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*a*b/c^3
```

### 3.266.8 Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.266.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx = \int x^3\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

# 3.267 $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.267.1 Optimal result

Integrand size = 28, antiderivative size = 405

$$\begin{aligned}\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}\\
&+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}\\
&+\frac{7b^2d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{1152c^3\sqrt{1+c^2x^2}}-\frac{bdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c\sqrt{1+c^2x^2}}\\
&-\frac{7bcdx^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{48\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{18\sqrt{1+c^2x^2}}\\
&+\frac{dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16c^2}+\frac{1}{8}dx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\\
&+\frac{1}{6}x^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{48bc^3\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-7/1152*b^2*d*x*(c^2*d*x^2
+d)^(1/2)/c^2+43/1728*b^2*d*x^3*(c^2*d*x^2+d)^(1/2)+1/108*b^2*c^2*d*x^5*(c
^2*d*x^2+d)^(1/2)+1/16*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/
8*d*x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*arcsinh(c*x)
*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/16*b*d*x^2*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/48*b*c*d*x^4*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/18*b*c^3*d*x^6*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/48*d*(a+b*arcsinh(c*x))^3*(c^2*d*
x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

### 3.267.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.25

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{864a^2cdx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+4032a^2c^3dx^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+2304a^2c^5dx^5\sqrt{}}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(864*a^2*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 4032*a^2*c^3*d*x^3*
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*d*x^5*Sqrt[1 + c^2*x^
2]*Sqrt[d + c^2*d*x^2] - 288*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 21
6*a*b*d*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 108*a*b*d*Sqrt[d + c^2*
d*x^2]*Cosh[4*ArcSinh[c*x]] - 24*a*b*d*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[
c*x]] - 864*a^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2
*d*x^2]] - 108*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 27*b^2*d*S
qrt[d + c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh
[6*ArcSinh[c*x]] + 12*b*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(18*b*Cosh[2*Ar
cSinh[c*x]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 36*a*S
inh[2*ArcSinh[c*x]] + 36*a*Sinh[4*ArcSinh[c*x]] + 12*a*Sinh[6*ArcSinh[c*x]
]) + 72*b*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-12*a - 3*b*Sinh[2*ArcSinh
[c*x]] + 3*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c^3*Sq
rt[1 + c^2*x^2])
```

### 3.267.3 Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.22, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {6223, 6218, 27, 363, 262, 262, 222, 6221, 6191, 262, 262, 222, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{ 6223}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\int x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6218}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-bc\int\frac{x^4\left(2c^2x^2+3\right)}{12\sqrt{c^2x^2+1}}dx+\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\int\frac{x^4\left(2c^2x^2+3\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\left(\frac{4}{3}\int\frac{x^4}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)+\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int\frac{x^2}{\sqrt{c^2x^2+1}}dx}{4c^2}\right)+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)+\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+\right.}{3\sqrt{c^2x^2+1}}$$
$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

---

3.267. $\int x^2(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{4c^2}\right)+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)+\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx)}{3\sqrt{c^2x^2+1}}$$
$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6221}$$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^3(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsi}\right.$$
$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\int\frac{x^4}{\sqrt{c^2x^2+1}}dx\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2}\right.$$
$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{262}$$

3.267. $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int \frac{x^2}{\sqrt{c^2x^2+1}}d}{4c^2}\right.\right.}{2\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 262

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}\right.}{}\right.\right.}{2\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 222

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsin}\right.}{}\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 6227

---

3.267. $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int x(a+b\text{arcsinh}(cx))dx}{c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 6191

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{c}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 262

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{c}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsi}}{2c^2}\right.}{4\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 222

3.267. $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsin}}{2c^3}\right.\right.}{c}\right.}{4\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2+\frac{\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcsinh}(cx))^3}{6bc^3}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(c\right.}{}\right.}{4\sqrt{c^2x^2+1}}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(c}{2c^3}\right.}{4c^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

input
```
Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/6 - (b*c*d*Sqrt[d + c^2
*d*x^2]*((x^4*(a + b*ArcSinh[c*x]))/4 + (c^2*x^6*(a + b*ArcSinh[c*x]))/6 -
 (b*c*((x^5*Sqrt[1 + c^2*x^2])/3 + (4*((x^3*Sqrt[1 + c^2*x^2])/(4*c^2) - (
3*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/(4*c^2)))/3))/12
))/(3*Sqrt[1 + c^2*x^2]) + (d*((x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^2)/4 - (b*c*Sqrt[d + c^2*d*x^2]*((x^4*(a + b*ArcSinh[c*x]))/4 - (b*c*((
x^3*Sqrt[1 + c^2*x^2])/(4*c^2) - (3*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSi
nh[c*x]/(2*c^3)))/(4*c^2)))/4))/(2*Sqrt[1 + c^2*x^2]) + (Sqrt[d + c^2*d*x^
2]*((x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*c^2) - (a + b*ArcSinh[
c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c
^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2))/c))/(4*Sqrt[1 + c^2*x^2])))/
2
```

3.267. $\int x^2(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.267.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.267. $\int x^2(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(351) = 702$.

Time = 0.36 (sec) , antiderivative size = 1552, normalized size of antiderivative = 3.83

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1552 |
| parts | Expression too large to display | 1552 |

input
```
int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a^2/c^2*x*(c^2*d*x^2+d)^(3/2)-1/1
6*a^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a^2/c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2
)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-1/48*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3*d+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^
7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*
c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(18*arcsinh(
c*x)^2-6*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8
*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2
)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d/c^3/(c^2*
x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+
2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d/c^3/(c^2*x^
2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*
c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+
1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*
c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*
x)^2+4*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*
c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/
2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(18*ar
csinh(c*x)^2+6*arcsinh(c*x)+1)*d/c^3/(c^2*x^2+1))+2*a*b*(-1/32*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d+1/2304*(d*(c^2*x^2+1)...
```

### 3.267.5 Fricas [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^2*d*x^4 + a^2*d*x^2 + (b^2*c^2*d*x^4 + b^2*d*x^2)*arcsinh(
c*x)^2 + 2*(a*b*c^2*d*x^4 + a*b*d*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d),
x)
```

### 3.267.6 Sympy [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \int x^2 \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\operatorname{asinh}(cx))^2 \, dx$$

input
```
integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

### 3.267.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.267.8 Giac [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}} (b\operatorname{arsinh}(cx)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

### 3.267.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 d x^2)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \int x^2 \left(a + b\,\text{asinh}(c\,x)\right)^2 \left(d\,c^2\,x^2 + d\right)^{3/2} \, dx$$

input
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

# 3.268 $\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.268.1 Optimal result

Integrand size = 26, antiderivative size = 267

$$\begin{aligned}\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{16b^2d\sqrt{d+c^2dx^2}}{75c^2}\\ &+\frac{8b^2d(1+c^2x^2)\sqrt{d+c^2dx^2}}{225c^2}+\frac{2b^2d(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{125c^2}\\ &-\frac{2bdx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}}\\ &-\frac{2bc^3dx^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d}\end{aligned}$$

output
```
1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/c^2/d+16/75*b^2*d*(c^2*d*x^2+
d)^(1/2)/c^2+8/225*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+2/125*b^2*d*(
c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*x*(a+b*arcsinh(c*x))*(c^2*d*x
^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-4/15*b*c*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.268.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.74

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{d\sqrt{d+c^2dx^2}\left(225a^2(1+c^2x^2)^3-30abcx\sqrt{1+c^2x^2}(15+10c^2x^2+3c^4x^4)+2b^2(149+\right.}{}$$

input
```
Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(225*a^2*(1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[1 + c^2*
x^2]*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 + 187*c^2*x^2 + 47*c^4*x^4
 + 9*c^6*x^6) + 30*b*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15 +
 10*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] + 225*b^2*(1 + c^2*x^2)^3*ArcSinh[c
*x]^2))/(1125*c^2*(1 + c^2*x^2))
```

### 3.268.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6213, 6199, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \; 6213$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d}-\frac{2bd\sqrt{c^2dx^2+d}\int\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))dx}{5c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6199$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{5c^2d}-\frac{2bd\sqrt{c^2dx^2+d}\left(-bc\int\frac{x(3c^4x^4+10c^2x^2+15)}{15\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\operatorname{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\operatorname{arcsinh}(cx))+x(a+b\operatorname{arcsin}\right.}{5c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d}-$$
$$\frac{2bd\sqrt{c^2dx^2+d}\left(-\frac{1}{15}bc\int\frac{x(3c^4x^4+10c^2x^2+15)}{\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arc}\right.}{5c\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 1576

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d}-$$
$$\frac{2bd\sqrt{c^2dx^2+d}\left(-\frac{1}{30}bc\int\frac{3c^4x^4+10c^2x^2+15}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsi}\right.}{5c\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 1140

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d}-$$
$$\frac{2bd\sqrt{c^2dx^2+d}\left(-\frac{1}{30}bc\int\left(3\left(c^2x^2+1\right)^{3/2}+4\sqrt{c^2x^2+1}+\frac{8}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\right.}{5c\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 2009

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2d}-$$
$$\frac{2bd\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6(c^2x^2+1)^{5/2}}{5c^2}+\right.\right.}{5c\sqrt{c^2x^2+1}}$$

input
```
Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*c^2*d) - (2*b*d*Sqrt[d +
 c^2*d*x^2]*(-1/30*(b*c*((16*Sqrt[1 + c^2*x^2])/c^2 + (8*(1 + c^2*x^2)^(3/
2))/(3*c^2) + (6*(1 + c^2*x^2)^(5/2))/(5*c^2))) + x*(a + b*ArcSinh[c*x]) +
 (2*c^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*x^5*(a + b*ArcSinh[c*x]))/5))/(
5*c*Sqrt[1 + c^2*x^2])
```

3.268. $\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.268.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.268.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. 2(233) = 466.

Time = 0.37 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.30

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1149 |
| parts | Expression too large to display | 1149 |

input
```
int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*(c^2*d*x^2+d)^(5/2)/c^2/d+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)
+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+
2)*d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arc
sinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^
2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/16
*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2
*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-
4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsin
h(c*x)^2+6*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*
(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)
^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh
(c*x)+2)*d/c^2/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6
+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c
^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/
96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+
3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(
c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^
2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)...
```

### 3.268.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.24

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\frac{225\left(b^2c^6dx^6+3\,b^2c^4dx^4+3\,b^2c^2dx^2+b^2d\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\,(15$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
1/1125*(225*(b^2*c^6*d*x^6 + 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 + b^2*d)*sq
rt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(15*a*b*c^6*d*x^6 +
45*a*b*c^4*d*x^4 + 45*a*b*c^2*d*x^2 + 15*a*b*d - (3*b^2*c^5*d*x^5 + 10*b^2
*c^3*d*x^3 + 15*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (9*(25*a^2 + 2*b^2)*c^6*d*x^6 + (675*a^2 + 94*b^2)*
c^4*d*x^4 + (675*a^2 + 374*b^2)*c^2*d*x^2 + (225*a^2 + 298*b^2)*d - 30*(3*
a*b*c^5*d*x^5 + 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c
^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

### 3.268.6 Sympy [F]

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

input
```
integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

### 3.268.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.86

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}b^2\operatorname{arsinh}\left(cx\right)^2}{5\,c^2d}$$
$$+\frac{2}{1125}\,b^2\left(\frac{9\sqrt{c^2x^2+1}c^2d^{\frac{5}{2}}x^4+38\sqrt{c^2x^2+1}d^{\frac{5}{2}}x^2+\frac{149\sqrt{c^2x^2+1}d^{\frac{5}{2}}}{c^2}}{d}-\frac{15\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)\text{a}}{cd}\right.$$
$$+\frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}ab\operatorname{arsinh}\left(cx\right)}{5\,c^2d}+\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}a^2}{5\,c^2d}-\frac{2\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)ab}{75\,cd}$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
1/5*(c^2*d*x^2 + d)^(5/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/1125*b^2*((9*sqrt
(c^2*x^2 + 1)*c^2*d^(5/2)*x^4 + 38*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 + 149*sqr
t(c^2*x^2 + 1)*d^(5/2)/c^2)/d - 15*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3
 + 15*d^(5/2)*x)*arcsinh(c*x)/(c*d)) + 2/5*(c^2*d*x^2 + d)^(5/2)*a*b*arcsi
nh(c*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2)*a^2/(c^2*d) - 2/75*(3*c^4*d^(5
/2)*x^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*a*b/(c*d)
```

## 3.268.8 Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## 3.268.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int x\,(a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

# 3.269 $\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx$



## 3.269.1 Optimal result

Integrand size = 25, antiderivative size = 294

$$\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{15}{64}b^2dx\sqrt{d+c^2dx^2} + \frac{1}{32}b^2dx(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{9b^2d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} - \frac{3bcdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{8bc\sqrt{1+c^2x^2}}$$

output
```
1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+15/64*b^2*d*x*(c^2*d*x^2+d)
^(1/2)+1/32*b^2*d*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*d*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)-9/64*b^2*d*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2
+1)^(1/2)-3/8*b*c*d*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+1/8*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2
)
```

### 3.269.2 Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{96a^2cdx\sqrt{1+c^2x^2}(5+2c^2x^2)\sqrt{d+c^2dx^2}+288a^2d^{3/2}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(96*a^2*c*d*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 288*
a^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 3
2*b^2*d*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcS
inh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 192*a*b*d*Sqrt[
d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh
[2*ArcSinh[c*x]])) - 12*a*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh
[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - b^2*d*Sqrt[d + c
^2*d*x^2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1
 + 8*ArcSinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/(768*c*Sqrt[1 + c^2*x^2])
```

### 3.269.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \; 6201$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\int\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \; 6200$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+$$
$$\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+$$
$$\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2d\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+$$
$$\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}}{2\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{222}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+$$
$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6198}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx)\right.}{\sqrt{c^2x^2}}\right.$$

$$\Big\downarrow \text{6213}$$

3.269. $\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int(c^2x^2+1)^{3/2}dx}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx)\right.}{\sqrt{c^2x^2}}\right.$$

$$\Big\downarrow \text{211}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx)\right.}{\sqrt{c^2x^2}}\right.$$

$$\Big\downarrow \text{211}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx)\right.}{\sqrt{c^2x^2}}\right.$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx)\right.}{\sqrt{c^2x^2}}\right.$$

input
```
Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

## 3.269. $\int\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*((x*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^3)/(6*b*c*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[d + c^2*d*x^2]*((x^2*(a + b*A
rcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^
3)))/2))/Sqrt[1 + c^2*x^2]))/4 - (b*c*d*Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x
*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 + c^2*x
^2])
```

### 3.269.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.269.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(254) = 508$.

Time = 0.26 (sec) , antiderivative size = 959, normalized size of antiderivative = 3.26

| method | result |
|---|---|
| default | $\frac{x\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} a^{2}}{4}+\frac{3 a^{2} d x \sqrt{c^{2} d\,x^{2}+d}}{8}+\frac{3 a^{2} d^{2} \ln\left(\frac{c^{2} d x}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{8 \sqrt{c^{2} d}}+b^{2}\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}(c x)^{3} d}{8 \sqrt{c^{2} x^{2}+1}\, c}+\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\,(8\right.$ |
| parts | $\frac{x\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} a^{2}}{4}+\frac{3 a^{2} d x \sqrt{c^{2} d\,x^{2}+d}}{8}+\frac{3 a^{2} d^{2} \ln\left(\frac{c^{2} d x}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{8 \sqrt{c^{2} d}}+b^{2}\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}(c x)^{3} d}{8 \sqrt{c^{2} x^{2}+1}\, c}+\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\,(8\right.$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2*
ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/8*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^3*d+1/512*(d*(c^2*x^2+1)
)^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x
^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d
/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)
^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d/c/(c
^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2
)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d/c/(c^2*x^
2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12
*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c
*x)^2+4*arcsinh(c*x)+1)*d/c/(c^2*x^2+1))+2*a*b*(3/16*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x
^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*
x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+
1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*
(-1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-
2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))*d/
c/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)
^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*...
```

## 3.269.5 Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

### 3.269.6 Sympy [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

### 3.269.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.269.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.269.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

# 3.270 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{x}\,dx$



## 3.270.1 Optimal result

Integrand size = 28, antiderivative size = 498

$$
\begin{aligned}
\int \frac{(d+c^2dx^2)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{x}\,dx &= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} \\
&- \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} \\
&+ \frac{2}{27}b^2d(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\,\mathrm{arcsinh}(cx)}{\sqrt{1+c^2x^2}} \\
&- \frac{2bcdx\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+ d\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))^2 + \frac{1}{3}(d+c^2dx^2)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2 \\
&- \frac{2d\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))^2\,\mathrm{arctanh}\left(e^{\mathrm{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&- \frac{2bd\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&+ \frac{2bd\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&+ \frac{2b^2d\sqrt{d+c^2dx^2}\,\mathrm{PolyLog}\left(3,-e^{\mathrm{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\
&- \frac{2b^2d\sqrt{d+c^2dx^2}\,\mathrm{PolyLog}\left(3,e^{\mathrm{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

output
```
1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+22/9*b^2*d*(c^2*d*x^2+d)^(1/2
)+2/27*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+d*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)-2*a*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*d*
x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/3*b*c*d*x*(a+b*arcs
inh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^3*d*x^3*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))^2*ar
ctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d*(
a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)+2*b*d*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b^2*d*polylog(3,-c*x-(c^2*x^2+1)^(
1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*d*polylog(3,c*x+(c^2*x^2
+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.270.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.04

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{1}{3}a^2d(4+c^2x^2)\sqrt{d+c^2dx^2} - \frac{2abd\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{9\sqrt{1+c^2x^2}} + a^2d^{3/2}\log(cx) - a^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{2abd\sqrt{d+c^2dx^2}\left(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}\right.}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

output
```
(a^2*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 - (2*a*b*d*Sqrt[d + c^2*d*x^2]
*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x
^2]) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] + (2*a*b*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*
x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-Ar
cSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x
])]))/Sqrt[1 + c^2*x^2] + (b^2*d*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 + c^2*x^2]
- 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh[c*x]^2*(
Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]) + 2*ArcSinh[c*x]*
(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]) + 2*(Poly
Log[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])])))/Sqrt[1 + c^2
*x^2] + (b^2*d*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]
^2) + (2 + 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x
+ Sinh[3*ArcSinh[c*x]])))/(108*Sqrt[1 + c^2*x^2])
```

## 3.270.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.70, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {6223, 6199, 27, 353, 53, 2009, 6221, 2009, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2bcd\sqrt{c^2dx^2+d}\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6199}$$

$$d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\int \frac{x\left(c^2x^2+3\right)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{3}bc\int \frac{x(c^2x^2+3)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{353}$$

$$d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int \frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{53}$$

$$d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int \left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))\right)}{3\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6221}$$

$$d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

---

3.270. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)-\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 6231

$$d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsi}\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 3042

$$d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2}}{}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 26

$$d\left(\frac{i\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2}}{}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$\downarrow$ 4670

3.270. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d\left(\frac{i\sqrt{c^2dx^2+d}\left(2ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int(a+b\text{arcsinh}(cx))\log\left(1+e^{\text{arcsi}}\right.\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3011}$$

$$d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)\right.}{}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2720}$$

$$d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcs}\right.\right.}{}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{7143}$$

$$d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-e^{\text{arcs}}\right.\right.\right.}{\sqrt{c^2x^2+}}\right.$$

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)\right)}{3\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

## 3.270. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

output
```
((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*d*Sqrt[d + c^2*d
*x^2]*(-1/6*(b*c*((4*Sqrt[1 + c^2*x^2])/c^2 + (2*(1 + c^2*x^2)^(3/2))/(3*c
^2))) + x*(a + b*ArcSinh[c*x]) + (c^2*x^3*(a + b*ArcSinh[c*x]))/3))/(3*Sqr
t[1 + c^2*x^2]) + d*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*b*c*S
qrt[d + c^2*d*x^2]*(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/Sqr
t[1 + c^2*x^2] + (I*Sqrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])^2*ArcT
anh[E^ArcSinh[c*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSin
h[c*x]]) + b*PolyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSinh[c*x]
)*PolyLog[2, E^ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/Sqrt[1 + c
^2*x^2])
```

### 3.270.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.270. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

## 3.270. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.270.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(491) = 982$.

Time = 0.35 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.11

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1053 |
| parts | Expression too large to display | 1053 |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c^2*d*x^2+d)^(3/2)*a^2+a^2*d*(c^2*d*x^2+d)^(1/2)-8/3*a*b*(d*(c^2*x^2+
1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c*x-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d+2*a*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-2/9*a*b*(d
*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c^3*x^3-a^2*d^(3/2)*ln((2*d+2*d^(1
/2)*(c^2*d*x^2+d)^(1/2))/x)-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*d+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-2*b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))*d+2/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*c^4*x^4+70/27*b^2*(d*(c^
2*x^2+1))^(1/2)*d/(c^2*x^2+1)*x^2*c^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d-2*b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*d+2/3*a*b*(
d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4+10/3*a*b*(d*(c^2*x
^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+2*a*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d-2*a*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d+8/3*a*b*(d*
(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)+68/27*b^2*(d*(c^2*x^2+1))^(1
/2)*d/(c^2*x^2+1)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,
-c*x-(c^2*x^2+1)^(1/2))*d+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*a...
```

### 3.270.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas"
)
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

### 3.270.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))^2}{x}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x, x)
```

### 3.270.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima"
)
```

output
```
-1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2
*d*x^2 + d)*d)*a^2 + integrate((c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^
2*x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))
/x, x)
```

### 3.270.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{3/2}}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x, x)
```

# 3.271 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$



## 3.271.1 Optimal result

Integrand size = 28, antiderivative size = 398

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx &= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}\\
&-\frac{5b^2cd\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\
&+bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\
&+\frac{3}{2}c^2dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}}\\
&-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+\frac{cd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{2b\sqrt{1+c^2x^2}}\\
&+\frac{2bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\\
&-\frac{b^2cd\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x+1/4*b^2*c^2*d*x*(c^2*d*x^2+d)^
(1/2)+3/2*c^2*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*arc
sinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b*c^3*d*x^2*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+c*d*(a+b*arcsinh(c*x))^2*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*c*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^
2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2
*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*d*polylog(2,
1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d*(
a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

### 3.271.2 Mathematica [A] (verified)

Time = 3.50 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.93

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \frac{12a^2d(-2+c^2x^2)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+24abd\sqrt{d+c^2dx^2}(-2}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
(12*a^2*d*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 24*a*b*d*
Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^
2 + 2*c*x*Log[c*x]) + 36*a^2*c*d^(3/2)*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqr
t[d]*Sqrt[d + c^2*d*x^2]] - 8*b^2*d*Sqrt[d + c^2*d*x^2]*(ArcSinh[c*x]*(3*S
qrt[1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]*(3 + ArcSinh[c*x]) - 6*c*
x*Log[1 - E^(-2*ArcSinh[c*x])]) + 3*c*x*PolyLog[2, E^(-2*ArcSinh[c*x])]) +
 b^2*c*d*x*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*A
rcSinh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 6*a*b*c*d*x*
Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] +
 Sinh[2*ArcSinh[c*x]])))/(24*x*Sqrt[1 + c^2*x^2])
```

### 3.271.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.96, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {6222, 6200, 6191, 262, 222, 6198, 6216, 211, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{ 6222}$$

$$3c^2d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{2bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{ 6200}$$

$$3c^2d\left(-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsin}\right.$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{ 6191}$$

$$3c^2d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2\right.$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{ 262}$$

$$3c^2d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}}{2\sqrt{c^2x^2+1}}\right.$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{ 222}$$

---

3.271. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$$\frac{2bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(\right.}{\sqrt{c}}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$\downarrow$ 6198

$$\frac{2bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$
$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x}}\right.$$

$\downarrow$ 6216

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int \sqrt{c^2x^2+1}dx+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))\right)}{\sqrt{c^2x^2+1}}-$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$
$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x}}\right.$$

$\downarrow$ 211

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))\right)}{\sqrt{c^2x^2+1}}-$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$
$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x}}\right.$$

$\downarrow$ 222

---

3.271. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$\downarrow$ 6190

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$\downarrow$ 25

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$\downarrow$ 3042

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

## 3.271. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$\Big\downarrow$ 26

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{i\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x}}\right.$$

$\Big\downarrow$ 4201

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{2}\left(c^2x^2+1\right)\left(\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x}}\right.$$

$\Big\downarrow$ 2620

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}}\right.\right.\right.}{b}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x}}\right.$$

$\Big\downarrow$ 2715

3.271. $\displaystyle\int\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$2bcd\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+b\text{arc}\right.\right.}{b}\right.$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$$\Big\downarrow \text{2838}$$

$$2bcd\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arcsi}\right.}{b}\right.$$

$$\sqrt{c^2x^2+1}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
-(((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x) + 3*c^2*d*((x*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[d + c^2*d*x^2]*((x^2*(a +
 b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(
2*c^3)))/2))/Sqrt[1 + c^2*x^2]) + (2*b*c*d*Sqrt[d + c^2*d*x^2]*(((1 + c^2*
x^2)*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x
]/(2*c)))/2 + (I*((-1/2*I)*(a + b*ArcSinh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*
ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)]) +
(b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/b))/Sqrt[1 + c^2*x^2]
```

## 3.271. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

### 3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

## 3.271. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## 3.271. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

### 3.271.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| default | $-\frac{a^2(c^2dx^2+d)^{\frac{5}{2}}}{dx}+a^2c^2x(c^2dx^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2dx^2+d}\,a^2c^2dx}{2}+\frac{3a^2c^2d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,(}{}$ |
| parts | $-\frac{a^2(c^2dx^2+d)^{\frac{5}{2}}}{dx}+a^2c^2x(c^2dx^2+d)^{\frac{3}{2}}+\frac{3\sqrt{c^2dx^2+d}\,a^2c^2dx}{2}+\frac{3a^2c^2d^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,(}{}$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(c^2*d*x^2+d)^(5/2)+a^2*c^2*x*(c^2*d*x^2+d)^(3/2)+3/2*(c^2*d*x^2+
d)^(1/2)*a^2*c^2*d*x+3/2*a^2*c^2*d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d
)^(1/2))/(c^2*d)^(1/2)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(
2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2*c^2-2*arcsinh(c*x)*c^3*x^3+2*arcsin
h(c*x)^3*x*c+c^2*x^2*(c^2*x^2+1)^(1/2)-4*arcsinh(c*x)^2*x*c+8*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))*x*c+8*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2)
)*x*c-4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-arcsinh(c*x)*c*x+8*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))*x*c+8*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x*c)*d+1/4*a*b*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(4*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^2*c^2-2*c^3*x^3+6*arcsinh(c*x)^2*x*c-8*arcsinh(c*x)*c*x+8*ln((c*x+(c^2
*x^2+1)^(1/2))^2-1)*x*c-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)*d
```

### 3.271.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

### 3.271.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^2}{x^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**2, x)
```

### 3.271.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.271.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

# 3.272 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$



## 3.272.1 Optimal result

Integrand size = 28, antiderivative size = 541

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx &= 2b^2c^2d\sqrt{d+c^2dx^2} \\ &- \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}} \\ &- \frac{bcd\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}} \\ &+ \frac{3}{2}c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2} \\ &- \frac{3c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &- \frac{b^2c^2d\sqrt{d+c^2dx^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}} \\ &- \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &+ \frac{3bc^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &+ \frac{3b^2c^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}} \\ &- \frac{3b^2c^2d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2+2*b^2*c^2*d*(c^2*d*x^2+d
)^(1/2)+3/2*c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-3*a*b*c^3*d*x*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b^2*c^3*d*x*arcsinh(c*x)*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x
/(c^2*x^2+1)^(1/2)+b*c^3*d*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x
^2+1)^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*d*arctanh((c^2*x^2+1)^(1/2))*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b*c^2*d*(a+b*arcsinh(c*x))*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3*b*c^2*d*
(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)+3*b^2*c^2*d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-3*b^2*c^2*d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.272.2 Mathematica [A] (verified)

Time = 7.79 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.43

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \left(a^2c^2d-\frac{a^2d}{2x^2}\right)\sqrt{d(1+c^2x^2)}+\frac{3}{2}a^2c^2d^{3/2}\log(x)-\frac{3}{2}a^2c^2d^{3/2}\log\Big($$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
(a^2*c^2*d - (a^2*d)/(2*x^2))*Sqrt[d*(1 + c^2*x^2)] + (3*a^2*c^2*d^(3/2)*L
og[x])/2 - (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/2 +
(2*a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x
] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x]
)]))/Sqrt[1 + c^2*x^2] + b^2*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(2 - (2*c*x*ArcSi
nh[c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[c*x]^2*(Log[1 - E^(
-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*ArcS
inh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))
/Sqrt[1 + c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-A
rcSinh[c*x])]))/Sqrt[1 + c^2*x^2]) + (a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-2*
Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x
]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] +
 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcS
inh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2
*x^2]) + (b^2*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*
x]/2] - ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E
^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Ta
nh[ArcSinh[c*x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*Ar
cSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*...
```

### 3.272.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.64, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6222, 6218, 25, 354, 90, 73, 221, 6221, 2009, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{6222}$$

$$\frac{3}{2}c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x^2}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6218}$$

$$\frac{3}{2}c^2 d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\left(-bc\int -\frac{1-c^2x^2}{x\sqrt{c^2x^2+1}}dx+c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{3}{2}c^2 d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\left(bc\int \frac{1-c^2x^2}{x\sqrt{c^2x^2+1}}dx+c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 354$$

$$\frac{3}{2}c^2 d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{2}bc\int \frac{1-c^2x^2}{x^2\sqrt{c^2x^2+1}}dx^2+c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 90$$

$$\frac{3}{2}c^2 d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{2}bc\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-2\sqrt{c^2x^2+1}\right)+c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 73$$

$$\frac{3}{2}c^2 d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{2}bc\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}-2\sqrt{c^2x^2+1}\right)+c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \ 221$$

---

3.272. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}dx$

$$\frac{3}{2}c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 6221}$$

$$\frac{3}{2}c^2d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(\right.$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{2}c^2d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)\right.}{\sqrt{c^2x^2+1}}\right.$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 6231}$$

$$\frac{3}{2}c^2d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bxa\right.}{\sqrt{c^2x^2}}\right.$$
$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

3.272. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$\frac{3}{2}c^2d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc}{}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc}{}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{4670}$$

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\left(2ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int(a+b\text{arcsinh}(cx))\log\left(1+e\right.\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx\right.\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2720}$$

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3.272. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \; 7143$$

$$\frac{3}{2}c^2d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-e\right.\right.\right.}{\sqrt{c^2}}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\left(c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-2\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}}-$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
-1/2*((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2 + (b*c*d*Sqrt[d +
c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/x) + c^2*x*(a + b*ArcSinh[c*x]) + (b*c*
(-2*Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/2))/Sqrt[1 + c^2*x^
2] + (3*c^2*d*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*b*c*Sqrt[d
+ c^2*d*x^2]*(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/Sqrt[1 +
c^2*x^2] + (I*Sqrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x])^2*ArcTanh[E^
ArcSinh[c*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]
]) + b*PolyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSinh[c*x])*Poly
Log[2, E^ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2
]))/2
```

### 3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

## 3.272. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.272. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.272.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| default | $a^2\left(-\frac{(c^2 d x^2+d)^{\frac{5}{2}}}{2d x^2}+\frac{3c^2\left(\frac{(c^2 d x^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2 d x^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right)\right)\right)}{2}\right)-\frac{3b^2\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(3,cx\right.}{\sqrt{c^2x^2+1}}$ |
| parts | $a^2\left(-\frac{(c^2 d x^2+d)^{\frac{5}{2}}}{2d x^2}+\frac{3c^2\left(\frac{(c^2 d x^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2 d x^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2 d x^2+d}}{x}\right)\right)\right)}{2}\right)-\frac{3b^2\sqrt{d(c^2x^2+1)}\ \mathrm{polylog}\left(3,cx\right.}{\sqrt{c^2x^2+1}}$ |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/d/x^2*(c^2*d*x^2+d)^(5/2)+3/2*c^2*(1/3*(c^2*d*x^2+d)^(3/2)+d*((c
^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x))))-3*b
^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2)
)*c^2*d+3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*
x^2+1)^(1/2))*c^2*d+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*x^2+1/2*
b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)^2-1/2*b^2*arcsinh
(c*x)^2*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)-2*b^2*(d*(c^2*x^2+1))^(1/2
)*c^3*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^
(1/2)*d/x/(c^2*x^2+1)^(1/2)*c+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1
)+b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^2-2*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2*d+
3/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c
^2*x^2+1)^(1/2))*c^2*d-3/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d-3*b^2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d+3*b
^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2
*x^2+1)^(1/2))*c^2*d+a*b*(d*(c^2*x^2+1))^(1/2)*(2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*x^2*c^2+3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-3*arcsin
h(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-2*c^3*x^3+3*polylog(2,c*x+(c^2*
x^2+1)^(1/2))*x^2*c^2-3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-arcsi...
```

### 3.272.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="frica
s")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

### 3.272.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^2}{x^3}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**3,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**3, x)
```

### 3.272.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxim
a")
```

output
```
-1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*
sqrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a^2 + integrate
((c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x
^2 + d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

### 3.272.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x^3} \,d x$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3, x)
```

# 3.273 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4} dx$



## 3.273.1 Optimal result

Integrand size = 28, antiderivative size = 378

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx &= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x}\\
&+\frac{b^2c^3d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{3\sqrt{1+c^2x^2}}-\frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2}\\
&-\frac{c^2d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{x}+\frac{4c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}}\\
&-\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}+\frac{c^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{3b\sqrt{1+c^2x^2}}\\
&+\frac{8bc^3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}\\
&-\frac{4b^2c^3d\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3-1/3*b^2*c^2*d*(c^2*d*x^2
+d)^(1/2)/x-c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+1/3*b^2*c^3*d
*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+4/3*c^3*d*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c^3*d*(a+b*arcsinh(c*x)
)^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+8/3*b*c^3*d*(a+b*arcsinh(c*x))
*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-4
/3*b^2*c^3*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+
d)^(1/2)/x^2
```

### 3.273.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.21

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x^4}\,dx = \frac{-abcdx\sqrt{d+c^2dx^2}-a^2d\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}-4a^2c^2dx^2\sqrt{1+}}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
(-(a*b*c*d*x*Sqrt[d + c^2*d*x^2]) - a^2*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d
*x^2] - 4*a^2*c^2*d*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - b^2*c^2*d*
x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + b*d*Sqrt[d + c^2*d*x^2]*(3*a*c
^3*x^3 - b*(-4*c^3*x^3 + Sqrt[1 + c^2*x^2] + 4*c^2*x^2*Sqrt[1 + c^2*x^2]))
*ArcSinh[c*x]^2 + b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + b*d*S
qrt[d + c^2*d*x^2]*ArcSinh[c*x]*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2]*(1 + 4*c
^2*x^2) + 8*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]) + 8*a*b*c^3*d*x^3*Sqrt
[d + c^2*d*x^2]*Log[c*x] + 3*a^2*c^3*d^(3/2)*x^3*Sqrt[1 + c^2*x^2]*Log[c*d
*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 4*b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*Po
lyLog[2, E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[1 + c^2*x^2])
```

### 3.273.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.58 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.10, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6222, 6217, 247, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838, 6220, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{6222}$$

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)(a+b\operatorname{arcsinh}(cx))}{x^3}dx}{3\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6217}$$

$$c^2d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{c^2dx^2+d}\left(c^2\int\frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}bc\int\frac{\sqrt{c^2x^2+1}}{x^2}dx-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}\right)}{3\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{247}$$

$$c^2d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{c^2dx^2+d}\left(c^2\int\frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{\sqrt{c^2x^2+1}}dx-\frac{\sqrt{c^2x^2+1}}{x}\right)-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}\right)}{3\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{222}$$

$$c^2d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{c^2dx^2+d}\left(c^2\int\frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(c\,\text{arcsinh}(cx)-\frac{\sqrt{c^2x^2+1}}{x}\right)\right)}{3\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6190}$$

$$c^2d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{c^2\int-\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc(\right.}{3\sqrt{c^2x^2+1}}-\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{25}$$

---

3.273. $\int\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{c^2\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(ca\right.\right.}{3\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$
$$\Big\downarrow \ 3042$$
$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{c^2\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\right.}{3\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$
$$\Big\downarrow \ 26$$
$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}-\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{2x^2}+\right.}{3\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$
$$\Big\downarrow \ 4201$$
$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+$$
$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}-\frac{(c^2x^2+1)(a+b}{2x}\right.}{3\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$
$$\Big\downarrow \ 2620$$

3.273. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx}{b}}\right.\right.\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2715

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+ba\right.\right.}{b}\right.}{3}$$

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2838

$$c^2d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6220

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2bc\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}\right)$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6190

3.273. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2c\sqrt{c^2dx^2+d}\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 25

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{2c\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+ba}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 3042

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}-\frac{2c\sqrt{c^2dx^2+d}\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 26

3.273. $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx}{b}\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 4201

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsi}\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2620

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcs}\right.\right.}{}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2715

3.273. $\int\frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

$$c^2d\left(\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right.\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2838

$$c^2d\left(\frac{c^2\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arc}\right.\right.}{\sqrt{c^2}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6198

$$c^2d\left(\frac{2ic\sqrt{c^2dx^2+d}\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}}\right.\right.\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{ic^2\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arc}\right.}{b}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

---

3.273. $\int\frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

output
```
-1/3*((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3 + (2*b*c*d*Sqrt[d
+ c^2*d*x^2]*(-1/2*((1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/x^2 + (b*c*(-(Sqrt
[1 + c^2*x^2]/x) + c*ArcSinh[c*x]))/2 + (I*c^2*((-1/2*I)*(a + b*ArcSinh[c*
x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2
*(a + b*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/b
))/(3*Sqrt[1 + c^2*x^2]) + c^2*d*(-((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/x) + (c*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*Sqrt[1 + c
^2*x^2]) + ((2*I)*c*Sqrt[d + c^2*d*x^2]*((-1/2*I)*(a + b*ArcSinh[c*x])^2 +
 (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b
*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/Sqrt[1 +
 c^2*x^2])
```

### 3.273.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

## 3.273. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6217
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1
 + c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

---

3.273. $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 6220
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/Sqrt[1 + c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x
], x] - Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]
Int[(f*x)^(m + 2)*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

### 3.273.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1928 vs. 2(350) = 700.

Time = 0.37 (sec) , antiderivative size = 1929, normalized size of antiderivative = 5.10

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1929 |
| parts | Expression too large to display | 1929 |

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^3*(3*arcsinh(c*x)^2*x^3*
c^3-8*arcsinh(c*x)*c^3*x^3+8*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-8*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*x)*
d+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1
)^(1/2))*c^3*d+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x
)^3*c^3*d+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*c^3*d-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)^2*c^3*d+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1
)/(c^2*x^2+1)^(1/2)*c^3-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*
x^2+1)*x^3*c^6-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(5/2)+2/3*a^2*c^4*x*(c^2*d*x^2+
d)^(3/2)+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1
-c*x-(c^2*x^2+1)^(1/2))*c^3*d-29/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-10/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4
*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*c^4+16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c
^4*x^4+9*c^2*x^2+1)*x^3*arcsinh(c*x)*c^6+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(
24*c^4*x^4+9*c^2*x^2+1)*x*arcsinh(c*x)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d
/(24*c^4*x^4+9*c^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*
d/(24*c^4*x^4+9*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2-20/3*b^2*(d*(c^2
*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+8*b^2*(d*(c^
2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+3*...
```

### 3.273.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="frica
s")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

### 3.273.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**4,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**4, x)
```

### 3.273.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.273.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

# 3.274 $\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.274.1 Optimal result

Integrand size = 28, antiderivative size = 625

$$\begin{aligned}
\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = & -\frac{160b^2d^2\sqrt{d+c^2dx^2}}{3969c^4} \\
& +\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{80b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{11907c^4} \\
& -\frac{4b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1323c^4}-\frac{50b^2d^2(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{27783c^4} \\
& +\frac{2b^2d^2(1+c^2x^2)^4\sqrt{d+c^2dx^2}}{729c^4}+\frac{4b^2d^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{63c^3\sqrt{1+c^2x^2}} \\
& -\frac{2bd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{189c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{21\sqrt{1+c^2x^2}} \\
& -\frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{441\sqrt{1+c^2x^2}} \\
& -\frac{2bc^5d^2x^9\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{81\sqrt{1+c^2x^2}}-\frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{63c^4} \\
& +\frac{d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{63c^2}+\frac{1}{21}d^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\
& +\frac{5}{63}dx^4(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{9}x^4(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2
\end{aligned}$$

output
```
5/63*d*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/9*x^4*(c^2*d*x^2+d)^
(5/2)*(a+b*arcsinh(c*x))^2-160/3969*b^2*d^2*(c^2*d*x^2+d)^(1/2)/c^4-80/119
07*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-4/1323*b^2*d^2*(c^2*x^2+1)^
2*(c^2*d*x^2+d)^(1/2)/c^4-50/27783*b^2*d^2*(c^2*x^2+1)^3*(c^2*d*x^2+d)^(1/
2)/c^4+2/729*b^2*d^2*(c^2*x^2+1)^4*(c^2*d*x^2+d)^(1/2)/c^4-2/63*d^2*(a+b*a
rcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/63*d^2*x^2*(a+b*arcsinh(c*x))^2*(
c^2*d*x^2+d)^(1/2)/c^2+1/21*d^2*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/
2)+4/63*a*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/63*b^2*d^2*x
*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/189*b*d^2*x^3*(a
+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/21*b*c*d^2*x^5*
(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-38/441*b*c^3*d^2*
x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/81*b*c^5*d^
2*x^9*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

## 3.274.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.44

$$\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{d^2\sqrt{d+c^2dx^2}\Big(3969a^2(1+c^2x^2)^4\left(-2+7c^2x^2\right)-126abcx\sqrt{1+c^2x^2}(-126+21c^2x^2+$$

input
```
Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(3969*a^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - 126*
a*b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 +
 49*c^8*x^8) + 2*b^2*(-6140 - 7039*c^2*x^2 + 106*c^4*x^4 + 2152*c^6*x^6 +
1490*c^8*x^8 + 343*c^10*x^10) - 126*b*(-63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x
^2) + b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x
^6 + 49*c^8*x^8))*ArcSinh[c*x] + 3969*b^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)
*ArcSinh[c*x]^2))/(250047*c^4*(1 + c^2*x^2))
```

### 3.274.3 Rubi [A] (verified)

Time = 4.22 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.23, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6223, 6218, 27, 1578, 1195, 2009, 6223, 6218, 27, 354, 86, 2009, 6221, 6191, 243, 53, 2009, 6227, 6191, 243, 53, 2009, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^2 dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2bcd^2\sqrt{c^2dx^2+d}\int x^4\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{9\sqrt{c^2x^2+1}}+\frac{5}{9}d\int x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6218}$$

$$\frac{5}{9}d\int x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-bc\int\frac{x^5(35c^4x^4+90c^2x^2+63)}{315\sqrt{c^2x^2+1}}dx+\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+\right.}{9\sqrt{c^2x^2+1}}$$
$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{9}d\int x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{315}bc\int\frac{x^5(35c^4x^4+90c^2x^2+63)}{\sqrt{c^2x^2+1}}dx+\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a\right.}{9\sqrt{c^2x^2+1}}$$
$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{1578}$$

$$\frac{5}{9}d\int x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{630}bc\int\frac{x^4(35c^4x^4+90c^2x^2+63)}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(\right.}{9\sqrt{c^2x^2+1}}$$
$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{1195}$$

$$\frac{5}{9}d\int x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-$$
$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{630}bc\int\left(\frac{35(c^2x^2+1)^{7/2}}{c^4}-\frac{50(c^2x^2+1)^{5/2}}{c^4}+\frac{3(c^2x^2+1)^{3/2}}{c^4}+\frac{4\sqrt{c^2x^2+1}}{c^4}+\frac{8}{c^4\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{9}c^4x^9(a}{9\sqrt{c^2x^2+1}}$$
$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 2009

$$\frac{5}{9}d\int x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 6223

$$\frac{5}{9}d\left(-\frac{2bcd\sqrt{c^2dx^2+d}\int x^4\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{7\sqrt{c^2x^2+1}}+\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}x^4(\right.$$
$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 6218

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\int\frac{x^5(5c^2x^2+7)}{35\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^2x^7(a+b\text{arcsinh}(cx\right.}{7\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 27

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{35}bc\int\frac{x^5(5c^2x^2+7)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^2x^7(a+b\text{arcsinh}(\right.}{7\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$
$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

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3.274. $\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\Big\downarrow \ 354$$

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{70}bc\int\frac{x^4(5c^2x^2+7)}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^2x^7(a+b\text{arcsinh}\right.}{7\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 86$$

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{70}bc\int\left(\frac{5(c^2x^2+1)^{5/2}}{c^4}-\frac{8(c^2x^2+1)^{3/2}}{c^4}+\frac{\sqrt{c^2x^2-}}{c^4}\right.\right.}{}\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 2009$$

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2\right.}{}\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 6221$$

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int x^4(a+b\text{arcsinh}(cx))dx}{5\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

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3.274. $\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$\downarrow$ 6191

$$\frac{5}{9}d\left(\frac{3}{7}d\left(-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{5}bc\int\frac{x^5}{\sqrt{c^2x^2+1}}dx\right)}{5\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\ldots\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+\ldots}{9c^6}\ldots\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 243

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int\frac{x^4}{\sqrt{c^2x^2+1}}dx^2\right)}{5\sqrt{c^2x^2+1}}+\ldots\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+\ldots}{9c^6}\ldots\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 53

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int\left(\frac{(c^2x^2+1)^{3/2}}{c^4}-2\ldots\right.\right.}{5\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+\ldots}{9c^6}\ldots\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{1}{5}x^5(a+b\ldots\right.}{\ldots}\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+\ldots}{9c^6}\ldots\right.\right.}{9\sqrt{c^2x^2+1}}$$

3.274. $\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$\downarrow$ 6227

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\int x^2(a+b\text{arcsinh}(cx))dx}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}\right)}{5\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 6191

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\int\frac{x^3}{\sqrt{c^2x^2+1}}dx\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(}{3c^2}\right.}{5\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 243

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx^2\right)}{3c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}}{3c^2}\right.}{5\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 53

3.274. $\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int\left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2\right)}{3c}+\frac{x^2\sqrt{c^2x^2-}}{}\right.}{5\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+\right.}{3c^4}\right.\right.}{3c}\right.}{5\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 6213

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\int(a+b\text{arcsinh}(cx))dx}{c}\right)}{3c^2}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a-\right.}{}\right.}{5\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\text{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\text{arcsinh}(cx))+\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

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3.274. $\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{9}x^4\left(c^2dx^2+d\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2-$$
$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{9}c^4x^9(a+b\operatorname{arcsinh}(cx))+\frac{2}{7}c^2x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{5}x^5(a+b\operatorname{arcsinh}(cx))-\frac{1}{630}bc\left(\frac{70(c^2x^2+}{9c^6}\right.\right.}{9\sqrt{c^2x^2+1}}$$

$$\frac{5}{9}d\left(\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^2x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{5}x^5(a+b\operatorname{arcsinh}(cx)\right.}{7\sqrt{c^2}}\right.$$

input
```
Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x^4*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/9 - (2*b*c*d^2*Sqrt[d +
 c^2*d*x^2]*(-1/630*(b*c*((16*Sqrt[1 + c^2*x^2])/c^6 + (8*(1 + c^2*x^2)^(3
/2))/(3*c^6) + (6*(1 + c^2*x^2)^(5/2))/(5*c^6) - (100*(1 + c^2*x^2)^(7/2))
/(7*c^6) + (70*(1 + c^2*x^2)^(9/2))/(9*c^6))) + (x^5*(a + b*ArcSinh[c*x]))
/5 + (2*c^2*x^7*(a + b*ArcSinh[c*x]))/7 + (c^4*x^9*(a + b*ArcSinh[c*x]))/9
))/(9*Sqrt[1 + c^2*x^2]) + (5*d*((x^4*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/7 - (2*b*c*d*Sqrt[d + c^2*d*x^2]*(-1/70*(b*c*((4*Sqrt[1 + c^2*x^
2])/c^6 + (2*(1 + c^2*x^2)^(3/2))/(3*c^6) - (16*(1 + c^2*x^2)^(5/2))/(5*c^
6) + (10*(1 + c^2*x^2)^(7/2))/(7*c^6))) + (x^5*(a + b*ArcSinh[c*x]))/5 + (
c^2*x^7*(a + b*ArcSinh[c*x]))/7))/(7*Sqrt[1 + c^2*x^2]) + (3*d*((x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*Sqrt[d + c^2*d*x^2]*(-1/
10*(b*c*((2*Sqrt[1 + c^2*x^2])/c^6 - (4*(1 + c^2*x^2)^(3/2))/(3*c^6) + (2*
(1 + c^2*x^2)^(5/2))/(5*c^6))) + (x^5*(a + b*ArcSinh[c*x]))/5))/(5*Sqrt[1
+ c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*((x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])^2)/(3*c^2) - (2*b*(-1/6*(b*c*((-2*Sqrt[1 + c^2*x^2])/c^4 + (2*(1 + c
^2*x^2)^(3/2))/(3*c^4))) + (x^3*(a + b*ArcSinh[c*x]))/3))/(3*c) - (2*((Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c^2 - (2*b*(a*x - (b*Sqrt[1 + c^2*x
^2])/c + b*x*ArcSinh[c*x]))/c))/(3*c^2)))/(5*Sqrt[1 + c^2*x^2])))/7))/9
```

## 3.274. $\int x^3(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx$

### 3.274.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## 3.274. $\int x^3(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\,dx$

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

## 3.274. $\int x^3(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.274.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2013 vs. $2(549) = 1098$.

Time = 0.40 (sec) , antiderivative size = 2014, normalized size of antiderivative = 3.22

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2014 |
| parts | Expression too large to display | 2014 |

input
```
int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b^2
*(1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1
/2)+704*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2
*x^2+1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(
c^2*x^2+1)^(1/2)+1)*(81*arcsinh(c*x)^2-18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2
+1)+3/175616*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2
)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^
2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arc
sinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4
*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh
(c*x)^2-6*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d^2/c^
4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1
)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2
+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2
+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3/175
616*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6
*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2
)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)
+2)*d^2/c^4/(c^2*x^2+1)+1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-2...
```

### 3.274.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.84

$$\int x^3\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{3969\left(7\,b^2c^{10}d^2x^{10}+26\,b^2c^8d^2x^8+34\,b^2c^6d^2x^6+16\,b^2c^4d^2x^4-b^2c^2d^2x^2-2\,b^2d^2\right)\sqrt{c^2d\cdots}}{\cdots}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
1/250047*(3969*(7*b^2*c^10*d^2*x^10 + 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*
x^6 + 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 - 2*b^2*d^2)*sqrt(c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 126*(441*a*b*c^10*d^2*x^10 + 1638*a*b*c
^8*d^2*x^8 + 2142*a*b*c^6*d^2*x^6 + 1008*a*b*c^4*d^2*x^4 - 63*a*b*c^2*d^2*
x^2 - 126*a*b*d^2 - (49*b^2*c^9*d^2*x^9 + 171*b^2*c^7*d^2*x^7 + 189*b^2*c^
5*d^2*x^5 + 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(
c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (343*(81*a^2 + 2*b^2)*c^10*d
^2*x^10 + 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152*b^2)*
c^6*d^2*x^6 + 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*b^2)*
c^2*d^2*x^2 - 2*(3969*a^2 + 6140*b^2)*d^2 - 126*(49*a*b*c^9*d^2*x^9 + 171*
a*b*c^7*d^2*x^7 + 189*a*b*c^5*d^2*x^5 + 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2
*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

## 3.274.6 Sympy [F(-1)]

Timed out.

$$\int x^3 (d + c^2 d x^2)^{5/2} (a + b\operatorname{arcsinh}(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.274.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.62

$$\int x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{63}\left(\frac{7\,(c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)b^2\,\text{arsinh}\,(cx)^2 + \frac{2}{63}\left(\frac{7\,(c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)ab\,\text{arsinh}\,(cx) + \frac{1}{63}\left(\frac{7\,(c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\,(c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)a^2 + \frac{2}{250047}b^2\left(\frac{343\sqrt{c^2x^2+1}c^6d^{\frac{5}{2}}x^8+1147\sqrt{c^2x^2+1}c^4d^{\frac{5}{2}}x^6+1005\sqrt{c^2x^2+1}c^2d^{\frac{5}{2}}x^4-899\sqrt{c^2x^2+1}d^{\frac{5}{2}}x^2}{c^2}\right. - \frac{2\left(49\,c^8d^{\frac{5}{2}}x^9+171\,c^6d^{\frac{5}{2}}x^7+189\,c^4d^{\frac{5}{2}}x^5+21\,c^2d^{\frac{5}{2}}x^3-126\,d^{\frac{5}{2}}x\right)ab}{3969\,c^3}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d
))*b^2*arcsinh(c*x)^2 + 2/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2
*d*x^2 + d)^(7/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2
)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 + 2/250047*b^2*((343*
sqrt(c^2*x^2 + 1)*c^6*d^(5/2)*x^8 + 1147*sqrt(c^2*x^2 + 1)*c^4*d^(5/2)*x^6
 + 1005*sqrt(c^2*x^2 + 1)*c^2*d^(5/2)*x^4 - 899*sqrt(c^2*x^2 + 1)*d^(5/2)*
x^2 - 6140*sqrt(c^2*x^2 + 1)*d^(5/2)/c^2)/c^2 - 63*(49*c^8*d^(5/2)*x^9 + 1
71*c^6*d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2
)*x)*arcsinh(c*x)/c^3) - 2/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^7
+ 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*a*b/c^3
```

### 3.274.8 Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.274.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b \text{arcsinh}(cx))^2 \, dx = \int x^3 \left(a+b \operatorname{asinh}(c x)\right)^2 \left(d c^2 x^2+d\right)^{5/2} \, dx$$

input
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

# 3.275 $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.275.1 Optimal result

Integrand size = 28, antiderivative size = 536

$$\begin{aligned}\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx &= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} \\ &+ \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824} + \frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2} \\ &+ \frac{359b^2d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{36864c^3\sqrt{1+c^2x^2}} - \frac{5bd^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{128c\sqrt{1+c^2x^2}} \\ &- \frac{59bcd^2x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{384\sqrt{1+c^2x^2}} \\ &- \frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{144\sqrt{1+c^2x^2}} \\ &- \frac{bc^5d^2x^8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{32\sqrt{1+c^2x^2}} \\ &+ \frac{5d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{128c^2} + \frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 \\ &+ \frac{5}{48}dx^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 \\ &+ \frac{1}{8}x^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2 - \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{384bc^3\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*x^3*(c^2*d*x^2+d)^
(5/2)*(a+b*arcsinh(c*x))^2-359/36864*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)/c^2+107
9/55296*b^2*d^2*x^3*(c^2*d*x^2+d)^(1/2)+209/13824*b^2*c^2*d^2*x^5*(c^2*d*x
^2+d)^(1/2)+1/256*b^2*c^4*d^2*x^7*(c^2*d*x^2+d)^(1/2)+5/128*d^2*x*(a+b*arc
sinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*x^3*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)+359/36864*b^2*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c
^2*x^2+1)^(1/2)-5/128*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(
c^2*x^2+1)^(1/2)-59/384*b*c*d^2*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-17/144*b*c^3*d^2*x^6*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-1/32*b*c^5*d^2*x^8*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)-5/384*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2
)/b/c^3/(c^2*x^2+1)^(1/2)
```

### 3.275.2 Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.15

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{d^2\Big(34560a^2cx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+271872a^2c^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+313344a}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(34560*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 271872*a^2*c^3
*x^3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 313344*a^2*c^5*x^5*Sqrt[1 + c
^2*x^2]*Sqrt[d + c^2*d*x^2] + 110592*a^2*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d
+ c^2*d*x^2] - 11520*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 13824*a*b*Sq
rt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 3456*a*b*Sqrt[d + c^2*d*x^2]*Cosh
[4*ArcSinh[c*x]] - 1536*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 216
*a*b*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] - 34560*a^2*Sqrt[d]*Sqrt[1 +
 c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6912*b^2*Sqrt[d + c^2
*d*x^2]*Sinh[2*ArcSinh[c*x]] + 864*b^2*Sqrt[d + c^2*d*x^2]*Sinh[4*ArcSinh[
c*x]] + 256*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] + 27*b^2*Sqrt[d +
 c^2*d*x^2]*Sinh[8*ArcSinh[c*x]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(
576*b*Cosh[2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*x]] - 64*b*Cosh[6*ArcS
inh[c*x]] - 9*b*Cosh[8*ArcSinh[c*x]] - 1152*a*Sinh[2*ArcSinh[c*x]] + 576*a
*Sinh[4*ArcSinh[c*x]] + 384*a*Sinh[6*ArcSinh[c*x]] + 72*a*Sinh[8*ArcSinh[c
*x]]) + 288*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-120*a - 48*b*Sinh[2*Arc
Sinh[c*x]] + 24*b*Sinh[4*ArcSinh[c*x]] + 16*b*Sinh[6*ArcSinh[c*x]] + 3*b*S
inh[8*ArcSinh[c*x]])))/(884736*c^3*Sqrt[1 + c^2*x^2])
```

### 3.275.3 Rubi [A] (verified)

Time = 3.73 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.39, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6223, 6218, 27, 1590, 27, 363, 262, 262, 222, 6223, 6218, 27, 363, 262, 262, 222, 6221, 6191, 262, 262, 222, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x^3\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{4\sqrt{c^2x^2+1}}+\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6218}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd^2\sqrt{c^2dx^2+d}\left(-bc\int\frac{x^4\left(3c^4x^4+8c^2x^2+6\right)}{24\sqrt{c^2x^2+1}}dx+\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arc}\right.}{4\sqrt{c^2x^2+1}}$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{24}bc\int\frac{x^4\left(3c^4x^4+8c^2x^2+6\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{a}\right.}{4\sqrt{c^2x^2+1}}$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{1590}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{24}bc\left(\frac{\int\frac{c^2x^4\left(43c^2x^2+48\right)}{\sqrt{c^2x^2+1}}dx}{8c^2}+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}\right.}{4\sqrt{c^2x^2+1}}$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{24}bc\left(\frac{1}{8}\int\frac{x^4\left(43c^2x^2+48\right)}{\sqrt{c^2x^2+1}}dx+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}\right.}{4\sqrt{c^2x^2+1}}$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{363}$$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\int\frac{x^4}{\sqrt{c^2x^2+1}}dx+\frac{43}{6}x^5\sqrt{c^2x^2+1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\right.}{4\sqrt{c^2x^2+1}}$$
$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

3.275. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int\frac{x^2}{\sqrt{c^2x^2+1}}dx}{4c^2}\right)+\frac{43}{6}x^5\sqrt{c^2x^2+1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right)+\frac{1}{8}c^4x^8(a+\right.}{4\sqrt{c^2x^2+1}}$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{4c^2}\right)+\frac{43}{6}x^5\sqrt{c^2x^2+1}\right)+\frac{3}{8}c^2x^7\sqrt{c^2x^2+1}\right.\right.}{4\sqrt{c^2x^2+1}}$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 222

$$\frac{5}{8}d\int x^2\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x^3}{}\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 6223

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\int x^3\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2d\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x^3}{}\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 6218

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3.275. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-bc\int\frac{x^4(2c^2x^2+3)}{12\sqrt{c^2x^2+1}}dx+\frac{1}{6}c^2x^6(a+b\text{arcsinh}(cx))\right.}{3\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(x\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 27

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\int\frac{x^4(2c^2x^2+3)}{\sqrt{c^2x^2+1}}dx+\frac{1}{6}c^2x^6(a+b\text{arcsinh}(c\right.}{3\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(x\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 363

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\left(\frac{4}{3}\int\frac{x^4}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}x^5\sqrt{c^2x^2+1}\right)+\frac{1}{6}c^2\right.}{3\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(x\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 262

3.275. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int\frac{x^2}{\sqrt{c^2x^2+1}}dx}{4c^2}\right)+\frac{1}{3}x^5\sqrt{}\right.\right.}{3\sqrt{c}}\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 262

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx-\frac{bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}}{2c^2}\right.}{4c^2}\right.\right.\right.}{}\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 222

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{1}{6}c^2x\right.}{}\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 6221

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3.275. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int x^3(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6191}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\int\frac{x^4}{\sqrt{c^2x^2+1}}dx\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int}{4}\right.\right.}{2\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{262}$$

3.275. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c}\ldots}{}\right.}{}\right.\right.}{2\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x}{}\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 222$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{4}x^4(a+b\text{a}\ldots\right.}{}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x}{}\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6227$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int x(a+b\text{arcsinh}(cx))dx}{c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx\ldots}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{x}{}\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6191$$

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3.275. $\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{c}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}\right.}{4\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(x\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 262

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{c}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\right.}{4\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(x\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 222

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{c^2dx^2+d}\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\text{a}\right.\right.}{c}\right.}{4\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(x\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

$\downarrow$ 6198

---

3.275. $\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{5}{8}d\left(\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2+\frac{\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcs}}{6}\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{8}c^4x^8(a+b\text{arcsinh}(cx))+\frac{1}{3}c^2x^6(a+b\text{arcsinh}(cx))+\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\right.\right.\right.\right.}{4\sqrt{c^2x^2+1}}$$

input
```
Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/8 - (b*c*d^2*Sqrt[d + c
^2*d*x^2]*((x^4*(a + b*ArcSinh[c*x]))/4 + (c^2*x^6*(a + b*ArcSinh[c*x]))/3
 + (c^4*x^8*(a + b*ArcSinh[c*x]))/8 - (b*c*((3*c^2*x^7*Sqrt[1 + c^2*x^2])/
8 + ((43*x^5*Sqrt[1 + c^2*x^2])/6 + (73*((x^3*Sqrt[1 + c^2*x^2])/(4*c^2) -
 (3*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/(4*c^2)))/6)/8
))/24))/(4*Sqrt[1 + c^2*x^2]) + (5*d*((x^3*(d + c^2*d*x^2)^(3/2)*(a + b*Ar
cSinh[c*x])^2)/6 - (b*c*d*Sqrt[d + c^2*d*x^2]*((x^4*(a + b*ArcSinh[c*x]))/
4 + (c^2*x^6*(a + b*ArcSinh[c*x]))/6 - (b*c*((x^5*Sqrt[1 + c^2*x^2])/3 + (
4*((x^3*Sqrt[1 + c^2*x^2])/(4*c^2) - (3*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - A
rcSinh[c*x]/(2*c^3)))/(4*c^2)))/3))/12))/(3*Sqrt[1 + c^2*x^2]) + (d*((x^3*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/4 - (b*c*Sqrt[d + c^2*d*x^2]*(
(x^4*(a + b*ArcSinh[c*x]))/4 - (b*c*((x^3*Sqrt[1 + c^2*x^2])/(4*c^2) - (3*
((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/(4*c^2)))/4))/(2*S
qrt[1 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*((x*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x])^2)/(2*c^2) - (a + b*ArcSinh[c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*
ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c
^3)))/2))/c))/(4*Sqrt[1 + c^2*x^2])))/2))/8
```

### 3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.275. $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.275.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. 2(468) = 936.

Time = 0.40 (sec) , antiderivative size = 2280, normalized size of antiderivative = 4.25

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2280 |
| parts | Expression too large to display | 2280 |

input
```
int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a^2/c^2*x*(c^2*d*x^2+d)^(5/2)-5/1
92*a^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a^2/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)
-5/128*a^2/c^2*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(
1/2)+b^2*(-5/384*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^
3*d^2+1/65536*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(
1/2)+320*c^7*x^7+256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^
2*x^2+1)^(1/2)+88*c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(
1/2))*(32*arcsinh(c*x)^2-8*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/6912*(d*(
c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c
^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^
2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2-6*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1
/2048*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*
x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2
-4*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*
x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2
-2*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)-1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*
x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2
+2*arcsinh(c*x)+1)*d^2/c^3/(c^2*x^2+1)+1/2048*(d*(c^2*x^2+1))^(1/2)*(8*c^5
*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*
c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)*d^2/c^3/(c^2...
```

### 3.275.5 Fricas [F]

$$\int x^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^6 + 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2
*x^6 + 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^
6 + 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

### 3.275.6 Sympy [F(-1)]

Timed out.

$$\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.275.7 Maxima [F(-2)]

Exception generated.

$$\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.275.8 Giac [F]

$$\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

### 3.275.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \int x^2 \left(a+b\,\text{asinh}(c\,x)\right)^2 \left(d\,c^2\,x^2+d\right)^{5/2} dx$$

input
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

# 3.276 $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.276.1 Optimal result

Integrand size = 26, antiderivative size = 366

$$\begin{aligned}\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{32b^2d^2\sqrt{d+c^2dx^2}}{245c^2}\\ &+\frac{16b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{735c^2}+\frac{12b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}}{1225c^2}\\ &+\frac{2b^2d^2(1+c^2x^2)^3\sqrt{d+c^2dx^2}}{343c^2}-\frac{2bd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7c\sqrt{1+c^2x^2}}\\ &-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7\sqrt{1+c^2x^2}}-\frac{6bc^3d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{35\sqrt{1+c^2x^2}}\\ &-\frac{2bc^5d^2x^7\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{49\sqrt{1+c^2x^2}}+\frac{(d+c^2dx^2)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}\end{aligned}$$

output
```
1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))^2/c^2/d+32/245*b^2*d^2*(c^2*d*x
^2+d)^(1/2)/c^2+16/735*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+12/1225
*b^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2+2/343*b^2*d^2*(c^2*x^2+1)^3
*(c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)/c/(c^2*x^2+1)^(1/2)-2/7*b*c*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-2/49*b*c^5*d^2*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.276.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.61

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{d^2\sqrt{d+c^2dx^2}\Big(3675a^2(1+c^2x^2)^4-210abcx\sqrt{1+c^2x^2}(35+35c^2x^2+21c^4x^4+5c^6x^6)}{}$$

input
```
Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(3675*a^2*(1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[1 +
c^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(2161 + 2918*c
^2*x^2 + 1108*c^4*x^4 + 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*(1 + c^2*x
^2)^4 - b*c*x*Sqrt[1 + c^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)
)*ArcSinh[c*x] + 3675*b^2*(1 + c^2*x^2)^4*ArcSinh[c*x]^2))/(25725*c^2*(1 +
 c^2*x^2))
```

### 3.276.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6213, 6199, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}-\frac{2bd^2\sqrt{c^2dx^2+d}\int\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))dx}{7c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6199}$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}-$$
$$\frac{2bd^2\sqrt{c^2dx^2+d}\Big(-bc\int\frac{x(5c^6x^6+21c^4x^4+35c^2x^2+35)}{35\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arcsinh}(cx))+c^2x^3(}{7c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}-\frac{2bd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{35}bc\int\frac{x(5c^6x^6+21c^4x^4+35c^2x^2+35)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arcsinh}(cx))+c^2x\right.}{7c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2331$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}-\frac{2bd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{70}bc\int\frac{5c^6x^6+21c^4x^4+35c^2x^2+35}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arcsinh}(cx))+c^2x^3\right.}{7c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2389$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}-\frac{2bd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{70}bc\int\left(5\left(c^2x^2+1\right)^{5/2}+6\left(c^2x^2+1\right)^{3/2}+8\sqrt{c^2x^2+1}+\frac{16}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(\right.}{7c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(c^2dx^2+d\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2d}-\frac{2bd^2\sqrt{c^2dx^2+d}\left(\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arcsinh}(cx))+c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(\right.}{7c\sqrt{c^2x^2+1}}$$

input

```
Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output

```
((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x])^2)/(7*c^2*d) - (2*b*d^2*Sqrt[d
 + c^2*d*x^2]*(-1/70*(b*c*((32*Sqrt[1 + c^2*x^2])/c^2 + (16*(1 + c^2*x^2)^
(3/2))/(3*c^2) + (12*(1 + c^2*x^2)^(5/2))/(5*c^2) + (10*(1 + c^2*x^2)^(7/2
))/(7*c^2))) + x*(a + b*ArcSinh[c*x]) + c^2*x^3*(a + b*ArcSinh[c*x]) + (3*
c^4*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*x^7*(a + b*ArcSinh[c*x]))/7))/(7*c*
Sqrt[1 + c^2*x^2])
```

## 3.276. $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.276.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.276.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1772 vs. 2(322) = 644.

Time = 0.37 (sec) , antiderivative size = 1773, normalized size of antiderivative = 4.84

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1773 |
| parts | Expression too large to display | 1773 |

input
```
int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^2*(c^2*d*x^2+d)^(7/2)/c^2/d+b^2*(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c
^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1
/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^
(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3200*
(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+
20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arc
sinh(c*x)^2-10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2*x^2+1))^(
1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c
^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsi
nh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c
^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1
/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^2/(
c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)
^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1
)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/439
04*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*
x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)
+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*...
```

### 3.276.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.22

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{3675\left(b^2c^8d^2x^8+4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4+4\,b^2c^2d^2x^2+b^2d^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c\right.}$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
1/25725*(3675*(b^2*c^8*d^2*x^8 + 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 + 4
*b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1
))^2 + 210*(35*a*b*c^8*d^2*x^8 + 140*a*b*c^6*d^2*x^6 + 210*a*b*c^4*d^2*x^4
 + 140*a*b*c^2*d^2*x^2 + 35*a*b*d^2 - (5*b^2*c^7*d^2*x^7 + 21*b^2*c^5*d^2*
x^5 + 35*b^2*c^3*d^2*x^3 + 35*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x
^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 +
12*(1225*a^2 + 71*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4
+ 4*(3675*a^2 + 1459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2 - 210*(5
*a*b*c^7*d^2*x^7 + 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 + 35*a*b*c*d^2*
x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

## 3.276.6 Sympy [F]

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x*(d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

## 3.276.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.75

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{(c^2dx^2+d)^{\frac{7}{2}}b^2\operatorname{arsinh}(cx)^2}{7\,c^2d}$$
$$+\frac{2\,(c^2dx^2+d)^{\frac{7}{2}}ab\operatorname{arsinh}(cx)}{7\,c^2d}$$
$$+\frac{2}{25725}\,b^2\left(\frac{75\sqrt{c^2x^2+1}c^4d^{\frac{7}{2}}x^6+351\sqrt{c^2x^2+1}c^2d^{\frac{7}{2}}x^4+757\sqrt{c^2x^2+1}d^{\frac{7}{2}}x^2+\frac{2161\sqrt{c^2x^2+1}d^{\frac{7}{2}}}{c^2}}{d}-\frac{105\left(5\,c^6\right.}{}\right.$$
$$+\frac{(c^2dx^2+d)^{\frac{7}{2}}a^2}{7\,c^2d}-\frac{2\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)ab}{245\,cd}$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
1/7*(c^2*d*x^2 + d)^(7/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/7*(c^2*d*x^2 + d)
^(7/2)*a*b*arcsinh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 + 1)*c^4*d
^(7/2)*x^6 + 351*sqrt(c^2*x^2 + 1)*c^2*d^(7/2)*x^4 + 757*sqrt(c^2*x^2 + 1)
*d^(7/2)*x^2 + 2161*sqrt(c^2*x^2 + 1)*d^(7/2)/c^2)/d - 105*(5*c^6*d^(7/2)*
x^7 + 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*arcsinh(c*x)
/(c*d)) + 1/7*(c^2*d*x^2 + d)^(7/2)*a^2/(c^2*d) - 2/245*(5*c^6*d^(7/2)*x^7
 + 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*a*b/(c*d)
```

### 3.276.8 Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.276.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int x\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

# 3.277 $\int \left(d+c^2 d x^2\right)^{5/2} (a+b\,\mathrm{arcsinh}(cx))^2\,dx$



## 3.277.1 Optimal result

Integrand size = 25, antiderivative size = 420

$$\int \left(d+c^2 d x^2\right)^{5/2} (a+b\,\mathrm{arcsinh}(cx))^2\,dx=\frac{245 b^2 d^2 x \sqrt{d+c^2 d x^2}}{1152}$$
$$+\frac{65 b^2 d^2 x (1+c^2x^2)\sqrt{d+c^2 d x^2}}{1728}+\frac{1}{108} b^2 d^2 x \left(1+c^2x^2\right)^2\sqrt{d+c^2 d x^2}$$
$$-\frac{115 b^2 d^2 \sqrt{d+c^2 d x^2}\,\mathrm{arcsinh}(cx)}{1152 c \sqrt{1+c^2x^2}}-\frac{5 b c d^2 x^2 \sqrt{d+c^2 d x^2}(a+b\,\mathrm{arcsinh}(cx))}{16\sqrt{1+c^2x^2}}$$
$$-\frac{5 b d^2 \left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2 d x^2}(a+b\,\mathrm{arcsinh}(cx))}{48 c}$$
$$-\frac{b d^2 \left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2 d x^2}(a+b\,\mathrm{arcsinh}(cx))}{18 c}$$
$$+\frac{5}{16} d^2 x \sqrt{d+c^2 d x^2}(a+b\,\mathrm{arcsinh}(cx))^2+\frac{5}{24} d x \left(d+c^2 d x^2\right)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2+\frac{1}{6} x \left(d+c^2 d x^2\right)^{5/2}(a+\mathrm{barcsi}$$

output
```
5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/6*x*(c^2*d*x^2+d)^(5/2
)*(a+b*arcsinh(c*x))^2+245/1152*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)+65/1728*b^2*
d^2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+1/108*b^2*d^2*x*(c^2*x^2+1)^2*(c^2*d
*x^2+d)^(1/2)-5/48*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d
)^(1/2)/c-1/18*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1
/2)/c+5/16*d^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-115/1152*b^2*d^2
*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d^2*x^2*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/48*d^2*(a+b*arcsin
h(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.277.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.19

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{d^2\left(9504a^2cx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+7488a^2c^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+2304a^2c^5x^5\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(9504*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 7488*a^2*c^3*x^
3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^
2]*Sqrt[d + c^2*d*x^2] + 1440*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 324
0*a*b*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 324*a*b*Sqrt[d + c^2*d*x^
2]*Cosh[4*ArcSinh[c*x]] - 24*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]]
+ 4320*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] + 1620*b^2*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sqrt[d +
c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh
[c*x]] - 12*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]]
 + 27*b*Cosh[4*ArcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*Arc
Sinh[c*x]] - 108*a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*
b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9
*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]])))/(13824*c*Sqrt[1 + c^2*
x^2])
```

### 3.277.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {6201, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \; 6201$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+\frac{5}{6}d\int\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6201

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\int\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{4}x\left(c^2dx^2\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6200

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2}}{}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6191

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\right.}{\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 262

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\right.\right.}{\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 222

3.277. $\int\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6198}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2}}{}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6213}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\int(c^2x^2+1)^{5/2}dx}{6c}\right)}{3\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int(c^2x^2+1)^{3/2}dx}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}a\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{211}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\int(c^2x^2+1)^{3/2}dx+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)}{3\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+\text{barcs}\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{211}$$

3.277. $\int\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)}{3\sqrt{c^2x^2+1}}+$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x(c^2dx^2+\right.$$

$$\frac{1}{6}x(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 211

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)}{3\sqrt{c^2x^2+1}}-$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x(c^2dx^2+\right.$$

$$\frac{1}{6}x(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 222

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)}{3\sqrt{c^2x^2+1}}+$$

$$\frac{1}{6}x(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{5}{6}d\left(\frac{1}{4}x(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

input
```
Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

## 3.277. $\int (d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

output
```
(x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 - (b*c*d^2*Sqrt[d + c^2
*d*x^2]*(((1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2) - (b*((x*(1 + c^2*
x^2)^(5/2))/6 + (5*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/
2 + ArcSinh[c*x]/(2*c)))/4))/6))/(6*c)))/(3*Sqrt[1 + c^2*x^2]) + (5*d*((x*
(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*((x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])
^3)/(6*b*c*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[d + c^2*d*x^2]*((x^2*(a + b*ArcS
inh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3))
)/2))/Sqrt[1 + c^2*x^2]))/4 - (b*c*d*Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2)^2
*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sq
rt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 + c^2*x^2]
)))/6
```

### 3.277.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.277.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(366) = 732$.

Time = 0.30 (sec) , antiderivative size = 1568, normalized size of antiderivative = 3.73

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1568 |
| parts | Expression too large to display | 1568 |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a^2*d^
2*x*(c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d
)^(1/2))/(c^2*d)^(1/2)+b^2*(5/48*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
*arcsinh(c*x)^3*d^2+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c
^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*
x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2-6*arcsin
h(c*x)+1)*d^2/c/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*
x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^
2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+15/256*(
d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2
+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+15/256*(d
*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+
1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+3/1024*(d*
(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2
*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2+4*arcsin
h(c*x)+1)*d^2/c/(c^2*x^2+1)+1/6912*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^
6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3
-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(18*arcsinh(c*x)^2+
6*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1))+2*a*b*(5/32*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d^2+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^...
```

### 3.277.5 Fricas [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

### 3.277.6 Sympy [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))^2\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

### 3.277.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.277.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.277.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

# 3.278 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$



## 3.278.1 Optimal result

Integrand size = 28, antiderivative size = 635

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2}-\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}$$
$$+\frac{74}{675}b^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}+\frac{2}{125}b^2d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}$$
$$-\frac{2b^2cd^2x\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}-\frac{16bcd^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}}$$
$$-\frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{45\sqrt{1+c^2x^2}}-\frac{2bc^5d^2x^5\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}$$
$$+d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2+\frac{1}{3}d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2$$
$$+\frac{1}{5}(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2-\frac{2d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}-\frac{2bd^2\sqrt{d+c^2}}{}$$

output
```
1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/5*(c^2*d*x^2+d)^(5/2)*(a+
b*arcsinh(c*x))^2+598/225*b^2*d^2*(c^2*d*x^2+d)^(1/2)+74/675*b^2*d^2*(c^2*
x^2+1)*(c^2*d*x^2+d)^(1/2)+2/125*b^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)
+d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-16/15*b*c*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-22/45*b*c^3*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-2/25*b*c^5*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-2*d^2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2
))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d^2*(a+b*arcsinh(c*x))*polylo
g(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*d^2*
(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)+2*b^2*d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

### 3.278.2 Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.19

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \frac{1}{15}a^2d^2\sqrt{d+c^2dx^2}(23+11c^2x^2+3c^4x^4)$$
$$-\frac{4abd^2\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)\right)}{9\sqrt{1+c^2x^2}}$$
$$+\frac{2abd^3\sqrt{1+c^2x^2}(30cx-5c^3x^3-9c^5x^5+15\sqrt{1+c^2x^2}(-2+c^2x^2+3c^4x^4)\,\text{arcsinh}(cx))}{225\sqrt{d+c^2dx^2}}$$
$$-\frac{b^2d^3\sqrt{1+c^2x^2}(480cx(-30+5c^2x^2+9c^4x^4)\,\text{arcsinh}(cx)+6750\sqrt{1+c^2x^2}(2+\text{arcsinh}(cx)^2)+125(2+9}{54000\sqrt{d+c^2dx^2}}$$
$$+a^2d^{5/2}\log(cx)-a^2d^{5/2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+\frac{2abd^3\sqrt{1+c^2x^2}(-cx+\sqrt{1+c^2x^2}\text{arcsinh}(cx)+\text{arcsinh}(}{}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

output
```
(a^2*d^2*Sqrt[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c^4*x^4))/15 - (4*a*b*d^
2*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x
]))/(9*Sqrt[1 + c^2*x^2]) + (2*a*b*d^3*Sqrt[1 + c^2*x^2]*(30*c*x - 5*c^3*x
^3 - 9*c^5*x^5 + 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c
*x]))/(225*Sqrt[d + c^2*d*x^2]) - (b^2*d^3*Sqrt[1 + c^2*x^2]*(480*c*x*(-30
 + 5*c^2*x^2 + 9*c^4*x^4)*ArcSinh[c*x] + 6750*Sqrt[1 + c^2*x^2]*(2 + ArcSi
nh[c*x]^2) + 125*(2 + 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 27*(2 + 25*
ArcSinh[c*x]^2)*Cosh[5*ArcSinh[c*x]]))/(54000*Sqrt[d + c^2*d*x^2]) + a^2*d
^(5/2)*Log[c*x] - a^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*
b*d^3*Sqrt[1 + c^2*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh
[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])]
 + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[d
 + c^2*d*x^2] + (b^2*d^3*Sqrt[1 + c^2*x^2]*(2*Sqrt[1 + c^2*x^2] - 2*c*x*Ar
cSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh[c*x]^2*(Log[1 - E^
(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]) + 2*ArcSinh[c*x]*(PolyLog[2
, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]) + 2*(PolyLog[3, -E^
(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])])))/Sqrt[d + c^2*d*x^2] +
(b^2*d^3*Sqrt[1 + c^2*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2
 + 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3
*ArcSinh[c*x]])))/(54*Sqrt[d + c^2*d*x^2])
```

### 3.278.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.83, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6223, 6199, 27, 1576, 1140, 2009, 6223, 6199, 27, 353, 53, 2009, 6221, 2009, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}dx$$

$$\Big\downarrow \ 6223$$

$$-\frac{2bcd^2\sqrt{c^2dx^2+d}\int \left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{5\sqrt{c^2x^2+1}}+ d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6199}$$

$$d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-bc\int \frac{x\left(3c^4x^4+10c^2x^2+15\right)}{15\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcs}\right.}{5\sqrt{c^2x^2+1}}$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{27}$$

$$d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{15}bc\int \frac{x\left(3c^4x^4+10c^2x^2+15\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{a}\right.}{5\sqrt{c^2x^2+1}}$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{1576}$$

$$d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{30}bc\int \frac{3c^4x^4+10c^2x^2+15}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arc}\right.}{5\sqrt{c^2x^2+1}}$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{1140}$$

$$d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{1}{30}bc\int \left(3\left(c^2x^2+1\right)^{3/2}+4\sqrt{c^2x^2+1}+\frac{8}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+\right.}{5\sqrt{c^2x^2+1}}$$
$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6223}$$

3.278. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d\left(-\frac{2bcd\sqrt{c^2dx^2+d}\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6199}$$

$$d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\int \frac{x\left(c^2x^2+3\right)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a\right.}{3\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{3}bc\int \frac{x\left(c^2x^2+3\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x\right.}{3\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{353}$$

$$d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int \frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(\right.}{3\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{53}$$

3.278. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+ba\right.}{3\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 2009

$$d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a+b\right.}{}\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 6221

$$d\left(d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(c\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 2009

$$d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(cx)\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 6231

---

3.278. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\Big(ax+bx\text{ar}}{\sqrt{c^2x^2}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\downarrow 3042$$

$$d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{}}{}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\downarrow 26$$

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{}}{}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\downarrow 4670$$

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(2ib\int (a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int (a+b\text{arcsinh}(cx))\log\left(1+e\right.\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\downarrow 3011$$

---

3.278. $\displaystyle\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)\left(a+b\text{arcsinh}(cx\right)\right.\right.}{}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2720}$$

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)\left(a+b\right.\right.\right.}{}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{7143}$$

$$d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-e\right.\right.\right.}{\sqrt{c^2}}\right.\right.$$

$$\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}\right.\right.}{5\sqrt{c^2x^2+1}}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

## 3.278. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

output
```
((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*d^2*Sqrt[d + c^2
*d*x^2]*(-1/30*(b*c*((16*Sqrt[1 + c^2*x^2])/c^2 + (8*(1 + c^2*x^2)^(3/2))/
(3*c^2) + (6*(1 + c^2*x^2)^(5/2))/(5*c^2))) + x*(a + b*ArcSinh[c*x]) + (2*
c^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*x^5*(a + b*ArcSinh[c*x]))/5))/(5*Sq
rt[1 + c^2*x^2]) + d*(((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/3 - (
2*b*c*d*Sqrt[d + c^2*d*x^2]*(-1/6*(b*c*((4*Sqrt[1 + c^2*x^2])/c^2 + (2*(1
+ c^2*x^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSinh[c*x]) + (c^2*x^3*(a + b*Arc
Sinh[c*x]))/3))/(3*Sqrt[1 + c^2*x^2]) + d*(Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2 - (2*b*c*Sqrt[d + c^2*d*x^2]*(a*x - (b*Sqrt[1 + c^2*x^2])/c +
b*x*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (I*Sqrt[d + c^2*d*x^2]*((2*I)*(a +
b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x]
)*PolyLog[2, -E^ArcSinh[c*x]]) + b*PolyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*
(-((a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSi
nh[c*x]])))/Sqrt[1 + c^2*x^2]))
```

### 3.278.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

## 3.278. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx$

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## 3.278. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx$

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.278.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1320 vs. 2(614) = 1228.

Time = 0.32 (sec) , antiderivative size = 1321, normalized size of antiderivative = 2.08

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1321 |
| parts | Expression too large to display | 1321 |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
2/125*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*c^6*x^6+532/3375*b^2*(d*(c
^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*c^4*x^4+9872/3375*b^2*(d*(c^2*x^2+1))^(1/
2)*d^2/(c^2*x^2+1)*x^2*c^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2-2*b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2-b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2
+1)^(1/2))*d^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*
ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-a^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d
)^(1/2))/x)+1/5*(c^2*d*x^2+d)^(5/2)*a^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*d^2+9394/3375*b^2*(d*(c^2*x^
2+1))^(1/2)*d^2/(c^2*x^2+1)+2/5*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*
arcsinh(c*x)*x^6*c^6+28/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsi
nh(c*x)*x^4*c^4+68/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*
x)*x^2*c^2+1/5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^
6*c^6-22/45*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x
^3*c^3-46/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
x*c-2/25*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5+2*a*b*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/
2))*d^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*
x+(c^2*x^2+1)^(1/2))*d^2-22/45*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1...
```

### 3.278.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas"
)
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

### 3.278.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))^2}{x}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x, x)
```

### 3.278.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima"
)
```

output
```
-1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2
*d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a^2 + integrate((c^2*d*x
^2 + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(5/
2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

### 3.278.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x, x)
```

# 3.279 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2}{x^2}\,dx$



## 3.279.1 Optimal result

Integrand size = 28, antiderivative size = 530

$$\begin{aligned}
\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx &= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} \\
&+ \frac{1}{32}b^2c^2d^2x(1+c^2x^2)\sqrt{d+c^2dx^2} - \frac{89b^2cd^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64\sqrt{1+c^2x^2}} \\
&- \frac{15bc^3d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&- \frac{1}{8}bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&+ \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{1+c^2x^2}} \\
&+ \frac{5}{4}c^2dx(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + \frac{5cd^2\sqrt{d+c^2dx^2}(a+b\text{arcsin}}{8b\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-(c^2*d*x^2+d)^(5/2)*(
a+b*arcsinh(c*x))^2/x+31/64*b^2*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+1/32*b^2*c^2
*d^2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))^2*(c^2
*d*x^2+d)^(1/2)-89/64*b^2*c*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-15/8*b*c^3*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)+c*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)+5/8*c*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+2
*b*c*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d^2*(a+b*arcsinh(c*x))*(c^2*x^2
+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

### 3.279.2 Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.04

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \frac{d^2\Big(-256a^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+288a^2c^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
(d^2*(-256*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 288*a^2*c^2*x^2*Sqr
t[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 64*a^2*c^4*x^4*Sqrt[1 + c^2*x^2]*Sqrt
[d + c^2*d*x^2] + 160*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 128*a*b
*c*x*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 4*a*b*c*x*Sqrt[d + c^2*d*x
^2]*Cosh[4*ArcSinh[c*x]] + 512*a*b*c*x*Sqrt[d + c^2*d*x^2]*Log[c*x] + 480*
a^2*c*Sqrt[d]*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]]
 - 256*b^2*c*x*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])] + 64*b^
2*c*x*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + b^2*c*x*Sqrt[d + c^2*d*x^
2]*Sinh[4*ArcSinh[c*x]] - 4*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(128*a*Sqrt
[1 + c^2*x^2] + 32*b*c*x*Cosh[2*ArcSinh[c*x]] + b*c*x*Cosh[4*ArcSinh[c*x]]
 - 128*b*c*x*Log[1 - E^(-2*ArcSinh[c*x])] - 64*a*c*x*Sinh[2*ArcSinh[c*x]]
- 4*a*c*x*Sinh[4*ArcSinh[c*x]]) + 8*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(
60*a*c*x + 32*b*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x
]] + b*c*x*Sinh[4*ArcSinh[c*x]])))/(256*x*Sqrt[1 + c^2*x^2])
```

### 3.279.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.17, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6222, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222, 6216, 211, 211, 222, 6216, 211, 222, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{6222}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+5c^2d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6201}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{4}x(c^2dx\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6200}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c}}{}\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6191}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\right.}{\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{262}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\right.}{\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{222}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6198}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}d\left(\frac{\sqrt{c^2d}}{}\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{6213}$$

3.279. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int (c^2x^2+1)^{3/2}dx}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{211}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{ar}\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{211}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+$$
$$5c^2d\left(-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2\right.\right.$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}$$

$$\Big\downarrow \text{222}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$
$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$$\Big\downarrow \text{6216}$$

---

3.279. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx - \frac{1}{4}bc\int \left(c^2x^2+1\right)^{3/2}dx + \frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))\right)}{\sqrt{c^2x^2+1}} - \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + 5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2} - \frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2}\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 211

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx - \frac{1}{4}bc\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx + \frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right) + \frac{1}{4}\left(c^2x^2+1\right)^2(a+\right.}{\sqrt{c^2x^2+1}} - \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + 5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2} - \frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2}\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 211

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx - \frac{1}{4}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx + \frac{1}{2}x\sqrt{c^2x^2+1}\right) + \frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right) + \frac{1}{4}\right.}{\sqrt{c^2x^2+1}} - \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x} + 5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2 - \frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2} - \frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2}\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 222

---

3.279. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\int \frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x\right.\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 6216

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int \sqrt{c^2x^2+1}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\left(c^2x^2+1\right)(a+\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 211

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 222

3.279. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\int \frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\ldots}\right.\right.}{4c\ldots}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 6190

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx)\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\ldots}\right.\right.}{4c\ldots}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 25

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))+\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\ldots}\right.\right.}{4c\ldots}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 3042

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3.279. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-\frac{\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}\right.}{\ldots}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\ldots}\right.\right.}{4c\ldots}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{i\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2(a+b\text{arcsinh}\right.}{\ldots}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\ldots}\right.\right.}{4c\ldots}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$$\Big\downarrow \text{4201}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{4}\left(c^2x^2+1\right)^2\right.}{\ldots}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2\ldots}\right.\right.}{4c\ldots}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$$\Big\downarrow \text{2620}$$

---

3.279. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

$$2bcd^2\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}}\right.\right.\right.}{b}\right.$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2}\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 2715

$$2bcd^2\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a+b\text{ar}\right.\right.}{b}\right.$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2}\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

$\downarrow$ 2838

$$2bcd^2\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a+b\text{arcs}\right.}{b}\right.$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x}+$$

$$5c^2d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2}\right.\right.}{4c}\right.}{2\sqrt{c^2x^2+1}}\right.$$

---

3.279. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

output
```
-(((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x) + 5*c^2*d*((x*(d + c^2
*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*((x*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^2)/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b
*c*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[d + c^2*d*x^2]*((x^2*(a + b*ArcSinh[c*x]
))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2))/Sq
rt[1 + c^2*x^2]))/4 - (b*c*d*Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2)^2*(a + b*
ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c
^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 + c^2*x^2])) + (2*
b*c*d^2*Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + ((1
+ c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 - (b*c*((x*Sqrt[1 + c^2*x^2])/2 + Arc
Sinh[c*x]/(2*c)))/2 - (b*c*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^
2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/4 + (I*((-1/2*I)*(a + b*ArcSinh[c*x])
^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a
 + b*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/b))/
Sqrt[1 + c^2*x^2]
```

### 3.279.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

## 3.279. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## 3.279. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx$

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.279.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| default | $-\frac{a^2(c^2dx^2+d)^{\frac{7}{2}}}{dx}+a^2c^2x(c^2dx^2+d)^{\frac{5}{2}}+\frac{5(c^2dx^2+d)^{\frac{3}{2}}a^2c^2dx}{4}+\frac{15a^2d^2\sqrt{c^2dx^2+d}\,c^2x}{8}+\frac{15a^2c^2d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d}\right.}{8\sqrt{c^2d}}$ |
| parts | $-\frac{a^2(c^2dx^2+d)^{\frac{7}{2}}}{dx}+a^2c^2x(c^2dx^2+d)^{\frac{5}{2}}+\frac{5(c^2dx^2+d)^{\frac{3}{2}}a^2c^2dx}{4}+\frac{15a^2d^2\sqrt{c^2dx^2+d}\,c^2x}{8}+\frac{15a^2c^2d^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d}\right.}{8\sqrt{c^2d}}$ |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(c^2*d*x^2+d)^(7/2)+a^2*c^2*x*(c^2*d*x^2+d)^(5/2)+5/4*(c^2*d*x^2+
d)^(3/2)*a^2*c^2*d*x+15/8*a^2*d^2*(c^2*d*x^2+d)^(1/2)*c^2*x+15/8*a^2*c^2*d
^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/64*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(16*(c^2*x^2+1)^(1/2)*arcsinh(c*x)
^2*x^4*c^4-8*arcsinh(c*x)*c^5*x^5+2*c^4*x^4*(c^2*x^2+1)^(1/2)+72*arcsinh(c
*x)^2*(c^2*x^2+1)^(1/2)*x^2*c^2-72*arcsinh(c*x)*c^3*x^3+40*arcsinh(c*x)^3*
x*c+33*c^2*x^2*(c^2*x^2+1)^(1/2)-64*arcsinh(c*x)^2*x*c+128*arcsinh(c*x)*ln
(1+c*x+(c^2*x^2+1)^(1/2))*x*c+128*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
*x*c-64*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-33*arcsinh(c*x)*c*x+128*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))*x*c+128*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x*c)*d^2
+1/64*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*(32*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)*x^4*c^4-8*c^5*x^5+144*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-
72*c^3*x^3+120*arcsinh(c*x)^2*x*c-128*arcsinh(c*x)*c*x+128*ln((c*x+(c^2*x^
2+1)^(1/2))^2-1)*x*c-128*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-33*c*x)*d^2
```

### 3.279.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

### 3.279.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))^2}{x^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**2, x)
```

### 3.279.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.279.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

# 3.280 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

## 3.280.1 Optimal result

Integrand size = 28, antiderivative size = 687

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{x^3}\,dx = \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2}$$
$$-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}$$
$$+\frac{2}{27}b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}-\frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}$$
$$-\frac{bcd^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{x\sqrt{1+c^2x^2}}+\frac{bc^3d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}}$$
$$-\frac{2bc^5d^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}$$
$$+\frac{5}{2}c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2$$
$$+\frac{5}{6}c^2d(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2-\frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{2x^2}$$
$$-\frac{5c^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$
$$-\frac{b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{1+c^2x^2}}$$
$$-\frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$
$$+\frac{5bc^2d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$
$$+\frac{5b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$
$$-\frac{5b^2c^2d^2\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{1+c^2x^2}}$$

output
```
5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-1/2*(c^2*d*x^2+d)^(5/2)
*(a+b*arcsinh(c*x))^2/x^2+40/9*b^2*c^2*d^2*(c^2*d*x^2+d)^(1/2)+2/27*b^2*c^
2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+5/2*c^2*d^2*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)-5*a*b*c^3*d^2*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*b
^2*c^3*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d^2*(a
+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)+1/3*b*c^3*d^2*x*(
a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^5*d^2*x^3*
(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*c^2*d^2*(a+b*ar
csinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-b^2*c^2*d^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x
^2+1)^(1/2)-5*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b*c^2*d^2*(a+b*arcsinh(c*x))*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b^2
*c^2*d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)-5*b^2*c^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

### 3.280.2 Mathematica [A] (verified)

Time = 8.04 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.44

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
Sqrt[d*(1 + c^2*x^2)]*((7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^4*d^
2*x^2)/3) + 2*a*b*c^2*d^2*(-1/9*(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/
Sqrt[1 + c^2*x^2] + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3)
+ (5*a^2*c^2*d^(5/2)*Log[x])/2 - (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d
*(1 + c^2*x^2)]])/2 + (4*a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[
1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcS
inh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - Pol
yLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + 2*b^2*c^2*d^2*Sqrt[d*(1 +
 c^2*x^2)]*(2 - (2*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 +
(ArcSinh[c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))
/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - Pol
yLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSi
nh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]) + (b^2*c^2*
d^2*Sqrt[d*(1 + c^2*x^2)]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2
+ 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*
ArcSinh[c*x]])))/(108*Sqrt[1 + c^2*x^2]) + (a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^
2)]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])
] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*S...
```

### 3.280.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.77, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.821, Rules used = {6222, 6218, 27, 1578, 1192, 25, 1467, 2009, 6223, 6199, 27, 353, 53, 2009, 6221, 2009, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{6222}$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x^2}dx}{\sqrt{c^2x^2+1}}+\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \; 6218$$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\left(-bc\int -\frac{-c^4x^4-6c^2x^2+3}{3x\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}bc\int \frac{-c^4x^4-6c^2x^2+3}{x\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \; 1578$$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{6}bc\int \frac{-c^4x^4-6c^2x^2+3}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2x^2+1}}-$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \; 1192$$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{b\int -\frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{c^2x^2+1}}{3c^3}+\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(c}{x}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \; 25$$

3.280. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{b\int \frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{c^2x^2+1}}{3c^3}+\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(c}{x}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 1467

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(-\frac{b\int \left(x^4c^4+\frac{3c^4}{1-x^4}+5c^4\right)d\sqrt{c^2x^2+1}}{3c^3}+\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}}{x}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 2009

$$\frac{5}{2}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}dx+$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 6223

$$\frac{5}{2}c^2d\left(-\frac{2bcd\sqrt{c^2dx^2+d}\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3\sqrt{c^2x^2+1}}+d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{3}\left(c^2dx^2\right.\right.$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 6199

---

3.280. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}dx$

$$\frac{5}{2}c^2d\left(d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\int\frac{x(c^2x^2+3)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+\right.}{3\sqrt{c^2x^2+1}}\right.$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 27

$$\frac{5}{2}c^2d\left(d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{3}bc\int\frac{x(c^2x^2+3)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))\right.}{3\sqrt{c^2x^2+1}}\right.$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 353

$$\frac{5}{2}c^2d\left(d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int\frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))\right.}{3\sqrt{c^2x^2+1}}\right.$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 53

$$\frac{5}{2}c^2d\left(d\int\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx-\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a\right.}{3\sqrt{c^2x^2+1}}\right.$$
$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$
$$\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 2009

---

3.280. $\int\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$\frac{5}{2}c^2d\left(d\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{x}dx+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^2x^3(a\right.}{}\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6221}$$

$$\frac{5}{2}c^2d\left(d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsin}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2x^2+1}}dx}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+bx\text{arcsinh}(\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6231}$$

---

3.280. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}dx$

$$\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{2bc\sqrt{c^2dx^2+d}\left(ax+\right.}{\sqrt{c}}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 3042

$$\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{c^2dx^2+d}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 26

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$\downarrow$ 4670

3.280. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(2ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int(a+b\text{arcsinh}(cx))\log\left(1\right.\right.}{\sqrt{c^2x^2+1}}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\downarrow 3011$$

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\downarrow 2720$$

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

$$\downarrow 7143$$

3.280. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

$$\frac{5}{2}c^2d\left(d\left(\frac{i\sqrt{c^2dx^2+d}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2\right.\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{c^2dx^2+d}\left(\frac{1}{3}c^4x^3(a+b\text{arcsinh}(cx))+2c^2x(a+b\text{arcsinh}(cx))-\frac{a+b\text{arcsinh}(cx)}{x}+\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)-\right.}{3c^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{2x^2}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

output
```
-1/2*((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2 + (b*c*d^2*Sqrt[d
+ c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/x) + 2*c^2*x*(a + b*ArcSinh[c*x]) + (
c^4*x^3*(a + b*ArcSinh[c*x]))/3 + (b*(-1/3*(c^4*x^6) - 5*c^4*Sqrt[1 + c^2*
x^2] - 3*c^4*ArcTanh[Sqrt[1 + c^2*x^2]]))/(3*c^3)))/Sqrt[1 + c^2*x^2] + (5
*c^2*d*(((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*d*Sqrt[d
 + c^2*d*x^2]*(-1/6*(b*c*((4*Sqrt[1 + c^2*x^2])/c^2 + (2*(1 + c^2*x^2)^(3/
2))/(3*c^2))) + x*(a + b*ArcSinh[c*x]) + (c^2*x^3*(a + b*ArcSinh[c*x]))/3)
)/(3*Sqrt[1 + c^2*x^2]) + d*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 -
(2*b*c*Sqrt[d + c^2*d*x^2]*(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*
x]))/Sqrt[1 + c^2*x^2] + (I*Sqrt[d + c^2*d*x^2]*((2*I)*(a + b*ArcSinh[c*x]
)^2*ArcTanh[E^ArcSinh[c*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -
E^ArcSinh[c*x]]) + b*PolyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcS
inh[c*x])*PolyLog[2, E^ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/Sq
rt[1 + c^2*x^2])))/2
```

### 3.280.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## 3.280. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

rule 6221
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]]   Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x]
, x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   I
nt[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

3.280. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx$

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.280.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.05

| method | result |
|---|---|
| default | $a^2\left(-\frac{(c^2dx^2+d)^{\frac{7}{2}}}{2dx^2}+\frac{5c^2\left(\frac{(c^2dx^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2dx^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b^2\sqrt{d(c^2}}{}$ |
| parts | $a^2\left(-\frac{(c^2dx^2+d)^{\frac{7}{2}}}{2dx^2}+\frac{5c^2\left(\frac{(c^2dx^2+d)^{\frac{5}{2}}}{5}+d\left(\frac{(c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{c^2dx^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{c^2dx^2+d}}{x}\right)\right)\right)\right)}{2}\right)+\frac{b^2\sqrt{d(c^2}}{}$ |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/d/x^2*(c^2*d*x^2+d)^(7/2)+5/2*c^2*(1/5*(c^2*d*x^2+d)^(5/2)+d*(1/
3*(c^2*d*x^2+d)^(3/2)+d*((c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(c^
2*d*x^2+d)^(1/2))/x)))))+1/54*b^2*(d*(c^2*x^2+1))^(1/2)*(18*(c^2*x^2+1)^(1
/2)*arcsinh(c*x)^2*x^4*c^4-12*arcsinh(c*x)*c^5*x^5+4*c^4*x^4*(c^2*x^2+1)^(
1/2)+126*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2*c^2+135*arcsinh(c*x)^2*ln(1-
c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-135*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/
2))*x^2*c^2-252*arcsinh(c*x)*c^3*x^3-270*arcsinh(c*x)*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))*x^2*c^2+270*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x^
2*c^2+244*c^2*x^2*(c^2*x^2+1)^(1/2)-108*arctanh(c*x+(c^2*x^2+1)^(1/2))*x^2
*c^2+270*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-270*polylog(3,c*x+(c^2*
x^2+1)^(1/2))*x^2*c^2-27*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-54*arcsinh(c*x)*
c*x)*d^2/(c^2*x^2+1)^(1/2)/x^2+1/9*a*b*(d*(c^2*x^2+1))^(1/2)*(6*arcsinh(c*
x)*(c^2*x^2+1)^(1/2)*x^4*c^4-2*c^5*x^5+42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x
^2*c^2+45*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-45*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2-42*c^3*x^3+45*polylog(2,c*x+(c^2*x^2+
1)^(1/2))*x^2*c^2-45*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2-9*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)-9*c*x)*d^2/(c^2*x^2+1)^(1/2)/x^2
```

## 3.280.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}\,(b\,\text{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

### 3.280.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))^2}{x^3}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**3,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**3, x)
```

### 3.280.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxim
a")
```

output
```
-1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
 5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d
*x^2 + d)^(7/2)/(d*x^2))*a^2 + integrate((c^2*d*x^2 + d)^(5/2)*b^2*log(c*x
 + sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c
^2*x^2 + 1))/x^3, x)
```

### 3.280.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x^3}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3, x)
```

# 3.281 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$



## 3.281.1 Optimal result

Integrand size = 28, antiderivative size = 561

$$\begin{aligned}
\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx &= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} \\
&- \frac{b^2c^2d^2(1+c^2x^2)\sqrt{d+c^2dx^2}}{3x} - \frac{23b^2c^3d^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{12\sqrt{1+c^2x^2}} \\
&- \frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&+ \frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\
&- \frac{bcd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3x^2} \\
&+ \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{7c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{1+c^2x^2}} \\
&- \frac{5c^2d(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3x} \\
&- \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3} + \frac{5c^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6b\sqrt{1+c^2x^2}} \\
&+ \frac{14bc^3d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}} \\
&- \frac{7b^2c^3d^2\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x-1/3*(c^2*d*x^2+d)^(5
/2)*(a+b*arcsinh(c*x))^2/x^3+7/12*b^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)-1/3*b^
2*c^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*
(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2+5/2*c^4*d^2*x*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)-23/12*b^2*c^3*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-5/2*b*c^5*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)+7/3*c^3*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)+5/6*c^3*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c
^2*x^2+1)^(1/2)+14/3*b*c^3*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^
(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/3*b^2*c^3*d^2*polylog(2,
1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+7/3*b*c
^3*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

### 3.281.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.10

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \frac{d^2\Big(-8abcx\sqrt{d+c^2dx^2}-8a^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}-56a^2c^2x^2\sqrt{\vphantom{x}}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
(d^2*(-8*a*b*c*x*Sqrt[d + c^2*d*x^2] - 8*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2] - 56*a^2*c^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 8*b^2*c^
2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 12*a^2*c^4*x^4*Sqrt[1 + c^2*
x^2]*Sqrt[d + c^2*d*x^2] + 20*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]
^3 - 6*a*b*c^3*x^3*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 112*a*b*c^3*
x^3*Sqrt[d + c^2*d*x^2]*Log[c*x] + 60*a^2*c^3*Sqrt[d]*x^3*Sqrt[1 + c^2*x^2
]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 56*b^2*c^3*x^3*Sqrt[d + c^2*d
*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])] + 3*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*
Sinh[2*ArcSinh[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(4*b*c*x + 8*a
*Sqrt[1 + c^2*x^2] + 56*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 3*b*c^3*x^3*Cosh[2*A
rcSinh[c*x]] - 56*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])] - 6*a*c^3*x^3*Sin
h[2*ArcSinh[c*x]]) + 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(30*a*c^3*x^3
- 4*b*(-7*c^3*x^3 + Sqrt[1 + c^2*x^2] + 7*c^2*x^2*Sqrt[1 + c^2*x^2]) + 3*b
*c^3*x^3*Sinh[2*ArcSinh[c*x]])))/(24*x^3*Sqrt[1 + c^2*x^2])
```

### 3.281.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{ 6222}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{x^3}dx}{3\sqrt{c^2x^2+1}}+\frac{5}{3}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 6217}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bc\int \frac{\left(c^2x^2+1\right)^{3/2}}{x^2}dx-\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{2x^2}\right)}{3\sqrt{c^2x^2+1}}+\frac{5}{3}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bc\left(3c^2\int \sqrt{c^2x^2+1}dx-\frac{\left(c^2x^2+1\right)^{3/2}}{x}\right)-\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsin}}{2x^2}}{3\sqrt{c^2x^2+1}}$$
$$\frac{5}{3}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\int \frac{\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))}{x}dx+\frac{1}{2}bc\left(3c^2\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)-\frac{\left(c^2x^2+1\right)^{3/2}}{x}\right)-}{3\sqrt{c^2x^2+1}}$$
$$\frac{5}{3}c^2d\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\int\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx-\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{2x^2}+\frac{1}{2}bc\left(3c^2\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2}\right.\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{5}{3}c^2d\int\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6216

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int\sqrt{c^2x^2+1}dx+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))\right)-\frac{(c^2x^2+1)^2(a+}{2}\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{5}{3}c^2d\int\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 211

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(c\right.\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{5}{3}c^2d\int\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 222

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)\right)\right.}{3\sqrt{c^2x^2+1}}$$

$$\frac{5}{3}c^2d\int\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6190

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{\int-\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}(c^2x^2+1)(a+b\text{arcsinh}\right.\right.}{}$$

$$\frac{5}{3}c^2d\int\frac{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 25

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3.281. $\int\frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

$$2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(-\frac{\int(a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx\right.\right.$$

$$\frac{5}{3}c^2d\int\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 3042$$

$$2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(-\frac{\int-i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arc}\right.\right.$$

$$\frac{5}{3}c^2d\int\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 26$$

$$2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arc}\right.\right.$$

$$\frac{5}{3}c^2d\int\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 4201$$

$$2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{2}\left(c^2x^2\right.\right.\right.$$

$$\frac{5}{3}c^2d\int\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2620$$

$$2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcs}}{b}}\right.\right.\right.}{b}\right.\right.$$

$$\frac{5}{3}c^2d\int\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

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3.281. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

$\downarrow$ 2715

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(\right.\right.}{b}\right.\right.}{}$$

$$\frac{5}{3}c^2d\int\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx-\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 2838

$$\frac{5}{3}c^2d\int\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x^2}dx+$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6222

$$\frac{5}{3}c^2d\left(3c^2d\int\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{2bcd\sqrt{c^2dx^2+d}\int\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/}}{}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\downarrow$ 6200

$$\frac{5}{3}c^2d\left(3c^2d\left(-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+\right.\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

3.281. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

$\Big\downarrow$ 6191

$$\frac{\frac{5}{3}c^2d\left(3c^2d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\right.\right.}{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 262

$$\frac{\frac{5}{3}c^2d\left(3c^2d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcs}}{\sqrt{c^2}}}{2\sqrt{c^2x^2+1}}\right.\right.}{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 222

$$\frac{\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\int\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}+3c^2d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(\right.\right.}{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 6198

3.281. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\int\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))}{x}dx}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{x}+3c^2d\left(\frac{\sqrt{c^2dx^2+d}(a}{6bc\sqrt{}}\right.\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{ PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{6216}$$

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(\int\frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\int\sqrt{c^2x^2+1}dx+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))\right)}{\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+\right.}{}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{ PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(\int\frac{a+b\text{arcsinh}(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{ PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{222}$$

3.281. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(\int\frac{a+b\text{arcsinh}(cx)}{x}dx+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 6190

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{\int-\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 25

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{\int(a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 3042

3.281. $\int\frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}dx$

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(-\frac{\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arc}\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{26}$$

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{i\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{b}+\frac{1}{2}\left(c^2x^2+1\right)(a+b\text{arc}\right.}{\sqrt{c^2x^2+1}}\right.$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.}{}$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{4201}$$

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3.281. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

$$\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{i\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{b}+\frac{1}{2}(c^2x^2\right.}{\sqrt{c^2x^2+1}}\right.$$

$$2bcd^2\sqrt{c^2dx^2+d}\left(2c^2\left(\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{1}{2}i(a-\right.}{b}\right.\right.$$

$$\frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{3x^3}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

output
```
$Aborted
```

### 3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.281. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6216
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSinh[c*x])/(2*p)), x] + (Simp[d
   Int[(d + e*x^2)^(p - 1)*((a + b*ArcSinh[c*x])/x), x], x] - Simp[b*c*(d^p
/(2*p))   Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
 && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6217
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSinh[c
*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1
 + c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

## 3.281. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx$

rule 6222
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m
 + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(
m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x
^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

## 3.281.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2296 vs. $2(511) = 1022$.

Time = 0.38 (sec) , antiderivative size = 2297, normalized size of antiderivative = 4.09

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2297 |
| parts | Expression too large to display | 2297 |

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x^3*(12*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)*x^4*c^4-6*c^5*x^5+30*arcsinh(c*x)^2*x^3*c^3-56*arcsinh(c*x)*
c^3*x^3+56*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3-56*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)*x^2*c^2-3*c^3*x^3-8*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-4*c*x)*d^2-1
/3*a^2/d/x^3*(c^2*d*x^2+d)^(7/2)+4/3*a^2*c^4*x*(c^2*d*x^2+d)^(5/2)-14/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d^2+14/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c
^3*d^2-7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6
-1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+5/3*
a^2*c^4*d*x*(c^2*d*x^2+d)^(3/2)+5/2*a^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)+5/2*
a^2*c^4*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-4/
3*a^2*c^2/d/x*(c^2*d*x^2+d)^(7/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^
4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^
6*d^2/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*x+
5/6*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c^3*d^2+14/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))*c^3*d^2+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x
)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3*d^2+21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63
*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+7/3*b^2*(d*(c^2*x^2+1))^(
1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3...
```

### 3.281.5 Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

### 3.281.6 Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\int \frac{\left(d(c^2x^2+1)\right)^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**4,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**4, x)
```

### 3.281.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.281.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

# 3.282 $\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx$



## 3.282.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{15\text{arcsinh}(ax)}{64a^5} + \frac{3x^2\text{arcsinh}(ax)}{8a^3} - \frac{x^4\text{arcsinh}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a^2} + \frac{\text{arcsinh}(ax)^3}{8a^5}$$

output
```
15/64*arcsinh(a*x)/a^5+3/8*x^2*arcsinh(a*x)/a^3-1/8*x^4*arcsinh(a*x)/a+1/8
*arcsinh(a*x)^3/a^5-15/64*x*(a^2*x^2+1)^(1/2)/a^4+1/32*x^3*(a^2*x^2+1)^(1/
2)/a^2-3/8*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^2*(
a^2*x^2+1)^(1/2)/a^2
```

## 3.282.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{ax\sqrt{1+a^2x^2}(-15+2a^2x^2) + (15+24a^2x^2-8a^4x^4)\,\text{arcsinh}(ax) + 8ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\,\text{arcsinh}(}{64a^5}$$

input
```
Integrate[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2) + (15 + 24*a^2*x^2 - 8*a^4*x^4)*A
rcSinh[a*x] + 8*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 + 8*
ArcSinh[a*x]^3)/(64*a^5)
```

## 3.282.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6227, 6191, 262, 262, 222, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int x^3\text{arcsinh}(ax)dx}{2a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\int \frac{x^4}{\sqrt{a^2x^2+1}}dx}{2a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\int \frac{x^2}{\sqrt{a^2x^2+1}}dx}{4a^2}\right)}{2a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{4a^2}\right)}{2a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}$$

$$\Big\downarrow \; 222$$

$$-\frac{3\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{4a^2}\right)}{2a}$$

$$\Big\downarrow \; 6227$$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int x\text{arcsinh}(ax)dx}{a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{4a^2}\right)}{2a}$$

$$\Big\downarrow \; 6191$$

$$-\frac{3\left(-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\int \frac{x^2}{\sqrt{a^2x^2+1}}dx}{a}-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{4a^2}\right)}{2a}$$

$$\Big\downarrow \; 262$$

$$-\frac{3\left(-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{a}-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{4a^2}\right)}{2a}$$

$$\Big\downarrow \; 222$$

---

3.282. $\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{a}\right)}{4a^2}+$$

$$\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{4a^2}\right)}{2a}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-$$

$$\frac{3\left(-\frac{\text{arcsinh}(ax)^3}{6a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{a}\right)}{4a^2}-$$

$$\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{4a^2}\right)}{2a}$$

input
```
Int[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*a^2) - ((x^4*ArcSinh[a*x])/4 - (
a*((x^3*Sqrt[1 + a^2*x^2])/(4*a^2) - (3*((x*Sqrt[1 + a^2*x^2])/(2*a^2) - A
rcSinh[a*x]/(2*a^3)))/(4*a^2)))/4)/(2*a) - (3*((x*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3) - ((x^2*ArcSinh[a*x])/2 - (a*((
x*Sqrt[1 + a^2*x^2])/(2*a^2) - ArcSinh[a*x]/(2*a^3)))/2)/a))/(4*a^2)
```

### 3.282.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.282. $\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.282.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| default | $\frac{16a^3x^3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}-8a^4x^4\operatorname{arcsinh}(ax)+2a^3x^3\sqrt{a^2x^2+1}-24\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax+24a^2x^2\operatorname{arcsinh}(ax)+8\operatorname{arcsinh}(a}{64a^5}$ |

input
```
int(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-8*a^4*x^4*arcsinh(a*x)+2
*a^3*x^3*(a^2*x^2+1)^(1/2)-24*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+24*a^2*
x^2*arcsinh(a*x)+8*arcsinh(a*x)^3-15*a*x*(a^2*x^2+1)^(1/2)+15*arcsinh(a*x)
)/a^5
```

### 3.282.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{8\left(2\,a^3x^3 - 3\,ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 8\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - \left(8\,a^4x^4 - 24\,a^2x^2 - 15\right)\log\left(ax+\sqrt{a^2x^2+1}\right) + \left(2\,a^3x^3 - 15\,ax\right)\sqrt{a^2x^2+1}}{64\,a^5}$$

input
```
integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/64*(8*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
^2 + 8*log(a*x + sqrt(a^2*x^2 + 1))^3 - (8*a^4*x^4 - 24*a^2*x^2 - 15)*log(
a*x + sqrt(a^2*x^2 + 1)) + (2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1))/a^5
```

### 3.282.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{x^4 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{x^4 \operatorname{asinh}(ax)}{8a} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a^2} + \frac{x^3\sqrt{a^2x^2+1}}{32a^2} + \frac{3x^2\operatorname{asinh}(ax)}{8a^3} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{8a^4} - \frac{15x\sqrt{a^2x^2+1}}{64a^4} + \frac{\operatorname{asinh}^3(ax)}{8a^5} + \frac{15\operatorname{asinh}(ax)}{64a^5} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**4*asinh(a*x)/(8*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)**2
/(4*a**2) + x**3*sqrt(a**2*x**2 + 1)/(32*a**2) + 3*x**2*asinh(a*x)/(8*a**3
) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(8*a**4) - 15*x*sqrt(a**2*x**2 +
 1)/(64*a**4) + asinh(a*x)**3/(8*a**5) + 15*asinh(a*x)/(64*a**5), Ne(a, 0)
), (0, True))
```

### 3.282.7 Maxima [F]

$$\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{arsinh}\,(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.282.8 Giac [F]

$$\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{arsinh}\,(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.283 $\int \frac{x^3\mathbf{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$



## 3.283.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = -\frac{14\sqrt{1+a^2x^2}}{9a^4}+\frac{2(1+a^2x^2)^{3/2}}{27a^4}+\frac{4x\text{arcsinh}(ax)}{3a^3}-\frac{2x^3\text{arcsinh}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{3a^2}$$

output
```
2/27*(a^2*x^2+1)^(3/2)/a^4+4/3*x*arcsinh(a*x)/a^3-2/9*x^3*arcsinh(a*x)/a-1
4/9*(a^2*x^2+1)^(1/2)/a^4-2/3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2
*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

## 3.283.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{2(-20+a^2x^2)\sqrt{1+a^2x^2}-6ax(-6+a^2x^2)\,\text{arcsinh}(ax)+9(-2+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{27a^4}$$

input
```
Integrate[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] - 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] +
 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(27*a^4)
```

### 3.283.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6227, 6191, 243, 53, 2009, 6213, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\int x^2\text{arcsinh}(ax)dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{3}a\int \frac{x^3}{\sqrt{a^2x^2+1}}dx\right)}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\int \frac{x^2}{\sqrt{a^2x^2+1}}dx^2\right)}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\int \left(\frac{\sqrt{a^2x^2+1}}{a^2}-\frac{1}{a^2\sqrt{a^2x^2+1}}\right)dx^2\right)}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2(a^2x^2+1)^{3/2}}{3a^4}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\right)}{3a}$$

$$\Big\downarrow \text{6213}$$

$$-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\int \text{arcsinh}(ax)dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2(a^2x^2+1)^{3/2}}{3a^4}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\right)}{3a}$$

$$\Big\downarrow \text{6187}$$

$$-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-a\int\frac{x}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2(a^2x^2+1)^{3/2}}{3a^4}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\right)}{3a}$$

$$\Big\downarrow \text{241}$$

$$\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2(a^2x^2+1)^{3/2}}{3a^4}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\right)}{3a}$$

input
```
Int[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a^2) - (2*(-1/6*(a*((-2*Sqrt[1 +
 a^2*x^2])/a^4 + (2*(1 + a^2*x^2)^(3/2))/(3*a^4))) + (x^3*ArcSinh[a*x])/3)
)/(3*a) - (2*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/a^2 - (2*(-(Sqrt[1 + a^2*
x^2]/a) + x*ArcSinh[a*x]))/a))/(3*a^2)
```

### 3.283.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

---

3.283. $\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.283.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| default | $\frac{9a^4x^4 \operatorname{arcsinh}(ax)^2-9\operatorname{arcsinh}(ax)^2a^2x^2-6a^3x^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}+2a^4x^4-38a^2x^2-18\operatorname{arcsinh}(ax)^2+36\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{27a^4\sqrt{a^2x^2+1}}$ |

input
```
int(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27/a^4/(a^2*x^2+1)^(1/2)*(9*a^4*x^4*arcsinh(a*x)^2-9*arcsinh(a*x)^2*a^2*
x^2-6*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+2*a^4*x^4-38*a^2*x^2-18*arcsi
nh(a*x)^2+36*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-40)
```

### 3.283.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx$$
$$= \frac{9\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\left(a^3x^3-6\,ax\right)\log\left(ax+\sqrt{a^2x^2+1}\right) + 2\sqrt{a^2x^2+1}(a^2x}{27\,a^4}$$

input
```
integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6
*(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1)*(a^2
*x^2 - 20))/a^4
```

### 3.283.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\, dx$$
$$= \begin{cases} -\frac{2x^3 \operatorname{asinh}(ax)}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^2} + \frac{2x^2\sqrt{a^2x^2+1}}{27a^2} + \frac{4x\operatorname{asinh}(ax)}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^4} - \frac{40\sqrt{a^2x^2+1}}{27a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-2*x**3*asinh(a*x)/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)*
*2/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)/(27*a**2) + 4*x*asinh(a*x)/(3*a**
3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**4) - 40*sqrt(a**2*x**2 + 1)
/(27*a**4), Ne(a, 0)), (0, True))
```

### 3.283.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)^2 + \frac{2\left(\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)}{27\,a^2} - \frac{2\,(a^2x^3-6\,x)\,\text{arsinh}\,(ax)}{9\,a^3}$$

input
```
integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^2 +
 2/27*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)/a^2 - 2/9*(a^2*x^
3 - 6*x)*arcsinh(a*x)/a^3
```

### 3.283.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3\,\text{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.284 $\int \frac{x^2 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx$



## 3.284.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{x^2 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{x\sqrt{1+a^2x^2}}{4a^2} - \frac{\text{arcsinh}(ax)}{4a^3} - \frac{x^2 \text{arcsinh}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2a^2} - \frac{\text{arcsinh}(ax)^3}{6a^3}$$

output
```
-1/4*arcsinh(a*x)/a^3-1/2*x^2*arcsinh(a*x)/a-1/6*arcsinh(a*x)^3/a^3+1/4*x*
(a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

## 3.284.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{3ax\sqrt{1+a^2x^2} - 3(1+2a^2x^2)\,\text{arcsinh}(ax) + 6ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 - 2\text{arcsinh}(ax)^3}{12a^3}$$

input
```
Integrate[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(3*a*x*Sqrt[1 + a^2*x^2] - 3*(1 + 2*a^2*x^2)*ArcSinh[a*x] + 6*a*x*Sqrt[1 +
 a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3)/(12*a^3)
```

### 3.284.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int x\text{arcsinh}(ax)dx}{a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\int \frac{x^2}{\sqrt{a^2x^2+1}}dx}{a}-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{a}-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{a}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{\text{arcsinh}(ax)^3}{6a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{a}$$

input
```
Int[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3) - ((
x^2*ArcSinh[a*x])/2 - (a*((x*Sqrt[1 + a^2*x^2])/(2*a^2) - ArcSinh[a*x]/(2*
a^3)))/2)/a
```

### 3.284.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.284. $\int \frac{x^2 \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

### 3.284.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $-\frac{-6\,\mathrm{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax+6a^2x^2\,\mathrm{arcsinh}(ax)+2\,\mathrm{arcsinh}(ax)^3-3ax\sqrt{a^2x^2+1}+3\,\mathrm{arcsinh}(ax)}{12a^3}$ | 69 |

input
```
int(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-6*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+6*a^2*x^2*arcsinh(a*x)+2*ar
csinh(a*x)^3-3*a*x*(a^2*x^2+1)^(1/2)+3*arcsinh(a*x))/a^3
```

### 3.284.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{x^2\mathrm{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{6\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\log\left(ax+\sqrt{a^2x^2+1}\right)^3+3\sqrt{a^2x^2+1}ax-3\left(2a^2x^2+1\right)\log\left(a\right.}{12\,a^3}$$

input
```
integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*log(a*x +
 sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)))/a^3
```

### 3.284.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{x^2\mathrm{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{x^2\,\mathrm{asinh}\,(ax)}{2a}+\frac{x\sqrt{a^2x^2+1}\,\mathrm{asinh}^2\,(ax)}{2a^2}+\frac{x\sqrt{a^2x^2+1}}{4a^2}-\frac{\mathrm{asinh}^3\,(ax)}{6a^3}-\frac{\mathrm{asinh}\,(ax)}{4a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**2*asinh(a*x)/(2*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(2
*a**2) + x*sqrt(a**2*x**2 + 1)/(4*a**2) - asinh(a*x)**3/(6*a**3) - asinh(a
*x)/(4*a**3), Ne(a, 0)), (0, True))
```

### 3.284.7 Maxima [F]

$$\int \frac{x^2 \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.284.8 Giac [F]

$$\int \frac{x^2 \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2 \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.285 $\int \frac{x \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx$



## 3.285.1 Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{x \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{2\sqrt{1+a^2x^2}}{a^2} - \frac{2x \text{arcsinh}(ax)}{a} + \frac{\sqrt{1+a^2x^2} \text{arcsinh}(ax)^2}{a^2}$$

output
```
-2*x*arcsinh(a*x)/a+2*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a*x)^2*(a^2*x^2+1)^(1/
2)/a^2
```

## 3.285.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} dx = \frac{2\sqrt{1+a^2x^2} - 2ax \text{arcsinh}(ax) + \sqrt{1+a^2x^2} \text{arcsinh}(ax)^2}{a^2}$$

input
```
Integrate[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2] - 2*a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
^2)/a^2
```

### 3.285.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6213, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{6213}$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\int \text{arcsinh}(ax)dx}{a}$$
$$\Big\downarrow \text{6187}$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-a\int \frac{x}{\sqrt{a^2x^2+1}}dx\right)}{a}$$
$$\Big\downarrow \text{241}$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}$$

input
```
Int[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/a^2 - (2*(-(Sqrt[1 + a^2*x^2]/a) + x*Ar
cSinh[a*x]))/a
```

#### 3.285.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.285.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{arcsinh}(ax)^2 a^2 x^2+\operatorname{arcsinh}(ax)^2-2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+2a^2x^2+2}{a^2\sqrt{a^2x^2+1}}$ | 64 |

input
```
int(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^2*a^2*x^2+arcsinh(a*x)^2-2*arcsinh(a
*x)*(a^2*x^2+1)^(1/2)*a*x+2*a^2*x^2+2)
```

### 3.285.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{x\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = -\frac{2\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\sqrt{a^2x^2+1}}{a^2}$$

input
```
integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(2*a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^
2*x^2 + 1))^2 - 2*sqrt(a^2*x^2 + 1))/a^2
```

### 3.285.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{2x\operatorname{asinh}(ax)}{a} + \frac{\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{a^2} + \frac{2\sqrt{a^2x^2+1}}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-2*x*asinh(a*x)/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**2/a**2 + 2*
sqrt(a**2*x**2 + 1)/a**2, Ne(a, 0)), (0, True))
```

### 3.285.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)^2}{a^2} - \frac{2\left(ax\,\text{arsinh}(ax) - \sqrt{a^2x^2+1}\right)}{a^2}$$

input
```
integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a^2 - 2*(a*x*arcsinh(a*x) - sqrt(a^2*x^2
+ 1))/a^2
```

### 3.285.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2} - \frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right) - \frac{\sqrt{a^2x^2+1}}{a}\right)}{a}$$

input
```
integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt
(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a)/a
```

### 3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x\,\mathrm{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.286 $\int \frac{\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$



## 3.286.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^3}{3a}$$

output
```
1/3*arcsinh(a*x)^3/a
```

## 3.286.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^3}{3a}$$

input
```
Integrate[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^3/(3*a)
```

## 3.286.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$
$$\Big\downarrow \; 6198$$
$$\frac{\text{arcsinh}(ax)^3}{3a}$$

input
```
Int[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^3/(3*a)
```

### 3.286.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.286.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{arcsinh}(ax)^3}{3a}$ | 12 |
| default | $\frac{\text{arcsinh}(ax)^3}{3a}$ | 12 |

input
```
int(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsinh(a*x)^3/a
```

### 3.286.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3\,a}$$

input
```
integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/a
```

### 3.286.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^3(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((asinh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

### 3.286.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arsinh}(ax)^3}{3\,a}$$

input
```
integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsinh(a*x)^3/a
```

### 3.286.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.286.9 Mupad [B] (verification not implemented)

Time = 3.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}(a\,x)^3}{3\,a}$$

input
```
int(asinh(a*x)^2/(a^2*x^2 + 1)^(1/2),x)
```

output
```
asinh(a*x)^3/(3*a)
```

# 3.287 $\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$



## 3.287.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = & -2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) \\ & - 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) \\ & + 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) \\ & + 2\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right) - 2\,\text{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right)\end{aligned}$$

output
```
-2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,
-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+2*
polylog(3,-a*x-(a^2*x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

## 3.287.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = & \,\text{arcsinh}(ax)^2\log\left(1-e^{-\text{arcsinh}(ax)}\right) - \text{arcsinh}(ax)^2\log\left(1+e^{-\text{arcsinh}(ax)}\right) \\ & + 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(ax)}\right) \\ & - 2\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{-\text{arcsinh}(ax)}\right) \\ & + 2\,\text{PolyLog}\left(3,-e^{-\text{arcsinh}(ax)}\right) - 2\,\text{PolyLog}\left(3,e^{-\text{arcsinh}(ax)}\right)\end{aligned}$$

input
```
Integrate[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] - ArcSinh[a*x]^2*Log[1 + E^(-Arc
Sinh[a*x])] + 2*ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*ArcSinh[a*
x]*PolyLog[2, E^(-ArcSinh[a*x])] + 2*PolyLog[3, -E^(-ArcSinh[a*x])] - 2*Po
lyLog[3, E^(-ArcSinh[a*x])]
```

### 3.287.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6231}$$

$$\int \frac{\text{arcsinh}(ax)^2}{ax}d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{3042}$$

$$\int i\text{arcsinh}(ax)^2\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{26}$$

$$i\int \text{arcsinh}(ax)^2\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{4670}$$

$$i\left(2i\int \text{arcsinh}(ax)\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-2i\int \text{arcsinh}(ax)\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2ia\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\right)+2i\left(\int \text{PolyLog}\left(2\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$i\left(-2i\left(\int e^{-\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\right)+2i\left(\int e\right.\right.$$

$$\Big\downarrow \; 7143$$

$$i\left(2i\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)-2i\left(\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\right)\right.$$

input
```
Int[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
I*((2*I)*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - (2*I)*(-(ArcSinh[a*x]*Po
lyLog[2, -E^ArcSinh[a*x]]) + PolyLog[3, -E^ArcSinh[a*x]]) + (2*I)*(-(ArcSi
nh[a*x]*PolyLog[2, E^ArcSinh[a*x]]) + PolyLog[3, E^ArcSinh[a*x]]))
```

### 3.287.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.287.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| default | $-\operatorname{arcsinh}(ax)^2 \ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 2\operatorname{arcsinh}(ax)\operatorname{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right) + 2\operatorname{pol}$ |

input
```
int(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-a*x-
(a^2*x^2+1)^(1/2))+2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^2*ln(1
-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-2*
polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

### 3.287.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^3 + x), x)
```

### 3.287.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^2(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**2/x/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**2/(x*sqrt(a**2*x**2 + 1)), x)
```

### 3.287.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.287.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.288 $\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$



## 3.288.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = -a\text{arcsinh}(ax)^2 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{x} + 2a\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) + a\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)$$

output
```
-a*arcsinh(a*x)^2+2*a*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+a*polyl
og(2,(a*x+(a^2*x^2+1)^(1/2))^2)-arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x
```

## 3.288.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = a\left(\text{arcsinh}(ax)\left(\text{arcsinh}(ax) - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{ax} + 2\log\left(1-e^{-2\text{arcsinh}(ax)}\right)\right) - \text{PolyLog}\left(2,e^{-2\text{arcsinh}(ax)}\right)\right)$$

input
```
Integrate[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
a*(ArcSinh[a*x]*(ArcSinh[a*x] - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(a*x) + 2
*Log[1 - E^(-2*ArcSinh[a*x])]) - PolyLog[2, E^(-2*ArcSinh[a*x])])
```

### 3.288.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6215, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{6215}$$
$$2a\int \frac{\text{arcsinh}(ax)}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}$$
$$\Big\downarrow \text{6190}$$
$$2a\int \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}+2a\int -i\text{arcsinh}(ax)\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)$$
$$\Big\downarrow \text{26}$$
$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\int \text{arcsinh}(ax)\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)$$
$$\Big\downarrow \text{4199}$$
$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(2i\int -\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)$$
$$\Big\downarrow \text{25}$$
$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)$$
$$\Big\downarrow \text{2620}$$
$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-$$
$$2ia\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\log\left(1-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right)-\frac{1}{2}i\text{arcsin}\right.$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)$$

input
```
Int[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x) - (2*I)*a*((-1/2*I)*ArcSinh[a*x]^2
 - (2*I)*(-1/2*(ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])]) - PolyLog[2, E^(
2*ArcSinh[a*x])]/4))
```

### 3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.288. $\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

### 3.288.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

| method | result |
|---|---|
| default | $\frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^2}{x} - 2a\operatorname{arcsinh}(ax)^2 + 2a\operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 2a\,\mathrm{polyl}$ |

input
```
int(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^2-2*a*arcsinh(a*x)^2+2*a*arcsinh(a*
x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*a*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*a*a
rcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2
))
```

### 3.288.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^4 + x^2), x)
```

### 3.288.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^2(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**2/x**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**2/(x**2*sqrt(a**2*x**2 + 1)), x)
```

### 3.288.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2
 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(sqrt(a^2*x^2
 + 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

### 3.288.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.289 $\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$



## 3.289.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = &-\frac{a\text{arcsinh}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2}\\ &+ a^2\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)\\ &+ a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\\ &- a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\\ &- a^2\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right) + a^2\,\text{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right)\end{aligned}$$

output
```
-a*arcsinh(a*x)/x+a^2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-a^2*ar
ctanh((a^2*x^2+1)^(1/2))+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2)
)-a^2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^
2*x^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^2*(a
^2*x^2+1)^(1/2)/x^2
```

### 3.289.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a^2\Bigg(-4\operatorname{arcsinh}(ax)\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) - \operatorname{arcsinh}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) - 4\operatorname{arcsinh}(ax)^2\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) + 4\operatorname{arcsinh}(ax)^2\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) + 8\log\left(\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\right) - 8\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) + 8\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right) - 8\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) + 8\operatorname{PolyLog}\left(3,e^{-\operatorname{arcsinh}(ax)}\right) - \operatorname{arcsinh}(ax)^2\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) + 4\operatorname{arcsinh}(ax)\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\Bigg)$$

input
```
Integrate[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

output
```
(a^2*(-4*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]^2*Csch[ArcSinh[a
*x]/2]^2 - 4*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]^2*
Log[1 + E^(-ArcSinh[a*x])] + 8*Log[Tanh[ArcSinh[a*x]/2]] - 8*ArcSinh[a*x]*
PolyLog[2, -E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x
])] - 8*PolyLog[3, -E^(-ArcSinh[a*x])] + 8*PolyLog[3, E^(-ArcSinh[a*x])] -
 ArcSinh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 + 4*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/
2]))/8
```

### 3.289.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6224, 6191, 243, 73, 221, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx + a\int \frac{\text{arcsinh}(ax)}{x^2}dx - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 6191}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx + a\left(a\int \frac{1}{x\sqrt{a^2x^2+1}}dx - \frac{\text{arcsinh}(ax)}{x}\right) - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx + a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2 - \frac{\text{arcsinh}(ax)}{x}\right) - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx + a\left(\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a} - \frac{\text{arcsinh}(ax)}{x}\right) - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx + a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{\text{arcsinh}(ax)}{x}\right) - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 6231}$$

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{ax}d\text{arcsinh}(ax) + a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{\text{arcsinh}(ax)}{x}\right) - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}a^2\int i\text{arcsinh}(ax)^2\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax) + a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{\text{arcsinh}(ax)}{x}\right) - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{2}ia^2\int \text{arcsinh}(ax)^2\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax) + a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{\text{arcsinh}(ax)}{x}\right) - \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 4670}$$

---

3.289. $\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$

$$-\frac{1}{2}ia^2\left(2i\int \text{arcsinh}(ax)\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-2i\int \text{arcsinh}(ax)\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right.$$
$$a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$\downarrow$ 3011

$$-\frac{1}{2}ia^2\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\right)+2i\left(\int \text{PolyI}\right.\right.$$
$$a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$\downarrow$ 2720

$$-\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\right)+2\right.$$
$$a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

$\downarrow$ 7143

$$-\frac{1}{2}ia^2\left(2i\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)-2i\left(\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(a}\right.\right.\right.$$
$$a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$

input
```
Int[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

output
```
-1/2*(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x^2 + a*(-(ArcSinh[a*x]/x) - a*Arc
Tanh[Sqrt[1 + a^2*x^2]]) - (I/2)*a^2*((2*I)*ArcSinh[a*x]^2*ArcTanh[E^ArcSi
nh[a*x]] - (2*I)*(-(ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]]) + PolyLog[3,
 -E^ArcSinh[a*x]]) + (2*I)*(-(ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]]) + P
olyLog[3, E^ArcSinh[a*x]]))
```

3.289. $\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$

### 3.289.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

---

## 3.289. $\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.289.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.73

| method | result |
|---|---|
| default | $-\frac{\operatorname{arcsinh}(ax)\left(a^2x^2\operatorname{arcsinh}(ax)+2ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)\right)}{2\sqrt{a^2x^2+1}\,x^2}+\frac{a^2\operatorname{arcsinh}(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+a^2\operatorname{arcsinh}(ax)$ |

input
```
int(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)*(a^2*x^2*arcsinh(a*x)+2*a*x*(a^2*x
^2+1)^(1/2)+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2
))+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(
a^2*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))-a^2*a
rcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)
^(1/2))-2*a^2*arctanh(a*x+(a^2*x^2+1)^(1/2))
```

## 3.289.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^5 + x^3), x)
```

## 3.289.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{x^3\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**2/x**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**2/(x**3*sqrt(a**2*x**2 + 1)), x)
```

### 3.289.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.289.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.290 $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$



## 3.290.1 Optimal result

Integrand size = 28, antiderivative size = 383

$$\begin{aligned}\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx =& -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}}+\frac{298b^2(1+c^2x^2)}{225c^6\sqrt{d+c^2dx^2}}-\frac{76b^2(1+c^2x^2)^2}{675c^6\sqrt{d+c^2dx^2}}\\&+\frac{2b^2(1+c^2x^2)^3}{125c^6\sqrt{d+c^2dx^2}}-\frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{15c^5\sqrt{d+c^2dx^2}}\\&+\frac{8bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{45c^3\sqrt{d+c^2dx^2}}\\&-\frac{2bx^5\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c\sqrt{d+c^2dx^2}}\\&+\frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^6d}\\&-\frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{15c^4d}\\&+\frac{x^4\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{5c^2d}\end{aligned}$$

output
```
298/225*b^2*(c^2*x^2+1)/c^6/(c^2*d*x^2+d)^(1/2)-76/675*b^2*(c^2*x^2+1)^2/c
^6/(c^2*d*x^2+d)^(1/2)+2/125*b^2*(c^2*x^2+1)^3/c^6/(c^2*d*x^2+d)^(1/2)-16/
15*a*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)-16/15*b^2*x*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+8/45*b*x^3*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/25*b*x^5*(a+b*arcsinh(c*x))*(c
^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+8/15*(a+b*arcsinh(c*x))^2*(c^2*d*x^2
+d)^(1/2)/c^6/d-4/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/
5*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

### 3.290.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.60

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-30abcx\sqrt{1+c^2x^2}(120-20c^2x^2+9c^4x^4)+225a^2(8+4c^2x^2-c^4x^4+3c^6x^6)+2b^2(2072+1936c^2x^2-}{}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(-30*a*b*c*x*Sqrt[1 + c^2*x^2]*(120 - 20*c^2*x^2 + 9*c^4*x^4) + 225*a^2*(8
 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6) + 2*b^2*(2072 + 1936*c^2*x^2 - 109*c^4
*x^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*
c^4*x^4) + 15*a*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6))*ArcSinh[c*x] + 225*
b^2*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*ArcSinh[c*x]^2)/(3375*c^6*Sqrt[d
 + c^2*d*x^2])
```

### 3.290.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6227, 6191, 243, 53, 2009, 6227, 6191, 243, 53, 2009, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}\,dx$$

$$\Big\downarrow \ 6227$$

$$-\frac{2b\sqrt{c^2x^2+1}\int x^4(a+b\text{arcsinh}(cx))dx}{5c\sqrt{c^2dx^2+d}}-\frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \ 6191$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{5}bc\int\frac{x^5}{\sqrt{c^2x^2+1}}dx\right)}{5c\sqrt{c^2dx^2+d}}-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{243}$$

$$-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{5c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int\frac{x^4}{\sqrt{c^2x^2+1}}dx^2\right)}{5c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{5c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\int\left(\frac{(c^2x^2+1)^{3/2}}{c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{1}{c^4\sqrt{c^2x^2+1}}\right)dx^2\right)}{5c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\int\frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6227}$$

$$-\frac{4\left(-\frac{2b\sqrt{c^2x^2+1}\int x^2(a+b\text{arcsinh}(cx))dx}{3c\sqrt{c^2dx^2+d}}-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6191}$$

---

3.290. $\int\frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}dx$

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\int \frac{x^3}{\sqrt{c^2x^2+1}}dx\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 243

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx^2\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 53

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsi}}{3c^2d}\right.}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2009

$$-\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{2\sqrt{c^2}}{c^4}\right.\right.}{3c\sqrt{c^2dx^2+d}}\right.}{5c^2}+\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}$$

---

3.290. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

$$\Big\downarrow \text{6213}$$

$$-\frac{4\left(-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\right.}{}\right.}{5c^2}$$

$$\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{5c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{5}x^5(a+b\text{arcsinh}(cx))-\frac{1}{10}bc\left(\frac{2(c^2x^2+1)^{5/2}}{5c^6}-\frac{4(c^2x^2+1)^{3/2}}{3c^6}+\frac{2\sqrt{c^2x^2+1}}{c^6}\right)\right)}{5c\sqrt{c^2dx^2+d}}-$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{5c^2}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(5*c^2*d) - (2*b*Sqrt[1 +
 c^2*x^2]*(-1/10*(b*c*((2*Sqrt[1 + c^2*x^2])/c^6 - (4*(1 + c^2*x^2)^(3/2))
/(3*c^6) + (2*(1 + c^2*x^2)^(5/2))/(5*c^6))) + (x^5*(a + b*ArcSinh[c*x]))/
5))/(5*c*Sqrt[d + c^2*d*x^2]) - (4*((x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(3*c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(-1/6*(b*c*((-2*Sqrt[1 + c^2
*x^2])/c^4 + (2*(1 + c^2*x^2)^(3/2))/(3*c^4))) + (x^3*(a + b*ArcSinh[c*x])
)/3))/(3*c*Sqrt[d + c^2*d*x^2]) - (2*((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x])^2)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a*x - (b*Sqrt[1 + c^2*x^2])/c +
 b*x*ArcSinh[c*x]))/(c*Sqrt[d + c^2*d*x^2])))/(3*c^2)))/(5*c^2)
```

3.290. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

### 3.290.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

---

## 3.290. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

## 3.290.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. 2(335) = 670.

Time = 0.33 (sec) , antiderivative size = 1227, normalized size of antiderivative = 3.20

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1227 |
| parts | Expression too large to display | 1227 |

input
```
int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d
)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*
(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)
^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh
(c*x)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^
3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2
-6*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c
*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1
)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*
x)^2+2*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^
4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*
arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^
(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*
x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*a
rcsinh(c*x)+2)/c^6/d/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c
^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2
)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2
+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/
16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(...
```

### 3.290.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.83

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{225\,(3\,b^2c^6x^6-b^2c^4x^4+4\,b^2c^2x^2+8\,b^2)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\,(45\,abc^6x^6-15\,abc^4x^4+}{}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
1/3375*(225*(3*b^2*c^6*x^6 - b^2*c^4*x^4 + 4*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 - 15*a*b*c
^4*x^4 + 60*a*b*c^2*x^2 + 120*a*b - (9*b^2*c^5*x^5 - 20*b^2*c^3*x^3 + 120*
b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1
)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 - (225*a^2 + 218*b^2)*c^4*x^4 + 4*(225*a
^2 + 968*b^2)*c^2*x^2 + 1800*a^2 + 4144*b^2 - 30*(9*a*b*c^5*x^5 - 20*a*b*c
^3*x^3 + 120*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^8*d*x^2 +
 c^6*d)
```

### 3.290.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^5\left(a+b\,\text{asinh}\,(cx)\right)^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.290.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.92

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2+d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)b^2\,\text{arsinh}\,(cx)^2$$
$$+ \frac{2}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2+d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)ab\,\text{arsinh}\,(cx)$$
$$+ \frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}x^4}{c^2d} - \frac{4\sqrt{c^2dx^2+d}x^2}{c^4d} + \frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)a^2$$
$$+ \frac{2}{3375}\,b^2\left(\frac{27\sqrt{c^2x^2+1}c^2x^4 - 136\sqrt{c^2x^2+1}x^2 + \frac{2072\sqrt{c^2x^2+1}}{c^2}}{c^4\sqrt{d}} - \frac{15\,(9\,c^4x^5 - 20\,c^2x^3 + 120\,x)\,\text{arsinh}\,(cx}{c^5\sqrt{d}}\right.$$
$$- \frac{2\,(9\,c^4x^5 - 20\,c^2x^3 + 120\,x)ab}{225\,c^5\sqrt{d}}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d
) + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b^2*arcsinh(c*x)^2 + 2/15*(3*sqrt(c^2*d
*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x
^2 + d)/(c^6*d))*a*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d
) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*a^2
 + 2/3375*b^2*((27*sqrt(c^2*x^2 + 1)*c^2*x^4 - 136*sqrt(c^2*x^2 + 1)*x^2 +
 2072*sqrt(c^2*x^2 + 1)/c^2)/(c^4*sqrt(d)) - 15*(9*c^4*x^5 - 20*c^2*x^3 +
120*x)*arcsinh(c*x)/(c^5*sqrt(d))) - 2/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x
)*a*b/(c^5*sqrt(d))
```

### 3.290.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^5\,\left(a+b\,\text{asinh}(c\,x)\right)^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

# 3.291 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$



## 3.291.1 Optimal result

Integrand size = 28, antiderivative size = 323

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx =& -\frac{15b^2x(1+c^2x^2)}{64c^4\sqrt{d+c^2dx^2}}+\frac{b^2x^3(1+c^2x^2)}{32c^2\sqrt{d+c^2dx^2}} \\ &+\frac{15b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{64c^5\sqrt{d+c^2dx^2}} \\ &+\frac{3bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c^3\sqrt{d+c^2dx^2}} \\ &-\frac{bx^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c\sqrt{d+c^2dx^2}} \\ &-\frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8c^4d} \\ &+\frac{x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4c^2d} \\ &+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{8bc^5\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-15/64*b^2*x*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+1/32*b^2*x^3*(c^2*x^2+1)/
c^2/(c^2*d*x^2+d)^(1/2)+15/64*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/(c^2*
d*x^2+d)^(1/2)+3/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x
^2+d)^(1/2)-1/8*b*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)
^(1/2)+1/8*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/(c^2*d*x^2+d)^(1/2
)-3/8*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/4*x^3*(a+b*arcsin
h(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

### 3.291.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.83

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{32a^2c\sqrt{d}x(1+c^2x^2)(-3+2c^2x^2)+96a^2\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+b^2\sqrt{d}\sqrt{1+c^2x^2}(32\text{ar}}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(32*a^2*c*Sqrt[d]*x*(1 + c^2*x^2)*(-3 + 2*c^2*x^2) + 96*a^2*Sqrt[d + c^2*d
*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*Sqrt[1 + c^2*
x^2]*(32*ArcSinh[c*x]^3 - 4*ArcSinh[c*x]*(-16*Cosh[2*ArcSinh[c*x]] + Cosh[
4*ArcSinh[c*x]]) - 32*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]] + 8*ArcS
inh[c*x]^2*(-8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + 4*a*b*Sqrt[
d]*Sqrt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*A
rcSinh[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]
])))/(256*c^5*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

### 3.291.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6227, 6191, 262, 262, 222, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \text{6227}$$
$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{4c^2}-\frac{b\sqrt{c^2x^2+1}\int x^3(a+b\text{arcsinh}(cx))dx}{2c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}$$
$$\Big\downarrow \text{6191}$$

$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{4c^2}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\int \frac{x^4}{\sqrt{c^2x^2+1}}dx\right)}{2c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}$$

$\Big\downarrow$ 262

$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{4c^2}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\int \frac{x^2}{\sqrt{c^2x^2+1}}dx}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}$$

$\Big\downarrow$ 262

$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{4c^2}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}$$

$\Big\downarrow$ 222

$$-\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 6227

$$-\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\int x(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}$$

3.291. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

$$\Big\downarrow \; 6191$$

$$-\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{c\sqrt{c^2dx^2+d}}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 262$$

$$-\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{c\sqrt{c^2dx^2+d}}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 222$$

$$-\frac{3\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{c\sqrt{c^2dx^2+d}}\right)}{4c^2}+\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2}-\frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 6198$$

---

3.291. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

$$\frac{x^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4c^2d} - \frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}} - \frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx)) - \frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2} - \frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{c\sqrt{c^2dx^2+d}}\right)}{4c^2}$$

$$\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{4}x^4(a+b\text{arcsinh}(cx)) - \frac{1}{4}bc\left(\frac{x^3\sqrt{c^2x^2+1}}{4c^2} - \frac{3\left(\frac{x\sqrt{c^2x^2+1}}{2c^2} - \frac{\text{arcsinh}(cx)}{2c^3}\right)}{4c^2}\right)\right)}{2c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*c^2*d) - (b*Sqrt[1 + c
^2*x^2]*((x^4*(a + b*ArcSinh[c*x]))/4 - (b*c*((x^3*Sqrt[1 + c^2*x^2])/(4*c
^2) - (3*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/(4*c^2)))
/4))/(2*c*Sqrt[d + c^2*d*x^2]) - (3*((x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c^3*
Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*((x^2*(a + b*ArcSinh[c*x]))/2
- (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2))/(c*Sqrt
[d + c^2*d*x^2])))/(4*c^2)
```

### 3.291.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.291. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.291.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(283) = 566$.

Time = 0.26 (sec) , antiderivative size = 992, normalized size of antiderivative = 3.07

| method | result |
|---|---|
| default | $\frac{a^2x^3\sqrt{c^2d\,x^2+d}}{4c^2d} - \frac{3a^2x\sqrt{c^2d\,x^2+d}}{8c^4d} + \frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^4\sqrt{c^2d}} + b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^3}{8\sqrt{c^2x^2+1}\,c^5d} + \frac{\sqrt{d(c^2x^2+1)}\left(8c^5x^5\right.}{}\right.$ |
| parts | $\frac{a^2x^3\sqrt{c^2d\,x^2+d}}{4c^2d} - \frac{3a^2x\sqrt{c^2d\,x^2+d}}{8c^4d} + \frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^4\sqrt{c^2d}} + b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(cx)^3}{8\sqrt{c^2x^2+1}\,c^5d} + \frac{\sqrt{d(c^2x^2+1)}\left(8c^5x^5\right.}{}\right.$ |

input
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(c^2*d*x^2+d)^(1/2)+
3/8*a^2/c^4*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^
2*(1/8*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^3+1/512*
(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*
c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)^2-4*arc
sinh(c*x)+1)/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2
*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsin
h(c*x)+1)/c^5/d/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^
2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c
*x)+1)/c^5/d/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*
(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)
^(1/2))*(8*arcsinh(c*x)^2+4*arcsinh(c*x)+1)/c^5/d/(c^2*x^2+1))+2*a*b*(3/16
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+1/256*(d*(c^
2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^
2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(c*x))/c^5/d/(c^
2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)-1/16*(d*(c
^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^
(1/2))*(1+2*arcsinh(c*x))/c^5/d/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8
*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(...
```

### 3.291.5 Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^4}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/sqrt(
c^2*d*x^2 + d), x)
```

### 3.291.6 Sympy [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^4(a + b\,\text{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2 + 1)}}\,dx$$

input
```
integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**4*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.291.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.291.8 Giac [F]

$$\int \frac{x^4(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^4}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^4/sqrt(c^2*d*x^2 + d), x)
```

### 3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

# 3.292 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$



## 3.292.1 Optimal result

Integrand size = 28, antiderivative size = 265

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx =& \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}-\frac{14b^2(1+c^2x^2)}{9c^4\sqrt{d+c^2dx^2}}\\&+\frac{2b^2(1+c^2x^2)^2}{27c^4\sqrt{d+c^2dx^2}}+\frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3\sqrt{d+c^2dx^2}}\\&-\frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c\sqrt{d+c^2dx^2}}\\&-\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d}\\&+\frac{x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^2d}\end{aligned}$$

output
```
-14/9*b^2*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^4/(
c^2*d*x^2+d)^(1/2)+4/3*a*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)+4/3
*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a
+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(
c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/3*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/c^2/d
```

### 3.292.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.66

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-6abcx(-6+c^2x^2)\sqrt{1+c^2x^2}+2b^2(-20-19c^2x^2+c^4x^4)+9a^2(-2-c^2x^2+c^4x^4)-6b\big(bcx(-6+c^2x^{2}}{27c^4\sqrt{d+c^2dx^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(-6*a*b*c*x*(-6 + c^2*x^2)*Sqrt[1 + c^2*x^2] + 2*b^2*(-20 - 19*c^2*x^2 + c
^4*x^4) + 9*a^2*(-2 - c^2*x^2 + c^4*x^4) - 6*b*(b*c*x*(-6 + c^2*x^2)*Sqrt[
1 + c^2*x^2] + a*(6 + 3*c^2*x^2 - 3*c^4*x^4))*ArcSinh[c*x] + 9*b^2*(-2 - c
^2*x^2 + c^4*x^4)*ArcSinh[c*x]^2)/(27*c^4*Sqrt[d + c^2*d*x^2])
```

### 3.292.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6227, 6191, 243, 53, 2009, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx$$
$$\Big\downarrow \; 6227$$
$$-\frac{2b\sqrt{c^2x^2+1}\int x^2(a+b\text{arcsinh}(cx))dx}{3c\sqrt{c^2dx^2+d}}-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}$$
$$\Big\downarrow \; 6191$$
$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\int \frac{x^3}{\sqrt{c^2x^2+1}}dx\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}$$
$$\Big\downarrow \; 243$$

$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx^2\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$\downarrow$ 53

$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}$$

$\downarrow$ 2009

$$-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)}{3c\sqrt{c^2dx^2+d}}$$

$\downarrow$ 6213

$$-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)}{3c\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2009

$$\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)}{3c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

3.292. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

output
```
(x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2*d) - (2*b*Sqrt[1 +
 c^2*x^2]*(-1/6*(b*c*((-2*Sqrt[1 + c^2*x^2])/c^4 + (2*(1 + c^2*x^2)^(3/2))
/(3*c^4))) + (x^3*(a + b*ArcSinh[c*x]))/3))/(3*c*Sqrt[d + c^2*d*x^2]) - (2
*((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*b*Sqrt[1 + c^2
*x^2]*(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/(c*Sqrt[d + c^2*
d*x^2])))/(3*c^2)
```

### 3.292.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.292. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.292.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(231) = 462$.

Time = 0.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.66

| method | result |
|---|---|
| default | $a^2\left(\frac{x^2\sqrt{c^2dx^2+d}}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}}{3d\,c^4}\right)+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\,\mathrm{arcsinh}(cx)^2-6\right.}{216c^4d(c^2x^2+1)}\right.$ |
| parts | $a^2\left(\frac{x^2\sqrt{c^2dx^2+d}}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}}{3d\,c^4}\right)+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1\right)\left(9\,\mathrm{arcsinh}(cx)^2-6\right.}{216c^4d(c^2x^2+1)}\right.$ |

input
```
int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b^2*
(1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*
x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/d/(
c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(ar
csinh(c*x)^2-2*arcsinh(c*x)+2)/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)
*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/d
/(c^2*x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^
(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x
)+2)/c^4/d/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3
*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(
c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)
^(1/2)+1)*(-1+arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^4/d/(c^2*x^2+1)+1/72*(d
*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x
*(c^2*x^2+1)^(1/2)+1)*(3*arcsinh(c*x)+1)/c^4/d/(c^2*x^2+1))
```

### 3.292.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{9\,(b^2c^4x^4-b^2c^2x^2-2\,b^2)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3\,abc^4x^4-3\,abc^2x^2-6\,ab-(b^2c^3x^3-6$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
1/27*(9*(b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*d*x^2 + d)*log(c*x +
sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 - 3*a*b*c^2*x^2 - 6*a*b - (b^2*c^3
*x^3 - 6*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - (9*a^2 + 38*b^2)*c^2*x^2 - 18*a^2
 - 40*b^2 - 6*(a*b*c^3*x^3 - 6*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2
+ d))/(c^6*d*x^2 + c^4*d)
```

### 3.292.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^3(a+b\,\text{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.292.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{1}{3}b^2\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx)^2$$
$$+\frac{2}{3}ab\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx)$$
$$+\frac{1}{3}a^2\left(\frac{\sqrt{c^2dx^2+d}x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)$$
$$+\frac{2}{27}b^2\left(\frac{\sqrt{c^2x^2+1}x^2-\frac{20\sqrt{c^2x^2+1}}{c^2}}{c^2\sqrt{d}}-\frac{3(c^2x^3-6x)\operatorname{arsinh}(cx)}{c^3\sqrt{d}}\right)-\frac{2(c^2x^3-6x)ab}{9c^3\sqrt{d}}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
1/3*b^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*
arcsinh(c*x)^2 + 2/3*a*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x
^2 + d)/(c^4*d))*arcsinh(c*x) + 1/3*a^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) -
 2*sqrt(c^2*d*x^2 + d)/(c^4*d)) + 2/27*b^2*((sqrt(c^2*x^2 + 1)*x^2 - 20*sq
rt(c^2*x^2 + 1)/c^2)/(c^2*sqrt(d)) - 3*(c^2*x^3 - 6*x)*arcsinh(c*x)/(c^3*s
qrt(d))) - 2/9*(c^2*x^3 - 6*x)*a*b/(c^3*sqrt(d))
```

### 3.292.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

# 3.293 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$



## 3.293.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{b^2x(1+c^2x^2)}{4c^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^3\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^2d} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{d+c^2dx^2}}$$

output
```
1/4*b^2*x*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1
)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/6*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b
/c^3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^
2/d
```

### 3.293.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{12a^2cx(d+c^2dx^2)-12a^2\sqrt{d}\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-6abd\sqrt{1+c^2x^2}(\cosh(2\text{arcsinh}(cx$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(12*a^2*c*x*(d + c^2*d*x^2) - 12*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
 + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6*a*b*d*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSin
h[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])) - b^2*d*Sq
rt[1 + c^2*x^2]*(4*ArcSinh[c*x]^3 + 6*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] -
3*(1 + 2*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(24*c^3*d*Sqrt[d + c^2*d*x
^2])
```

### 3.293.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \text{6227}$$
$$-\frac{b\sqrt{c^2x^2+1}\int x(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}$$
$$\Big\downarrow \text{6191}$$
$$-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{c\sqrt{c^2dx^2+d}}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{c\sqrt{c^2dx^2+d}}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c^3*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^
2*x^2]*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2)
 - ArcSinh[c*x]/(2*c^3)))/2))/(c*Sqrt[d + c^2*d*x^2])
```

### 3.293.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

---

3.293. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.293.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(178) = 356$.

Time = 0.23 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.48

| method | result |
|---|---|
| default | $\frac{a^2x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c^3d}+\frac{\sqrt{d(c^2x^2+1)}\,\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2c\right.}{16c}\right.$ |
| parts | $\frac{a^2x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c^3d}+\frac{\sqrt{d(c^2x^2+1)}\,\left(2c^3x^3+2c^2x^2\sqrt{c^2x^2+1}+2c\right.}{16c}\right.$ |

input
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a^2/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(
c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(-1/6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/c^3/d*arcsinh(c*x)^3+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2
*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2-2*arcsin
h(c*x)+1)/c^3/d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^
2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(2*arcsinh(c*x)^2+2*arcsinh(c
*x)+1)/c^3/d/(c^2*x^2+1))+2*a*b*(-1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^3/d*arcsinh(c*x)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(
c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x
^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*
c*x-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1))
```

## 3.293.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^2}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/sqrt(
c^2*d*x^2 + d), x)
```

## 3.293.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^2\left(a+b\,\text{asinh}\left(cx\right)\right)^2}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.293.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.293.8 Giac [F]

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^2}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^2/sqrt(c^2*d*x^2 + d), x)
```

### 3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x^2\,(a + b\,\text{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

# 3.294 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$



## 3.294.1 Optimal result

Integrand size = 26, antiderivative size = 138

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^2\sqrt{d+c^2dx^2}}-\frac{2b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^2d}$$

output
```
2*b^2*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-2*a*b*x*(c^2*x^2+1)^(1/2)/c/(c^2
*d*x^2+d)^(1/2)-2*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/
2)+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

## 3.294.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}-2b\left(bcx-a\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)+b^2\sqrt{1+c^2x^2}\right.}{c^2d\sqrt{1+c^2x^2}}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + 2*b^2*Sqrt[1 +
c^2*x^2] - 2*b*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + b^2*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x]^2))/(c^2*d*Sqrt[1 + c^2*x^2])
```

## 3.294.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*b*Sqrt[1 + c^2*x
^2]*(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/(c*Sqrt[d + c^2*d*
x^2])
```

### 3.294.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.294.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(126) = 252$.

Time = 0.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.14

| method | result |
|---|---|
| default | $\frac{a^2\sqrt{c^2d\,x^2+d}}{c^2d} + b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+2\right)}{2c^2d(c^2x^2+1)} + \frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+}\right.}{2c^2}\right.$ |
| parts | $\frac{a^2\sqrt{c^2d\,x^2+d}}{c^2d} + b^2\left(\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\operatorname{arcsinh}(cx)^2-2\operatorname{arcsinh}(cx)+2\right)}{2c^2d(c^2x^2+1)} + \frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+}\right.}{2c^2}\right.$ |

input
```
int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/c^2/d*(c^2*d*x^2+d)^(1/2)+b^2*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*
(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1)+1
/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2
+2*arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1))+2*a*b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^
2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/c^2/
d/(c^2*x^2+1))
```

### 3.294.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

$$\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{(b^2c^2x^2+b^2)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\left(abc^2x^2-\sqrt{c^2x^2+1}b^2cx+ab\right)\sqrt{c^2dx^2+d}\log\left(cx+\right.}{c^4dx^2+c^2d}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
((b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 +
2*(a*b*c^2*x^2 - sqrt(c^2*x^2 + 1)*b^2*c*x + a*b)*sqrt(c^2*d*x^2 + d)*log(
c*x + sqrt(c^2*x^2 + 1)) + ((a^2 + 2*b^2)*c^2*x^2 - 2*sqrt(c^2*x^2 + 1)*a*
b*c*x + a^2 + 2*b^2)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

### 3.294.6 Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x(a+b\operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2+1)}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.294.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = -2\,b^2\left(\frac{x\operatorname{arsinh}(cx)}{c\sqrt{d}} - \frac{\sqrt{c^2x^2+1}}{c^2\sqrt{d}}\right) - \frac{2\,abx}{c\sqrt{d}} + \frac{\sqrt{c^2dx^2+d}b^2\operatorname{arsinh}(cx)^2}{c^2d} + \frac{2\sqrt{c^2dx^2+d}ab\operatorname{arsinh}(cx)}{c^2d} + \frac{\sqrt{c^2dx^2+d}a^2}{c^2d}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-2*b^2*(x*arcsinh(c*x)/(c*sqrt(d)) - sqrt(c^2*x^2 + 1)/(c^2*sqrt(d))) - 2*
a*b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b^2*arcsinh(c*x)^2/(c^2*d) + 2*sqr
t(c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2 + d)*a^2/(c^2*d
)
```

### 3.294.8 Giac [F]

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{\left(b\,\text{arsinh}\left(cx\right) + a\right)^2 x}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x/sqrt(c^2*d*x^2 + d), x)
```

### 3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{x\,\left(a + b\,\text{asinh}(c\,x)\right)^2}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

# 3.295 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$



## 3.295.1 Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+c^2dx^2}}$$

output
```
1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

## 3.295.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. $2(47) = 94$.

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\frac{\frac{3ab\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^3}{\sqrt{d+c^2dx^2}}+\frac{3a^2\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{\sqrt{d}}}{3c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

output
```
((3*a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]^3)/Sqrt[d + c^2*d*x^2] + (3*a^2*ArcTanh[(c*Sqrt[
d]*x)/Sqrt[d + c^2*d*x^2]])/Sqrt[d])/(3*c)
```

## 3.295.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + c^2*d*x^2])
```

### 3.295.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.295.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(41) = 82.

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.55

| method | result | size |
|---|---|---|
| default | $\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,cd}+\frac{ab\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{\sqrt{c^2x^2+1}\,cd}$ | 120 |
| parts | $\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,cd}+\frac{ab\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{\sqrt{c^2x^2+1}\,cd}$ | 120 |

input
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^3+a*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

### 3.295.5 Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\sqrt{c^2dx^2 + d}}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(c^2*d*x^2 +
d), x)
```

### 3.295.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\, dx = \int \frac{(a + b \operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2 + 1)}}\, dx$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.295.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + c^2dx^2}}\, dx = \frac{b^2 \operatorname{arsinh}(cx)^3}{3\, c\sqrt{d}} + \frac{ab \operatorname{arsinh}(cx)^2}{c\sqrt{d}} + \frac{a^2 \operatorname{arsinh}(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/3*b^2*arcsinh(c*x)^3/(c*sqrt(d)) + a*b*arcsinh(c*x)^2/(c*sqrt(d)) + a^2*
arcsinh(c*x)/(c*sqrt(d))
```

### 3.295.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/sqrt(c^2*d*x^2 + d), x)
```

### 3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2), x)
```

# 3.296 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx$



## 3.296.1 Optimal result

Integrand size = 28, antiderivative size = 223

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = & -\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ & -\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ & +\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ & +\frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ & -\frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(
c^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*polylog(2,c
*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b^2*polylog(
3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2*b^2*poly
log(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

### 3.296.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = \frac{a^2\log(cx)}{\sqrt{d}} - \frac{a^2\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{\sqrt{d}}$$
$$+ \frac{2ab\sqrt{1+c^2x^2}\left(\operatorname{arcsinh}(cx)\left(\log\left(1-e^{-\operatorname{arcsinh}(cx)}\right)-\log\left(1+e^{-\operatorname{arcsinh}(cx)}\right)\right)+\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(cx)}\right)\right.}{\sqrt{d+c^2dx^2}}$$
$$+ \frac{b^2\sqrt{1+c^2x^2}\left(\operatorname{arcsinh}(cx)^2\log\left(1-e^{-\operatorname{arcsinh}(cx)}\right)-\operatorname{arcsinh}(cx)^2\log\left(1+e^{-\operatorname{arcsinh}(cx)}\right)+2\operatorname{arcsinh}(cx)\operatorname{Pol}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d
] + (2*a*b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - L
og[1 + E^(-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E
^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[1 + c^2*x^2]*(ArcSinh[
c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x
])] + 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyL
og[2, E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3,
 E^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2]
```

### 3.296.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.56, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \text{6231}$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{cx}d\operatorname{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{i\sqrt{c^2x^2+1}\left(2ib\int (a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int (a+b\text{arcsinh}(cx))\log\left(1+e^{\text{arcsinh}(cx)}\right)\right.}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+2ib\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$\frac{i\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(c}\right.\right.\right.}{\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(I*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]]
 - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]]) + b*PolyLo
g[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, E^ArcS
inh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2]
```

3.296. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx$

### 3.296.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.296. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx$

### 3.296.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(250) = 500$.

Time = 0.28 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.45

| method | result |
|---|---|
| default | $-\frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{\sqrt{d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 \ln\left(1+cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\frac{2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\operatorname{polylog}(2}{\sqrt{c^2x^2+1}\,d}\right.$ |
| parts | $-\frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{\sqrt{d}}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 \ln\left(1+cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\frac{2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)\operatorname{polylog}(2}{\sqrt{c^2x^2+1}\,d}\right.$ |

input
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b^2*(-(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,-c*x-(c^2
*x^2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,-c*x-
(c^2*x^2+1)^(1/2))+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^
2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*
arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)/d*polylog(3,c*x+(c^2*x^2+1)^(1/2)))+2*a*b*((d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)
)-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/
2)))
```

### 3.296.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}\,x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^2*d*x^3 + d*x), x)
```

### 3.296.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^2}{x\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.296.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-a^2*arcsinh(1/(c*abs(x)))/sqrt(d) + integrate(b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(sqrt(c^2*d*x^2 + d)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt
(c^2*d*x^2 + d)*x), x)
```

### 3.296.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x), x)
```

### 3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{d+c^2dx^2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.297 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx$



## 3.297.1 Optimal result

Integrand size = 28, antiderivative size = 167

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = & \frac{c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}} \\ & - \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{dx} \\ & + \frac{2bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\ & - \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*c*(a+b*ar
csinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2
+d)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(
c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x
```

### 3.297.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{b^2\left(-1-c^2x^2+cx\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)^2-2b\text{arcsinh}(cx)\left(a+ac^2x^2-bcx\sqrt{1+c^2x^2}\log\left(1-e^{-2\text{arcsinh}}\right.\right.}{x\sqrt{d+c^2dx^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(b^2*(-1 - c^2*x^2 + c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - 2*b*ArcSinh[c
*x]*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c*x])])
 - a*(a + a*c^2*x^2 - 2*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x]) - b^2*c*x*Sqrt[1
 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(x*Sqrt[d + c^2*d*x^2])
```

### 3.297.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6215, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \ 6215$$
$$\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x}dx}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}$$
$$\Big\downarrow \ 6190$$
$$\frac{2c\sqrt{c^2x^2+1}\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}$$
$$\Big\downarrow \ 25$$

$$-\frac{2c\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}$$

$\Big\downarrow$ 3042

$$-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}-\frac{2c\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 26

$$-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 4201

$$-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i(a+b\text{arcsinh}(cx))^2\right)}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2620

$$-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-2\cdots}\right.\right.\right.}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2715

$$-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}-\frac{1}{2}b(a\cdots\right.\right.}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2838

3.297. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx$

$$-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcsinh}(cx))\log\left(1+e^{-\frac{2(a+b\text{arcsinh}(cx))}{b}+\frac{2a}{b}-i\pi}\right)\right)\right.}{\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
-((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x)) + ((2*I)*c*Sqrt[1 +
c^2*x^2]*((-1/2*I)*(a + b*ArcSinh[c*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[
c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c*x]))/b)]) + (b^2*Pol
yLog[2, -a - b*ArcSinh[c*x]])/4)))/Sqrt[d + c^2*d*x^2]
```

### 3.297.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

### 3.297.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(171) = 342$.

Time = 0.26 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.86

| method | result |
|---|---|
| default | $-\frac{a^2\sqrt{c^2 d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)^2}{(c^2x^2+1)dx}-\frac{2\sqrt{d(c^2x^2+1)}\operatorname{arcsinh}(cx)^2c}{\sqrt{c^2x^2+1}\,d}+\frac{2\sqrt{d(c^2x^2+1)}}{\quad}\right.$ |
| parts | $-\frac{a^2\sqrt{c^2 d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-cx\sqrt{c^2x^2+1}+1\right)\operatorname{arcsinh}(cx)^2}{(c^2x^2+1)dx}-\frac{2\sqrt{d(c^2x^2+1)}\operatorname{arcsinh}(cx)^2c}{\sqrt{c^2x^2+1}\,d}+\frac{2\sqrt{d(c^2x^2+1)}}{\quad}\right.$ |

input
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(c^2*d*x^2+d)^(1/2)+b^2*(-(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2
*x^2+1)^(1/2)+1)*arcsinh(c*x)^2/(c^2*x^2+1)/d/x-2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*c+2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+2*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c)+2
*a*b*(-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*c-(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*arcsinh(c*x)/(c^2*x^2+1)/d
/x+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-
1)*c)
```

## 3.297.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}\,x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^2*d*x^4 + d*x^2), x)
```

## 3.297.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{x^2\sqrt{d(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.297.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^2}{\sqrt{c^2dx^2 + d}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*log(x^
2 + 1/c^2))*a*b*c/d + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c
^2*d*x^2 + d)*x^2), x) - 2*sqrt(c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(d*x) - sq
rt(c^2*d*x^2 + d)*a^2/(d*x)
```

### 3.297.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^2}{\sqrt{c^2dx^2 + d}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

### 3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{x^2\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.298 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx$



## 3.298.1 Optimal result

Integrand size = 28, antiderivative size = 360

$$
\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx = & -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{x\sqrt{d+c^2dx^2}} \\
& -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2dx^2} \\
& +\frac{c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
& -\frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{\sqrt{d+c^2dx^2}} \\
& +\frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
& -\frac{bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
& -\frac{b^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}} \\
& +\frac{b^2c^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{\sqrt{d+c^2dx^2}}
\end{aligned}
$$

output
```
-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*a
rcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2
+d)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+
d)^(1/2)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x
^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^
2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-b^2*c^2*polylog(3,-c
*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+b^2*c^2*polylo
g(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*
arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^2
```

## 3.298.2 Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-\frac{4a^2\sqrt{d+c^2dx^2}}{x^2} - 4a^2c^2\sqrt{d}\log(x) + 4a^2c^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{2abc^2d^2(1+c^2x^2)^{3/2}\left(-2\coth\left(\frac{1}{2}\text{arcsinh}(cx\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

output
```
((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 - 4*a^2*c^2*Sqrt[d]*Log[x] + 4*a^2*c^2*S
qrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*d^2*(1 + c^2*x^2)
^(3/2)*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*
ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSi
nh[c*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*
x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d +
 c^2*d*x^2)^(3/2) + (b^2*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-4*ArcSinh[c*x]*Coth
[ArcSinh[c*x]/2] - ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]^
2*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])]
 + 8*Log[Tanh[ArcSinh[c*x]/2]] - 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*
x])] + 8*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 8*PolyLog[3, -E^(-Ar
cSinh[c*x])] + 8*PolyLog[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSi
nh[c*x]/2]^2 + 4*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2)
)/(8*d)
```

### 3.298.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.63, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6224, 6191, 243, 73, 221, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{1}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx+\frac{bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x^2}dx}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{1}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx+\frac{bc\sqrt{c^2x^2+1}\left(bc\int \frac{1}{x\sqrt{c^2x^2+1}}dx-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx+\frac{bc\sqrt{c^2x^2+1}\left(\frac{1}{2}bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx+\frac{bc\sqrt{c^2x^2+1}\left(\frac{b\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c}-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx+\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\,\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 6231

$$-\frac{c^2\sqrt{c^2x^2+1}\int \frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 3042

$$-\frac{c^2\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 26

$$-\frac{ic^2\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{2\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 4670

$$-\frac{ic^2\sqrt{c^2x^2+1}\left(2ib\int (a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int (a+b\text{arcsinh}(cx))\log\left(1+e^{\text{arcsi}}\right.\right.}{2\sqrt{c^2dx^2+d}}$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 3011

$$-\frac{ic^2\sqrt{c^2x^2+1}\left(-2ib\left(b\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+\right.}{}$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 2720

$$-\frac{ic^2\sqrt{c^2x^2+1}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsi}\right.\right.}{}$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{x}-b\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2dx^2}$$

$\Big\downarrow$ 7143

---

3.298. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx$

$$-\frac{ic^2\sqrt{c^2x^2+1}\left(2i\mathrm{arctanh}\left(e^{\mathrm{arcsinh}(cx)}\right)(a+b\mathrm{arcsinh}(cx))^2-2ib\left(b\,\mathrm{PolyLog}\left(3,-e^{\mathrm{arcsinh}(cx)}\right)-\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsi}}\right.\right.\right.}{2\sqrt{c^2dx^2}}$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-\frac{a+b\mathrm{arcsinh}(cx)}{x}-bc\mathrm{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\mathrm{arcsinh}(cx))^2}{2dx^2}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

output
```
-1/2*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x^2) + (b*c*Sqrt[1 +
c^2*x^2]*(-((a + b*ArcSinh[c*x])/x) - b*c*ArcTanh[Sqrt[1 + c^2*x^2]]))/Sqr
t[d + c^2*d*x^2] - ((I/2)*c^2*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x]
)^2*ArcTanh[E^ArcSinh[c*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -
E^ArcSinh[c*x]]) + b*PolyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcS
inh[c*x])*PolyLog[2, E^ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/Sq
rt[d + c^2*d*x^2]
```

### 3.298.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.298. $\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.298. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.298.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(373) = 746$.

Time = 0.31 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.17

| method | result |
|---|---|
| default | $-\frac{a^2\sqrt{c^2 d\,x^2+d}}{2d\,x^2}+\frac{a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b^2\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)\operatorname{arcsinh}(cx)\sqrt{d(c^2x^2+}}{2x^2d(c^2x^2+1)}\right.$ |
| parts | $-\frac{a^2\sqrt{c^2 d\,x^2+d}}{2d\,x^2}+\frac{a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b^2\left(-\frac{\left(\operatorname{arcsinh}(cx)c^2x^2+2cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)\operatorname{arcsinh}(cx)\sqrt{d(c^2x^2+}}{2x^2d(c^2x^2+1)}\right.$ |

input
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a^2*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(
c^2*d*x^2+d)^(1/2))/x)+b^2*(-1/2*(arcsinh(c*x)*c^2*x^2+2*c*x*(c^2*x^2+1)^(
1/2)+arcsinh(c*x))*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)+1/
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x
^2+1)^(1/2))*c^2+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*po
lylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)/d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))
*c^2+(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,c*x+(c^2*x^2+1)^(
1/2))*c^2-2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arctanh(c*x+(c^2*x^2
+1)^(1/2))*c^2)+2*a*b*(-1/2*(arcsinh(c*x)*c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+ar
csinh(c*x))*(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)-1/2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*
c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c
^2*x^2+1)^(1/2))*c^2+1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog
(2,-c*x-(c^2*x^2+1)^(1/2))*c^2)
```

### 3.298.5 Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^3\sqrt{d + c^2dx^2}}\, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{c^2dx^2 + d}x^3}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^2*d*x^5 + d*x^3), x)
```

### 3.298.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{x^3\sqrt{d(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.298.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a^2
+ integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^3) +
 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

### 3.298.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

### 3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3\sqrt{d+c^2dx^2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^3\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.299 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx$



## 3.299.1 Optimal result

Integrand size = 28, antiderivative size = 299

$$
\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx = & -\frac{b^2c^2(1+c^2x^2)}{3x\sqrt{d+c^2dx^2}}-\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3x^2\sqrt{d+c^2dx^2}} \\
& -\frac{2c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+c^2dx^2}} \\
& -\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx^3} \\
& +\frac{2c^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3dx} \\
& -\frac{4bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1-e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{d+c^2dx^2}} \\
& +\frac{2b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(cx)}\right)}{3\sqrt{d+c^2dx^2}}
\end{aligned}
$$

output
```
-1/3*b^2*c^2*(c^2*x^2+1)/x/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*c^3*(a+b*arcsinh(c*x))^2*(c^
2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-4/3*b*c^3*(a+b*arcsinh(c*x))*ln(1-1/(c*
x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*b^2*c^3*
polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/
2)-1/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^3+2/3*c^2*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x
```

### 3.299.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-a^2+a^2c^2x^2-b^2c^2x^2+2a^2c^4x^4-b^2c^4x^4-abcx\sqrt{1+c^2x^2}+b^2\left(-1+c^2x^2+2c^4x^4-2c^3x^3\sqrt{1+c^2x^2}\right)}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(-a^2 + a^2*c^2*x^2 - b^2*c^2*x^2 + 2*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x*
Sqrt[1 + c^2*x^2] + b^2*(-1 + c^2*x^2 + 2*c^4*x^4 - 2*c^3*x^3*Sqrt[1 + c^2
*x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(b*c*x*Sqrt[1 + c^2*x^2] - 2*a*(-1
+ c^2*x^2 + 2*c^4*x^4) + 4*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSi
nh[c*x])]) - 4*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[c*x] + 2*b^2*c^3*x^3*Sqrt
[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[d + c^2*d*x^2])
```

### 3.299.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6224, 6191, 242, 6215, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{c^2dx^2+d}}\,dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{2}{3}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{c^2dx^2+d}}dx+\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x^3}dx}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{2}{3}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{c^2dx^2+d}}dx+\frac{2bc\sqrt{c^2x^2+1}\left(\frac{1}{2}bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2}{3}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\sqrt{c^2dx^2+d}}dx+\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{6215}$$

$$-\frac{2}{3}c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x}dx}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}\right)+\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{6190}$$

$$-\frac{2}{3}c^2\left(\frac{2c\sqrt{c^2x^2+1}\int -\left((a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b}{dx}\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{3}c^2\left(-\frac{2c\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+\text{barc}}{dx}\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}c^2\left(-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}-\frac{2c\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{c^2dx^2+d}}\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

3.299. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx$

$$\Big\downarrow \text{26}$$

$$-\frac{2}{3}c^2\left(-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(cx))}{b}\right.}{\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{2}{3}c^2\left(-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\int\frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}(a+b\text{arcsinh}(cx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}}d(a+b\text{arcsin}\right.}{\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2}{3}c^2\left(-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}-i\pi}\right)d(a+b\text{arcsin}\right.\right.}{}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2}{3}c^2\left(-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(cx))}{b}+i\pi}\log\left(1+e^{\frac{2a}{b}-\frac{2(a+ba}{}}\right.\right.\right.}{}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{2838}$$

---

3.299. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx$

$$-\frac{2}{3}c^2\left(-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{dx}+\frac{2ic\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-a-b\text{arcsinh}(cx))-\frac{1}{2}b(a+b\text{arcs}\right.\right.}{\sqrt{c^2d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3dx^3}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

output
```
-1/3*(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x^3) + (2*b*c*Sqrt[1
+ c^2*x^2]*(-1/2*(b*c*Sqrt[1 + c^2*x^2])/x - (a + b*ArcSinh[c*x])/(2*x^2))
)/(3*Sqrt[d + c^2*d*x^2]) - (2*c^2*(-((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x])^2)/(d*x)) + ((2*I)*c*Sqrt[1 + c^2*x^2]*((-1/2*I)*(a + b*ArcSinh[c*x]
)^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(
a + b*ArcSinh[c*x]))/b)]) + (b^2*PolyLog[2, -a - b*ArcSinh[c*x]])/4)))/Sqr
t[d + c^2*d*x^2]))/3
```

### 3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## 3.299. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

### 3.299.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1505 vs. $2(281) = 562$.

Time = 0.31 (sec) , antiderivative size = 1506, normalized size of antiderivative = 5.04

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1506 |
| parts | Expression too large to display | 1506 |

input
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/3/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*c^2/d/x*(c^2*d*x^2+d)^(1/2))+b^2*(
d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^2*(c^2*x^2+1)^(1/2)*c^5+1
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)^2*c^
4-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*c
^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*arcsinh(c*x)^
2*c^2+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*c^3
-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*c^3*(c^2*x^2+1)^(
1/2)+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8-1/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*c^6-2/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+2*c^2*x^2-1)/d/x^3*arcsinh(c*x)^2-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3-4/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1/2)*arcsinh(c*x
)^2*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*c^3+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d
*x^3*arcsinh(c*x)^2*c^6+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2
-1)/d*x^3*arcsinh(c*x)*c^6+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(4*arcsinh(c*x)*c
^3*x^3-4*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*x^3*c^3+4*arcsinh(c*x)*(c^2*x^...
```

### 3.299.5 Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{\sqrt{c^2dx^2 + dx^4}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^2*d*x^6 + d*x^4), x)
```

### 3.299.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4\sqrt{d + c^2dx^2}}\,dx = \int \frac{(a + b\operatorname{asinh}(cx))^2}{x^4\sqrt{d(c^2x^2 + 1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

### 3.299.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^4\sqrt{d + c^2dx^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{\sqrt{c^2dx^2 + dx^4}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/3*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*a*b*c + 2/3*a*b*(2*sqrt(c^2*
d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a^2
*(2*sqrt(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3)) + b^2*int
egrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

### 3.299.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+dx^4}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

### 3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^4\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.300 $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.300.1 Optimal result

Integrand size = 28, antiderivative size = 515

$$\begin{aligned}\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx &= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} \\
&- \frac{32b^2(1+c^2x^2)}{9c^6d\sqrt{d+c^2dx^2}} + \frac{2b^2(1+c^2x^2)^2}{27c^6d\sqrt{d+c^2dx^2}} \\
&+ \frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d\sqrt{d+c^2dx^2}} - \frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^5d\sqrt{d+c^2dx^2}} \\
&- \frac{2bx^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} \\
&- \frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^2} + \frac{4x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^4d^2} \\
&+ \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}} \\
&- \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}} \\
&+ \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^6d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-32/9*b^2*(c^2*x^2+1)/c^6/d/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^6
/d/(c^2*d*x^2+d)^(1/2)-x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+
16/3*a*b*x*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+16/3*b^2*x*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2*b*x*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arctan(c
*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*
polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d
)^(1/2)+2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6
/d/(c^2*d*x^2+d)^(1/2)-8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^
2+4/3*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

### 3.300.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.83

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{-72a^2-94b^2-36a^2c^2x^2-92b^2c^2x^2+9a^2c^4x^4+2b^2c^4x^4+90abcx\sqrt{1+c^2x^2}$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(-72*a^2 - 94*b^2 - 36*a^2*c^2*x^2 - 92*b^2*c^2*x^2 + 9*a^2*c^4*x^4 + 2*b^
2*c^4*x^4 + 90*a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]
 - 144*a*b*ArcSinh[c*x] - 72*a*b*c^2*x^2*ArcSinh[c*x] + 18*a*b*c^4*x^4*Arc
Sinh[c*x] + 90*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt
[1 + c^2*x^2]*ArcSinh[c*x] - 72*b^2*ArcSinh[c*x]^2 - 36*b^2*c^2*x^2*ArcSin
h[c*x]^2 + 9*b^2*c^4*x^4*ArcSinh[c*x]^2 + 108*a*b*Sqrt[1 + c^2*x^2]*ArcTan
[Tanh[ArcSinh[c*x]/2]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 -
 I/E^ArcSinh[c*x]] + (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E
^ArcSinh[c*x]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*
x]] + (54*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(27*c^6*d
*Sqrt[d + c^2*d*x^2])
```

### 3.300.3 Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6225, 6227, 243, 53, 2009, 6191, 243, 53, 2009, 6213, 2009, 6227, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{\left(c^2dx^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^4(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6227}$$

$$\frac{4\left(-\frac{2b\sqrt{c^2x^2+1}\int x^2(a+b\text{arcsinh}(cx))dx}{3c\sqrt{c^2dx^2+d}}-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{b\int \frac{x^3}{\sqrt{c^2x^2+1}}dx}{3c}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{243}$$

$$\frac{4\left(-\frac{2b\sqrt{c^2x^2+1}\int x^2(a+b\text{arcsinh}(cx))dx}{3c\sqrt{c^2dx^2+d}}-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{b\int \frac{x^2}{\sqrt{c^2x^2+1}}dx^2}{6c}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{53}$$

$$\frac{4\left(-\frac{2b\sqrt{c^2x^2+1}\int x^2(a+b\text{arcsinh}(cx))dx}{3c\sqrt{c^2dx^2+d}}-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{b\int\left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2}{6c}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(-\frac{2b\sqrt{c^2x^2+1}\int x^2(a+b\text{arcsinh}(cx))dx}{3c\sqrt{c^2dx^2+d}}-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6191}$$

$$\frac{4\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{3}bc\int\frac{x^3}{\sqrt{c^2x^2+1}}dx\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{243}$$

---

3.300. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx^2\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 53$$

$$\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\int \left(\frac{\sqrt{c^2x^2+1}}{c^2}-\frac{1}{c^2\sqrt{c^2x^2+1}}\right)dx^2\right)}{3c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}+\frac{4\left(-\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)}{3c\sqrt{c^2dx^2+d}}\right)}{c^2d}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 6213$$

---

3.300. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{4\left(-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}+\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x^3(a+b\text{arc}\right.}{}\right.}{c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$\big\downarrow$ 2009

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

$\big\downarrow$ 6227

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-\frac{\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}-\frac{b\int\frac{x}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

$\big\downarrow$ 241

3.300. $\int\frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)}{6c}\right)}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

$\downarrow$ 6204

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{2(c^2x^2+1)^{3/2}}{3c^4}-\frac{2\sqrt{c^2}}{c}\right.}{6c}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

$\downarrow$ 3042

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}}{c^2}+\frac{x^3(a+b\text{arcsinh}(cx))}{3c^2}-\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

3.300. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}dx$

$\downarrow$ 4668

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3}}{c^2}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

$\downarrow$ 2715

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsi}}\right.}{c^3}}{c^2}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

$\downarrow$ 2838

3.300. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3}}{c^2}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{4\left(\frac{x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{3c^2d}-\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{3c^2}-\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{3}x\right.}{}\right.}{c^2d}$$

input
```
Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((x^4*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) + (4*((x^2*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2*d) - (2*b*Sqrt[1 + c^2*x^2
]*(-1/6*(b*c*((-2*Sqrt[1 + c^2*x^2])/c^4 + (2*(1 + c^2*x^2)^(3/2))/(3*c^4)
)) + (x^3*(a + b*ArcSinh[c*x]))/3))/(3*c*Sqrt[d + c^2*d*x^2]) - (2*((Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a
*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/(c*Sqrt[d + c^2*d*x^2]))
)/(3*c^2)))/(c^2*d) + (2*b*Sqrt[1 + c^2*x^2]*(-1/6*(b*((-2*Sqrt[1 + c^2*x^
2])/c^4 + (2*(1 + c^2*x^2)^(3/2))/(3*c^4)))/c + (x^3*(a + b*ArcSinh[c*x]))
/(3*c^2) - (-((b*Sqrt[1 + c^2*x^2])/c^3) + (x*(a + b*ArcSinh[c*x]))/c^2 -
(2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^Arc
Sinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/c^3)/c^2))/(c*d*Sqrt[d + c^
2*d*x^2])
```

### 3.300.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

---

3.300. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.300.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| default | $a^2\left(\frac{x^4}{3c^2d\sqrt{c^2d\,x^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)}{3c^2}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\left(9\operatorname{arcsinh}(cx)^2x^4c^4-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3\right.}{}$ |
| parts | $a^2\left(\frac{x^4}{3c^2d\sqrt{c^2d\,x^2+d}}-\frac{4\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)}{3c^2}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\left(9\operatorname{arcsinh}(cx)^2x^4c^4-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3\right.}{}$ |

input
```
int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3/c^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1
/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2)))+1/27*b^2*(d*(c^2*x^2+1))^(1/2)*(9*arcsin
h(c*x)^2*x^4*c^4-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+2*c^4*x^4-36*arc
sinh(c*x)^2*x^2*c^2-54*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x
^2+1)^(1/2)))+54*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^
(1/2)))+90*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)-92*c^2*x^2-54*I*(c^2*x^2+1)^
(1/2)*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+54*I*(c^2*x^2+1)^(1/2)*dilog(1-I*
(c*x+(c^2*x^2+1)^(1/2)))-72*arcsinh(c*x)^2-94)/c^6/d^2/(c^2*x^2+1)+2/9*a*b
*(d*(c^2*x^2+1))^(1/2)*(3*arcsinh(c*x)*c^4*x^4-c^3*x^3*(c^2*x^2+1)^(1/2)-1
2*arcsinh(c*x)*c^2*x^2-9*I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)+9
*I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)+15*c*x*(c^2*x^2+1)^(1/2)-
24*arcsinh(c*x))/c^6/d^2/(c^2*x^2+1)
```

## 3.300.5 Fricas [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(
c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

## 3.300.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^5(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.300.7 Maxima [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
1/3*a^2*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*
d) - 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*(b^2*c^4*sqrt(d)*x^4 - 4*b^2*c^2
*sqrt(d)*x^2 - 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^8*d^2*x^2 + c^6*d^2) + integrate(2/3*((4*b^2*c^3*x^3 + (3*a*b*c^5
- b^2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) + (3*b^2*c^4*x^4 + (3*a*b*c^6 -
b^2*c^6)*x^6 + 12*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c
^2*x^2 + 1))/(c^10*d^(3/2)*x^5 + 2*c^8*d^(3/2)*x^3 + c^6*d^(3/2)*x + (c^9*
d^(3/2)*x^4 + 2*c^7*d^(3/2)*x^2 + c^5*d^(3/2))*sqrt(c^2*x^2 + 1)), x)
```

### 3.300.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

# 3.301 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.301.1 Optimal result

Integrand size = 28, antiderivative size = 400

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx &= \frac{b^2x(1+c^2x^2)}{4c^4d\sqrt{d+c^2dx^2}} \\ &- \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c^3d\sqrt{d+c^2dx^2}} \\ &- \frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^5d\sqrt{d+c^2dx^2}} \\ &+ \frac{3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2c^4d^2} - \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc^5d\sqrt{d+c^2dx^2}} \\ &- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}} \\ &- \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^5d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
1/4*b^2*x*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^3*(a+b*arcsinh(c*x))^2/c
^2/d/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2
*d*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*
d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^
(1/2)-1/2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d/(c^2*d*x^2+d)^(1/
2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2
)/c^5/d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2
*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/c^4/d^2
```

### 3.301.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.72

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{4a^2c\sqrt{d}x(3+c^2x^2)-12a^2\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+b^2\sqrt{d}(8}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(4*a^2*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a^2*Sqrt[d + c^2*d*x^2]*Log[c*d*x +
Sqrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*(8*c*x*ArcSinh[c*x]^2 + 8*Sqrt[
1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*(-4*ArcS
inh[c*x]^3 - 2*ArcSinh[c*x]*(Cosh[2*ArcSinh[c*x]] + 8*Log[1 + E^(-2*ArcSin
h[c*x])]) + 2*ArcSinh[c*x]^2*(-4 + Sinh[2*ArcSinh[c*x]]) + Sinh[2*ArcSinh[
c*x]])) + 2*a*b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(6*ArcSinh
[c*x]^2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[c*x]*Sinh[
2*ArcSinh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

### 3.301.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {6225, 6227, 262, 222, 6191, 262, 222, 6198, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\int \frac{x^3(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6227}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{b\int \frac{x^2}{\sqrt{c^2x^2+1}}dx}{2c}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}\right)}{cd\sqrt{c^2dx^2+d}}+\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\int x(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 262$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)}{2c}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}\right)}{cd\sqrt{c^2dx^2+d}}+\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\int x(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 222$$

$$\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\int x(a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}-\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 6191$$

---

3.301. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{c\sqrt{c^2dx^2+d}}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 262$$

$$\frac{3\left(-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{c\sqrt{c^2dx^2+d}}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 222$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}+\frac{3\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{2c^2}+\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 6198$$

---

3.301. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}\right.}{c^2d}$$

$\downarrow$ 6212

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^4}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}\right.}{c^2d}$$

$\downarrow$ 3042

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int-i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}\right.}{c^2d}$$

$\downarrow$ 26

---

3.301. $\int\frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{i\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)}{2c}\right)}{cd\sqrt{c^2dx^2+d}}-$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}}{c^2d}$$

$$\Big\downarrow \text{4201}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}+\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2}-\frac{b\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\text{ar}\right.}{2c}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}}{c^2d}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}+\frac{x^2(}{}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}}{c^2d}$$

$$\Big\downarrow \text{2715}$$

3.301. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}\right.}{c^2d}$$

$\downarrow$ 2838

$$-\frac{x^3(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+$$

$$\frac{3\left(\frac{x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2c^2d}-\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{6bc^3\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.}{c\sqrt{c^2dx^2+d}}\right.}{c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}+\frac{x^2(a+b\text{arcsi}}{2c^2}\right.}{cd\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((x^3*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) + (3*((x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x])^3)/(6*b*c^3*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*(
(x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSi
nh[c*x]/(2*c^3)))/2))/(c*Sqrt[d + c^2*d*x^2])))/(c^2*d) + (2*b*Sqrt[1 + c^
2*x^2]*((x^2*(a + b*ArcSinh[c*x]))/(2*c^2) - (b*((x*Sqrt[1 + c^2*x^2])/(2*
c^2) - ArcSinh[c*x]/(2*c^3)))/(2*c) + (I*(((-1/2*I)*(a + b*ArcSinh[c*x])^2
)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*Pol
yLog[2, -E^(2*ArcSinh[c*x])])/4)))/c^4))/(c*d*Sqrt[d + c^2*d*x^2])
```

3.301. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

### 3.301.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.301. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.301. $\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

## 3.301.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.45

| method | result |
|---|---|
| default | $\frac{a^2x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3a^2x}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\left(-2\,\mathrm{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,x^3c^3+2\ \mathrm{arcsin}\right.}{}$ |
| parts | $\frac{a^2x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3a^2x}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\left(-2\,\mathrm{arcsinh}(cx)^2\sqrt{c^2x^2+1}\,x^3c^3+2\ \mathrm{arcsin}\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(c^2*d*x^2+d)^(1/2)-
3/2*a^2/c^4/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-
1/4*b^2/(c^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)*(-2*arcsinh(c*x)^2*(c^2*x^
2+1)^(1/2)*x^3*c^3+2*arcsinh(c*x)*c^4*x^4+2*arcsinh(c*x)^3*x^2*c^2-c^3*x^3
*(c^2*x^2+1)^(1/2)-4*arcsinh(c*x)^2*x^2*c^2+8*arcsinh(c*x)*ln(1+(c*x+(c^2*
x^2+1)^(1/2))^2)*x^2*c^2-6*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c*x+3*arcsinh(
c*x)*c^2*x^2+4*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2+2*arcsinh(c*x
)^3-c*x*(c^2*x^2+1)^(1/2)-4*arcsinh(c*x)^2+8*arcsinh(c*x)*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)+arcsinh(c*x)+4*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2))/c^5/
d^2-1/4*a*b/(c^2*x^2+1)^(3/2)*(d*(c^2*x^2+1))^(1/2)*(-4*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)*x^3*c^3+2*c^4*x^4+6*arcsinh(c*x)^2*x^2*c^2-8*arcsinh(c*x)*c^2
*x^2+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-12*arcsinh(c*x)*c*x*(c^2*x^
2+1)^(1/2)+3*c^2*x^2+6*arcsinh(c*x)^2-8*arcsinh(c*x)+8*ln(1+(c*x+(c^2*x^2+
1)^(1/2))^2)+1)/c^5/d^2
```

## 3.301.5 Fricas [F]

$$\int \frac{x^4(a+b\mathrm{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(
c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.301.6 Sympy [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^4(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.301.7 Maxima [F]

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
1/2*a^2*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d)
 - 3*arcsinh(c*x)/(c^5*d^(3/2))) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^
2 + 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^2 + d)^(3/2), x)
```

### 3.301.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

# 3.302 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.302.1 Optimal result

Integrand size = 28, antiderivative size = 383

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = &-\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}+\frac{2b^2(1+c^2x^2)}{c^4d\sqrt{d+c^2dx^2}}\\&-\frac{4b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{c^3d\sqrt{d+c^2dx^2}}+\frac{2bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c^3d\sqrt{d+c^2dx^2}}\\&-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{c^4d^2}\\&-\frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}\\&+\frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}-\frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^4d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
2*b^2*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^2*(a+b*arcsinh(c*x))^2/c^2/d
/(c^2*d*x^2+d)^(1/2)-4*a*b*x*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4
*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*x*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsin
h(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/d/(c^2*d*x^2+d
)^(1/2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^
4/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*
x^2+1)^(1/2)/c^4/d/(c^2*d*x^2+d)^(1/2)+2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d
)^(1/2)/c^4/d^2
```

### 3.302.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.83

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{2a^2+2b^2+a^2c^2x^2+2b^2c^2x^2-2abcx\sqrt{1+c^2x^2}+4ab\text{arcsinh}(cx)+2abc^2x^2}{}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(2*a^2 + 2*b^2 + a^2*c^2*x^2 + 2*b^2*c^2*x^2 - 2*a*b*c*x*Sqrt[1 + c^2*x^2]
 + 4*a*b*ArcSinh[c*x] + 2*a*b*c^2*x^2*ArcSinh[c*x] - 2*b^2*c*x*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x] + 2*b^2*ArcSinh[c*x]^2 + b^2*c^2*x^2*ArcSinh[c*x]^2 -
4*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 +
c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x
^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*P
olyLog[2, (-I)/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/
E^ArcSinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

### 3.302.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.69, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6225, 6213, 2009, 6227, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}\,dx$$

$$\Big\downarrow \; 6225$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 6213$$

$$\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

$$\Big\downarrow \text{241}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

$$\Big\downarrow \text{6204}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

$$\Big\downarrow \text{3042}$$

3.302. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{cd\sqrt{c^2dx^2+d}} -$$
$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

$\downarrow$ 4668

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}}{c^3}\right.}{cd\sqrt{c^2dx^2+d}}$$
$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

$\downarrow$ 2715

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{a}}\right.}{c^3}\right.}{cd\sqrt{c^2dx^2+d}}$$
$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

$\downarrow$ 2838

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3}+\frac{x(a+b\text{arcsinh}}{c^2}\right.}{cd\sqrt{c^2dx^2+d}}$$
$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}$$

input
```
Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

---

3.302. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

output
```
-((x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) + (2*((Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a*x
 - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/(c*Sqrt[d + c^2*d*x^2])))/
(c^2*d) + (2*b*Sqrt[1 + c^2*x^2]*(-((b*Sqrt[1 + c^2*x^2])/c^3) + (x*(a + b
*ArcSinh[c*x]))/c^2 - (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b
*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/c^3))
/(c*d*Sqrt[d + c^2*d*x^2])
```

### 3.302.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.302. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.302.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| default | $a^2\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\left(\operatorname{arcsinh}(cx)^2x^2c^2+2i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)\right.}{}$ |
| parts | $a^2\left(\frac{x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{c^2d\,x^2+d}}\right)+\frac{b^2\sqrt{d(c^2x^2+1)}\left(\operatorname{arcsinh}(cx)^2x^2c^2+2i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)\right.}{}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2/d/c^4/(c^2*d*x^2+d)^(1/2))+b^2*(d*(c^
2*x^2+1))^(1/2)*(arcsinh(c*x)^2*x^2*c^2+2*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)
*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-I
*(c*x+(c^2*x^2+1)^(1/2)))-2*arcsinh(c*x)*c*x*(c^2*x^2+1)^(1/2)+2*c^2*x^2-2
*I*(c^2*x^2+1)^(1/2)*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*(c^2*x^2+1)^(1
/2)*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2*arcsinh(c*x)^2+2)/c^4/d^2/(c^2*x^
2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)*(arcsinh(c*x)*c^2*x^2+I*(c^2*x^2+1)^(1/2)
*ln(c*x+(c^2*x^2+1)^(1/2)-I)-I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+
I)-c*x*(c^2*x^2+1)^(1/2)+2*arcsinh(c*x))/c^4/d^2/(c^2*x^2+1)
```

### 3.302.5 Fricas [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(
c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.302.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^3(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.302.7 Maxima [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
-2*a*b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + 2*a*b*(x^2/(sqrt(
c^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a^2*
(x^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d)) + b^2*((
c^2*x^2 + 2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*x^2 + 1)*c^4*d^(3/2)
) - integrate(2*(c^4*x^4 + 3*c^2*x^2 + (c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1)
 + 2)*log(c*x + sqrt(c^2*x^2 + 1))/((c^5*d^(3/2)*x^2 + c^3*d^(3/2))*(c^2*x
^2 + 1) + (c^6*d^(3/2)*x^3 + c^4*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x))
```

### 3.302.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

input
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

# 3.303 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.303.1 Optimal result

Integrand size = 28, antiderivative size = 233

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{d+c^2dx^2}}+\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c^3d\sqrt{d+c^2dx^2}}$$

output
```
-x*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^
2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2
+1)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+b^2*polylog
(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

### 3.303.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{-3a^2cdx - 3abd\left(2cx\text{arcsinh}(cx) - \sqrt{1+c^2x^2}(\text{arcsinh}(cx)^2 + \log\left(1+c^2x^2\right)\right)}{}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(-3*a^2*c*d*x - 3*a*b*d*(2*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(ArcSinh[c
*x]^2 + Log[1 + c^2*x^2])) + 3*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x +
 Sqrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*d*(ArcSinh[c*x]*(-3*c*x*ArcSinh[c*x] +
 Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(3 + ArcSinh[c*x]) + 6*Log[1 + E^(-2*ArcS
inh[c*x])])) - 3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c
^3*d^2*Sqrt[d + c^2*d*x^2])
```

### 3.303.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6225, 6198, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}\,dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}} + \frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}} - \frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}} + \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6212}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2ib\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{2ib\sqrt{c^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{ar}}{}\right.}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(c}\right.\right.}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2838}$$

3.303. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

$$-\frac{x(a+b\operatorname{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\operatorname{arcsinh}(cx)}+1\right)(a+b\operatorname{arcsinh}(cx))+\frac{1}{4}b\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)\right)-\frac{i(a+b\operatorname{arcsinh}(cx))^2}{2b}\right)}{c^3d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((x*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) + (Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c^3*d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b*Sq
rt[1 + c^2*x^2]*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcS
inh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x
])])/4)))/(c^3*d*Sqrt[d + c^2*d*x^2])
```

### 3.303.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

### 3.303.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(231) = 462$.

Time = 0.25 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.05

| method | result |
|---|---|
| default | $-\frac{a^2 x}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{a^2 \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{c^2 d\sqrt{c^2 d}}+\frac{b^2 \sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b^2 \sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 x}{c^2 d^2(c^2x^2+1)}-\frac{b^2 \sqrt{d(c^2x}{c^3 d}$ |
| parts | $-\frac{a^2 x}{c^2 d\sqrt{c^2 d\,x^2+d}}+\frac{a^2 \ln\left(\frac{c^2 dx}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{c^2 d\sqrt{c^2 d}}+\frac{b^2 \sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,c^3 d^2}-\frac{b^2 \sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2 x}{c^2 d^2(c^2x^2+1)}-\frac{b^2 \sqrt{d(c^2x}{c^3 d}$ |

input
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a^2/c^2/d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d
*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/c^3/d^2*arcsinh(c*x)^3-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^2/d^2
/(c^2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^3/d^2/(c^2*x^2+1
)^(1/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/c^3/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+a*b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x
)/c^2/d^2/(c^2*x^2+1)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/
d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

## 3.303.5 Fricas [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(
c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

## 3.303.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^2(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.303.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
-a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + integr
ate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x
^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.303.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

# 3.304 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.304.1 Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}}$$

output
```
-(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arc
tan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*x^2+d)^(1/2)-2*I
*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*
x^2+d)^(1/2)+2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2
)/c^2/d/(c^2*d*x^2+d)^(1/2)
```

### 3.304.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.15

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx =$$
$$-\frac{a^2+2ab\text{arcsinh}(cx)+b^2\text{arcsinh}(cx)^2-4ab\sqrt{1+c^2x^2}\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)+2ib^2\sqrt{1+c^2x^2}\text{arcs}}{}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((a^2 + 2*a*b*ArcSinh[c*x] + b^2*ArcSinh[c*x]^2 - 4*a*b*Sqrt[1 + c^2*x^2]
*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*L
og[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1
+ I/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSin
h[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c^2*d
*Sqrt[d + c^2*d*x^2]))
```

### 3.304.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6213, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6204}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 4668

$$-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2715

$$-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(c}\right.}{c^2d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2838

$$-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right.}{c^2d\sqrt{c^2dx^2+d}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(c^2*d*Sqrt[d + c^2*d*x^2])) + (2*b*Sqrt[1 + c^2*
x^2]*(2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*
E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]]))/(c^2*d*Sqrt[d + c^2*d
*x^2])
```

### 3.304.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.304.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.69

| method | result |
|---|---|
| default | $-\frac{a^2}{c^2 d\sqrt{c^2 d\,x^2+d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\left(2i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)-2i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\ln\left(1-i\left(cx+\sqrt{c^2\right.\right.\right.}{c^2d^2(c^2}$ |
| parts | $-\frac{a^2}{c^2 d\sqrt{c^2 d\,x^2+d}}-\frac{b^2\sqrt{d(c^2x^2+1)}\left(2i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)-2i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(cx)\ln\left(1-i\left(cx+\sqrt{c^2\right.\right.\right.}{c^2d^2(c^2}$ |

input
```
int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2/c^2/d/(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)*(2*I*(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*(c^2*x^2+1)^(1/2)*dilog(1+
I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*(c^2*x^2+1)^(1/2)*dilog(1-I*(c*x+(c^2*x^2+1
)^(1/2)))+arcsinh(c*x)^2)/c^2/d^2/(c^2*x^2+1)-2*a*b*(d*(c^2*x^2+1))^(1/2)*
(-I*(c^2*x^2+1)^(1/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)+I*(c^2*x^2+1)^(1/2)*ln(c
*x+(c^2*x^2+1)^(1/2)-I)+arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)
```

## 3.304.5 Fricas [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x)
+ a^2*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

## 3.304.6 Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.304.7 Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-a^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2*x^2
+ 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*
d*x^2 + d)^(3/2), x)
```

### 3.304.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

# 3.305 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.305.1 Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}$$

output
```
x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2
+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^
2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)
```

## 3.305.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)^2 + 2b\text{arcsinh}(cx)\left(acx - b\sqrt{1+c^2x^2}\log\left(1\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*
c*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2
*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2])
```

### 3.305.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6202, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6212}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+$$
$$\frac{2ib\sqrt{c^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 2620$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcs}}{2}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 2715$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 2838$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{cd\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b*Sqrt[1 + c^2
*x^2]*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*
Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/
(c*d*Sqrt[d + c^2*d*x^2])
```

### 3.305.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

### 3.305.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| default | $\frac{a^2 x}{d\sqrt{c^2 d\, x^2+d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\, \operatorname{arcsinh}(cx)^2 x}{d^2(c^2x^2+1)}+\frac{b^2\sqrt{d(c^2x^2+1)}\, \operatorname{arcsinh}(cx)^2}{c\, d^2\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d(c^2x^2+1)}\, \operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x\right.\right.}{\sqrt{c^2x^2+1}\, c\, d^2}$ |
| parts | $\frac{a^2 x}{d\sqrt{c^2 d\, x^2+d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\, \operatorname{arcsinh}(cx)^2 x}{d^2(c^2x^2+1)}+\frac{b^2\sqrt{d(c^2x^2+1)}\, \operatorname{arcsinh}(cx)^2}{c\, d^2\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d(c^2x^2+1)}\, \operatorname{arcsinh}(cx)\ln\left(1+\left(cx+\sqrt{c^2x\right.\right.}{\sqrt{c^2x^2+1}\, c\, d^2}$ |

input
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*x/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/d^2/(
c^2*x^2+1)*x+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c/d^2/(c^2*x^2+1)^(1
/2)-2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*arcsinh(c*x)*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d
^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x
^2+1))^(1/2)/c/d^2*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d
^2/(c^2*x^2+1)*x-2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2)
```

### 3.305.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.305.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.305.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2), x) + 2
*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a^2*x/(sqrt(c^2*d*x^2 + d)*d
) - a*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

### 3.305.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2), x)
```

# 3.306 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx$



## 3.306.1 Optimal result

Integrand size = 28, antiderivative size = 412

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^{3/2}}\,dx &= \frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} \\ &- \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &- \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &+ \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &+ \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\ &+ \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsinh(c*x))*arctan(c
*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsi
nh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d
)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+
1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)
))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x
^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x
))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2
)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d
)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*
x^2+d)^(1/2)
```

### 3.306.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.38

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{a^2d+a^2\sqrt{d}\sqrt{d+c^2dx^2}\log(cx)-a^2\sqrt{d}\sqrt{d+c^2dx^2}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right)+}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
(a^2*d + a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*x] - a^2*Sqrt[d]*Sqrt[d + c
^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2*a*b*d*(ArcSinh[c*x] - 2
*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]*Log[1 - E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 +
 E^(-ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - S
qrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])]) + b^2*d*(ArcSinh[c*x]^2 +
Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] + (2*I)*Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
^2*Log[1 + E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2
, -E^(-ArcSinh[c*x])] + (2*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[
c*x]] - (2*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*Sqrt[1 +
c^2*x^2]*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*
PolyLog[3, -E^(-ArcSinh[c*x])] - 2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSin
h[c*x])]))/(d^2*Sqrt[d + c^2*d*x^2])
```

### 3.306.3 Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.59, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {6226, 6204, 3042, 4668, 2715, 2838, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{6226}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6204}$$

$$-\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{2b\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsi}}\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}}\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d} - \frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx)) - ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right) + ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 6231

$$\frac{\sqrt{c^2x^2+1}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{cx}d\operatorname{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}} - \frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx)) - ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right) + ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 3042

$$\frac{\sqrt{c^2x^2+1}\int i(a+b\operatorname{arcsinh}(cx))^2\csc(i\operatorname{arcsinh}(cx))d\operatorname{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}} - \frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx)) - ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right) + ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 26

$$\frac{i\sqrt{c^2x^2+1}\int (a+b\operatorname{arcsinh}(cx))^2\csc(i\operatorname{arcsinh}(cx))d\operatorname{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}} - \frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx)) - ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right) + ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 4670

$$\frac{i\sqrt{c^2x^2+1}\left(2ib\int (a+b\operatorname{arcsinh}(cx))\log\left(1-e^{\operatorname{arcsinh}(cx)}\right)d\operatorname{arcsinh}(cx) - 2ib\int (a+b\operatorname{arcsinh}(cx))\log\left(1+e^{\operatorname{arcsinh}(cx)}\right)\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx)) - ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right) + ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$\downarrow$ 3011

3.306. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+2ib\right.}{}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(\right.\right.}{}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx}\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{i\sqrt{c^2x^2+1}\left(2i\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2-2ib\left(b\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(c}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
(a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(2
*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSi
nh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]]))/(d*Sqrt[d + c^2*d*x^2]) + (I
*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]] -
 (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]]) + b*PolyLog[
3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSin
h[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/(d*Sqrt[d + c^2*d*x^2])
```

## 3.306. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx$

### 3.306.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.306. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{3/2}}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.306.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x\,(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

### 3.306.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

### 3.306.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}\left(cx\right)\right)^2}{x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

### 3.306.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-a^2*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + integra
te(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x) + 2*a*b*lo
g(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

### 3.306.8 Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x), x)
```

### 3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2}{x\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.307 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx$



## 3.307.1 Optimal result

Integrand size = 28, antiderivative size = 305

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx &= -\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{d+c^2dx^2}}\\ &- \frac{2c^2x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}\\ &- \frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\ &+ \frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\ &+ \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))^2
/d/(c^2*d*x^2+d)^(1/2)-2*c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d
*x^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*
(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+4*b*c*(a+b*arcsinh(c*x))*ln(1+(c*x
+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b^2*c*polyl
og(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b
^2*c*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d
)^(1/2)
```

### 3.307.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2 (d+c^2dx^2)^{3/2}}\,dx =$$
$$-\frac{a^2+2a^2c^2x^2+2ab\text{arcsinh}(cx)+4abc^2x^2\text{arcsinh}(cx)+b^2\text{arcsinh}(cx)^2+2b^2c^2x^2\text{arcsinh}(cx)^2-2b^2cx\sqrt{1}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
-((a^2 + 2*a^2*c^2*x^2 + 2*a*b*ArcSinh[c*x] + 4*a*b*c^2*x^2*ArcSinh[c*x] +
 b^2*ArcSinh[c*x]^2 + 2*b^2*c^2*x^2*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E
^(-2*ArcSinh[c*x])] - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(
-2*ArcSinh[c*x])] - 2*a*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x] - a*b*c*x*Sqrt[1
+ c^2*x^2]*Log[1 + c^2*x^2] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*
ArcSinh[c*x])] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])]
)/(d*x*Sqrt[d + c^2*d*x^2]))
```

### 3.307.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.87, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6224, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2 (c^2dx^2+d)^{3/2}}\,dx$$

$$\Big\downarrow \text{6224}$$

$$-2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx+\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6202}$$

$$-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)+\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 6212$$

$$-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int\frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)+\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int-i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 26$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 4201$$

$$-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\int\frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{cd\sqrt{c^2dx^2+d}}\right)+\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 2620$$

---

3.307. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}dx$

$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2715

$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2838

$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$\downarrow$ 6214

$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$\downarrow$ 5984

$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{4bc\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

---

3.307. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}dx$

$$\Big\downarrow \; 3042$$

$$\frac{4bc\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-$$
$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 26$$

$$\frac{4ibc\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-$$
$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 4670$$

$$\frac{4ibc\sqrt{c^2x^2+1}\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}\left(e^{2\text{ar}}\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 2715$$

$$\frac{4ibc\sqrt{c^2x^2+1}\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{a}}\right.}{d\sqrt{c^2dx^2+d}}$$
$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 2838$$

3.307. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx$

$$\frac{4ibc\sqrt{c^2x^2+1}\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{arcsi}\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcs}\right.\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(d*x*Sqrt[d + c^2*d*x^2])) - 2*c^2*((x*(a + b*Arc
Sinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b*Sqrt[1 + c^2*x^2]*(((-1/2
*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*
ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c*d*Sqrt[d +
 c^2*d*x^2])) + ((4*I)*b*c*Sqrt[1 + c^2*x^2]*(I*(a + b*ArcSinh[c*x])*ArcTa
nh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcSinh[c*x])] - (I/4)*b
*PolyLog[2, E^(2*ArcSinh[c*x])]))/(d*Sqrt[d + c^2*d*x^2])
```

### 3.307.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.307. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

### 3.307.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. 2(317) = 634.

Time = 0.30 (sec) , antiderivative size = 1117, normalized size of antiderivative = 3.66

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1117 |
| parts | Expression too large to display | 1117 |

input
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/2))-b^2*(arcsin
h(c*x)^2-4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^3*
c^3-2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x*c-2
*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x*c-2*(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x*c-4*(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3+4*arcsinh(c*x)*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c
*x+(c^2*x^2+1)^(1/2))*x^3*c^3+4*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*x
^4*c^4+4*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^4*c^4-2*(c^2*x^2+1)^(1
/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^3*c^3-4*(c^2*x^2+1)^(1/2)*poly
log(2,-c*x-(c^2*x^2+1)^(1/2))*x^3*c^3-4*(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c
^2*x^2+1)^(1/2))*x^3*c^3-(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/
2))^2)*x*c-2*(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x*c-2*(c^
2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x*c+4*arcsinh(c*x)*ln(1+(c
*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x
^2*c^2+4*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2+4*arcsinh(c*x)*l
n(1+c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)
*x^4*c^4+4*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*x^4*c^4+4*polylog(2,c*x+(c^2*
x^2+1)^(1/2))*x^4*c^4+4*polylog(2,c*x+(c^2*x^2+1)^(1/2))*x^2*c^2+4*polylog
(2,-c*x-(c^2*x^2+1)^(1/2))*x^2*c^2)*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2...
```

### 3.307.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

### 3.307.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^2\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

### 3.307.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
a*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - 2*(2*c^2*x/(sqrt(c^2
*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a*b*arcsinh(c*x) - (2*c^2*x/
(sqrt(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a^2 + b^2*integrate
(log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

### 3.307.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

### 3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{x^2\,(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.308 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx$



## 3.308.1 Optimal result

Integrand size = 28, antiderivative size = 573

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx &= -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{dx\sqrt{d+c^2dx^2}} \\
&-\frac{3c^2(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+c^2dx^2}}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{d+c^2dx^2}} \\
&+\frac{4bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&+\frac{3c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&-\frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d\sqrt{d+c^2dx^2}} \\
&+\frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&-\frac{2ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&+\frac{2ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&-\frac{3bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&-\frac{3b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}} \\
&+\frac{3b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-3/2*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsinh(c*x))
^2/d/x^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x/
(c^2*d*x^2+d)^(1/2)+4*b*c^2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arcsinh(c*x))^2*arct
anh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^2
*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*b*c^
2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d
/(c^2*d*x^2+d)^(1/2)-2*I*b^2*c^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^
2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*c^2*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-3*b*c^2*(a+b*arcsinh(c
*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1
/2)-3*b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d
*x^2+d)^(1/2)+3*b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)
/d/(c^2*d*x^2+d)^(1/2)
```

## 3.308.2 Mathematica [A] (verified)

Time = 7.42 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx = \sqrt{d(1+c^2x^2)}\left(-\frac{a^2}{2d^2x^2}-\frac{a^2c^2}{d^2(1+c^2x^2)}\right)$$
$$-\frac{3a^2c^2\log(x)}{2d^{3/2}}+\frac{3a^2c^2\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{2d^{3/2}}$$
$$+\frac{abc^2\left(-8\text{arcsinh}(cx)+16\sqrt{1+c^2x^2}\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)-2\sqrt{1+c^2x^2}\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\sqrt{1}\right.}{}$$
$$+\frac{b^2c^2\left(-8\text{arcsinh}(cx)^2-4\sqrt{1+c^2x^2}\text{arcsinh}(cx)\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2\text{csch}^2\left(\frac{1}{2}\text{arcsin}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^2*x^2) - (a^2*c^2)/(d^2*(1 + c^2*x^2)))
 - (3*a^2*c^2*Log[x])/(2*d^(3/2)) + (3*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 +
 c^2*x^2)]])/(2*d^(3/2)) + (a*b*c^2*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2
]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2]
- Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-A
rcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tan
h[ArcSinh[c*x]/2]))/(4*d*Sqrt[d*(1 + c^2*x^2)]) + (b^2*c^2*(-8*ArcSinh[c*x
]^2 - 4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2
*x^2]*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
*Log[1 - I/E^ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 +
 I/E^ArcSinh[c*x]] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 + E^(-ArcSi
nh[c*x])] + 8*Sqrt[1 + c^2*x^2]*Log[Tanh[ArcSinh[c*x]/2]] - 24*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x
^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2,
I/E^ArcSinh[c*x]] + 24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, E^(-ArcSi
nh[c*x])] - 24*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^(-ArcSinh[c*x])] + 24*Sq...
```

### 3.308.3 Rubi [A] (verified)

Time = 4.18 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.71, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.821$, Rules used = {6224, 6224, 243, 73, 221, 6204, 3042, 4668, 2715, 2838, 6226, 6204, 3042, 4668, 2715, 2838, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(c^2dx^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{3}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx+\frac{bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x^2(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6224}$$

$$-\frac{3}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx+\frac{bc\sqrt{c^2x^2+1}\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\sqrt{c^2x^2+1}}dx-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{3}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx+\frac{bc\sqrt{c^2x^2+1}\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{3}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx+\frac{bc\sqrt{c^2x^2+1}\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)+\frac{b\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c}-\frac{a+b\text{arcsinh}(cx)}{x}\right)}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{bc\sqrt{c^2x^2+1}\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx\right)-\frac{a+b\text{arcsinh}(cx)}{x}-b\,\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6204}$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-c\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)-\frac{a+b\text{arcsinh}(cx)}{x}-b\,\text{carctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 3042}$$

---

3.308. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx$

$$\frac{bc\sqrt{c^2x^2+1}\left(-c\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)-\frac{a+b\text{arcsinh}(cx)}{x}-b c\text{arctanh}\left(\sqrt{c^2x^2+\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{3}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 4668

$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{ar}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{3}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2715

$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcs}}\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{3}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2838

$$-\frac{3}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx+$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 6226

$$-\frac{3}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}\right)+$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 6204

3.308. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}dx$

$$-\frac{3}{2}c^2\left(-\frac{2b\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}\right)+$$
$$\frac{bc\sqrt{c^2x^2+1}\Big(-c\big(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}\ldots}\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 3042

$$-\frac{3}{2}c^2\left(\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{2b\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{(a+b\text{arc}\ldots}{d\sqrt{c^2\ldots}}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\Big(-c\big(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}\ldots}\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 4668

$$-\frac{3}{2}c^2\left(-\frac{2b\sqrt{c^2x^2+1}\Big(-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\,\text{arcta}\ldots}{d\sqrt{c^2dx^2+d}}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\Big(-c\big(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}\ldots}\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2715

$$-\frac{3}{2}c^2\left(-\frac{2b\sqrt{c^2x^2+1}\Big(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)\ldots}{d\sqrt{c^2dx^2+d}}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\Big(-c\big(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}\ldots}\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2838

3.308. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx$

$$-\frac{3}{2}c^2\left(\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(c}\right.\right.}{d\sqrt{c^2dx^2+d}}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \text{6231}$$

$$-\frac{3}{2}c^2\left(\frac{\sqrt{c^2x^2+1}\int \frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{Po}\right.}{d\sqrt{c^2dx^2}}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}c^2\left(\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+\right.}{}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \text{26}$$

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+\right.}{}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \text{4670}$$

3.308. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx$

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\left(2ib\int(a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int(a+b\text{arcsinh}(cx))\log\left(1+e^{a}\right.\right.}{d\sqrt{c^2dx^2+d}}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 3011

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx)\right.\right.}{}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2720

$$-\frac{3}{2}c^2\left(\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+ba\right.\right.}{}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

$\downarrow$ 7143

$$-\frac{3}{2}c^2\left(-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{a}\right.\right.}{d\sqrt{c^2dx^2+d}}\right.$$
$$\frac{bc\sqrt{c^2x^2+1}\left(-c\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

3.308. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx$

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(d*x^2*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[1 + c^
2*x^2]*(-((a + b*ArcSinh[c*x])/x) - b*c*ArcTanh[Sqrt[1 + c^2*x^2]] - c*(2*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSin
h[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])))/(d*Sqrt[d + c^2*d*x^2]) - (3
*c^2*((a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x
^2]*(2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E
^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]]))/(d*Sqrt[d + c^2*d*x^2]
) + (I*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c
*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]]) + b*Po
lyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, E^
ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/(d*Sqrt[d + c^2*d*x^2])))
/2
```

### 3.308.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.308. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.308. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{3/2}}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.308.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^3\,(c^2 d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

### 3.308.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

### 3.308.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}\,(cx)\right)^2}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

### 3.308.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) -
 1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/
((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

## 3.308.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^3 (d + c^2dx^2)^{3/2}} \, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}x^3} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

## 3.308.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{x^3 (d + c^2dx^2)^{3/2}} \, dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{x^3\,(d\,c^2\,x^2 + d)^{3/2}} \, dx$$

input
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.309 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx$



## 3.309.1 Optimal result

Integrand size = 28, antiderivative size = 452

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx &= -\frac{b^2c^2(1+c^2x^2)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3dx^2\sqrt{d+c^2dx^2}}\\
&- \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2(a+b\text{arcsinh}(cx))^2}{3dx\sqrt{d+c^2dx^2}}\\
&+ \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}} + \frac{8c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d\sqrt{d+c^2dx^2}}\\
&+ \frac{20bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}}\\
&- \frac{16bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}}\\
&- \frac{b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{d\sqrt{d+c^2dx^2}}\\
&- \frac{5b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-1/3*b^2*c^2*(c^2*x^2+1)/d/x/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))^2/
d/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(
1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arc
sinh(c*x))*(c^2*x^2+1)^(1/2)/d/x^2/(c^2*d*x^2+d)^(1/2)+8/3*c^3*(a+b*arcsin
h(c*x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+20/3*b*c^3*(a+b*arcsinh(
c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)
^(1/2)-16/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*
x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/
2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-5/3*b^2*c^3*polylog(2,(c*x+
(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

### 3.309.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{-a^2+4a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4-b^2c^4x^4-abcx\sqrt{1+c^2x^2}-2ab\operatorname{arcsinh}(cx)}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
(-a^2 + 4*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*
x*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh[c*x] + 8*a*b*c^2*x^2*ArcSinh[c*x] + 16
*a*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b^2*A
rcSinh[c*x]^2 + 4*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSinh[c*x]^
2 - 8*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - 10*b^2*c^3*x^3*Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 6*b^2*c^3*x^3*Sqrt
[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 10*a*b*c^3*x^3*S
qrt[1 + c^2*x^2]*Log[c*x] - 3*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^
2] + 3*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 5*
b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*d*x^3*Sq
rt[d + c^2*d*x^2])
```

### 3.309.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6224, 6224, 242, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4 (c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{6224}$$

$$-\frac{4}{3}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2 (c^2dx^2+d)^{3/2}}dx+\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x^3(c^2x^2+1)}dx}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6224}$$

$$-\frac{4}{3}c^2\left(-2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx+\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}\right)+\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{4}{3}c^2\left(-2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx+\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\sqrt{c^2dx^2+d}}\right)+\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{4}{3}c^2\left(-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)+\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$\downarrow$ 6212

$$-\frac{4}{3}c^2\left(-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int\frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)+\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsi}}{x(c^2x^2}}{d\sqrt{c^2dx^2+d}}\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$\downarrow$ 3042

$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-$$
$$\frac{4}{3}c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int-i(a+b\text{arcsinh}(cx))\tan(ia}{cd\sqrt{c^2dx^2+d}}\right.\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$\downarrow$ 26

$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-$$
$$\frac{4}{3}c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\tan(iarc}{cd\sqrt{c^2dx^2+d}}\right.\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$\downarrow$ 4201

$$-\frac{4}{3}c^2\left(-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\int\frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(c}{2b}\right.}{cd\sqrt{c^2dx^2+d}}\right.\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2620

---

3.309. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}dx$

$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-$$
$$\frac{4}{3}c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)\right.\right.}{}\right.\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-$$
$$\frac{4}{3}c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)\right.\right.}{}\right.\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{4}{3}c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)\right.\right.}{}\right.\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6214}$$

$$-\frac{4}{3}c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsi}}\right.\right.\right.}{}\right.\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(c^2\left(-\int\frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{5984}$$

3.309. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx$

$$-\frac{4}{3}c^2\left(\frac{4bc\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2}}{\phantom{x}}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int (a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4}{3}c^2\left(\frac{4bc\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c}}{\phantom{x}}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{4}{3}c^2\left(\frac{4ibc\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-2c^2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2}}{\phantom{x}}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2ic^2\int (a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)-\frac{a+b\text{arcsinh}(cx)}{2x^2}-\frac{bc\sqrt{c^2x^2+1}}{2x}\right)}{3d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{4}{3}c^2\left(\frac{4ibc\sqrt{c^2x^2+1}\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arcta}\right.}{d\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2ic^2\left(\frac{1}{2}ib\int \log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctan}\right.\right.}{3d\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2715}$$

---

3.309. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx$

$$-\frac{4}{3}c^2\left(\frac{4ibc\sqrt{c^2x^2+1}\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right.\right.}{d\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2ic^2\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right.\right.\right.}{3d\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2ic^2\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,\right.\right.\right.}{3d\sqrt{c^2dx^2+d}}$$

$$\frac{4}{3}c^2\left(\frac{4ibc\sqrt{c^2x^2+1}\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,\right.\right.}{d\sqrt{c^2dx^2+d}}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])^2/(d*x^3*Sqrt[d + c^2*d*x^2]) + (2*b*c*Sqrt[1 +
c^2*x^2]*(-1/2*(b*c*Sqrt[1 + c^2*x^2])/x - (a + b*ArcSinh[c*x])/(2*x^2) -
(2*I)*c^2*(I*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*Po
lyLog[2, -E^(2*ArcSinh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])])))/
(3*d*Sqrt[d + c^2*d*x^2]) - (4*c^2*(-((a + b*ArcSinh[c*x])^2/(d*x*Sqrt[d +
 c^2*d*x^2])) - 2*c^2*((x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2])
+ ((2*I)*b*Sqrt[1 + c^2*x^2]*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*
(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(
2*ArcSinh[c*x])])/4)))/(c*d*Sqrt[d + c^2*d*x^2])) + ((4*I)*b*c*Sqrt[1 + c^
2*x^2]*(I*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyL
og[2, -E^(2*ArcSinh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])]))/(d*S
qrt[d + c^2*d*x^2])))/3
```

3.309. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx$

### 3.309.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.309. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx$

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

### 3.309.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2607 vs. $2(438) = 876$.

Time = 0.37 (sec) , antiderivative size = 2608, normalized size of antiderivative = 5.77

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2608 |
| parts | Expression too large to display | 2608 |

input
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*c^5+40/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*
x^2-1)/d^2*x^5*c^8-7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d
^2*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*c^2+1
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)^
2-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-c*x-(c^2
*x^2+1)^(1/2))*c^3+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ar
csinh(c*x)^2*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2
*c^3*(c^2*x^2+1)^(1/2)-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^
2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3+32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^4*x^4+7*c^2*x^2-1)/d^2*x^7*c^10-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3+a^2*(-1/3/d/x^3/(c^2*d*x^
2+d)^(1/2)-4/3*c^2*(-1/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2/d*x/(c^2*d*x^2+d)^(1/
2)))-64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2*x
^2+1)*c^8-32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(
c^2*x^2+1)*c^6+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2
*x^3*arcsinh(c*x)*c^6+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1
)/d^2*x*(c^2*x^2+1)*c^4+16*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-
1)/d^2*x*arcsinh(c*x)*c^4+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*
x^2-1)/d^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-8*a*b*(d*(c^2*x^2+1))^(1/...
```

### 3.309.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

### 3.309.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^4\,(d\,(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)
```

### 3.309.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1
/(sqrt(c^2*d*x^2 + d)*d*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/((c^2*d*x^2 + d)^(3/2)*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((
c^2*d*x^2 + d)^(3/2)*x^4), x)
```

### 3.309.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

### 3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^4\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)), x)
```

# 3.310 $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.310.1 Optimal result

Integrand size = 28, antiderivative size = 512

$$\begin{aligned}\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}}-\frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}}\\
&+\frac{2b^2(1+c^2x^2)}{c^6d^2\sqrt{d+c^2dx^2}}-\frac{16b^2x\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{bx^3(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}\\
&+\frac{11bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}\\
&-\frac{4x^2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{3c^6d^3}\\
&-\frac{22b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}\\
&+\frac{11ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}\\
&-\frac{11ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^6d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-1/3*x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^6/d^2/(c
^2*d*x^2+d)^(1/2)+2*b^2*(c^2*x^2+1)/c^6/d^2/(c^2*d*x^2+d)^(1/2)-4/3*x^2*(a
+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*x^3*(a+b*arcsinh(c*x)
)/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-16/3*a*b*x*(c^2*x^2+1)^(1/
2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-16/3*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c
^5/d^2/(c^2*d*x^2+d)^(1/2)+11/3*b*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
^5/d^2/(c^2*d*x^2+d)^(1/2)-22/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1
)^(1/2))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2)+11/3*I*b^2*polylog(
2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2
)-11/3*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^
2/(c^2*d*x^2+d)^(1/2)+8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

### 3.310.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.65

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\Big(a^2(8+12c^2x^2+3c^4x^4)+ab\Big(2(8+12c^2x^2+3c^4x^4)\,\text{arcsinh}(cx)$$

input
```
Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(a^2*(8 + 12*c^2*x^2 + 3*c^4*x^4) + a*b*(2*(8 + 12*c^
2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(c*x*(5 + 6*c^2*x^2) +
 22*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) + b^2*(c*x*Sqrt[1 + c^2*x
^2]*ArcSinh[c*x] - 6*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - ArcSinh[c*x]^2
 + 3*(1 + c^2*x^2)^2*(2 + ArcSinh[c*x]^2) + (1 + c^2*x^2)*(1 + 6*ArcSinh[c
*x]^2) + (11*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]
] - Log[1 + I/E^ArcSinh[c*x]]) + (11*I)*(1 + c^2*x^2)^(3/2)*(PolyLog[2, (-
I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSinh[c*x]])))/(3*c^6*d^3*(1 + c^2*
x^2)^2)
```

### 3.310.3 Rubi [A] (verified)

Time = 2.56 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6225, 6225, 243, 53, 2009, 6213, 2009, 6227, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}+\frac{4\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx}{3c^2d}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6225}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2}+\frac{b\int \frac{x^3}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{4\left(\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2}+\frac{b\int \frac{x^2}{\left(c^2x^2+1\right)^{3/2}}dx^2}{4c}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{4\left(\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2}+\frac{b\int \left(\frac{1}{c^2\sqrt{c^2x^2+1}}-\frac{1}{c^2\left(c^2x^2+1\right)^{3/2}}\right)dx^2}{4c}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{4\left(\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{2\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6213}$$

$$\frac{4\left(\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))dx}{c\sqrt{c^2dx^2+d}}\right)}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\int \frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

3.310. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{4\left(\frac{2b\sqrt{c^2x^2+1}\int\frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\frac{2b\sqrt{c^2x^2+1}\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{}}{}\right.}{c\sqrt{c^2dx^2+d}}\right.}{c^2d}\right.}{3c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\int\frac{x^2(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{2c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 6227

$$\frac{4\left(\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}-\frac{b\int\frac{x}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx)}{c^2d}\right.}{}\right.}{3c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}-\frac{b\int\frac{x}{\sqrt{c^2x^2+1}}dx}{c}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}\right)}{2c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4c}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 241

$$\frac{4\left(\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{c^2d}-\right.}{}\right.}{3c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\left(-\frac{\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c^2}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{2c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2x^2+1}}\right)}{4c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

3.310. $\int\frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}dx$

$\Big\downarrow$ 6204

$$4\left(\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsi}}{c^2d}\right.}{}\right.$$
$$\overline{3c^2d}$$
$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\left(-\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{2c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{2\sqrt{c^2x^2+1}}{c^4}+\frac{2}{c^4\sqrt{c^2}}\right.}{4c}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$
$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\Big\downarrow$ 3042

$$4\left(\frac{2b\sqrt{c^2x^2+1}\left(-\frac{\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{cd\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\right.$$
$$\overline{3c^2d}$$
$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\left(-\frac{\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{c^2x^2+1}}{c^3}\right)}{2c^2}-\frac{x^3(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$
$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\Big\downarrow$ 4668

3.310. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

$$4\left(\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c^3}+\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\left(-\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx\right.}{c^3}\right.}{2c^2}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2715

$$4\left(\frac{2b\sqrt{c^2x^2+1}\left(-\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(\right.}{c^3}\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\left(-\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcs}\right.}{c^3}\right.}{2c^2}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$4\left(\frac{2b\sqrt{c^2x^2+1}\left(-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\sqrt{}}{}\right.}{cd\sqrt{c^2dx^2+d}}\right.$$

$$3c^2d$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{3\left(-\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c^3}+\frac{x(a+b\text{arcsinh}(cx))}{c^2}-\right.}{2c^2}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^4(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

3.310. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

input
```
Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
-1/3*(x^4*(a + b*ArcSinh[c*x])^2)/(c^2*d*(d + c^2*d*x^2)^(3/2)) + (2*b*Sqr
t[1 + c^2*x^2]*((b*(2/(c^4*Sqrt[1 + c^2*x^2]) + (2*Sqrt[1 + c^2*x^2])/c^4)
)/(4*c) - (x^3*(a + b*ArcSinh[c*x]))/(2*c^2*(1 + c^2*x^2)) + (3*(-((b*Sqrt
[1 + c^2*x^2])/c^3) + (x*(a + b*ArcSinh[c*x]))/c^2 - (2*(a + b*ArcSinh[c*x
])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*Poly
Log[2, I*E^ArcSinh[c*x]])/c^3))/(2*c^2)))/(3*c*d^2*Sqrt[d + c^2*d*x^2]) +
(4*(-((x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) + (2*((Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*
(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/(c*Sqrt[d + c^2*d*x^2]
)))/(c^2*d) + (2*b*Sqrt[1 + c^2*x^2]*(-((b*Sqrt[1 + c^2*x^2])/c^3) + (x*(a
 + b*ArcSinh[c*x]))/c^2 - (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] -
 I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/c
^3))/(c*d*Sqrt[d + c^2*d*x^2])))/(3*c^2*d)
```

### 3.310.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

## 3.310. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.310. $\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.310.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(475) = 950$.

Time = 0.40 (sec) , antiderivative size = 1042, normalized size of antiderivative = 2.04

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1042 |
| parts | Expression too large to display | 1042 |

input
```
int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(x^4/c^2/d/(c^2*d*x^2+d)^(3/2)-4/c^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2
/3/d/c^4/(c^2*d*x^2+d)^(3/2)))-11/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-11/3*I*b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/c^6/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*b^2*(d*(
c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)+b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c
^2*x^2+1)*arcsinh(c*x)^2*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^4
/d^3*arcsinh(c*x)^2*x^2+11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c^6/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*b^2*(d*(c^2*x^2+1)
)^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(3/2)/c^5/d^3*arcsinh(c*x)*x-11/3*I*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2))
)+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*x^2+b^2*(d*(c^2*x^2+1))^
(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^2/c^6/d^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^4/d^3*x^2
+5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^6/d^3*arcsinh(c*x)^2+2*a*b*
(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(c*x)*x^2-2*a*b*(d*(c^2*x
^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/c^6/d
^3/(c^2*x^2+1)*arcsinh(c*x)+4*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^4/
d^3*arcsinh(c*x)*x^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/c^5/d
^3*x+10/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/c^6/d^3*arcsinh(c*x)+...
```

## 3.310.5 Fricas [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(
c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.310.6 Sympy [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^5(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.310.7 Maxima [F]

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^5}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
1/3*a^2*(3*x^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/
2)*c^4*d) + 8/((c^2*d*x^2 + d)^(3/2)*c^6*d)) + 1/3*(3*b^2*c^4*sqrt(d)*x^4
+ 12*b^2*c^2*sqrt(d)*x^2 + 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt
(c^2*x^2 + 1))^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + integrate(-2/3
*((12*b^2*c^3*x^3 - 3*(a*b*c^5 - b^2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) +
 (15*b^2*c^4*x^4 - 3*(a*b*c^6 - b^2*c^6)*x^6 + 20*b^2*c^2*x^2 + 8*b^2)*sqr
t(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^12*d^(5/2)*x^7 + 3*c^10*d^
(5/2)*x^5 + 3*c^8*d^(5/2)*x^3 + c^6*d^(5/2)*x + (c^11*d^(5/2)*x^6 + 3*c^9*
d^(5/2)*x^4 + 3*c^7*d^(5/2)*x^2 + c^5*d^(5/2))*sqrt(c^2*x^2 + 1)), x)
```

### 3.310.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^5\,(a+b\,\operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

# 3.311 $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.311.1 Optimal result

Integrand size = 28, antiderivative size = 398

$$\begin{aligned}\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} \\ &\quad -\frac{bx^2(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^4d^2\sqrt{d+c^2dx^2}} \\ &\quad -\frac{4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^5d^2\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc^5d^2\sqrt{d+c^2dx^2}} \\ &\quad +\frac{8b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}} \\ &\quad +\frac{4b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^5d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-1/3*x^3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/c^4/d^2/
(c^2*d*x^2+d)^(1/2)-x*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3
*b*x^2*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/
3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-4/3*(a+b*
arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arc
sinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^(1/2)+8/3*b*(a+b*ar
csinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2
*d*x^2+d)^(1/2)+4/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^
(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)
```

### 3.311.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.90

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{-a^2c\sqrt{d}x(3+4c^2x^2)+ab\sqrt{d}\left(\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)-8cx(1+c^2x^2)\right.}{}$$

input
```
Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(-(a^2*c*Sqrt[d]*x*(3 + 4*c^2*x^2)) + a*b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c
*x*ArcSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(
3*ArcSinh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 3*a^2*(1 + c^2*x^2)*Sqrt[d + c^2
*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - b^2*Sqrt[d]*(c*x + c^3*
x^3 - Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 3*c*x*ArcSinh[c*x]^2 + 4*c^3*x^3*Ar
cSinh[c*x]^2 - 4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - (1 + c^2*x^2)^(3/2)*
ArcSinh[c*x]^3 - 8*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[
c*x])] + 4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^5*d
^(5/2)*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

### 3.311.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6225, 6225, 252, 222, 6198, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}\,dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6225}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{b\int \frac{x^2}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{\frac{2b\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 252

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{b\left(\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{c^2}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{\frac{2b\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 222

$$\frac{\frac{2b\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{c^2d}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6198

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{\frac{2b\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}}{c^2d}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6212

3.311. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}dx$

$$\frac{\frac{2b\sqrt{c^2x^2+1}\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{2ib\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc^3d\sqrt{c^2dx^2+d}}}{c^2d}+\frac{2b\sqrt{c^2x^2+1}\left(-\frac{i\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{2ib\sqrt{c^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}(a+b\text{arcsin}}{3bc^3d\sqrt{c^2dx^2+}}}{c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx}{c^3}\right.}{2}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

3.311. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{-\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^3d\sqrt{c^2dx^2+d}}-\frac{x(a+ba}{c^2d}}{c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}-\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}}{c^3d\sqrt{c^2dx^2+d}}}{c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arcsin}}{2b}}{c^4}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{x^3(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}+$$

$$\frac{-\frac{x(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))}{2b}}{c^3d\sqrt{c^2dx^2+d}}}{c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}-\frac{x^2(a+b\text{arc}}{2c^2(c^2}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

3.311. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

output
```
-1/3*(x^3*(a + b*ArcSinh[c*x])^2)/(c^2*d*(d + c^2*d*x^2)^(3/2)) + (2*b*Sqr
t[1 + c^2*x^2]*(-1/2*(x^2*(a + b*ArcSinh[c*x]))/(c^2*(1 + c^2*x^2)) + (b*(
-(x/(c^2*Sqrt[1 + c^2*x^2])) + ArcSinh[c*x]/c^3))/(2*c) - (I*(((-1/2*I)*(a
 + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSin
h[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/c^4))/(3*c*d^2*Sqrt
[d + c^2*d*x^2]) + (-((x*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2
])) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c^3*d*Sqrt[d + c^2*d
*x^2]) - ((2*I)*b*Sqrt[1 + c^2*x^2]*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b +
 (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[
2, -E^(2*ArcSinh[c*x])])/4)))/(c^3*d*Sqrt[d + c^2*d*x^2]))/(c^2*d)
```

### 3.311.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

## 3.311. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

## 3.311. $\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

## 3.311.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(370) = 740.

Time = 0.31 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| default | $-\frac{a^2x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{a^2x}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(\operatorname{arcsinh}(cx)^3x^4c^4-4\operatorname{arcsinh}\right.}{}$ |
| parts | $-\frac{a^2x^3}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{a^2x}{c^4d^2\sqrt{c^2d\,x^2+d}}+\frac{a^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^4d^2\sqrt{c^2d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\sqrt{c^2x^2+1}\,\left(\operatorname{arcsinh}(cx)^3x^4c^4-4\operatorname{arcsinh}\right.}{}$ |

input
```
int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-a^2/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a
^2/c^4/d^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3
*b^2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+
1)/c^5/d^3*(arcsinh(c*x)^3*x^4*c^4-4*arcsinh(c*x)^2*x^4*c^4+8*arcsinh(c*x)
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4-4*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2
)*x^3*c^3+4*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+c^4*x^4+2*arcsin
h(c*x)^3*x^2*c^2-c^3*x^3*(c^2*x^2+1)^(1/2)-8*arcsinh(c*x)^2*x^2*c^2+16*arc
sinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-3*arcsinh(c*x)^2*(c^2*x^
2+1)^(1/2)*c*x+arcsinh(c*x)*c^2*x^2+8*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)*x^2*c^2+2*c^2*x^2+arcsinh(c*x)^3-c*x*(c^2*x^2+1)^(1/2)-4*arcsinh(c*x)^2+
8*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+arcsinh(c*x)+4*polylog(2,-(
c*x+(c^2*x^2+1)^(1/2))^2)+1)+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/
2)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/c^5/d^3*(3*arcsinh(c*x)^2*x^4*c^4-8*arc
sinh(c*x)*c^4*x^4+8*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^4*c^4-8*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)*x^3*c^3+6*arcsinh(c*x)^2*x^2*c^2-16*arcsinh(c*x)*c^2*x^
2+16*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-6*arcsinh(c*x)*c*x*(c^2*x^2+1
)^(1/2)+c^2*x^2+3*arcsinh(c*x)^2-8*arcsinh(c*x)+8*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)+1)
```

## 3.311.5 Fricas [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^4}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(
c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

## 3.311.6 Sympy [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^4\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

## 3.311.7 Maxima [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^4}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^
4*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a^
2 + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2)
 + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.311.8 Giac [F]

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^4}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^4\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

# 3.312 $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.312.1 Optimal result

Integrand size = 28, antiderivative size = 307

$$\begin{aligned}\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}}\\ &- \frac{bx(a+b\text{arcsinh}(cx))}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}\\ &- \frac{2(a+b\text{arcsinh}(cx))^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{10b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}}\\ &- \frac{5ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^4d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-1/3*x^2*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/c^4/d^2/(c
^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3
*b*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+10/3
*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/
d^2/(c^2*d*x^2+d)^(1/2)-5/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c
^2*x^2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)+5/3*I*b^2*polylog(2,I*(c*x+(c^
2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)
```

### 3.312.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{-a^2(2+3c^2x^2)+ab\left(-2(2+3c^2x^2)\operatorname{arcsinh}(cx)+\sqrt{1+c^2x^2}\left(-cx+10(1+\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(-(a^2*(2 + 3*c^2*x^2)) + a*b*(-2*(2 + 3*c^2*x^2)*ArcSinh[c*x] + Sqrt[1 +
c^2*x^2]*(-(c*x) + 10*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) - b^2*(
1 + c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 2*ArcSinh[c*x]^2 + 3*c^
2*x^2*ArcSinh[c*x]^2 + (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] - (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh
[c*x]] + (5*I)*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (5*I)
*(1 + c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^4*d^2*(1 + c^2*x^
2)*Sqrt[d + c^2*d*x^2])
```

### 3.312.3 Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6225, 6213, 6204, 3042, 4668, 2715, 2838, 6225, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\operatorname{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}\,dx$$

$$\Big\downarrow \text{6225}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}+\frac{2\int \frac{x(a+b\operatorname{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx}{3c^2d}-\frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6213}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{cd\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 6204

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}+\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^2d\sqrt{c^2dx^2+d}}\right)}{3c^2d}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 4668

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arc}}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 2715

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{a}}\right.}{c^2d\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}-\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 2838

3.312. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{x^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}+$$

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 6225$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2c^2}+\frac{b\int \frac{x}{(c^2x^2+1)^{3/2}}dx}{2c}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+$$

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 241$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2c^2}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}-\frac{b}{2c^3\sqrt{c^2x^2+1}}\right)}{3cd^2\sqrt{c^2dx^2+d}}+$$

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 6204$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c^3}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}-\frac{b}{2c^3\sqrt{c^2x^2+1}}\right)}{3cd^2\sqrt{c^2dx^2+d}}+$$

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}}{3c^2d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 3042$$

---

3.312. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c^3}-\frac{x(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}-\frac{b}{2c^3\sqrt{c^2x^2+1}}\right)}{3cd^2\sqrt{c^2dx^2+d}}+$$

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 4668

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(c}{2c^3}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 2715

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcs}}\right.}{2c^3}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 2838

$$\frac{2\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(c}\right.\right.}{c^2d\sqrt{c^2dx^2+d}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c^3}-\frac{x(a+b\text{arcsinh}(c}{2c^2(c^2x^2+1)}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$$\frac{x^2(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

---

3.312. $\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

input
```
Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
-1/3*(x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*(d + c^2*d*x^2)^(3/2)) + (2*b*Sqr
t[1 + c^2*x^2]*(-1/2*b/(c^3*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/
(2*c^2*(1 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I
*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*
c^3)))/(3*c*d^2*Sqrt[d + c^2*d*x^2]) + (2*(-((a + b*ArcSinh[c*x])^2/(c^2*d
*Sqrt[d + c^2*d*x^2])) + (2*b*Sqrt[1 + c^2*x^2]*(2*(a + b*ArcSinh[c*x])*Ar
cTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2
, I*E^ArcSinh[c*x]]))/(c^2*d*Sqrt[d + c^2*d*x^2])))/(3*c^2*d)
```

### 3.312.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

### 3.312.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(294) = 588.

Time = 0.20 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.29

| method | result |
|---|---|
| default | $a^2\left(-\frac{x^2}{c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{2}{3d\,c^4(c^2d\,x^2+d)^{\frac{3}{2}}}\right)-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2x^2}{(c^2x^2+1)^2d^3c^2}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{3(c^2x^2+1)^{\frac{3}{2}}d^3c^3}-\frac{b^2\sqrt{d(c^2}{3(c^2x^2}$ |
| parts | $a^2\left(-\frac{x^2}{c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{2}{3d\,c^4(c^2d\,x^2+d)^{\frac{3}{2}}}\right)-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2x^2}{(c^2x^2+1)^2d^3c^2}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{3(c^2x^2+1)^{\frac{3}{2}}d^3c^3}-\frac{b^2\sqrt{d(c^2}{3(c^2x^2}$ |

input
```
int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(c^2*d*x^2+d)^(3/2))-b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)^2*x^2-1/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d^3/c^3*arcsinh(c*x)*x-1/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*x^2-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^2/d^3/c^4*arcsinh(c*x)^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
2/d^3/c^4-5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*arcsin
h(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/c^4/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-5/3*I*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1+I*(c*x+(c^2*x^
2+1)^(1/2)))+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dil
og(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/
d^3/c^2*arcsinh(c*x)*x^2-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d
^3/c^3*x-4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^4*arcsinh(c*x)+
5/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+
1)^(1/2)+I)-5/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c
*x+(c^2*x^2+1)^(1/2)-I)
```

## 3.312.5 Fricas [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(
c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

## 3.312.6 Sympy [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^3(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.312.7 Maxima [F]

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^3}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2)
)) - 2/3*a*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/
2)*c^4*d))*arcsinh(c*x) - 1/3*a^2*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2
/((c^2*d*x^2 + d)^(3/2)*c^4*d)) + b^2*integrate(x^3*log(c*x + sqrt(c^2*x^2
 + 1))^2/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.312.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^3\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

# 3.313 $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.313.1 Optimal result

Integrand size = 28, antiderivative size = 312

$$\begin{aligned}\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{bx^2(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}} + \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3c^3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c^3d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
1/3*x^3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*x/c^2/d^2/(c^2*
d*x^2+d)^(1/2)+1/3*b*x^2*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d
*x^2+d)^(1/2)-1/3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)
^(1/2)+1/3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1
/2)-2/3*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(
1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)
```

### 3.313.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b^2cx + a^2c^3x^3 + b^2c^3x^3 - ab\sqrt{1+c^2x^2} - b^2\left(-c^3x^3 + \sqrt{1+c^2x^2} + c^2x^2\sqrt{1+\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(b^2*c*x + a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 + c^2*x^2] - b^2*(-(c^3*
x^3) + Sqrt[1 + c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*A
rcSinh[c*x]*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)^(3/2)*
Log[1 + E^(-2*ArcSinh[c*x])]) - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - a
*b*c^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + b^2*(1 + c^2*x^2)^(3/2)*Po
lyLog[2, -E^(-2*ArcSinh[c*x])])/(3*c^3*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^
2])
```

### 3.313.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.67, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {6215, 6225, 252, 222, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}\,dx$$

$$\Big\downarrow \ 6215$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}} - \frac{2bc\sqrt{c^2x^2+1}\int \frac{x^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 6225$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{b\int\frac{x^2}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}+\frac{b\left(\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{c^2}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{c^2}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6212}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int\frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int-i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 26}$$

---

3.313. $\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}dx$

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{i\int(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(cx)}{c^3}-\frac{x}{c^2\sqrt{c^2x^2+1}}\right)}{2c}\right)}{3d^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 4201

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\int\frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}-\frac{x^2(a+b\text{arcsinh}(cx))}{2c^2(c^2x^2+1)}+\frac{b\left(\frac{\text{arcsinh}(c}{c^3}\right.}{}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2620

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}-\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2715

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arcsi}}{2b}\right.}{c^4}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2838

$$\frac{x^3(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(-\frac{i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c^4}-\frac{x^2(a+b\text{ar}}{2c^2(c}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

## 3.313. $\int\frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

output
```
(x^3*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 +
 c^2*x^2]*(-1/2*(x^2*(a + b*ArcSinh[c*x]))/(c^2*(1 + c^2*x^2)) + (b*(-(x/(
c^2*Sqrt[1 + c^2*x^2])) + ArcSinh[c*x]/c^3))/(2*c) - (I*(((-1/2*I)*(a + b*
ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x
])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/c^4))/(3*d^2*Sqrt[d + c^
2*d*x^2])
```

### 3.313.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## 3.313. $\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6225
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
   Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - S
imp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(
m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

### 3.313.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. 2(292) = 584.

Time = 0.29 (sec) , antiderivative size = 2352, normalized size of antiderivative = 7.54

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2352 |
| parts | Expression too large to display | 2352 |

3.313. $\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

input
```
int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
*c^4/d^3*arcsinh(c*x)*x^7-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+1
0*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^6-1/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*arcs
inh(c*x)*x^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4
+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)*x^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^
8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7+b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5-1/3*b^2*(d*(c
^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2
*x^2+1)^(1/2)-1/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3/c^3*poly
log(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8
+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*x)*x^3+2/3*b^2/(c^2*x^2+1
)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3/c^3*arcsinh(c*x)^2+2/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3+
1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
/d^3*arcsinh(c*x)^2*x^3-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+1
0*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*x^4-2/3*b^2/(c^2*x^2+1)^(1/
2)*(d*(c^2*x^2+1))^(1/2)/d^3/c^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^
2+1)/c/d^3*x^2*(c^2*x^2+1)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x...
```

### 3.313.5 Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(
c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.313.6 Sympy [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^2(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.313.7 Maxima [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5
/2))) + 2/3*a*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2
)*c^2*d))*arcsinh(c*x) + 1/3*a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^
2*d*x^2 + d)^(3/2)*c^2*d)) + b^2*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1)
)^2/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.313.8 Giac [F]

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

# 3.314 $\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.314.1 Optimal result

Integrand size = 26, antiderivative size = 270

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(d+c^2dx^2)^{3/2}}+\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c^2d^2\sqrt{d+c^2dx^2}}$$

output
```
-1/3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^2/d^2/(c^2*d
*x^2+d)^(1/2)+1/3*b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^
2+d)^(1/2)+2/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2
+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2
+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*I*b^2*polylo
g(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/
2)
```

### 3.314.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.94

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{-a^2+ab\left(-2\text{arcsinh}(cx)+\sqrt{1+c^2x^2}\left(cx+2(1+c^2x^2)\arctan\left(\tanh\left(\tfrac{1}{2}\text{arcsin}\right.\right.\right.\right.}{}$$

input
```
Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(-a^2 + a*b*(-2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*(c*x + 2*(1 + c^2*x^2)*Ar
cTan[Tanh[ArcSinh[c*x]/2]])) + b^2*(1 + c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*Ar
cSinh[c*x] - ArcSinh[c*x]^2 - I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I
/E^ArcSinh[c*x]] + I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[
c*x]] - I*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] + I*(1 + c^2
*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2))
```

### 3.314.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6213, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}\,dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{3cd^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6203}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\Big\downarrow$ 6204

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3cd^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3cd^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 4668

$$-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(c}{2c}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2715

$$-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcs}}\right.}{2c}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2838

$$-\frac{(a+b\text{arcsinh}(cx))^2}{3c^2d\,(c^2dx^2+d)^{3/2}}+\frac{2b\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(c}{2(c^2x^2+1)}\right.}{3cd^2\sqrt{c^2dx^2+d}}$$

input
```
Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

## 3.314. $\int\frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

output
```
-1/3*(a + b*ArcSinh[c*x])^2/(c^2*d*(d + c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1 +
c^2*x^2]*(b/(2*c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*(1 + c^2
*x^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (
-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c)))/(3*c*d^2*S
qrt[d + c^2*d*x^2])
```

### 3.314.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.314.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(261) = 522$.

Time = 0.29 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.19

| method | result |
|---|---|
| default | $-\frac{a^2}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{3(c^2x^2+1)^{\frac{3}{2}}d^3c}+\frac{b^2\sqrt{d(c^2x^2+1)}\,x^2}{3(c^2x^2+1)^2d^3}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{3(c^2x^2+1)^2d^3c^2}+\frac{b^2\sqrt{d(c^2x^2+1)}}{3(c^2x^2+1)^2d^3c^2}$ |
| parts | $-\frac{a^2}{3c^2d(c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)x}{3(c^2x^2+1)^{\frac{3}{2}}d^3c}+\frac{b^2\sqrt{d(c^2x^2+1)}\,x^2}{3(c^2x^2+1)^2d^3}-\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(cx)^2}{3(c^2x^2+1)^2d^3c^2}+\frac{b^2\sqrt{d(c^2x^2+1)}}{3(c^2x^2+1)^2d^3c^2}$ |

input
```
int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(3/2)/d^3/c*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/
d^3*x^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)^2
+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2-1/3*I*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^
(1/2)))+1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(
c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/c^2/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*I*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))
+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(3/2)/d^3/c*x-2/3*a*b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^2/d^3/c^2*arcsinh(c*x)+1/3*I*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-1/3*I*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

### 3.314.5 Fricas [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2 x}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x)
+ a^2*x)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.314.6 Sympy [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x\left(a+b\,\text{asinh}\left(cx\right)\right)^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.314.7 Maxima [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2 x}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*a^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))
/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.314.8 Giac [F]

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

# 3.315 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.315.1 Optimal result

Integrand size = 25, antiderivative size = 292

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx =& -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}}+\frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}}+\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+c^2dx^2}} \\
&-\frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}} \\
&-\frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

output
```
1/3*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/d^2/(c^2*d*x^2+
d)^(1/2)+2/3*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*arc
sinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*(a+b*arcsinh(c*
x))^2*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)-4/3*b*(a+b*arcsinh(c*x))
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/
2)-2/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(
c^2*d*x^2+d)^(1/2)
```

### 3.315.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{a^2cx(3+2c^2x^2)+ab\left((6cx+4c^3x^3)\operatorname{arcsinh}(cx)+\sqrt{1+c^2x^2}(1-2(1+c^2x^2)\,\mathrm{l}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(a^2*c*x*(3 + 2*c^2*x^2) + a*b*((6*c*x + 4*c^3*x^3)*ArcSinh[c*x] + Sqrt[1
+ c^2*x^2]*(1 - 2*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - b^2*(c*x + c^3*x^3 -
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]^2 - 2*c*x*(1 + c^2*x^2)*
ArcSinh[c*x]^2 + 2*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[
1 + E^(-2*ArcSinh[c*x])]) - 2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSin
h[c*x])]))/(3*d^2*(c + c^3*x^2)*Sqrt[d + c^2*d*x^2])
```

### 3.315.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}\,dx$$

$$\Big\downarrow \; 6203$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx}{3d}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 6202$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\operatorname{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\operatorname{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

---

3.315. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+$$
$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{a}}{}}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$
$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2715

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+$$
$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsin}}\right.}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$
$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+$$
$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx}{2b}}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$
$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6213

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{b\int\frac{1}{(c^2x^2+1)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+$$
$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx}{2b}}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$
$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 208

---

3.315. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+$$
$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx)}{2b}}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$
$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 + c
^2*x^2]*((b*x)/(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*(1 +
c^2*x^2))))/(3*d^2*Sqrt[d + c^2*d*x^2]) + (2*((x*(a + b*ArcSinh[c*x])^2)/(
d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b*Sqrt[1 + c^2*x^2]*(((-1/2*I)*(a + b*ArcS
inh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])
/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c*d*Sqrt[d + c^2*d*x^2])))/
(3*d)
```

### 3.315.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.315. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.315. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

### 3.315.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2130 vs. $2(274) = 548$.

Time = 0.28 (sec) , antiderivative size = 2131, normalized size of antiderivative = 7.30

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2131 |
| parts | Expression too large to display | 2131 |

input
```
int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*x^
2*(c^2*x^2+1)^(1/2)-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11
*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2+4/3*b^2*(d*(c^2*x^2+1))
^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)*arcsinh
(c*x)+17/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c
^2/d^3*arcsinh(c*x)^2*x^3+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^
4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*arcsinh(c*x)*x-16/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3-4/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*arcsinh(c
*x)*x^7+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*
c^4/d^3*(c^2*x^2+1)*x^5+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+
11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)^2*x^5-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3
*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x^5+b^2*(d*(c^2*x^2
+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*(c^2*x^2+1)^(1/2)*x
^4+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d
^3*(c^2*x^2+1)*x^3-4/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3/c*a
rcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*x^7-3*b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5-13/3*b^2*(d*(c^2*x^2+1))
^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*x^3+4/3*b^2*(d*(c^2*...
```

## 3.315.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

## 3.315.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

## 3.315.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(
5/2))) + 2/3*a*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)
*d))*arcsinh(c*x) + 1/3*a^2*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2
 + d)^(3/2)*d)) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2
+ d)^(5/2), x)
```

### 3.315.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2), x)
```

# 3.316 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$



## 3.316.1 Optimal result

Integrand size = 28, antiderivative size = 518

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{bcx(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} + \frac{(a+b\text{arcsinh}(cx))^2}{d^2\sqrt{d+c^2dx^2}} \\ &- \frac{14b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\ &- \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{7ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\ &- \frac{7ib^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
1/3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/d^2/(c^2*d*x^2+d)^(
1/2)+(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*x*(a+b*arcsinh(c
*x))/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-14/3*b*(a+b*arcsinh(c*x))*a
rctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(
a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(
c^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+7/3*I*b^2*polylog(2,-I*(c*x+(c
^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-7/3*I*b^2*poly
log(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
+2*b*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)
/d^2/(c^2*d*x^2+d)^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+
1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

### 3.316.2 Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\frac{a^2(4+3c^2x^2)\sqrt{d+c^2dx^2}}{(1+c^2x^2)^2} + 3a^2\sqrt{d}\log(cx) - 3a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{abd^2(1\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
((a^2*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 3*a^2*Sqrt[d]
*Log[c*x] - 3*a^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (a*b*d^2*
(1 + c^2*x^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^
2)^(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x
]/2]] + 6*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 +
 E^(-ArcSinh[c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-
ArcSinh[c*x])]))/(d + c^2*d*x^2)^(3/2) + (b^2*d^2*(1 + c^2*x^2)^(3/2)*(-(1
/Sqrt[1 + c^2*x^2]) - (c*x*ArcSinh[c*x])/(1 + c^2*x^2) + ArcSinh[c*x]^2/(1
 + c^2*x^2)^(3/2) + (3*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]^
2*Log[1 - E^(-ArcSinh[c*x])] + (7*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]
] - (7*I)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Log[1
+ E^(-ArcSinh[c*x])] + 6*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + (7*
I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (7*I)*PolyLog[2, I/E^ArcSinh[c*x]] -
6*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 6*PolyLog[3, -E^(-ArcSinh[c
*x])] - 6*PolyLog[3, E^(-ArcSinh[c*x])]))/(d + c^2*d*x^2)^(3/2))/(3*d^3)
```

### 3.316.3 Rubi [A] (verified)

Time = 4.06 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.81, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6226, 6203, 241, 6204, 3042, 4668, 2715, 2838, 6226, 6204, 3042, 4668, 2715, 2838, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6226}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6203}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x}{\left(c^2x^2+1\right)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2\left(c^2x^2+1\right)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2\left(c^2x^2+1\right)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6204}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2\left(c^2x^2+1\right)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 4668

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsin}}{2c}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2715

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e\right.}{2c}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}-$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6226

---

3.316. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{-\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}}{d}-$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6204

$$\frac{-\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}}{d}-$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{2b\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}}{d}-$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 4668

$$\frac{-\frac{2b\sqrt{c^2x^2+1}\left(-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right.}{d\sqrt{c^2dx^2+d}}}{d}$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2715

---

3.316. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{d}{}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{d}{}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6231

$$\frac{\sqrt{c^2x^2+1}\int\frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{Po}\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{d}{}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{d}{}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 26

---

3.316. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}dx$

$$\frac{\frac{i\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-\right.\right.}{d\sqrt{c^2dx^2+d}}}{d}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 4670

$$\frac{i\sqrt{c^2x^2+1}\left(2ib\int (a+b\text{arcsinh}(cx))\log\left(1-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-2ib\int (a+b\text{arcsinh}(cx))\log\left(1+e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2i\text{arcta}\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 3011

$$\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+2ib\left(b\int \text{PolyLog}\left(2,e^{\text{arcsinh}(cx}\right.\right.\right.}{d\sqrt{c^2dx^2+d}}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2720

$$\frac{i\sqrt{c^2x^2+1}\left(-2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)+2ib\left(b\int e^{-\text{arcsinh}}\right.\right.}{d\sqrt{c^2dx}}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 7143

---

3.316. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))-ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)+ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)\right)}{d\sqrt{c^2dx^2+d}}+\frac{i\sqrt{c^2x^2+1}\left(2i\operatorname{arcta}\right.}{}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)(a+b\operatorname{arcsinh}(cx))-ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)+ib\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\operatorname{arcsinh}(}{2(c^2x^2+1)}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\operatorname{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
(a + b*ArcSinh[c*x])^2/(3*d*(d + c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 + c^2*x
^2]*(b/(2*c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*(1 + c^2*x^2)
) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E
^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c)))/(3*d^2*Sqrt[d +
 c^2*d*x^2]) + ((a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt
[1 + c^2*x^2]*(2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog
[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]]))/(d*Sqrt[d +
c^2*d*x^2]) + (I*Sqrt[1 + c^2*x^2]*((2*I)*(a + b*ArcSinh[c*x])^2*ArcTanh[E
^ArcSinh[c*x]] - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x
]]) + b*PolyLog[3, -E^ArcSinh[c*x]]) + (2*I)*b*(-((a + b*ArcSinh[c*x])*Pol
yLog[2, E^ArcSinh[c*x]]) + b*PolyLog[3, E^ArcSinh[c*x]])))/(d*Sqrt[d + c^2
*d*x^2]))/d
```

### 3.316.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.316. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.316. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$

### 3.316.4 Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{x\,(c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

### 3.316.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

### 3.316.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\,(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{x\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.316.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*a^2*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) -
1/((c^2*d*x^2 + d)^(3/2)*d)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^
2/((c^2*d*x^2 + d)^(5/2)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x
^2 + d)^(5/2)*x), x)
```

### 3.316.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x), x)
```

### 3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{x\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.317 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$



## 3.317.1 Optimal result

Integrand size = 28, antiderivative size = 421

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx &= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} \\
&- \frac{(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} - \frac{4c^2x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} \\
&- \frac{8c^2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{8c\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&- \frac{4bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{16bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{5b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{b^2c\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

output
```
-(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))
^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2*x/d^2/(c^2*d*x^2+d)^(1/2)-8/3*c^2*x*(
a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2
/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-8/3*c*(a+b*arcsinh(c*x))^2*(c^2*x^2
+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+16/3*b*c*(a+
b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2
*d*x^2+d)^(1/2)+5/3*b^2*c*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1
)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)
*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

## 3.317.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx =$$

$$-\frac{3a^2+12a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4-b^2c^4x^4+abcx\sqrt{1+c^2x^2}+6ab\text{arcsinh}(cx)+24abc^2x^2\text{arcsinh}(cx)+}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(3*a^2 + 12*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 +
 a*b*c*x*Sqrt[1 + c^2*x^2] + 6*a*b*ArcSinh[c*x] + 24*a*b*c^2*x^2*ArcSinh[c
*x] + 16*a*b*c^4*x^4*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
 + 3*b^2*ArcSinh[c*x]^2 + 12*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*Ar
cSinh[c*x]^2 - 8*b^2*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - 6*b^2*c*x*(1
 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 10*b^2*c*x*(
1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 6*a*b*c*x*(
1 + c^2*x^2)^(3/2)*Log[c*x] - 5*a*b*c*x*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^
2] + 5*b^2*c*x*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 3*b^
2*c*x*(1 + c^2*x^2)^(3/2)*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*(d + c^2*d
*x^2)^(3/2))
```

### 3.317.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.30 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.04, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6224, 6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208, 6226, 208, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6224}$$

$$\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6203}$$

$$-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2dx^2+d\right)^{3/2}}dx}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}\right)+\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b}{3d\left(c^2\right.}\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)^2}dx}{d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int\frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int-i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d}{cd\sqrt{c^2dx^2+d}}\right.}{3d}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 26$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))da}{cd\sqrt{c^2dx^2+d}}\right.}{3d}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 4201$$

$$-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\int\frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}da\right.}{cd\sqrt{c^2dx^2+d}}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 2620$$

---

3.317. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}\right.\right.}{}\right.}{3d}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 2715

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}\right.\right.}{}\right.}{}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 2838

$$-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsi}\right.\right.}{}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 6213

3.317. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

$$-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{b\int\frac{1}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(\right.\right.\right.}{}\right)}{}\right)$$

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 208

$$\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right)}{3}\right)$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6226

$$\frac{2bc\sqrt{c^2x^2+1}\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx-\frac{1}{2}bc\int\frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}\right)}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right)}{3}\right)$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 208

3.317. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}dx$

$$\frac{2bc\sqrt{c^2x^2+1}\left(\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 6214

$$\frac{2bc\sqrt{c^2x^2+1}\left(\int \frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 5984

$$\frac{2bc\sqrt{c^2x^2+1}\left(2\int(a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 3042

---

3.317. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{2bc\sqrt{c^2x^2+1}\left(2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 26

$$\frac{2bc\sqrt{c^2x^2+1}\left(2i\int(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 4670

$$\frac{2bc\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arctanh}(e\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 2715

3.317. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{2bc\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{2bc\sqrt{c^2x^2+1}\left(2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2\text{ar}}\right.\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+\right.\right.}{}\right.}{3}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{dx\left(c^2dx^2+d\right)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(d*x*(d + c^2*d*x^2)^(3/2))) - 4*c^2*((x*(a + b*A
rcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 + c^2*x^2]*((b
*x)/(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*(1 + c^2*x^2))))
/(3*d^2*Sqrt[d + c^2*d*x^2]) + (2*((x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d +
c^2*d*x^2]) + ((2*I)*b*Sqrt[1 + c^2*x^2]*(((-1/2*I)*(a + b*ArcSinh[c*x])^2
)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*Pol
yLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c*d*Sqrt[d + c^2*d*x^2])))/(3*d)) + (2
*b*c*Sqrt[1 + c^2*x^2]*(-1/2*(b*c*x)/Sqrt[1 + c^2*x^2] + (a + b*ArcSinh[c*
x])/(2*(1 + c^2*x^2)) + (2*I)*(I*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh
[c*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcSinh[c*x])] - (I/4)*b*PolyLog[2, E^(
2*ArcSinh[c*x])])))/(d^2*Sqrt[d + c^2*d*x^2])
```

3.317. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

### 3.317.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

## 3.317. $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.317. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

### 3.317.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3512 vs. $2(411) = 822$.

Time = 0.35 (sec) , antiderivative size = 3513, normalized size of antiderivative = 8.34

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3513 |
| parts | Expression too large to display | 3513 |

input
```
int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^
4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+272/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^
6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-
40*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c
^8-160/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3
*x^5*c^6-29*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/
d^3*x^3*c^4-5*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9
)/d^3*x*c^2-9*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9
)/d^3/x*arcsinh(c*x)^2-3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+2
6*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c+5/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)*c+a^2*(-1/d/x/(c^2*d*x^2+d)^(3/2)-4*c^2*(1/3/d*x/(c^2*d*x^2+d)^(3/2)+2/3
/d^2*x/(c^2*d*x^2+d)^(1/2)))+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*
c^4*x^4+26*c^2*x^2+9)/d^3*x^4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5+40*b^2*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2
+1)*arcsinh(c*x)*c^4+136/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4
+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-8/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)...
```

### 3.317.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

### 3.317.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^2\,(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.317.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a^2*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/
2)*d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + integrate(b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1
))/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

### 3.317.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

### 3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.318 $\int \frac{(a+b\,\mathrm{arcsinh}(cx))^2}{x^3 (d+c^2dx^2)^{5/2}}\,dx$

### 3.318.1 Optimal result

Integrand size = 28, antiderivative size = 687

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3 (d+c^2dx^2)^{5/2}}\,dx = \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(a+b\text{arcsinh}(cx))}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{6d(d+c^2dx^2)^{3/2}} - \frac{(a+b\text{arcsinh}(cx))^2}{2dx^2(d+c^2dx^2)^{3/2}} - \frac{5c^2(a+b\text{arcsinh}(cx))^2}{2d^2\sqrt{d+c^2dx^2}} + \frac{26bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{5c^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\text{arctanh}\left(e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{b^2c^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{13ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} + \frac{13ib^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{5bc^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} - \frac{5b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}} + \frac{5b^2c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(3,e^{\text{arcsinh}(cx)}\right)}{d^2\sqrt{d+c^2dx^2}}$$

output
```
-5/6*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/2*(a+b*arcsinh(c*x))
^2/d/x^2/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2/d^2/(c^2*d*x^2+d)^(1/2)-5/2*c^2*(
a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))/d^2/x/(
c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*x*(a+b*arcsinh(c*x))/d^2/(c
^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+26/3*b*c^2*(a+b*arcsinh(c*x))*arctan(c
*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*c^2*(a+b
*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2
*d*x^2+d)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(
c^2*d*x^2+d)^(1/2)+5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-13/3*I*b^2*c^2*polylog(2,-
I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+13/3*
I*b^2*c^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*
d*x^2+d)^(1/2)-5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))
*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5*b^2*c^2*polylog(3,-c*x-(c^2*x
^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*b^2*c^2*polylog(3
,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

## 3.318.2 Mathematica [A] (verified)

Time = 7.78 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx = \sqrt{d(1+c^2x^2)}\left(-\frac{a^2}{2d^3x^2}-\frac{a^2c^2}{3d^3(1+c^2x^2)^2}-\frac{2a^2c^2}{d^3(1+c^2x^2)}\right)$$
$$-\frac{5a^2c^2\log(x)}{2d^{5/2}}+\frac{5a^2c^2\log\left(d+\sqrt{d}\sqrt{d(1+c^2x^2)}\right)}{2d^{5/2}}$$
$$+\frac{abc^2\left(\frac{4cx}{\sqrt{1+c^2x^2}}-48\text{arcsinh}(cx)-\frac{8\text{arcsinh}(cx)}{1+c^2x^2}+104\sqrt{1+c^2x^2}\arctan\left(\tanh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)-6\sqrt{1+c^2x^2}\right.}{}$$
$$+\frac{b^2c^2\left(8+\frac{8cx\text{arcsinh}(cx)}{\sqrt{1+c^2x^2}}-48\text{arcsinh}(cx)^2-\frac{8\text{arcsinh}(cx)^2}{1+c^2x^2}-12\sqrt{1+c^2x^2}\text{arcsinh}(cx)\coth\left(\frac{1}{2}\text{arcsinh}(cx)\right)-\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^3*x^2) - (a^2*c^2)/(3*d^3*(1 + c^2*x^2)
^2) - (2*a^2*c^2)/(d^3*(1 + c^2*x^2))) - (5*a^2*c^2*Log[x])/(2*d^(5/2)) +
(5*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(2*d^(5/2)) + (a*b*c^2*
((4*c*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[c*x])/(1 + c^2*x
^2) + 104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*Sqrt[1 + c^2*
x^2]*Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[
c*x]/2]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] +
 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*Sqrt[1
+ c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*PolyLog[2
, E^(-ArcSinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/
2]^2 + 6*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/(12*d^2*Sqrt[d*(1 + c^2*
x^2)]) + (b^2*c^2*(8 + (8*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 48*ArcSinh
[c*x]^2 - (8*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 12*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x]*Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Csch[ArcSin
h[c*x]/2]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x]
)] - (104*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + (1
04*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + 60*Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^
2]*Log[Tanh[ArcSinh[c*x]/2]] - 120*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[
2, -E^(-ArcSinh[c*x])] - (104*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^Ar...
```

### 3.318.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6224}$$

$$\frac{bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x^2\left(c^2x^2+1\right)^2}dx}{d^2\sqrt{c^2dx^2+d}}-\frac{5}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6224}$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^2}dx+bc\int \frac{1}{x\left(c^2x^2+1\right)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{x\left(c^2x^2+1\right)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5}{2}c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)^{3/2}}dx^2-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 61

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{1}{2}bc\left(\int\frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+\frac{2}{\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 73

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx+\frac{1}{2}bc\left(\frac{2\int\frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+\frac{2}{\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 221

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 6203

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int\frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2}}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$
$$\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 241

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2+1}}-2\text{ar}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$
$$\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 6204

---

3.318. $\displaystyle\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}+\frac{1}{2}bc\left(\frac{2}{\sqrt{c^2x^2\ldots}}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{\int(a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsi}\ldots}{x(c^2x^2\ldots}\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 4668

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{-ib\int\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{ar}\ldots}{2c}\right.\right.}{d^2\sqrt{c^2dx^2+\ldots}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 2715

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\,\text{arct}\ldots}{2c}\right.\right.}{d\ldots}$$

$$\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 2838

$$-\frac{5}{2}c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{5/2}}dx+$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}\ldots}{2(c^2\ldots}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\big\downarrow$ 6226

3.318. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}dx$

$$-\frac{5}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}\right)+$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 6203

$$-\frac{5}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int\frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 241

$$-\frac{5}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}+\frac{(a+ba}{3d\,(c^2}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 6204

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3.318. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

$$-\frac{5}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x(c^2dx^2+d)^{3/2}}dx}{d}+\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{5}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a}{}}{}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 4668

$$-\frac{5}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+}{2c}\right.}{3d^2\sqrt{c^2dx^2+d}}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2715

3.318. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

$$-\frac{5}{2}c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2}{2c}\right.}{3d^2\sqrt{c^2dx^2+d}}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{5}{2}c^2\left(\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x\left(c^2dx^2+d\right)^{3/2}}dx}{d}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyL}}{2c}\right.}{3d^2\sqrt{c^2dx^2+d}}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6226

$$-\frac{5}{2}c^2\left(\frac{-\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}}{d}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan\left(e^{\text{arcsinh}(c}\right.}{}\right.}{}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6204

---

3.318. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}dx$

$$-\frac{5}{2}c^2\left(\frac{-\frac{2b\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}}{d}-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{2\arctan}{}\right.}{}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{5}{2}c^2\left(\frac{\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{2b\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}+\frac{(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}}{d}-\frac{2bc}{}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 4668

$$-\frac{5}{2}c^2\left(\frac{-\frac{2b\sqrt{c^2x^2+1}\left(-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcs}\right.}{d\sqrt{c^2dx^2+d}}}{d}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2715

3.318. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

$$-\frac{5}{2}c^2\left(-\frac{\frac{2b\sqrt{c^2x^2+1}\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\,\text{arctan}\right.}{d\sqrt{c^2dx^2+d}}}{d}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{5}{2}c^2\left(\frac{\frac{\int\frac{(a+b\text{arcsinh}(cx))^2}{x\sqrt{c^2dx^2+d}}dx}{d}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{a}}\right.\right.}{d\sqrt{c^2dx^2+d}}}{d}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6231

$$-\frac{5}{2}c^2\left(\frac{\frac{\sqrt{c^2x^2+1}\int\frac{(a+b\text{arcsinh}(cx))^2}{cx}d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(c}\right.\right.}{d\sqrt{c^2dx^2+d}}}{d}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 3042

3.318. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}dx$

$$-\frac{5}{2}c^2\left(\frac{\frac{\sqrt{c^2x^2+1}\int i(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\text{ Poly}}{d\sqrt{c^2dx^2-}}}{d}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\text{ PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\text{ PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{5}{2}c^2\left(\frac{\frac{i\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))^2\csc(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\text{ Poly}}{d\sqrt{c^2dx^2+}}}{d}\right.$$

$$\frac{bc\sqrt{c^2x^2+1}\left(-3c^2\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\text{ PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\text{ PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{ar}}{2(c^2}\right.\right.}{d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2dx^2\left(c^2dx^2+d\right)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

### 3.318.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.318. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

## 3.318. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

## 3.318. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3(d+c^2dx^2)^{5/2}}\,dx$

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

## 3.318.4 Maple [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

## 3.318.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

## 3.318.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.318.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
1/6*a^2*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d
)*d^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2)
) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^
3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

### 3.318.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

### 3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{x^3\,\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.319 $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$



## 3.319.1 Optimal result

Integrand size = 28, antiderivative size = 506

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} \\
&- \frac{bc(a+b\text{arcsinh}(cx))}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a+b\text{arcsinh}(cx))^2}{3dx^3(d+c^2dx^2)^{3/2}} \\
&+ \frac{2c^2(a+b\text{arcsinh}(cx))^2}{dx(d+c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\text{arcsinh}(cx))^2}{3d(d+c^2dx^2)^{3/2}} \\
&+ \frac{16c^4x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{16c^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} \\
&+ \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&- \frac{32bc^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&- \frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}} \\
&- \frac{8b^2c^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(cx)}\right)}{3d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

output
```
-1/3*(a+b*arcsinh(c*x))^2/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x
))^2/d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d
)^(3/2)-1/3*b^2*c^2/d^2/x/(c^2*d*x^2+d)^(1/2)-2/3*b^2*c^4*x/d^2/(c^2*d*x^2
+d)^(1/2)+16/3*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*
(a+b*arcsinh(c*x))/d^2/x^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+16/3*c^3*
(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+32/3*b*c^3*
(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^
2/(c^2*d*x^2+d)^(1/2)-32/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*
b^2*c^3*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*
x^2+d)^(1/2)
```

### 3.319.2 Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4 (d+c^2dx^2)^{5/2}} \, dx = \frac{\frac{a^2(-1+6c^2x^2+24c^4x^4+16c^6x^6)}{x^3} - \frac{ab\left(-2(-1+6c^2x^2+24c^4x^4+16c^6x^6)\text{arcsinh}(cx)+cx\sqrt{1+c^2x^2}(1+1\right.}{x^3}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

output
```
((a^2*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6))/x^3 - (a*b*(-2*(-1 + 6*c
^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x] + c*x*Sqrt[1 + c^2*x^2]*(1
+ 16*(c^2*x^2 + c^4*x^4)*Log[c*x] + 8*(c^2*x^2 + c^4*x^4)*Log[1 + c^2*x^2]
)))/x^3 + b^2*c^3*(1 + c^2*x^2)^(3/2)*(-((c*x)/Sqrt[1 + c^2*x^2]) - Sqrt[1
 + c^2*x^2]/(c*x) - ArcSinh[c*x]/(c^2*x^2) + ArcSinh[c*x]/(1 + c^2*x^2) -
16*ArcSinh[c*x]^2 + (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^(3/2) + (8*c*x*ArcS
inh[c*x]^2)/Sqrt[1 + c^2*x^2] - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c^3*x^
3) + (8*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*x) - 16*ArcSinh[c*x]*Log[1 -
E^(-2*ArcSinh[c*x])] - 16*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] + 8*Po
lyLog[2, -E^(-2*ArcSinh[c*x])] + 8*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(3*d*
(d + c^2*d*x^2)^(3/2))
```

### 3.319.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.37, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6224, 6224, 245, 208, 6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208, 6226, 208, 6214, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4 \left(c^2dx^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{6224}$$

$$\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x^3(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}-2c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{x^2 \left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3 \left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6224}$$

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{dx \left(c^2dx^2+d\right)^{3/2}}\right)+\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)^{3/2}}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3 \left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{245}$$

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2dx^2+d\right)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{dx \left(c^2dx^2+d\right)^{3/2}}\right)+\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx+\frac{1}{2}bc\left(-2c^2\int \frac{1}{(c^2x^2+1)^{3/2}}dx-\frac{1}{x\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3 \left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\int\frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}dx-\frac{(a+b\text{arcsinh}(cx))^2}{dx\,(c^2dx^2+d)^{3/2}}\right)+\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 6203

$$-2c^2\left(-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\int\frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}\right)+\frac{2bc\sqrt{c}}{}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 6202

$$-2c^2\left(-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\,(c^2dx^2+d)^{3/2}}$$

$\downarrow$ 6212

$$-2c^2\left(-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int\frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}}{cd\sqrt{c^2dx^2+d}}\right.}{3d}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\,(c^2dx^2+d)^{3/2}}$$

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3.319. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

$\downarrow$ 3042

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-$$
$$2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{}}{}\right.}{}\right.\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 26

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-$$
$$2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib}{}\right.}{}\right.\right.$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 4201

$$-2c^2\left(-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\int\frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsin}}{1+e^{2\text{arcsinh}(cx)}}\right.}{cd\sqrt{c^2}}\right.}{3d}\right.\right.$$
$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-$$
$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2620

3.319. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-$$

$$2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib}{\cdots}\right.}{\cdots}\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2715

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-$$

$$2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib}{\cdots}\right.}{\cdots}\right.\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 2838

$$-2c^2\left(-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int \frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6213

3.319. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

$$-2c^2\left(-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{b\int\frac{1}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e\right.\right.\right.}{}\right)}{}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 208

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(\right.}{d\sqrt{c^2dx^2+d}}}{}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)^2}dx-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x^2+1}}\right)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6226

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx-\frac{1}{2}bc\int\frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}\right)}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2}}\right.}{3d^2\sqrt{c^2}}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx-\frac{1}{2}bc\int\frac{1}{(c^2x^2+1)^{3/2}}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+}}\right.\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 208

---

3.319. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}dx$

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{a}}{2c}\right.}{3d^2\sqrt{c^2dx^2+d}}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{x(c^2x^2+1)}dx+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2+1}}-\frac{1}{x\sqrt{c^2x}}\right.\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 6214

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(\int\frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-4c^2\left(-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x}}\right.}{3d^2\sqrt{c^2}}\right.\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(\int\frac{a+b\text{arcsinh}(cx)}{cx\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{arcsinh}(cx)}{2x^2(c^2x^2+1)}+\frac{1}{2}bc\left(-\frac{2c^2x}{\sqrt{c^2x^2}}\right.\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 5984

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(2\int(a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-4c^2\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(2\int(a+b\text{arcsinh}(cx))\text{csch}(2\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)-\frac{a+b\text{ar}}{2x^2(}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\downarrow$ 3042

3.319. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}dx$

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-4c^2\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(2\int i(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)-\frac{a+ba}{2x^2}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 26

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(2i\int(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)}{d^2\sqrt{c^2dx^2+d}}-4c^2\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(2i\int(a+b\text{arcsinh}(cx))\csc(2i\text{arcsinh}(cx))d\text{arcsinh}(cx)+\frac{a+b\text{arcsinh}(cx)}{2(c^2x^2+1)}-\frac{bcx}{2\sqrt{c^2x^2+1}}\right)-\frac{a+ba}{2x^2}\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 4670

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arc}\right.\right.}{d^2\sqrt{c^2dx^2+d}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(2i\left(\frac{1}{2}ib\int\log\left(1-e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\frac{1}{2}ib\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+i\text{arc}\right.\right.\right.}{3d^2\sqrt{}}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$\Big\downarrow$ 2715

---

3.319. $\int\frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

$$-2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsin}\right.\right.\right.}{d^2\sqrt{c^2dx^2+c}}\right.$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(2i\left(\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1-e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}-\frac{1}{4}ib\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}\right.\right.\right.\right.}{}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

$$\downarrow \text{2838}$$

$$\frac{2bc\sqrt{c^2x^2+1}\left(-2c^2\left(2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\right.\right.\right.}{3d^2\sqrt{c^2dx^2+d}}$$

$$2c^2\left(\frac{2bc\sqrt{c^2x^2+1}\left(2i\left(i\text{arctanh}\left(e^{2\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(\right.\right.\right.}{d^2\sqrt{c^2dx^2+d}}\right.$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

## 3.319. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

output
```
-1/3*(a + b*ArcSinh[c*x])^2/(d*x^3*(d + c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1
+ c^2*x^2]*((b*c*(-(1/(x*Sqrt[1 + c^2*x^2])) - (2*c^2*x)/Sqrt[1 + c^2*x^2]
))/2 - (a + b*ArcSinh[c*x])/(2*x^2*(1 + c^2*x^2)) - 2*c^2*(-1/2*(b*c*x)/Sq
rt[1 + c^2*x^2] + (a + b*ArcSinh[c*x])/(2*(1 + c^2*x^2)) + (2*I)*(I*(a + b
*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcS
inh[c*x])] - (I/4)*b*PolyLog[2, E^(2*ArcSinh[c*x])]))))/(3*d^2*Sqrt[d + c^
2*d*x^2]) - 2*c^2*(-((a + b*ArcSinh[c*x])^2/(d*x*(d + c^2*d*x^2)^(3/2))) -
 4*c^2*((x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) - (2*b*c*Sq
rt[1 + c^2*x^2]*((b*x)/(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c
^2*(1 + c^2*x^2))))/(3*d^2*Sqrt[d + c^2*d*x^2]) + (2*((x*(a + b*ArcSinh[c*
x])^2)/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b*Sqrt[1 + c^2*x^2]*(((-1/2*I)*(a
+ b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh
[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c*d*Sqrt[d + c^2*d*
x^2])))/(3*d)) + (2*b*c*Sqrt[1 + c^2*x^2]*(-1/2*(b*c*x)/Sqrt[1 + c^2*x^2]
+ (a + b*ArcSinh[c*x])/(2*(1 + c^2*x^2)) + (2*I)*(I*(a + b*ArcSinh[c*x])*A
rcTanh[E^(2*ArcSinh[c*x])] + (I/4)*b*PolyLog[2, -E^(2*ArcSinh[c*x])] - (I/
4)*b*PolyLog[2, E^(2*ArcSinh[c*x])]))))/(d^2*Sqrt[d + c^2*d*x^2]))
```

### 3.319.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

## 3.319. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

---

3.319. $\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6214
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[1/d   Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

## 3.319. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx$

rule 6226
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1
))   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Simp[
b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

### 3.319.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3505 vs. 2(484) = 968.

Time = 0.37 (sec) , antiderivative size = 3506, normalized size of antiderivative = 6.93

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3506 |
| parts | Expression too large to display | 3506 |

input
```
int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
344/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x
^2-1)/d^3*x^3*arcsinh(c*x)^2*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^
8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)*c^6+22/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^
2*c^5*(c^2*x^2+1)^(1/2)+12*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^
6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)^2*c^4-16/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x
)*c^4-6*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2
*x^2-1)/d^3/x*arcsinh(c*x)^2*c^2-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3-16/3*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/
2))^2)*c^3+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^
4+10*c^2*x^2-1)/d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-4*b^2*(d*(c^2*x^2
+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*(c^2*x^2+1)
^(1/2)*arcsinh(c*x)*c^3-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+256/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*arcsinh(c*x
)*c^14-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10
*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)*c^12+896/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c
^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*arcsinh(c*x)*c^12+1/...
```

### 3.319.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

### 3.319.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{x^4(d(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(5/2)), x)
```

### 3.319.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^
2*d^(5/2)*x^4 + d^(5/2)*x^2)) + 2/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) +
8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/
((c^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*
d*x^2 + d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 +
d)^(3/2)*d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a^2 + b^2*integrate(log(c
*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

### 3.319.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

### 3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^4\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)), x)
```

# 3.320 $\int \frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx$



## 3.320.1 Optimal result

Integrand size = 21, antiderivative size = 366

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx =& -\frac{x}{3c^3\sqrt{c+a^2cx^2}}-\frac{x}{30c^3(1+a^2x^2)\sqrt{c+a^2cx^2}}\\&+\frac{\text{arcsinh}(ax)}{10ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}+\frac{4\text{arcsinh}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}\\&+\frac{x\text{arcsinh}(ax)^2}{5c(c+a^2cx^2)^{5/2}}+\frac{4x\text{arcsinh}(ax)^2}{15c^2(c+a^2cx^2)^{3/2}}+\frac{8x\text{arcsinh}(ax)^2}{15c^3\sqrt{c+a^2cx^2}}\\&+\frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{15ac^3\sqrt{c+a^2cx^2}}-\frac{16\sqrt{1+a^2x^2}\text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}}\\&-\frac{8\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{15ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/5*x*arcsinh(a*x)^2/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^2/c^2/(a^2*
c*x^2+c)^(3/2)-1/3*x/c^3/(a^2*c*x^2+c)^(1/2)-1/30*x/c^3/(a^2*x^2+1)/(a^2*c
*x^2+c)^(1/2)+1/10*arcsinh(a*x)/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2
)+8/15*x*arcsinh(a*x)^2/c^3/(a^2*c*x^2+c)^(1/2)+4/15*arcsinh(a*x)/a/c^3/(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/
a/c^3/(a^2*c*x^2+c)^(1/2)-16/15*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^
2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)-8/15*polylog(2,-(a*x+(a^2*x
^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

### 3.320.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \frac{ax\left(-10-\frac{1}{1+a^2x^2}\right)+\left(-16\sqrt{1+a^2x^2}+\frac{2ax(15+20a^2x^2+8a^4x^4)}{(1+a^2x^2)^2}\right)\operatorname{arcsinh}(ax)^2+\frac{\operatorname{arcsinh}}{}}{30ac^3\sqrt{c}}$$

input

```
Integrate[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

output

```
(a*x*(-10 - (1 + a^2*x^2)^(-1)) + (-16*Sqrt[1 + a^2*x^2] + (2*a*x*(15 + 20
*a^2*x^2 + 8*a^4*x^4))/(1 + a^2*x^2)^2)*ArcSinh[a*x]^2 + (ArcSinh[a*x]*(11
 + 8*a^2*x^2 - 32*(1 + a^2*x^2)^2*Log[1 + E^(-2*ArcSinh[a*x])]))/(1 + a^2*
x^2)^(3/2) + 16*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[a*x])])/(30*a*
c^3*Sqrt[c + a^2*c*x^2])
```

### 3.320.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6203, 6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(a^2cx^2+c)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\operatorname{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\int \frac{\operatorname{arcsinh}(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{5c}+\frac{x\operatorname{arcsinh}(ax)^2}{5c\,(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\int \frac{\text{arcsinh}(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\text{arcsinh}(ax)^2}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)^2}{5c\,(a^2cx^2+c)^{5/2}}$$

$\downarrow$ 6202

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^2}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)^2}{5c\,(a^2cx^2+c)^{5/2}}$$

$\downarrow$ 6212

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{ax\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^2}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)^2}{5c\,(a^2cx^2+c)^{5/2}}$$

$\downarrow$ 3042

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int -i\text{arcsinh}(ax)\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^2}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)^2}{5c\,(a^2cx^2+c)^{5/2}}$$

$\downarrow$ 26

3.320. $\int \frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx$

$$\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\int \text{arcsinh}(ax)\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)}{3c(a^2cx^2+c)^{3/}}\right.}{5c} - \frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}} + \frac{x\text{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)}{1+e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}\right.}{5c} - \frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}} + \frac{x\text{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)\log\left(e^{2\text{arcsinh}(ax)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{arcsinh}(ax)}\right)da\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.}{5c} - \frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}} + \frac{x\text{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{2715}$$

---

3.320. $\int \frac{\text{arcsinh}(ax)^2}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)\log\left(e^{2\text{arcsinh}(ax)}+1\right)-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\log\left(1+e^{2\cdots}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.}{5c}$$

$$\frac{x\text{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$\downarrow$ 2838

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^2}dx}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsinh}(ax)\log\left(e^{2\text{arcsinh}(ax)}+1\cdots\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.}{5c}$$

$$\frac{x\text{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$\downarrow$ 6213

$$-\frac{2a\sqrt{a^2x^2+1}\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^{5/2}}dx}{4a}-\frac{\text{arcsinh}(ax)}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^{3/2}}dx}{2a}-\frac{\text{arcsinh}(ax)}{2a^2\left(a^2x^2+1\right)}\right)}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsinh}(ax)\log\cdots\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.}{5c}$$

$$\frac{x\text{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$\downarrow$ 208

3.320. $\int \frac{\text{arcsinh}(ax)^2}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

$$-\frac{2a\sqrt{a^2x^2+1}\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^{5/2}}dx}{4a}-\frac{\operatorname{arcsinh}(ax)}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\left(\frac{x}{2a\sqrt{a^2x^2+1}}-\frac{\operatorname{arcsinh}(ax)}{2a^2\left(a^2x^2+1\right)}\right)}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\operatorname{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)+\frac{1}{2}\operatorname{arcsinh}(ax)\log\left(e^{2\operatorname{arc}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}}{3c}}{5c}$$

$$\frac{x\operatorname{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$\Big\downarrow$ 209

$$-\frac{2a\sqrt{a^2x^2+1}\left(\frac{\frac{2}{3}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}}dx+\frac{x}{3\left(a^2x^2+1\right)^{3/2}}}{4a}-\frac{\operatorname{arcsinh}(ax)}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\left(\frac{x}{2a\sqrt{a^2x^2+1}}-\frac{\operatorname{arcsinh}(ax)}{2a^2\left(a^2x^2+1\right)}\right)}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\operatorname{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)+\frac{1}{2}\operatorname{arcsinh}(ax)\log\left(e^{2\operatorname{arc}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}}{3c}}{5c}$$

$$\frac{x\operatorname{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$\Big\downarrow$ 208

$$-\frac{2a\sqrt{a^2x^2+1}\left(\frac{\frac{2x}{3\sqrt{a^2x^2+1}}+\frac{x}{3\left(a^2x^2+1\right)^{3/2}}}{4a}-\frac{\operatorname{arcsinh}(ax)}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{2a\sqrt{a^2x^2+1}\left(\frac{x}{2a\sqrt{a^2x^2+1}}-\frac{\operatorname{arcsinh}(ax)}{2a^2\left(a^2x^2+1\right)}\right)}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\operatorname{arcsinh}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)+\frac{1}{2}\operatorname{arcsinh}(ax)\log\left(e^{2\operatorname{arc}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}}{3c}}{5c}$$

$$\frac{x\operatorname{arcsinh}(ax)^2}{5c\left(a^2cx^2+c\right)^{5/2}}$$

input
```
Int[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

## 3.320. $\int \frac{\operatorname{arcsinh}(ax)^2}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

output
```
(x*ArcSinh[a*x]^2)/(5*c*(c + a^2*c*x^2)^(5/2)) - (2*a*Sqrt[1 + a^2*x^2]*((
x/(3*(1 + a^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + a^2*x^2]))/(4*a) - ArcSinh[a
*x]/(4*a^2*(1 + a^2*x^2)^2)))/(5*c^3*Sqrt[c + a^2*c*x^2]) + (4*((x*ArcSinh
[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (2*a*Sqrt[1 + a^2*x^2]*(x/(2*a*Sqrt
[1 + a^2*x^2]) - ArcSinh[a*x]/(2*a^2*(1 + a^2*x^2))))/(3*c^2*Sqrt[c + a^2*
c*x^2]) + (2*((x*ArcSinh[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 +
 a^2*x^2]*((-1/2*I)*ArcSinh[a*x]^2 + (2*I)*((ArcSinh[a*x]*Log[1 + E^(2*Arc
Sinh[a*x])])/2 + PolyLog[2, -E^(2*ArcSinh[a*x])]/4)))/(a*c*Sqrt[c + a^2*c*
x^2])))/(3*c)))/(5*c)
```

### 3.320.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## 3.320. $\int \frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

---

## 3.320. $\int \frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx$

### 3.320.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| default | $\frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(-64\ \text{arcsinh}(ax)a^8x^8-64\ \text{arcsinh}(ax)\sqrt{a^2}\right.}{}$ |

input
```
int(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x
^3-16*a^2*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*arcsinh(a
*x)*a^8*x^8-64*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^7*x^7-32*a^8*x^8-32*(a^2*x
^2+1)^(1/2)*a^7*x^7-280*arcsinh(a*x)*a^6*x^6-248*arcsinh(a*x)*(a^2*x^2+1)^
(1/2)*a^5*x^5-142*a^6*x^6-126*x^5*a^5*(a^2*x^2+1)^(1/2)+80*a^4*x^4*arcsinh
(a*x)^2-456*a^4*x^4*arcsinh(a*x)-340*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2
)-265*a^4*x^4-156*a^3*x^3*(a^2*x^2+1)^(1/2)+190*arcsinh(a*x)^2*a^2*x^2-328
*a^2*x^2*arcsinh(a*x)-165*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-235*a^2*x^2-6
2*a*x*(a^2*x^2+1)^(1/2)+128*arcsinh(a*x)^2-88*arcsinh(a*x)-80)/(40*a^10*x^
10+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4+16/15/(a^2*x^
2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2-16/15/(a^2*x^2+1)^(1
/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^
2)-8/15/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*polylog(2,-(a*x+(a^2
*x^2+1)^(1/2))^2)
```

### 3.320.5 Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx=\int \frac{\text{arsinh}\,(ax)^2}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^2/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

### 3.320.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}^2(ax)}{(c(a^2x^2+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**2/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(asinh(a*x)**2/(c*(a**2*x**2 + 1))**(7/2), x)
```

### 3.320.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^2/(a^2*c*x^2 + c)^(7/2), x)
```

### 3.320.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{(c\,a^2\,x^2+c)^{7/2}}\,dx$$

input
```
int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2),x)
```

output
```
int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2), x)
```

# 3.321 $\int x^m (d+c^2dx^2)^{5/2} (a+b\,\mathrm{arcsinh}(cx))^2 \, dx$

### 3.321.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$
\begin{aligned}
&\int x^m (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{10b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3(6+m)} \\
&+ \frac{2b^2c^2d^2(52+15m+m^2)\,x^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^2(6+m)^3} + \frac{2b^2c^4d^2x^{5+m}\sqrt{d+c^2dx^2}}{(6+m)^3} \\
&- \frac{30bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&- \frac{10bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)\,(8+6m+m^2)\,\sqrt{1+c^2x^2}} - \frac{2bcd^2x^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(12+8m+m^2)\,\sqrt{1+c^2x^2}} \\
&- \frac{10bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)^2(6+m)\sqrt{1+c^2x^2}} - \frac{4bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&- \frac{2bc^5d^2x^{6+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(6+m)^2\sqrt{1+c^2x^2}} + \frac{15d^2x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{(6+m)\,(8+6m+m^2)} \\
&+ \frac{5dx^{1+m}(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{(4+m)(6+m)} + \frac{x^{1+m}(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{6+m} \\
&+ \frac{30b^2c^2d^2x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)(4+m)(6+m)\sqrt{1+c^2x^2}} \\
&+ \frac{10b^2c^2d^2(10+3m)x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^3(6+m)\sqrt{1+c^2x^2}} \\
&+ \frac{2b^2c^2d^2(264+130m+15m^2)\,x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^2(6+m)^3\sqrt{1+c^2x^2}} \\
&+ \frac{15d^3\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)}{(6+m)\,(8+6m+m^2)}
\end{aligned}
$$

output
```
5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)/(6+m)+x^(1+m)*(
c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/(6+m)+10*b^2*c^2*d^2*x^(3+m)*(c^2*
d*x^2+d)^(1/2)/(4+m)^3/(6+m)+2*b^2*c^2*d^2*(m^2+15*m+52)*x^(3+m)*(c^2*d*x^
2+d)^(1/2)/(4+m)^2/(6+m)^3+2*b^2*c^4*d^2*x^(5+m)*(c^2*d*x^2+d)^(1/2)/(6+m)
^3+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+
8)-30*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)
/(6+m)/(c^2*x^2+1)^(1/2)-10*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c*d^2*x^(2+m)*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)/(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-10*b*c^3*d^2*x^
(4+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)/(c^2*x^2+1)^(1/
2)-4*b*c^3*d^2*x^(4+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)/(6+m)/
(c^2*x^2+1)^(1/2)-2*b*c^5*d^2*x^(6+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)/(6+m)^2/(c^2*x^2+1)^(1/2)+10*b^2*c^2*d^2*(10+3*m)*x^(3+m)*hypergeom([1/
2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^3/(6+m)/(m^2
+5*m+6)/(c^2*x^2+1)^(1/2)+30*b^2*c^2*d^2*x^(3+m)*hypergeom([1/2, 3/2+1/2*m
],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+7*m+12)/(c^
2*x^2+1)^(1/2)+2*b^2*c^2*d^2*(15*m^2+130*m+264)*x^(3+m)*hypergeom([1/2, 3/
2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)^3/(m^2+5*
m+6)/(c^2*x^2+1)^(1/2)+15*d^3*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d
*x^2+d)^(1/2),x)/(6+m)/(m^2+6*m+8)
```

## 3.321.2 Mathematica [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx=\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx$$

input
```
Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2, x]
```

### 3.321.3 Rubi [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6223, 6218, 1590, 27, 363, 278, 6223, 6218, 363, 278, 6223, 6191, 278, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(c^2 dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2bcd^2\sqrt{c^2dx^2+d}\int x^{m+1}\left(c^2x^2+1\right)^2 (a+b\text{arcsinh}(cx))dx}{(m+6)\sqrt{c^2x^2+1}}+\frac{5d\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^2dx}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^2}{m+6}$$

$$\Big\downarrow \text{6218}$$

$$\frac{5d\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^2dx}{m+6}-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-bc\int \frac{x^{m+2}\left(\frac{c^4x^4}{m+6}+\frac{2c^2x^2}{m+4}+\frac{1}{m+2}\right)}{\sqrt{c^2x^2+1}}dx+\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+ba}{m}\right.}{(m+6)\sqrt{c^2x^2+1}}\\ \frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^2}{m+6}$$

$$\Big\downarrow \text{1590}$$

$$\frac{5d\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^2dx}{m+6}-\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-bc\left(\frac{\int \frac{c^2x^{m+2}\left(\frac{c^2\left(m^2+15m+52\right)x^2}{(m+4)(m+6)}+\frac{m+6}{m+2}\right)}{\sqrt{c^2x^2+1}}dx}{c^2(m+6)}+\frac{c^2\sqrt{c^2x^2+1}x^{m+5}}{(m+6)^2}\right)+\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(}{}\right.}{(m+6)\sqrt{c^2x^2+1}}\\ \frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^2}{m+6}$$

$$\Big\downarrow \text{27}$$

$$\frac{5d\int x^m\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-bc\left(\frac{\int\frac{x^{m+2}\left(\frac{c^2\left(m^2+15m+52\right)x^2}{(m+4)(m+6)}+\frac{m+6}{m+2}\right)}{\sqrt{c^2x^2+1}}dx}{m+6}+\frac{c^2\sqrt{c^2x^2+1}x^{m+5}}{(m+6)^2}\right)+\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+\cdots}{\cdots}\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}$$

$\downarrow$ 363

$$\frac{5d\int x^m\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(-bc\left(\frac{\frac{\left(15m^2+130m+264\right)\int\frac{x^{m+2}}{\sqrt{c^2x^2+1}}dx}{(m+2)(m+4)^2(m+6)}+\frac{\left(m^2+15m+52\right)\sqrt{c^2x^2+1}x^{m+3}}{(m+4)^2(m+6)}}{m+6}+\frac{c^2\sqrt{c^2x^2+1}x^{m+5}}{(m+6)^2}\right)+\frac{c^4x^{m+6}(a+b\text{arcsin}\cdots}{m+6}\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}$$

$\downarrow$ 278

$$\frac{5d\int x^m\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{m+6}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^{m}\cdots}{(\cdots}\right.\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$\downarrow$ 6223

$$\frac{5d\left(-\frac{2bcd\sqrt{c^2dx^2+d}\int x^{m+1}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{(m+4)\sqrt{c^2x^2+1}}+\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsi}\cdots}{m+4}\right.}{m+6}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^{m}\cdots}{(\cdots}\right.\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$\downarrow$ 6218

3.321. $\int x^m(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5d\left(-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\int \frac{x^{m+2}\left(\frac{c^2x^2}{m+4}+\frac{1}{m+2}\right)}{\sqrt{c^2x^2+1}}dx+\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}+\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcs}}{m+4}\right.}{m+6}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^m}{(}\right.\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{363}$$

$$\frac{5d\left(-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\left(\frac{(3m+10)\int\frac{x^{m+2}}{\sqrt{c^2x^2+1}}dx}{(m+2)(m+4)^2}+\frac{\sqrt{c^2x^2+1}x^{m+3}}{(m+4)^2}\right)+\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}+\frac{3d\int x^m\sqrt{c^2}}{}\right.}{m+6}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^m}{(}\right.\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{5d\left(\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx}{m+4}-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{(3m+10)x^{m+3}\text{ Hy}}{}\right.\right.}{(m+4)\sqrt{c^2x^2+1}}\right.}{m+6}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^m}{(}\right.\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6223}$$

3.321. $\int x^m\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5d\left(\frac{3d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int x^{m+1}(a+b\text{arcsinh}(cx))dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}\right)}{m+4}-\frac{2bcd\sqrt{c^2dx^2+d}\left(\right.}{}\right.}{}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^{m}}{(}\right.\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$\downarrow$ 6191

$$\frac{5d\left(\frac{3d\left(-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bc\int \frac{x^{m+2}}{\sqrt{c^2x^2+1}}dx}{m+2}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}\right)}{m+4}-\right.}{}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^{m}}{(}\right.\right.}{(m+6)\sqrt{c^2x^2+1}}$$

$\downarrow$ 278

3.321. $\int x^m\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{5d\left(\frac{3d\left(\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx \cdots}}{\cdots}\right)}{m+4}\cdots\right)}{\cdots}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^{m}\cdots}{(\cdots}\cdots\right)\right)}{(m+6)\sqrt{c^2x^2+1}}$$

$\downarrow$ 6239

$$\frac{5d\left(\frac{3d\left(\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx \cdots}}{\cdots}\right)}{m+4}\cdots\right)}{\cdots}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{m+6}-$$

$$\frac{2bcd^2\sqrt{c^2dx^2+d}\left(\frac{c^4x^{m+6}(a+b\text{arcsinh}(cx))}{m+6}+\frac{2c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{\left(15m^2+130m+264\right)x^{m}\cdots}{(\cdots}\cdots\right)\right)}{(m+6)\sqrt{c^2x^2+1}}$$

input
```
Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

3.321. $\int x^m(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.321.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

## 3.321. $\int x^m (d + c^2 d x^2)^{5/2} (a + b\text{arcsinh}(cx))^2 \, dx$

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.321.4 Maple [N/A] (verified)

Not integrable

Time = 1.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^{m} \left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}d x$$

input
```
int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

### 3.321.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.75

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx=\int { {\left(c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m} \,d x }$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*
b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

### 3.321.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.321.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^m \, dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

### 3.321.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.321.9 Mupad [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^2\,dx=\int x^m\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

input
```
int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

# 3.322 $\int x^m(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.322.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\begin{aligned}\int x^m\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{2b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}}{(4+m)^3}\\ &-\frac{6bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2(4+m)\sqrt{1+c^2x^2}}-\frac{2bcdx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(8+6m+m^2)\sqrt{1+c^2x^2}}\\ &-\frac{2bc^3dx^{4+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(4+m)^2\sqrt{1+c^2x^2}}\\ &+\frac{3dx^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{8+6m+m^2}+\frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{4+m}\\ &+\frac{6b^2c^2dx^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)(4+m)\sqrt{1+c^2x^2}}\\ &+\frac{2b^2c^2d(10+3m)x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)(3+m)(4+m)^3\sqrt{1+c^2x^2}}\\ &+\frac{3d^2\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)}{8+6m+m^2}\end{aligned}$$

output
```
x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)+2*b^2*c^2*d*x^(3+m)
*(c^2*d*x^2+d)^(1/2)/(4+m)^3+3*d*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d
)^(1/2)/(m^2+6*m+8)-6*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)
/(2+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-2*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c^3*d*x^(4+m)*(a+b*arcsinh
(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*d*(10+3*m)*
x^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/
2)/(4+m)^3/(m^2+5*m+6)/(c^2*x^2+1)^(1/2)+6*b^2*c^2*d*x^(3+m)*hypergeom([1/
2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+7*m+1
2)/(c^2*x^2+1)^(1/2)+3*d^2*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^
2+d)^(1/2),x)/(m^2+6*m+8)
```

### 3.322.2 Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx=\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx$$

input
```
Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2, x]
```

### 3.322.3 Rubi [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6223, 6218, 363, 278, 6223, 6191, 278, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \; 6223$$

$$-\frac{2bcd\sqrt{c^2dx^2+d}\int x^{m+1}\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{(m+4)\sqrt{c^2x^2+1}}+\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

$$\Big\downarrow \text{6218}$$

$$-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\int\frac{x^{m+2}\left(\frac{c^2x^2}{m+4}+\frac{1}{m+2}\right)}{\sqrt{c^2x^2+1}}dx+\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}+\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

$$\Big\downarrow \text{363}$$

$$-\frac{2bcd\sqrt{c^2dx^2+d}\left(-bc\left(\frac{(3m+10)\int\frac{x^{m+2}}{\sqrt{c^2x^2+1}}dx}{(m+2)(m+4)^2}+\frac{\sqrt{c^2x^2+1}x^{m+3}}{(m+4)^2}\right)+\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}\right)}{(m+4)\sqrt{c^2x^2+1}}+\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx}{m+4}+\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

$$\Big\downarrow \text{278}$$

$$\frac{3d\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx}{m+4}-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{(3m+10)x^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{c^2x^2+1}}\ \frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

$$\Big\downarrow \text{6223}$$

$$\frac{3d\left(-\frac{2bc\sqrt{c^2dx^2+d}\int x^{m+1}(a+b\text{arcsinh}(cx))dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{d\int\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}\right)}{m+4}-\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{(3m+10)x^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{c^2x^2+1}}\ \frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

$$\Big\downarrow \text{6191}$$

---

3.322. $\int x^m\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$\frac{3d\left(-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bc\int\frac{x^{m+2}}{\sqrt{c^2x^2+1}}dx}{m+2}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{d\int\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}\right.}{m+4}$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{(3m+10)x^{m+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{c^2x^2+1}}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

$\downarrow$ 278

$$\frac{3d\left(\frac{d\int\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2\cdots}}{}\right.}{m+4}$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{(3m+10)x^{m+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{c^2x^2+1}}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

$\downarrow$ 6239

$$\frac{3d\left(\frac{d\int\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2\cdots}}{}\right.}{m+4}$$

$$\frac{2bcd\sqrt{c^2dx^2+d}\left(\frac{c^2x^{m+4}(a+b\text{arcsinh}(cx))}{m+4}+\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-bc\left(\frac{(3m+10)x^{m+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{c^2x^2+1}}$$

$$\frac{x^{m+1}\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{m+4}$$

input
```
Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

3.322. $\int x^m\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.322.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6218
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 +
 c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& IGtQ[p, 0]
```

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.322. $\int x^m (d + c^2 d x^2)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx$

### 3.322.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^{m} \left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}d x$$

input
```
int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

### 3.322.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int x^m \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx=\int { {\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m} \,d x }$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

### 3.322.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx=\text{Timed out}$$

input
```
integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.322.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

### 3.322.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.322.9 Mupad [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^2\,dx=\int x^m\,(a+b\,\text{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

# 3.323 $\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.323.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\begin{aligned}&\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx\\&=-\frac{2bcx^{2+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2+m}\\&\quad+\frac{2b^2c^2x^{3+m}\sqrt{d+c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(2+m)^2(3+m)\sqrt{1+c^2x^2}}\\&\quad+\frac{d\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)}{2+m}\end{aligned}$$

output
```
x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(2+m)-2*b*c*x^(2+m)*(a+b*
arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*x^(3
+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(
2+m)^2/(3+m)/(c^2*x^2+1)^(1/2)+d*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^
2*d*x^2+d)^(1/2),x)/(2+m)
```

### 3.323.2 Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$

input
```
Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

### 3.323.3 Rubi [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6223, 6191, 278, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6223}$$

$$-\frac{2bc\sqrt{c^2dx^2+d}\int x^{m+1}(a+b\text{arcsinh}(cx))dx}{(m+2)\sqrt{c^2x^2+1}}+\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bc\int \frac{x^{m+2}}{\sqrt{c^2x^2+1}}dx}{m+2}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}$$

$$\Big\downarrow \text{278}$$

$$\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}$$

$\Big\downarrow$ 6239

$$\frac{d\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx}{m+2}-\frac{2bc\sqrt{c^2dx^2+d}\left(\frac{x^{m+2}(a+b\text{arcsinh}(cx))}{m+2}-\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{m+2}$$

input
```
Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

### 3.323.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.323. $\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

rule 6223
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*
x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Simp[b*c*(n/(f*(m
 + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 +
c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.323.4 Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^{m}\sqrt{c^{2} d\,x^{2}+d}\,\left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}d x$$

input
```
int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

## 3.323.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int x^m \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{c^{2} d x^{2} + d}{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m} \,d x$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)*x^m, x)
```

### 3.323.6 Sympy [N/A]

Not integrable

Time = 21.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^m\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

### 3.323.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

### 3.323.8 Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.323.9 Mupad [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int x^m\,(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

# 3.324 $\int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$



## 3.324.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \mathrm{Int}\left(\frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}},x\right)$$

output
```
Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

## 3.324.2 Mathematica [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{x^m(a+b\,\mathrm{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

### 3.324.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2+d}}dx$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
$Aborted
```

#### 3.324.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.324.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}{\sqrt{c^2d\,x^2+d}}dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

### 3.324.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*x^m/sqrt(c^2*d*x^
2 + d), x)
```

### 3.324.6 Sympy [N/A]

Not integrable

Time = 7.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m(a+b\,\text{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**m*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

### 3.324.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

### 3.324.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

### 3.324.9 Mupad [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

# 3.325 $\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$



## 3.325.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

## 3.325.2 Mathematica [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

## 3.325.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 6239}$$

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

### 3.325.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.325.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m(a+b\ \text{arcsinh}\,(cx))^2}{(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

### 3.325.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

### 3.325.6 Sympy [N/A]

Not integrable

Time = 9.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{x^m(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

### 3.325.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.325.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

### 3.325.9 Mupad [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

# 3.326 $\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$



## 3.326.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

## 3.326.2 Mathematica [N/A]

Not integrable

Time = 4.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

## 3.326.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}dx$$

input
```
Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
$Aborted
```

### 3.326.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.326.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^{m} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}d x$$

input
```
int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

output
```
int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

### 3.326.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

### 3.326.6 Sympy [N/A]

Not integrable

Time = 165.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{x^m(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

### 3.326.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.326.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

### 3.326.9 Mupad [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{x^m\,(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

# 3.327 $\int \frac{x^m \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$



## 3.327.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \operatorname{Int}\left(\frac{x^m \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}},x\right)$$

output
```
Unintegrable(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

## 3.327.2 Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

input
```
Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

### 3.327.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}} \, dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}} dx$$

input
```
Int[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

output
```
$Aborted
```

#### 3.327.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.327.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m \operatorname{arcsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}}dx$$

input
```
int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

### 3.327.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.327.6 Sympy [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}^2(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x**m*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*asinh(a*x)**2/sqrt(a**2*x**2 + 1), x)
```

### 3.327.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.327.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \,\text{arsinh}\,(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

### 3.327.9 Mupad [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \,\text{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.328 $\int (c+a^2cx^2)^3 \operatorname{arcsinh}(ax)^3\,dx$



## 3.328.1 Optimal result

Integrand size = 19, antiderivative size = 359

$$\int (c+a^2cx^2)^3 \operatorname{arcsinh}(ax)^3\,dx = -\frac{413312c^3\sqrt{1+a^2x^2}}{128625a} - \frac{30256c^3(1+a^2x^2)^{3/2}}{385875a} - \frac{2664c^3(1+a^2x^2)^{5/2}}{214375a} - \frac{6c^3(1+a^2x^2)^{7/2}}{2401a} + \frac{4322c^3x\operatorname{arcsinh}(ax)}{1225} + \frac{1514a^2c^3x^3\operatorname{arcsinh}(ax)}{3675} + \frac{702a^4c^3x^5\operatorname{arcsinh}(ax)}{6125} + \frac{6}{343}a^6c^3x^7\operatorname{arcsinh}(ax) - \frac{48c^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{8c^3(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{35a} - \frac{18c^3(1+a^2x^2)^{5/}}{175}$$

output
```
-30256/385875*c^3*(a^2*x^2+1)^(3/2)/a-2664/214375*c^3*(a^2*x^2+1)^(5/2)/a-
6/2401*c^3*(a^2*x^2+1)^(7/2)/a+4322/1225*c^3*x*arcsinh(a*x)+1514/3675*a^2*
c^3*x^3*arcsinh(a*x)+702/6125*a^4*c^3*x^5*arcsinh(a*x)+6/343*a^6*c^3*x^7*a
rcsinh(a*x)-8/35*c^3*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-18/175*c^3*(a^2*x^
2+1)^(5/2)*arcsinh(a*x)^2/a-3/49*c^3*(a^2*x^2+1)^(7/2)*arcsinh(a*x)^2/a+16
/35*c^3*x*arcsinh(a*x)^3+8/35*c^3*x*(a^2*x^2+1)*arcsinh(a*x)^3+6/35*c^3*x*
(a^2*x^2+1)^2*arcsinh(a*x)^3+1/7*c^3*x*(a^2*x^2+1)^3*arcsinh(a*x)^3-413312
/128625*c^3*(a^2*x^2+1)^(1/2)/a-48/35*c^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)
/a
```

### 3.328.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.47

$$\int \left(c+a^2cx^2\right)^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$= \frac{c^3\left(-2\sqrt{1+a^2x^2}(22329151+747937a^2x^2+134541a^4x^4+16875a^6x^6)+210ax(226905+26495a^2x^2+73\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

output
```
(c^3*(-2*Sqrt[1 + a^2*x^2]*(22329151 + 747937*a^2*x^2 + 134541*a^4*x^4 + 1
6875*a^6*x^6) + 210*a*x*(226905 + 26495*a^2*x^2 + 7371*a^4*x^4 + 1125*a^6*
x^6)*ArcSinh[a*x] - 11025*Sqrt[1 + a^2*x^2]*(2161 + 757*a^2*x^2 + 351*a^4*
x^4 + 75*a^6*x^6)*ArcSinh[a*x]^2 + 385875*a*x*(35 + 35*a^2*x^2 + 21*a^4*x^
4 + 5*a^6*x^6)*ArcSinh[a*x]^3))/(13505625*a)
```

### 3.328.3 Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.57, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.947$, Rules used = {6201, 27, 6201, 6201, 6187, 6213, 6187, 241, 6199, 27, 353, 53, 1576, 1140, 2009, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^3 \left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3}{7}ac^3\int x\left(a^2x^2+1\right)^{5/2}\operatorname{arcsinh}(ax)^2dx+\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{7}ac^3\int x\left(a^2x^2+1\right)^{5/2}\operatorname{arcsinh}(ax)^2dx+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3}{7}ac^3\int x\left(a^2x^2+1\right)^{5/2}\operatorname{arcsinh}(ax)^2dx+$$
$$\frac{6}{7}c^3\left(-\frac{3}{5}a\int x\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2dx+\frac{4}{5}\int\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^3\right)+$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3}{7}ac^3\int x\left(a^2x^2+1\right)^{5/2}\operatorname{arcsinh}(ax)^2dx+$$
$$\frac{6}{7}c^3\left(-\frac{3}{5}a\int x\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2dx+\frac{4}{5}\left(-a\int x\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2dx+\frac{2}{3}\int\operatorname{arcsinh}(ax)^3dx+\frac{1}{3}x\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{6187}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\operatorname{arcsinh}(ax)^3-3a\int\frac{x\operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx\right)-a\int x\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\operatorname{arcsinh}(\right.\right.$$
$$\frac{3}{7}ac^3\int x\left(a^2x^2+1\right)^{5/2}\operatorname{arcsinh}(ax)^2dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{6213}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\int\operatorname{arcsinh}(ax)dx}{a}\right)\right)-a\left(\frac{\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax}{3a^2}\right.\right.\right.$$
$$\frac{3}{7}ac^3\left(\frac{\left(a^2x^2+1\right)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\int\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)dx}{7a}\right)+$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{6187}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\operatorname{arcsinh}(ax)-a\int\frac{x}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)\right)-a\left(\frac{\left(a^2x^2-\right.}{}\right.\right.\right.$$
$$\frac{3}{7}ac^3\left(\frac{\left(a^2x^2+1\right)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\int\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)dx}{7a}\right)+$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{241}$$

3.328. $\int\left(c+a^2cx^2\right)^3\operatorname{arcsinh}(ax)^3\,dx$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(-a\left(\frac{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\int(a^2x^2+1)\operatorname{arcsinh}(ax)dx}{3a}\right)+\frac{1}{3}x(a^2x^2+1)\operatorname{arcsinh}(ax)^3+\frac{2}{3}\left(x\right.\right.\right.$$
$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\int(a^2x^2+1)^3\operatorname{arcsinh}(ax)dx}{7a}\right)+$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3$$

$\downarrow$ 6199

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(-a\left(\frac{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-a\int\frac{x(a^2x^2+3)}{3\sqrt{a^2x^2+1}}dx+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}x(a\right.\right.$$
$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-a\int\frac{x(5a^6x^6+21a^4x^4+35a^2x^2+35)}{35\sqrt{a^2x^2+1}}dx+\frac{1}{7}a^6x^7\operatorname{arcsinh}(ax)+\frac{3}{5}a^4x^5\operatorname{arcsinh}(ax\right.}{7a}\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3$$

$\downarrow$ 27

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(-a\left(\frac{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{3}a\int\frac{x(a^2x^2+3)}{\sqrt{a^2x^2+1}}dx+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}x(\right.\right.$$
$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{35}a\int\frac{x(5a^6x^6+21a^4x^4+35a^2x^2+35)}{\sqrt{a^2x^2+1}}dx+\frac{1}{7}a^6x^7\operatorname{arcsinh}(ax)+\frac{3}{5}a^4x^5\operatorname{arcsinh}(\right.}{7a}\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3$$

$\downarrow$ 353

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(-a\left(\frac{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\frac{a^2x^2+3}{\sqrt{a^2x^2+1}}dx^2+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}x(\right.\right.$$
$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{35}a\int\frac{x(5a^6x^6+21a^4x^4+35a^2x^2+35)}{\sqrt{a^2x^2+1}}dx+\frac{1}{7}a^6x^7\operatorname{arcsinh}(ax)+\frac{3}{5}a^4x^5\operatorname{arcsinh}(\right.}{7a}\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3$$

$\downarrow$ 53

---

3.328. $\int(c+a^2cx^2)^3\operatorname{arcsinh}(ax)^3\,dx$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(-a\left(\frac{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\left(\sqrt{a^2x^2+1}+\frac{2}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsin}\right.}{3a}\right.\right.\right.$$

$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{35}a\int\frac{x(5a^6x^6+21a^4x^4+35a^2x^2+35)}{\sqrt{a^2x^2+1}}dx+\frac{1}{7}a^6x^7\operatorname{arcsinh}(ax)+\frac{3}{5}a^4x^5\operatorname{arcsinh}(\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{1576}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(-a\left(\frac{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\left(\sqrt{a^2x^2+1}+\frac{2}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsin}\right.}{3a}\right.\right.\right.$$

$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{35}a\int\frac{x(5a^6x^6+21a^4x^4+35a^2x^2+35)}{\sqrt{a^2x^2+1}}dx+\frac{1}{7}a^6x^7\operatorname{arcsinh}(ax)+\frac{3}{5}a^4x^5\operatorname{arcsinh}(\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{1140}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(-a\left(\frac{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\left(\sqrt{a^2x^2+1}+\frac{2}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsin}\right.}{3a}\right.\right.\right.$$

$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{35}a\int\frac{x(5a^6x^6+21a^4x^4+35a^2x^2+35)}{\sqrt{a^2x^2+1}}dx+\frac{1}{7}a^6x^7\operatorname{arcsinh}(ax)+\frac{3}{5}a^4x^5\operatorname{arcsinh}(\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\operatorname{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{35}a\int\frac{x(5a^6x^6+21a^4x^4+35a^2x^2+35)}{\sqrt{a^2x^2+1}}dx+\frac{1}{7}a^6x^7\operatorname{arcsinh}(ax)+\frac{3}{5}a^4x^5\operatorname{arcsinh}\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{arcsinh}(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(a^2x^2+1)^2\operatorname{arcsinh}(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(a^2x^2+1)\operatorname{arcsinh}(ax)^3+\frac{2}{3}\left(x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}}{a^2}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2331}$$

3.328. $\int\left(c+a^2cx^2\right)^3\operatorname{arcsinh}(ax)^3\,dx$

$$-\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\,\text{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{70}a\int\frac{5a^6x^6+21a^4x^4+35a^2x^2+35}{\sqrt{a^2x^2+1}}dx^2+\frac{1}{7}a^6x^7\text{arcsinh}(ax)+\frac{3}{5}a^4x^5\text{arcsinh}(\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(a^2x^2+1)^3\,\text{arcsinh}(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(a^2x^2+1)^2\,\text{arcsinh}(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(a^2x^2+1)\,\text{arcsinh}(ax)^3+\frac{2}{3}\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}}{a^2}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2389}$$

$$-\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\,\text{arcsinh}(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{70}a\int\left(5(a^2x^2+1)^{5/2}+6(a^2x^2+1)^{3/2}+8\sqrt{a^2x^2+1}+\frac{16}{\sqrt{a^2x^2+1}}\right)dx\right.}{}\right.$$

$$\frac{1}{7}c^3x(a^2x^2+1)^3\,\text{arcsinh}(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(a^2x^2+1)^2\,\text{arcsinh}(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(a^2x^2+1)\,\text{arcsinh}(ax)^3+\frac{2}{3}\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}}{a^2}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}c^3x(a^2x^2+1)^3\,\text{arcsinh}(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(a^2x^2+1)^2\,\text{arcsinh}(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(a^2x^2+1)\,\text{arcsinh}(ax)^3+\frac{2}{3}\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}}{a^2}\right.\right.\right.\right.$$

$$\frac{3}{7}ac^3\left(\frac{(a^2x^2+1)^{7/2}\,\text{arcsinh}(ax)^2}{7a^2}-\frac{2\left(\frac{1}{7}a^6x^7\text{arcsinh}(ax)+\frac{3}{5}a^4x^5\text{arcsinh}(ax)+a^2x^3\text{arcsinh}(ax)-\frac{1}{70}a\left(\frac{10(a^2x^2}{7a^2}\right.\right.}{7a}\right.$$

input
```
Int[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

## 3.328. $\int(c+a^2cx^2)^3\,\text{arcsinh}(ax)^3\,dx$

output
```
(c^3*x*(1 + a^2*x^2)^3*ArcSinh[a*x]^3)/7 - (3*a*c^3*(((1 + a^2*x^2)^(7/2)*
ArcSinh[a*x]^2)/(7*a^2) - (2*(-1/70*(a*((32*Sqrt[1 + a^2*x^2])/a^2 + (16*(
1 + a^2*x^2)^(3/2))/(3*a^2) + (12*(1 + a^2*x^2)^(5/2))/(5*a^2) + (10*(1 +
a^2*x^2)^(7/2))/(7*a^2))) + x*ArcSinh[a*x] + a^2*x^3*ArcSinh[a*x] + (3*a^4
*x^5*ArcSinh[a*x])/5 + (a^6*x^7*ArcSinh[a*x])/7))/(7*a)))/7 + (6*c^3*((x*(
1 + a^2*x^2)^2*ArcSinh[a*x]^3)/5 - (3*a*(((1 + a^2*x^2)^(5/2)*ArcSinh[a*x]
^2)/(5*a^2) - (2*(-1/30*(a*((16*Sqrt[1 + a^2*x^2])/a^2 + (8*(1 + a^2*x^2)^
(3/2))/(3*a^2) + (6*(1 + a^2*x^2)^(5/2))/(5*a^2))) + x*ArcSinh[a*x] + (2*a
^2*x^3*ArcSinh[a*x])/3 + (a^4*x^5*ArcSinh[a*x])/5))/(5*a)))/5 + (4*((x*(1
+ a^2*x^2)*ArcSinh[a*x]^3)/3 - a*(((1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(3*
a^2) - (2*(-1/6*(a*((4*Sqrt[1 + a^2*x^2])/a^2 + (2*(1 + a^2*x^2)^(3/2))/(3
*a^2))) + x*ArcSinh[a*x] + (a^2*x^3*ArcSinh[a*x])/3))/(3*a)) + (2*(x*ArcSi
nh[a*x]^3 - 3*a*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/a^2 - (2*(-(Sqrt[1 + a
^2*x^2]/a) + x*ArcSinh[a*x]))/a)))/3))/5))/7
```

### 3.328.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.328. $\int (c + a^2cx^2)^3 \operatorname{arcsinh}(ax)^3 \, dx$

### 3.328.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| derivativedivides | $\frac{c^3\left(1929375\,\mathrm{arcsinh}(ax)^3a^7x^7-826875\,\mathrm{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,a^6x^6+8103375a^5x^5\,\mathrm{arcsinh}(ax)^3+236250\,\mathrm{arcsinh}(ax)a^7\right.}{}$ |
| default | $\frac{c^3\left(1929375\,\mathrm{arcsinh}(ax)^3a^7x^7-826875\,\mathrm{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,a^6x^6+8103375a^5x^5\,\mathrm{arcsinh}(ax)^3+236250\,\mathrm{arcsinh}(ax)a^7\right.}{}$ |

input
```
int((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13505625/a*c^3*(1929375*arcsinh(a*x)^3*a^7*x^7-826875*arcsinh(a*x)^2*(a^
2*x^2+1)^(1/2)*a^6*x^6+8103375*a^5*x^5*arcsinh(a*x)^3+236250*arcsinh(a*x)*
a^7*x^7-3869775*a^4*x^4*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-33750*x^6*a^6*(a^
2*x^2+1)^(1/2)+13505625*a^3*x^3*arcsinh(a*x)^3+1547910*a^5*x^5*arcsinh(a*x
)-8345925*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-269082*a^4*x^4*(a^2*x^2
+1)^(1/2)+13505625*a*x*arcsinh(a*x)^3+5563950*a^3*x^3*arcsinh(a*x)-2382502
5*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1495874*a^2*x^2*(a^2*x^2+1)^(1/2)+47650
050*a*x*arcsinh(a*x)-44658302*(a^2*x^2+1)^(1/2))
```

### 3.328.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.69

$$\int \left(c+a^2cx^2\right)^3 \mathrm{arcsinh}(ax)^3\,dx$$
$$= \frac{385875\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-11025\left(75\,a^6c^3x^6+351\,a^4c^3\right.}{}$$

input
```
integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
1/13505625*(385875*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a
*c^3*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 11025*(75*a^6*c^3*x^6 + 351*a^4*c
^3*x^4 + 757*a^2*c^3*x^2 + 2161*c^3)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*
x^2 + 1))^2 + 210*(1125*a^7*c^3*x^7 + 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3
 + 226905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(16875*a^6*c^3*x^6 + 1
34541*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 + 22329151*c^3)*sqrt(a^2*x^2 + 1))/
a
```

### 3.328.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.99

$$\int \left(c+a^2cx^2\right)^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$=\begin{cases} \frac{a^6c^3x^7\operatorname{asinh}^3(ax)}{7}+\frac{6a^6c^3x^7\operatorname{asinh}(ax)}{343}-\frac{3a^5c^3x^6\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{49}-\frac{6a^5c^3x^6\sqrt{a^2x^2+1}}{2401}+\frac{3a^4c^3x^5\operatorname{asinh}^3(ax)}{5}+\frac{702a^4c^3x^5\operatorname{as}}{6125} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**3*asinh(a*x)**3,x)
```

output
```
Piecewise((a**6*c**3*x**7*asinh(a*x)**3/7 + 6*a**6*c**3*x**7*asinh(a*x)/34
3 - 3*a**5*c**3*x**6*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/49 - 6*a**5*c**3*x*
*6*sqrt(a**2*x**2 + 1)/2401 + 3*a**4*c**3*x**5*asinh(a*x)**3/5 + 702*a**4*
c**3*x**5*asinh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)*asinh(a
*x)**2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)/1500625 + a**2*c**3
*x**3*asinh(a*x)**3 + 1514*a**2*c**3*x**3*asinh(a*x)/3675 - 757*a*c**3*x**
2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/1225 - 1495874*a*c**3*x**2*sqrt(a**2*x
**2 + 1)/13505625 + c**3*x*asinh(a*x)**3 + 4322*c**3*x*asinh(a*x)/1225 - 2
161*c**3*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a
**2*x**2 + 1)/(13505625*a), Ne(a, 0)), (0, True))
```

### 3.328.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.77

$$\int \left(c+a^2cx^2\right)^3 \operatorname{arcsinh}(ax)^3\,dx =$$
$$-\frac{1}{1225}\left(75\sqrt{a^2x^2+1}a^4c^3x^6+351\sqrt{a^2x^2+1}a^2c^3x^4+757\sqrt{a^2x^2+1}c^3x^2+\frac{2161\sqrt{a^2x^2+1}c^3}{a^2}\right)a\operatorname{arsin}$$
$$+\frac{1}{35}\left(5a^6c^3x^7+21a^4c^3x^5+35a^2c^3x^3+35c^3x\right)\operatorname{arsinh}(ax)^3$$
$$-\frac{2}{13505625}\left(16875\sqrt{a^2x^2+1}a^4c^3x^6+134541\sqrt{a^2x^2+1}a^2c^3x^4+747937\sqrt{a^2x^2+1}c^3x^2+\frac{22329151\sqrt{}}{a}\right.$$

input
```
integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
-1/1225*(75*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 351*sqrt(a^2*x^2 + 1)*a^2*c^3*
x^4 + 757*sqrt(a^2*x^2 + 1)*c^3*x^2 + 2161*sqrt(a^2*x^2 + 1)*c^3/a^2)*a*ar
csinh(a*x)^2 + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*
c^3*x)*arcsinh(a*x)^3 - 2/13505625*(16875*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 +
134541*sqrt(a^2*x^2 + 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 + 1)*c^3*x^2 +
22329151*sqrt(a^2*x^2 + 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 + 7371*a^4*c^3*
x^5 + 26495*a^2*c^3*x^3 + 226905*c^3*x)*arcsinh(a*x)/a)*a
```

### 3.328.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.328.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^3 \operatorname{arcsinh}(ax)^3 \, dx = \int \operatorname{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3 \,dx$$

input
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^3, x)
```

# 3.329 $\int (c + a^2cx^2)^2 \operatorname{arcsinh}(ax)^3 \, dx$



## 3.329.1 Optimal result

Integrand size = 19, antiderivative size = 265

$$\begin{aligned}
&\int (c + a^2cx^2)^2 \operatorname{arcsinh}(ax)^3 \, dx \\
&= -\frac{4144c^2\sqrt{1+a^2x^2}}{1125a} - \frac{272c^2(1+a^2x^2)^{3/2}}{3375a} - \frac{6c^2(1+a^2x^2)^{5/2}}{625a} + \frac{298}{75}c^2x\operatorname{arcsinh}(ax) \\
&\quad + \frac{76}{225}a^2c^2x^3\operatorname{arcsinh}(ax) + \frac{6}{125}a^4c^2x^5\operatorname{arcsinh}(ax) - \frac{8c^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{5a} \\
&\quad - \frac{4c^2(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{15a} - \frac{3c^2(1+a^2x^2)^{5/2}\operatorname{arcsinh}(ax)^2}{25a} \\
&\quad + \frac{8}{15}c^2x\operatorname{arcsinh}(ax)^3 + \frac{4}{15}c^2x(1+a^2x^2)\operatorname{arcsinh}(ax)^3 + \frac{1}{5}c^2x(1+a^2x^2)^2\operatorname{arcsinh}(ax)^3
\end{aligned}$$

output
```
-272/3375*c^2*(a^2*x^2+1)^(3/2)/a-6/625*c^2*(a^2*x^2+1)^(5/2)/a+298/75*c^2
*x*arcsinh(a*x)+76/225*a^2*c^2*x^3*arcsinh(a*x)+6/125*a^4*c^2*x^5*arcsinh(
a*x)-4/15*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-3/25*c^2*(a^2*x^2+1)^(5/2
)*arcsinh(a*x)^2/a+8/15*c^2*x*arcsinh(a*x)^3+4/15*c^2*x*(a^2*x^2+1)*arcsin
h(a*x)^3+1/5*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)^3-4144/1125*c^2*(a^2*x^2+1)^
(1/2)/a-8/5*c^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

### 3.329.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int (c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^3 \, dx$$
$$= \frac{c^2\left(-2\sqrt{1+a^2x^2}(31841+842a^2x^2+81a^4x^4)+30ax(2235+190a^2x^2+27a^4x^4)\operatorname{arcsinh}(ax)-225\sqrt{1+\right.}{16875a}$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

output
```
(c^2*(-2*Sqrt[1 + a^2*x^2]*(31841 + 842*a^2*x^2 + 81*a^4*x^4) + 30*a*x*(22
35 + 190*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(149 +
 38*a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a*x*(15 + 10*a^2*x^2 + 3*a^
4*x^4)*ArcSinh[a*x]^3))/(16875*a)
```

### 3.329.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {6201, 27, 6201, 6187, 6213, 6187, 241, 6199, 27, 353, 53, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^3 \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \; 6201$$

$$-\frac{3}{5}ac^2\int x\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2dx+\frac{4}{5}c\int c\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{5}ac^2\int x\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2dx+\frac{4}{5}c^2\int \left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^3$$

$$\Big\downarrow \; 6201$$

$$-\frac{3}{5}ac^2\int x\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2dx+$$
$$\frac{4}{5}c^2\left(-a\int x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2dx+\frac{2}{3}\int \text{arcsinh}(ax)^3dx+\frac{1}{3}x\left(a^2x^2+1\right)\text{arcsinh}(ax)^3\right)+$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\text{arcsinh}(ax)^3-3a\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx\right)-a\int x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\text{arcsinh}(ax)$$
$$\frac{3}{5}ac^2\int x\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\int \text{arcsinh}(ax)dx}{a}\right)\right)-a\left(\frac{\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2}{3a^2}\right.\right.$$
$$\frac{3}{5}ac^2\left(\frac{\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\int \left(a^2x^2+1\right)^2\text{arcsinh}(ax)dx}{5a}\right)+$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-a\int \frac{x}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)\right)-a\left(\frac{\left(a^2x^2+1\right)}{}\right.\right.$$
$$\frac{3}{5}ac^2\left(\frac{\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\int \left(a^2x^2+1\right)^2\text{arcsinh}(ax)dx}{5a}\right)+$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow \text{ 241}$$

$$\frac{4}{5}c^2\left(-a\left(\frac{\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\int \left(a^2x^2+1\right)\text{arcsinh}(ax)dx}{3a}\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{arcsinh}(ax)^3+\frac{2}{3}\left(x\text{arc}\right.\right.$$
$$\frac{3}{5}ac^2\left(\frac{\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\int \left(a^2x^2+1\right)^2\text{arcsinh}(ax)dx}{5a}\right)+$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow \text{ 6199}$$

3.329. $\int \left(c+a^2cx^2\right)^2\text{arcsinh}(ax)^3\,dx$

$$\frac{4}{5}c^2\left(-a\left(\frac{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-a\int\frac{x(a^2x^2+3)}{3\sqrt{a^2x^2+1}}dx+\frac{1}{3}a^2x^3\text{arcsinh}(ax)+x\text{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}x(a^2x^2\right.$$
$$\frac{3}{5}ac^2\left(\frac{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\left(-a\int\frac{x(3a^4x^4+10a^2x^2+15)}{15\sqrt{a^2x^2+1}}dx+\frac{1}{5}a^4x^5\text{arcsinh}(ax)+\frac{2}{3}a^2x^3\text{arcsinh}(ax)+x\text{ar}\right.}{5a}\right.$$
$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow 27$$

$$\frac{4}{5}c^2\left(-a\left(\frac{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{3}a\int\frac{x(a^2x^2+3)}{\sqrt{a^2x^2+1}}dx+\frac{1}{3}a^2x^3\text{arcsinh}(ax)+x\text{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}x(a^2x\right.$$
$$\frac{3}{5}ac^2\left(\frac{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\left(-\frac{1}{15}a\int\frac{x(3a^4x^4+10a^2x^2+15)}{\sqrt{a^2x^2+1}}dx+\frac{1}{5}a^4x^5\text{arcsinh}(ax)+\frac{2}{3}a^2x^3\text{arcsinh}(ax)+x\right.}{5a}\right.$$
$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow 353$$

$$\frac{4}{5}c^2\left(-a\left(\frac{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\frac{a^2x^2+3}{\sqrt{a^2x^2+1}}dx^2+\frac{1}{3}a^2x^3\text{arcsinh}(ax)+x\text{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}x(a^2x\right.$$
$$\frac{3}{5}ac^2\left(\frac{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\left(-\frac{1}{15}a\int\frac{x(3a^4x^4+10a^2x^2+15)}{\sqrt{a^2x^2+1}}dx+\frac{1}{5}a^4x^5\text{arcsinh}(ax)+\frac{2}{3}a^2x^3\text{arcsinh}(ax)+x\right.}{5a}\right.$$
$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow 53$$

$$\frac{4}{5}c^2\left(-a\left(\frac{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\left(\sqrt{a^2x^2+1}+\frac{2}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{3}a^2x^3\text{arcsinh}(ax)+x\text{arcsinh}(a\right.}{3a}\right.\right.$$
$$\frac{3}{5}ac^2\left(\frac{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\left(-\frac{1}{15}a\int\frac{x(3a^4x^4+10a^2x^2+15)}{\sqrt{a^2x^2+1}}dx+\frac{1}{5}a^4x^5\text{arcsinh}(ax)+\frac{2}{3}a^2x^3\text{arcsinh}(ax)+x\right.}{5a}\right.$$
$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow 1576$$

---

3.329. $\int(c+a^2cx^2)^2\,\text{arcsinh}(ax)^3\,dx$

$$\frac{4}{5}c^2\left(-a\left(\frac{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\left(\sqrt{a^2x^2+1}+\frac{2}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{3}a^2x^3\text{arcsinh}(ax)+x\text{arcsinh}(a\right.}{3a}\right.\right.$$

$$\frac{3}{5}ac^2\left(\frac{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\left(-\frac{1}{30}a\int\frac{3a^4x^4+10a^2x^2+15}{\sqrt{a^2x^2+1}}dx^2+\frac{1}{5}a^4x^5\text{arcsinh}(ax)+\frac{2}{3}a^2x^3\text{arcsinh}(ax)+xa\right.}{5a}\right.$$

$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow \text{1140}$$

$$\frac{4}{5}c^2\left(-a\left(\frac{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\left(\sqrt{a^2x^2+1}+\frac{2}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{3}a^2x^3\text{arcsinh}(ax)+x\text{arcsinh}(a\right.}{3a}\right.\right.$$

$$\frac{3}{5}ac^2\left(\frac{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\left(-\frac{1}{30}a\int\left(3(a^2x^2+1)^{3/2}+4\sqrt{a^2x^2+1}+\frac{8}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{5}a^4x^5\text{arcsinh}(a\right.}{5a}\right.$$

$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{arcsinh}(ax)^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{arcsinh}(ax)^3+$$

$$\frac{4}{5}c^2\left(\frac{1}{3}x(a^2x^2+1)\text{arcsinh}(ax)^3+\frac{2}{3}\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+}}{a}\right.}{a}\right.\right.\right.$$

$$\frac{3}{5}ac^2\left(\frac{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)^2}{5a^2}-\frac{2\left(\frac{1}{5}a^4x^5\text{arcsinh}(ax)+\frac{2}{3}a^2x^3\text{arcsinh}(ax)-\frac{1}{30}a\left(\frac{6(a^2x^2+1)^{5/2}}{5a^2}+\frac{8(a^2x^2+1)^{3/2}}{3a^2}\right.\right.}{5a}\right.$$

input
```
Int[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

output
```
(c^2*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/5 - (3*a*c^2*(((1 + a^2*x^2)^(5/2)*
ArcSinh[a*x]^2)/(5*a^2) - (2*(-1/30*(a*((16*Sqrt[1 + a^2*x^2]))/a^2 + (8*(1
 + a^2*x^2)^(3/2))/(3*a^2) + (6*(1 + a^2*x^2)^(5/2))/(5*a^2))) + x*ArcSinh
[a*x] + (2*a^2*x^3*ArcSinh[a*x])/3 + (a^4*x^5*ArcSinh[a*x])/5))/(5*a)))/5
+ (4*c^2*((x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/3 - a*(((1 + a^2*x^2)^(3/2)*Arc
Sinh[a*x]^2)/(3*a^2) - (2*(-1/6*(a*((4*Sqrt[1 + a^2*x^2]))/a^2 + (2*(1 + a^
2*x^2)^(3/2))/(3*a^2))) + x*ArcSinh[a*x] + (a^2*x^3*ArcSinh[a*x])/3))/(3*a
)) + (2*(x*ArcSinh[a*x]^3 - 3*a*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/a^2 -
(2*(-(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]))/a)))/3))/5
```

3.329. $\int (c+a^2cx^2)^2 \text{arcsinh}(ax)^3\,dx$

### 3.329.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## 3.329. $\int (c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^3\,dx$

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.329.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| derivativedivides | $c^2\left(3375a^5x^5\operatorname{arcsinh}(ax)^3-2025a^4x^4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+11250a^3x^3\operatorname{arcsinh}(ax)^3+810a^5x^5\operatorname{arcsinh}(ax)-8550a^2x\right.$ |
| default | $c^2\left(3375a^5x^5\operatorname{arcsinh}(ax)^3-2025a^4x^4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+11250a^3x^3\operatorname{arcsinh}(ax)^3+810a^5x^5\operatorname{arcsinh}(ax)-8550a^2x\right.$ |

input
```
int((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16875/a*c^2*(3375*a^5*x^5*arcsinh(a*x)^3-2025*a^4*x^4*arcsinh(a*x)^2*(a^
2*x^2+1)^(1/2)+11250*a^3*x^3*arcsinh(a*x)^3+810*a^5*x^5*arcsinh(a*x)-8550*
a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-162*a^4*x^4*(a^2*x^2+1)^(1/2)+168
75*a*x*arcsinh(a*x)^3+5700*a^3*x^3*arcsinh(a*x)-33525*arcsinh(a*x)^2*(a^2*
x^2+1)^(1/2)-1684*a^2*x^2*(a^2*x^2+1)^(1/2)+67050*a*x*arcsinh(a*x)-63682*(
a^2*x^2+1)^(1/2))
```

### 3.329.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.77

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3\,dx$$
$$= \frac{1125\left(3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-225\left(9\,a^4c^2x^4+38\,a^2c^2x^2+149\,c^2\right)\sqrt{a^2x^2}}{}$$

input
```
integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
1/16875*(1125*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt
(a^2*x^2 + 1))^3 - 225*(9*a^4*c^2*x^4 + 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2
*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(27*a^5*c^2*x^5 + 190*a^3*c^
2*x^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(81*a^4*c^2*x^4 + 8
42*a^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 + 1))/a
```

### 3.329.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.99

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3\,dx$$
$$= \begin{cases} \frac{a^4c^2x^5\operatorname{asinh}^3(ax)}{5}+\frac{6a^4c^2x^5\operatorname{asinh}(ax)}{125}-\frac{3a^3c^2x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25}-\frac{6a^3c^2x^4\sqrt{a^2x^2+1}}{625}+\frac{2a^2c^2x^3\operatorname{asinh}^3(ax)}{3}+\frac{76a^2c^2x^3\operatorname{asi}}{225} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**2*asinh(a*x)**3,x)
```

output
```
Piecewise((a**4*c**2*x**5*asinh(a*x)**3/5 + 6*a**4*c**2*x**5*asinh(a*x)/12
5 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/25 - 6*a**3*c**2*x*
*4*sqrt(a**2*x**2 + 1)/625 + 2*a**2*c**2*x**3*asinh(a*x)**3/3 + 76*a**2*c*
*2*x**3*asinh(a*x)/225 - 38*a*c**2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/
75 - 1684*a*c**2*x**2*sqrt(a**2*x**2 + 1)/16875 + c**2*x*asinh(a*x)**3 + 2
98*c**2*x*asinh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(75*a
) - 63682*c**2*sqrt(a**2*x**2 + 1)/(16875*a), Ne(a, 0)), (0, True))
```

### 3.329.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.79

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3 \, dx$$
$$= -\frac{1}{75}\left(9\sqrt{a^2x^2+1}a^2c^2x^4+38\sqrt{a^2x^2+1}c^2x^2+\frac{149\sqrt{a^2x^2+1}c^2}{a^2}\right)a\,\operatorname{arsinh}(ax)^2$$
$$+\frac{1}{15}\left(3\,a^4c^2x^5+10\,a^2c^2x^3+15\,c^2x\right)\operatorname{arsinh}(ax)^3$$
$$-\frac{2}{16875}\left(81\sqrt{a^2x^2+1}a^2c^2x^4+842\sqrt{a^2x^2+1}c^2x^2-\frac{15\left(27\,a^4c^2x^5+190\,a^2c^2x^3+2235\,c^2x\right)\operatorname{arsinh}(ax)}{a}\right.$$

input
```
integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
-1/75*(9*sqrt(a^2*x^2 + 1)*a^2*c^2*x^4 + 38*sqrt(a^2*x^2 + 1)*c^2*x^2 + 14
9*sqrt(a^2*x^2 + 1)*c^2/a^2)*a*arcsinh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 + 10*a
^2*c^2*x^3 + 15*c^2*x)*arcsinh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 + 1)*a^2*
c^2*x^4 + 842*sqrt(a^2*x^2 + 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 + 190*a^2*c^2
*x^3 + 2235*c^2*x)*arcsinh(a*x)/a + 31841*sqrt(a^2*x^2 + 1)*c^2/a^2)*a
```

### 3.329.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.329.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^2 \operatorname{arcsinh}(ax)^3 \, dx = \int \operatorname{asinh}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2 \, dx$$

input
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^2, x)
```

# 3.330 $\int (c+a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$



## 3.330.1 Optimal result

Integrand size = 17, antiderivative size = 153

$$\begin{aligned}\int (c+a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx = &-\frac{40c\sqrt{1+a^2x^2}}{9a}-\frac{2c(1+a^2x^2)^{3/2}}{27a} \\ &+\frac{14}{3}cx\operatorname{arcsinh}(ax)+\frac{2}{9}a^2cx^3\operatorname{arcsinh}(ax) \\ &-\frac{2c\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a}-\frac{c(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)^2}{3a} \\ &+\frac{2}{3}cx\operatorname{arcsinh}(ax)^3+\frac{1}{3}cx(1+a^2x^2)\operatorname{arcsinh}(ax)^3\end{aligned}$$

output
```
-2/27*c*(a^2*x^2+1)^(3/2)/a+14/3*c*x*arcsinh(a*x)+2/9*a^2*c*x^3*arcsinh(a*
x)-1/3*c*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a+2/3*c*x*arcsinh(a*x)^3+1/3*c*x
*(a^2*x^2+1)*arcsinh(a*x)^3-40/9*c*(a^2*x^2+1)^(1/2)/a-2*c*arcsinh(a*x)^2*
(a^2*x^2+1)^(1/2)/a
```

## 3.330.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$\int (c+a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$$
$$=\frac{c\left(-2\sqrt{1+a^2x^2}(61+a^2x^2)+6ax(21+a^2x^2)\operatorname{arcsinh}(ax)-9\sqrt{1+a^2x^2}(7+a^2x^2)\operatorname{arcsinh}(ax)^2+9ax(3\right.}{27a}$$

input
```
Integrate[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

output
```
(c*(-2*Sqrt[1 + a^2*x^2]*(61 + a^2*x^2) + 6*a*x*(21 + a^2*x^2)*ArcSinh[a*x
] - 9*Sqrt[1 + a^2*x^2]*(7 + a^2*x^2)*ArcSinh[a*x]^2 + 9*a*x*(3 + a^2*x^2)
*ArcSinh[a*x]^3))/(27*a)
```

### 3.330.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {6201, 6187, 6213, 6187, 241, 6199, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^3 \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \ 6201$$

$$-ac\int x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2dx+\frac{2}{3}c\int \text{arcsinh}(ax)^3dx+\frac{1}{3}cx\left(a^2x^2+1\right)\text{arcsinh}(ax)^3$$

$$\Big\downarrow \ 6187$$

$$\frac{2}{3}c\left(x\text{arcsinh}(ax)^3-3a\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx\right)-ac\int x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2dx+\frac{1}{3}cx\left(a^2x^2+1\right)\text{arcsinh}(ax)^3$$

$$\Big\downarrow \ 6213$$

$$\frac{2}{3}c\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\int \text{arcsinh}(ax)dx}{a}\right)\right)-ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\int \left(a^2x^2+1\right)\text{arcsinh}(ax)dx}{3a}\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\text{arcsinh}(ax)^3$$

$$\Big\downarrow \ 6187$$

$$\frac{2}{3}c\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-a\int \frac{x}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)\right)-ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\int \left(a^2x^2+1\right)\text{arcsinh}(ax)dx}{3a}\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\text{arcsinh}(ax)^3$$

$$\Big\downarrow \ 241$$

$$-ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\int\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)dx}{3a}\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3+\frac{2}{3}c\left(x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\operatorname{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\right)$$

$$\Big\downarrow \ 6199$$

$$-ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-a\int\frac{x\left(a^2x^2+3\right)}{3\sqrt{a^2x^2+1}}dx+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3+\frac{2}{3}c\left(x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\operatorname{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\right)$$

$$\Big\downarrow \ 27$$

$$-ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{3}a\int\frac{x\left(a^2x^2+3\right)}{\sqrt{a^2x^2+1}}dx+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3+\frac{2}{3}c\left(x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\operatorname{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\right)$$

$$\Big\downarrow \ 353$$

$$-ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\operatorname{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\frac{a^2x^2+3}{\sqrt{a^2x^2+1}}dx^2+\frac{1}{3}a^2x^3\operatorname{arcsinh}(ax)+x\operatorname{arcsinh}(ax)\right)}{3a}\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3+\frac{2}{3}c\left(x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\operatorname{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\right)$$

$$\Big\downarrow \ 53$$

---

3.330. $\int\left(c+a^2cx^2\right)\operatorname{arcsinh}(ax)^3\,dx$

$$-ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{6}a\int\left(\sqrt{a^2x^2+1}+\frac{2}{\sqrt{a^2x^2+1}}\right)dx^2+\frac{1}{3}a^2x^3\text{arcsinh}(ax)+x\text{arcsinh}(ax)\right)}{3a}\right)$$

$$\frac{1}{3}cx\left(a^2x^2+1\right)\text{arcsinh}(ax)^3+$$
$$\frac{2}{3}c\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\right)$$

$\downarrow$ 2009

$$\frac{1}{3}cx\left(a^2x^2+1\right)\text{arcsinh}(ax)^3+$$
$$\frac{2}{3}c\left(x\text{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\right)-$$
$$ac\left(\frac{\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{1}{3}a^2x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2\left(a^2x^2+1\right)^{3/2}}{3a^2}+\frac{4\sqrt{a^2x^2+1}}{a^2}\right)+x\text{arcsinh}(ax)\right)}{3a}\right)$$

input
```
Int[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

output
```
(c*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/3 - a*c*(((1 + a^2*x^2)^(3/2)*ArcSinh[a
*x]^2)/(3*a^2) - (2*(-1/6*(a*((4*Sqrt[1 + a^2*x^2])/a^2 + (2*(1 + a^2*x^2)
^(3/2))/(3*a^2))) + x*ArcSinh[a*x] + (a^2*x^3*ArcSinh[a*x])/3))/(3*a)) + (
2*c*(x*ArcSinh[a*x]^3 - 3*a*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/a^2 - (2*(
-(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]))/a)))/3
```

### 3.330.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.330. $\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$

### 3.330.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| derivativedivides | $\frac{c\left(9a^3x^3\operatorname{arcsinh}(ax)^3-9a^2x^2\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+27ax\operatorname{arcsinh}(ax)^3+6a^3x^3\operatorname{arcsinh}(ax)-63\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+}\right.}{27a}$ |
| default | $\frac{c\left(9a^3x^3\operatorname{arcsinh}(ax)^3-9a^2x^2\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+27ax\operatorname{arcsinh}(ax)^3+6a^3x^3\operatorname{arcsinh}(ax)-63\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+}\right.}{27a}$ |

input
```
int((a^2*c*x^2+c)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/27/a*c*(9*a^3*x^3*arcsinh(a*x)^3-9*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1
/2)+27*a*x*arcsinh(a*x)^3+6*a^3*x^3*arcsinh(a*x)-63*arcsinh(a*x)^2*(a^2*x^
2+1)^(1/2)-2*a^2*x^2*(a^2*x^2+1)^(1/2)+126*a*x*arcsinh(a*x)-122*(a^2*x^2+1
)^(1/2))
```

### 3.330.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int (c+a^2cx^2)\operatorname{arcsinh}(ax)^3\,dx$$
$$=\frac{9\,(a^3cx^3+3\,acx)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-9\,(a^2cx^2+7\,c)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+6\,(a^3cx^3+}{27\,a}$$

input
```
integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
1/27*(9*(a^3*c*x^3 + 3*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*(a^2*c*x^
2 + 7*c)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*c*x^3 +
 21*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(a^2*c*x^2 + 61*c)*sqrt(a^2*x^
2 + 1))/a
```

### 3.330.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$$
$$= \begin{cases} \frac{a^2cx^3 \operatorname{asinh}^3(ax)}{3} + \frac{2a^2cx^3 \operatorname{asinh}(ax)}{9} - \frac{acx^2\sqrt{a^2x^2+1} \operatorname{asinh}^2(ax)}{3} - \frac{2acx^2\sqrt{a^2x^2+1}}{27} + cx \operatorname{asinh}^3(ax) + \frac{14cx \operatorname{asinh}(ax)}{3} - \frac{7c}{} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)*asinh(a*x)**3,x)
```

output
```
Piecewise((a**2*c*x**3*asinh(a*x)**3/3 + 2*a**2*c*x**3*asinh(a*x)/9 - a*c*
x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/3 - 2*a*c*x**2*sqrt(a**2*x**2 + 1)/
27 + c*x*asinh(a*x)**3 + 14*c*x*asinh(a*x)/3 - 7*c*sqrt(a**2*x**2 + 1)*asi
nh(a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 + 1)/(27*a), Ne(a, 0)), (0, True))
```

### 3.330.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx$$
$$= -\frac{1}{3} \left( \sqrt{a^2x^2 + 1}cx^2 + \frac{7\sqrt{a^2x^2 + 1}c}{a^2} \right) a \operatorname{arsinh}(ax)^2 + \frac{1}{3} (a^2cx^3 + 3\,cx) \operatorname{arsinh}(ax)^3$$
$$- \frac{2}{27} \left( \sqrt{a^2x^2 + 1}cx^2 - \frac{3(a^2cx^3 + 21\,cx) \operatorname{arsinh}(ax)}{a} + \frac{61\sqrt{a^2x^2 + 1}c}{a^2} \right) a$$

input
```
integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
-1/3*(sqrt(a^2*x^2 + 1)*c*x^2 + 7*sqrt(a^2*x^2 + 1)*c/a^2)*a*arcsinh(a*x)^
2 + 1/3*(a^2*c*x^3 + 3*c*x)*arcsinh(a*x)^3 - 2/27*(sqrt(a^2*x^2 + 1)*c*x^2
 - 3*(a^2*c*x^3 + 21*c*x)*arcsinh(a*x)/a + 61*sqrt(a^2*x^2 + 1)*c/a^2)*a
```

### 3.330.8 Giac [F(-2)]

Exception generated.

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.330.9 Mupad [F(-1)]

Timed out.

$$\int (c + a^2cx^2) \operatorname{arcsinh}(ax)^3 \, dx = \int \operatorname{asinh}(a\,x)^3 \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(asinh(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(asinh(a*x)^3*(c + a^2*c*x^2), x)
```

# 3.331 $\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx$



## 3.331.1 Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \frac{2\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac} - \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{3i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{ac} - \frac{6i\,\text{PolyLog}\left(4,-ie^{\text{arcsinh}(ax)}\right)}{ac} + \frac{6i\,\text{PolyLog}\left(4,ie^{\text{arcsinh}(ax)}\right)}{ac}$$

output
```
2*arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c-3*I*arcsinh(a*x)^2*poly
log(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+3*I*arcsinh(a*x)^2*polylog(2,I*(a*x+
(a^2*x^2+1)^(1/2)))/a/c+6*I*arcsinh(a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/
2)))/a/c-6*I*arcsinh(a*x)*polylog(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c-6*I*pol
ylog(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+6*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1
/2)))/a/c
```

### 3.331.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx$$
$$= \frac{-\operatorname{arcsinh}(ax)^3\log\left(1+\frac{ae^{\operatorname{arcsinh}(ax)}}{\sqrt{-a^2}}\right)+\operatorname{arcsinh}(ax)^3\log\left(1+\frac{\sqrt{-a^2}e^{\operatorname{arcsinh}(ax)}}{a}\right)+3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2\right.}{}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

output
```
(-(ArcSinh[a*x]^3*Log[1 + (a*E^ArcSinh[a*x])/Sqrt[-a^2]]) + ArcSinh[a*x]^3
*Log[1 + (Sqrt[-a^2]*E^ArcSinh[a*x])/a] + 3*ArcSinh[a*x]^2*PolyLog[2, (a*E
^ArcSinh[a*x])/Sqrt[-a^2]] - 3*ArcSinh[a*x]^2*PolyLog[2, (Sqrt[-a^2]*E^Arc
Sinh[a*x])/a] - 6*ArcSinh[a*x]*PolyLog[3, (a*E^ArcSinh[a*x])/Sqrt[-a^2]] +
 6*ArcSinh[a*x]*PolyLog[3, (Sqrt[-a^2]*E^ArcSinh[a*x])/a] + 6*PolyLog[4, (
a*E^ArcSinh[a*x])/Sqrt[-a^2]] - 6*PolyLog[4, (Sqrt[-a^2]*E^ArcSinh[a*x])/a
])/(Sqrt[-a^2]*c)
```

### 3.331.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6204, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 6204}$$
$$\frac{\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}d\operatorname{arcsinh}(ax)}{ac}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \operatorname{arcsinh}(ax)^3\csc\left(i\operatorname{arcsinh}(ax)+\frac{\pi}{2}\right)d\operatorname{arcsinh}(ax)}{ac}$$
$$\Big\downarrow \text{ 4668}$$

$$\frac{-3i\int \operatorname{arcsinh}(ax)^2\log\left(1-ie^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)+3i\int \operatorname{arcsinh}(ax)^2\log\left(1+ie^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)+2a}{ac}$$

$\downarrow$ 3011

$$3i\left(2\int \operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)-\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)\right)-3i\left(2\int \operatorname{ar}\right.$$

$\downarrow$ 7163

$$3i\left(2\left(\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)-\int \operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)\right)-\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\right.$$

$\downarrow$ 2720

$$3i\left(2\left(\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)-\int e^{-\operatorname{arcsinh}(ax)}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)de^{\operatorname{arcsinh}(ax)}\right)-\operatorname{arcsinh}(ax\right.$$

$\downarrow$ 7143

$$2\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)+3i\left(2\left(\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)-\operatorname{PolyLog}\left(4,-ie^{\operatorname{arcsinh}(ax)}\right)\right)-\right.$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

output
```
(2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]] + (3*I)*(-(ArcSinh[a*x]^2*PolyLog
[2, (-I)*E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]
] - PolyLog[4, (-I)*E^ArcSinh[a*x]])) - (3*I)*(-(ArcSinh[a*x]^2*PolyLog[2,
 I*E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]] - PolyL
og[4, I*E^ArcSinh[a*x]])))/(a*c)
```

### 3.331.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## 3.331. $\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx$

### 3.331.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(a x\right)^{3}}{a^{2} c \,x^{2}+c}d x$$

input
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

### 3.331.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{3}}{a^{2}cx^{2}+c}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

### 3.331.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{\operatorname{asinh}^{3}\left(ax\right)}{a^{2}x^{2}+1}\,dx}{c}$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(asinh(a*x)**3/(a**2*x**2 + 1), x)/c
```

### 3.331.7 Maxima [F]

$$\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{a^2cx^2+c}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

### 3.331.8 Giac [F]

$$\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{a^2cx^2+c}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

### 3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{\text{asinh}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2), x)
```

# 3.332 $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx$



## 3.332.1 Optimal result

Integrand size = 19, antiderivative size = 294

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx &= \frac{3\operatorname{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\operatorname{arcsinh}(ax)^3}{2c^2(1+a^2x^2)} - \frac{6\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac^2}\\
&\quad + \frac{\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2}\\
&\quad - \frac{3i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^2}\\
&\quad - \frac{3i\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)}{2ac^2}\\
&\quad + \frac{3i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2}\\
&\quad - \frac{3i\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2}\\
&\quad - \frac{3i\operatorname{PolyLog}\left(4,-ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2} + \frac{3i\operatorname{PolyLog}\left(4,ie^{\operatorname{arcsinh}(ax)}\right)}{ac^2}\end{aligned}$$

output
```
1/2*x*arcsinh(a*x)^3/c^2/(a^2*x^2+1)-6*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)
^(1/2))/a/c^2+arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^2+3*I*polyl
og(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3/2*I*arcsinh(a*x)^2*polylog(2,-I*(
a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c
^2+3/2*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3*I*arc
sinh(a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*arcsinh(a*x)*pol
ylog(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(4,-I*(a*x+(a^2*x^2+1)^
(1/2)))/a/c^2+3*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3/2*arcsinh(a
*x)^2/a/c^2/(a^2*x^2+1)^(1/2)
```

### 3.332.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx =$$

$$-\frac{i\Big(7\pi^4+8i\pi^3\operatorname{arcsinh}(ax)+24\pi^2\operatorname{arcsinh}(ax)^2+\frac{192i\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}-32i\pi\operatorname{arcsinh}(ax)^3+\frac{64iax\operatorname{arcsinh}(ax)^3}{1+a^2x^2}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

output
```
((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 + (
(192*I)*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2] - (32*I)*Pi*ArcSinh[a*x]^3 + ((6
4*I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 16*ArcSinh[a*x]^4 - 384*ArcSinh[a
*x]*Log[1 - I/E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[a*x]] + 384
*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E
^ArcSinh[a*x]] - (96*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*A
rcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E
^ArcSinh[a*x]] + (96*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I
)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 64*ArcSinh[a*x]^3*Log[1 + I*E^ArcSinh[a
*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]] - 48*(8 + Pi^2 - (
4*I)*Pi*ArcSinh[a*x] - 4*ArcSinh[a*x]^2)*PolyLog[2, (-I)/E^ArcSinh[a*x]] +
 384*PolyLog[2, I/E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^A
rcSinh[a*x]] - 48*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a
*x]*PolyLog[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*
x]] + 384*ArcSinh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*
PolyLog[3, (-I)*E^ArcSinh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]]
+ 384*PolyLog[4, (-I)/E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]
]))/(a*c^2)
```

### 3.332.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.86, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {6203, 27, 6204, 3042, 4668, 3011, 6213, 6204, 3042, 4668, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx}{2c^2}+\frac{\int \frac{\text{arcsinh}(ax)^3}{c(a^2x^2+1)}dx}{2c}+\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx}{2c^2}+\frac{\int \frac{\text{arcsinh}(ax)^3}{a^2x^2+1}dx}{2c^2}+\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 6204}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx}{2c^2}+\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{2ac^2}+\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx}{2c^2}+\frac{\int \text{arcsinh}(ax)^3\csc\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)}{2ac^2}+\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 4668}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx}{2c^2}+\frac{-3i\int \text{arcsinh}(ax)^2\log\left(1-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+3i\int \text{arcsinh}(ax)^2\log\left(1+ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2a}{2ac^2}$$
$$\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx}{2c^2}+$$
$$3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right)-3i\left(2\int \text{ar}\right.$$
$$\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\downarrow \text{6213}$$

$$-\frac{3a\left(\frac{2\int \frac{\text{arcsinh}(ax)}{a^2x^2+1}dx}{a}-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}\right)}{2c^2}+$$
$$3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right)-3i\left(2\int \text{ar}\right.$$
$$\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\downarrow \text{6204}$$

$$-\frac{3a\left(\frac{2\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{a^2}-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}\right)}{2c^2}+$$
$$3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right)-3i\left(2\int \text{ar}\right.$$
$$\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\downarrow \text{3042}$$

$$-\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\int \text{arcsinh}(ax)\csc\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)}{a^2}\right)}{2c^2}+$$
$$3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right)-3i\left(2\int \text{ar}\right.$$
$$\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

$$\downarrow \text{4668}$$

$$-\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(-i\int \log\left(1-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+i\int \log\left(1+ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2\text{arcsinh}(ax)\arctan\left(e^{\text{arc}}\right.\right.}{a^2}\right.}{2c^2}$$
$$3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right)-3i\left(2\int \text{ar}\right.$$
$$\frac{x\text{arcsinh}(ax)^3}{2c^2(a^2x^2+1)}$$

---

3.332. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx$

$\big\downarrow$ 2715

$$-\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(-i\int e^{-\text{arcsinh}(ax)}\log\left(1-ie^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}+i\int e^{-\text{arcsinh}(ax)}\log\left(1+ie^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}+2\right.}{a^2}\right.}{2c^2}$$

$$\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right)-3i\left(2\int \text{ar}\right.}{}$$

$$\frac{x\text{arcsinh}(ax)^3}{2c^2\left(a^2x^2+1\right)}$$

$\big\downarrow$ 2838

$$\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right)-3i\left(2\int \text{ar}\right.}{}$$

$$\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(2\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)\right)}{a^2}\right)}{2c^2}+$$

$$\frac{x\text{arcsinh}(ax)^3}{2c^2\left(a^2x^2+1\right)}$$

$\big\downarrow$ 7163

$$\frac{3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)-\int \text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\text{arcsinh}(ax)^2\,\text{PolyLog}\right.}{}$$

$$\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(2\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)\right)}{a^2}\right)}{2c^2}+$$

$$\frac{x\text{arcsinh}(ax)^3}{2c^2\left(a^2x^2+1\right)}$$

$\big\downarrow$ 2720

$$\frac{3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)-\int e^{-\text{arcsinh}(ax)}\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}\right)-\text{arcsinh}(ax\right.}{}$$

$$\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(2\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)\right)}{a^2}\right)}{2c^2}+$$

$$\frac{x\text{arcsinh}(ax)^3}{2c^2\left(a^2x^2+1\right)}$$

$\big\downarrow$ 7143

---

3.332. $\int \frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$

$$-\frac{3a\left(-\frac{\operatorname{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(2\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\operatorname{arcsinh}(ax)}\right)\right)}{a^2}\right)}{2c^2}+\frac{x\operatorname{arcsinh}(ax)^3}{2c^2\left(a^2x^2+1\right)}+$$

$$\frac{2\operatorname{arcsinh}(ax)^3\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)+3i\left(2\left(\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)-\text{PolyLog}\left(4,-ie^{\operatorname{arcsinh}(ax)}\right)\right)-\right.}{}$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcSinh[a*x]^3)/(2*c^2*(1 + a^2*x^2)) - (3*a*(-(ArcSinh[a*x]^2/(a^2*Sqr
t[1 + a^2*x^2])) + (2*(2*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]] - I*PolyLog[2
, (-I)*E^ArcSinh[a*x]] + I*PolyLog[2, I*E^ArcSinh[a*x]]))/a^2))/(2*c^2) +
(2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]] + (3*I)*(-(ArcSinh[a*x]^2*PolyLog
[2, (-I)*E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]
] - PolyLog[4, (-I)*E^ArcSinh[a*x]])) - (3*I)*(-(ArcSinh[a*x]^2*PolyLog[2,
 I*E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]] - PolyL
og[4, I*E^ArcSinh[a*x]])))/(2*a*c^2)
```

### 3.332.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.332. $\int \frac{\operatorname{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.332. $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.332.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2c\,x^2+c)^2}dx$$

input
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

### 3.332.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

### 3.332.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{asinh}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(asinh(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

### 3.332.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

### 3.332.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

### 3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^2}\,dx=\int \frac{\mathrm{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^2,x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^2, x)
```

# 3.333 $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3} dx$



## 3.333.1 Optimal result

Integrand size = 19, antiderivative size = 409

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3} dx = &-\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\text{arcsinh}(ax)}{4c^3(1+a^2x^2)} + \frac{\text{arcsinh}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} \\ &+ \frac{9\text{arcsinh}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\text{arcsinh}(ax)^3}{4c^3(1+a^2x^2)^2} \\ &+ \frac{3x\text{arcsinh}(ax)^3}{8c^3(1+a^2x^2)} - \frac{5\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)}{ac^3} \\ &+ \frac{3\text{arcsinh}(ax)^3\arctan\left(e^{\text{arcsinh}(ax)}\right)}{4ac^3} + \frac{5i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{2ac^3} \\ &- \frac{9i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)}{8ac^3} \\ &- \frac{5i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{2ac^3} + \frac{9i\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)}{8ac^3} \\ &+ \frac{9i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)}{4ac^3} \\ &- \frac{9i\text{arcsinh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(ax)}\right)}{4ac^3} \\ &- \frac{9i\,\text{PolyLog}\left(4,-ie^{\text{arcsinh}(ax)}\right)}{4ac^3} + \frac{9i\,\text{PolyLog}\left(4,ie^{\text{arcsinh}(ax)}\right)}{4ac^3}\end{aligned}$$

output
```
-1/4*x*arcsinh(a*x)/c^3/(a^2*x^2+1)+1/4*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(
3/2)+1/4*x*arcsinh(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arcsinh(a*x)^3/c^3/(a^2*
x^2+1)-5*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3+3/4*arcsinh(a*x)
^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3+9/4*I*polylog(4,I*(a*x+(a^2*x^2+1)^
(1/2)))/a/c^3-9/8*I*arcsinh(a*x)^2*polylog(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a
/c^3+5/2*I*polylog(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+9/4*I*arcsinh(a*x)*
polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-5/2*I*polylog(2,I*(a*x+(a^2*x^
2+1)^(1/2)))/a/c^3+9/8*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)
))/a/c^3-9/4*I*arcsinh(a*x)*polylog(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-9/4
*I*polylog(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-1/4/a/c^3/(a^2*x^2+1)^(1/2)
+9/8*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(1/2)
```

## 3.333.2 Mathematica [A] (verified)

Time = 3.74 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.60

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx =$$

$$-\frac{i\left(21\pi^4 - \frac{128i}{\sqrt{1+a^2x^2}} + 24i\pi^3\text{arcsinh}(ax) - \frac{128iax\text{arcsinh}(ax)}{1+a^2x^2} + 72\pi^2\text{arcsinh}(ax)^2 + \frac{128i\text{arcsinh}(ax)^2}{(1+a^2x^2)^{3/2}} + \frac{576ia}{\sqrt{}}\right.}{}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

output
```
((-1/512*I)*(21*Pi^4 - (128*I)/Sqrt[1 + a^2*x^2] + (24*I)*Pi^3*ArcSinh[a*x
] - ((128*I)*a*x*ArcSinh[a*x])/(1 + a^2*x^2) + 72*Pi^2*ArcSinh[a*x]^2 + ((
128*I)*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + ((576*I)*ArcSinh[a*x]^2)/Sqrt
[1 + a^2*x^2] - (96*I)*Pi*ArcSinh[a*x]^3 + ((128*I)*a*x*ArcSinh[a*x]^3)/(1
 + a^2*x^2)^2 + ((192*I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 48*ArcSinh[a*
x]^4 - 1280*ArcSinh[a*x]*Log[1 - I/E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[1 + I
/E^ArcSinh[a*x]] + 1280*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 144*Pi^2*
ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] - (288*I)*Pi*ArcSinh[a*x]^2*Log[1 +
 I/E^ArcSinh[a*x]] - 192*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 144*Pi
^2*ArcSinh[a*x]*Log[1 - I*E^ArcSinh[a*x]] + (288*I)*Pi*ArcSinh[a*x]^2*Log[
1 - I*E^ArcSinh[a*x]] - (24*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 192*ArcSin
h[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSi
nh[a*x])/4]] - 16*(80 + 9*Pi^2 - (36*I)*Pi*ArcSinh[a*x] - 36*ArcSinh[a*x]^
2)*PolyLog[2, (-I)/E^ArcSinh[a*x]] + 1280*PolyLog[2, I/E^ArcSinh[a*x]] + 5
76*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 144*Pi^2*PolyLog[2, I*
E^ArcSinh[a*x]] + (576*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*x]] + (
576*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 1152*ArcSinh[a*x]*PolyLog[3, (
-I)/E^ArcSinh[a*x]] - 1152*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]] -
(576*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 1152*PolyLog[4, (-I)/E^ArcSinh[a
*x]] + 1152*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c^3)
```

### 3.333.3 Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.947, Rules used = {6203, 27, 6203, 6204, 3042, 4668, 3011, 6213, 6203, 241, 6204, 3042, 4668, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{5/2}}dx}{4c^3}+\frac{3\int \frac{\text{arcsinh}(ax)^3}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\text{arcsinh}(ax)^3}{4c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{5/2}}dx}{4c^3}+\frac{3\int \frac{\text{arcsinh}(ax)^3}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{5/2}}dx}{4c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx+\frac{1}{2}\int \frac{\text{arcsinh}(ax)^3}{a^2x^2+1}dx+\frac{x\text{arcsinh}(ax)^3}{2(a^2x^2+1)}\right)}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 6204}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{5/2}}dx}{4c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx+\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{2a}+\frac{x\text{arcsinh}(ax)^3}{2(a^2x^2+1)}\right)}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{5/2}}dx}{4c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx+\frac{\int \text{arcsinh}(ax)^3\csc\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)}{2a}+\frac{x\text{arcsinh}(ax)^3}{2(a^2x^2+1)}\right)}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{3\left(-\frac{3}{2}a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx+\frac{-3i\int \text{arcsinh}(ax)^2\log\left(1-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+3i\int \text{arcsinh}(ax)^2\log\left(1+ie^{\text{arcsinh}(ax)}\right)d\text{arcs}}{2a}\right.}{4c^3}$$

$$\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{5/2}}dx}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 3011}$$

---

3.333. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}dx$

$$3\left(-\frac{3}{2}a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{3/2}}dx+\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)\right.}{}\right.$$

$$\frac{3a\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^{5/2}}dx}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 6213

$$3\left(-\frac{3}{2}a\left(\frac{2\int \frac{\text{arcsinh}(ax)}{a^2x^2+1}dx}{a}-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}\right)+\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyL}\right.}{}\right.$$

$$\frac{3a\left(\frac{2\int \frac{\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^2}dx}{3a}-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}\right)}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 6203

$$3\left(-\frac{3}{2}a\left(\frac{2\int \frac{\text{arcsinh}(ax)}{a^2x^2+1}dx}{a}-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}\right)+\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyL}\right.}{}\right.$$

$$\frac{3a\left(\frac{2\left(\frac{1}{2}\int \frac{\text{arcsinh}(ax)}{a^2x^2+1}dx-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^{3/2}}dx+\frac{x\text{arcsinh}(ax)}{2\left(a^2x^2+1\right)}\right)}{3a}-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}\right)}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 241

$$3\left(-\frac{3}{2}a\left(\frac{2\int \frac{\text{arcsinh}(ax)}{a^2x^2+1}dx}{a}-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}\right)+\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyL}\right.}{}\right.$$

$$\frac{3a\left(\frac{2\left(\frac{1}{2}\int \frac{\text{arcsinh}(ax)}{a^2x^2+1}dx+\frac{x\text{arcsinh}(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{2a\sqrt{a^2x^2+1}}\right)}{3a}-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}\right)}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 6204

3.333. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(-\frac{3}{2}a\left(\frac{2\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{a^2}-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}\right)+\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsin}\right.}{}\right.}{}$$

$$\frac{3a\left(\frac{2\left(\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{2a}+\frac{x\text{arcsinh}(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{2a\sqrt{a^2x^2+1}}\right)}{3a}-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}\right)}{4c^3}+\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 3042

$$\frac{3\left(-\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\int \text{arcsinh}(ax)\csc\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)}{a^2}\right)+\frac{3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right.\right.}{}\right.}{}$$

$$\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}+\frac{2\left(\frac{\int \text{arcsinh}(ax)\csc\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)}{2a}+\frac{x\text{arcsinh}(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{2a\sqrt{a^2x^2+1}}\right)}{3a}\right)}{4c^3}+$$

$$\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 4668

$$\frac{3\left(-\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(-i\int \log\left(1-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+i\int \log\left(1+ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2\text{arcsinh}(ax)\arctan\left(\right.\right.}{a^2}\right.\right.}{}$$

$$\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}+\frac{2\left(\frac{-i\int \log\left(1-ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+i\int \log\left(1+ie^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(a}\right.}{2a}\right.}{3a}\right.}{4c^3}$$

$$\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 2715

3.333. $\int \frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx$

$$\frac{3\left(-\frac{3}{2}a\left(-\frac{\operatorname{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(-i\int e^{-\operatorname{arcsinh}(ax)}\log\left(1-ie^{\operatorname{arcsinh}(ax)}\right)de^{\operatorname{arcsinh}(ax)}+i\int e^{-\operatorname{arcsinh}(ax)}\log\left(1+ie^{\operatorname{arcsinh}(ax)}\right)de^{\operatorname{arcsinh}(ax)}\right.}{a^2}\right.\right.}{3a\left(-\frac{\operatorname{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}+\frac{2\left(\frac{-i\int e^{-\operatorname{arcsinh}(ax)}\log\left(1-ie^{\operatorname{arcsinh}(ax)}\right)de^{\operatorname{arcsinh}(ax)}+i\int e^{-\operatorname{arcsinh}(ax)}\log\left(1+ie^{\operatorname{arcsinh}(ax)}\right)de^{\operatorname{arcsinh}(ax)}+2\operatorname{arcs}}{2a}\right.}{3a}\right.}$$

$$4c^3$$

$$\frac{x\operatorname{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 2838

$$\frac{3\left(\frac{3i\left(2\int\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)-\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)\right)-3i\left(2\int\operatorname{arcsinh}(ax)\operatorname{PolyLog}\right.}{2a}\right.}{3a\left(-\frac{\operatorname{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}+\frac{2\left(\frac{x\operatorname{arcsinh}(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{2a\sqrt{a^2x^2+1}}+\frac{2\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)-i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)+i\operatorname{PolyLog}\left(2,ie^{\operatorname{arc}}\right.}{2a}\right.}{3a}\right.}$$

$$4c^3$$

$$\frac{x\operatorname{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 7163

$$\frac{3\left(\frac{3i\left(2\left(\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)-\int\operatorname{PolyLog}\left(3,-ie^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)\right)-\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)\right.}{}\right.}{3a\left(-\frac{\operatorname{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}+\frac{2\left(\frac{x\operatorname{arcsinh}(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{2a\sqrt{a^2x^2+1}}+\frac{2\operatorname{arcsinh}(ax)\arctan\left(e^{\operatorname{arcsinh}(ax)}\right)-i\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(ax)}\right)+i\operatorname{PolyLog}\left(2,ie^{\operatorname{arc}}\right.}{2a}\right.}{3a}\right.}$$

$$4c^3$$

$$\frac{x\operatorname{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 2720

3.333. $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx$

$$3\left(\frac{3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)-\int e^{-\text{arcsinh}(ax)}\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}\right)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-ie\right.\right.}{}\right.$$

$$\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}+\frac{2\left(\frac{x\text{arcsinh}(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{2a\sqrt{a^2x^2+1}}+\frac{2\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{arc}}\right.}{2a}\right.}{3a}\right.}{4c^3}$$

$$\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 7143

$$-\frac{3a\left(-\frac{\text{arcsinh}(ax)^2}{3a^2(a^2x^2+1)^{3/2}}+\frac{2\left(\frac{x\text{arcsinh}(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{2a\sqrt{a^2x^2+1}}+\frac{2\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{a}}\right.}{2a}\right.}{3a}\right.}{4c^3}$$

$$\frac{3\left(-\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{a^2\sqrt{a^2x^2+1}}+\frac{2\left(2\text{arcsinh}(ax)\arctan\left(e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(ax)}\right)\right)}{a^2}\right)+\frac{x\text{ar}}{2}\right.}{}$$

$$\frac{x\text{arcsinh}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

output
```
(x*ArcSinh[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) - (3*a*(-1/3*ArcSinh[a*x]^2/(a^
2*(1 + a^2*x^2)^(3/2)) + (2*(1/(2*a*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x])/
(2*(1 + a^2*x^2)) + (2*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]] - I*PolyLog[2,
(-I)*E^ArcSinh[a*x]] + I*PolyLog[2, I*E^ArcSinh[a*x]])/(2*a)))/(3*a)))/(4*
c^3) + (3*((x*ArcSinh[a*x]^3)/(2*(1 + a^2*x^2)) - (3*a*(-(ArcSinh[a*x]^2/(
a^2*Sqrt[1 + a^2*x^2])) + (2*(2*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]] - I*Po
lyLog[2, (-I)*E^ArcSinh[a*x]] + I*PolyLog[2, I*E^ArcSinh[a*x]]))/a^2))/2 +
 (2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]] + (3*I)*(-(ArcSinh[a*x]^2*PolyLo
g[2, (-I)*E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x
]] - PolyLog[4, (-I)*E^ArcSinh[a*x]])) - (3*I)*(-(ArcSinh[a*x]^2*PolyLog[2
, I*E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]] - Poly
Log[4, I*E^ArcSinh[a*x]])))/(2*a)))/(4*c^3)
```

## 3.333. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx$

### 3.333.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

## 3.333. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.333.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2cx^2+c)^3}dx$$

input
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

output
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

### 3.333.5 Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3} dx = \int \frac{\text{arsinh}(ax)^3}{(a^2cx^2+c)^3} dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3
), x)
```

### 3.333.6 Sympy [F]

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3} dx = \frac{\int \frac{\text{asinh}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} dx}{c^3}$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(asinh(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**
3
```

### 3.333.7 Maxima [F]

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3} dx = \int \frac{\text{arsinh}(ax)^3}{(a^2cx^2+c)^3} dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

### 3.333.8 Giac [F]

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{\operatorname{arsinh}{(ax)}^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

### 3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{\text{asinh}(a\,x)^3}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^3,x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^3, x)
```

# 3.334 $\int (c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx$



## 3.334.1 Optimal result

Integrand size = 21, antiderivative size = 509

$$\int (c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx = -\frac{865ac^2x^2\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}} - \frac{65a^3c^2x^4\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}$$
$$-\frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}}{216a} + \frac{245}{384}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)$$
$$+\frac{65}{576}c^2x(1+a^2x^2)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{1}{36}c^2x(1+a^2x^2)^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)$$
$$-\frac{115c^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{768a\sqrt{1+a^2x^2}} - \frac{15ac^2x^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32\sqrt{1+a^2x^2}}$$
$$-\frac{5c^2(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{32a} - \frac{c^2(1+a^2x^2)^{5/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{12a}$$
$$+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3 + \frac{5}{24}cx(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^3 + \frac{1}{6}x(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^3 + \frac{5c^2\sqrt{c+}}{6}$$

output
```
5/24*c*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+1/6*x*(a^2*c*x^2+c)^(5/2)*arcs
inh(a*x)^3-1/216*c^2*(a^2*x^2+1)^(5/2)*(a^2*c*x^2+c)^(1/2)/a+245/384*c^2*x
*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+65/576*c^2*x*(a^2*x^2+1)*arcsinh(a*x)*(a
^2*c*x^2+c)^(1/2)+1/36*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2
)-5/32*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-1/12*c^2
*(a^2*x^2+1)^(5/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+5/16*c^2*x*arcsinh
(a*x)^3*(a^2*c*x^2+c)^(1/2)-865/2304*a*c^2*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^
2+1)^(1/2)-65/2304*a^3*c^2*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-115/7
68*c^2*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/32*a*c^2*
x^2*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+5/64*c^2*arcsinh(
a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

### 3.334.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.35

$$\int (c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx = \frac{c^2\sqrt{c+a^2cx^2}(4320\operatorname{arcsinh}(ax)^4 - 9720\cosh(2\operatorname{arcsinh}(ax)) - 243\cosh(4\operatorname{arcsi}}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(4320*ArcSinh[a*x]^4 - 9720*Cosh[2*ArcSinh[a*x]]
- 243*Cosh[4*ArcSinh[a*x]] - 8*Cosh[6*ArcSinh[a*x]] - 72*ArcSinh[a*x]^2*(2
70*Cosh[2*ArcSinh[a*x]] + 27*Cosh[4*ArcSinh[a*x]] + 2*Cosh[6*ArcSinh[a*x]]
) + 288*ArcSinh[a*x]^3*(45*Sinh[2*ArcSinh[a*x]] + 9*Sinh[4*ArcSinh[a*x]] +
 Sinh[6*ArcSinh[a*x]]) + 12*ArcSinh[a*x]*(1620*Sinh[2*ArcSinh[a*x]] + 81*S
inh[4*ArcSinh[a*x]] + 4*Sinh[6*ArcSinh[a*x]])))/(55296*a*Sqrt[1 + a^2*x^2]
)
```

### 3.334.3 Rubi [A] (verified)

Time = 4.20 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.16, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.048, Rules used = {6201, 6201, 6200, 6191, 6198, 6213, 6201, 241, 244, 2009, 6200, 15, 6198, 6201, 244, 2009, 6200, 15, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^3 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\text{arcsinh}(ax)^3dx+\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \text{6201}$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{5}{6}c\left(-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\text{arcsinh}(ax)^3dx+\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2\right.\right.$$
$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{5}{6}c\left(-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \text{a}}{2\sqrt{a^2x}}\right.\right.$$
$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{5}{6}c\left(-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}\right.}{2\sqrt{a^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \; 6198$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \; 6213$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\int\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)dx}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \; 6201$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\int\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)dx-\frac{1}{6}a\int x\left(a^2x^2+1\right)^2dx+\frac{1}{6}x\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \; 241$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\int\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)dx+\frac{1}{6}x\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)-\frac{\left(a^2x^2+1\right)^3}{36a}}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \; 244$$

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3.334. $\int\left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\int(a^2x^2+1)^{3/2}\text{arcsinh}(ax)dx+\frac{1}{6}x(a^2x^2+1)^{5/2}\text{arcsinh}(ax)-\frac{(a^2x^2+1)^3}{36a}}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\int(a^2x^2+1)^{3/2}\text{arcsinh}(ax)dx+\frac{1}{6}x(a^2x^2+1)^{5/2}\text{arcsinh}(ax)-\frac{(a^2x^2+1)^3}{36a}}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\int(a^2x^2+1)^{3/2}\text{arcsinh}(ax)dx+\frac{1}{6}x(a^2x^2+1)^{5/2}\text{arcsinh}(ax)-\frac{(a^2x^2+1)^3}{36a}}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow \text{15}$$

3.334. $\int\left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\int (a^2x^2+1)^{3/2}\text{arcsinh}(ax)dx+\frac{1}{6}x(a^2x^2+1)^{5/2}\text{arcsinh}(ax)-\frac{(a^2x^2+1)^3}{36a}}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$\big\downarrow$ 6198

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\int (a^2x^2+1)^{3/2}\text{arcsinh}(ax)dx+\frac{1}{6}x(a^2x^2+1)^{5/2}\text{arcsinh}(ax)-\frac{(a^2x^2+1)^3}{36a}}{3a}\right)}{2\sqrt{a^2x^2+1}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$\big\downarrow$ 6201

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\left(\frac{3}{4}\int \sqrt{a^2x^2+1}\text{arcsinh}(ax)dx-\frac{1}{4}a\int x(a^2x^2+1)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)\right)+\frac{1}{6}}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$\big\downarrow$ 244

3.334. $\int \left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\left(\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx-\frac{1}{4}a\int(a^2x^3+x)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)\right)+\frac{1}{6}x}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow\ 2009$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\left(\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)\right)+\frac{1}{6}x(a}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow\ 6200$$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx-\frac{a\int xdx}{2}+\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)\right)+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsi}}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$$\Big\downarrow\ 15$$

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3.334. $\int\left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx$

$$-\frac{ac^2\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx+\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)-\frac{ax^2}{4}\right)+\frac{1}{4}x\left(a^2x^2+1\right)^{3/2}\text{arcsinh}}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$\downarrow$ 6198

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.\right.$$

$$\frac{ac^2\left(\frac{\left(a^2x^2+1\right)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{1}{6}x\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)+\frac{5}{6}\left(\frac{1}{4}x\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}}{4a}\right.\right.}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$\downarrow$ 6227

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(}{8a}\right.\right.$$

$$\frac{ac^2\left(\frac{\left(a^2x^2+1\right)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{1}{6}x\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)+\frac{5}{6}\left(\frac{1}{4}x\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}}{4a}\right.\right.}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$\downarrow$ 15

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3.334. $\int\left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax}{8a\sqrt{a}}\right.\right.$$

$$\frac{ac^2\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{1}{6}x(a^2x^2+1)^{5/2}\text{arcsinh}(ax)+\frac{5}{6}\left(\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}}{4a}\right.\right.}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}$$

$\downarrow$ 6198

$$-\frac{ac^2\left(\frac{(a^2x^2+1)^3\text{arcsinh}(ax)^2}{6a^2}-\frac{\frac{1}{6}x(a^2x^2+1)^{5/2}\text{arcsinh}(ax)+\frac{5}{6}\left(\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsi}}{}\right.\right.}{3a}\right.}{2\sqrt{a^2x^2+1}}$$

$$\frac{1}{6}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{5/2}+$$

$$\frac{5}{6}c\left(\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(\right.}{2a}\right.}{4\sqrt{a^2x^2+1}}\right.$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

3.334. $\int\left(c+a^2cx^2\right)^{5/2}\text{arcsinh}(ax)^3\,dx$

output
```
(x*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3)/6 - (a*c^2*Sqrt[c + a^2*c*x^2]*((
(1 + a^2*x^2)^3*ArcSinh[a*x]^2)/(6*a^2) - (-1/36*(1 + a^2*x^2)^3/a + (x*(1
 + a^2*x^2)^(5/2)*ArcSinh[a*x])/6 + (5*(-1/4*(a*(x^2/2 + (a^2*x^4)/4)) + (
x*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])/4 + (3*(-1/4*(a*x^2) + (x*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x])/2 + ArcSinh[a*x]^2/(4*a)))/4))/6)/(3*a)))/(2*Sqrt[1 +
a^2*x^2]) + (5*c*((x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/4 + (3*c*((x*Sq
rt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)
/(8*a*Sqrt[1 + a^2*x^2]) - (3*a*Sqrt[c + a^2*c*x^2]*((x^2*ArcSinh[a*x]^2)/
2 - a*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a
*x]^2/(4*a^3))))/(2*Sqrt[1 + a^2*x^2])))/4 - (3*a*c*Sqrt[c + a^2*c*x^2]*((
(1 + a^2*x^2)^2*ArcSinh[a*x]^2)/(4*a^2) - (-1/4*(a*(x^2/2 + (a^2*x^4)/4))
+ (x*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])/4 + (3*(-1/4*(a*x^2) + (x*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x])/2 + ArcSinh[a*x]^2/(4*a)))/4)/(2*a)))/(4*Sqrt[1 + a
^2*x^2])))/6
```

### 3.334.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.334. $\int (c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx$

### 3.334.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.58

| method | result |
|---|---|
| default | $\frac{5\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)^4c^2}{64\sqrt{a^2x^2+1}\,a}+\frac{\sqrt{c(a^2x^2+1)}\,\left(32a^7x^7+32x^6a^6\sqrt{a^2x^2+1}+64a^5x^5+48a^4x^4\sqrt{a^2x^2+1}+38a^3x^3+18a^2x^2\sqrt{a^2x^2+1}\right.}{13824a(a^2x^2+1)}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
5/64*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c^2+1/13824*
(c*(a^2*x^2+1))^(1/2)*(32*a^7*x^7+32*x^6*a^6*(a^2*x^2+1)^(1/2)+64*a^5*x^5+
48*a^4*x^4*(a^2*x^2+1)^(1/2)+38*a^3*x^3+18*a^2*x^2*(a^2*x^2+1)^(1/2)+6*a*x
+(a^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3-18*arcsinh(a*x)^2+6*arcsinh(a*x)-1)
*c^2/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5+8*a^4*x^4*(a^2*
x^2+1)^(1/2)+12*a^3*x^3+8*a^2*x^2*(a^2*x^2+1)^(1/2)+4*a*x+(a^2*x^2+1)^(1/2
))*(32*arcsinh(a*x)^3-24*arcsinh(a*x)^2+12*arcsinh(a*x)-3)*c^2/a/(a^2*x^2+
1)+15/512*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a
*x+(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)
*c^2/a/(a^2*x^2+1)+15/512*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*
x^2+1)^(1/2)+2*a*x-(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6
*arcsinh(a*x)+3)*c^2/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5
-8*a^4*x^4*(a^2*x^2+1)^(1/2)+12*a^3*x^3-8*a^2*x^2*(a^2*x^2+1)^(1/2)+4*a*x-
(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3+24*arcsinh(a*x)^2+12*arcsinh(a*x)+3)
*c^2/a/(a^2*x^2+1)+1/13824*(c*(a^2*x^2+1))^(1/2)*(32*a^7*x^7-32*x^6*a^6*(a
^2*x^2+1)^(1/2)+64*a^5*x^5-48*a^4*x^4*(a^2*x^2+1)^(1/2)+38*a^3*x^3-18*a^2*
x^2*(a^2*x^2+1)^(1/2)+6*a*x-(a^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3+18*arcsi
nh(a*x)^2+6*arcsinh(a*x)+1)*c^2/a/(a^2*x^2+1)
```

### 3.334.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arcsinh(a
*x)^3, x)
```

### 3.334.6 Sympy [F(-1)]

Timed out.

$$\int \left(c + a^2 c x^2\right)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*asinh(a*x)**3,x)
```

output
```
Timed out
```

### 3.334.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c + a^2 c x^2\right)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.334.8 Giac [F(-2)]

Exception generated.

$$\int \left(c + a^2 c x^2\right)^{5/2} \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.334.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\operatorname{arcsinh}(ax)^3\,dx=\int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input

```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output

```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

# 3.335 $\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^3\,dx$



## 3.335.1 Optimal result

Integrand size = 21, antiderivative size = 348

$$\begin{aligned}\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^3\,dx = &-\frac{51acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}-\frac{3a^3cx^4\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}\\&+\frac{45}{64}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)+\frac{3}{32}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)\\&-\frac{27c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{128a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16\sqrt{1+a^2x^2}}\\&-\frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^3\\&+\frac{1}{4}x(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^3+\frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^4}{32a\sqrt{1+a^2x^2}}\end{aligned}$$

output
```
1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+45/64*c*x*arcsinh(a*x)*(a^2*c*x^2
+c)^(1/2)+3/32*c*x*(a^2*x^2+1)*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)-3/16*c*(a^
2*x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^3
*(a^2*c*x^2+c)^(1/2)-51/128*a*c*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-
3/128*a^3*c*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-27/128*c*arcsinh(a*x
)^2*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/16*a*c*x^2*arcsinh(a*x)^2*(a
^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/32*c*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1
/2)/a/(a^2*x^2+1)^(1/2)
```

### 3.335.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.39

$$\int (c + a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \frac{c\sqrt{c+a^2cx^2}(96\operatorname{arcsinh}(ax)^4 - 24\operatorname{arcsinh}(ax)^2(16\cosh(2\operatorname{arcsinh}(ax)) + \cosh(}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(96*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*(16*Cosh[2*A
rcSinh[a*x]] + Cosh[4*ArcSinh[a*x]]) - 3*(64*Cosh[2*ArcSinh[a*x]] + Cosh[4
*ArcSinh[a*x]]) + 32*ArcSinh[a*x]^3*(8*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSi
nh[a*x]]) + 12*ArcSinh[a*x]*(32*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]
])))/(1024*a*Sqrt[1 + a^2*x^2])
```

### 3.335.3 Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6201, 6200, 6191, 6198, 6213, 6201, 244, 2009, 6200, 15, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \; 6201$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\operatorname{arcsinh}(ax)^3dx+\frac{1}{4}x\operatorname{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 6200$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}\right)+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\right.$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6213}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\int\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)dx}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6201}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx-\frac{1}{4}a\int x\left(a^2x^2+1\right)dx+\frac{1}{4}x\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

---

3.335. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^3\,dx$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx-\frac{1}{4}a\int(a^2x^3+x)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6200}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx-\frac{a\int xdx}{2}+\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)\right)+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(a}{2a}\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx+\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)-\frac{ax^2}{4}\right)+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)-}{2a}\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

---

3.335. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^3\,dx$

$\downarrow$ 6198

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-$$

$$\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$\downarrow$ 6227

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4}{8a\sqrt{a^2x}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-$$

$$\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$\downarrow$ 15

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a}}{8a\sqrt{a^2x^2-}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-$$

$$\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$\downarrow$ 6198

3.335. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^3\,dx$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}-\frac{3a\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+}}{}\right.\right.}{2\sqrt{a^2x^2+1}}\right.$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

output
```
(x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/4 + (3*c*((x*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)/(8*a*Sqrt[1 + a^2
*x^2]) - (3*a*Sqrt[c + a^2*c*x^2]*((x^2*ArcSinh[a*x]^2)/2 - a*(-1/4*x^2/a
+ (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4*a^3))))/(
2*Sqrt[1 + a^2*x^2])))/4 - (3*a*c*Sqrt[c + a^2*c*x^2]*(((1 + a^2*x^2)^2*Ar
cSinh[a*x]^2)/(4*a^2) - (-1/4*(a*(x^2/2 + (a^2*x^4)/4)) + (x*(1 + a^2*x^2)
^(3/2)*ArcSinh[a*x])/4 + (3*(-1/4*(a*x^2) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a
*x])/2 + ArcSinh[a*x]^2/(4*a)))/4)/(2*a)))/(4*Sqrt[1 + a^2*x^2])
```

### 3.335.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.335. $\int (c + a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx$

### 3.335.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| default | $\frac{3\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)^4 c}{32\sqrt{a^2x^2+1}\,a}+\frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5+8a^4x^4\sqrt{a^2x^2+1}+12a^3x^3+8a^2x^2\sqrt{a^2x^2+1}+4ax+\sqrt{a^2x^2+1}\right)\left(32\operatorname{arcsinh}(a\right.}{2048(a^2x^2+1)a}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c+1/2048*(c*
(a^2*x^2+1))^(1/2)*(8*a^5*x^5+8*a^4*x^4*(a^2*x^2+1)^(1/2)+12*a^3*x^3+8*a^2
*x^2*(a^2*x^2+1)^(1/2)+4*a*x+(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3-24*arcs
inh(a*x)^2+12*arcsinh(a*x)-3)*c/(a^2*x^2+1)/a+1/32*(c*(a^2*x^2+1))^(1/2)*(
2*a^3*x^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*x^2+1)^(1/2))*(4*arcsinh(
a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c/(a^2*x^2+1)/a+1/32*(c*(a^2*x^2
+1))^(1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x-(a^2*x^2+1)^(1/2))
*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)*c/(a^2*x^2+1)/a+1/20
48*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+12*a^3*x^3
-8*a^2*x^2*(a^2*x^2+1)^(1/2)+4*a*x-(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3+2
4*arcsinh(a*x)^2+12*arcsinh(a*x)+3)*c/(a^2*x^2+1)/a
```

### 3.335.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^3, x)
```

### 3.335.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{asinh}^3(ax) \, dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**3, x)
```

### 3.335.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.335.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.335.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2} \, dx$$

input
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

# 3.336 $\int \sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)^3\,dx$



## 3.336.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)^3\,dx &= -\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}}+\frac{3}{4}x\sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)\\ &\quad-\frac{3\sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)^2}{8a\sqrt{1+a^2x^2}}-\frac{3ax^2\sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)^2}{4\sqrt{1+a^2x^2}}\\ &\quad+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)^3+\frac{\sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)^4}{8a\sqrt{1+a^2x^2}}\end{aligned}$$

output
```
3/4*x*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+1/2*x*arcsinh(a*x)^3*(a^2*c*x^2+c)^
(1/2)-3/8*a*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/8*arcsinh(a*x)^2*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/4*a*x^2*arcsinh(a*x)^2*(a^2*c*x^2
+c)^(1/2)/(a^2*x^2+1)^(1/2)+1/8*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(a^2*
x^2+1)^(1/2)
```

## 3.336.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$\int \sqrt{c+a^2cx^2}\,\mathrm{arcsinh}(ax)^3\,dx$$

$$=\frac{\sqrt{c\left(1+a^2x^2\right)}\left(-3(1+2\mathrm{arcsinh}(ax)^2)\cosh(2\mathrm{arcsinh}(ax))+2\mathrm{arcsinh}(ax)\left(\mathrm{arcsinh}(ax)^3+(3+2\mathrm{arcsinh}(\right.\right.}{16a\sqrt{1+a^2x^2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(-3*(1 + 2*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]] + 2
*ArcSinh[a*x]*(ArcSinh[a*x]^3 + (3 + 2*ArcSinh[a*x]^2)*Sinh[2*ArcSinh[a*x]
])))/(16*a*Sqrt[1 + a^2*x^2])
```

## 3.336.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6200, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{6200}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{6227}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$$\downarrow 15$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$$\downarrow 6198$$

$$\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}-\frac{3a\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)\sqrt{a^2cx^2+c}}{2\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x]^4)/(8*a*Sqrt[1 + a^2*x^2]) - (3*a*Sqrt[c + a^2*c*x^2]*((x^2*ArcSinh[a*x
]^2)/2 - a*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcS
inh[a*x]^2/(4*a^3))))/(2*Sqrt[1 + a^2*x^2])
```

### 3.336.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.336. $\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx$

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.336.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| default | $\frac{\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(ax)^4}{8\sqrt{a^2x^2+1}\,a}+\frac{\sqrt{c(a^2x^2+1)}\left(2a^3x^3+2a^2x^2\sqrt{a^2x^2+1}+2ax+\sqrt{a^2x^2+1}\right)\left(4\operatorname{arcsinh}(ax)^3-6\operatorname{arcsinh}(ax)^2+6\operatorname{arcsinh}\right.}{32(a^2x^2+1)a}$ |

input
```
int(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4+1/32*(c*(a^2*
x^2+1))^(1/2)*(2*a^3*x^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*x^2+1)^(1/
2))*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)/(a^2*x^2+1)/a+1/3
2*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x-(a^2*
x^2+1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)/(a^2*x^
2+1)/a
```

### 3.336.5 Fricas [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx = \int \sqrt{a^2cx^2+c}\,\text{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3, x)
```

### 3.336.6 Sympy [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\,\text{asinh}^3\left(ax\right)dx$$

input
```
integrate(asinh(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**3, x)
```

### 3.336.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.336.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.336.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx=\int \text{asinh}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

# 3.337 $\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$



## 3.337.1 Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^4}{4a\sqrt{c+a^2cx^2}}$$

output
```
1/4*arcsinh(a*x)^4*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

## 3.337.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^4}{4a\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c*(1 + a^2*x^2)])
```

## 3.337.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

### 3.337.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.337.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{c(a^2x^2+1)}\,\text{arcsinh}(ax)^4}{4\sqrt{a^2x^2+1}\,ac}$ | 39 |

input
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c*arcsinh(a*x)^4
```

### 3.337.5 Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

### 3.337.6 Sympy [F]

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\text{asinh}^3\,(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(asinh(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

### 3.337.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.35

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\text{arsinh}\,(ax)^4}{4\,a\sqrt{c}}$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*arcsinh(a*x)^4/(a*sqrt(c))
```

### 3.337.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

### 3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\operatorname{asinh}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

# 3.338 $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$



## 3.338.1 Optimal result

Integrand size = 21, antiderivative size = 218

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{x\text{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}}\\&-\frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}\\&-\frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}\\&+\frac{3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(1/2)+arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a/
c/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^
2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)*polylog(2,-(a*x+(a^2
*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+3/2*polylog(3,
-(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

### 3.338.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.61

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2ax\text{arcsinh}(ax)^3 - 2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\left(\text{arcsinh}(ax)+3\log\left(1+e^{-2\text{arcsinh}(ax)}\right)\right)}{2ac}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2*a*x*ArcSinh[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*(ArcSinh[a*x] +
 3*Log[1 + E^(-2*ArcSinh[a*x])]) + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLo
g[2, -E^(-2*ArcSinh[a*x])] + 3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*ArcSinh
[a*x])])/(2*a*c*Sqrt[c*(1 + a^2*x^2)])
```

### 3.338.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6202, 6212, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}} - \frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{6212}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}} - \frac{3\sqrt{a^2x^2+1}\int \frac{ax\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}} - \frac{3\sqrt{a^2x^2+1}\int -i\text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{26}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\int \text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}$$

$\downarrow$ 4201

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1+e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)}{ac\sqrt{a^2cx^2+c}}$$

$\downarrow$ 2620

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+$$
$$\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2\text{arcsinh}(ax)}+1\right)-\int \text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\frac{1}{3}i\text{arcs}\right.}{ac\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3011

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+$$
$$\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcs}\right.\right.}{ac\sqrt{a^2cx^2+c}}$$

$\downarrow$ 2720

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+$$
$$\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}$$

$\downarrow$ 7143

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+$$
$$\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2\text{arc}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(x*ArcSinh[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*((-1
/3*I)*ArcSinh[a*x]^3 + (2*I)*((ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])
/2 + (ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/2 - PolyLog[3, -E^(2*A
rcSinh[a*x])]/4)))/(a*c*Sqrt[c + a^2*c*x^2])
```

## 3.338. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

### 3.338.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

## 3.338. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.338.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| default | $\frac{\sqrt{c(a^2x^2+1)}\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^3}{a\,c^2(a^2x^2+1)}+\frac{2\sqrt{c(a^2x^2+1)}\operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}\,a\,c^2}-\frac{3\sqrt{c(a^2x^2+1)}\operatorname{arcsinh}(ax)^2\ln\left(1+\left(ax+\sqrt{a^2x^2+1}\right.\right.}{\sqrt{a^2x^2+1}\,a\,c^2}$ |

input
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2+1))^(1/2)*(a*x-(a^2*x^2+1)^(1/2))*arcsinh(a*x)^3/a/c^2/(a^2*x^
2+1)+2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)^3-3/(a^2
*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^
2+1)^(1/2))^2)-3/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x
)*polylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+
1))^(1/2)/a/c^2*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

### 3.338.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
 c^2), x)
```

### 3.338.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

### 3.338.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.338.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\mathrm{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

# 3.339 $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$



## 3.339.1 Optimal result

Integrand size = 21, antiderivative size = 363

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx &= -\frac{x\text{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\text{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}\\&+\frac{x\text{arcsinh}(ax)^3}{3c(c+a^2cx^2)^{3/2}}+\frac{2x\text{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}\\&-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^2\sqrt{c+a^2cx^2}}\\&-\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}\\&+\frac{\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/3*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(3/2)-x*arcsinh(a*x)/c^2/(a^2*c*x^2+c
)^(1/2)+2/3*x*arcsinh(a*x)^3/c^2/(a^2*c*x^2+c)^(1/2)+1/2*arcsinh(a*x)^2/a/
c^2/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(
1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2
))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x
^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)*polylog(2,-(a*x+(a^2*
x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+polylog(3,-(a
*x+(a^2*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

### 3.339.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.54

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(1+a^2x^2)^{3/2}\left(-\frac{6ax\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}+\frac{3\text{arcsinh}(ax)^2}{1+a^2x^2}-4\text{arcsinh}(ax)^3+\frac{2ax\text{arcsinh}(ax)^3}{(1+a^2x^2)^{3/2}}+4\right.}{}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output
```
((1 + a^2*x^2)^(3/2)*((-6*a*x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2] + (3*ArcSinh
[a*x]^2)/(1 + a^2*x^2) - 4*ArcSinh[a*x]^3 + (2*a*x*ArcSinh[a*x]^3)/(1 + a^
2*x^2)^(3/2) + (4*a*x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2] - 12*ArcSinh[a*x]^
2*Log[1 + E^(-2*ArcSinh[a*x])] + 3*Log[1 + a^2*x^2] + 12*ArcSinh[a*x]*Poly
Log[2, -E^(-2*ArcSinh[a*x])] + 6*PolyLog[3, -E^(-2*ArcSinh[a*x])]))/(6*a*c
*(c + a^2*c*x^2)^(3/2))
```

### 3.339.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6203, 6202, 6212, 3042, 26, 4201, 2620, 3011, 2720, 6213, 6202, 240, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}\right)}{3c}+$$
$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\int \frac{ax\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\int -i\text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\int \text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1+e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2\text{arcsinh}(ax)}+1\right)-\int \text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\frac{1}{3}i\text{arc}}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3011}$$

3.339. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsi}\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 2720

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 6213

$$-\frac{a\sqrt{a^2x^2+1}\left(\frac{\int \frac{\text{arcsinh}(ax)}{(a^2x^2+1)^{3/2}}dx}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 6202

$$-\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-a\int \frac{x}{a^2x^2+1}dx}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 240

---

3.339. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}}{3c}$$

$$\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-\frac{\log\left(a^2x^2+1\right)}{2a}}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 7143$$

$$-\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-\frac{\log\left(a^2x^2+1\right)}{2a}}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)-\frac{1}{4}\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2\text{arc}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^2)^(3/2)) - (a*Sqrt[1 + a^2*x^2]*(-1/
2*ArcSinh[a*x]^2/(a^2*(1 + a^2*x^2)) + ((x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2]
 - Log[1 + a^2*x^2]/(2*a))/a))/(c^2*Sqrt[c + a^2*c*x^2]) + (2*((x*ArcSinh[
a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*((-1/3*I)*ArcSi
nh[a*x]^3 + (2*I)*((ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/2 + (ArcSi
nh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/2 - PolyLog[3, -E^(2*ArcSinh[a*x]
)]/4)))/(a*c*Sqrt[c + a^2*c*x^2])))/(3*c)
```

### 3.339.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

3.339. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

## 3.339. $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.339.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| default | $\frac{\sqrt{c(a^2x^2+1)}\left(2a^3x^3-2a^2x^2\sqrt{a^2x^2+1}+3ax-2\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)\left(-6a^4x^4\operatorname{arcsinh}(ax)-6a^3x^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}-6a^4x^4-\cdots\right.}{6(3a^6x^6\cdots}$ |

input
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*(a^2*x^2+1))^(1/2)*(2*a^3*x^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+3*a*x-2*(
a^2*x^2+1)^(1/2))*arcsinh(a*x)*(-6*a^4*x^4*arcsinh(a*x)-6*a^3*x^3*arcsinh(
a*x)*(a^2*x^2+1)^(1/2)-6*a^4*x^4-6*a^3*x^3*(a^2*x^2+1)^(1/2)+6*arcsinh(a*x
)^2*a^2*x^2-12*a^2*x^2*arcsinh(a*x)-9*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-1
8*a^2*x^2-6*a*x*(a^2*x^2+1)^(1/2)+8*arcsinh(a*x)^2-6*arcsinh(a*x)-12)/(3*a
^6*x^6+10*a^4*x^4+11*a^2*x^2+4)/a/c^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^
(1/2)/a/c^3*ln(a*x+(a^2*x^2+1)^(1/2))+1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^
(1/2)/a/c^3*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+4/3/(a^2*x^2+1)^(1/2)*(c*(a^2*
x^2+1))^(1/2)/a/c^3*arcsinh(a*x)^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/
2)/a/c^3*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-2/(a^2*x^2+1)^(1/2
)*(c*(a^2*x^2+1))^(1/2)/a/c^3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/
2))^2)+1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^3*polylog(3,-(a*x+(a^
2*x^2+1)^(1/2))^2)
```

## 3.339.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
 3*a^2*c^3*x^2 + c^3), x)
```

## 3.339.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

### 3.339.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

### 3.339.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

# 3.340 $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx$



## 3.340.1 Optimal result

Integrand size = 21, antiderivative size = 515

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx &= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\text{arcsinh}(ax)}{c^3\sqrt{c+a^2cx^2}} \\
&- \frac{x\text{arcsinh}(ax)}{10c^3(1+a^2x^2)\sqrt{c+a^2cx^2}} + \frac{3\text{arcsinh}(ax)^2}{20ac^3(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}} \\
&+ \frac{2\text{arcsinh}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\text{arcsinh}(ax)^3}{5c(c+a^2cx^2)^{5/2}} \\
&+ \frac{4x\text{arcsinh}(ax)^3}{15c^2(c+a^2cx^2)^{3/2}} + \frac{8x\text{arcsinh}(ax)^3}{15c^3\sqrt{c+a^2cx^2}} + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{15ac^3\sqrt{c+a^2cx^2}} \\
&- \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\log\left(1+e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^3\sqrt{c+a^2cx^2}} \\
&- \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}} \\
&+ \frac{4\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)}{5ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/5*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^3/c^2/(a^2*
c*x^2+c)^(3/2)-x*arcsinh(a*x)/c^3/(a^2*c*x^2+c)^(1/2)-1/10*x*arcsinh(a*x)/
c^3/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2)+3/20*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^
(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a*x)^3/c^3/(a^2*c*x^2+c)^(1/2)-1/
20/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/5*arcsinh(a*x)^2/a/c^3/(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/
a/c^3/(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^
2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x^2+
1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x
^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+4/5*polylog(3,
-(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

### 3.340.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \frac{-\frac{3}{\sqrt{1+a^2x^2}} - 60ax\operatorname{arcsinh}(ax) - \frac{6ax\operatorname{arcsinh}(ax)}{1+a^2x^2} + \frac{9\operatorname{arcsinh}(ax)^2}{(1+a^2x^2)^{3/2}} + \frac{24\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}} + 32a}{}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

output
```
(-3/Sqrt[1 + a^2*x^2] - 60*a*x*ArcSinh[a*x] - (6*a*x*ArcSinh[a*x])/(1 + a^
2*x^2) + (9*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + (24*ArcSinh[a*x]^2)/Sqrt
[1 + a^2*x^2] + 32*a*x*ArcSinh[a*x]^3 + (12*a*x*ArcSinh[a*x]^3)/(1 + a^2*x
^2)^2 + (16*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 32*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^3 - 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(-2*ArcSinh[a*x]
)] + 30*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2] + 96*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]*PolyLog[2, -E^(-2*ArcSinh[a*x])] + 48*Sqrt[1 + a^2*x^2]*PolyLog[3, -E
^(-2*ArcSinh[a*x])])/(60*a*c^3*Sqrt[c + a^2*c*x^2])
```

### 3.340.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.89, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6203, 6203, 6202, 6212, 3042, 26, 4201, 2620, 3011, 2720, 6213, 6202, 240, 6203, 241, 6202, 240, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^{5/2}}dx}{5c}+\frac{x\text{arcsinh}(ax)^3}{5c(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)^3}{5c(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{ 6202}$$

$$-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)^3}{5c(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{ 6212}$$

$$-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\int \frac{ax\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c(a^2cx^2+c)^{3/2}}\right)}{5c}+\frac{x\text{arcsinh}(ax)^3}{5c(a^2cx^2+c)^{5/2}}$$

---

3.340. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{\left(a^2x^2+1\right)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\int -i\text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(a}{3c(a^2cx^2+c)}\right.}{5c}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{\left(a^2x^2+1\right)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\int \text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax}{3c(a^2cx^2+c)^3}\right.}{5c}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{\left(a^2x^2+1\right)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1+e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}\right.}{5c}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{2620}$$

3.340. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx$

$$\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{\left(a^2x^2+1\right)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2\text{arcsinh}(ax)}+1\right)-\int \text{arcsinh}(ax)\log\left(1+e^{2\text{ar}\cdots}\right)\cdots\right)\cdots\right)}{ac\sqrt{a^2cx^2+c}}\cdots\right)}{3c}\cdots\right)}{5c}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \; 3011$$

$$\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{\left(a^2x^2+1\right)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+\frac{1}{2}\text{arcsinh}(ax)\text{Po}\cdots\right)\cdots\right)}{ac\sqrt{\cdots}}\cdots\right)}{3c}\cdots\right)}{5c}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \; 2720$$

$$\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{\left(a^2x^2+1\right)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{a}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{3\cdots}\cdots\right)}{5c}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^3}dx}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \; 6213$$

---

3.340. $\int \frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

$$-\frac{3a\sqrt{a^2x^2+1}\left(\frac{\int\frac{\text{arcsinh}(ax)}{(a^2x^2+1)^{5/2}}dx}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+$$

$$4\left(-\frac{a\sqrt{a^2x^2+1}\left(\frac{\int\frac{\text{arcsinh}(ax)}{(a^2x^2+1)^{3/2}}dx}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)\right.\right.}{}\right.}{}\right.$$

$$5c$$

$$\frac{x\text{arcsinh}(ax)^3}{5c(a^2cx^2+c)^{5/2}}$$

$$\downarrow 6202$$

$$-\frac{3a\sqrt{a^2x^2+1}\left(\frac{\int\frac{\text{arcsinh}(ax)}{(a^2x^2+1)^{5/2}}dx}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+$$

$$4\left(-\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-a\int\frac{x}{a^2x^2+1}dx}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\ldots}\right.\right.\right.}{}\right.}{}\right.$$

$$\frac{x\text{arcsinh}(ax)^3}{5c(a^2cx^2+c)^{5/2}}$$

$$\downarrow 240$$

$$-\frac{3a\sqrt{a^2x^2+1}\left(\frac{\int\frac{\text{arcsinh}(ax)}{(a^2x^2+1)^{5/2}}dx}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+$$

$$4\left(\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.$$

$$\frac{x\text{arcsinh}(ax)^3}{5c(a^2cx^2+c)^{5/2}}$$

---

3.340. $\int\frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}dx$

$$\Big\downarrow \; 6203$$

$$-\frac{3a\sqrt{a^2x^2+1}\left(\frac{\frac{2}{3}\int \frac{\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^{3/2}}dx-\frac{1}{3}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\text{arcsinh}(ax)}{3\left(a^2x^2+1\right)^{3/2}}}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+$$

$$4\left(\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.$$

$$\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \; 241$$

$$-\frac{3a\sqrt{a^2x^2+1}\left(\frac{\frac{2}{3}\int \frac{\text{arcsinh}(ax)}{\left(a^2x^2+1\right)^{3/2}}dx+\frac{x\text{arcsinh}(ax)}{3\left(a^2x^2+1\right)^{3/2}}+\frac{1}{6a\left(a^2x^2+1\right)}}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+$$

$$4\left(\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.$$

$$\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \; 6202$$

$$-\frac{3a\sqrt{a^2x^2+1}\left(\frac{\frac{2}{3}\left(\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{x\text{arcsinh}(ax)}{3\left(a^2x^2+1\right)^{3/2}}+\frac{1}{6a\left(a^2x^2+1\right)}}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+$$

$$4\left(\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.$$

$$\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

3.340. $\int \frac{\text{arcsinh}(ax)^3}{\left(c+a^2cx^2\right)^{7/2}}dx$

$\Big\downarrow$ 240

$$4\left(\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}\right.$$

$$\frac{3a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{3\left(a^2x^2+1\right)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-\frac{\log\left(a^2x^2+1\right)}{2a}\right)+\frac{1}{6a\left(a^2x^2+1\right)}}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

$\Big\downarrow$ 7143

$$-\frac{3a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{3\left(a^2x^2+1\right)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-\frac{\log\left(a^2x^2+1\right)}{2a}\right)+\frac{1}{6a\left(a^2x^2+1\right)}}{2a}-\frac{\text{arcsinh}(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{5c^3\sqrt{a^2cx^2+c}}+$$

$$\frac{4\left(-\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-\frac{\log\left(a^2x^2+1\right)}{2a}}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2\left(a^2x^2+1\right)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsin}}\right.\right.\right.}{}\right.}{}\right.}{5c}$$

$$\frac{x\text{arcsinh}(ax)^3}{5c\left(a^2cx^2+c\right)^{5/2}}$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

---

3.340. $\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx$

output
```
(x*ArcSinh[a*x]^3)/(5*c*(c + a^2*c*x^2)^(5/2)) - (3*a*Sqrt[1 + a^2*x^2]*(-
1/4*ArcSinh[a*x]^2/(a^2*(1 + a^2*x^2)^2) + (1/(6*a*(1 + a^2*x^2)) + (x*Arc
Sinh[a*x])/(3*(1 + a^2*x^2)^(3/2)) + (2*((x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2
] - Log[1 + a^2*x^2]/(2*a)))/3)/(2*a)))/(5*c^3*Sqrt[c + a^2*c*x^2]) + (4*(
(x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^2)^(3/2)) - (a*Sqrt[1 + a^2*x^2]*(-1/
2*ArcSinh[a*x]^2/(a^2*(1 + a^2*x^2)) + ((x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2]
 - Log[1 + a^2*x^2]/(2*a))/a))/(c^2*Sqrt[c + a^2*c*x^2]) + (2*((x*ArcSinh[
a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*((-1/3*I)*ArcSi
nh[a*x]^3 + (2*I)*((ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/2 + (ArcSi
nh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/2 - PolyLog[3, -E^(2*ArcSinh[a*x]
)]/4)))/(a*c*Sqrt[c + a^2*c*x^2])))/(3*c)))/(5*c)
```

#### 3.340.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## 3.340. $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

## 3.340. $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.340.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 888, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| default | $\frac{\sqrt{c(a^2x^2+1)}\left(8a^5x^5-8a^4x^4\sqrt{a^2x^2+1}+20a^3x^3-16a^2x^2\sqrt{a^2x^2+1}+15ax-8\sqrt{a^2x^2+1}\right)\left(24-1590a^4x^4\operatorname{arcsinh}(ax)-1368a^4x^4\operatorname{arcsi}\right.}{}$ |

input
```
int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(c*(a^2*x^2+1))^(1/2)*(8*a^5*x^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*a^3*x
^3-16*a^2*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(24-1590*a^4*x
^4*arcsinh(a*x)-1368*a^4*x^4*arcsinh(a*x)^2-1410*a^2*x^2*arcsinh(a*x)+105*
a^3*x^3*(a^2*x^2+1)^(1/2)+160*a^4*x^4*arcsinh(a*x)^3+45*a*x*(a^2*x^2+1)^(1
/2)-744*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^5*x^5-1020*a^3*x^3*arcsinh(a*x)
^2*(a^2*x^2+1)^(1/2)-495*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+256*arcsinh(
a*x)^3-480*arcsinh(a*x)-264*arcsinh(a*x)^2+96*a^2*x^2-192*arcsinh(a*x)^2*(
a^2*x^2+1)^(1/2)*a^7*x^7-192*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^7*x^7-840*ar
csinh(a*x)^2*a^6*x^6+84*x^5*a^5*(a^2*x^2+1)^(1/2)-984*arcsinh(a*x)^2*a^2*x
^2+144*a^4*x^4-936*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-372*arcsinh(a*x)
*(a^2*x^2+1)^(1/2)*a*x-756*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^5*x^5+96*a^6*x
^6+24*(a^2*x^2+1)^(1/2)*a^7*x^7+24*a^8*x^8-192*arcsinh(a*x)*a^8*x^8-852*ar
csinh(a*x)*a^6*x^6-192*arcsinh(a*x)^2*a^8*x^8+380*arcsinh(a*x)^3*a^2*x^2)/
(40*a^10*x^10+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-2/
(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*ln(a*x+(a^2*x^2+1)^(1/2))+1/
(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))
^2)+16/15/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^3-8/5
/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2*ln(1+(a*x+(a
^2*x^2+1)^(1/2))^2)-8/5/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcs
inh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+4/5/(a^2*x^2+1)^(1/2)*(c...
```

## 3.340.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

### 3.340.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{(c(a^2x^2+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(7/2), x)
```

### 3.340.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(7/2), x)
```

### 3.340.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{7/2}}\,dx=\int \frac{\operatorname{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{7/2}}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2), x)
```

# 3.341 $\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx$



## 3.341.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx = \text{Int}\left(\frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}, x\right)$$

output
```
Unintegrable(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

## 3.341.2 Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx = \int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx$$

input
```
Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

## 3.341.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}} \, dx$$

$$\Big\downarrow \text{6239}$$

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

input
```
Int[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
$Aborted
```

### 3.341.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.341.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m \operatorname{arcsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}}dx$$

input
```
int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

### 3.341.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.341.6 Sympy [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}^3(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x**m*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*asinh(a*x)**3/sqrt(a**2*x**2 + 1), x)
```

### 3.341.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.341.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.341.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.342 $\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx$



## 3.342.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} dx = & \frac{45x^2}{128a^3} - \frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{64a^4} \\ & + \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{32a^2} + \frac{45\text{arcsinh}(ax)^2}{128a^5} + \frac{9x^2\text{arcsinh}(ax)^2}{16a^3} \\ & - \frac{3x^4\text{arcsinh}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{8a^4} \\ & + \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a^2} + \frac{3\text{arcsinh}(ax)^4}{32a^5}\end{aligned}$$

output
```
45/128*x^2/a^3-3/128*x^4/a+45/128*arcsinh(a*x)^2/a^5+9/16*x^2*arcsinh(a*x)
^2/a^3-3/16*x^4*arcsinh(a*x)^2/a+3/32*arcsinh(a*x)^4/a^5-45/64*x*arcsinh(a
*x)*(a^2*x^2+1)^(1/2)/a^4+3/32*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2-3/8*
x*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^
(1/2)/a^2
```

### 3.342.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.65

$$\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{45a^2x^2 - 3a^4x^4 + 6ax\sqrt{1+a^2x^2}(-15+2a^2x^2)\,\text{arcsinh}(ax) + (45+72a^2x^2-24a^4x^4)\,\text{arcsinh}(ax)^2 + 16a}{128a^5}$$

input
```
Integrate[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
(45*a^2*x^2 - 3*a^4*x^4 + 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcSin
h[a*x] + (45 + 72*a^2*x^2 - 24*a^4*x^4)*ArcSinh[a*x]^2 + 16*a*x*Sqrt[1 + a
^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 12*ArcSinh[a*x]^4)/(128*a^5)
```

### 3.342.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6227, 6191, 6227, 15, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$
$$\Big\downarrow \; 6227$$
$$-\frac{3\int \frac{x^2 \text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{4a^2} - \frac{3\int x^3 \text{arcsinh}(ax)^2dx}{4a} + \frac{x^3\sqrt{a^2x^2+1}\,\text{arcsinh}(ax)^3}{4a^2}$$
$$\Big\downarrow \; 6191$$
$$-\frac{3\int \frac{x^2 \text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{4a^2} - \frac{3\left(\frac{1}{4}x^4 \text{arcsinh}(ax)^2 - \frac{1}{2}a\int \frac{x^4 \text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{4a} + \frac{x^3\sqrt{a^2x^2+1}\,\text{arcsinh}(ax)^3}{4a^2}$$
$$\Big\downarrow \; 6227$$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\text{arcsinh}(ax)^2dx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int x^3dx}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}\right)\right)}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\text{arcsinh}(ax)^2dx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2a}-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6227}$$

---

3.342. $\displaystyle\int \frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

$$-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}$$

$$\Big\downarrow \text{6198}$$

$$\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}-\frac{3\left(-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a}\right)}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{3\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}$$

3.342. $\int \frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

input
```
Int[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
(x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(4*a^2) - (3*((x^4*ArcSinh[a*x]^2)/
4 - (a*(-1/16*x^4/a + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*(-
1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)))/(4*a^2)))/2))/(4*a) - (3*((x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*
a^2) - ArcSinh[a*x]^4/(8*a^3) - (3*((x^2*ArcSinh[a*x]^2)/2 - a*(-1/4*x^2/a
 + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4*a^3))))/
(2*a)))/(4*a^2)
```

### 3.342.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.342.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| default | $\frac{32a^3x^3 \operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}-24a^4x^4 \operatorname{arcsinh}(ax)^2+12a^3x^3 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}-3a^4x^4-48 \operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\, ax+72 \operatorname{arcsi}}{128a^5}$ |

input
```
int(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(32*a^3*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-24*a^4*x^4*arcsinh(a*x)
^2+12*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-3*a^4*x^4-48*arcsinh(a*x)^3*(
a^2*x^2+1)^(1/2)*a*x+72*arcsinh(a*x)^2*a^2*x^2+12*arcsinh(a*x)^4-90*arcsin
h(a*x)*(a^2*x^2+1)^(1/2)*a*x+45*a^2*x^2+45*arcsinh(a*x)^2+45)/a^5
```

### 3.342.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89

$$\int \frac{x^4 \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{3\,a^4x^4 - 16\,(2\,a^3x^3 - 3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 45\,a^2x^2 - 12\,\log\left(ax+\sqrt{a^2x^2+1}\right)^4 + \cdots}{128 \cdots}$$

input
```
integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/128*(3*a^4*x^4 - 16*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqr
t(a^2*x^2 + 1))^3 - 45*a^2*x^2 - 12*log(a*x + sqrt(a^2*x^2 + 1))^4 + 3*(8*
a^4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(2*a^3*x^3 -
 15*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^5
```

### 3.342.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$\int \frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{3x^4\operatorname{asinh}^2(ax)}{16a} - \frac{3x^4}{128a} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a^2} + \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a^2} + \frac{9x^2\operatorname{asinh}^2(ax)}{16a^3} + \frac{45x^2}{128a^3} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^3(a}{8a^4} \\ 0 \end{cases}$$

input
```
integrate(x**4*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-3*x**4*asinh(a*x)**2/(16*a) - 3*x**4/(128*a) + x**3*sqrt(a**2*
x**2 + 1)*asinh(a*x)**3/(4*a**2) + 3*x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(
32*a**2) + 9*x**2*asinh(a*x)**2/(16*a**3) + 45*x**2/(128*a**3) - 3*x*sqrt(
a**2*x**2 + 1)*asinh(a*x)**3/(8*a**4) - 45*x*sqrt(a**2*x**2 + 1)*asinh(a*x
)/(64*a**4) + 3*asinh(a*x)**4/(32*a**5) + 45*asinh(a*x)**2/(128*a**5), Ne(
a, 0)), (0, True))
```

### 3.342.7 Maxima [F]

$$\int \frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.342.8 Giac [F]

$$\int \frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^4\,\text{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.343 $\int \frac{x^3\textbf{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$



## 3.343.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{40x}{9a^3}-\frac{2x^3}{27a}-\frac{40\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{9a^2}+\frac{2x\text{arcsinh}(ax)^2}{a^3}-\frac{x^3\text{arcsinh}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^4}+\frac{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3a^2}$$

output
```
40/9*x/a^3-2/27*x^3/a+2*x*arcsinh(a*x)^2/a^3-1/3*x^3*arcsinh(a*x)^2/a-40/9
*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+2/9*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)
/a^2-2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(a*x)^3*(a^2*
x^2+1)^(1/2)/a^2
```

## 3.343.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{-2ax(-60+a^2x^2)+6(-20+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)-9ax(-6+a^2x^2)\,\text{arcsinh}(ax)^2+9(-2+a^2}{27a^4}$$

input
```
Integrate[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
(-2*a*x*(-60 + a^2*x^2) + 6*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
 - 9*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 + 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2
]*ArcSinh[a*x]^3)/(27*a^4)
```

### 3.343.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6227, 6191, 6213, 6187, 6213, 24, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2\text{arcsinh}(ax)^2dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{2\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6213}$$

$$-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\int \text{arcsinh}(ax)^2dx}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6187}$$

$$-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6213}$$

$$-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}\right)}{3a^2}$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}\right)}{3a^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}\right)}{3a^2}$$

$$\Big\downarrow \text{ 6213}$$

3.343. $\int\frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

$$-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}\right)}{3a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}\right)}{a}$$

input
```
Int[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*a^2) - ((x^3*ArcSinh[a*x]^2)/3 -
 (2*a*(-1/9*x^3/a + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2) - (2*(-(x
/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2))/(3*a^2)))/3)/a - (2*((Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^3)/a^2 - (3*(x*ArcSinh[a*x]^2 - 2*a*(-(x/a) + (Sqr
t[1 + a^2*x^2]*ArcSinh[a*x])/a^2)))/a))/(3*a^2)
```

### 3.343.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.343. $\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.343.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| default | $\frac{9a^4x^4 \operatorname{arcsinh}(ax)^3 - 9\operatorname{arcsinh}(ax)^3 a^2x^2 - 9a^3x^3 \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1} + 6a^4x^4 \operatorname{arcsinh}(ax) - 114a^2x^2 \operatorname{arcsinh}(ax) - 2a^3x^3\sqrt{a^2x^2+1}}{27a^4\sqrt{a^2x^2+1}}$ |

input
```
int(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

3.343. $\int \frac{x^3 \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\, dx$

output
```
1/27/a^4/(a^2*x^2+1)^(1/2)*(9*a^4*x^4*arcsinh(a*x)^3-9*arcsinh(a*x)^3*a^2*
x^2-9*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+6*a^4*x^4*arcsinh(a*x)-114*
a^2*x^2*arcsinh(a*x)-2*a^3*x^3*(a^2*x^2+1)^(1/2)-18*arcsinh(a*x)^3+54*arcs
inh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x-120*arcsinh(a*x)+120*a*x*(a^2*x^2+1)^(1/2
))
```

### 3.343.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{2\,a^3x^3 - 9\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 9\,(a^3x^3-6\,ax)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\,\sqrt{a}}{27\,a^4}$$

input
```
integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(2*a^3*x^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^
2 + 1))^3 + 9*(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^
2*x^2 + 1)*(a^2*x^2 - 20)*log(a*x + sqrt(a^2*x^2 + 1)) - 120*a*x)/a^4
```

### 3.343.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{x^3\operatorname{asinh}^2(ax)}{3a} - \frac{2x^3}{27a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^2} + \frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^2} + \frac{2x\operatorname{asinh}^2(ax)}{a^3} + \frac{40x}{9a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^4} - \\ 0 \end{cases}$$

input
```
integrate(x**3*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**3*asinh(a*x)**2/(3*a) - 2*x**3/(27*a) + x**2*sqrt(a**2*x**2
 + 1)*asinh(a*x)**3/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a*
*2) + 2*x*asinh(a*x)**2/a**3 + 40*x/(9*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh
(a*x)**3/(3*a**4) - 40*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**4), Ne(a, 0)),
 (0, True))
```

### 3.343.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.83

$$\int \frac{x^3 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)^3 + \frac{2}{27}a\left(\frac{3\left(\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)\text{arsinh}\,(ax)}{a^3} - \frac{a^2x^3-60\,x}{a^4}\right) - \frac{(a^2x^3-6\,x)\,\text{arsinh}\,(ax)^2}{3\,a^3}$$

input
```
integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^3 +
 2/27*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)*arcsinh(a*x)
/a^3 - (a^2*x^3 - 60*x)/a^4) - 1/3*(a^2*x^3 - 6*x)*arcsinh(a*x)^2/a^3
```

### 3.343.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x^3\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.344 $\int \frac{x^2 \textbf{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$



## 3.344.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{3x^2}{8a}+\frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{4a^2}-\frac{3\text{arcsinh}(ax)^2}{8a^3} -\frac{3x^2\text{arcsinh}(ax)^2}{4a}+\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2a^2}-\frac{\text{arcsinh}(ax)^4}{8a^3}$$

output
```
-3/8*x^2/a-3/8*arcsinh(a*x)^2/a^3-3/4*x^2*arcsinh(a*x)^2/a-1/8*arcsinh(a*x
)^4/a^3+3/4*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^3*(a^2
*x^2+1)^(1/2)/a^2
```

## 3.344.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{3a^2x^2-6ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)+(3+6a^2x^2)\,\text{arcsinh}(ax)^2-4ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3+\text{arcsinh}}{8a^3}$$

input
```
Integrate[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
-1/8*(3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*A
rcSinh[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + ArcSinh[a*x]^4)/a
^3
```

## 3.344.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6227, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\text{arcsinh}(ax)^2dx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2a}-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6227}$$

$$-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a}$$

input
```
Int[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*a^2) - ArcSinh[a*x]^4/(8*a^3) - (3
*((x^2*ArcSinh[a*x]^2)/2 - a*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*
x])/(2*a^2) - ArcSinh[a*x]^2/(4*a^3))))/(2*a)
```

### 3.344.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.344. $\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

## 3.344.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $-\frac{-4 \operatorname{arcsinh}(ax)^3 \sqrt{a^2x^2+1}\, ax+6 \operatorname{arcsinh}(ax)^2 a^2x^2+\operatorname{arcsinh}(ax)^4-6 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\, ax+3a^2x^2+3 \operatorname{arcsinh}(ax)^2+3}{8a^3}$ | 84 |

input
```
int(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a*x+6*arcsinh(a*x)^2*a^2*x^2+arc
sinh(a*x)^4-6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^
2+3)/a^3
```

## 3.344.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

$$\int \frac{x^2 \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\, dx$$
$$= \frac{4\sqrt{a^2x^2+1}ax \log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 3\,a^2x^2 - \log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 6\sqrt{a^2x^2+1}ax \log\left(ax+\sqrt{a^2\right.}{8\,a^3}$$

input
```
integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a^2*x^2 -
log(a*x + sqrt(a^2*x^2 + 1))^4 + 6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^
2*x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

### 3.344.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{3x^2\,\text{asinh}^2(ax)}{4a} - \frac{3x^2}{8a} + \frac{x\sqrt{a^2x^2+1}\,\text{asinh}^3(ax)}{2a^2} + \frac{3x\sqrt{a^2x^2+1}\,\text{asinh}(ax)}{4a^2} - \frac{\text{asinh}^4(ax)}{8a^3} - \frac{3\,\text{asinh}^2(ax)}{8a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-3*x**2*asinh(a*x)**2/(4*a) - 3*x**2/(8*a) + x*sqrt(a**2*x**2 +
 1)*asinh(a*x)**3/(2*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) -
 asinh(a*x)**4/(8*a**3) - 3*asinh(a*x)**2/(8*a**3), Ne(a, 0)), (0, True))
```

### 3.344.7 Maxima [F]

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.344.8 Giac [F]

$$\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.345 $\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$



## 3.345.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{6x}{a} + \frac{6\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^2} - \frac{3x\text{arcsinh}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2}$$

output
```
-6*x/a-3*x*arcsinh(a*x)^2/a+6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a
*x)^3*(a^2*x^2+1)^(1/2)/a^2
```

## 3.345.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{-6ax + 6\sqrt{1+a^2x^2}\text{arcsinh}(ax) - 3ax\text{arcsinh}(ax)^2 + \sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a^2}$$

input
```
Integrate[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
(-6*a*x + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 3*a*x*ArcSinh[a*x]^2 + Sqrt[1
 + a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

### 3.345.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6213, 6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\int \text{arcsinh}(ax)^2dx}{a}$$

$$\Big\downarrow \text{6187}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{a}$$

$$\Big\downarrow \text{6213}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)\right)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}$$

input
```
Int[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2 - (3*(x*ArcSinh[a*x]^2 - 2*a*(-(x/a
) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2)))/a
```

#### 3.345.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.345.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{arcsinh}(ax)^3 a^2x^2+\operatorname{arcsinh}(ax)^3-3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax+6a^2x^2\operatorname{arcsinh}(ax)+6\operatorname{arcsinh}(ax)-6ax\sqrt{a^2x^2+1}}{a^2\sqrt{a^2x^2+1}}$ | 90 |

input
```
int(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^3*a^2*x^2+arcsinh(a*x)^3-3*arcsinh(a
*x)^2*(a^2*x^2+1)^(1/2)*a*x+6*a^2*x^2*arcsinh(a*x)+6*arcsinh(a*x)-6*a*x*(a
^2*x^2+1)^(1/2))
```

### 3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.44

$$\int \frac{x\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx =$$
$$-\frac{3\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3+6\,ax-6\,\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2\right.}}{a^2}$$

input
```
integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(3*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(
a^2*x^2 + 1))^3 + 6*a*x - 6*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
)/a^2
```

### 3.345.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} -\frac{3x\,\text{asinh}^2\,(ax)}{a} - \frac{6x}{a} + \frac{\sqrt{a^2x^2+1}\,\text{asinh}^3\,(ax)}{a^2} + \frac{6\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-3*x*asinh(a*x)**2/a - 6*x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**
3/a**2 + 6*sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True))
```

### 3.345.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = -\frac{3\,x\,\text{arsinh}\,(ax)^2}{a} + \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^3}{a^2} - \frac{6\left(x - \frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)}{a}\right)}{a}$$

input
```
integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-3*x*arcsinh(a*x)^2/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a^2 - 6*(x - sqrt
(a^2*x^2 + 1)*arcsinh(a*x)/a)/a
```

### 3.345.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.58

$$\int \frac{x\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2} - \frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)\right)}{a}$$

input
```
integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt
(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1
))/a^2))/a
```

### 3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\mathrm{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.346 $\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx$



## 3.346.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx = \frac{\text{arcsinh}(ax)^4}{4a}$$

output
```
1/4*arcsinh(a*x)^4/a
```

## 3.346.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx = \frac{\text{arcsinh}(ax)^4}{4a}$$

input
```
Integrate[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^4/(4*a)
```

## 3.346.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{\operatorname{arcsinh}(ax)^4}{4a}$$

input
```
Int[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^4/(4*a)
```

### 3.346.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.346.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}(ax)^4}{4a}$ | 12 |
| default | $\frac{\operatorname{arcsinh}(ax)^4}{4a}$ | 12 |

input
```
int(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*arcsinh(a*x)^4/a
```

### 3.346.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{4\,a}$$

input
```
integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*log(a*x + sqrt(a^2*x^2 + 1))^4/a
```

### 3.346.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((asinh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

### 3.346.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arsinh}\,(ax)^4}{4\,a}$$

input
```
integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*arcsinh(a*x)^4/a
```

### 3.346.8 Giac [F]

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

### 3.346.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{asinh}(a\,x)^4}{4\,a}$$

input
```
int(asinh(a*x)^3/(a^2*x^2 + 1)^(1/2),x)
```

output
```
asinh(a*x)^4/(4*a)
```

# 3.347 $\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$



## 3.347.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = & -2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) \\ & - 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) \\ & + 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) \\ & + 6\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right) \\ & - 6\text{arcsinh}(ax)\,\text{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right) \\ & - 6\,\text{PolyLog}\left(4,-e^{\text{arcsinh}(ax)}\right) + 6\,\text{PolyLog}\left(4,e^{\text{arcsinh}(ax)}\right)\end{aligned}$$

output
```
-2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(
2,-a*x-(a^2*x^2+1)^(1/2))+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2)
)+6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(
3,a*x+(a^2*x^2+1)^(1/2))-6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+6*polylog(4,a
*x+(a^2*x^2+1)^(1/2))
```

### 3.347.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}\big(\pi^4 - 2\operatorname{arcsinh}(ax)^4 - 8\operatorname{arcsinh}(ax)^3\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) + 8\operatorname{arcsinh}(ax)^3\log\left(1-e^{\operatorname{arcsinh}(ax)}\right) + 24\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) + 24\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) + 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) - 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,-e^{-\operatorname{arcsinh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\big)$$

input
```
Integrate[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
(Pi^4 - 2*ArcSinh[a*x]^4 - 8*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] + 8
*ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*ArcSinh[a*x]^2*PolyLog[2, -E^
(-ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 48*ArcSi
nh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]*PolyLog[3, E^ArcS
inh[a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 48*PolyLog[4, E^ArcSinh[a*
x]])/8
```

### 3.347.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6231, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \; 6231$$

$$\int \frac{\operatorname{arcsinh}(ax)^3}{ax}d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \; 3042$$

$$\int i\text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{26}$$

$$i\int \text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{4670}$$

$$i\left(3i\int \text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int \text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(-3i\left(2\int \text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\right)+3i\right($$

$$\Big\downarrow \text{7163}$$

$$i\left(-3i\left(2\left(\text{arcsinh}(ax)\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\text{arcsinh}(ax)^2\text{P}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$i\left(-3i\left(2\left(\text{arcsinh}(ax)\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int e^{-\text{arcsinh}(ax)}\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}\right)-\text{arcs}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(2i\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)-3i\left(2\left(\text{arcsinh}(ax)\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\text{PolyLog}\left(4,-e^{\text{arcsinh}(ax)}\right)\right.\right.\right.$$

input
```
Int[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
I*((2*I)*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - (3*I)*(-(ArcSinh[a*x]^2*
PolyLog[2, -E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]]
 - PolyLog[4, -E^ArcSinh[a*x]])) + (3*I)*(-(ArcSinh[a*x]^2*PolyLog[2, E^Ar
cSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] - PolyLog[4, E^A
rcSinh[a*x]])))
```

## 3.347. $\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$

### 3.347.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.347. $\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.347.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.93

| method | result |
|---|---|
| default | $-\operatorname{arcsinh}(ax)^3 \ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 3\operatorname{arcsinh}(ax)^2 \operatorname{polylog}\left(2, -ax-\sqrt{a^2x^2+1}\right) + 6\ \mathrm{ar}$ |

input
```
int(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2,-a*
x-(a^2*x^2+1)^(1/2))+6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*po
lylog(4,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))
+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(
3,a*x+(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

### 3.347.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^3 + x), x)
```

### 3.347.6 Sympy [F]

$$\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{asinh}^3\,(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**3/x/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**3/(x*sqrt(a**2*x**2 + 1)), x)
```

### 3.347.7 Maxima [F]

$$\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.347.8 Giac [F]

$$\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.348 $\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx$



## 3.348.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = -a\text{arcsinh}(ax)^3 - \frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{x} + 3a\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right) + 3a\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)$$

output
```
-a*arcsinh(a*x)^3+3*a*arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3*a*a
rcsinh(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*a*polylog(3,(a*x+(a^2
*x^2+1)^(1/2))^2)-arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x
```

## 3.348.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \frac{1}{8}a\left(i\pi^3 - 8\text{arcsinh}(ax)^3 - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ax} + 24\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right) + 24\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - 12\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)\right)$$

input
```
Integrate[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
(a*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(a*x)
 + 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 24*ArcSinh[a*x]*PolyLog
[2, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcSinh[a*x])]))/8
```

### 3.348.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6215, 6190, 3042, 26, 4199, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6215}$$

$$3a\int \frac{\text{arcsinh}(ax)^2}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}$$

$$\Big\downarrow \text{6190}$$

$$3a\int \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{ax}d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}+3a\int -i\text{arcsinh}(ax)^2\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-3ia\int \text{arcsinh}(ax)^2\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{4199}$$

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-3ia\left(2i\int -\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-3ia\left(-2i\int\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)$$

$\downarrow$ 2620

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-$$
$$3ia\left(-2i\left(\int\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right)-\frac{1}{3}i\text{arcsinh}$$

$\downarrow$ 3011

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-$$
$$3ia\left(-2i\left(\frac{1}{2}\int\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)^2\right.\right.$$

$\downarrow$ 2720

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-$$
$$3ia\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}-\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\right.\right.$$

$\downarrow$ 7143

$$-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-$$
$$3ia\left(-2i\left(-\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)+\frac{1}{4}\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsi}}\right.\right.\right.$$

input
```
Int[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x) - (3*I)*a*((-1/3*I)*ArcSinh[a*x]^3
 - (2*I)*(-1/2*(ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])]) - (ArcSinh[a*x
]*PolyLog[2, E^(2*ArcSinh[a*x])])/2 + PolyLog[3, E^(2*ArcSinh[a*x])]/4))
```

### 3.348.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.348. $\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

## 3.348. $\int \frac{\text{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx$

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.348.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| default | $\frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^3}{x} - 2a\operatorname{arcsinh}(ax)^3 + 3a\operatorname{arcsinh}(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 6a\operatorname{arcsi}$ |

input
```
int(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^3-2*a*arcsinh(a*x)^3+3*a*arcsinh(a*
x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+
1)^(1/2))-6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a*arcsinh(a*x)^2*ln(1-a*
x-(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*a
*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

## 3.348.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^4 + x^2), x)
```

### 3.348.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**3/x**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**3/(x**2*sqrt(a**2*x**2 + 1)), x)
```

### 3.348.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2
 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(sqrt(a^2*x
^2 + 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

### 3.348.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx=\int \frac{{\mathrm{asinh}\left(a\,x\right)}^3}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.349 $\int \frac{\text{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$



## 3.349.1 Optimal result

Integrand size = 23, antiderivative size = 210

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = &-\frac{3a\text{arcsinh}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{2x^2}\\ &-6a^2\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)\\ &+a^2\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)-3a^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\\ &+\frac{3}{2}a^2\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\\ &+3a^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\\ &-\frac{3}{2}a^2\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\\ &-3a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)\\ &+3a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right)\\ &+3a^2\,\text{PolyLog}\left(4,-e^{\text{arcsinh}(ax)}\right)-3a^2\,\text{PolyLog}\left(4,e^{\text{arcsinh}(ax)}\right)\end{aligned}$$

output
```
-3/2*a*arcsinh(a*x)^2/x-6*a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+
a^2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^
2*x^2+1)^(1/2))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3
*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^2*polylog(2,a*x
+(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3
*a^2*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a
^2*x^2+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^3
*(a^2*x^2+1)^(1/2)/x^2
```

### 3.349.2 Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{a\left(-a\pi^4x + 2ax\operatorname{arcsinh}(ax)^4 - 12ax\operatorname{arcsinh}(ax)^2\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) - 2ax\operatorname{arcsinh}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(\right.\right.}{}$$

input
```
Integrate[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

output
```
(a*(-(a*Pi^4*x) + 2*a*x*ArcSinh[a*x]^4 - 12*a*x*ArcSinh[a*x]^2*Coth[ArcSin
h[a*x]/2] - 2*a*x*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]/2]^2 + 48*a*x*ArcSinh[a
*x]*Log[1 - E^(-ArcSinh[a*x])] - 48*a*x*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a
*x])] + 8*a*x*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 8*a*x*ArcSinh[a*
x]^3*Log[1 - E^ArcSinh[a*x]] - 24*a*x*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^
(-ArcSinh[a*x])] - 48*a*x*PolyLog[2, E^(-ArcSinh[a*x])] - 24*a*x*ArcSinh[a
*x]^2*PolyLog[2, E^ArcSinh[a*x]] - 48*a*x*ArcSinh[a*x]*PolyLog[3, -E^(-Arc
Sinh[a*x])] + 48*a*x*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 48*a*x*Poly
Log[4, -E^(-ArcSinh[a*x])] - 48*a*x*PolyLog[4, E^ArcSinh[a*x]] + 12*a*x*Ar
cSinh[a*x]^2*Tanh[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2])
)/(16*x)
```

### 3.349.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6224, 6191, 6231, 3042, 26, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{a^2x^2+1}}\,dx$$
$$\Big\downarrow \; 6224$$
$$-\frac{1}{2}a^2\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{a^2x^2+1}}dx + \frac{3}{2}a\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}dx - \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{2x^2}$$
$$\Big\downarrow \; 6191$$

$$\frac{3}{2}a\left(2a\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^2}{x}\right)-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{a^2x^2+1}}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}$$

$\downarrow$ 6231

$$-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^3}{ax}d\text{arcsinh}(ax)+\frac{3}{2}a\left(2a\int \frac{\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\text{arcsinh}(ax)^2}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}$$

$\downarrow$ 3042

$$-\frac{1}{2}a^2\int i\text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)+\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2a\int i\text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}$$

$\downarrow$ 26

$$-\frac{1}{2}ia^2\int \text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)+\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\int \text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}$$

$\downarrow$ 4670

$$-\frac{1}{2}ia^2\left(3i\int \text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int \text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(a\right.$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\left(i\int \log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-i\int \log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2i\text{arc}\right.\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}$$

$\downarrow$ 2715

$$-\frac{1}{2}ia^2\left(3i\int \text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int \text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(a\right.$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\left(i\int e^{-\text{arcsinh}(ax)}\log\left(1-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}\log\left(1+e^{\text{arcsinh}(ax}\right.\right.\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}$$

$\downarrow$ 2838

3.349. $\int \frac{\text{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

$$-\frac{1}{2}ia^2\left(3i\int \text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int \text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(a\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsi}}\right.\right.\right.$$

$\downarrow$ 3011

$$-\frac{1}{2}ia^2\left(-3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\right)\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsi}}\right.\right.\right.$$

$\downarrow$ 7163

$$-\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\text{arcsinh}(a\right.\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsi}}\right.\right.\right.$$

$\downarrow$ 2720

$$-\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int e^{-\text{arcsinh}(ax)}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}\right)-\right.\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsi}}\right.\right.\right.$$

$\downarrow$ 7143

$$-\frac{1}{2}ia^2\left(2i\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)-3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\text{PolyLog}\left(4,-e^{\text{arcsin}}\right.\right.\right.\right.$$
$$\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsi}}\right.\right.\right.$$

input
```
Int[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

3.349. $\int \frac{\text{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

output
```
-1/2*(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x^2 + (3*a*(-(ArcSinh[a*x]^2/x) +
(2*I)*a*((2*I)*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + I*PolyLog[2, -E^ArcS
inh[a*x]] - I*PolyLog[2, E^ArcSinh[a*x]])))/2 - (I/2)*a^2*((2*I)*ArcSinh[a
*x]^3*ArcTanh[E^ArcSinh[a*x]] - (3*I)*(-(ArcSinh[a*x]^2*PolyLog[2, -E^ArcS
inh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] - PolyLog[4, -E^A
rcSinh[a*x]])) + (3*I)*(-(ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]]) + 2*(
ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] - PolyLog[4, E^ArcSinh[a*x]])))
```

### 3.349.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

## 3.349. $\int \frac{\text{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

---

## 3.349. $\int \frac{\text{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

### 3.349.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.80

| method | result |
|---|---|
| default | $-\frac{\operatorname{arcsinh}(ax)^2\left(a^2x^2\operatorname{arcsinh}(ax)+3ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)\right)}{2\sqrt{a^2x^2+1}\,x^2}+\frac{a^2\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+\frac{3a^2\operatorname{arcsinh}(ax)^2\,\mathrm{po}}{}$ |

input
```
int(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)^2*(a^2*x^2*arcsinh(a*x)+3*a*x*(a^2
*x^2+1)^(1/2)+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1
/2))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-3*a^2*arcsin
h(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2+1)^
(1/2))-1/2*a^2*arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(
a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+3*a^2*arcsinh(a*x)*polylog(3,a*x+(
a^2*x^2+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x
)*ln(1+a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^
2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(2,a*x+(a^2*x^2+1)
^(1/2))
```

### 3.349.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^5 + x^3), x)
```

### 3.349.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**3/x**3/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**3/(x**3*sqrt(a**2*x**2 + 1)), x)
```

### 3.349.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.349.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

### 3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{asinh}(a\,x)^3}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.350 $\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)} dx$



## 3.350.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)} dx = \frac{35c^3\text{Chi}(\text{arcsinh}(ax))}{64a} + \frac{21c^3\text{Chi}(3\text{arcsinh}(ax))}{64a} + \frac{7c^3\text{Chi}(5\text{arcsinh}(ax))}{64a} + \frac{c^3\text{Chi}(7\text{arcsinh}(ax))}{64a}$$

output
```
35/64*c^3*Chi(arcsinh(a*x))/a+21/64*c^3*Chi(3*arcsinh(a*x))/a+7/64*c^3*Chi
(5*arcsinh(a*x))/a+1/64*c^3*Chi(7*arcsinh(a*x))/a
```

## 3.350.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)} dx = \frac{c^3(35\text{Chi}(\text{arcsinh}(ax)) + 21\text{Chi}(3\text{arcsinh}(ax)) + 7\text{Chi}(5\text{arcsinh}(ax)) + \text{Chi}(7\text{arcsinh}(ax)))}{64a}$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

output
```
(c^3*(35*CoshIntegral[ArcSinh[a*x]] + 21*CoshIntegral[3*ArcSinh[a*x]] + 7*
CoshIntegral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]]))/(64*a)
```

### 3.350.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\text{arcsinh}(ax)}\,dx$$

$$\Big\downarrow \; 6206$$

$$\frac{c^3\int \frac{(a^2x^2+1)^{7/2}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{c^3\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^7}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \; 3793$$

$$\frac{c^3\int \left(\frac{21\cosh(3\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{7\cosh(5\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{\cosh(7\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{35\sqrt{a^2x^2+1}}{64\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^3\left(\frac{35}{64}\text{Chi}(\text{arcsinh}(ax))+\frac{21}{64}\text{Chi}(3\text{arcsinh}(ax))+\frac{7}{64}\text{Chi}(5\text{arcsinh}(ax))+\frac{1}{64}\text{Chi}(7\text{arcsinh}(ax))\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

output
```
(c^3*((35*CoshIntegral[ArcSinh[a*x]])/64 + (21*CoshIntegral[3*ArcSinh[a*x]
])/64 + (7*CoshIntegral[5*ArcSinh[a*x]])/64 + CoshIntegral[7*ArcSinh[a*x]]
/64))/a
```

### 3.350.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.350.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{c^3(35\ \mathrm{Chi}(\mathrm{arcsinh}(ax))+21\ \mathrm{Chi}(3\ \mathrm{arcsinh}(ax))+7\ \mathrm{Chi}(5\ \mathrm{arcsinh}(ax))+\mathrm{Chi}(7\ \mathrm{arcsinh}(ax)))}{64a}$ | 42 |
| default | $\frac{c^3(35\ \mathrm{Chi}(\mathrm{arcsinh}(ax))+21\ \mathrm{Chi}(3\ \mathrm{arcsinh}(ax))+7\ \mathrm{Chi}(5\ \mathrm{arcsinh}(ax))+\mathrm{Chi}(7\ \mathrm{arcsinh}(ax)))}{64a}$ | 42 |

input
```
int((a^2*c*x^2+c)^3/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(35*Chi(arcsinh(a*x))+21*Chi(3*arcsinh(a*x))+7*Chi(5*arcsinh(a*
x))+Chi(7*arcsinh(a*x)))
```

### 3.350.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x),
 x)
```

### 3.350.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{asinh}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/asinh(a*x),x)
```

output
```
c**3*(Integral(3*a**2*x**2/asinh(a*x), x) + Integral(3*a**4*x**4/asinh(a*x
), x) + Integral(a**6*x**6/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

### 3.350.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

### 3.350.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

### 3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\operatorname{asinh}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^3/asinh(a*x),x)
```

output
```
int((c + a^2*c*x^2)^3/asinh(a*x), x)
```

# 3.351 $\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)} dx$



## 3.351.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)} dx = \frac{5c^2\text{Chi}(\text{arcsinh}(ax))}{8a} + \frac{5c^2\text{Chi}(3\text{arcsinh}(ax))}{16a} + \frac{c^2\text{Chi}(5\text{arcsinh}(ax))}{16a}$$

output
```
5/8*c^2*Chi(arcsinh(a*x))/a+5/16*c^2*Chi(3*arcsinh(a*x))/a+1/16*c^2*Chi(5*
arcsinh(a*x))/a
```

## 3.351.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)} dx = \frac{c^2(10\text{Chi}(\text{arcsinh}(ax)) + 5\text{Chi}(3\text{arcsinh}(ax)) + \text{Chi}(5\text{arcsinh}(ax)))}{16a}$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

output
```
(c^2*(10*CoshIntegral[ArcSinh[a*x]] + 5*CoshIntegral[3*ArcSinh[a*x]] + Cos
hIntegral[5*ArcSinh[a*x]]))/(16*a)
```

### 3.351.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{c^2\int \frac{(a^2x^2+1)^{5/2}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c^2\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^5}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{c^2\int \left(\frac{5\cosh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\cosh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{5\sqrt{a^2x^2+1}}{8\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\left(\frac{5}{8}\text{Chi}(\text{arcsinh}(ax))+\frac{5}{16}\text{Chi}(3\text{arcsinh}(ax))+\frac{1}{16}\text{Chi}(5\text{arcsinh}(ax))\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

output
```
(c^2*((5*CoshIntegral[ArcSinh[a*x]])/8 + (5*CoshIntegral[3*ArcSinh[a*x]])/
16 + CoshIntegral[5*ArcSinh[a*x]]/16))/a
```

### 3.351.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.351.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{c^{2}\left(10\,\mathrm{Chi}\left(\operatorname{arcsinh}\left(a x\right)\right)+5\,\mathrm{Chi}\left(3\,\operatorname{arcsinh}\left(a x\right)\right)+\mathrm{Chi}\left(5\,\operatorname{arcsinh}\left(a x\right)\right)\right)}{16 a}$ | 33 |
| default | $\frac{c^{2}\left(10\,\mathrm{Chi}\left(\operatorname{arcsinh}\left(a x\right)\right)+5\,\mathrm{Chi}\left(3\,\operatorname{arcsinh}\left(a x\right)\right)+\mathrm{Chi}\left(5\,\operatorname{arcsinh}\left(a x\right)\right)\right)}{16 a}$ | 33 |

input
```
int((a^2*c*x^2+c)^2/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(10*Chi(arcsinh(a*x))+5*Chi(3*arcsinh(a*x))+Chi(5*arcsinh(a*x))
)
```

### 3.351.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x), x)
```

### 3.351.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/asinh(a*x),x)
```

output
```
c**2*(Integral(2*a**2*x**2/asinh(a*x), x) + Integral(a**4*x**4/asinh(a*x),
 x) + Integral(1/asinh(a*x), x))
```

### 3.351.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

### 3.351.8 Giac [F]

$$\int \frac{\left(c+a^2cx^2\right)^2}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{\left(a^2cx^2+c\right)^2}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

### 3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^2}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{\left(c\,a^2\,x^2+c\right)^2}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^2/asinh(a*x),x)
```

output
```
int((c + a^2*c*x^2)^2/asinh(a*x), x)
```

# 3.352 $\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx$



## 3.352.1 Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx=\frac{3c\operatorname{Chi}(\operatorname{arcsinh}(ax))}{4a}+\frac{c\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{4a}$$

output
```
3/4*c*Chi(arcsinh(a*x))/a+1/4*c*Chi(3*arcsinh(a*x))/a
```

## 3.352.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx=\frac{c(3\operatorname{Chi}(\operatorname{arcsinh}(ax))+\operatorname{Chi}(3\operatorname{arcsinh}(ax)))}{4a}$$

input
```
Integrate[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

output
```
(c*(3*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]))/(4*a)
```

### 3.352.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\text{arcsinh}(ax)}\,dx$$
$$\Big\downarrow \text{6206}$$
$$\frac{c\int \frac{(a^2x^2+1)^{3/2}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{c\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^3}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$
$$\Big\downarrow \text{3793}$$
$$\frac{c\int \left(\frac{\cosh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}+\frac{3\sqrt{a^2x^2+1}}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c\left(\frac{3}{4}\text{Chi}(\text{arcsinh}(ax))+\frac{1}{4}\text{Chi}(3\text{arcsinh}(ax))\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

output
```
(c*((3*CoshIntegral[ArcSinh[a*x]])/4 + CoshIntegral[3*ArcSinh[a*x]]/4))/a
```

### 3.352.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.352.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{c(3\,\mathrm{Chi}(\mathrm{arcsinh}(ax))+\mathrm{Chi}(3\,\mathrm{arcsinh}(ax)))}{4a}$ | 22 |
| default | $\frac{c(3\,\mathrm{Chi}(\mathrm{arcsinh}(ax))+\mathrm{Chi}(3\,\mathrm{arcsinh}(ax)))}{4a}$ | 22 |

input
```
int((a^2*c*x^2+c)/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(3*Chi(arcsinh(a*x))+Chi(3*arcsinh(a*x)))
```

### 3.352.5 Fricas [F]

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)}\,dx = \int \frac{a^2cx^2+c}{\text{arsinh}\,(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

### 3.352.6 Sympy [F]

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)}\,dx = c\left(\int \frac{a^2x^2}{\text{asinh}\,(ax)}\,dx + \int \frac{1}{\text{asinh}\,(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/asinh(a*x),x)
```

output
```
c*(Integral(a**2*x**2/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

### 3.352.7 Maxima [F]

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)}\,dx = \int \frac{a^2cx^2+c}{\text{arsinh}\,(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

### 3.352.8 Giac [F]

$$\int \frac{c + a^2cx^2}{\text{arcsinh}(ax)}\,dx = \int \frac{a^2cx^2 + c}{\text{arsinh}\,(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

### 3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c + a^2cx^2}{\text{arcsinh}(ax)}\,dx = \int \frac{c\,a^2\,x^2 + c}{\text{asinh}\,(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)/asinh(a*x),x)
```

output
```
int((c + a^2*c*x^2)/asinh(a*x), x)
```

# 3.353 $\int \frac{1}{(c+a^2cx^2)\,\mathbf{arcsinh}(ax)}\,dx$



## 3.353.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \operatorname{Int}\left(\frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)},x\right)$$

output
```
Unintegrable(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

## 3.353.2 Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

## 3.353.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{\text{arcsinh}(ax)\,(a^2cx^2+c)}dx$$

input
```
Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

### 3.353.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.353.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2c\,x^2+c)\,\text{arcsinh}\,(ax)}dx$$

input
```
int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

output
```
int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

### 3.353.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

### 3.353.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \frac{\int \frac{1}{a^2x^2\,\text{asinh}\,(ax)+\text{asinh}\,(ax)}\,dx}{c}$$

input
```
integrate(1/(a**2*c*x**2+c)/asinh(a*x),x)
```

output
```
Integral(1/(a**2*x**2*asinh(a*x) + asinh(a*x)), x)/c
```

### 3.353.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

### 3.353.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

### 3.353.9 Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)), x)
```

# 3.354 $\int \frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)} dx$



## 3.354.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)} dx = \text{Int}\left(\frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)}, x\right)$$

output
```
Unintegrable(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

## 3.354.2 Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)} dx = \int \frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)} dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

## 3.354.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)\left(a^2cx^2+c\right)^2}\,dx$$

$$\Big\downarrow \; 6209$$

$$\int \frac{1}{\text{arcsinh}(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

### 3.354.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.354.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^2\text{arcsinh}\left(ax\right)}dx$$

input
```
int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

output
```
int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

### 3.354.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)), x)
```

### 3.354.6 Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{asinh}(ax)+2a^2x^2 \operatorname{asinh}(ax)+\operatorname{asinh}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/asinh(a*x),x)
```

output
```
Integral(1/(a**4*x**4*asinh(a*x) + 2*a**2*x**2*asinh(a*x) + asinh(a*x)), x
)/c**2
```

### 3.354.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

### 3.354.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

### 3.354.9 Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)^2), x)
```

# 3.355 $\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx$



## 3.355.1 Optimal result

Integrand size = 27, antiderivative size = 206

$$\begin{aligned}\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)}\,dx = &-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5}\\ &-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}\\ &+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5}\\ &+\frac{\log(a+b\,\text{arcsinh}(cx))}{16bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5}\\ &+\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}\\ &-\frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^5}\end{aligned}$$

output
```
-1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5-1/16*Chi(4*(a+b*arcsin
h(c*x))/b)*cosh(4*a/b)/b/c^5+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/
b/c^5+1/16*ln(a+b*arcsinh(c*x))/b/c^5+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sin
h(2*a/b)/b/c^5+1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^5-1/32*Shi
(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^5
```

### 3.355.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\text{arcsi}\right.\right.}{}$$

input
```
Integrate[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) - 2*Cosh[(4*a)/b]*C
oshIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b +
ArcSinh[c*x])] + 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*
(a/b + ArcSinh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x]
)] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^5)
```

### 3.355.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^5}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{1}{16(a+b\text{arcsinh}(cx))}\right)d(a}{bc^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{16}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right.$$

input
```
Int[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
(-1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]) - (Cosh[(4
*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/16 + (Cosh[(6*a)/b]*CoshI
ntegral[(6*(a + b*ArcSinh[c*x]))/b])/32 + Log[a + b*ArcSinh[c*x]]/16 + (Si
nh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/32 + (Sinh[(4*a)/b]*
SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/16 - (Sinh[(6*a)/b]*SinhIntegral
[(6*(a + b*ArcSinh[c*x]))/b])/32)/(b*c^5)
```

### 3.355.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.355.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $-\frac{e^{\frac{6a}{b}}\text{Ei}_1\left(6\text{ arcsinh}(cx)+\frac{6a}{b}\right)-2e^{\frac{4a}{b}}\text{Ei}_1\left(4\text{ arcsinh}(cx)+\frac{4a}{b}\right)-e^{\frac{2a}{b}}\text{Ei}_1\left(2\text{ arcsinh}(cx)+\frac{2a}{b}\right)-e^{-\frac{2a}{b}}\text{Ei}_1\left(-2\text{ arcsinh}(cx)-\frac{2a}{b}\right)-2e^{-}}{64c^5b}$ |

3.355. $\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$

input
```
int(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-2*exp(4*a/b)*Ei(1,4*arcsinh(c
*x)+4*a/b)-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-exp(-2*a/b)*Ei(1,-2*arcsi
nh(c*x)-2*a/b)-2*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(-6*a/b)*Ei(1,
-6*arcsinh(c*x)-6*a/b)-4*ln(a+b*arcsinh(c*x)))/c^5/b
```

### 3.355.5 Fricas [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^4}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

### 3.355.6 Sympy [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^4\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**4*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**4*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

### 3.355.7 Maxima [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^4}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

### 3.355.8 Giac [F]

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^4}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

### 3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^4\,\sqrt{c^2\,x^2+1}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

# 3.356 $\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.356.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^4} + \frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^4} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^4} - \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^4}$$

output
```
-1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/16*cosh(3*a/b)*Shi(3*(a+b
*arcsinh(c*x))/b)/b/c^4+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^4
+1/8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4+1/16*Chi(3*(a+b*arcsinh(c*x
))/b)*sinh(3*a/b)/b/c^4-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4
```

### 3.356.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{2\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right.}{}$$

input
```
Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
(2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcS
inh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)
/b] - 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - Cosh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSin
h[c*x])])/(16*b*c^4)
```

### 3.356.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}\,dx$$
$$\Big\downarrow \; 6234$$
$$\frac{\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$
$$\Big\downarrow \; 25$$
$$-\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$
$$\Big\downarrow \; 5971$$

$$-\frac{\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\text{c}}{bc^4}$$

input
```
Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
((CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/8 + (CoshIntegral[(3*(a
+ b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/16 - (CoshIntegral[(5*(a + b*ArcSinh[
c*x]))/b]*Sinh[(5*a)/b])/16 - (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])
/b])/8 - (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16 + (Co
sh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16)/(b*c^4)
```

### 3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.356. $\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$

### 3.356.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| default | $\frac{e^{\frac{5a}{b}} \mathrm{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)-e^{\frac{3a}{b}} \mathrm{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)-2 e^{\frac{a}{b}} \mathrm{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)+2 e^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)+e^{-\frac{3a}{b}} \mathrm{Ei}_1(-3}{32c^4 b}$ |

input
```
int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/32*(exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-exp(3*a/b)*Ei(1,3*arcsinh(c*x)
+3*a/b)-2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a
/b)+exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcsinh(c*
x)-5*a/b))/c^4/b
```

### 3.356.5 Fricas [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

### 3.356.6 Sympy [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{x^3\sqrt{c^2x^2+1}}{a+b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

### 3.356.7 Maxima [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

### 3.356.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3\,\sqrt{c^2\,x^2+1}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

# 3.357 $\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.357.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3}-\frac{\log(a+b\text{arcsinh}(cx))}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^3}$$

output
```
1/8*Chi(4*(a+b*arcsinh(c*x))/b)*cosh(4*a/b)/b/c^3-1/8*ln(a+b*arcsinh(c*x))
/b/c^3-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3
```

## 3.357.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-\log(a+b\text{arcsinh}(cx))-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{8bc^3}$$

input
```
Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
(Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*
x]] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])])/(8*b*c^3)
```

### 3.357.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}-\frac{1}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{8}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\log(a+b\text{arcsinh}(cx))}{bc^3}$$

input
```
Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
((Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8 - Log[a + b*Ar
cSinh[c*x]]/8 - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8
)/(b*c^3)
```

### 3.357.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.357.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $-\frac{{\mathrm e}^{\frac{4 a}{b}} \operatorname{Ei}_{1}\left(4 \,\operatorname{arcsinh}\left(c x \right)+\frac{4 a}{b}\right)+{\mathrm e}^{-\frac{4 a}{b}} \operatorname{Ei}_{1}\left(-4 \,\operatorname{arcsinh}\left(c x \right)-\frac{4 a}{b}\right)+2 \ln \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)}{16 c^{3} b}$ | 67 |

input
```
int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+exp(-4*a/b)*Ei(1,-4*arcsinh(c
*x)-4*a/b)+2*ln(a+b*arcsinh(c*x)))/c^3/b
```

## 3.357.5 Fricas [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

### 3.357.6 Sympy [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

### 3.357.7 Maxima [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

### 3.357.8 Giac [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

### 3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

input
```
int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

# 3.358 $\int \frac{x\sqrt{1+c^2x^2}}{a+b\,\mathbf{arcsinh}(cx)}\,dx$



## 3.358.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = -\frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^2} - \frac{\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^2} + \frac{\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^2}$$

output
```
1/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+1/4*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^2-1/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-1/4*Ch
i(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2
```

## 3.358.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{-\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+}{4bc^2}$$

input
```
Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
(-(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + C
osh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^2)
```

### 3.358.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh}{bc^2}$$

input
```
Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
(-1/4*(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b]) - (CoshIntegral[(3*
(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/4 + (Cosh[a/b]*SinhIntegral[(a + b
*ArcSinh[c*x])/b])/4 + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x])
)/b])/4)/(b*c^2)
```

### 3.358.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.358.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)+e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)-e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)-e^{-\frac{3a}{b}} \operatorname{Ei}_1\left(-3 \operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}$ | 100 |

input
```
int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*(exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+exp(a/b)*Ei(1,arcsinh(c*x)+a/b)
-exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
)/c^2/b
```

### 3.358.5 Fricas [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

### 3.358.6 Sympy [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

### 3.358.7 Maxima [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

### 3.358.8 Giac [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

### 3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x\,\sqrt{c^2\,x^2+1}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

# 3.359 $\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.359.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc}+\frac{\log(a+b\text{arcsinh}(cx))}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc}$$

output
```
1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/2*ln(a+b*arcsinh(c*x))/b
/c-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c
```

## 3.359.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\log(a+b\text{arcsinh}(cx))-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{2bc}$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Log[a + b*ArcSinh[c*
x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c)
```

### 3.359.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^2}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}+\frac{1}{2(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{2}\log(a+b\text{arcsinh}(cx))}{bc}$$

input
```
Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/2 + Log[a + b*Ar
cSinh[c*x]]/2 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/2
)/(b*c)
```

### 3.359.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.359.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $-\frac{e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)+e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)-2\ln\left(a+b \operatorname{arcsinh}(cx)\right)}{4bc}$ | 67 |

input
```
int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*
x)-2*a/b)-2*ln(a+b*arcsinh(c*x)))/b/c
```

### 3.359.5 Fricas [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

### 3.359.6 Sympy [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}}{a+b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

### 3.359.7 Maxima [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

### 3.359.8 Giac [F]

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

### 3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)), x)
```

# 3.360 $\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx$



## 3.360.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = -\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b} + \text{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b-Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/
b+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

## 3.360.2 Mathematica [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])), x]
```

## 3.360.3 Rubi [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x(a+b\operatorname{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6235}$$
$$\int \left(\frac{c^2x}{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}+\frac{1}{x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\int \frac{1}{x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}dx-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b}$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.360.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

### 3.360.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x\,(a+b\ \mathrm{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

### 3.360.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\mathrm{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*x), x)
```

### 3.360.6 Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\mathrm{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{x\,(a+b\,\mathrm{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))), x)
```

### 3.360.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x), x)
```

### 3.360.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.360.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x\,\left(a+b\,\text{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))), x)
```

# 3.361 $\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$



## 3.361.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \frac{c\log(a+b\text{arcsinh}(cx))}{b}+\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
c*ln(a+b*arcsinh(c*x))/b+Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)
^(1/2),x)
```

## 3.361.2 Mathematica [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])), x]
```

## 3.361.3 Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x^2(a+b\text{arcsinh}(cx))}dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{c^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx+\frac{c\log(a+b\text{arcsinh}(cx))}{b}$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.361.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

### 3.361.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
```

### 3.361.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

### 3.361.6 Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^2\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))), x)
```

### 3.361.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

### 3.361.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

### 3.361.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^2\,\left(a+b\,\text{asinh}\,(c\,x)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))), x)
```

# 3.362 $\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$



## 3.362.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

## 3.362.2 Mathematica [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

## 3.362.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x^3(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{\sqrt{c^2x^2+1}}{x^3(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.362.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.362.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

### 3.362.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

### 3.362.6 Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{x^3\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))), x)
```

### 3.362.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^3), x)
```

### 3.362.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.362.9 Mupad [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))), x)
```

# 3.363 $\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))} dx$



## 3.363.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))} dx = \text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}, x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

## 3.363.2 Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))} dx = \int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))} dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

## 3.363.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$
$$\Big\downarrow \; 6239$$
$$\int \frac{\sqrt{c^2x^2+1}}{x^4(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.363.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.363.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

### 3.363.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

### 3.363.6 Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^4\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))), x)
```

### 3.363.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

### 3.363.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

### 3.363.9 Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^4\,\left(a+b\,\text{asinh}\,(c\,x)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))), x)
```

# 3.364 $\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.364.1 Optimal result

Integrand size = 27, antiderivative size = 245

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^4} + \frac{3\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^4} - \frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^4} - \frac{\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} + \frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^4}$$

output
```
-3/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-3/64*cosh(3*a/b)*Shi(3*(a+
b*arcsinh(c*x))/b)/b/c^4+1/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^
4+1/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^4+3/64*Chi((a+b*arcsinh
(c*x))/b)*sinh(a/b)/b/c^4+3/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c
^4-1/64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4-1/64*Chi(7*(a+b*arcs
inh(c*x))/b)*sinh(7*a/b)/b/c^4
```

### 3.364.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{3\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+3\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}\right.\right.}{}$$

input
```
Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*
a)/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] - 3*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + Ar
cSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(
7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^4)
```

### 3.364.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c^2x^2+1\right)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \ 6234$$

$$\frac{\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$

$$\Big\downarrow \ 5971$$

$$-\frac{\int \left(\frac{\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{3\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64(a+b\text{arcsinh}(cx))}\right.}{bc^4}$$

$\big\downarrow$ 2009

$$\frac{\frac{3}{64}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{3}{64}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{64}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{64}}{}$$

input
```
Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
((3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/64 + (3*CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/64 - (CoshIntegral[(5*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(5*a)/b])/64 - (CoshIntegral[(7*(a + b*ArcSinh[c*x]))/
b]*Sinh[(7*a)/b])/64 - (3*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/
64 - (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/64 + (Cosh
[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/64 + (Cosh[(7*a)/b]*Si
nhIntegral[(7*(a + b*ArcSinh[c*x]))/b])/64)/(b*c^4)
```

### 3.364.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.364. $\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$

### 3.364.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{e^{\frac{7a}{b}} \operatorname{Ei}_1\left(7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)+e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)-3 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)-3 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)+3 e^{-\frac{a}{b}} \operatorname{Ei}_1(-}{128c^4b}$ |

input
```
int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/128*(exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+exp(5*a/b)*Ei(1,5*arcsinh(c*x
)+5*a/b)-3*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3*exp(a/b)*Ei(1,arcsinh(c
*x)+a/b)+3*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+3*exp(-3*a/b)*Ei(1,-3*arcsinh
(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-exp(-7*a/b)*Ei(1,-7*a
rcsinh(c*x)-7*a/b))/c^4/b
```

### 3.364.5 Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

### 3.364.6 Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

### 3.364.7 Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)*x^3/(b*arcsinh(c*x) + a), x)
```

### 3.364.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{3/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

# 3.365 $\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.365.1 Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\log(a+b\text{arcsinh}(cx))}{16bc^3} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^3} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^3}$$

output
```
-1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/16*Chi(4*(a+b*arcsin
h(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/
b/c^3-1/16*ln(a+b*arcsinh(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sin
h(2*a/b)/b/c^3-1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi
(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^3
```

### 3.365.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\text{co}$$

input
```
Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) + 2*Cosh[(4*a)/b]*C
oshIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b +
ArcSinh[c*x])] - 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*
(a/b + ArcSinh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x]
)] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^3)
```

### 3.365.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c^2x^2+1\right)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}-\frac{1}{16(a+b\text{arcsinh}(cx))}\right)d(a}{bc^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right.$$

input
```
Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(-1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]) + (Cosh[(4
*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/16 + (Cosh[(6*a)/b]*CoshI
ntegral[(6*(a + b*ArcSinh[c*x]))/b])/32 - Log[a + b*ArcSinh[c*x]]/16 + (Si
nh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/32 - (Sinh[(4*a)/b]*
SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/16 - (Sinh[(6*a)/b]*SinhIntegral
[(6*(a + b*ArcSinh[c*x]))/b])/32)/(b*c^3)
```

### 3.365.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.365.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $-\frac{e^{\frac{6a}{b}}\text{Ei}_1\left(6\text{ arcsinh}(cx)+\frac{6a}{b}\right)+2e^{\frac{4a}{b}}\text{Ei}_1\left(4\text{ arcsinh}(cx)+\frac{4a}{b}\right)-e^{\frac{2a}{b}}\text{Ei}_1\left(2\text{ arcsinh}(cx)+\frac{2a}{b}\right)-e^{-\frac{2a}{b}}\text{Ei}_1\left(-2\text{ arcsinh}(cx)-\frac{2a}{b}\right)+2e^{-}}{64c^3b}$ |

input
```
int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)+2*exp(4*a/b)*Ei(1,4*arcsinh(c
*x)+4*a/b)-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-exp(-2*a/b)*Ei(1,-2*arcsi
nh(c*x)-2*a/b)+2*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(-6*a/b)*Ei(1,
-6*arcsinh(c*x)-6*a/b)+4*ln(a+b*arcsinh(c*x)))/c^3/b
```

### 3.365.5 Fricas [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

### 3.365.6 Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

### 3.365.7 Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

### 3.365.8 Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

### 3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2\,(c^2\,x^2+1)^{3/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

# 3.366 $\int \frac{x(1+c^2x^2)^{3/2}}{a+b\textbf{arcsinh}(cx)}\,dx$



## 3.366.1 Optimal result

Integrand size = 25, antiderivative size = 183

$$\begin{aligned}\int \frac{x(1+c^2x^2)^{3/2}}{a+b\operatorname{arcsinh}(cx)}\,dx &= -\frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^2} \\ &- \frac{3\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^2} \\ &- \frac{\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^2} \\ &+ \frac{3\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^2} + \frac{\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^2}\end{aligned}$$

output
```
1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+3/16*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^2+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2-
1/8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-3/16*Chi(3*(a+b*arcsinh(c*x)
)/b)*sinh(3*a/b)/b/c^2-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2
```

### 3.366.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{-2\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-3\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}(5(}{}$$

input
```
Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(-2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + A
rcSinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5
*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + A
rcSinh[c*x])])/(16*b*c^2)
```

### 3.366.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c^2x^2+1\right)^{3/2}}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{3}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{8}}{bc^2}$$

input
```
Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(-1/8*(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b]) - (3*CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/16 - (CoshIntegral[(5*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[
c*x])/b])/8 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/1
6 + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16)/(b*c^2)
```

### 3.366.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.366. $\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$

### 3.366.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| default | $\frac{e^{\frac{5a}{b}} \mathrm{Ei}_1 (5 \operatorname{arcsinh}(cx)+\frac{5a}{b})+3 e^{\frac{3a}{b}} \mathrm{Ei}_1 (3 \operatorname{arcsinh}(cx)+\frac{3a}{b})+2 e^{\frac{a}{b}} \mathrm{Ei}_1 (\operatorname{arcsinh}(cx)+\frac{a}{b})-2 e^{-\frac{a}{b}} \mathrm{Ei}_1 (-\operatorname{arcsinh}(cx)-\frac{a}{b})-3 e^{-\frac{3a}{b}} \mathrm{Ei}_1 (}{32c^2b}$ |

input
```
int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/32*(exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3*exp(3*a/b)*Ei(1,3*arcsinh(c*
x)+3*a/b)+2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-2*exp(-a/b)*Ei(1,-arcsinh(c*x)
-a/b)-3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcsin
h(c*x)-5*a/b))/c^2/b
```

### 3.366.5 Fricas [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

### 3.366.6 Sympy [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

### 3.366.7 Maxima [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)*x/(b*arcsinh(c*x) + a), x)
```

### 3.366.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x\,(c^2\,x^2+1)^{3/2}}{a+b\operatorname{asinh}(c\,x)}\,dx$$

input
```
int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

# 3.367 $\int \frac{(1+c^2x^2)^{3/2}}{a+b\textbf{arcsinh}(cx)}\,dx$



## 3.367.1 Optimal result

Integrand size = 24, antiderivative size = 144

$$\begin{aligned}\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx &= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} \\ &+ \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc} + \frac{3\log(a+b\text{arcsinh}(cx))}{8bc} \\ &- \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc}\end{aligned}$$

output
```
1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/8*Chi(4*(a+b*arcsinh(c*x
))/b)*cosh(4*a/b)/b/c+3/8*ln(a+b*arcsinh(c*x))/b/c-1/2*Shi(2*(a+b*arcsinh(
c*x))/b)*sinh(2*a/b)/b/c-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c
```

## 3.367.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+3\log}{}$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*Cosh
Integral[4*(a/b + ArcSinh[c*x])] + 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcSinh[c*x])])/(8*b*c)
```

## 3.367.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2x^2+1\right)^{3/2}}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^4}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}+\frac{3}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{8}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}}{bc}$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

3.367. $\int \frac{\left(1+c^2x^2\right)^{3/2}}{a+b\text{arcsinh}(cx)}dx$

output
```
((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/2 + (Cosh[(4*a)/
b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8 + (3*Log[a + b*ArcSinh[c*x]
])/8 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/2 - (Sinh[
(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8)/(b*c)
```

### 3.367.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.367.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $-\frac{e^{\frac{4a}{b}} \mathrm{Ei}_1\left(4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)+4\,e^{\frac{2a}{b}} \mathrm{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)+4\,e^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)+e^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4 \operatorname{arcsinh}(cx)-\frac{4a}{b}\right)-}{16bc}$ |

input
```
int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+4*exp(2*a/b)*Ei(1,2*arcsinh(c
*x)+2*a/b)+4*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+exp(-4*a/b)*Ei(1,-4*a
rcsinh(c*x)-4*a/b)-6*ln(a+b*arcsinh(c*x)))/b/c
```

### 3.367.5 Fricas [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

### 3.367.6 Sympy [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

### 3.367.7 Maxima [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

### 3.367.8 Giac [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

### 3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2\,x^2+1)^{3/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)), x)
```

# 3.368 $\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))} \, dx$



## 3.368.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))} \, dx = -\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b} - \frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b} + \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b} + \text{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}, x\right)$$

output
```
5/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+1/4*cosh(3*a/b)*Shi(3*(a+b*arcsi
nh(c*x))/b)/b-5/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b-1/4*Chi(3*(a+b*arc
sinh(c*x))/b)*sinh(3*a/b)/b+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1
)^(1/2),x)
```

### 3.368.2 Mathematica [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])), x]
```

### 3.368.3 Rubi [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{2c^2x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{1}{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{c^4x^3}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b}$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.368.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

## 3.368.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\,\operatorname{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
```

## 3.368.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\operatorname{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b*x*arcsinh(c*x) + a*x), x)
```

## 3.368.6 Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))), x)
```

## 3.368.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x), x)
```

## 3.368.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.368.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))), x)
```

# 3.369 $\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\textbf{arcsinh}(cx))}\,dx$



## 3.369.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2b} + \frac{3c\log(a+b\text{arcsinh}(cx))}{2b} - \frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2b} + \text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
1/2*c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+3/2*c*ln(a+b*arcsinh(c*x))
/b-1/2*c*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b+Unintegrable(1/x^2/(a+b
*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

## 3.369.2 Mathematica [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

### 3.369.3 Rubi [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2x^2+1\right)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{2c^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{c^4x^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx+\frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2b}-\frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2b}+\frac{3c\log(a+b\text{arcsinh}(cx))}{2b}$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.369.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

## 3.369.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2 (a+b \operatorname{arcsinh}(cx))}dx$$

input
```
int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
```

## 3.369.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

### 3.369.6 Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))), x)
```

### 3.369.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

### 3.369.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

### 3.369.9 Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))), x)
```

# 3.370 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\,\text{arcsinh}(cx))}\,dx$



## 3.370.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

## 3.370.2 Mathematica [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

## 3.370.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \text{ 6239}$$

$$\int \frac{(c^2x^2+1)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.370.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.370.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

### 3.370.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

### 3.370.6 Sympy [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))), x)
```

### 3.370.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

### 3.370.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.370.9 Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))), x)
```

# 3.371 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\,\mathbf{arcsinh}(cx))}\,dx$



## 3.371.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\,\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^4(a+b\,\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

## 3.371.2 Mathematica [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\,\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\,\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

## 3.371.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$
$$\Big\downarrow \; 6239$$
$$\int \frac{(c^2x^2+1)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.371.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.371.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

### 3.371.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

### 3.371.6 Sympy [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))), x)
```

### 3.371.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

### 3.371.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

### 3.371.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2\,x^2+1)^{3/2}}{x^4\,\left(a+b\,\text{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))), x)
```

# 3.372 $\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.372.1 Optimal result

Integrand size = 27, antiderivative size = 245

$$\begin{aligned}\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx &= \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{128bc^4} \\ &+ \frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32bc^4} \\ &- \frac{3\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{256bc^4} - \frac{\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{9a}{b}\right)}{256bc^4} \\ &- \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128bc^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32bc^4} \\ &+ \frac{3\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4} + \frac{\cosh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256bc^4}\end{aligned}$$

output
```
-3/128*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/32*cosh(3*a/b)*Shi(3*(a
+b*arcsinh(c*x))/b)/b/c^4+3/256*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/
c^4+1/256*cosh(9*a/b)*Shi(9*(a+b*arcsinh(c*x))/b)/b/c^4+3/128*Chi((a+b*arc
sinh(c*x))/b)*sinh(a/b)/b/c^4+1/32*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)
/b/c^4-3/256*Chi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/b/c^4-1/256*Chi(9*(a+
b*arcsinh(c*x))/b)*sinh(9*a/b)/b/c^4
```

### 3.372.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{6\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+8\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\text{Chi}(7(}{}$$

input
```
Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(6*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 8*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - 3*CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(
7*a)/b] - CoshIntegral[9*(a/b + ArcSinh[c*x])]*Sinh[(9*a)/b] - 6*Cosh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - 8*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
ArcSinh[c*x])] + 3*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + Co
sh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(256*b*c^4)
```

### 3.372.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c^2x^2+1\right)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \; 6234$$

$$\frac{\int -\frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$

$$\Big\downarrow \; 5971$$

$$-\frac{\int \left(\frac{\sinh\left(\frac{9a}{b}-\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256(a+b\text{arcsinh}(cx))}+\frac{3\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256(a+b\text{arcsinh}(cx))}-\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}-\frac{3\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{128(a+b\text{arcsinh}(cx))}\right.}{bc^4}$$

$\downarrow$ 2009

$$\frac{\frac{3}{128}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{32}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{256}\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)-}{}$$

input
```
Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
((3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/128 + (CoshIntegral[(3
*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/32 - (3*CoshIntegral[(7*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(7*a)/b])/256 - (CoshIntegral[(9*(a + b*ArcSinh[c*x])
)/b]*Sinh[(9*a)/b])/256 - (3*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/128 - (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/32 + (3*
Cosh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSinh[c*x]))/b])/256 + (Cosh[(9*a)/
b]*SinhIntegral[(9*(a + b*ArcSinh[c*x]))/b])/256)/(b*c^4)
```

### 3.372.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.372.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{e^{\frac{9a}{b}} \operatorname{Ei}_1\left(9 \operatorname{arcsinh}(cx)+\frac{9a}{b}\right)+3 e^{\frac{7a}{b}} \operatorname{Ei}_1\left(7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)-8 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)-6 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)+6 e^{-\frac{a}{b}} \operatorname{Ei}_1(\cdots}{512c^4b}$ |

input
```
int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/512*(exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)+3*exp(7*a/b)*Ei(1,7*arcsinh(c
*x)+7*a/b)-8*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-6*exp(a/b)*Ei(1,arcsinh
(c*x)+a/b)+6*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+8*exp(-3*a/b)*Ei(1,-3*arcsi
nh(c*x)-3*a/b)-3*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-exp(-9*a/b)*Ei(1,
-9*arcsinh(c*x)-9*a/b))/c^4/b
```

### 3.372.5 Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a
), x)
```

### 3.372.6 Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

### 3.372.7 Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)*x^3/(b*arcsinh(c*x) + a), x)
```

### 3.372.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{5/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

# 3.373 $\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\mathbf{arcsinh}(cx)}\,dx$



## 3.373.1 Optimal result

Integrand size = 27, antiderivative size = 268

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\operatorname{arcsinh}(cx)}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} + \frac{\cosh\left(\frac{8a}{b}\right)\operatorname{Chi}\left(\frac{8(a+b\operatorname{arcsinh}(cx))}{b}\right)}{128bc^3} - \frac{5\log(a+b\operatorname{arcsinh}(cx))}{128bc^3} + \frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc^3} - \frac{\sinh\left(\frac{8a}{b}\right)\operatorname{Shi}\left(\frac{8(a+b\operatorname{arcsinh}(cx))}{b}\right)}{128bc^3}$$

output
```
-1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/32*Chi(4*(a+b*arcsin
h(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/
b/c^3+1/128*Chi(8*(a+b*arcsinh(c*x))/b)*cosh(8*a/b)/b/c^3-5/128*ln(a+b*arc
sinh(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3-1/32*S
hi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi(6*(a+b*arcsinh(c*x))
/b)*sinh(6*a/b)/b/c^3-1/128*Shi(8*(a+b*arcsinh(c*x))/b)*sinh(8*a/b)/b/c^3
```

## 3.373.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.74

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+4}{}$$

input
```
Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 4*Cosh[(4*a)/b]*C
oshIntegral[4*(a/b + ArcSinh[c*x])] + 4*Cosh[(6*a)/b]*CoshIntegral[6*(a/b
+ ArcSinh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcSinh[c*x])] - 5*
Log[a + b*ArcSinh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*
x])] - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*Sinh[(6*a)
/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - Sinh[(8*a)/b]*SinhIntegral[8*(a
/b + ArcSinh[c*x])])/(128*b*c^3)
```

## 3.373.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c^2x^2+1\right)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \textbf{6234}$$

$$\frac{\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \textbf{5971}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{8a}{b}-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx)}{b}\right.}{32(a+b\text{arcsinh}(cx))}\right.}{bc^3}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{1}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right.$$

input
```
Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(-1/32*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]) + (Cosh[(4
*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/32 + (Cosh[(6*a)/b]*CoshI
ntegral[(6*(a + b*ArcSinh[c*x]))/b])/32 + (Cosh[(8*a)/b]*CoshIntegral[(8*(
a + b*ArcSinh[c*x]))/b])/128 - (5*Log[a + b*ArcSinh[c*x]])/128 + (Sinh[(2*
a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/32 - (Sinh[(4*a)/b]*SinhIn
tegral[(4*(a + b*ArcSinh[c*x]))/b])/32 - (Sinh[(6*a)/b]*SinhIntegral[(6*(a
 + b*ArcSinh[c*x]))/b])/32 - (Sinh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcSinh
[c*x]))/b])/128)/(b*c^3)
```

### 3.373.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.373. $\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$

### 3.373.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $-\frac{e^{\frac{8a}{b}} \operatorname{Ei}_1\left(8 \operatorname{arcsinh}(cx)+\frac{8a}{b}\right)+4 e^{\frac{6a}{b}} \operatorname{Ei}_1\left(6 \operatorname{arcsinh}(cx)+\frac{6a}{b}\right)+4 e^{\frac{4a}{b}} \operatorname{Ei}_1\left(4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)-4 e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)-4 e^{-}}{}$ |

input
```
int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/256*(exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)+4*exp(6*a/b)*Ei(1,6*arcsinh(
c*x)+6*a/b)+4*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-4*exp(2*a/b)*Ei(1,2*ar
csinh(c*x)+2*a/b)-4*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+4*exp(-4*a/b)*
Ei(1,-4*arcsinh(c*x)-4*a/b)+4*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)+exp(
-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)+10*ln(a+b*arcsinh(c*x)))/c^3/b
```

### 3.373.5 Fricas [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a
), x)
```

### 3.373.6 Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2(c^2x^2+1)^{\frac{5}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

### 3.373.7 Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

### 3.373.8 Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

### 3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^2\,(c^2\,x^2+1)^{5/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

# 3.374 $\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.374.1 Optimal result

Integrand size = 25, antiderivative size = 245

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = -\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64bc^2} - \frac{9\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64bc^2} - \frac{5\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64bc^2} - \frac{\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64bc^2} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2} + \frac{\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^2}$$

output
```
5/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+9/64*cosh(3*a/b)*Shi(3*(a+b
*arcsinh(c*x))/b)/b/c^2+5/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2
+1/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^2-5/64*Chi((a+b*arcsinh(
c*x))/b)*sinh(a/b)/b/c^2-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^
2-5/64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2-1/64*Chi(7*(a+b*arcsi
nh(c*x))/b)*sinh(7*a/b)/b/c^2
```

### 3.374.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{-5\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-9\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-5\operatorname{Chi}(5}{}$$

input
```
Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
(-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 9*CoshIntegral[3*(a/b + A
rcSinh[c*x])]*Sinh[(3*a)/b] - 5*CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[
(5*a)/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] + 5*Cosh[a/b
]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcSinh[c*x])] + 5*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + C
osh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^2)
```

### 3.374.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c^2x^2+1\right)^{5/2}}{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \ 6234$$

$$\frac{\int -\frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{a+b\operatorname{arcsinh}(cx)}d(a+b\operatorname{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{a+b\operatorname{arcsinh}(cx)}d(a+b\operatorname{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \ 5971$$

$$-\frac{\int \left(\frac{\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{5\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{5\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64(a+b\text{arcsinh}(cx))}\right.}{bc^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{5}{64}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{9}{64}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{5}{64}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-}{}$$

input
```
Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
((-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/64 - (9*CoshIntegral[
(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/64 - (5*CoshIntegral[(5*(a + b*
ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/64 - (CoshIntegral[(7*(a + b*ArcSinh[c*x]
))/b]*Sinh[(7*a)/b])/64 + (5*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/64 + (9*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/64 + (5
*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/64 + (Cosh[(7*a)/
b]*SinhIntegral[(7*(a + b*ArcSinh[c*x]))/b])/64)/(b*c^2)
```

### 3.374.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

---

3.374. $\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$

### 3.374.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{e^{\frac{7a}{b}} \operatorname{Ei}_1\left(7 \operatorname{arcsinh}(cx)+\frac{7a}{b}\right)+5 e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)+9 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)+5 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)-5 e^{-\frac{a}{b}} \operatorname{Ei}_1(-}{128c^2b}$ |

input
```
int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/128*(exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+5*exp(5*a/b)*Ei(1,5*arcsinh(c
*x)+5*a/b)+9*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+5*exp(a/b)*Ei(1,arcsinh
(c*x)+a/b)-5*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-9*exp(-3*a/b)*Ei(1,-3*arcsi
nh(c*x)-3*a/b)-5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-exp(-7*a/b)*Ei(1,
-7*arcsinh(c*x)-7*a/b))/c^2/b
```

### 3.374.5 Fricas [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
 x)
```

### 3.374.6 Sympy [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x(c^2x^2+1)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

### 3.374.7 Maxima [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)*x/(b*arcsinh(c*x) + a), x)
```

### 3.374.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x\,(c^2\,x^2+1)^{5/2}}{a+b\,\operatorname{asinh}(c\,x)}\,dx$$

input
```
int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

# 3.375 $\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.375.1 Optimal result

Integrand size = 24, antiderivative size = 206

$$\begin{aligned}\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx &= \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} \\ &+ \frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} + \frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} \\ &+ \frac{5\log(a+b\text{arcsinh}(cx))}{16bc} - \frac{15\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32bc} \\ &- \frac{3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} - \frac{\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32bc}\end{aligned}$$

output
```
15/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+3/16*Chi(4*(a+b*arcsinh(
c*x))/b)*cosh(4*a/b)/b/c+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/c+
5/16*ln(a+b*arcsinh(c*x))/b/c-15/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b
)/b/c-3/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c-1/32*Shi(6*(a+b*arc
sinh(c*x))/b)*sinh(6*a/b)/b/c
```

### 3.375.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\text{co}$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 6*Cosh[(4*a)/b]*C
oshIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b +
ArcSinh[c*x])] + 10*Log[a + b*ArcSinh[c*x]] - 15*Sinh[(2*a)/b]*SinhIntegra
l[2*(a/b + ArcSinh[c*x])] - 6*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c)
```

### 3.375.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^6}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{15\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{5}{16(a+b\text{arcsinh}(cx))}\right)}{bc}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{15}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{3}{16}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)+}{}$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
((15*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/32 + (3*Cosh[
(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/16 + (Cosh[(6*a)/b]*Cos
hIntegral[(6*(a + b*ArcSinh[c*x]))/b])/32 + (5*Log[a + b*ArcSinh[c*x]])/16
 - (15*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/32 - (3*Sin
h[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/16 - (Sinh[(6*a)/b]*S
inhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/32)/(b*c)
```

### 3.375.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

3.375. $\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$

### 3.375.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $-\frac{e^{\frac{6a}{b}} \operatorname{Ei}_1\left(6 \operatorname{arcsinh}(cx)+\frac{6a}{b}\right)+6 e^{\frac{4a}{b}} \operatorname{Ei}_1\left(4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)+15 e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)+15 e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)+}{64bc}$ |

input
```
int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)+6*exp(4*a/b)*Ei(1,4*arcsinh(c
*x)+4*a/b)+15*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+15*exp(-2*a/b)*Ei(1,-2
*arcsinh(c*x)-2*a/b)+6*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(-6*a/b)
*Ei(1,-6*arcsinh(c*x)-6*a/b)-20*ln(a+b*arcsinh(c*x)))/b/c
```

### 3.375.5 Fricas [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
 x)
```

### 3.375.6 Sympy [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

### 3.375.7 Maxima [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

### 3.375.8 Giac [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

### 3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2\,x^2+1)^{5/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)), x)
```

# 3.376 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx$



## 3.376.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\begin{aligned}\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx &= -\frac{11\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b}\\ &\quad -\frac{7\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b}-\frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b}\\ &\quad +\frac{11\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b}+\frac{7\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b}\\ &\quad +\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b}+\text{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)\end{aligned}$$

output
```
11/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+7/16*cosh(3*a/b)*Shi(3*(a+b*arc
sinh(c*x))/b)/b+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b-11/8*Chi((a
+b*arcsinh(c*x))/b)*sinh(a/b)/b-7/16*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/
b)/b-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b+Unintegrable(1/x/(a+b*
arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

### 3.376.2 Mathematica [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])), x]
```

### 3.376.3 Rubi [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{3c^2x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{1}{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{c^6x^5}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{}{\sqrt{c^2x^2+1}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx-\frac{11\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b}-\frac{7\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b}+\frac{11\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b}+\frac{7\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b}+\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b}$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.376.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

## 3.376.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\,\operatorname{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

## 3.376.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a
*x), x)
```

### 3.376.6 Sympy [N/A]

Not integrable

Time = 5.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))), x)
```

### 3.376.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x), x)
```

### 3.376.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.376.9 Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))}\,dx=\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))), x)
```

# 3.377 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$



## 3.377.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\begin{aligned}\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx &= \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b} \\ &+ \frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b} + \frac{15c\log(a+b\text{arcsinh}(cx))}{8b} \\ &- \frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b} - \frac{c\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b} \\ &+ \text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)\end{aligned}$$

output
```
c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+1/8*c*Chi(4*(a+b*arcsinh(c*x))
/b)*cosh(4*a/b)/b+15/8*c*ln(a+b*arcsinh(c*x))/b-c*Shi(2*(a+b*arcsinh(c*x))
/b)*sinh(2*a/b)/b-1/8*c*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b+Unintegr
able(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

### 3.377.2 Mathematica [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

### 3.377.3 Rubi [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{3c^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{c^6x^4}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}+\frac{}{\sqrt{c^2x^2+}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx+\frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b}+\frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b}-\frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b}-\frac{c\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b}+\frac{15c\log(a+b\text{arcsinh}(cx))}{8b}$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.377.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

## 3.377.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\,(a+b\,\mathrm{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
```

## 3.377.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\mathrm{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\mathrm{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) +
 a*x^2), x)
```

## 3.377.6 Sympy [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))), x)
```

## 3.377.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

## 3.377.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

### 3.377.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))), x)
```

# 3.378 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$



## 3.378.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

## 3.378.2 Mathematica [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

## 3.378.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \text{6239}$$

$$\int \frac{(c^2x^2+1)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.378.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.378.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

### 3.378.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) +
 a*x^3), x)
```

### 3.378.6 Sympy [N/A]

Not integrable

Time = 4.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))), x)
```

### 3.378.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

### 3.378.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.378.9 Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))), x)
```

# 3.379 $\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\,\mathrm{arcsinh}(cx))}\,dx$



## 3.379.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\,\mathrm{arcsinh}(cx))}\,dx = \mathrm{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^4(a+b\,\mathrm{arcsinh}(cx))},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

## 3.379.2 Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\,\mathrm{arcsinh}(cx))}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\,\mathrm{arcsinh}(cx))}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

## 3.379.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{(c^2x^2+1)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.379.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.379.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\,(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

output
```
int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

### 3.379.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) +
 a*x^4), x)
```

### 3.379.6 Sympy [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x)),x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))), x)
```

### 3.379.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

### 3.379.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

### 3.379.9 Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))}\,dx=\int \frac{(c^2\,x^2+1)^{5/2}}{x^4\,\left(a+b\,\text{asinh}\,(c\,x)\right)}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))), x)
```

# 3.380 $\int \frac{x^4}{\sqrt{1+a^2x^2}\,\textbf{arcsinh}(ax)}\,dx$



## 3.380.1 Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = -\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^5}+\frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{8a^5}+\frac{3\log(\operatorname{arcsinh}(ax))}{8a^5}$$

output
```
-1/2*Chi(2*arcsinh(a*x))/a^5+1/8*Chi(4*arcsinh(a*x))/a^5+3/8*ln(arcsinh(a*
x))/a^5
```

## 3.380.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \frac{-4\operatorname{Chi}(2\operatorname{arcsinh}(ax))+\operatorname{Chi}(4\operatorname{arcsinh}(ax))+3\log(\operatorname{arcsinh}(ax))}{8a^5}$$

input
```
Integrate[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(-4*CoshIntegral[2*ArcSinh[a*x]] + CoshIntegral[4*ArcSinh[a*x]] + 3*Log[Ar
cSinh[a*x]])/(8*a^5)
```

### 3.380.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6234, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int \frac{a^4x^4}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(i\text{arcsinh}(ax))^4}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^5}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(-\frac{\cosh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}+\frac{\cosh(4\text{arcsinh}(ax))}{8\text{arcsinh}(ax)}+\frac{3}{8\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\text{Chi}(2\text{arcsinh}(ax))+\frac{1}{8}\text{Chi}(4\text{arcsinh}(ax))+\frac{3}{8}\log(\text{arcsinh}(ax))}{a^5}$$

input
```
Int[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(-1/2*CoshIntegral[2*ArcSinh[a*x]] + CoshIntegral[4*ArcSinh[a*x]]/8 + (3*L
og[ArcSinh[a*x]])/8)/a^5
```

### 3.380.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.380.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| default | $\frac{3\ln(\operatorname{arcsinh}(ax))-4\operatorname{Chi}(2\operatorname{arcsinh}(ax))+\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{8a^5}$ | 30 |

input
```
int(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(3*ln(arcsinh(a*x))-4*Chi(2*arcsinh(a*x))+Chi(4*arcsinh(a*x)))/a^5
```

### 3.380.5 Fricas [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.380.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

input
```
integrate(x**4/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

### 3.380.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.380.8 Giac [F]

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^4}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.380.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^4}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.381 $\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$



## 3.381.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = -\frac{3\text{Shi}(\text{arcsinh}(ax))}{4a^4} + \frac{\text{Shi}(3\text{arcsinh}(ax))}{4a^4}$$

output
```
-3/4*Shi(arcsinh(a*x))/a^4+1/4*Shi(3*arcsinh(a*x))/a^4
```

## 3.381.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{-3\text{Shi}(\text{arcsinh}(ax)) + \text{Shi}(3\text{arcsinh}(ax))}{4a^4}$$

input
```
Integrate[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(-3*SinhIntegral[ArcSinh[a*x]] + SinhIntegral[3*ArcSinh[a*x]])/(4*a^4)
```

### 3.381.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6234, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int \frac{a^3x^3}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{i\sin(i\text{arcsinh}(ax))^3}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sin(i\text{arcsinh}(ax))^3}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{3793}$$

$$\frac{i\int \left(\frac{3iax}{4\text{arcsinh}(ax)}-\frac{i\sinh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{3}{4}i\text{Shi}(\text{arcsinh}(ax))-\frac{1}{4}i\text{Shi}(3\text{arcsinh}(ax))\right)}{a^4}$$

input
```
Int[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(I*(((3*I)/4)*SinhIntegral[ArcSinh[a*x]] - (I/4)*SinhIntegral[3*ArcSinh[a*
x]]))/a^4
```

### 3.381.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.381.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $-\frac{3\,\mathrm{Shi}(\mathrm{arcsinh}(ax))-\mathrm{Shi}(3\,\mathrm{arcsinh}(ax))}{4a^4}$ | 23 |

input
```
int(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(3*Shi(arcsinh(a*x))-Shi(3*arcsinh(a*x)))/a^4
```

### 3.381.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.381.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\sqrt{a^2x^2+1}\,\operatorname{asinh}\left(ax\right)}\,dx$$

input
```
integrate(x**3/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

### 3.381.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.381.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.381.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^3}{\text{asinh}\left(a\,x\right)\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.382 $\int \frac{x^2}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx$



## 3.382.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx=\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^3}-\frac{\log(\operatorname{arcsinh}(ax))}{2a^3}$$

output
```
1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

## 3.382.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx=\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))-\log(\operatorname{arcsinh}(ax))}{2a^3}$$

input
```
Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

### 3.382.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int \frac{a^2x^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{\sin(i\text{arcsinh}(ax))^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sin(i\text{arcsinh}(ax))^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{1}{2\text{arcsinh}(ax)}-\frac{\cosh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\text{Chi}(2\text{arcsinh}(ax))-\frac{1}{2}\log(\text{arcsinh}(ax))}{a^3}$$

input
```
Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(CoshIntegral[2*ArcSinh[a*x]]/2 - Log[ArcSinh[a*x]]/2)/a^3
```

#### 3.382.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.382.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(\operatorname{arcsinh}(ax))-\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^3}$ | 21 |

input
```
int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(arcsinh(a*x))-Chi(2*arcsinh(a*x)))/a^3
```

### 3.382.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx=\int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.382.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx=\int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

input
```
integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

### 3.382.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx=\int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.382.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.382.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.383 $\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$



## 3.383.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^3} - \frac{\log(\text{arcsinh}(ax))}{2a^3}$$

output
```
1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

## 3.383.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(ax)) - \log(\text{arcsinh}(ax))}{2a^3}$$

input
```
Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

### 3.383.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{\int \frac{a^2x^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\frac{\sin(i\text{arcsinh}(ax))^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sin(i\text{arcsinh}(ax))^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{\int \left(\frac{1}{2\text{arcsinh}(ax)}-\frac{\cosh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\text{Chi}(2\text{arcsinh}(ax))-\frac{1}{2}\log(\text{arcsinh}(ax))}{a^3}$$

input
```
Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
(CoshIntegral[2*ArcSinh[a*x]]/2 - Log[ArcSinh[a*x]]/2)/a^3
```

### 3.383.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.383.4 Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(\operatorname{arcsinh}(ax))-\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^3}$ | 21 |

input
```
int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(arcsinh(a*x))-Chi(2*arcsinh(a*x)))/a^3
```

### 3.383.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.383.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

input
```
integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

### 3.383.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.383.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x^2}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.384 $\int \frac{x}{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)} \, dx$



## 3.384.1 Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{x}{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)} \, dx = \frac{\text{Shi}(\text{arcsinh}(ax))}{a^2}$$

output
```
Shi(arcsinh(a*x))/a^2
```

## 3.384.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)} \, dx = \frac{\text{Shi}(\text{arcsinh}(ax))}{a^2}$$

input
```
Integrate[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
SinhIntegral[ArcSinh[a*x]]/a^2
```

## 3.384.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6234, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int \frac{ax}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(\text{arcsinh}(ax))}{a^2}$$

input
```
Int[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
SinhIntegral[ArcSinh[a*x]]/a^2
```

### 3.384.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.384.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{a^2}$ | 10 |

input
```
int(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
Shi(arcsinh(a*x))/a^2
```

## 3.384.5 Fricas [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.384.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

input
```
integrate(x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

### 3.384.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.384.8 Giac [F]

$$\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{x}{\mathrm{asinh}\left(a\,x\right)\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.385 $\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$



## 3.385.1 Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\log(\text{arcsinh}(ax))}{a}$$

output
```
ln(arcsinh(a*x))/a
```

## 3.385.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\log(\text{arcsinh}(ax))}{a}$$

input
```
Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
Log[ArcSinh[a*x]]/a
```

## 3.385.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6197}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6197}$$

$$\frac{\log(\text{arcsinh}(ax))}{a}$$

input
```
Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
Log[ArcSinh[a*x]]/a
```

### 3.385.3.1 Defintions of rubi rules used

rule 6197
```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

## 3.385.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(\text{arcsinh}(ax))}{a}$ | 10 |
| default | $\frac{\ln(\text{arcsinh}(ax))}{a}$ | 10 |

input
```
int(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(arcsinh(a*x))/a
```

### 3.385.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)}{a}$$

input
```
integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
log(log(a*x + sqrt(a^2*x^2 + 1)))/a
```

### 3.385.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\log\left(\text{asinh}\left(ax\right)\right)}{a}$$

input
```
integrate(1/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
log(asinh(a*x))/a
```

### 3.385.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\log\left(\text{arsinh}\left(ax\right)\right)}{a}$$

input
```
integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
log(arcsinh(a*x))/a
```

### 3.385.8 Giac [F]

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

### 3.385.9 Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \frac{\ln\left(\text{asinh}(a\,x)\right)}{a}$$

input
```
int(1/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
log(asinh(a*x))/a
```

# 3.386 $\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$



## 3.386.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \text{Int}\left(\frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)},x\right)$$

output
```
Unintegrable(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

## 3.386.2 Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

## 3.386.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{1}{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

input
```
Int[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

### 3.386.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.386.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x \,\text{arcsinh}\left(a x \right) \sqrt{a^{2} x^{2}+1}}d x$$

input
```
int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

output
```
int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

### 3.386.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)/((a^2*x^3 + x)*arcsinh(a*x)), x)
```

### 3.386.6 Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

input
```
integrate(1/x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

### 3.386.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

### 3.386.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

### 3.386.9 Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.387 $\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$



## 3.387.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)},x\right)$$

output
```
Unintegrable(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

## 3.387.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

## 3.387.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{1}{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx$$

input
```
Int[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

### 3.387.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.387.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}dx$$

input
```
int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

output
```
int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

### 3.387.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x^2\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)/((a^2*x^4 + x^2)*arcsinh(a*x)), x)
```

### 3.387.6 Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

input
```
integrate(1/x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

### 3.387.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x^2\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

### 3.387.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}x^2\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

### 3.387.9 Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\,\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.388 $\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.388.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=-\frac{5\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^6}+\frac{5\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bc^6}-\frac{\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bc^6}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^6}-\frac{5\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^6}+\frac{\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^6}$$

output
```
5/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^6-5/16*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^6+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^6-
5/8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^6+5/16*Chi(3*(a+b*arcsinh(c*x)
)/b)*sinh(3*a/b)/b/c^6-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^6
```

### 3.388.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx =$$
$$-\frac{10\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-5\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\text{Chi}\left(5\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sin}{}$$

input
```
Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
-1/16*(10*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a
/b + ArcSinh[c*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c*x])]*S
inh[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*Cosh[(3*a
)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(
a/b + ArcSinh[c*x])])/(b*c^6)
```

### 3.388.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6234, 25, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6234}$$
$$\frac{\int -\frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^6}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int \frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^6}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^5}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^6}$$

$$\Big\downarrow \; 26$$

$$\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^5}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^6}$$

$$\Big\downarrow \; 3793$$

$$\frac{i\int \left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{5i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{5i\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^6}$$

$$\Big\downarrow \; 2009$$

$$\frac{i\left(\frac{5}{8}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{5}{16}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}i\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right) \ldots}{bc \ldots}$$

input
```
Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
(I*(((5*I)/8)*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b] - ((5*I)/16)*
CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b] + (I/16)*CoshIntegr
al[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b] - ((5*I)/8)*Cosh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b] + ((5*I)/16)*Cosh[(3*a)/b]*SinhIntegral[(3*(
a + b*ArcSinh[c*x]))/b] - (I/16)*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcS
inh[c*x]))/b]))/(b*c^6)
```

### 3.388.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.388. $\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.388.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| default | $\frac{e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)-5 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)+10 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)-10 e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)+5 e^{-\frac{3a}{b}} \operatorname{Ei}}{32c^6 b}$ |

input
```
int(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*(exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-5*exp(3*a/b)*Ei(1,3*arcsinh(c*
x)+3*a/b)+10*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-10*exp(-a/b)*Ei(1,-arcsinh(c*
x)-a/b)+5*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-exp(-5*a/b)*Ei(1,-5*arcs
inh(c*x)-5*a/b))/c^6/b
```

### 3.388.5 Fricas [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^5}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^5/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
 a), x)
```

### 3.388.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^5}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**5/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**5/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

### 3.388.7 Maxima [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^5}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.388.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^5}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.389 $\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.389.1 Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^5} + \frac{3\log(a+b\text{arcsinh}(cx))}{8bc^5} + \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^5} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc^5}$$

output
```
-1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5+1/8*Chi(4*(a+b*arcsinh(
c*x))/b)*cosh(4*a/b)/b/c^5+3/8*ln(a+b*arcsinh(c*x))/b/c^5+1/2*Shi(2*(a+b*a
rcsinh(c*x))/b)*sinh(2*a/b)/b/c^5-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a
/b)/b/c^5
```

### 3.389.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = -\frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-3\log(a+b\text{arcsinh}(cx))-}{8bc^5}$$

input
```
Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
-1/8*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Cosh[(4*a)/b]
*CoshIntegral[4*(a/b + ArcSinh[c*x])] - 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh
[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Sinh[(4*a)/b]*SinhIntegra
l[4*(a/b + ArcSinh[c*x])])/(b*c^5)
```

### 3.389.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6234, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \; 6234$$

$$\frac{\int \frac{\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^4}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^5}$$

$$\Big\downarrow \; 3793$$

$$\frac{\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}+\frac{3}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{8}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-}{bc^5}$$

input
```
Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
(-1/2*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]) + (Cosh[(4*
a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8 + (3*Log[a + b*ArcSinh[c
*x]])/8 + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/2 - (Si
nh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8)/(b*c^5)
```

### 3.389.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.389.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $-\frac{e^{-\frac{4a}{b}} \operatorname{Ei}_1\left(-4 \operatorname{arcsinh}(cx)-\frac{4a}{b}\right)+e^{\frac{4a}{b}} \operatorname{Ei}_1\left(4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)-4 e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)-4 e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)-}{16c^5 b}$ |

input
```
int(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+exp(4*a/b)*Ei(1,4*arcsinh(c
*x)+4*a/b)-4*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-4*exp(-2*a/b)*Ei(1,-2*a
rcsinh(c*x)-2*a/b)-6*ln(a+b*arcsinh(c*x)))/c^5/b
```

## 3.389.5 Fricas [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^4/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
 a), x)
```

## 3.389.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^4}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**4/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

### 3.389.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.389.8 Giac [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^4}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.390 $\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.390.1 Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^4} - \frac{\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^4} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^4} + \frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^4}$$

output
```
-3/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4+1/4*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^4+3/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4-1/4*C
hi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^4
```

### 3.390.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx$$
$$= \frac{3\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right) -}{4bc^4}$$

input
```
Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
(3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - CoshIntegral[3*(a/b + ArcS
inh[c*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] +
Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^4)
```

### 3.390.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6234, 25, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6234}$$
$$\frac{\int -\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{a+b\operatorname{arcsinh}(cx)}d(a+b\operatorname{arcsinh}(cx))}{bc^4}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int \frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{a+b\operatorname{arcsinh}(cx)}d(a+b\operatorname{arcsinh}(cx))}{bc^4}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$

$\Big\downarrow$ 26

$$-\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^4}$$

$\Big\downarrow$ 3793

$$-\frac{i\int \left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}-\frac{i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^4}$$

$\Big\downarrow$ 2009

$$-\frac{i\left(\frac{3}{4}i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}i\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\right.}{bc^4}$$

input
```
Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
((-I)*(((3*I)/4)*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b] - (I/4)*Co
shIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b] - ((3*I)/4)*Cosh[a/b]
*SinhIntegral[(a + b*ArcSinh[c*x])/b] + (I/4)*Cosh[(3*a)/b]*SinhIntegral[(
3*(a + b*ArcSinh[c*x]))/b]))/(b*c^4)
```

### 3.390.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.390. $\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.390.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(cx)+\frac{3a}{b}\right)-3 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)+3 e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)-e^{-\frac{3a}{b}} \operatorname{Ei}_1\left(-3 \operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^4 b}$ | 101 |

input
```
int(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3*exp(a/b)*Ei(1,arcsinh(c*x)+a/
b)+3*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b))/c^4/b
```

## 3.390.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^3}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
 a), x)
```

### 3.390.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**3/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

### 3.390.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^3}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.390.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^3}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.391 $\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.391.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3} - \frac{\log(a+b\text{arcsinh}(cx))}{2bc^3} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3}$$

output
```
1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3-1/2*ln(a+b*arcsinh(c*x))
/b/c^3-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3
```

## 3.391.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right) - \log(a+b\text{arcsinh}(cx)) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{2bc^3}$$

input
```
Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*
x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^3)
```

### 3.391.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int \frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{1}{2(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{2}\log(a+b\text{arcsinh}(cx))}{bc^3}$$

input
```
Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/2 - Log[a + b*Ar
cSinh[c*x]]/2 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/2
)/(b*c^3)
```

### 3.391.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.391.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $-\frac{e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)+e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)+2\ln(a+b \operatorname{arcsinh}(cx))}{4c^3b}$ | 67 |

input
```
int(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*
x)-2*a/b)+2*ln(a+b*arcsinh(c*x)))/c^3/b
```

### 3.391.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
 a), x)
```

### 3.391.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

### 3.391.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.391.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} \, dx = \int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} \, dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}} \, dx$$

input
```
int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.392 $\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.392.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=-\frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc^2}$$

output
```
cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2-Chi((a+b*arcsinh(c*x))/b)*sinh(a
/b)/b/c^2
```

## 3.392.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=-\frac{\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc^2}$$

input
```
Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
-((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
 + ArcSinh[c*x]])/(b*c^2))
```

### 3.392.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \text{3784}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{bc^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{bc^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{bc^2}$$

$$\Big\downarrow \ 26$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{bc^2}$$

$$\Big\downarrow \ 3779$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{bc^2}$$

$$\Big\downarrow \ 3782$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{bc^2}$$

input
```
Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
(I*(I*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b]))/(b*c^2)
```

## 3.392. $\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$

### 3.392.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.392. $\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$

### 3.392.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

| method | result | size |
|---|---|---|
| default | $\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)-e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2c^2b}$ | 53 |

input
```
int(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))/c^
2/b
```

### 3.392.5 Fricas [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a
), x)
```

### 3.392.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

### 3.392.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.392.8 Giac [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.393 $\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} dx$



## 3.393.1 Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} dx = \frac{\log(a+b\text{arcsinh}(cx))}{bc}$$

output
```
ln(a+b*arcsinh(c*x))/b/c
```

## 3.393.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} dx = \frac{\log(a+b\text{arcsinh}(cx))}{bc}$$

input
```
Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Log[a + b*ArcSinh[c*x]]/(b*c)
```

## 3.393.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6197}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

$$\Big\downarrow \text{ 6197}$$

$$\frac{\log(a+b\text{arcsinh}(cx))}{bc}$$

input
```
Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Log[a + b*ArcSinh[c*x]]/(b*c)
```

### 3.393.3.1 Defintions of rubi rules used

rule 6197
```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

## 3.393.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(a+b\,\text{arcsinh}(cx))}{bc}$ | 17 |
| default | $\frac{\ln(a+b\,\text{arcsinh}(cx))}{bc}$ | 17 |

input
```
int(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*arcsinh(c*x))/b/c
```

### 3.393.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\frac{\log\left(b\log\left(cx+\sqrt{c^2x^2+1}\right)+a\right)}{bc}$$

input
```
integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
log(b*log(c*x + sqrt(c^2*x^2 + 1)) + a)/(b*c)
```

### 3.393.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\begin{cases}\frac{x}{a} & \text{for } b=0\wedge c=0\\ \frac{\operatorname{asinh}(cx)}{ac} & \text{for } b=0\\ \frac{x}{a} & \text{for } c=0\\ \frac{\log\left(\frac{a}{b}+\operatorname{asinh}(cx)\right)}{bc} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), (asinh(c*x)/(a*c), Eq(b, 0)), (x/a,
Eq(c, 0)), (log(a/b + asinh(c*x))/(b*c), True))
```

### 3.393.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\frac{\log\left(b\,\text{arsinh}\left(cx\right)+a\right)}{bc}$$

input
```
integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
log(b*arcsinh(c*x) + a)/(b*c)
```

### 3.393.8 Giac [F]

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.393.9 Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\ln\left(a+b\,\text{asinh}(c\,x)\right)}{b\,c}$$

input
```
int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
log(a + b*asinh(c*x))/(b*c)
```

# 3.394 $\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.394.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

## 3.394.2 Mathematica [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

## 3.394.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))} \, dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{1}{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.394.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.394.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x \left(a+b \,\operatorname{arcsinh}\left(c x \right)\right) \sqrt{c^{2} x^{2}+1}}d x$$

input
```
int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

output
```
int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

## 3.394.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^3 + a*x + (b*c^2*x^3 + b*x)*arcsinh(c*
x)), x)
```

## 3.394.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x\,(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(1/x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

## 3.394.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x), x)
```

### 3.394.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.394.9 Mupad [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.395 $\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$



## 3.395.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx=\operatorname{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

## 3.395.2 Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx=\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

## 3.395.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.395.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.395.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+b\,\text{arcsinh}\left(cx\right)\right)\sqrt{c^2x^2+1}}dx$$

input
```
int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

output
```
int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

### 3.395.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^4 + a*x^2 + (b*c^2*x^4 + b*x^2)*arcsin
h(c*x)), x)
```

### 3.395.6 Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x^2\left(a+b\,\text{asinh}\,(cx)\right)\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(1/x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

### 3.395.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

### 3.395.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

### 3.395.9 Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{x^2\,\left(a+b\,\text{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.396 $\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.396.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.396.2 Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

## 3.396.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{x^2}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.396.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.396.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.396.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b
*c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

### 3.396.6 Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^2}{(a+b\operatorname{asinh}(cx))(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**2/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.396.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.396.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.396.9 Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))\,\left(c^2\,x^2+1\right)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.397 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.397.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.397.2 Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

## 3.397.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \ 6239$$
$$\int \frac{x}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.397.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.397.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.397.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c
^2*x^2 + b)*arcsinh(c*x) + a), x)
```

### 3.397.6 Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.397.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.397.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.397.9 Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x}{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(c^2\,x^2+1\right)^{3/2}}\,dx$$

input
```
int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.398 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.398.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.398.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

## 3.398.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.398.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.398.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.398.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^2
*x^2 + b)*arcsinh(c*x) + a), x)
```

### 3.398.6 Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.398.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.398.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.398.9 Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(c^2\,x^2+1\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.399 $\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.399.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.399.2 Mathematica [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

## 3.399.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

$$\Big\downarrow \text{6239}$$

$$\int \frac{1}{x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.399.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.399.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\ \text{arcsinh}\left(cx\right)\right)}dx$$

input
```
int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.399.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^5 + 2*a*c^2*x^3 + a*x + (b*c^4*x^5 + 2
*b*c^2*x^3 + b*x)*arcsinh(c*x)), x)
```

### 3.399.6 Sympy [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x\,(a+b\,\text{asinh}\,(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(x*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.399.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x), x)
```

### 3.399.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.399.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{1}{x\,(a+b\operatorname{asinh}(c\,x))\,(c^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.400 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.400.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.400.2 Mathematica [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

## 3.400.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))} dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{1}{x^2 (c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))} dx$$

input
```
Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.400.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.400.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (c^2x^2+1)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))} dx$$

input
```
int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.400.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^2 (1 + c^2x^2)^{3/2} (a + b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2x^2 + 1)^{\frac{3}{2}} (b \,\text{arsinh}\,(cx) + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^6 + 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 +
 2*b*c^2*x^4 + b*x^2)*arcsinh(c*x)), x)
```

### 3.400.6 Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2 (1 + c^2x^2)^{3/2} (a + b\text{arcsinh}(cx))} \, dx = \int \frac{1}{x^2 (a + b \,\text{asinh}\,(cx)) (c^2x^2 + 1)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(x**2*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.400.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 + c^2x^2)^{3/2} (a + b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2x^2 + 1)^{\frac{3}{2}} (b \,\text{arsinh}\,(cx) + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima"
)
```

output
```
integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

### 3.400.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 + c^2x^2)^{3/2} (a + b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2x^2 + 1)^{\frac{3}{2}}(b \operatorname{arsinh}(cx) + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

### 3.400.9 Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 + c^2x^2)^{3/2} (a + b\text{arcsinh}(cx))} \, dx = \int \frac{1}{x^2 \left(a + b \operatorname{asinh}(c\,x)\right) \left(c^2\,x^2 + 1\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.401 $\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.401.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Int}\left(\frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)},x\right)$$

output
```
Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

## 3.401.2 Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

input
```
Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

## 3.401.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}x^m}{a+b\operatorname{arcsinh}(cx)}\,dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{(c^2x^2+1)^{5/2}x^m}{a+b\operatorname{arcsinh}(cx)}dx$$

input
```
Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

### 3.401.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.401.4 Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m(c^2x^2+1)^{\frac{5}{2}}}{a+b\,\operatorname{arcsinh}(cx)}dx$$

input
```
int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

### 3.401.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) +
 a), x)
```

### 3.401.6 Sympy [N/A]

Not integrable

Time = 178.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(c^2x^2+1)^{\frac{5}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**m*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

### 3.401.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)*x^m/(b*arcsinh(c*x) + a), x)
```

### 3.401.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.401.9 Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^m\,\left(c^2\,x^2+1\right)^{5/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

# 3.402 $\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.402.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Int}\left(\frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)},x\right)$$

output
```
Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.402.2 Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$$

input
```
Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

## 3.402.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2x^2+1\right)^{3/2}x^m}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{ 6239}$$

$$\int \frac{\left(c^2x^2+1\right)^{3/2}x^m}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

### 3.402.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.402.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\,\text{arcsinh}\left(cx\right)}dx$$

input
```
int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.402.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

### 3.402.6 Sympy [N/A]

Not integrable

Time = 14.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

### 3.402.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

### 3.402.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (1 + c^2 x^2)^{3/2}}{a + b\text{arcsinh}(cx)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.402.9 Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m (1 + c^2 x^2)^{3/2}}{a + b\text{arcsinh}(cx)} \, dx = \int \frac{x^m \left(c^2 x^2 + 1\right)^{3/2}}{a + b \operatorname{asinh}(c x)} \, dx$$

input
```
int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

# 3.403 $\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.403.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Int}\left(\frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)},x\right)$$

output
```
Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

## 3.403.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx$$

input
```
Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

## 3.403.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}x^m}{a+b\text{arcsinh}(cx)} \, dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{\sqrt{c^2x^2+1}x^m}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

### 3.403.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.403.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m\sqrt{c^2x^2+1}}{a+b\,\text{arcsinh}\,(cx)}dx$$

input
```
int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

### 3.403.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^m}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

### 3.403.6 Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{x^m\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\left(cx\right)}\,dx$$

input
```
integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

### 3.403.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2x^2+1}x^m}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

### 3.403.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.403.9 Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{x^m\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

output
```
int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

# 3.404 $\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$



## 3.404.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \operatorname{Int}\left(\frac{x^m}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

## 3.404.2 Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx$$

input
```
Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

## 3.404.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{x^m}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.404.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.404.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\,\left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

input
```
int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

### 3.404.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
 a), x)
```

### 3.404.6 Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**m/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

### 3.404.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.404.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

### 3.404.9 Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{x^m}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.405 $\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.405.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.405.2 Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

## 3.405.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{x^m}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.405.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.405.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.405.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b
*c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

### 3.405.6 Sympy [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(cx))\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(x**m/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.405.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.405.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.405.9 Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(c\,x))\,(c^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.406 $\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)^2} \, dx$



## 3.406.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)^2} \, dx = -\frac{c^3(1+a^2x^2)^{7/2}}{a\text{arcsinh}(ax)} + \frac{35c^3\text{Shi}(\text{arcsinh}(ax))}{64a} + \frac{63c^3\text{Shi}(3\text{arcsinh}(ax))}{64a} + \frac{35c^3\text{Shi}(5\text{arcsinh}(ax))}{64a} + \frac{7c^3\text{Shi}(7\text{arcsinh}(ax))}{64a}$$

output
```
-c^3*(a^2*x^2+1)^(7/2)/a/arcsinh(a*x)+35/64*c^3*Shi(arcsinh(a*x))/a+63/64*
c^3*Shi(3*arcsinh(a*x))/a+35/64*c^3*Shi(5*arcsinh(a*x))/a+7/64*c^3*Shi(7*a
rcsinh(a*x))/a
```

## 3.406.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)^2} \, dx = \frac{c^3\left(-64(1+a^2x^2)^{7/2} + 35\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax)) + 63\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax)) + 35\text{arcsinh}(a\right.}{64a\text{arcsinh}(ax)}$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

output
```
(c^3*(-64*(1 + a^2*x^2)^(7/2) + 35*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
 + 63*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 35*ArcSinh[a*x]*SinhInte
gral[5*ArcSinh[a*x]] + 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]]))/(64*a
*ArcSinh[a*x])
```

### 3.406.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^3}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{6205}$$

$$7ac^3\int \frac{x\left(a^2x^2+1\right)^{5/2}}{\text{arcsinh}(ax)}dx-\frac{c^3\left(a^2x^2+1\right)^{7/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{6234}$$

$$\frac{7c^3\int \frac{ax\left(a^2x^2+1\right)^3}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{c^3\left(a^2x^2+1\right)^{7/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{7c^3\int \left(\frac{5ax}{64\text{arcsinh}(ax)}+\frac{9\sinh(3\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{5\sinh(5\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{\sinh(7\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}-\frac{c^3\left(a^2x^2+1\right)^{7/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7c^3\left(\frac{5}{64}\text{Shi}(\text{arcsinh}(ax))+\frac{9}{64}\text{Shi}(3\text{arcsinh}(ax))+\frac{5}{64}\text{Shi}(5\text{arcsinh}(ax))+\frac{1}{64}\text{Shi}(7\text{arcsinh}(ax))\right)}{a}-\frac{c^3\left(a^2x^2+1\right)^{7/2}}{a\text{arcsinh}(ax)}$$

input
```
Int[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

3.406. $\int \frac{\left(c+a^2cx^2\right)^3}{\text{arcsinh}(ax)^2}dx$

output
```
-((c^3*(1 + a^2*x^2)^(7/2))/(a*ArcSinh[a*x])) + (7*c^3*((5*SinhIntegral[Ar
cSinh[a*x]])/64 + (9*SinhIntegral[3*ArcSinh[a*x]])/64 + (5*SinhIntegral[5*
ArcSinh[a*x]])/64 + SinhIntegral[7*ArcSinh[a*x]]/64))/a
```

### 3.406.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.406.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $\frac{c^3\left(35 \operatorname{Shi}(\operatorname{arcsinh}(ax)) \operatorname{arcsinh}(ax)+63 \operatorname{Shi}(3 \operatorname{arcsinh}(ax)) \operatorname{arcsinh}(ax)+35 \operatorname{Shi}(5 \operatorname{arcsinh}(ax)) \operatorname{arcsinh}(ax)+7 \operatorname{Shi}(7 a\right.}{64a \operatorname{arcsinh}(}$ |
| default | $\frac{c^3\left(35 \operatorname{Shi}(\operatorname{arcsinh}(ax)) \operatorname{arcsinh}(ax)+63 \operatorname{Shi}(3 \operatorname{arcsinh}(ax)) \operatorname{arcsinh}(ax)+35 \operatorname{Shi}(5 \operatorname{arcsinh}(ax)) \operatorname{arcsinh}(ax)+7 \operatorname{Shi}(7 a\right.}{64a \operatorname{arcsinh}(}$ |

input
```
int((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(35*Shi(arcsinh(a*x))*arcsinh(a*x)+63*Shi(3*arcsinh(a*x))*arcsi
nh(a*x)+35*Shi(5*arcsinh(a*x))*arcsinh(a*x)+7*Shi(7*arcsinh(a*x))*arcsinh(
a*x)-35*(a^2*x^2+1)^(1/2)-21*cosh(3*arcsinh(a*x))-7*cosh(5*arcsinh(a*x))-c
osh(7*arcsinh(a*x)))/arcsinh(a*x)
```

### 3.406.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x)^
2, x)
```

### 3.406.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/asinh(a*x)**2,x)
```

output
```
c**3*(Integral(3*a**2*x**2/asinh(a*x)**2, x) + Integral(3*a**4*x**4/asinh(
a*x)**2, x) + Integral(a**6*x**6/asinh(a*x)**2, x) + Integral(asinh(a*x)**
(-2), x))
```

### 3.406.7 Maxima [F]

$$\int \frac{\left(c+a^2cx^2\right)^3}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{\left(a^2cx^2+c\right)^3}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x +
(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*sqrt(a
^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x
^2 + 1))) + integrate((7*a^10*c^3*x^10 + 29*a^8*c^3*x^8 + 46*a^6*c^3*x^6 +
 34*a^4*c^3*x^4 + 11*a^2*c^3*x^2 + c^3 + (7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 +
 18*a^4*c^3*x^4 + 4*a^2*c^3*x^2 - c^3)*(a^2*x^2 + 1) + 7*(2*a^9*c^3*x^9 +
7*a^7*c^3*x^7 + 9*a^5*c^3*x^5 + 5*a^3*c^3*x^3 + a*c^3*x)*sqrt(a^2*x^2 + 1)
)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a
^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

### 3.406.8 Giac [F]

$$\int \frac{\left(c+a^2cx^2\right)^3}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{\left(a^2cx^2+c\right)^3}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arcsinh(a*x)^2, x)
```

### 3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^3}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{\left(c\,a^2\,x^2+c\right)^3}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^3/asinh(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^3/asinh(a*x)^2, x)
```

# 3.407 $\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2} \, dx$



## 3.407.1 Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2} \, dx = -\frac{c^2(1+a^2x^2)^{5/2}}{a\operatorname{arcsinh}(ax)}+\frac{5c^2\operatorname{Shi}(\operatorname{arcsinh}(ax))}{8a}+\frac{15c^2\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{16a}+\frac{5c^2\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{16a}$$

output
```
-c^2*(a^2*x^2+1)^(5/2)/a/arcsinh(a*x)+5/8*c^2*Shi(arcsinh(a*x))/a+15/16*c^
2*Shi(3*arcsinh(a*x))/a+5/16*c^2*Shi(5*arcsinh(a*x))/a
```

## 3.407.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2} \, dx = \frac{c^2\left(-16(1+a^2x^2)^{5/2}+10\operatorname{arcsinh}(ax)\operatorname{Shi}(\operatorname{arcsinh}(ax))+15\operatorname{arcsinh}(ax)\operatorname{Shi}(3\operatorname{arcsinh}(ax))+5\operatorname{arcsinh}(ax\right.}{16a\operatorname{arcsinh}(ax)}$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

output
```
(c^2*(-16*(1 + a^2*x^2)^(5/2) + 10*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
 + 15*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 5*ArcSinh[a*x]*SinhInteg
ral[5*ArcSinh[a*x]]))/(16*a*ArcSinh[a*x])
```

### 3.407.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{6205}$$

$$5ac^2\int \frac{x(a^2x^2+1)^{3/2}}{\text{arcsinh}(ax)}dx-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{6234}$$

$$\frac{5c^2\int \frac{ax(a^2x^2+1)^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{5c^2\int \left(\frac{ax}{8\text{arcsinh}(ax)}+\frac{3\sinh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\sinh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5c^2\left(\frac{1}{8}\text{Shi}(\text{arcsinh}(ax))+\frac{3}{16}\text{Shi}(3\text{arcsinh}(ax))+\frac{1}{16}\text{Shi}(5\text{arcsinh}(ax))\right)}{a}-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

input
```
Int[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

output
```
-((c^2*(1 + a^2*x^2)^(5/2))/(a*ArcSinh[a*x])) + (5*c^2*(SinhIntegral[ArcSi
nh[a*x]]/8 + (3*SinhIntegral[3*ArcSinh[a*x]])/16 + SinhIntegral[5*ArcSinh[
a*x]]/16))/a
```

### 3.407.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.407.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $\frac{c^2\left(10\ \mathrm{Shi}(\mathrm{arcsinh}(ax))\ \mathrm{arcsinh}(ax)+15\ \mathrm{Shi}(3\ \mathrm{arcsinh}(ax))\ \mathrm{arcsinh}(ax)+5\ \mathrm{Shi}(5\ \mathrm{arcsinh}(ax))\ \mathrm{arcsinh}(ax)-5\cosh(3\ \mathrm{ar}\right.}{16a\ \mathrm{arcsinh}(ax)}$ |
| default | $\frac{c^2\left(10\ \mathrm{Shi}(\mathrm{arcsinh}(ax))\ \mathrm{arcsinh}(ax)+15\ \mathrm{Shi}(3\ \mathrm{arcsinh}(ax))\ \mathrm{arcsinh}(ax)+5\ \mathrm{Shi}(5\ \mathrm{arcsinh}(ax))\ \mathrm{arcsinh}(ax)-5\cosh(3\ \mathrm{ar}\right.}{16a\ \mathrm{arcsinh}(ax)}$ |

input
```
int((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(10*Shi(arcsinh(a*x))*arcsinh(a*x)+15*Shi(3*arcsinh(a*x))*arcsi
nh(a*x)+5*Shi(5*arcsinh(a*x))*arcsinh(a*x)-5*cosh(3*arcsinh(a*x))-cosh(5*a
rcsinh(a*x))-10*(a^2*x^2+1)^(1/2))/arcsinh(a*x)
```

### 3.407.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x)^2, x)
```

### 3.407.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2/asinh(a*x)**2, x) + Integral(a**4*x**4/asinh(a*
x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

### 3.407.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x + (a^6*c^2*x^6 + 3
*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^8*c^2
*x^8 + 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 + 8*a^2*c^2*x^2 + (5*a^6*c^2*x^6 +
9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*(a^2*x^2 + 1) + c^2 + 5*(2*a^7*c^2*x^
7 + 5*a^5*c^2*x^5 + 4*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2*x^2 + 1))/((a^4*x^4
+ (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1)
+ 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

### 3.407.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2} \, dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arcsinh(a*x)^2, x)
```

### 3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2} \, dx = \int \frac{(c\,a^2\,x^2+c)^2}{\operatorname{asinh}(a\,x)^2} \, dx$$

input
```
int((c + a^2*c*x^2)^2/asinh(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^2/asinh(a*x)^2, x)
```

# 3.408 $\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2}\,dx$



## 3.408.1 Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2}\,dx = -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)}+\frac{3c\text{Shi}(\text{arcsinh}(ax))}{4a}+\frac{3c\text{Shi}(3\text{arcsinh}(ax))}{4a}$$

output
```
-c*(a^2*x^2+1)^(3/2)/a/arcsinh(a*x)+3/4*c*Shi(arcsinh(a*x))/a+3/4*c*Shi(3*
arcsinh(a*x))/a
```

## 3.408.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2}\,dx = \frac{c\left(-4(1+a^2x^2)^{3/2}+3\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax))+3\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax))\right)}{4a\text{arcsinh}(ax)}$$

input
```
Integrate[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

output
```
(c*(-4*(1 + a^2*x^2)^(3/2) + 3*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]] + 3
*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]]))/(4*a*ArcSinh[a*x])
```

### 3.408.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\text{arcsinh}(ax)^2}\,dx$$

$$\Big\downarrow \text{ 6205}$$

$$3ac\int \frac{x\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}dx-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{3c\int \frac{ax(a^2x^2+1)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{3c\int \left(\frac{ax}{4\text{arcsinh}(ax)}+\frac{\sinh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3c\left(\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))+\frac{1}{4}\text{Shi}(3\text{arcsinh}(ax))\right)}{a}-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\text{arcsinh}(ax)}$$

input
```
Int[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

output
```
-((c*(1 + a^2*x^2)^(3/2))/(a*ArcSinh[a*x])) + (3*c*(SinhIntegral[ArcSinh[a
*x]]/4 + SinhIntegral[3*ArcSinh[a*x]]/4))/a
```

### 3.408.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.408.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{c\left(3\,\mathrm{Shi}(\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)+3\,\mathrm{Shi}(3\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)-3\sqrt{a^2x^2+1}-\cosh(3\operatorname{arcsinh}(ax))\right)}{4a\operatorname{arcsinh}(ax)}$ | 60 |
| default | $\frac{c\left(3\,\mathrm{Shi}(\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)+3\,\mathrm{Shi}(3\operatorname{arcsinh}(ax))\operatorname{arcsinh}(ax)-3\sqrt{a^2x^2+1}-\cosh(3\operatorname{arcsinh}(ax))\right)}{4a\operatorname{arcsinh}(ax)}$ | 60 |

input
```
int((a^2*c*x^2+c)/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(3*Shi(arcsinh(a*x))*arcsinh(a*x)+3*Shi(3*arcsinh(a*x))*arcsinh(a*
x)-3*(a^2*x^2+1)^(1/2)-cosh(3*arcsinh(a*x)))/arcsinh(a*x)
```

## 3.408.5 Fricas [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

## 3.408.6 Sympy [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{asinh}^2\left(ax\right)}\,dx + \int \frac{1}{\operatorname{asinh}^2\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/asinh(a*x)**2,x)
```

output
```
c*(Integral(a**2*x**2/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

## 3.408.7 Maxima [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 + 2*a^2*c*x^2 + c)*sqrt(a^2
*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2
 + 1))) + integrate((3*a^6*c*x^6 + 7*a^4*c*x^4 + 5*a^2*c*x^2 + (3*a^4*c*x^
4 + 2*a^2*c*x^2 - c)*(a^2*x^2 + 1) + 3*(2*a^5*c*x^5 + 3*a^3*c*x^3 + a*c*x)
*sqrt(a^2*x^2 + 1) + c)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*
(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

### 3.408.8 Giac [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

### 3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{c\,a^2\,x^2+c}{\operatorname{asinh}(a\,x)^2}\,dx$$

input
```
int((c + a^2*c*x^2)/asinh(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)/asinh(a*x)^2, x)
```

# 3.409 $\int \frac{1}{(c+a^2cx^2)\text{arcsinh}(ax)^2}\,dx$



## 3.409.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = -\frac{1}{ac\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)} - \frac{a\text{Int}\left(\frac{x}{(1+a^2x^2)^{3/2}\text{arcsinh}(ax)},x\right)}{c}$$

output
```
-1/a/c/arcsinh(a*x)/(a^2*x^2+1)^(1/2)-a*Unintegrable(x/(a^2*x^2+1)^(3/2)/a
rcsinh(a*x),x)/c
```

## 3.409.2 Mathematica [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2), x]
```

## 3.409.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2 (a^2cx^2+c)}dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{a\int \frac{x}{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)}dx}{c}-\frac{1}{ac\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{a\int \frac{x}{(a^2x^2+1)^{3/2}\operatorname{arcsinh}(ax)}dx}{c}-\frac{1}{ac\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

output
```
$Aborted
```

### 3.409.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.409.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2 c\,x^2+c)\operatorname{arcsinh}(ax)^2}dx$$

input
```
int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

output
```
int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

## 3.409.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

## 3.409.6 Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx = \frac{\int \frac{1}{a^2x^2\operatorname{asinh}^2(ax)+\operatorname{asinh}^2(ax)}\,dx}{c}$$

input
```
integrate(1/(a**2*c*x**2+c)/asinh(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**2*asinh(a*x)**2 + asinh(a*x)**2), x)/c
```

### 3.409.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.89

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*x + sqrt(a^2*x^2 + 1))/((a^3*c*x^2 + sqrt(a^2*x^2 + 1)*a^2*c*x + a*c)*
log(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^4*x^4 + (a^2*x^2 + 1)^2 + (2*
a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^6*c*x^6 + 3*a^4*c*x^4 + 3*a^2*c*
x^2 + (a^4*c*x^4 + a^2*c*x^2)*(a^2*x^2 + 1) + 2*(a^5*c*x^5 + 2*a^3*c*x^3 +
 a*c*x)*sqrt(a^2*x^2 + 1) + c)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

### 3.409.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

### 3.409.9 Mupad [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{1}{\operatorname{asinh}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)), x)
```

# 3.410 $\int \frac{1}{(c+a^2cx^2)^2\text{arcsinh}(ax)^2}\,dx$



## 3.410.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)^2}\,dx = -\frac{1}{ac^2(1+a^2x^2)^{3/2}\,\text{arcsinh}(ax)} - \frac{3a\text{Int}\left(\frac{x}{(1+a^2x^2)^{5/2}\text{arcsinh}(ax)},x\right)}{c^2}$$

output
```
-1/a/c^2/(a^2*x^2+1)^(3/2)/arcsinh(a*x)-3*a*Unintegrable(x/(a^2*x^2+1)^(5/
2)/arcsinh(a*x),x)/c^2
```

## 3.410.2 Mathematica [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{(c+a^2cx^2)^2\,\text{arcsinh}(ax)^2}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2), x]
```

### 3.410.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^2 \left(a^2cx^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{3a\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)}dx}{c^2}-\frac{1}{ac^2\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{3a\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\text{arcsinh}(ax)}dx}{c^2}-\frac{1}{ac^2\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

output
```
$Aborted
```

#### 3.410.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.410.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{2} \operatorname{arcsinh}\left(a x \right)^{2}}d x$$

input
```
int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

output
```
int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

### 3.410.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{2} \operatorname{arsinh}\left(a x\right)^{2}} \,d x$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)^2), x)
```

### 3.410.6 Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \frac{\int \frac{1}{a^{4} x^{4} \operatorname{asinh}^{2}{\left(a x \right)} + 2 a^{2} x^{2} \operatorname{asinh}^{2}{\left(a x \right)} + \operatorname{asinh}^{2}{\left(a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**4*asinh(a*x)**2 + 2*a**2*x**2*asinh(a*x)**2 + asinh(a*
x)**2), x)/c**2
```

### 3.410.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 325, normalized size of antiderivative = 17.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2} \, dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*x + sqrt(a^2*x^2 + 1))/((a^5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2 + (a^4*c^
2*x^3 + a^2*c^2*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))) - inte
grate((3*a^4*x^4 + 2*a^2*x^2 + (3*a^2*x^2 + 1)*(a^2*x^2 + 1) + 3*(2*a^3*x^
3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^8*c^2*x^8 + 4*a^6*c^2*x^6 + 6*a^4*c^2*
x^4 + 4*a^2*c^2*x^2 + (a^6*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a^2*x^2
 + 1) + c^2 + 2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*sq
rt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

### 3.410.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2} \, dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)^2), x)
```

### 3.410.9 Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)^2\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2), x)
```

# 3.411 $\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.411.1 Optimal result

Integrand size = 27, antiderivative size = 213

$$\begin{aligned}\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = &-\frac{x^3(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4}\\&-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4}\\&+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4}\\&+\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^4}\\&+\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4}\\&-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^4}\end{aligned}$$

output
```
-x^3*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))-1/8*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2/c^4-3/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/16*Ch
i(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+1/8*Shi((a+b*arcsinh(c*x))/b
)*sinh(a/b)/b^2/c^4+3/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5
/16*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^4
```

### 3.411.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{\frac{16bc^3x^3}{a+b\operatorname{arcsinh}(cx)}+\frac{16bc^5x^5}{a+b\operatorname{arcsinh}(cx)}+2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx\right.\right.}{}$$

input
```
Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*((16*b*c^3*x^3)/(a + b*ArcSinh[c*x]) + (16*b*c^5*x^5)/(a + b*ArcSinh
[c*x]) + 2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcSinh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegral[5*(a/b +
 ArcSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 3*Sinh[(3
*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral
[5*(a/b + ArcSinh[c*x])])/(b^2*c^4)
```

### 3.411.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6229, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c^2x^2+1}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{5c\int \frac{x^4}{a+b\operatorname{arcsinh}(cx)}dx}{b}+\frac{3\int \frac{x^2}{a+b\operatorname{arcsinh}(cx)}dx}{bc}-\frac{x^3\left(c^2x^2+1\right)}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$\Big\downarrow \text{6195}$$

$$\frac{5\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^4}+$$

$$\frac{3\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{x^3(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 5971

$$\frac{3\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^4}+$$

$$\frac{5\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^4}-$$

$$\frac{x^3(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 2009

$$\frac{3\left(-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\text{si}\right.}{b^2c^4}$$

$$\frac{5\left(\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{3}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\right.}{b^2c^4}$$

$$\frac{x^3(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((x^3*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-1/4*(Cosh[a/b]*Co
shIntegral[(a + b*ArcSinh[c*x])/b]) + (Cosh[(3*a)/b]*CoshIntegral[(3*(a +
b*ArcSinh[c*x]))/b])/4 + (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/
4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/4))/(b^2*c^4)
 + (5*((Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/8 - (3*Cosh[(3*a)/
b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16 + (Cosh[(5*a)/b]*CoshInteg
ral[(5*(a + b*ArcSinh[c*x]))/b])/16 - (Sinh[a/b]*SinhIntegral[(a + b*ArcSi
nh[c*x])/b])/8 + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b]
)/16 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16))/(b^2*
c^4)
```

3.411. $\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.411.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

## 3.411.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(201) = 402$.

Time = 0.40 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.97

| method | result |
|---|---|
| default | $-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^4b(a+b\,\operatorname{arcsinh}(cx))}-\frac{5\,\mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}_1\left(5\,\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b^2}+\frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2\cdots}}{32c^4b(a+b\,\mathrm{a}\cdots}$ |

input
```
int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*
x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/32/c^4/b^
2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+1/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2
+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/32/c^4/b^2*e
xp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^4/b/(
a+b*arcsinh(c*x))+1/16/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/b^
2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b
)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+1/32/c^4/b^2*(
4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsinh(c
*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*b*c
*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^4/b^2*(16*b*c^5*x^5+16*(
c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*a
rcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*
x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

### 3.411.5 Fricas [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^3/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2), x)
```

### 3.411.6 Sympy [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{x^3\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

### 3.411.7 Maxima [F]

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*
b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*
x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*
c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^4*x^6 + 11*c^2*x^4 + 3*x^2)
*(c^2*x^2 + 1) + (5*c^5*x^7 + 9*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b
*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^
4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b
^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3
 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.411.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3\,\sqrt{c^2\,x^2+1}}{\left(a+b\,\text{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

# 3.412 $\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$



## 3.412.1 Optimal result

Integrand size = 27, antiderivative size = 93

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = -\frac{x^2(1+c^2x^2)}{bc(a+b\mathrm{arcsinh}(cx))} - \frac{\mathrm{Chi}\left(\frac{4(a+b\mathrm{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\mathrm{arcsinh}(cx))}{b}\right)}{2b^2c^3}$$

output
```
-x^2*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh
(c*x))/b)/b^2/c^3-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3
```

## 3.412.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \frac{-\frac{2bc^2x^2(1+c^2x^2)}{a+b\mathrm{arcsinh}(cx)} - \mathrm{Chi}\left(4\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right) + \cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\right)}{2b^2c^3}$$

input
```
Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
((-2*b*c^2*x^2*(1 + c^2*x^2))/(a + b*ArcSinh[c*x]) - CoshIntegral[4*(a/b +
 ArcSinh[c*x])]*Sinh[(4*a)/b] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSin
h[c*x])])/(2*b^2*c^3)
```

## 3.412.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.16, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {6229, 6195, 25, 5971, 27, 2009, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c^2x^2+1}}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{4c\int \frac{x^3}{a+b\text{arcsinh}(cx)}dx}{b}+\frac{2\int \frac{x}{a+b\text{arcsinh}(cx)}dx}{bc}-\frac{x^2(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6195}$$

$$\frac{4\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}+\frac{2\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$
$$\frac{4\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}-\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$
$$\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 27

$$-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$
$$\frac{4\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}-\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$
$$\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 2009

$$-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}+$$
$$\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}$$
$$\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 3042

$$-\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}+$$
$$\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}$$
$$\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 26

---

3.412. $\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

$$\frac{\frac{i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}+\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}}{\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}}$$

$\downarrow$ 3784

$$\frac{\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(c\right.}{b^2c^3}\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}}{\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}}$$

$\downarrow$ 26

$$\frac{\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx\right.}{b^2c^3}\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}}{\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}}$$

$\downarrow$ 3042

$$\frac{\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsin}\right.}{b^2c^3}\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}}{\frac{x^2\left(c^2x^2+1\right)}{bc(a+b\text{arcsinh}(cx))}}$$

$\downarrow$ 26

---

3.412. $\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

$$\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx\right.}{b^2c^3}$$

$$\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}$$

$$\frac{x^2(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \; 3779$$

$$\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c^3}+$$

$$\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}$$

$$\frac{x^2(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \; 3782$$

$$\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c^3}+$$

$$\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^3}$$

$$\frac{x^2(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((x^2*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) + (I*(I*CoshIntegral[(2*
(a + b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(
a + b*ArcSinh[c*x]))/b]))/(b^2*c^3) + (4*((CoshIntegral[(2*(a + b*ArcSinh[
c*x]))/b]*Sinh[(2*a)/b])/4 - (CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]*Sin
h[(4*a)/b])/8 - (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/4
 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8))/(b^2*c^3)
```

3.412. $\int\frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.412.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

## 3.412. $\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

### 3.412.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.55

| method | result |
|---|---|
| default | $-\frac{4b\,c^4x^4+4b\,c^2x^2+\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)-\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)+\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}\right.}{4c^3b^2(a+b\,\mathrm{arcsinh}(cx))}$ |

input
```
int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b*c^4*x^4+4*b*c^2*x^2+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*ar
csinh(c*x)-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)+exp(-4*a/b
)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*a)/c
^3/b^2/(a+b*arcsinh(c*x))
```

### 3.412.5 Fricas [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^2/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2), x)
```

### 3.412.6 Sympy [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^2\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

### 3.412.7 Maxima [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*
b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*
x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*
c^3*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^4*x^5 + 4*c^2*x^3 + x)*(c^2*
x^2 + 1) + (4*c^5*x^6 + 7*c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x
^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c
^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2
*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b
*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.412.8 Giac [F]

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

### 3.412.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2\,\sqrt{c^2\,x^2+1}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

# 3.413 $\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.413.1 Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x(1+c^2x^2)}{bc(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^2}$$

output
```
-x*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a
/b)/b^2/c^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2-1/4*Shi((a
+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh
(3*a/b)/b^2/c^2
```

### 3.413.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\frac{4bcx}{a+b\text{arcsinh}(cx)}+\frac{4bc^3x^3}{a+b\text{arcsinh}(cx)}-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{4b^2c^2}$$

input
```
Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/4*((4*b*c*x)/(a + b*ArcSinh[c*x]) + (4*b*c^3*x^3)/(a + b*ArcSinh[c*x])
- Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegra
l[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3
*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^2)
```

### 3.413.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {6229, 6189, 3042, 3784, 26, 3042, 26, 3779, 3782, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c^2x^2+1}}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}+\frac{\int \frac{1}{a+b\text{arcsinh}(cx)}dx}{bc}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 3784

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 26

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 3042

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 26

$$\frac{i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 3779

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

---

3.413. $\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

$$\Big\downarrow \ 3782$$

$$\frac{3c\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 6195$$

$$\frac{3\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 5971$$

$$\frac{3\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}-\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 2009$$

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}+\frac{3\left(-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\text{si}\right.}{b^2c^2}$$

$$\frac{x(c^2x^2+1)}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((x*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[
(a + b*ArcSinh[c*x])/b] - Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/
(b^2*c^2) + (3*(-1/4*(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b]) + (C
osh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/4 + (Sinh[a/b]*Sinh
Integral[(a + b*ArcSinh[c*x])/b])/4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a +
b*ArcSinh[c*x]))/b])/4))/(b^2*c^2)
```

## 3.413. $\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.413.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

## 3.413. $\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

## 3.413.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(141) = 282$.

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.44

| method | result |
|---|---|
| default | $-\frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{-\sqrt{c^2x^2+1}+cx}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right.}{8c^2b^2}$ |

input
```
int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b
/(a+b*arcsinh(c*x))-3/8/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8*
(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*arcsinh(c*x))-1/8/c^2/b^2*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)-1/8/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(
-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+
b*arcsinh(c*x))-1/8/c^2/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*a
rcsinh(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*
x)-3*a/b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

## 3.413.5 Fricas [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}\,x}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2), x)
```

## 3.413.6 Sympy [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

## 3.413.7 Maxima [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^3 + x)*(c^2*x^2 + 1) + (c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*
c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((3*(c^
2*x^2 + 1)^(3/2)*c^3*x^3 + (6*c^4*x^4 + 5*c^2*x^2 + 1)*(c^2*x^2 + 1) + (3*
c^5*x^5 + 5*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 +
1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*
c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2
 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2
*x^2 + 1)), x)
```

### 3.413.8 Giac [F]

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}x}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a)^2, x)
```

### 3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x\,\sqrt{c^2\,x^2+1}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

# 3.414 $\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.414.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{1+c^2x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c}$$

output
```
(-c^2*x^2-1)/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b
)/b^2/c-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c
```

## 3.414.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\frac{-\frac{b+bc^2x^2}{a+b\text{arcsinh}(cx)}-\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{b^2c}$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
(-((b + b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c
*x])]*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/
(b^2*c)
```

## 3.414.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6205, 6195, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{2c\int \frac{x}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{2\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 3042

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}-\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$\downarrow$ 26

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}+\frac{i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$\downarrow$ 3784

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(c\right.}{b^2c}$$

$\downarrow$ 26

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx\right.}{b^2c}$$

$\downarrow$ 3042

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsin}\right.}{b^2c}$$

$\downarrow$ 26

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(ca\right.}{b^2c}$$

$\downarrow$ 3779

---

3.414. $\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{c^2x^2+1}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c}$$

input
```
Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((1 + c^2*x^2)/(b*c*(a + b*ArcSinh[c*x]))) + (I*(I*CoshIntegral[(2*(a + b
*ArcSinh[c*x]))/b]*Sinh[(2*a)/b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*
ArcSinh[c*x]))/b]))/(b^2*c)
```

### 3.414.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

3.414. $\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

## 3.414.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| default | $-\frac{2b\,c^{2}x^{2}+{\mathrm e}^{-\frac{2a}{b}}\,\mathrm{Ei}_{1}\left(-2\,\operatorname{arcsinh}\left(cx\right)-\frac{2a}{b}\right)b\,\operatorname{arcsinh}\left(cx\right)-{\mathrm e}^{\frac{2a}{b}}\,\mathrm{Ei}_{1}\left(2\,\operatorname{arcsinh}\left(cx\right)+\frac{2a}{b}\right)b\,\operatorname{arcsinh}\left(cx\right)+{\mathrm e}^{-\frac{2a}{b}}\,\mathrm{Ei}_{1}\left(-2\,\operatorname{arcsinh}\left(cx\right)-\frac{2a}{b}}{2c\,b^{2}\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)}$ |

input
```
int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*c^2*x^2+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-e
xp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)+exp(-2*a/b)*Ei(1,-2*ar
csinh(c*x)-2*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a+2*b)/c/b^2/(a+
b*arcsinh(c*x))
```

## 3.414.5 Fricas [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)
, x)
```

## 3.414.6 Sympy [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

## 3.414.7 Maxima [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt
(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2
*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^2*x^2 - 1)*(c^
2*x^2 + 1)^(3/2) + 2*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*
x^2 + 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a
*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^
2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.414.8 Giac [F]

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)^2, x)
```

### 3.414.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+c^2x^2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{c^2\,x^2+1}}{(a+b\,\mathrm{asinh}\,(c\,x))^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2, x)
```

# 3.415 $\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.415.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1+c^2x^2}{bcx(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2}-\frac{\text{Int}\left(\frac{1}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

output
```
(-c^2*x^2-1)/b/c/x/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/
b^2-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2-Unintegrable(1/x^2/(a+b*arcsin
h(c*x)),x)/b/c
```

## 3.415.2 Mathematica [N/A]

Not integrable

Time = 10.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2), x]
```

### 3.415.3 Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6229, 6189, 3042, 3784, 26, 3042, 26, 3779, 3782, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6229}$$

$$-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}+\frac{c\int \frac{1}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6196}$$

$$-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2}-\frac{c^2x^2+1}{bcx(a+b\text{arcsinh}(cx))}$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

---

3.415. $\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.415.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

3.415. $\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

## 3.415.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

## 3.415.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2 x}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
a^2*x), x)
```

### 3.415.6 Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\text{asinh}\,(cx)\right)^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))**2), x)
```

### 3.415.7 Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 14.52

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\text{arsinh}\,(cx)+a\right)^2x}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt
(c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*x^3 -
2*c*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 - c^2*x^2 - 1)*(c^2*x^2 + 1) + (c^
5*x^5 + c^3*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x
^4 + 2*a*b*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4
+ 2*b^2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2
*x^2 + 1)), x)
```

### 3.415.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.415.9 Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2), x)
```

# 3.416 $\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.416.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{1+c^2x^2}{bcx^2(a+b\text{arcsinh}(cx))}-\frac{2\text{Int}\left(\frac{1}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

output
```
(-c^2*x^2-1)/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(
c*x)),x)/b/c
```

## 3.416.2 Mathematica [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

### 3.416.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6228, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6228}$$

$$-\frac{2\int \frac{1}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx^2(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6196}$$

$$-\frac{2\int \frac{1}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{c^2x^2+1}{bcx^2(a+b\text{arcsinh}(cx))}$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.416.3.1 Defintions of rubi rules used

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6228
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
- 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && EqQ[m + 2*p + 1, 0]
```

### 3.416.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\ \operatorname{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

### 3.416.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x
) + a^2*x^2), x)
```

### 3.416.6 Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\,\text{asinh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))**2), x)
```

### 3.416.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 378, normalized size of antiderivative = 14.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt
(c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b
^2*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate((3*(c^2*
x^2 + 1)^(3/2)*c*x + 2*(2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 + 2*a*b*c^3*x^5 +
 a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b^2*c^3*x^5 + b^
2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 + 1)), x)
```

### 3.416.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

### 3.416.9 Mupad [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

# 3.417 $\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.417.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

## 3.417.2 Mathematica [N/A]

Not integrable

Time = 16.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

## 3.417.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{\sqrt{c^2x^2+1}}{x^3(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.417.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.417.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

### 3.417.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x
) + a^2*x^3), x)
```

### 3.417.6 Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^3\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))**2), x)
```

### 3.417.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 403, normalized size of antiderivative = 14.93

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt
(c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b
^2*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((c^3*x^
3 + 4*c*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 + 7*c^2*x^2 + 3)*(c^2*x^2 + 1)
 + (c^5*x^5 + 3*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^
2 + 1)*a*b*c^3*x^6 + 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 +
 1)*b^2*c^3*x^6 + 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c
^2*x^5)*sqrt(c^2*x^2 + 1)), x)
```

## 3.417.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## 3.417.9 Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

# 3.418 $\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))^2} dx$



## 3.418.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))^2} dx = \operatorname{Int}\left(\frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))^2}, x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

## 3.418.2 Mathematica [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))^2} dx = \int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\operatorname{arcsinh}(cx))^2} dx$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

### 3.418.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{x^4(a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{\sqrt{c^2x^2+1}}{x^4(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.418.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.418.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\ \text{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

### 3.418.5 Fricas [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x
) + a^2*x^4), x)
```

### 3.418.6 Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\,\text{asinh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))**2), x)
```

### 3.418.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 406, normalized size of antiderivative = 15.04

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt
(c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b
^2*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((2*c^3*
x^3 + 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^4*x^4 + 5*c^2*x^2 + 2)*(c^2*x^2
+ 1) + (2*c^5*x^5 + 5*c^3*x^3 + 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (
c^2*x^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2
*x^2 + 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c
^2*x^6)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 +
 a*b*c^2*x^6)*sqrt(c^2*x^2 + 1)), x)
```

### 3.418.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

### 3.418.9 Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{x^4\,\left(a+b\,\text{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

# 3.419 $\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$



## 3.419.1 Optimal result

Integrand size = 27, antiderivative size = 277

$$\begin{aligned}\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))}\\ &-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4}-\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}\\ &+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}+\frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}\\ &+\frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^4}+\frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}\\ &-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}-\frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^4}\end{aligned}$$

output
```
-x^3*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-3/64*Chi((a+b*arcsinh(c*x))/b)*c
osh(a/b)/b^2/c^4-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/64
*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+7/64*Chi(7*(a+b*arcsinh(c
*x))/b)*cosh(7*a/b)/b^2/c^4+3/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c
^4+9/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5/64*Shi(5*(a+b*ar
csinh(c*x))/b)*sinh(5*a/b)/b^2/c^4-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7
*a/b)/b^2/c^4
```

### 3.419.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.44

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-64bc^3x^3-128bc^5x^5-64bc^7x^7-3(a+b\text{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsin}\right.}{}$$

input
```
Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
(-64*b*c^3*x^3 - 128*b*c^5*x^5 - 64*b*c^7*x^7 - 3*(a + b*ArcSinh[c*x])*Cos
h[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a
)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + 5*a*Cosh[(5*a)/b]*CoshIntegral
[5*(a/b + ArcSinh[c*x])] + 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(
a/b + ArcSinh[c*x])] + 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x
])] + 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])]
+ 3*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*b*ArcSinh[c*x]*Sinh[a
/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*SinhIntegral[3*(a
/b + ArcSinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcSinh[c*x])] - 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] -
 5*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 7*a
*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Sin
h[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b^2*c^4*(a + b*ArcSin
h[c*x]))
```

### 3.419.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6229, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c^2x^2+1\right)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6229}$$

$$\frac{3\int \frac{x^2(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{bc}+\frac{7c\int \frac{x^4(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x^3\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 6234

$$\frac{7\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^4}+\frac{3\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{x^3\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 5971

$$\frac{3\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^4}+\frac{7\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{64(a+b\text{arcsinh}(cx))}\right.}{b^2c^4}$$

$$\frac{x^3\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 2009

$$\frac{3\left(-\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\right.}{b^2c^4}$$

$$\frac{7\left(\frac{3}{64}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{3}{64}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{64}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{}$$

$$\frac{x^3\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

3.419. $\int \frac{x^3\left(1+c^2x^2\right)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

output
```
-((x^3*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-1/8*(Cosh[a/b]*
CoshIntegral[(a + b*ArcSinh[c*x])/b]) + (Cosh[(3*a)/b]*CoshIntegral[(3*(a
+ b*ArcSinh[c*x]))/b])/16 + (Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[
c*x]))/b])/16 + (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/8 - (Sinh
[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16 - (Sinh[(5*a)/b]*Si
nhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16))/(b^2*c^4) + (7*((3*Cosh[a/b]*
CoshIntegral[(a + b*ArcSinh[c*x])/b])/64 - (3*Cosh[(3*a)/b]*CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b])/64 - (Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*Arc
Sinh[c*x]))/b])/64 + (Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcSinh[c*x]))/
b])/64 - (3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/64 + (3*Sinh[(
3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/64 + (Sinh[(5*a)/b]*Sinh
Integral[(5*(a + b*ArcSinh[c*x]))/b])/64 - (Sinh[(7*a)/b]*SinhIntegral[(7*
(a + b*ArcSinh[c*x]))/b])/64))/(b^2*c^4)
```

### 3.419.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.419. $\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.419.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(261) = 522.

Time = 0.31 (sec) , antiderivative size = 958, normalized size of antiderivative = 3.46

| method | result | size |
|---|---|---|
| default | Expression too large to display | 958 |

input
```
int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^
2*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(
1/2))/c^4/(a+b*arcsinh(c*x))/b-7/128/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x
)+7*a/b)-1/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*
x^2*(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/
128/c^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/128*(4*c^3*x^3-4*c^2*x
^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+9/1
28/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/128*(-(c^2*x^2+1)^(1/2)
+c*x)/c^4/b/(a+b*arcsinh(c*x))+3/128/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/
b)+3/128/c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-a
rcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
+3/128/c^4/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*E
i(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp
(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/128/c^4/b^2*(
16*b*c^5*x^5+16*(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1
/2)*b*c^2*x^2+5*arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*E
i(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b
*arcsinh(c*x))-1/128/c^4/b^2*(64*b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+
112*b*c^5*x^5+80*(c^2*x^2+1)^(1/2)*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(
1/2)*b*c^2*x^2+7*arcsinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b...
```

### 3.419.5 Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arc
sinh(c*x) + a^2), x)
```

### 3.419.6 Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

### 3.419.7 Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^4*x^7 + 2*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^5*x^8 + 2*c^3*x^6 + c*x^4
)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
 + 1))) + integrate(((7*c^5*x^7 + 9*c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2)
 + (14*c^6*x^8 + 27*c^4*x^6 + 16*c^2*x^4 + 3*x^2)*(c^2*x^2 + 1) + (7*c^7*x
^9 + 18*c^5*x^7 + 15*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 +
(c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^
2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*s
qrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*
x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.419.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.419.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{3/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

# 3.420 $\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.420.1 Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^2(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c^3} - \frac{3\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} + \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3}$$

output
```
-x^2*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcs
inh(c*x))/b)/b^2/c^3+1/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3
/16*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsin
h(c*x))/b)*sinh(2*a/b)/b^2/c^3-1/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)
/b^2/c^3-3/16*Chi(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3
```

### 3.420.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.40

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx =$$
$$-\frac{16bc^2x^2+32bc^4x^4+16bc^6x^6-(a+b\operatorname{arcsinh}(cx))\mathrm{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+4(a+b\operatorname{arcsinh}(cx$$

input
```
Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*(16*b*c^2*x^2 + 32*b*c^4*x^4 + 16*b*c^6*x^6 - (a + b*ArcSinh[c*x])*C
oshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 4*(a + b*ArcSinh[c*x])
*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshIntegral[6*(
a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[6*(a/b
+ ArcSinh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Arc
Sinh[c*x])] + b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c
*x])] - 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*b*ArcSi
nh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a
)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]
*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(b^2*c^3*(a + b*ArcSinh[c*x]))
```

### 3.420.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6229, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c^2x^2+1\right)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \ 6229$$

$$\frac{2\int \frac{x\left(c^2x^2+1\right)}{a+b\operatorname{arcsinh}(cx)}dx}{bc}+\frac{6c\int \frac{x^3\left(c^2x^2+1\right)}{a+b\operatorname{arcsinh}(cx)}dx}{b}-\frac{x^2\left(c^2x^2+1\right)^2}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$\Big\downarrow \ 6234$$

$$\frac{6\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}+\frac{2\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 25

$$-\frac{6\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 5971

$$-\frac{6\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}-\frac{3\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 2009

$$\frac{2\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right.}{b^2c^3}$$

$$\frac{6\left(\frac{3}{32}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{32}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{32}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right.}{b^2c^3}$$

$$\frac{x^2\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

3.420. $\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

output
```
-((x^2*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (2*(-1/4*(CoshIntegr
al[(2*(a + b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b]) - (CoshIntegral[(4*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(4*a)/b])/8 + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*A
rcSinh[c*x]))/b])/4 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))
/b])/8))/(b^2*c^3) + (6*((3*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]*Sinh[
(2*a)/b])/32 - (CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b]*Sinh[(6*a)/b])/32
 - (3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/32 + (Cosh[(
6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/32))/(b^2*c^3)
```

### 3.420.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.420. $\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.420.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.68

| method | result |
|---|---|
| default | $\frac{-32 b\, c^{6} x^{6}-64 b\, c^{4} x^{4}-32 b\, c^{2} x^{2}+{\mathrm e}^{-\frac{2 a}{b}} \operatorname{Ei}_{1}\left(-2 \operatorname{arcsinh}\left(c x \right)-\frac{2 a}{b}\right) b \,\operatorname{arcsinh}\left(c x \right)-4 \,{\mathrm e}^{-\frac{4 a}{b}} \operatorname{Ei}_{1}\left(-4 \operatorname{arcsinh}\left(c x \right)-\frac{4 a}{b}\right) b \,\operatorname{arcsinh}\left(c x \right)+3 \,{\mathrm e}^{\frac{6 a}{b}}}{}$ |

input
```
int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-32*b*c^6*x^6-64*b*c^4*x^4-32*b*c^2*x^2+exp(-2*a/b)*Ei(1,-2*arcsinh(
c*x)-2*a/b)*b*arcsinh(c*x)-4*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arc
sinh(c*x)+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)*b*arcsinh(c*x)+4*exp(4*a
/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)-exp(2*a/b)*Ei(1,2*arcsinh(c*
x)+2*a/b)*b*arcsinh(c*x)-3*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b*arcsi
nh(c*x)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a-4*exp(-4*a/b)*Ei(1,-4*ar
csinh(c*x)-4*a/b)*a+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)*a+4*exp(4*a/b)
*Ei(1,4*arcsinh(c*x)+4*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-3*ex
p(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a)/c^3/b^2/(a+b*arcsinh(c*x))
```

### 3.420.5 Fricas [F]

$$\int \frac{x^2 (1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{{\left(c^{2} x^{2} + 1\right)}^{\frac{3}{2}} x^{2}}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2}}\,{d x}$$

input
```
integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arc
sinh(c*x) + a^2), x)
```

### 3.420.6 Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

### 3.420.7 Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^4*x^6 + 2*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^5*x^7 + 2*c^3*x^5 + c*x^3
)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
 + 1))) + integrate(((6*c^5*x^6 + 7*c^3*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) +
 2*(6*c^6*x^7 + 11*c^4*x^5 + 6*c^2*x^3 + x)*(c^2*x^2 + 1) + 3*(2*c^7*x^8 +
 5*c^5*x^6 + 4*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2
 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b
^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(
c^2*x^2 + 1)), x)
```

## 3.420.8 Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

## 3.420.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^2\,(c^2\,x^2+1)^{3/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

# 3.421 $\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2} dx$



## 3.421.1 Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2} dx = -\frac{x(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^2} - \frac{9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^2}$$

output
```
-x*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+1/8*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2/c^2+9/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+5/16*Ch
i(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2-1/8*Shi((a+b*arcsinh(c*x))/b
)*sinh(a/b)/b^2/c^2-9/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-5
/16*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^2
```

### 3.421.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{16bcx+32bc^3x^3+16bc^5x^5-2(a+b\operatorname{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)-9(a+b\operatorname{arcsinh}(cx))\operatorname{co}}{}$$

input
```
Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*(16*b*c*x + 32*b*c^3*x^3 + 16*b*c^5*x^5 - 2*(a + b*ArcSinh[c*x])*Cos
h[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a
)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 5*a*Cosh[(5*a)/b]*CoshIntegral
[5*(a/b + ArcSinh[c*x])] - 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(
a/b + ArcSinh[c*x])] + 2*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 2*
b*ArcSinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*
SinhIntegral[3*(a/b + ArcSinh[c*x])] + 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])] + 5*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b +
 ArcSinh[c*x])])/(b^2*c^2*(a + b*ArcSinh[c*x]))
```

### 3.421.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6229, 6206, 3042, 3793, 2009, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c^2x^2+1)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{\int \frac{c^2x^2+1}{a+b\operatorname{arcsinh}(cx)}dx}{bc}+\frac{5c\int \frac{x^2(c^2x^2+1)}{a+b\operatorname{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)^2}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$\Big\downarrow \text{6206}$$

$$\frac{\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{5c\int \frac{x^2(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^3}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{5c\int \frac{x^2(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 3793

$$\frac{\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{5c\int \frac{x^2(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 2009

$$\frac{5c\int \frac{x^2(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{b}+\frac{\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\right.}{b^2c^2}$$
$$\frac{x\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 6234

$$\frac{5\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\right.}{b^2c^2}$$
$$\frac{x\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 5971

3.421. $\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

$$\frac{5\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^2}+$$

$$\frac{\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\right.}{b^2c^2}$$

$$\frac{x\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 2009

$$\frac{\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\right.}{b^2c^2}$$

$$\frac{5\left(-\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\right.}{b^2c^2}$$

$$\frac{x\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((x*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + ((3*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/4 + (Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Ar
cSinh[c*x]))/b])/4 - (3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/4
- (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/4)/(b^2*c^2) +
(5*(-1/8*(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b]) + (Cosh[(3*a)/b]
*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16 + (Cosh[(5*a)/b]*CoshIntegra
l[(5*(a + b*ArcSinh[c*x]))/b])/16 + (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh
[c*x])/b])/8 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16
 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16))/(b^2*c^2)
```

### 3.421.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

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rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

---

3.421. $\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.421.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(201) = 402.

Time = 0.27 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.97

| method | result |
|---|---|
| default | $-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{5\,\mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}_1\left(5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^2b^2}-\frac{3\left(4c^3x^3-4c^2x^2\sqrt{c^2\ldots}\right.}{32c^2b(a+b\ldots}$ |

input
```
int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*
x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-5/32/c^2/b^
2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2
+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-9/32/c^2/b^2*e
xp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(
a+b*arcsinh(c*x))-1/16/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/b^
2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b
)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/32/c^2/b^2*(
4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsinh(c
*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*b*c
*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^2/b^2*(16*b*c^5*x^5+16*(
c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*a
rcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c*
x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

### 3.421.5 Fricas [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsi
nh(c*x) + a^2), x)
```

### 3.421.6 Sympy [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x(c^2x^2+1)^{\frac{3}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

### 3.421.7 Maxima [F]

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{(b\,\text{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^4*x^5 + 2*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^5*x^6 + 2*c^3*x^4 + c*x^2)*
sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
 1))) + integrate((5*(c^5*x^5 + c^3*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^6*x^6
 + 17*c^4*x^4 + 8*c^2*x^2 + 1)*(c^2*x^2 + 1) + (5*c^7*x^7 + 12*c^5*x^5 + 9
*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*
x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2
*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(
c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)),
 x)
```

### 3.421.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.421.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{x\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

# 3.422 $\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2} dx$



## 3.422.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2} dx = -\frac{(1+c^2x^2)^2}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c}$$

output
```
-(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))
/b)/b^2/c+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c-Chi(2*(a+b*arc
sinh(c*x))/b)*sinh(2*a/b)/b^2/c-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b
)/b^2/c
```

## 3.422.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2} dx = \frac{-\frac{2b(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)} - 2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx\right.\right.}{}$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
((-2*b*(1 + c^2*x^2)^2)/(a + b*ArcSinh[c*x]) - 2*CoshIntegral[2*(a/b + Arc
Sinh[c*x])]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a
)/b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b
]*SinhIntegral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c)
```

### 3.422.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{4c\int \frac{x(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{4\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{4\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{(c^2x^2+1)^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right.}{b^2c}$$

$$\frac{\left(c^2x^2+1\right)^2}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((1 + c^2*x^2)^2/(b*c*(a + b*ArcSinh[c*x]))) + (4*(-1/4*(CoshIntegral[(2*
(a + b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b]) - (CoshIntegral[(4*(a + b*ArcSinh[
c*x]))/b]*Sinh[(4*a)/b])/8 + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/4 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8
))/(b^2*c)
```

### 3.422.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_^2))^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.422.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| default | $-\frac{4b\,c^4x^4+8b\,c^2x^2+{\mathrm e}^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)+2\,{\mathrm e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)b\,\mathrm{arcsinh}(cx)-{\mathrm e}^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsin}\right.}{}$ |

input
```
int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b*c^4*x^4+8*b*c^2*x^2+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*ar
csinh(c*x)+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-exp(4*
a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)-2*exp(2*a/b)*Ei(1,2*arcsinh
(c*x)+2*a/b)*b*arcsinh(c*x)+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+2*ex
p(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a-exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a
/b)*a-2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a+4*b)/c/b^2/(a+b*arcsinh(c*
x))
```

### 3.422.5 Fricas [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2), x)
```

### 3.422.6 Sympy [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

### 3.422.7 Maxima [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2
*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1
))) + integrate(((4*c^4*x^4 + 3*c^2*x^2 - 1)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^
5*x^5 + 3*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (4*c^6*x^6 + 9*c^4*x^4 + 6*c^2*x^
2 + 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b
*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2
+ b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^
2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.422.8 Giac [F]

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a)^2, x)
```

### 3.422.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{\left(a+b\,\operatorname{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2, x)
```

# 3.423 $\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.423.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^2}{bcx(a+b\text{arcsinh}(cx))} + \frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2} - \frac{9\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2} - \frac{\text{Int}\left(\frac{1+c^2x^2}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

output
```
-(c^2*x^2+1)^2/b/c/x/(a+b*arcsinh(c*x))+9/4*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2-9/4*Shi((a+b*arc
sinh(c*x))/b)*sinh(a/b)/b^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^
2-Unintegrable((c^2*x^2+1)/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

### 3.423.2 Mathematica [N/A]

Not integrable

Time = 7.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

### 3.423.3 Rubi [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6229, 6206, 3042, 3793, 2009, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6229}$$

$$\frac{3c\int \frac{c^2x^2+1}{a+b\text{arcsinh}(cx)}dx}{b} - \frac{\int \frac{c^2x^2+1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc} - \frac{(c^2x^2+1)^2}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{3\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2} - \frac{\int \frac{c^2x^2+1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc} - \frac{(c^2x^2+1)^2}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^3}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{c^2x^2+1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{\left(c^2x^2+1\right)^2}{bcx(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 3793

$$\frac{3\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{c^2x^2+1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{\left(c^2x^2+1\right)^2}{bcx(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 2009

$$-\frac{\int \frac{c^2x^2+1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}+\frac{3\left(\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\text{sinl}\right.}{b^2}$$

$$\frac{\left(c^2x^2+1\right)^2}{bcx(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 6239

$$-\frac{\int \frac{c^2x^2+1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}+\frac{3\left(\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\text{sinl}\right.}{b^2}$$

$$\frac{\left(c^2x^2+1\right)^2}{bcx(a+b\text{arcsinh}(cx))}$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

## 3.423. $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.423.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.423. $\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.423.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\operatorname{arcsinh}(cx))^2}dx$$

input
```
int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
```

### 3.423.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\operatorname{arsinh}(cx)+a)^2x}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x)
+ a^2*x), x)
```

### 3.423.6 Sympy [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))**2), x)
```

### 3.423.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 433, normalized size of antiderivative = 16.04

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x +
(b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2
*x^2 + 1))) + integrate(((3*c^5*x^5 + c^3*x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2)
 + (6*c^6*x^6 + 7*c^4*x^4 - 1)*(c^2*x^2 + 1) + 3*(c^7*x^7 + 2*c^5*x^5 + c^
3*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b
*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^
3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1))
, x)
```

### 3.423.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.423.9 Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2), x)
```

# 3.424 $\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.424.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^2}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\text{Int}\left(\frac{1+c^2x^2}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc} + \frac{2c\text{Int}\left(\frac{1+c^2x^2}{x(a+b\text{arcsinh}(cx))},x\right)}{b}$$

output
```
-(c^2*x^2+1)^2/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)/x^3/(
a+b*arcsinh(c*x)),x)/b/c+2*c*Unintegrable((c^2*x^2+1)/x/(a+b*arcsinh(c*x))
,x)/b
```

## 3.424.2 Mathematica [N/A]

Not integrable

Time = 3.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

### 3.424.3 Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6229, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{2c\int \frac{c^2x^2+1}{x(a+b\text{arcsinh}(cx))}dx}{b}-\frac{2\int \frac{c^2x^2+1}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^2}{bcx^2(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6239}$$

$$\frac{2c\int \frac{c^2x^2+1}{x(a+b\text{arcsinh}(cx))}dx}{b}-\frac{2\int \frac{c^2x^2+1}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^2}{bcx^2(a+b\text{arcsinh}(cx))}$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.424.3.1 Defintions of rubi rules used

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.424.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{2} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}d x$$

input
```
int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

## 3.424.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{2}}\,{d x}$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c
*x) + a^2*x^2), x)
```

### 3.424.6 Sympy [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

### 3.424.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 443, normalized size of antiderivative = 16.41

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2
+ (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt
(c^2*x^2 + 1))) + integrate(((2*c^5*x^5 - c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(
3/2) + 2*(2*c^6*x^6 + c^4*x^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (2*c^7*x^7
+ 3*c^5*x^5 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3
*x^5 + 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^
5 + 2*b^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2
 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c
^2*x^2 + 1)), x)
```

### 3.424.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

### 3.424.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2\,x^2+1)^{3/2}}{x^2\,(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

# 3.425 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$



## 3.425.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\,\mathrm{arcsinh}(cx))^2}\,dx = \mathrm{Int}\left(\frac{(1+c^2x^2)^{3/2}}{x^3(a+b\,\mathrm{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

## 3.425.2 Mathematica [N/A]

Not integrable

Time = 12.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\,\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

### 3.425.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{(c^2x^2+1)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.425.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.425.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

### 3.425.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c
*x) + a^2*x^3), x)
```

### 3.425.6 Sympy [N/A]

Not integrable

Time = 3.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

### 3.425.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 441, normalized size of antiderivative = 16.33

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3
+ (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt
(c^2*x^2 + 1))) + integrate(((c^5*x^5 - 3*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(
3/2) + (2*c^6*x^6 - 3*c^4*x^4 - 8*c^2*x^2 - 3)*(c^2*x^2 + 1) + (c^7*x^7 -
3*c^3*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3
*x^6 + 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^
6 + 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2
 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c
^2*x^2 + 1)), x)
```

### 3.425.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.425.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(c^2\,x^2+1)^{3/2}}{x^3\,(a+b\,\mathrm{asinh}\,(c\,x))^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

# 3.426 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\textbf{arcsinh}(cx))^2}\,dx$



## 3.426.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^2}{bcx^4(a+b\text{arcsinh}(cx))} - \frac{4\text{Int}\left(\frac{1+c^2x^2}{x^5(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

output
```
-(c^2*x^2+1)^2/b/c/x^4/(a+b*arcsinh(c*x))-4*Unintegrable((c^2*x^2+1)/x^5/(
a+b*arcsinh(c*x)),x)/b/c
```

## 3.426.2 Mathematica [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

## 3.426.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6228, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6228}$$

$$-\frac{4\int \frac{c^2x^2+1}{x^5(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^2}{bcx^4(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{4\int \frac{c^2x^2+1}{x^5(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^2}{bcx^4(a+b\text{arcsinh}(cx))}$$

input
```
Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.426.3.1 Defintions of rubi rules used

rule 6228
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
- 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && EqQ[m + 2*p + 1, 0]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.426. $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$

## 3.426.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4 (a+b \operatorname{arcsinh}(cx))^2} dx$$

input
```
int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

## 3.426.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2} dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b \operatorname{arsinh}(cx)+a)^2 x^4} dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^2*x^2 + 1)^(3/2)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c
*x) + a^2*x^4), x)
```

## 3.426.6 Sympy [N/A]

Not integrable

Time = 5.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2} dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^4 (a+b \operatorname{asinh}(cx))^2} dx$$

input
```
integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

### 3.426.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 428, normalized size of antiderivative = 15.85

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4
+ (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt
(c^2*x^2 + 1))) - integrate((5*(c^3*x^3 + c*x)*(c^2*x^2 + 1)^(3/2) + 4*(2*
c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + 3*(c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 +
 a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^
2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1)), x)
```

### 3.426.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

### 3.426.9 Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

# 3.427 $\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.427.1 Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^3(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} + \frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} + \frac{9\cosh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128b^2c^4} + \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32b^2c^4} - \frac{21\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4} - \frac{9\sinh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256b^2c^4}$$

output
```
-x^3*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-3/128*Chi((a+b*arcsinh(c*x))/b)*
cosh(a/b)/b^2/c^4-3/32*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+21/
256*Chi(7*(a+b*arcsinh(c*x))/b)*cosh(7*a/b)/b^2/c^4+9/256*Chi(9*(a+b*arcsi
nh(c*x))/b)*cosh(9*a/b)/b^2/c^4+3/128*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/
b^2/c^4+3/32*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-21/256*Shi(7*
(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/b^2/c^4-9/256*Shi(9*(a+b*arcsinh(c*x))/b
)*sinh(9*a/b)/b^2/c^4
```

### 3.427.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.47

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx =$$
$$-\frac{256bc^3x^3+768bc^5x^5+768bc^7x^7+256bc^9x^9+6(a+b\text{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+24(a}{}$$

input
```
Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/256*(256*b*c^3*x^3 + 768*b*c^5*x^5 + 768*b*c^7*x^7 + 256*b*c^9*x^9 + 6*
(a + b*ArcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 24*(a +
b*ArcSinh[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 21*a*
Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 21*b*ArcSinh[c*x]*Cos
h[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 9*a*Cosh[(9*a)/b]*CoshIn
tegral[9*(a/b + ArcSinh[c*x])] - 9*b*ArcSinh[c*x]*Cosh[(9*a)/b]*CoshIntegr
al[9*(a/b + ArcSinh[c*x])] - 6*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]
] - 6*b*ArcSinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 24*a*Sin
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 24*b*ArcSinh[c*x]*Sinh[(
3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 21*a*Sinh[(7*a)/b]*SinhInte
gral[7*(a/b + ArcSinh[c*x])] + 21*b*ArcSinh[c*x]*Sinh[(7*a)/b]*SinhIntegra
l[7*(a/b + ArcSinh[c*x])] + 9*a*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSin
h[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*
x])])/(b^2*c^4*(a + b*ArcSinh[c*x]))
```

### 3.427.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6229, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c^2x^2+1\right)^{5/2}}{(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{6229}$$

$$\frac{3\int \frac{x^2(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{bc}+\frac{9c\int \frac{x^4(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x^3(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6234}$$

$$\frac{9\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^4}+\frac{3\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{x^3(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$\frac{3\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{5\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{64(a+b\text{arcsinh}(cx))}\right.}{b^2c^4}$$

$$\frac{9\int \left(\frac{\cosh\left(\frac{9a}{b}-\frac{9(a+b\text{arcsinh}(cx))}{b}\right)}{256(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{256(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c}{b}\right.}{64(a+b\text{arcsinh}(cx))}\right.}{b^2c^4}$$

$$\frac{x^3(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{5}{64}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{64}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{3}{64}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\right.}{}$$

$$\frac{9\left(\frac{3}{128}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{64}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{64}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\right.}{}$$

$$\frac{x^3(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

3.427. $\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

output
```
-((x^3*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (3*((-5*Cosh[a/b]*Co
shIntegral[(a + b*ArcSinh[c*x])/b])/64 + (Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/64 + (3*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSi
nh[c*x]))/b])/64 + (Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcSinh[c*x]))/b]
)/64 + (5*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/64 - (Sinh[(3*a)
/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/64 - (3*Sinh[(5*a)/b]*SinhIn
tegral[(5*(a + b*ArcSinh[c*x]))/b])/64 - (Sinh[(7*a)/b]*SinhIntegral[(7*(a
 + b*ArcSinh[c*x]))/b])/64))/(b^2*c^4) + (9*((3*Cosh[a/b]*CoshIntegral[(a
+ b*ArcSinh[c*x])/b])/128 - (Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[
c*x]))/b])/64 - (Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b])/6
4 + (Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcSinh[c*x]))/b])/256 + (Cosh[(
9*a)/b]*CoshIntegral[(9*(a + b*ArcSinh[c*x]))/b])/256 - (3*Sinh[a/b]*SinhI
ntegral[(a + b*ArcSinh[c*x])/b])/128 + (Sinh[(3*a)/b]*SinhIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/64 + (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c
*x]))/b])/64 - (Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSinh[c*x]))/b])/25
6 - (Sinh[(9*a)/b]*SinhIntegral[(9*(a + b*ArcSinh[c*x]))/b])/256))/(b^2*c^
4)
```

### 3.427.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

3.427. $\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.427.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(261) = 522$.

Time = 0.34 (sec) , antiderivative size = 1070, normalized size of antiderivative = 3.86

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1070 |

input
```
int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/512*(256*c^9*x^9-256*c^8*x^8*(c^2*x^2+1)^(1/2)+576*c^7*x^7-448*c^6*x^6*
(c^2*x^2+1)^(1/2)+432*c^5*x^5-240*c^4*x^4*(c^2*x^2+1)^(1/2)+120*c^3*x^3-40
*c^2*x^2*(c^2*x^2+1)^(1/2)+9*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsinh(c*x))
/b-9/512/c^4/b^2*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)-3/512*(64*c^7*x^7-6
4*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2*x^2+1)^(1/2)+56*c^
3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsi
nh(c*x))/b-21/512/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/64*(4*c^
3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcs
inh(c*x))+3/64/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/256*(-(c^2*
x^2+1)^(1/2)+c*x)/c^4/b/(a+b*arcsinh(c*x))+3/256/c^4/b^2*exp(a/b)*Ei(1,arc
sinh(c*x)+a/b)+3/256/c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/
b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*a
rcsinh(c*x))+1/64/c^4/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arc
sinh(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)
-3*a/b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/51
2/c^4/b^2*(64*b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^6+112*b*c^5*x^5+80*(c
^2*x^2+1)^(1/2)*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)^(1/2)*b*c^2*x^2+7*ar
csinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x
)-7*a/b)*exp(-7*a/b)*a+7*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/5
12/c^4/b^2*(256*b*c^9*x^9+256*(c^2*x^2+1)^(1/2)*b*c^8*x^8+576*b*c^7*x^7...
```

### 3.427.5 Fricas [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
 + 2*a*b*arcsinh(c*x) + a^2), x)
```

### 3.427.6 Sympy [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3(c^2x^2+1)^{\frac{5}{2}}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

### 3.427.7 Maxima [F]

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^6*x^9 + 3*c^4*x^7 + 3*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^7*x^10 + 3*c^
5*x^8 + 3*c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2
+ 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*
c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((9*c^7*x^9 + 20*c^5*x^7 + 13
*c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + 3*(6*c^8*x^10 + 17*c^6*x^8 + 17*
c^4*x^6 + 7*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (9*c^9*x^11 + 31*c^7*x^9 + 39*c
^5*x^7 + 21*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2
+ 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^
2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x
^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c
^2*x^2 + 1)), x)
```

### 3.427.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3\,(c^2\,x^2+1)^{5/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

# 3.428 $\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.428.1 Optimal result

Integrand size = 27, antiderivative size = 281

$$\begin{aligned}\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{x^2(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} \\ &+ \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8b^2c^3} \\ &- \frac{3\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{8a}{b}\right)}{16b^2c^3} \\ &- \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8b^2c^3} \\ &+ \frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3} + \frac{\cosh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^3}\end{aligned}$$

output
```
-x^2*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcs
inh(c*x))/b)/b^2/c^3+1/8*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3
/16*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*cosh(8*a/b)*Shi(8
*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b
)/b^2/c^3-1/8*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3-3/16*Chi(6*(
a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3-1/16*Chi(8*(a+b*arcsinh(c*x))/b)*
sinh(8*a/b)/b^2/c^3
```

### 3.428.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.47

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx =$$

$$-\,16bc^2x^2 + 48bc^4x^4 + 48bc^6x^6 + 16bc^8x^8 - (a+b\operatorname{arcsinh}(cx))\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) + 2(a+b$$

input
```
Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*(16*b*c^2*x^2 + 48*b*c^4*x^4 + 48*b*c^6*x^6 + 16*b*c^8*x^8 - (a + b*
ArcSinh[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 2*(a +
b*ArcSinh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*C
oshIntegral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshI
ntegral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + a*CoshIntegral[8*(a/b + Ar
cSinh[c*x])]*Sinh[(8*a)/b] + b*ArcSinh[c*x]*CoshIntegral[8*(a/b + ArcSinh[
c*x])]*Sinh[(8*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])
] + b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 2*
a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Co
sh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhI
ntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhInteg
ral[6*(a/b + ArcSinh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSin
h[c*x])] - b*ArcSinh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x]
)])/(b^2*c^3*(a + b*ArcSinh[c*x]))
```

### 3.428.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.43, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6229, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c^2x^2+1\right)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \; 6229$$

$$\frac{2\int \frac{x(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{bc}+\frac{8c\int \frac{x^3(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x^2(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 6234

$$\frac{8\int -\frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}+\frac{2\int -\frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 25

$$-\frac{8\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 5971

$$-\frac{8\int \left(\frac{\sinh\left(\frac{8a}{b}-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{3\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsin}}{b}\right.}{64(a+b\text{arcsinh}(cx}\right.}{b^2c^3}$$

$$\frac{2\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$

$$\frac{x^2(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 2009

$$\frac{2\left(-\frac{5}{32}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{32}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right.\right.}{}$$

$$\frac{8\left(\frac{3}{64}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{64}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{64}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right.\right.}{}$$

$$\frac{x^2(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

3.428. $\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

output
```
-((x^2*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (2*((-5*CoshIntegral
[(2*(a + b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/32 - (CoshIntegral[(4*(a + b*A
rcSinh[c*x]))/b]*Sinh[(4*a)/b])/8 - (CoshIntegral[(6*(a + b*ArcSinh[c*x]))
/b]*Sinh[(6*a)/b])/32 + (5*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*
x]))/b])/32 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8 +
 (Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/32))/(b^2*c^3) +
 (8*((3*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/64 + (Cosh
Integral[(4*(a + b*ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/64 - (CoshIntegral[(6*
(a + b*ArcSinh[c*x]))/b]*Sinh[(6*a)/b])/64 - (CoshIntegral[(8*(a + b*ArcSi
nh[c*x]))/b]*Sinh[(8*a)/b])/128 - (3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*
ArcSinh[c*x]))/b])/64 - (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]
))/b])/64 + (Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/64 +
(Cosh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcSinh[c*x]))/b])/128))/(b^2*c^3)
```

### 3.428.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

---

3.428. $\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.428.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| default | $\frac{-96 b c^6 x^6 - 2 e^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4 \operatorname{arcsinh}(cx) - \frac{4a}{b}\right) b \operatorname{arcsinh}(cx) + e^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2 \operatorname{arcsinh}(cx) - \frac{2a}{b}\right) b \operatorname{arcsinh}(cx) - e^{\frac{2a}{b}} \mathrm{Ei}_1\left(2 \operatorname{arcsinh}(cx) + \cdots\right.}{}$ |

input
```
int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-96*b*c^6*x^6-2*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arcsinh(c*
x)+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-exp(2*a/b)*Ei(1,
2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)+2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/
b)*b*arcsinh(c*x)-exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)*b*arcsinh(c*x)+e
xp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)*b*arcsinh(c*x)+3*exp(6*a/b)*Ei(1,6*ar
csinh(c*x)+6*a/b)*b*arcsinh(c*x)-3*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)
*b*arcsinh(c*x)-96*b*c^4*x^4-32*b*c^2*x^2+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)
+6*a/b)*a-3*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a+exp(-2*a/b)*Ei(1,-2*
arcsinh(c*x)-2*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-exp(-8*a/b)*
Ei(1,-8*arcsinh(c*x)-8*a/b)*a+exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)*a-2*ex
p(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a+2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4
*a/b)*a-32*b*c^8*x^8)/c^3/b^2/(a+b*arcsinh(c*x))
```

### 3.428.5 Fricas [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
 + 2*a*b*arcsinh(c*x) + a^2), x)
```

### 3.428.6 Sympy [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^2(c^2x^2+1)^{\frac{5}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

### 3.428.7 Maxima [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^6*x^8 + 3*c^4*x^6 + 3*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^7*x^9 + 3*c^5
*x^7 + 3*c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 +
 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c
)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((8*c^7*x^8 + 17*c^5*x^6 + 10*
c^3*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(8*c^8*x^9 + 22*c^6*x^7 + 21*c^4*
x^5 + 8*c^2*x^3 + x)*(c^2*x^2 + 1) + (8*c^9*x^10 + 27*c^7*x^8 + 33*c^5*x^6
 + 17*c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a
*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2
 + 1)), x)
```

### 3.428.8 Giac [F]

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

### 3.428.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2\,(c^2\,x^2+1)^{5/2}}{(a+b\,\operatorname{asinh}(c\,x))^2}\,dx$$

input
```
int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

# 3.429 $\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.429.1 Optimal result

Integrand size = 25, antiderivative size = 275

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x(1+c^2x^2)^3}{bc(a+b\text{arcsinh}(cx))} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2} + \frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} + \frac{25\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} + \frac{7\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} - \frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c^2} - \frac{27\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} - \frac{25\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2} - \frac{7\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c^2}$$

output
```
-x*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+5/64*Chi((a+b*arcsinh(c*x))/b)*cos
h(a/b)/b^2/c^2+27/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+25/64
*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2+7/64*Chi(7*(a+b*arcsinh(c
*x))/b)*cosh(7*a/b)/b^2/c^2-5/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c
^2-27/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-25/64*Shi(5*(a+b*
arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^2-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh
(7*a/b)/b^2/c^2
```

### 3.429.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.47

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx =$$
$$-\frac{64bcx+192bc^3x^3+192bc^5x^5+64bc^7x^7-5(a+b\text{arcsinh}(cx))\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-27(a+ba}{}$$

input
```
Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/64*(64*b*c*x + 192*b*c^3*x^3 + 192*b*c^5*x^5 + 64*b*c^7*x^7 - 5*(a + b*
ArcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 27*(a + b*ArcSi
nh[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 25*a*Cosh[(5
*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 25*b*ArcSinh[c*x]*Cosh[(5*a)
/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Cosh[(7*a)/b]*CoshIntegral[
7*(a/b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a
/b + ArcSinh[c*x])] + 5*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*b
*ArcSinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 27*a*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 27*b*ArcSinh[c*x]*Sinh[(3*a)/b]
*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 25*a*Sinh[(5*a)/b]*SinhIntegral[5*
(a/b + ArcSinh[c*x])] + 25*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/
b + ArcSinh[c*x])] + 7*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])
] + 7*b*ArcSinh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(
b^2*c^2*(a + b*ArcSinh[c*x]))
```

### 3.429.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6229, 6206, 3042, 3793, 2009, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c^2x^2+1)^{5/2}}{(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{6229}$$

$$\frac{\int \frac{(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{bc}+\frac{7c\int \frac{x^2(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 6206$$

$$\frac{\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{7c\int \frac{x^2(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^5}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{7c\int \frac{x^2(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 3793$$

$$\frac{\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{5\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{5\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{7c\int \frac{x^2(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 2009$$

$$\frac{7c\int \frac{x^2(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}+\frac{\frac{5}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{5}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\frac{5}{8}\text{s}}{b^2c^2}-\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 6234$$

$$\frac{7\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{\frac{5}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{5}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\frac{5}{8}\text{s}}{b^2c^2}-\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 5971$$

---

3.429. $\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}dx$

$$\frac{7\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}-\frac{5\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{64(a+b\text{arcsinh}(cx))}}{b^2c^2}$$

$$\frac{\frac{5}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{5}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\frac{5}{8}}{b^2c^2}$$

$$\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\downarrow$ 2009

$$\frac{\frac{5}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{5}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\frac{5}{8}}{b^2c^2}$$

$$7\left(-\frac{5}{64}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{64}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{3}{64}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\right.$$

$$\frac{x(c^2x^2+1)^3}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((x*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + ((5*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/8 + (5*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*
ArcSinh[c*x]))/b])/16 + (Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c*x]
))/b])/16 - (5*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/8 - (5*Sinh
[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16 - (Sinh[(5*a)/b]*Si
nhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16)/(b^2*c^2) + (7*((-5*Cosh[a/b]*
CoshIntegral[(a + b*ArcSinh[c*x])/b])/64 + (Cosh[(3*a)/b]*CoshIntegral[(3*
(a + b*ArcSinh[c*x]))/b])/64 + (3*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*Arc
Sinh[c*x]))/b])/64 + (Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcSinh[c*x]))/
b])/64 + (5*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/64 - (Sinh[(3*
a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/64 - (3*Sinh[(5*a)/b]*Sinh
Integral[(5*(a + b*ArcSinh[c*x]))/b])/64 - (Sinh[(7*a)/b]*SinhIntegral[(7*
(a + b*ArcSinh[c*x]))/b])/64))/(b^2*c^2)
```

## 3.429. $\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.429.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

---

## 3.429. $\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.429.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(259) = 518$.

Time = 0.28 (sec) , antiderivative size = 958, normalized size of antiderivative = 3.48

| method | result | size |
|---|---|---|
| default | Expression too large to display | 958 |

input
```
int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^
2*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(
1/2))/c^2/(a+b*arcsinh(c*x))/b-7/128/c^2/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x
)+7*a/b)-5/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*
x^2*(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-25
/128/c^2/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-9/128*(4*c^3*x^3-4*c^2*
x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-27
/128/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/128*(-(c^2*x^2+1)^(1/
2)+c*x)/c^2/b/(a+b*arcsinh(c*x))-5/128/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+
a/b)-5/128/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,
-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x
))-9/128/c^2/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)
*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*e
xp(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-5/128/c^2/b^2
*(16*b*c^5*x^5+16*(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^
(1/2)*b*c^2*x^2+5*arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5
*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a
+b*arcsinh(c*x))-1/128/c^2/b^2*(64*b*c^7*x^7+64*(c^2*x^2+1)^(1/2)*b*c^6*x^
6+112*b*c^5*x^5+80*(c^2*x^2+1)^(1/2)*b*c^4*x^4+56*b*c^3*x^3+24*(c^2*x^2+1)
^(1/2)*b*c^2*x^2+7*arcsinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)...
```

### 3.429.5 Fricas [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
 2*a*b*arcsinh(c*x) + a^2), x)
```

### 3.429.6 Sympy [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x(c^2x^2+1)^{\frac{5}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

### 3.429.7 Maxima [F]

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*x^7 + 3*c^4*x^5 + 3*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^7*x^8 + 3*c^5*x
^6 + 3*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1
)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*
log(c*x + sqrt(c^2*x^2 + 1))) + integrate((7*(c^7*x^7 + 2*c^5*x^5 + c^3*x^
3)*(c^2*x^2 + 1)^(3/2) + (14*c^8*x^8 + 37*c^6*x^6 + 33*c^4*x^4 + 11*c^2*x^
2 + 1)*(c^2*x^2 + 1) + (7*c^9*x^9 + 23*c^7*x^7 + 27*c^5*x^5 + 13*c^3*x^3 +
 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*
b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x
^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt
(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.429.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.429.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{x\,(c^2\,x^2+1)^{5/2}}{(a+b\,\operatorname{asinh}(c\,x))^2}\,dx$$

input
```
int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

# 3.430 $\int \frac{(1+c^2x^2)^{5/2}}{(a+b\,\text{arcsinh}(cx))^2}\,dx$



## 3.430.1 Optimal result

Integrand size = 24, antiderivative size = 216

$$\begin{aligned}\int \frac{(1+c^2x^2)^{5/2}}{(a+b\,\text{arcsinh}(cx))^2}\,dx &= -\frac{(1+c^2x^2)^3}{bc(a+b\,\text{arcsinh}(cx))} \\ &\quad-\frac{15\,\text{Chi}\left(\frac{2(a+b\,\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c}-\frac{3\,\text{Chi}\left(\frac{4(a+b\,\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c} \\ &\quad-\frac{3\,\text{Chi}\left(\frac{6(a+b\,\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c}+\frac{15\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\,\text{arcsinh}(cx))}{b}\right)}{16b^2c} \\ &\quad+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\,\text{arcsinh}(cx))}{b}\right)}{4b^2c}+\frac{3\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\,\text{arcsinh}(cx))}{b}\right)}{16b^2c}\end{aligned}$$

output
```
-(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+15/16*cosh(2*a/b)*Shi(2*(a+b*arcsinh
(c*x))/b)/b^2/c+3/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c+3/16*cos
h(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c-15/16*Chi(2*(a+b*arcsinh(c*x))/
b)*sinh(2*a/b)/b^2/c-3/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c-3/1
6*Chi(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c
```

### 3.430.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.44

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx =$$
$$-\frac{16b+48bc^2x^2+48bc^4x^4+16bc^6x^6+15(a+b\operatorname{arcsinh}(cx))\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+12(a+ba}{}$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*(16*b + 48*b*c^2*x^2 + 48*b*c^4*x^4 + 16*b*c^6*x^6 + 15*(a + b*ArcSi
nh[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 12*(a + b*Ar
cSinh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshI
ntegral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshInteg
ral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[(2*a)/b]*SinhIntegra
l[2*(a/b + ArcSinh[c*x])] - 15*b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcSinh[c*x])] - 12*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c*x])] - 12*b*ArcSinh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x
])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh
[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(b^2*c*(a + b*Ar
cSinh[c*x]))
```

### 3.430.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{6205}$$
$$\frac{6c\int \frac{x(c^2x^2+1)^2}{a+b\operatorname{arcsinh}(cx)}dx}{b}-\frac{(c^2x^2+1)^3}{bc(a+b\operatorname{arcsinh}(cx))}$$
$$\Big\downarrow \text{6234}$$

$$\frac{6\int -\frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\left(c^2x^2+1\right)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 25

$$-\frac{6\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\left(c^2x^2+1\right)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 5971

$$-\frac{6\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\frac{\left(c^2x^2+1\right)^3}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 2009

$$\frac{6\left(-\frac{5}{32}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{32}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\text{arcsinh}(cx))}{b}\right.\right.}{}$$

$$\frac{\left(c^2x^2+1\right)^3}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((1 + c^2*x^2)^3/(b*c*(a + b*ArcSinh[c*x]))) + (6*((-5*CoshIntegral[(2*(a
 + b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/32 - (CoshIntegral[(4*(a + b*ArcSinh
[c*x]))/b]*Sinh[(4*a)/b])/8 - (CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b]*Si
nh[(6*a)/b])/32 + (5*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b
])/32 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8 + (Cosh
[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/32))/(b^2*c)
```

## 3.430. $\int \frac{\left(1+c^2x^2\right)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.430.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.430.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.73

| method | result |
|---|---|
| default | $\frac{-32 b c^{6} x^{6}-96 b c^{4} x^{4}-96 b c^{2} x^{2}+3 \mathrm{e}^{\frac{6 a}{b}} \mathrm{Ei}_{1}\left(6 \,\mathrm{arcsinh}(c x)+\frac{6 a}{b}\right) b \,\mathrm{arcsinh}(c x)+12 \mathrm{e}^{\frac{4 a}{b}} \mathrm{Ei}_{1}\left(4 \,\mathrm{arcsinh}(c x)+\frac{4 a}{b}\right) b \,\mathrm{arcsinh}(c x)+15 \mathrm{e}^{\frac{2 a}{b}} \mathrm{E}}{}$ |

input
```
int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-32*b*c^6*x^6-96*b*c^4*x^4-96*b*c^2*x^2+3*exp(6*a/b)*Ei(1,6*arcsinh(
c*x)+6*a/b)*b*arcsinh(c*x)+12*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcs
inh(c*x)+15*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)-3*exp(-6*
a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b*arcsinh(c*x)-15*exp(-2*a/b)*Ei(1,-2*arc
sinh(c*x)-2*a/b)*b*arcsinh(c*x)-12*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)
*b*arcsinh(c*x)+3*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)*a+12*exp(4*a/b)*Ei
(1,4*arcsinh(c*x)+4*a/b)*a+15*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-3*ex
p(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a-15*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x
)-2*a/b)*a-12*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a-32*b)/c/b^2/(a+b*a
rcsinh(c*x))
```

## 3.430.5 Fricas [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
 2*a*b*arcsinh(c*x) + a^2), x)
```

## 3.430.6 Sympy [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

### 3.430.7 Maxima [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x
^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*lo
g(c*x + sqrt(c^2*x^2 + 1))) + integrate(((6*c^6*x^6 + 11*c^4*x^4 + 4*c^2*x
^2 - 1)*(c^2*x^2 + 1)^(3/2) + 6*(2*c^7*x^7 + 5*c^5*x^5 + 4*c^3*x^3 + c*x)*
(c^2*x^2 + 1) + (6*c^8*x^8 + 19*c^6*x^6 + 21*c^4*x^4 + 9*c^2*x^2 + 1)*sqrt
(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 +
a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(
b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2
*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.430.8 Giac [F]

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a)^2, x)
```

### 3.430.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2\,x^2+1)^{5/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2, x)
```

# 3.431 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.431.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\begin{aligned}\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{(1+c^2x^2)^3}{bcx(a+b\text{arcsinh}(cx))} \\ &+ \frac{25\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2} + \frac{25\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} \\ &+ \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} - \frac{25\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2} \\ &- \frac{25\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} \\ &- \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2} - \frac{\text{Int}\left(\frac{(1+c^2x^2)^2}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}\end{aligned}$$

output
```
-(c^2*x^2+1)^3/b/c/x/(a+b*arcsinh(c*x))+25/8*Chi((a+b*arcsinh(c*x))/b)*cos
h(a/b)/b^2+25/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2+5/16*Chi(5*(a
+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2-25/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b^2-25/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2-5/16*Shi(5*(a+b*a
rcsinh(c*x))/b)*sinh(5*a/b)/b^2-Unintegrable((c^2*x^2+1)^2/x^2/(a+b*arcsin
h(c*x)),x)/b/c
```

### 3.431.2 Mathematica [N/A]

Not integrable

Time = 9.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

### 3.431.3 Rubi [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6229, 6206, 3042, 3793, 2009, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{5c\int \frac{(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{\int \frac{(c^2x^2+1)^2}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^3}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6206}$$

$$\frac{5\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{(c^2x^2+1)^2}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^3}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^5}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{(c^2x^2+1)^2}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^3}{bcx(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 3793

$$\frac{5\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{5\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{5\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2}-\frac{\int \frac{(c^2x^2+1)^2}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^3}{bcx(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 2009

$$-\frac{\int \frac{(c^2x^2+1)^2}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}+\frac{5\left(\frac{5}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{5}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\right.}{b^2}$$
$$\frac{(c^2x^2+1)^3}{bcx(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 6239

$$-\frac{\int \frac{(c^2x^2+1)^2}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}+\frac{5\left(\frac{5}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{5}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\right.}{b^2}$$
$$\frac{(c^2x^2+1)^3}{bcx(a+b\text{arcsinh}(cx))}$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

3.431. $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.431.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.431. $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx$

### 3.431.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\,\mathrm{arcsinh}\,(cx))^2}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

### 3.431.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\mathrm{arsinh}\,(cx)+a)^2x}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2
 + 2*a*b*x*arcsinh(c*x) + a^2*x), x)
```

### 3.431.6 Sympy [N/A]

Not integrable

Time = 7.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))**2), x)
```

### 3.431.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 480, normalized size of antiderivative = 17.78

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)^2x}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x
^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2
*c*x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^7*x^7 + 8*c^5*x^5 +
c^3*x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (10*c^8*x^8 + 23*c^6*x^6 + 15*c^4*x
^4 + c^2*x^2 - 1)*(c^2*x^2 + 1) + 5*(c^9*x^9 + 3*c^7*x^7 + 3*c^5*x^5 + c^3
*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*
c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3
*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)),
 x)
```

### 3.431.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\operatorname{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.431.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2), x)
```

# 3.432 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.432.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{(1+c^2x^2)^3}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\text{Int}\left(\frac{(1+c^2x^2)^2}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc} + \frac{4c\text{Int}\left(\frac{(1+c^2x^2)^2}{x(a+b\text{arcsinh}(cx))},x\right)}{b}$$

output
```
-(c^2*x^2+1)^3/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)^2/x^3
/(a+b*arcsinh(c*x)),x)/b/c+4*c*Unintegrable((c^2*x^2+1)^2/x/(a+b*arcsinh(c
*x)),x)/b
```

## 3.432.2 Mathematica [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

## 3.432.3 Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6229, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \ 6229$$

$$\frac{4c\int \frac{(c^2x^2+1)^2}{x(a+b\text{arcsinh}(cx))}dx}{b}-\frac{2\int \frac{(c^2x^2+1)^2}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^3}{bcx^2(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ 6239$$

$$\frac{4c\int \frac{(c^2x^2+1)^2}{x(a+b\text{arcsinh}(cx))}dx}{b}-\frac{2\int \frac{(c^2x^2+1)^2}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{(c^2x^2+1)^3}{bcx^2(a+b\text{arcsinh}(cx))}$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.432.3.1 Defintions of rubi rules used

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.432.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

### 3.432.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)
^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2), x)
```

### 3.432.6 Sympy [N/A]

Not integrable

Time = 8.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

### 3.432.7 Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 491, normalized size of antiderivative = 18.19

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x
^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b
^2*c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^7*x^7 + 5*c^5*x^
5 - 2*c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^8*x^8 + 8*c^6*x^6 + 3*
c^4*x^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (4*c^9*x^9 + 11*c^7*x^7 + 9*c^5*x
^5 + c^3*x^3 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^
3*x^5 + 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x
^5 + 2*b^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^
2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(
c^2*x^2 + 1)), x)
```

### 3.432.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

### 3.432.9 Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2\,x^2+1)^{5/2}}{x^2\,(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

# 3.433 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.433.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

## 3.433.2 Mathematica [N/A]

Not integrable

Time = 12.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

## 3.433.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6239}$$

$$\int \frac{(c^2x^2+1)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.433.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.433.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

### 3.433.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)
^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3), x)
```

### 3.433.6 Sympy [N/A]

Not integrable

Time = 7.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

### 3.433.7 Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 492, normalized size of antiderivative = 18.22

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x
^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b
^2*c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((3*c^7*x^7 + 2*c^5*x^
5 - 5*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2) + 3*(2*c^8*x^8 + 3*c^6*x^6 - c^
4*x^4 - 3*c^2*x^2 - 1)*(c^2*x^2 + 1) + (3*c^9*x^9 + 7*c^7*x^7 + 3*c^5*x^5
- 3*c^3*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c
^3*x^6 + 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*
x^6 + 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x
^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt
(c^2*x^2 + 1)), x)
```

### 3.433.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.433.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

3.433. $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.434 $\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.434.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

## 3.434.2 Mathematica [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

## 3.434.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{(c^2x^2+1)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.434.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.434.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\,(a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

### 3.434.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)
^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4), x)
```

### 3.434.6 Sympy [N/A]

Not integrable

Time = 11.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{x^4\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

### 3.434.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 491, normalized size of antiderivative = 18.19

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x
^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b
^2*c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^7*x^7 - c^5*x^5
- 8*c^3*x^3 - 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^8*x^8 + c^6*x^6 - 6*c^4*
x^4 - 7*c^2*x^2 - 2)*(c^2*x^2 + 1) + (2*c^9*x^9 + 3*c^7*x^7 - 3*c^5*x^5 -
7*c^3*x^3 - 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3
*x^7 + 2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^
7 + 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2
 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c
^2*x^2 + 1)), x)
```

### 3.434.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

input
```
integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

### 3.434.9 Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2\,x^2+1)^{5/2}}{x^4\,(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2),x)
```

output
```
int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

3.434. $\int \frac{(1+c^2x^2)^{5/2}}{x^4(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.435 $\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.435.1 Optimal result

Integrand size = 27, antiderivative size = 204

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^5}{bc(a+b\text{arcsinh}(cx))} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^6} - \frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} + \frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} - \frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^6} + \frac{15\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6} - \frac{5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^6}$$

output
```
-x^5/b/c/(a+b*arcsinh(c*x))+5/8*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^
6-15/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^6+5/16*Chi(5*(a+b*ar
csinh(c*x))/b)*cosh(5*a/b)/b^2/c^6-5/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b^2/c^6+15/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^6-5/16*Shi(5*
(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^6
```

## 3.435.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{x^5}{bc(a+b\operatorname{arcsinh}(cx))}$$
$$+\frac{5\left(2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(5\left(\frac{a}{b}+\operatorname{arcsi}\right.\right.\right.}{}$$

input
```
Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*(2*Cosh[a/b]*CoshIntegral[a/b + Arc
Sinh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(
5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/
b + ArcSinh[c*x]] + 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] -
 Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])]))/(16*b^2*c^6)
```

## 3.435.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6233, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6233}$$

$$\frac{5\int \frac{x^4}{a+b\operatorname{arcsinh}(cx)}dx}{bc}-\frac{x^5}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{5\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{a+b\operatorname{arcsinh}(cx)}d(a+b\operatorname{arcsinh}(cx))}{b^2c^6}-\frac{x^5}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{5\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^6}-\frac{x^5}{bc(a+b\text{arcsinh}(cx))}$$

$\big\downarrow$ 2009

$$\frac{5\left(\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{3}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)-\right.}{b^2c^6}$$

$$\frac{x^5}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*((Cosh[a/b]*CoshIntegral[(a + b*Arc
Sinh[c*x])/b])/8 - (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/
b])/16 + (Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16 - (Si
nh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/8 + (3*Sinh[(3*a)/b]*SinhInt
egral[(3*(a + b*ArcSinh[c*x]))/b])/16 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/16))/(b^2*c^6)
```

### 3.435.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

3.435. $\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

## 3.435.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(192) = 384$.

Time = 0.27 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.10

| method | result |
|---|---|
| default | $-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^6b(a+b\,\mathrm{arcsinh}(cx))}-\frac{5\,\mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}_1\left(5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^6b^2}+\frac{\frac{5c^3x^3}{8}-\frac{5c^2x^2\sqrt{c^2x^2}}{8}}{c^6b(a+b\,\mathrm{ar}}$ |

input
```
int(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*
x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))-5/32/c^6/b^
2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+5/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2
+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))+15/32/c^6/b^2*
exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/16*(-(c^2*x^2+1)^(1/2)+c*x)/c^6/b/
(a+b*arcsinh(c*x))-5/16/c^6/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/b
^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/
b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+5/32/c^6/b^2*
(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*arcsinh(c*x)*Ei(1,-3*arcsinh(
c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*a+3*b*
c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^6/b^2*(16*b*c^5*x^5+16*
(c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x^3+12*(c^2*x^2+1)^(1/2)*b*c^2*x^2+5*
arcsinh(c*x)*Ei(1,-5*arcsinh(c*x)-5*a/b)*exp(-5*a/b)*b+5*Ei(1,-5*arcsinh(c
*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

### 3.435.5 Fricas [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^5}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(
c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.435.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^5}{(a+b\,\text{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**5/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**5/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.435.7 Maxima [F]

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^5}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-(c^3*x^8 + c*x^6 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*
c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1))
+ integrate((5*c^5*x^9 + 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 + 4*c*x^5)*(c^2
*x^2 + 1) + 5*(2*c^4*x^8 + 3*c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 +
 1)^(3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*
x^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (
b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.435.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^5}{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.436 $\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.436.1 Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^4}{bc(a+b\text{arcsinh}(cx))} + \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5} - \frac{\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^5}$$

output
```
-x^4/b/c/(a+b*arcsinh(c*x))-cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^
5+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^5+Chi(2*(a+b*arcsinh(c
*x))/b)*sinh(2*a/b)/b^2/c^5-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^
2/c^5
```

### 3.436.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$
$$= \frac{-\frac{2bc^4x^4}{a+b\text{arcsinh}(cx)} + 2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right) - 2\cosh\left(\frac{2a}{b}\right.}{2b^2c^5}$$

input
```
Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
((-2*b*c^4*x^4)/(a + b*ArcSinh[c*x]) + 2*CoshIntegral[2*(a/b + ArcSinh[c*x
])]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2
*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhIn
tegral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c^5)
```

### 3.436.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6233, 6195, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \; 6233$$

$$\frac{4\int \frac{x^3}{a+b\text{arcsinh}(cx)}dx}{bc} - \frac{x^4}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \; 6195$$

$$\frac{4\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^5} - \frac{x^4}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \; 25$$

$$-\frac{4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^5}-\frac{x^4}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 5971

$$-\frac{4\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}-\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^5}-\frac{x^4}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 2009

$$\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\right.}{b^2c^5}$$
$$\frac{x^4}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(x^4/(b*c*(a + b*ArcSinh[c*x]))) + (4*((CoshIntegral[(2*(a + b*ArcSinh[c*
x]))/b]*Sinh[(2*a)/b])/4 - (CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]*Sinh[
(4*a)/b])/8 - (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/4 +
 (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8))/(b^2*c^5)
```

### 3.436.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.436. $\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

## 3.436.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| default | $-\frac{4b\,c^4x^4+\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)-\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)b\,\mathrm{arcsinh}(cx)+2\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)b}{}$ |

input
```
int(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b*c^4*x^4+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*b*arcsinh(c*x)-e
xp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)*b*arcsinh(c*x)+2*exp(2*a/b)*Ei(1,2*ar
csinh(c*x)+2*a/b)*b*arcsinh(c*x)-2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
*b*arcsinh(c*x)+exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)*a-exp(4*a/b)*Ei(1,
4*arcsinh(c*x)+4*a/b)*a+2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a-2*exp(-2
*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a)/c^5/b^2/(a+b*arcsinh(c*x))
```

## 3.436.5 Fricas [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^4}{\sqrt{c^2x^2+1}{\left(b\,\text{arsinh}\left(cx\right)+a\right)}^2}\,dx$$

input
```
integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(
c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.436.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{(a+b\operatorname{asinh}(cx))^2\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**4/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.436.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-(c^3*x^7 + c*x^5 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*
c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1))
+ integrate((4*c^5*x^8 + 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 + 3*c*x^4)*(c^2*
x^2 + 1) + 4*(2*c^4*x^7 + 3*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 +
1)^(3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x
^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b
^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.436.8 Giac [F]

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^4}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.437 $\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\,\text{arcsinh}(cx))^2}\,dx$



## 3.437.1 Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\,\text{arcsinh}(cx))^2}\,dx = -\frac{x^3}{bc(a+b\,\text{arcsinh}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\,\text{arcsinh}(cx))}{b}\right)}{4b^2c^4} + \frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)}{4b^2c^4} - \frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\,\text{arcsinh}(cx))}{b}\right)}{4b^2c^4}$$

output
```
-x^3/b/c/(a+b*arcsinh(c*x))-3/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^
4+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+3/4*Shi((a+b*arcsinh
(c*x))/b)*sinh(a/b)/b^2/c^4-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^
2/c^4
```

### 3.437.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^3}{bc(a+b\text{arcsinh}(cx))} + \frac{3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(c$$

$$4b^2c^4$$

input
```
Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(x^3/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-(Cosh[a/b]*CoshIntegral[a/b + Arc
Sinh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/
b]*SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
ArcSinh[c*x])]))/(4*b^2*c^4)
```

### 3.437.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6233, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6233}$$

$$\frac{3\int \frac{x^2}{a+b\text{arcsinh}(cx)}dx}{bc} - \frac{x^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6195}$$

$$\frac{3\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^4} - \frac{x^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$\frac{3\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{x^3}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\text{si}\right.}{b^2c^4}$$

$$\frac{x^3}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(x^3/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-1/4*(Cosh[a/b]*CoshIntegral[(a +
b*ArcSinh[c*x])/b]) + (Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))
/b])/4 + (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/4 - (Sinh[(3*a)/
b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/4))/(b^2*c^4)
```

### 3.437.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

## 3.437.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(134) = 268$.

Time = 0.26 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.56

| method | result |
|---|---|
| default | $-\frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}}{8c^4b(a+b\,\operatorname{arcsinh}(cx))}-\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^4b^2}+\frac{-\frac{3\sqrt{c^2x^2+1}}{8}+\frac{3cx}{8}}{c^4b(a+b\,\operatorname{arcsinh}(cx))}+\frac{3\,\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right.}{8c^4b^2}$ |

input
```
int(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b
/(a+b*arcsinh(c*x))-3/8/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/8*
(-(c^2*x^2+1)^(1/2)+c*x)/c^4/b/(a+b*arcsinh(c*x))+3/8/c^4/b^2*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)+3/8/c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(
-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+
b*arcsinh(c*x))-1/8/c^4/b^2*(4*b*c^3*x^3+4*(c^2*x^2+1)^(1/2)*b*c^2*x^2+3*a
rcsinh(c*x)*Ei(1,-3*arcsinh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arcsinh(c*
x)-3*a/b)*exp(-3*a/b)*a+3*b*c*x+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

## 3.437.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(
c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.437.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**3/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.437.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*
c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1))
+ integrate((3*c^5*x^7 + 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 + 2*c*x^3)*(c^2*
x^2 + 1) + 3*(2*c^4*x^6 + 3*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 +
1)^(3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x
^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b
^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.437.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.438 $\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.438.1 Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^2}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^3}$$

output
```
-x^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^
3-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^3
```

## 3.438.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{bc^2x^2}{a+b\text{arcsinh}(cx)} - \text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) + \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{b^2c^3}$$

input
```
Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
(-((b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*x])
]*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b^2
*c^3)
```

### 3.438.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {6233, 6195, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6233}$$

$$\frac{2\int \frac{x}{a+b\text{arcsinh}(cx)}dx}{bc}-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6195}$$

$$\frac{2\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}-\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}$$

$$\Big\downarrow \; 26$$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^3}$$

$$\Big\downarrow \; 3784$$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(c\right.}{b^2c^3}$$

$$\Big\downarrow \; 26$$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx\right.}{b^2c^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsin}\right.}{b^2c^3}$$

$$\Big\downarrow \; 26$$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(ca\right.}{b^2c^3}$$

$$\Big\downarrow \; 3779$$

3.438. $\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c^3}$$

$$\Big\downarrow \; 3782$$

$$-\frac{x^2}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c^3}$$

input
```
Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(x^2/(b*c*(a + b*ArcSinh[c*x]))) + (I*(I*CoshIntegral[(2*(a + b*ArcSinh[c
*x]))/b]*Sinh[(2*a)/b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*
x]))/b]))/(b^2*c^3)
```

### 3.438.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

3.438. $\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

### 3.438.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| default | $-\frac{2b\,c^{2}x^{2}+\mathrm{e}^{-\frac{2a}{b}}\operatorname{Ei}_{1}\left(-2\operatorname{arcsinh}(cx)-\frac{2a}{b}\right)b\operatorname{arcsinh}(cx)-\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}_{1}\left(2\operatorname{arcsinh}(cx)+\frac{2a}{b}\right)b\operatorname{arcsinh}(cx)+\mathrm{e}^{-\frac{2a}{b}}\operatorname{Ei}_{1}\left(-2\operatorname{arcsinh}(cx)-\frac{2a}{b}\right.}{2c^{3}b^{2}(a+b\operatorname{arcsinh}(cx))}$ |

input
```
int(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*c^2*x^2+exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b*arcsinh(c*x)-e
xp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*b*arcsinh(c*x)+exp(-2*a/b)*Ei(1,-2*ar
csinh(c*x)-2*a/b)*a-exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)*a)/c^3/b^2/(a+b*
arcsinh(c*x))
```

### 3.438.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(
c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.438.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.438.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-(c^3*x^5 + c*x^3 + (c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*
c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1))
+ integrate((2*c^5*x^6 + 5*c^3*x^4 + 3*c*x^2 + (2*c^3*x^4 + c*x^2)*(c^2*x^
2 + 1) + 2*(2*c^4*x^5 + 3*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(
3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 +
 1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c
^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.438.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.439 $\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.439.1 Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x}{bc(a+b\text{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}$$

output
```
-x/b/c/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^2-Shi(
(a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2
```

## 3.439.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{bcx}{a+b\text{arcsinh}(cx)} + \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right) - \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b^2c^2}$$

input
```
Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
(-((b*c*x)/(a + b*ArcSinh[c*x])) + Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*
x]] - Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c^2)
```

### 3.439.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6233, 6189, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6233}$$

$$\frac{\int \frac{1}{a+b\text{arcsinh}(cx)}dx}{bc}-\frac{x}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}-\frac{x}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x}{bc(a+b\text{arcsinh}(cx))}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{x}{bc(a+b\text{arcsinh}(cx))}+$$
$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}-$$
$$\frac{x}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{x}{bc(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{x}{bc(a+b\text{arcsinh}(cx))}+\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}$$

$$\Big\downarrow \; 3779$$

$$-\frac{x}{bc(a+b\text{arcsinh}(cx))}+\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c^2}$$

$$\Big\downarrow \; 3782$$

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c^2}-\frac{x}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(x/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c
*x])/b] - Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c^2)
```

### 3.439.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.439. $\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

### 3.439.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(73) = 146$.

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.07

| method | result |
|---|---|
| default | $-\frac{-\sqrt{c^2x^2+1}+cx}{2c^2b(a+b\ \mathrm{arcsinh}(cx))}-\frac{\mathrm{e}^{\frac{a}{b}}\ \mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2c^2b^2}-\frac{\mathrm{arcsinh}(cx)\ \mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)\mathrm{e}^{-\frac{a}{b}}b+\mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)\mathrm{e}^{-\frac{a}{b}}a+}{2c^2b^2(a+b\ \mathrm{arcsinh}(cx))}$ |

input
```
int(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(c^2*x^2+1)^(1/2)+c*x)/c^2/b/(a+b*arcsinh(c*x))-1/2/c^2/b^2*exp(a/b
)*Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)
*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+b*c*x+(c^2*x^2+1)^(1/2)*b
)/(a+b*arcsinh(c*x))
```

### 3.439.5 Fricas [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*
x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.439.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))^2\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(x/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.439.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^
2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) +
integrate((c^5*x^5 + (c^2*x^2 + 1)*c^3*x^3 + 3*c^3*x^3 + 2*c*x + (2*c^4*x^
4 + 3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^2 + 2
*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^
2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x
^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4
 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

## 3.439.8 Giac [F]

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

## 3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.440 $\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.440.1 Optimal result

Integrand size = 24, antiderivative size = 18

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{1}{bc(a+b\text{arcsinh}(cx))}$$

output
```
-1/b/c/(a+b*arcsinh(c*x))
```

## 3.440.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=-\frac{1}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(1/(b*c*(a + b*ArcSinh[c*x])))
```

## 3.440.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6198}$$

$$-\frac{1}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(1/(b*c*(a + b*ArcSinh[c*x])))
```

### 3.440.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.440.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{bc(a+b\,\text{arcsinh}(cx))}$ | 19 |
| default | $-\frac{1}{bc(a+b\,\text{arcsinh}(cx))}$ | 19 |

input
```
int(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/c/(a+b*arcsinh(c*x))
```

### 3.440.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{b^2c\log\left(cx+\sqrt{c^2x^2+1}\right)+abc}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/(b^2*c*log(c*x + sqrt(c^2*x^2 + 1)) + a*b*c)
```

### 3.440.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 2.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for } b=0 \wedge c=0 \\ \frac{\operatorname{asinh}(cx)}{a^2c} & \text{for } b=0 \\ \frac{x}{a^2} & \text{for } c=0 \\ -\frac{1}{abc+b^2c\operatorname{asinh}(cx)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), (asinh(c*x)/(a**2*c), Eq(b, 0)),
(x/a**2, Eq(c, 0)), (-1/(a*b*c + b**2*c*asinh(c*x)), True))
```

### 3.440.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{(b\,\text{arsinh}(cx)+a)bc}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/((b*arcsinh(c*x) + a)*b*c)
```

### 3.440.8 Giac [F]

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.440.9 Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{c\,\text{asinh}\,(c\,x)\;b^2+a\,c\,b}$$

input
```
int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
-1/(b^2*c*asinh(c*x) + a*b*c)
```

# 3.441 $\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.441.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bcx(a+b\text{arcsinh}(cx))} - \frac{\text{Int}\left(\frac{1}{x^2(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

output
```
-1/b/c/x/(a+b*arcsinh(c*x))-Unintegrable(1/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

## 3.441.2 Mathematica [N/A]

Not integrable

Time = 6.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

### 3.441.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6233, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6233}$$

$$-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{1}{bcx(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6196}$$

$$-\frac{\int \frac{1}{x^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{1}{bcx(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.441.3.1 Defintions of rubi rules used

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

### 3.441.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^{2}\sqrt{c^{2}x^{2}+1}}dx$$

input
```
int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

output
```
int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

### 3.441.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.78

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\sqrt{c^{2}x^{2}+1}\left(b\,\operatorname{arsinh}\left(cx\right)+a\right)^{2}x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^3 + a^2*x + (b^2*c^2*x^3 + b^2*x)*ar
csinh(c*x)^2 + 2*(a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x)
```

### 3.441.6 Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\left(a+b\,\operatorname{asinh}\left(cx\right)\right)^{2}\sqrt{c^{2}x^{2}+1}}\,dx$$

input
```
integrate(1/x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.441.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 416, normalized size of antiderivative = 15.41

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*
x^2 + 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integr
ate((c^5*x^5 + c^3*x^3 + (c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*
c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^4 + 2*(a*b*
c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^4 +
2*(b^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4
 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x
^6 + 2*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

### 3.441.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.441.9 Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}} \,d x$$

input
```
int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.442 $\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.442.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bcx^2(a+b\text{arcsinh}(cx))} - \frac{2\text{Int}\left(\frac{1}{x^3(a+b\text{arcsinh}(cx))},x\right)}{bc}$$

output
```
-1/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(c*x)),x)/b
/c
```

## 3.442.2 Mathematica [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

### 3.442.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6233, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6233}$$

$$-\frac{2\int \frac{1}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{1}{bcx^2(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6196}$$

$$-\frac{2\int \frac{1}{x^3(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{1}{bcx^2(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.442.3.1 Defintions of rubi rules used

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

### 3.442.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (a + b \operatorname{arcsinh}(cx))^2 \sqrt{c^2x^2+1}} dx$$

input
```
int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

output
```
int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

### 3.442.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)^2x^2} dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^4 + a^2*x^2 + (b^2*c^2*x^4 + b^2*x^2
)*arcsinh(c*x)^2 + 2*(a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

### 3.442.6 Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{x^2 (a+b\operatorname{asinh}(cx))^2 \sqrt{c^2x^2+1}} dx$$

input
```
integrate(1/x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.442.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 427, normalized size of antiderivative = 15.81

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*
x^2 + 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - in
tegrate((2*c^5*x^5 + 3*c^3*x^3 + (2*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x +
 2*(2*c^4*x^4 + 3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b
*c^3*x^5 + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3
/2)*b^2*c^3*x^5 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^5*x
^7 + 2*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + (a*b*c^5*x^7 + 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

### 3.442.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

### 3.442.9 Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.443 $\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx$



## 3.443.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{x^3}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.443.2 Mathematica [N/A]

Not integrable

Time = 53.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.443.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))^2} dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{x^3}{(c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))^2} dx$$

input
```
Int[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.443.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.443.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(c^2x^2+1)^{\frac{3}{2}} (a+b\,\text{arcsinh}\,(cx))^2} dx$$

input
```
int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.443.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 + 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*
x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.443.6 Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.443.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 475, normalized size of antiderivative = 17.59

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^3}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)
*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((c^5*x^7 +
 5*c^3*x^5 + 4*c*x^3 + (c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 + 7*
c^2*x^4 + 3*x^2)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2
+ 1)^(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + (
(b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c
^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c
^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7
*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.443.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.443.9 Mupad [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}(c\,x))^2\,(c^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.444 $\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx$



## 3.444.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx = -\frac{x^2}{bc(1+c^2x^2)(a+b\,\text{arcsinh}(cx))}+\frac{2\text{Int}\left(\frac{x}{(1+c^2x^2)^2(a+b\,\text{arcsinh}(cx))},x\right)}{bc}$$

output
```
-x^2/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))+2*Unintegrable(x/(c^2*x^2+1)^2/(a+
b*arcsinh(c*x)),x)/b/c
```

## 3.444.2 Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\,\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

### 3.444.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6228, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6228}$$
$$\frac{2\int \frac{x}{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{x^2}{bc(c^2x^2+1)(a+b\text{arcsinh}(cx))}$$
$$\Big\downarrow \text{ 6239}$$
$$\frac{2\int \frac{x}{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}dx}{bc}-\frac{x^2}{bc(c^2x^2+1)(a+b\text{arcsinh}(cx))}$$

input
```
Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.444.3.1 Defintions of rubi rules used

rule 6228
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
- 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && EqQ[m + 2*p + 1, 0]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.444.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))^2}dx$$

input
```
int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.444.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 + 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*
x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.444.6 Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\mathrm{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.444.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 447, normalized size of antiderivative = 16.56

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)
*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((3*c^3*x^4
 + (c^2*x^2 + 1)*c*x^2 + 3*c*x^2 + 2*(2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((
a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^
4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 +
 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b
^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log
(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 +
 a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.444.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.444.9 Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}(c\,x))^2\,(c^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.445 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.445.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.445.2 Mathematica [N/A]

Not integrable

Time = 54.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.445.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{x}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.445.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.445.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.445.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.12

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 +
 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

### 3.445.6 Sympy [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.445.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 461, normalized size of antiderivative = 18.44

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b
^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^5 + (
c^2*x^2 + 1)*c^3*x^3 - c^3*x^3 - 2*c*x + (2*c^4*x^4 - c^2*x^2 - 1)*sqrt(c^
2*x^2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*
x^5 + 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x
^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2
*x^2 + 1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3
*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.445.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.445.9 Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}(c\,x))^2\,(c^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.446 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.446.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bc(1+c^2x^2)(a+b\text{arcsinh}(cx))} - \frac{2c\text{Int}\left(\frac{x}{(1+c^2x^2)^2(a+b\text{arcsinh}(cx))},x\right)}{b}$$

output
```
-1/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))-2*c*Unintegrable(x/(c^2*x^2+1)^2/(a+
b*arcsinh(c*x)),x)/b
```

## 3.446.2 Mathematica [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

### 3.446.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{2c\int \frac{x}{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}dx}{b}-\frac{1}{bc(c^2x^2+1)(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{2c\int \frac{x}{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}dx}{b}-\frac{1}{bc(c^2x^2+1)(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.446.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.446.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))^2}dx$$

input
```
int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.446.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 + 2
*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2
+ a*b)*arcsinh(c*x)), x)
```

### 3.446.6 Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.446.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 446, normalized size of antiderivative = 18.58

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c
^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^4*x^4 + c^2
*x^2 + (2*c^2*x^2 + 1)*(c^2*x^2 + 1) + 2*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 +
1) - 1)/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^5*x^5
+ 2*a*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^4*x^4 + b^2*c^2*x^2)*(c
^2*x^2 + 1)^(3/2) + 2*(b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1
) + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x
^2 + a*b)*sqrt(c^2*x^2 + 1)), x)
```

### 3.446.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.446.9 Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

input
```
int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.447 $\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.447.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.447.2 Mathematica [N/A]

Not integrable

Time = 35.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.447.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{1}{x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.447.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.447.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^{2}}dx$$

input
```
int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.447.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.96

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx = \int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^5 + 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4
*x^5 + 2*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^5 + 2*a*b*c^2*
x^3 + a*b*x)*arcsinh(c*x)), x)
```

### 3.447.6 Sympy [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx = \int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.447.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 477, normalized size of antiderivative = 17.67

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx = \int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x^2 + 1)*b^2
*c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x
^5 + 3*c^3*x^3 + (3*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (6*c^4*x^4 + 5*c^2*x^
2 + 1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2)
 + 2*(a*b*c^6*x^7 + 2*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b^2*c^5
*x^6 + b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^7 + 2*b^2*c^4*x^5 +
 b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^7*x^8 + 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4
 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x
^8 + 3*a*b*c^5*x^6 + 3*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

### 3.447.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.447.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.448 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.448.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.448.2 Mathematica [N/A]

Not integrable

Time = 15.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.448.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{1}{x^2 (c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.448.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.448.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (c^2x^2+1)^{\frac{3}{2}} (a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.448.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.19

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2} dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^6 + 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c
^4*x^6 + 2*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^6 + 2*a*b*
c^2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

### 3.448.6 Sympy [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{x^2 (a+b\,\text{asinh}\,(cx))^2 (c^2x^2+1)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.448.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 485, normalized size of antiderivative = 17.96

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2 x^2} dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x^2 + 1)*b^2
*c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((4*c
^5*x^5 + 5*c^3*x^3 + (4*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(4*c^4*x^
4 + 4*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^
2 + 1)^(3/2) + 2*(a*b*c^6*x^8 + 2*a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1)
 + ((b^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^8 + 2*b
^2*c^4*x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^7*x^9 + 3*b^2*c^5*x^7 + 3
*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
+ (a*b*c^7*x^9 + 3*a*b*c^5*x^7 + 3*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 +
 1)), x)
```

### 3.448.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

### 3.448.9 Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{3/2} (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2\,(a+b\,\text{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

3.448. $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.449 $\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\mathbf{arcsinh}(cx))^2}\,dx$



## 3.449.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.449.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

## 3.449.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6239}$$

$$\int \frac{x^3}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.449.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.449.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(c^2x^2+1)^{\frac{5}{2}}(a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.449.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.07

$$\int \frac{x^3}{(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{x^3}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2} dx$$

input
```
integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^
2 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a
^2 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)),
x)
```

### 3.449.6 Sympy [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(cx))^2 (c^2x^2+1)^{\frac{5}{2}}} dx$$

input
```
integrate(x**3/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

### 3.449.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 584, normalized size of antiderivative = 21.63

$$\int \frac{x^3}{(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{x^3}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2} dx$$

input
```
integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1)
+ ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2
+ b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 +
2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^7 - 3*c^3*x^5
 - 4*c*x^3 + (c^3*x^5 - 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 - 5*c^2*x^4 -
3*x^2)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^
2*x^2 + 1)^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^
2*x)*(c^2*x^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2
 + 1)^(3/2) + 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*
(c^2*x^2 + 1) + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x
^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8
 + 4*a*b*c^7*x^6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1
)), x)
```

### 3.449.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.449.9 Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\text{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{5/2}}\,dx$$

input
```
int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

output
```
int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.449. $\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.450 $\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.450.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.450.2 Mathematica [N/A]

Not integrable

Time = 10.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.450.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{x^2}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.450.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.450.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\text{arcsinh}\,(cx)\right)^2}dx$$

input
```
int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.450.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.07

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^
2 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a
^2 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)),
x)
```

### 3.450.6 Sympy [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

### 3.450.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 585, normalized size of antiderivative = 21.67

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1)
+ ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2
+ b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 +
2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^5*x^6 - c^3*x^4
 - 3*c*x^2 + (2*c^3*x^4 - c*x^2)*(c^2*x^2 + 1) + 2*(2*c^4*x^5 - c^2*x^3 -
x)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^
2 + 1)^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)
*(c^2*x^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1
)^(3/2) + 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2
*x^2 + 1) + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 +
 b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4
*a*b*c^7*x^6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)),
x)
```

### 3.450.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^2/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.450.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{5/2}}\,dx$$

input
```
int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

output
```
int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.450. $\int \frac{x^2}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.451 $\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.451.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.451.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

## 3.451.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{x}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.451.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.451.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x}{(c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.451.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.40

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
+ (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.451.6 Sympy [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

### 3.451.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 568, normalized size of antiderivative = 22.72

$$\int \frac{x}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*
a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5 + 3*(c^2*x^
2 + 1)*c^3*x^3 + c^3*x^3 - 2*c*x + (6*c^4*x^4 + c^2*x^2 - 1)*sqrt(c^2*x^2
+ 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2
*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2
*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2
*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^6 + 6
*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

### 3.451.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.451.9 Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{5/2}}\,dx$$

input
```
int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

output
```
int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.451. $\int \frac{x}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.452 $\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.452.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{1}{bc(1+c^2x^2)^2(a+b\text{arcsinh}(cx))} - \frac{4c\text{Int}\left(\frac{x}{(1+c^2x^2)^3(a+b\text{arcsinh}(cx))},x\right)}{b}$$

output
```
-1/b/c/(c^2*x^2+1)^2/(a+b*arcsinh(c*x))-4*c*Unintegrable(x/(c^2*x^2+1)^3/(
a+b*arcsinh(c*x)),x)/b
```

## 3.452.2 Mathematica [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

### 3.452.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6205}$$

$$-\frac{4c\int \frac{x}{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}dx}{b}-\frac{1}{bc(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{4c\int \frac{x}{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}dx}{b}-\frac{1}{bc(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.452.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.452.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(c^{2} x^{2}+1\right)^{\frac{5}{2}} \left(a+b \operatorname{arcsinh}\left(c x \right)\right)^{2}}d x$$

input
```
int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.452.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.58

$$\int \frac{1}{(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx=\int { \frac {1}{{\left (c^{2} x^{2} + 1\right )}^{\frac {5}{2}} {\left (b \operatorname {arsinh}\left (c x\right ) + a\right )}^{2}} \,d x }$$

input
```
integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
(b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 +
 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.452.6 Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx=\int \frac {1}{\left (a + b \operatorname {asinh}{\left (c x \right )}\right )^{2} \left (c^{2} x^{2} + 1\right )^{\frac {5}{2}}}\, dx$$

input
```
integrate(1/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

## 3.452.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 554, normalized size of antiderivative = 23.08

$$\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^
2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*
c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*
c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((4*c^4*x^4 + 3*c^2*x^2 + (
4*c^2*x^2 + 1)*(c^2*x^2 + 1) + 4*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) - 1)/
((a*b*c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^7*x^7 + 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c
^6*x^6 + 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^7*x^7
 + 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^2*c^8*x^8 +
 4*b^2*c^6*x^6 + 6*b^2*c^4*x^4 + 4*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^8*x^8 + 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4
 + 4*a*b*c^2*x^2 + a*b)*sqrt(c^2*x^2 + 1)), x)
```

## 3.452.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.452.9 Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

# 3.453 $\int \frac{1}{x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.453.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x\,(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{x\,(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.453.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{x\,(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

## 3.453.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (c^2x^2+1)^{5/2} (a+b\text{arcsinh}(cx))^2} dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{1}{x (c^2x^2+1)^{5/2} (a+b\text{arcsinh}(cx))^2} dx$$

input
```
Int[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.453.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.453.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x (c^2x^2+1)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))^2} dx$$

input
```
int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.453.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 5.15

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^7 + 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 +
a^2*x + (b^2*c^6*x^7 + 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)
^2 + 2*(a*b*c^6*x^7 + 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 + a*b*x)*arcsinh(c*x))
, x)
```

### 3.453.6 Sympy [N/A]

Not integrable

Time = 8.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

### 3.453.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 584, normalized size of antiderivative = 21.63

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((
b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 +
b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 +
2*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((5*c^5*x^5 + 5*c^3
*x^3 + (5*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (10*c^4*x^4 + 7*c^2*x^2 + 1)*sq
rt(c^2*x^2 + 1))/((a*b*c^7*x^8 + 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1
)^(3/2) + 2*(a*b*c^8*x^9 + 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 + a*b*c^2*x^3)*(c
^2*x^2 + 1) + ((b^2*c^7*x^8 + 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c^2*x^2 + 1)^(
3/2) + 2*(b^2*c^8*x^9 + 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*
x^2 + 1) + (b^2*c^9*x^10 + 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6 + 4*b^2*c^3*x^4 +
 b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^1
0 + 4*a*b*c^7*x^8 + 6*a*b*c^5*x^6 + 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^
2 + 1)), x)
```

### 3.453.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.453.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(c^2\,x^2+1\right)}^{5/2}}\,dx$$

input
```
int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.453. $\int \frac{1}{x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.454 $\int \frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.454.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.454.2 Mathematica [N/A]

Not integrable

Time = 11.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.454.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (c^2x^2+1)^{5/2} (a+b\text{arcsinh}(cx))^2} dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{1}{x^2 (c^2x^2+1)^{5/2} (a+b\text{arcsinh}(cx))^2} dx$$

input
```
Int[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.454.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.454.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (c^2x^2+1)^{\frac{5}{2}} (a+b\,\text{arcsinh}\,(cx))^2} dx$$

input
```
int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.454.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.37

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(c^2x^2+1)^{\frac{5}{2}} (b \operatorname{arsinh}(cx)+a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^8 + 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 +
a^2*x^2 + (b^2*c^6*x^8 + 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 + b^2*x^2)*arcsinh(
c*x)^2 + 2*(a*b*c^6*x^8 + 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 + a*b*x^2)*arcsinh
(c*x)), x)
```

### 3.454.6 Sympy [N/A]

Not integrable

Time = 6.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{x^2 (a+b \operatorname{asinh}(cx))^2 (c^2x^2+1)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

### 3.454.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 592, normalized size of antiderivative = 21.93

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(c^2x^2+1)^{\frac{5}{2}} (b \operatorname{arsinh}(cx)+a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxim
a")
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((
b^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 +
b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6
+ 2*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((6*c^5*x^5 + 7
*c^3*x^3 + 3*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + c*x + 2*(6*c^4*x^4 + 5*c^2*
x^2 + 1)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^9 + 2*a*b*c^5*x^7 + a*b*c^3*x^5)*(
c^2*x^2 + 1)^(3/2) + 2*(a*b*c^8*x^10 + 3*a*b*c^6*x^8 + 3*a*b*c^4*x^6 + a*b
*c^2*x^4)*(c^2*x^2 + 1) + ((b^2*c^7*x^9 + 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(c^
2*x^2 + 1)^(3/2) + 2*(b^2*c^8*x^10 + 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6 + b^2*c
^2*x^4)*(c^2*x^2 + 1) + (b^2*c^9*x^11 + 4*b^2*c^7*x^9 + 6*b^2*c^5*x^7 + 4*
b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
 (a*b*c^9*x^11 + 4*a*b*c^7*x^9 + 6*a*b*c^5*x^7 + 4*a*b*c^3*x^5 + a*b*c*x^3
)*sqrt(c^2*x^2 + 1)), x)
```

## 3.454.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

### 3.454.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1+c^2x^2)^{5/2} (a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

# 3.455 $\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.455.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.455.2 Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

## 3.455.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2x^2+1\right)^{5/2}x^m}{(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{\left(c^2x^2+1\right)^{5/2}x^m}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

### 3.455.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.455.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.455.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)
^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

### 3.455.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.455.7 Maxima [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 537, normalized size of antiderivative = 19.89

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2x^2+1)^{\frac{5}{2}}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^7*x^7 + 3*c
^5*x^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x
^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b
^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^7*(m + 6)*x^7 + c^5*(3
*m + 11)*x^5 + c^3*(3*m + 4)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m +
(2*c^8*(m + 6)*x^8 + c^6*(7*m + 30)*x^6 + 3*c^4*(3*m + 8)*x^4 + c^2*(5*m +
 6)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^9*(m + 6)*x^9 + c^7*(4*m + 19)*x^7 + 3
*c^5*(2*m + 7)*x^5 + c^3*(4*m + 9)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^
m)/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b
^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*
c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2
*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

### 3.455.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.455.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{5/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

# 3.456 $\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.456.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.456.2 Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

## 3.456.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2x^2+1)^{3/2} x^m}{(a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{(c^2x^2+1)^{3/2} x^m}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

### 3.456.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.456.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m (c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.456.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^2*x^2 + 1)^(3/2)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x)
+ a^2), x)
```

### 3.456.6 Sympy [N/A]

Not integrable

Time = 34.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m(c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

### 3.456.7 Maxima [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 480, normalized size of antiderivative = 17.78

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^5*x^5 + 2*c^3*x^3 + c*x
)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*
c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2
*x^2 + 1))) + integrate(((c^5*(m + 4)*x^5 + c^3*(2*m + 3)*x^3 + c*(m - 1)*
x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^6*(m + 4)*x^6 + c^4*(5*m + 12)*x^4 + 4*c
^2*(m + 1)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^7*(m + 4)*x^7 + 3*c^5*(m + 3)*x
^5 + 3*c^3*(m + 2)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5
+ (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^
2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^
2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 +
a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

## 3.456.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## 3.456.9 Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

---

3.456. $\int \frac{x^m (1+c^2x^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.457 $\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.457.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.457.2 Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

### 3.457.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}x^m}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6239}$$

$$\int \frac{\sqrt{c^2x^2+1}x^m}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

#### 3.457.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.457.4 Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m\sqrt{c^2x^2+1}}{(a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.457.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2), x)
```

### 3.457.6 Sympy [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m\sqrt{c^2x^2+1}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

### 3.457.7 Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 424, normalized size of antiderivative = 15.70

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2x^2+1}x^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^2 + 1)^2*x^m + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^
2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)
*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*(m + 2
)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^4*(m + 2)*x^4 + c^2*(3
*m + 2)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^5*(m + 2)*x^5 + c^3*(2*m + 3)*x^3
+ c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3
*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3
+ 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x
^2 + 1)), x)
```

### 3.457.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.457.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

output
```
int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

# 3.458 $\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.458.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{x^m}{bc(a+b\text{arcsinh}(cx))} + \frac{m\text{Int}\left(\frac{x^{-1+m}}{a+b\text{arcsinh}(cx)},x\right)}{bc}$$

output
```
-x^m/b/c/(a+b*arcsinh(c*x))+m*Unintegrable(x^(-1+m)/(a+b*arcsinh(c*x)),x)/
b/c
```

## 3.458.2 Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

### 3.458.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6233, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6233}$$

$$\frac{m\int \frac{x^{m-1}}{a+b\text{arcsinh}(cx)}dx}{bc}-\frac{x^m}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6196}$$

$$\frac{m\int \frac{x^{m-1}}{a+b\text{arcsinh}(cx)}dx}{bc}-\frac{x^m}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

#### 3.458.3.1 Defintions of rubi rules used

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

### 3.458.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\,\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}dx$$

input
```
int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.458.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.70

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(
c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.458.6 Sympy [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**m/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

### 3.458.7 Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 441, normalized size of antiderivative = 16.33

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^2 + 1)^(3/2)*x^m + (c^3*x^3 + c*x)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x
+ ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(
c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + inte
grate(((c^3*m*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*m*x^4 + 3*c^2*
m*x^2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*m*x^5 + c^3*(2*m + 1)*x^3 + c*(m +
 1)*x)*x^m)/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 + a*b*c^2*x^
2)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 + b^2
*c^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a*b*c^3*x^3 + a
*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.458.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.458.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.459 $\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2} dx$



## 3.459.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2} dx = \text{Int}\left(\frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}, x\right)$$

output
```
Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.459.2 Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2} dx = \int \frac{x^m}{(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2} dx$$

input
```
Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.459.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{x^m}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.459.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.459.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.459.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 + 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*
x^2 + a*b)*arcsinh(c*x)), x)
```

### 3.459.6 Sympy [N/A]

Not integrable

Time = 11.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

### 3.459.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 506, normalized size of antiderivative = 18.74

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 +
1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m
 - 2)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 2)*x^4 + c^2*(3*m
 - 2)*x^2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 2)*x^5 + c^3*(2*m - 1)*x^
3 + c*(m + 1)*x)*x^m)/((a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2
*(a*b*c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b^2*c^5*x^5
 + b^2*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^6 + 2*b^2*c^4*x^4 + b^2
*c^2*x^2)*(c^2*x^2 + 1) + (b^2*c^7*x^7 + 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 + b
^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^7 + 3
*a*b*c^5*x^5 + 3*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.459.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{(c^2x^2+1)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.459.9 Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{x^m}{(a+b\,\text{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

output
```
int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

# 3.460 $\int \frac{x^m}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.460.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.460.2 Mathematica [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.460.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{6239}$$
$$\int \frac{x^m}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.460.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.460.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}(a+b\,\text{arcsinh}\,(cx))^2}dx$$

input
```
int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.460.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.07

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^
2 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a
^2 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)),
x)
```

### 3.460.6 Sympy [N/A]

Not integrable

Time = 82.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(cx))^2\,(c^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**m/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

### 3.460.7 Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 613, normalized size of antiderivative = 22.70

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1
) + ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^
2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4
+ 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m - 4)*x^3
+ c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 4)*x^4 + c^2*(3*m - 4)*x^2
+ m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 4)*x^5 + c^3*(2*m - 3)*x^3 + c*(m +
 1)*x)*x^m)/((a*b*c^7*x^7 + 2*a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/
2) + 2*(a*b*c^8*x^8 + 3*a*b*c^6*x^6 + 3*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^
2 + 1) + ((b^2*c^7*x^7 + 2*b^2*c^5*x^5 + b^2*c^3*x^3)*(c^2*x^2 + 1)^(3/2)
+ 2*(b^2*c^8*x^8 + 3*b^2*c^6*x^6 + 3*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 +
 1) + (b^2*c^9*x^9 + 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 + 4*b^2*c^3*x^3 + b^2*c
*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^9 + 4*a*b
*c^7*x^7 + 6*a*b*c^5*x^5 + 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.460.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(c^2x^2+1)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^m/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.460.9 Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{x^m}{(a+b\,\text{asinh}\,(c\,x))^2\,(c^2\,x^2+1)^{5/2}}\,dx$$

input
```
int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

output
```
int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

3.460. $\int \frac{x^m}{(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.461 $\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx$



## 3.461.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arcsinh}(ax)^2}$$

output
```
-1/2/a/arcsinh(a*x)^2
```

## 3.461.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arcsinh}(ax)^2}$$

input
```
Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

output
```
-1/2*1/(a*ArcSinh[a*x]^2)
```

## 3.461.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}\,dx$$

$$\Big\downarrow \text{ 6198}$$

$$-\frac{1}{2a\text{arcsinh}(ax)^2}$$

input
```
Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

output
```
-1/2*1/(a*ArcSinh[a*x]^2)
```

### 3.461.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.461.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{2a\,\text{arcsinh}(ax)^2}$ | 12 |
| default | $-\frac{1}{2a\,\text{arcsinh}(ax)^2}$ | 12 |

input
```
int(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/arcsinh(a*x)^2
```

### 3.461.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\log\left(ax+\sqrt{a^2x^2+1}\right)^2}$$

input
```
integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

### 3.461.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\,\text{asinh}^2\,(ax)}$$

input
```
integrate(1/asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

output
```
-1/(2*a*asinh(a*x)**2)
```

### 3.461.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\,\text{arsinh}\,(ax)^2}$$

input
```
integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2/(a*arcsinh(a*x)^2)
```

### 3.461.8 Giac [F]

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2x^2+1}\,\text{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3), x)
```

### 3.461.9 Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\,\text{asinh}\left(a\,x\right)^2}$$

input
```
int(1/(asinh(a*x)^3*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
-1/(2*a*asinh(a*x)^2)
```

# 3.462 $\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.462.1 Optimal result

Integrand size = 26, antiderivative size = 254

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2dx^3(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} - \frac{3de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{de^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}$$

output
```
-3/32*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^4-3/32*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*
2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/32*d*exp(6*a/b)*erf(6^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/32*d*erfi(6^(
1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*
a/b)-2*d*x^3*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

### 3.462.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.91

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-\frac{6a}{b}}\left(\sqrt{6}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)-3\sqrt{2}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arcsin}}{b}}\right.}{}$$

input
```
Integrate[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d*(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSinh[
c*x]))/b] - 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/
2, (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcSinh
[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] - Sqrt[6]*E^((12*a)/b)*Sqrt[
a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[c*x]))/b] - 8*E^((6*a)/b)
*Sinh[2*ArcSinh[c*x]]^3))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.462.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6229, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c^2dx^2+d)}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6229}$$

$$\frac{6d\int \frac{x^2\sqrt{c^2x^2+1}}{\sqrt{a+b\operatorname{arcsinh}(cx)}}dx}{bc}+\frac{12cd\int \frac{x^4\sqrt{c^2x^2+1}}{\sqrt{a+b\operatorname{arcsinh}(cx)}}dx}{b}-\frac{2dx^3\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{12d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^4}+$$
$$\frac{6d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{2dx^3\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\downarrow$ 5971

$$\frac{6d\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}-\frac{1}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^4}+$$
$$\frac{12d\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{1}{16\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b^2c^4}$$
$$\frac{2dx^3\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\downarrow$ 2009

$$\frac{6d\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^4}+$$
$$\frac{12d\left(-\frac{1}{64}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arc}}}{\sqrt{b}}\right.\right.}{}$$
$$\frac{2dx^3\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

3.462. $\int \frac{x^3\left(d+c^2dx^2\right)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

output
```
(-2*d*x^3*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (6*d*(-1/4
*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a +
b*ArcSinh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSi
nh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b))))/(b^2*c^4) + (12*d*(Sqrt[a + b*ArcSi
nh[c*x]]/8 - (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/64 - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/64 + (Sqrt[b]*E^((6*a)/b)*Sqrt[Pi/6]*Erf[(Sqrt[6]*
Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((2*a)/b)) + (Sqrt[b]*Sq
rt[Pi/6]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((6*a)/b)
)))/(b^2*c^4)
```

### 3.462.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.462. $\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

### 3.462.4 Maple [F]

$$\int \frac{x^3(c^2d\,x^2+d)}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.462.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.462.6 Sympy [F]

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{x^3}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx + \int \frac{c^2x^5}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

input
```
integrate(x**3*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asin
h(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*
asinh(c*x))*asinh(c*x)), x))
```

### 3.462.7 Maxima [F]

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)x^3}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.462.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.462.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^3\,(d\,c^2\,x^2+d)}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

# 3.463 $\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.463.1 Optimal result

Integrand size = 26, antiderivative size = 335

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2dx^2(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{de^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} - \frac{de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} + \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{de^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}$$

output
```
1/8*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3-
1/8*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)
-1/16*d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*P
i^(1/2)/b^(3/2)/c^3+1/16*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*
3^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-1/16*d*exp(5*a/b)*erf(5^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/16*d*erfi(5^(
1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(5*
a/b)-2*d*x^2*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

### 3.463.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.30

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-5\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{5a}{b}}-e^{\frac{5a}{b}+2\text{arcsinh}(cx)}+2e^{\frac{5a}{b}+4\text{arcsinh}(cx)}+2e^{\frac{5a}{b}+6\text{arcsinh}(cx)}\right.}{}$$

input
```
Integrate[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d*(-E^((5*a)/b) - E^((5*a)/b + 2*ArcSinh[c*x]) + 2*E^((5*a)/b + 4*ArcSinh
[c*x]) + 2*E^((5*a)/b + 6*ArcSinh[c*x]) - E^((5*a)/b + 8*ArcSinh[c*x]) - E
^((5*a)/b + 10*ArcSinh[c*x]) - 2*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + A
rcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*x])*S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + S
qrt[3]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
1/2, (-3*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b + 5*ArcSinh[c*x])*Sqrt[-(
(a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E
^((8*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*
ArcSinh[c*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c^3*E^(5*(a/b + A
rcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.463.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6229, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c^2dx^2+d\right)}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6229}$$

$$\frac{4d\int \frac{x\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}+\frac{10cd\int \frac{x^3\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2dx^2\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{10d\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}+$$
$$\frac{4d\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2dx^2\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\big\downarrow$ 25

$$-\frac{10d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$
$$\frac{4d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2dx^2\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\big\downarrow$ 5971

$$-\frac{10d\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$
$$\frac{4d\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$
$$\frac{2dx^2\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\big\downarrow$ 2009

$$\frac{4d\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsi}}}{\sqrt{b}}\right.\right.}{b^2c^3}$$
$$\frac{10d\left(\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsi}}}{\sqrt{b}}\right.\right.}{}$$
$$\frac{2dx^2\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

## 3.463. $\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

output
```
(-2*d*x^2*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (4*d*(-1/8
*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) - (Sqrt[
b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*E^(a/b))
 + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
8*E^((3*a)/b))))/(b^2*c^3) + (10*d*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/16 + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5
]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqr
t[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((
5*a)/b))))/(b^2*c^3)
```

### 3.463.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

---

3.463. $\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.463.4 Maple [F]

$$\int \frac{x^2(c^2d\,x^2+d)}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.463.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.463.6 Sympy [F]

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{x^2}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx + \int \frac{c^2x^4}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

input
```
integrate(x**2*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asin
h(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*
asinh(c*x))*asinh(c*x)), x))
```

### 3.463.7 Maxima [F]

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.463.8 Giac [F]

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.463.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x^2\,(d\,c^2\,x^2+d)}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

# 3.464 $\int \frac{x(d+c^2dx^2)}{(a+b\,\mathbf{arcsinh}(cx))^{3/2}}\,dx$



## 3.464.1 Optimal result

Integrand size = 24, antiderivative size = 236

$$\int \frac{x(d+c^2dx^2)}{(a+b\,\mathrm{arcsinh}(cx))^{3/2}}\,dx = -\frac{2dx(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\,\mathrm{arcsinh}(cx)}} + \frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2} + \frac{de^{-\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}$$

output
```
1/4*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(3/2)/c^2+1/4*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d*exp(4*a/b)*erf(2*(a+b*arcsinh(c
*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d*erfi(2*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)-2*d*x*(c^2*x^2+1)^(3/2)/b/c/
(a+b*arcsinh(c*x))^(1/2)
```

### 3.464.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96

$$\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\right.}{}$$

input
```
Integrate[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d*(Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b
] + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a
+ b*ArcSinh[c*x]))/b] - E^((4*a)/b)*(Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSin
h[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + Ar
cSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] + 2*Sinh[2*ArcSinh[c*x]
] + Sinh[4*ArcSinh[c*x]])))/(4*b*c^2*E^((4*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.464.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6229, 6206, 3042, 3793, 2009, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c^2dx^2+d)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{2d\int \frac{\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}+\frac{8cd\int \frac{x^2\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6206}$$

$$\frac{2d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{8cd\int \frac{x^2\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{8cd\int \frac{x^2\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\Big\downarrow$ 3793

$$\frac{2d\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{1}{2\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{8cd\int \frac{x^2\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\Big\downarrow$ 2009

$$\frac{8cd\int \frac{x^2\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\Big\downarrow$ 6234

$$\frac{8d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\Big\downarrow$ 5971

---

3.464. $\int \frac{x\left(d+c^2dx^2\right)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

$$\frac{8d\int\left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}-\frac{1}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\downarrow$ 2009

$$\frac{8d\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}+\frac{2d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}-\frac{2dx\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d*x*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (8*d*(-1/4*S
qrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh
[c*x]])/Sqrt[b]])/(32*E^((4*a)/b))))/(b^2*c^2) + (2*d*(Sqrt[a + b*ArcSinh[
c*x]] + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(4*E^((2*a)/b))))/(b^2*c^2)
```

### 3.464.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.464. $\int\frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.464. $\int \frac{x(d+c^2dx^2)}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

### 3.464.4 Maple [F]

$$\int \frac{x(c^2d\,x^2+d)}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.464.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d+c^2dx^2)}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.464.6 Sympy [F]

$$\int \frac{x(d+c^2dx^2)}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{x}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx + \int \frac{c^2x^3}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

input
```
integrate(x*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c
*x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asi
nh(c*x))*asinh(c*x)), x))
```

### 3.464.7 Maxima [F]

$$\int \frac{x(d + c^2dx^2)}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2 + d)x}{(b\,\text{arsinh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.464.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(d + c^2dx^2)}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.464.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d + c^2dx^2)}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x\,(d\,c^2\,x^2 + d)}{(a + b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

# 3.465 $\int \frac{d+c^2dx^2}{(a+b\,\mathbf{arcsinh}(cx))^{3/2}}\,dx$



## 3.465.1 Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \frac{d+c^2dx^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{3de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{3de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

output
```
-3/4*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+3
/4*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/
4*d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/b^(3/2)/c+1/4*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)
*Pi^(1/2)/b^(3/2)/c/exp(3*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x)
)^(1/2)
```

### 3.465.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.29

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{de^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{3a}{b}}-3e^{\frac{3a}{b}+2\text{arcsinh}(cx)}-3e^{\frac{3a}{b}+4\text{arcsinh}(cx)}-e^{\frac{3a}{b}+6\text{arcsinh}(cx)}\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d*(-E^((3*a)/b) - 3*E^((3*a)/b + 2*ArcSinh[c*x]) - 3*E^((3*a)/b + 4*ArcSi
nh[c*x]) - E^((3*a)/b + 6*ArcSinh[c*x]) + 3*E^((4*a)/b + 3*ArcSinh[c*x])*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcS
inh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*
x]))/b] + 3*E^((2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x]
)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(4*b*c
*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.465.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2dx^2+d}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{6cd\int \frac{x\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{6d\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{6d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 5971$$

$$-\frac{6d\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{6d\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c}}{\sqrt{b}}\right.\right.}{b^2c}$$

$$\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (6*d*(-1/8*(Sq
rt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) - (Sqrt[b]*E
^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/8 +
 (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*E^(a/b)) + (
Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(8*E^
((3*a)/b))))/(b^2*c)
```

### 3.465.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.465. $\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.465.4 Maple [F]

$$\int \frac{c^2 d\,x^2 + d}{\left(a + b\,\operatorname{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}dx$$

input
```
int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.465.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{d + c^2dx^2}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.465.6 Sympy [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{c^2x^2}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d*(Integral(c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))
*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asi
nh(c*x))*asinh(c*x)), x))
```

### 3.465.7 Maxima [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{c^2dx^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

## 3.465.8 Giac [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{c^2dx^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

## 3.465.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{d\,c^2\,x^2+d}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

# 3.466 $\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.466.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d(1+c^2x^2)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{2d\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}},x\right)}{bc}$$

output
```
1/2*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(3/2)+1/2*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(3/2)/exp(2*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/x/(a+b*arcsinh(c*x)
)^(1/2)-2*d*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^(1/2),
x)/b/c
```

### 3.466.2 Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

### 3.466.3 Rubi [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6229, 6206, 3042, 3793, 2009, 6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2dx^2+d}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{4cd\int \frac{\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b} - \frac{2d\int \frac{\sqrt{c^2x^2+1}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc} - \frac{2d\left(c^2x^2+1\right)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6206}$$

$$\frac{4d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2} - \frac{2d\int \frac{\sqrt{c^2x^2+1}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc} - \frac{2d\left(c^2x^2+1\right)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4d\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{2d\int \frac{\sqrt{c^2x^2+1}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2d(c^2x^2+1)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{4d\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{1}{2\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2}-\frac{2d\int \frac{\sqrt{c^2x^2+1}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2d(c^2x^2+1)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2d\int \frac{\sqrt{c^2x^2+1}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}+\frac{4d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2}-\frac{2d(c^2x^2+1)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6235}$$

$$-\frac{2d\int \left(\frac{c^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}+\frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}\right)dx}{bc}+\frac{4d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2}-\frac{2d(c^2x^2+1)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 2009}$$

3.466. $\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

$$-\frac{2d\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{2c\sqrt{a+b\text{arcsinh}(cx)}}{b}\right)}{bc}+\frac{4d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2}-\frac{2d(c^2x^2+1)^{3/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

### 3.466.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

3.466. $\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

### 3.466.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{c^{2} d \,x^{2}+d}{x \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

input
```
int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.466.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.466.6 Sympy [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{c^2x^2}{ax\sqrt{a+b\,\text{asinh}\,(cx)}+bx\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{ax\sqrt{a+b\,\text{asinh}\,(cx)}+bx\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)/x/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d*(Integral(c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asinh(c
*x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a
 + b*asinh(c*x))*asinh(c*x)), x))
```

### 3.466.7 Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{c^2dx^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

### 3.466.8 Giac [F(-2)]

Exception generated.

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.466.9 Mupad [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{d+c^2dx^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{d\,c^2\,x^2+d}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

input
```
int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)),x)
```

output
```
int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)), x)
```

# 3.467 $\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.467.1 Optimal result

Integrand size = 28, antiderivative size = 474

$$\begin{aligned}
\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d^2x^3(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} \\
&- \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} \\
&+ \frac{d^2e^{\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} \\
&- \frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} \\
&+ \frac{d^2e^{-\frac{8a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{2\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}
\end{aligned}$$

output
```
-3/64*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*exp(8*a/b)*erf(2*2^(1/2)*(a+b*arcsinh(c*x))
^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4-3/64*d^2*erfi(2^(1/2)*(a+b*ar
csinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/64*d^
2*erfi(2*2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2
)/c^4/exp(8*a/b)-1/32*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/b^(3/2)/c^4-1/32*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*
Pi^(1/2)/b^(3/2)/c^4/exp(4*a/b)+1/64*d^2*exp(6*a/b)*erf(6^(1/2)*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*erfi(6^(1/2)
*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)
-2*d^2*x^3*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

### 3.467.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.97

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{d^2e^{-\frac{8a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)+\sqrt{6}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsin}}{b}}\right.}{}$$

input
```
Integrate[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d^2*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-8*(a + b*ArcSin
h[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/
2, (-6*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x]
)/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[2]*E^((6*a)/b)*Sqrt
[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sq
rt[2]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c
*x]))/b] + 2*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*Ar
cSinh[c*x]))/b] - Sqrt[6]*E^((14*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2,
 (6*(a + b*ArcSinh[c*x]))/b] - Sqrt[2]*E^((16*a)/b)*Sqrt[a/b + ArcSinh[c*x
]]*Gamma[1/2, (8*(a + b*ArcSinh[c*x]))/b] + 6*E^((8*a)/b)*Sinh[2*ArcSinh[c
*x]] + 2*E^((8*a)/b)*Sinh[4*ArcSinh[c*x]] - 2*E^((8*a)/b)*Sinh[6*ArcSinh[c
*x]] - E^((8*a)/b)*Sinh[8*ArcSinh[c*x]]))/(64*b*c^4*E^((8*a)/b)*Sqrt[a + b
*ArcSinh[c*x]])
```

### 3.467.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6229, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c^2dx^2+d)^2}{(a+b\text{arcsinh}(cx))^{3/2}} \, dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{6d^2\int \frac{x^2(c^2x^2+1)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}+\frac{16cd^2\int \frac{x^4(c^2x^2+1)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^2x^3(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{16d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^4}+\frac{6d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{2d^2x^3(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{16d^2\int \left(\frac{\cosh\left(\frac{8a}{b}-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{128\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3}{128\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^4}+\frac{6d^2\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}-\frac{1}{16\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b^2c^4}$$

$$\frac{2d^2x^3(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$16d^2\left(-\frac{1}{128}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{512}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{8a}{b}}\operatorname{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{128}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a}}{}\right.\right.$$
$$b^2c^4$$
$$6d^2\left(\frac{1}{64}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\operatorname{erf}\left(\frac{\sqrt{6}\sqrt{a+b\operatorname{arcsi}}}{\sqrt{b}}\right.\right.$$

$$\frac{2d^2x^3\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

input
```
Int[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d^2*x^3*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (16*d^2*
((3*Sqrt[a + b*ArcSinh[c*x]])/64 - (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sq
rt[a + b*ArcSinh[c*x]])/Sqrt[b]])/128 + (Sqrt[b]*E^((8*a)/b)*Sqrt[Pi/2]*Er
f[(2*Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/512 - (Sqrt[b]*Sqrt[Pi]*E
rfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E^((4*a)/b)) + (Sqrt[b]*Sq
rt[Pi/2]*Erfi[(2*Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(512*E^((8*a)
/b))))/(b^2*c^4) + (6*d^2*(-1/8*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*E^
((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 +
 (Sqrt[b]*E^((6*a)/b)*Sqrt[Pi/6]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/64 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/(64*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
*x]])/Sqrt[b]])/(64*E^((2*a)/b)) + (Sqrt[b]*Sqrt[Pi/6]*Erfi[(Sqrt[6]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((6*a)/b))))/(b^2*c^4)
```

### 3.467.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.467.4 Maple [F]

$$\int \frac{x^3 (c^2 d\, x^2 + d)^2}{(a + b \,\operatorname{arcsinh}(cx))^{\frac{3}{2}}} dx$$

input
```
int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

## 3.467.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 (d + c^2 d x^2)^2}{(a + b\operatorname{arcsinh}(cx))^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.467.6 Sympy [F]

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x^3}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx + \int \frac{2c^2x^5}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx + \int \frac{c^4x^7}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

input
```
integrate(x**3*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*a
sinh(c*x)), x) + Integral(2*c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a
 + b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*asinh(
c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

### 3.467.7 Maxima [F]

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x^3}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^2*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.467.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.467.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^3\,(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

# 3.468 $\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.468.1 Optimal result

Integrand size = 28, antiderivative size = 457

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d^2x^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{3d^2e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{d^2e^{\frac{7a}{b}}\sqrt{7\pi}\,\text{erf}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{3d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{d^2e^{-\frac{7a}{b}}\sqrt{7\pi}\,\text{erfi}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$

output
```
5/64*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c
^3-5/64*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/ex
p(a/b)-1/64*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-3/64*d^2*exp(5*a/b)*erf(
5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+3/6
4*d^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3
/2)/c^3/exp(5*a/b)-1/64*d^2*exp(7*a/b)*erf(7^(1/2)*(a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(7^(1/2)*(a+b*arcsinh
(c*x))^(1/2)/b^(1/2))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(7*a/b)-2*d^2*x^2*(c
^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

### 3.468.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.26

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx =$$

$$-\frac{d^2e^{-7\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(e^{\frac{7a}{b}}+3e^{\frac{7a}{b}+2\text{arcsinh}(cx)}+e^{\frac{7a}{b}+4\text{arcsinh}(cx)}-5e^{\frac{7a}{b}+6\text{arcsinh}(cx)}-5e^{\frac{7a}{b}+8\text{arcsinh}(cx)}+e^{\frac{7a}{b}+10\text{arcs}}\right.}{}$$

input
```
Integrate[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
-1/64*(d^2*(E^((7*a)/b) + 3*E^((7*a)/b + 2*ArcSinh[c*x]) + E^((7*a)/b + 4*
ArcSinh[c*x]) - 5*E^((7*a)/b + 6*ArcSinh[c*x]) - 5*E^((7*a)/b + 8*ArcSinh[
c*x]) + E^((7*a)/b + 10*ArcSinh[c*x]) + 3*E^((7*a)/b + 12*ArcSinh[c*x]) +
E^((7*a)/b + 14*ArcSinh[c*x]) + 5*E^((8*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b +
ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] - Sqrt[7]*E^(7*ArcSinh[c*x])*
Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-7*(a + b*ArcSinh[c*x]))/b] -
3*Sqrt[5]*E^((2*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gam
ma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] - Sqrt[3]*E^((4*a)/b + 7*ArcSinh[c*x]
)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b]
+ 5*E^((6*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2
, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((10*a)/b + 7*ArcSinh[c*x])*Sqrt[
a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*E^(
(12*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*A
rcSinh[c*x]))/b] - Sqrt[7]*E^(7*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSi
nh[c*x]]*Gamma[1/2, (7*(a + b*ArcSinh[c*x]))/b]))/(b*c^3*E^(7*(a/b + ArcSi
nh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.468.3 Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {6229, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c^2 d x^2+d\right)^2}{(a+b \text{arcsinh}(c x))^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6229}$$

$$\frac{4 d^2 \int \frac{x \left(c^2 x^2+1\right)^{3/2}}{\sqrt{a+b \text{arcsinh}(c x)}}dx}{b c}+\frac{14 c d^2 \int \frac{x^3 \left(c^2 x^2+1\right)^{3/2}}{\sqrt{a+b \text{arcsinh}(c x)}}dx}{b}-\frac{2 d^2 x^2 \left(c^2 x^2+1\right)^{5/2}}{b c \sqrt{a+b \text{arcsinh}(c x)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{14 d^2 \int -\frac{\cosh ^4\left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right) \sinh ^3\left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)}{\sqrt{a+b \text{arcsinh}(c x)}}d(a+b \text{arcsinh}(c x))}{b^2 c^3}+\frac{4 d^2 \int -\frac{\cosh ^4\left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right) \sinh \left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)}{\sqrt{a+b \text{arcsinh}(c x)}}d(a+b \text{arcsinh}(c x))}{b^2 c^3}-\frac{2 d^2 x^2 \left(c^2 x^2+1\right)^{5/2}}{b c \sqrt{a+b \text{arcsinh}(c x)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{14 d^2 \int \frac{\cosh ^4\left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right) \sinh ^3\left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)}{\sqrt{a+b \text{arcsinh}(c x)}}d(a+b \text{arcsinh}(c x))}{b^2 c^3}-\frac{4 d^2 \int \frac{\cosh ^4\left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right) \sinh \left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)}{\sqrt{a+b \text{arcsinh}(c x)}}d(a+b \text{arcsinh}(c x))}{b^2 c^3}-\frac{2 d^2 x^2 \left(c^2 x^2+1\right)^{5/2}}{b c \sqrt{a+b \text{arcsinh}(c x)}}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{14d^2\int\left(\frac{\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}-\frac{3\sinh\left(\frac{a}{b}-\frac{a+b\text{arcs}}{b}\right.}{64\sqrt{a+b\text{arcsin}}}}{b^2c^3}$$

$$\frac{4d^2\int\left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-$$

$$\frac{2d^2x^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\downarrow$ 2009

$$\frac{4d^2\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{ar}}}{\sqrt{b}}\right.\right.}{}$$

$$\frac{14d^2\left(\frac{3}{128}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{128}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{128}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b}}{}\right.\right.}{}$$

$$\frac{2d^2x^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

3.468. $\int\frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

output
```
(-2*d^2*x^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (4*d^2*(
-1/16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) - (
Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh
[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqr
t[b]])/(16*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh
[c*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/(b^2*c^3) + (14*d^2*((
3*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/128 + (S
qrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/128 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh
[c*x]])/Sqrt[b]])/128 - (Sqrt[b]*E^((7*a)/b)*Sqrt[Pi/7]*Erf[(Sqrt[7]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/128 - (3*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*A
rcSinh[c*x]]/Sqrt[b]])/(128*E^(a/b)) - (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*S
qrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]
*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E^((5*a)/b)) + (Sq
rt[b]*Sqrt[Pi/7]*Erfi[(Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E^
((7*a)/b))))/(b^2*c^3)
```

### 3.468.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.468. $\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.468.4 Maple [F]

$$\int \frac{x^2 (c^2 d\,x^2 + d)^2}{(a + b\,\operatorname{arcsinh}(cx))^{\frac{3}{2}}} dx$$

input
```
int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.468.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 (d + c^2 d x^2)^2}{(a + b\operatorname{arcsinh}(cx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.468.6 Sympy [F]

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x^2}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{2c^2x^4}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{c^4x^6}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

input
```
integrate(x**2*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*a
sinh(c*x)), x) + Integral(2*c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a
 + b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*asinh(
c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

### 3.468.7 Maxima [F]

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.468.8 Giac [F]

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.468.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x^2\,(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

# 3.469 $\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.469.1 Optimal result

Integrand size = 26, antiderivative size = 358

$$\begin{aligned}\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d^2x(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}\\
&+\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}\\
&+\frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}\\
&+\frac{5d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}\end{aligned}$$

output
```
5/32*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/c^2+5/32*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d^2*erfi(2*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+1/32*d^2*exp(6*a/b
)*erf(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c
^2+1/32*d^2*erfi(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2
)/b^(3/2)/c^2/exp(6*a/b)-2*d^2*x*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^
(1/2)
```

### 3.469.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx =$$
$$-\frac{d^2e^{-\frac{6a}{b}}\left(-\sqrt{6}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)-8e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arcsinh}(cx))}{b}\right.}{}$$

input
```
Integrate[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
-1/32*(d^2*(-(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a +
b*ArcSinh[c*x]))/b]) - 8*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma
[1/2, (-4*(a + b*ArcSinh[c*x]))/b] - 5*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*A
rcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 5*Sqrt[2]*E^((8
*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + 8
*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))
/b] + Sqrt[6]*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*A
rcSinh[c*x]))/b] + 10*E^((6*a)/b)*Sinh[2*ArcSinh[c*x]] + 8*E^((6*a)/b)*Sin
h[4*ArcSinh[c*x]] + 2*E^((6*a)/b)*Sinh[6*ArcSinh[c*x]]))/(b*c^2*E^((6*a)/b
)*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.469.3 Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.57, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6229, 6206, 3042, 3793, 2009, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c^2dx^2+d\right)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{2d^2\int \frac{\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}+\frac{12cd^2\int \frac{x^2\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6206}$$

$$\frac{2d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^4}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{2d^2\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{12cd^2\int \frac{x^2\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}+\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsi}}}{\sqrt{b}}\right.\right.}{b^2c^2}$$

$$\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6234}$$

3.469. $\int \frac{x\left(d+c^2dx^2\right)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

$$\frac{12d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}+$$

$$\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsi}}}{\sqrt{b}}\right.\right.}{b^2c^2}$$

$$\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 5971$$

$$\frac{12d^2\int\left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}-\frac{1}{16\sqrt{a+b\text{arcsinh}(cx)}}\right.}{b^2c^2}$$

$$\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsi}}}{\sqrt{b}}\right.\right.}{b^2c^2}$$

$$\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsi}}}{\sqrt{b}}\right.\right.}{b^2c^2}$$

$$\frac{12d^2\left(\frac{1}{64}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arc}}}{\sqrt{b}}\right.\right.}{}$$

$$\frac{2d^2x\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

3.469. $\int \frac{x\left(d+c^2dx^2\right)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

output
```
(-2*d^2*x*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (2*d^2*((3
*Sqrt[a + b*ArcSinh[c*x]])/4 + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sq
rt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*E^((2*a)/b))))/(b^2*c^
2) + (12*d^2*(-1/8*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi
]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*E^((2*a)/b)*Sqr
t[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 + (Sqrt[b]*E^(
(6*a)/b)*Sqrt[Pi/6]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 +
(Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((4*a)
/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(64*E^((2*a)/b)) + (Sqrt[b]*Sqrt[Pi/6]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh
[c*x]])/Sqrt[b]])/(64*E^((6*a)/b))))/(b^2*c^2)
```

### 3.469.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.469. $\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.469.4 Maple [F]

$$\int \frac{x(c^2d\,x^2+d)^2}{(a+b\,\operatorname{arcsinh}(cx))^{\frac{3}{2}}}dx$$

input
```
int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.469.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.469.6 Sympy [F]

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{x}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{2c^2x^3}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx + \int \frac{c^4x^5}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

input
```
integrate(x*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asin
h(c*x)), x) + Integral(2*c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

### 3.469.7 Maxima [F]

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2x}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^2*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.469.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.469.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x\,(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

# 3.470 $\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.470.1 Optimal result

Integrand size = 25, antiderivative size = 346

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{5d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} - \frac{d^2e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} + \frac{5d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$

output
```
-5/8*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c
+5/8*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b
)-5/16*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2
)*Pi^(1/2)/b^(3/2)/c+5/16*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*3^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(3*a/b)-1/16*d^2*exp(5*a/b)*erf(5^(1/2)*(
a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c+1/16*d^2*erfi(
5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(5
*a/b)-2*d^2*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

### 3.470.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.27

$$\int \frac{(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{d^2e^{-5\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}\left(-e^{\frac{5a}{b}}-5e^{\frac{5a}{b}+2\operatorname{arcsinh}(cx)}-10e^{\frac{5a}{b}+4\operatorname{arcsinh}(cx)}-10e^{\frac{5a}{b}+6\operatorname{arcsin}\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d^2*(-E^((5*a)/b) - 5*E^((5*a)/b + 2*ArcSinh[c*x]) - 10*E^((5*a)/b + 4*Ar
cSinh[c*x]) - 10*E^((5*a)/b + 6*ArcSinh[c*x]) - 5*E^((5*a)/b + 8*ArcSinh[c
*x]) - E^((5*a)/b + 10*ArcSinh[c*x]) + 10*E^((6*a)/b + 5*ArcSinh[c*x])*Sqr
t[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSin
h[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]
))/b] + 5*Sqrt[3]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])
/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 10*E^((4*a)/b + 5*ArcSinh[c
*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]
 + 5*Sqrt[3]*E^((8*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1
/2, (3*(a + b*ArcSinh[c*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c*E
^(5*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.470.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{10cd^2\int \frac{x(c^2x^2+1)^{3/2}}{\sqrt{a+b\operatorname{arcsinh}(cx)}}dx}{b}-\frac{2d^2(c^2x^2+1)^{5/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{10d^2\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{10d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 5971$$

$$-\frac{10d^2\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx)}{b^2c}$$
$$\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{10d^2\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+ba}}{\sqrt{}}\right.\right.}{}$$
$$\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (10*d^2*(-1/
16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) - (Sqr
t[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b
]])/(16*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/(b^2*c)
```

## 3.470. $\int \frac{\left(d+c^2dx^2\right)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

### 3.470.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.470.4 Maple [F]

$$\int \frac{(c^2 d\,x^2 + d)^2}{(a + b\,\operatorname{arcsinh}(cx))^{\frac{3}{2}}}\,dx$$

input
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.470.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.470.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{2c^2x^2}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{c^4x^4}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(2*c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(
c*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*asinh(c*x)) + b*s
qrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

### 3.470.7 Maxima [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.470.8 Giac [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.470.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2), x)
```

# 3.471 $\int \frac{(d+c^2dx^2)^2}{x(a+b\textbf{arcsinh}(cx))^{3/2}}\,dx$



## 3.471.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d^2(1+c^2x^2)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}\\
&+\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}-\frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}}\\
&+\frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{d^2e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}\\
&-\frac{d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{d^2e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}\\
&-\frac{2d^2\text{Int}\left(\frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}},x\right)}{bc}\end{aligned}$$

output
```
3/4*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)+3/4*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x
))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)+1/4*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(3/2)/exp(4*a/b)-2*d^2*(c^2*x^2+1)^(5/2)/b/c/x/(a+b*a
rcsinh(c*x))^(1/2)-2*d^2*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh
(c*x))^(1/2),x)/b/c
```

### 3.471.2 Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

### 3.471.3 Rubi [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6229, 6206, 3042, 3793, 2009, 6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6229}$$

$$\frac{8cd^2\int \frac{(c^2x^2+1)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^2\int \frac{(c^2x^2+1)^{3/2}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2d^2(c^2x^2+1)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6206}$$

$$\frac{8d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{2d^2\int \frac{(c^2x^2+1)^{3/2}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2d^2(c^2x^2+1)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8d^2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^4}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2}-\frac{2d^2\int \frac{(c^2x^2+1)^{3/2}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2d^2(c^2x^2+1)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{8d^2\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2}-\frac{2d^2\int \frac{(c^2x^2+1)^{3/2}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2d^2(c^2x^2+1)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^2\int \frac{(c^2x^2+1)^{3/2}}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}+\frac{8d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.\right.}{b^2}$$

$$\frac{2d^2(c^2x^2+1)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6235}$$

$$-\frac{2d^2\int \left(\frac{x^2c^4}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2c^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}+\frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}\right)dx}{bc}+\frac{8d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.\right.}{b^2}$$

$$\frac{2d^2(c^2x^2+1)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

---

3.471. $\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

$$-\frac{2d^2\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{\sqrt{\frac{\pi}{2}}ce^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}ce^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}}+\frac{3c\sqrt{a}}{}\right.}{bc}$$

$$\frac{8d^2\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.\right.}{b^2}$$

$$\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bcx\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

### 3.471.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

3.471. $\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

### 3.471.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} d \,x^{2}+d \right)^{2}}{x \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

input
```
int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.471.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.471.6 Sympy [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.68

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d^2\left(\int \frac{2c^2x^2}{ax\sqrt{a+b\,\text{asinh}\,(cx)}+bx\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{c^4x^4}{ax\sqrt{a+b\,\text{asinh}\,(cx)}+bx\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{ax\sqrt{a+b\,\text{asinh}\,(cx)}+bx\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2/x/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(2*c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*asinh(c*x))
 + b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b
*asinh(c*x)) + b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

### 3.471.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^2/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

### 3.471.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.471.9 Mupad [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^2}{x(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{{\left(d\,c^2\,x^2+d\right)}^2}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)),x)
```

output
```
int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)), x)
```

# 3.472 $\int (c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx$



## 3.472.1 Optimal result

Integrand size = 23, antiderivative size = 319

$$\int (c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx = \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} + \frac{1}{4}x(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)} + \frac{c\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$

output
```
1/4*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*er
f(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*
x^2+1)^(1/2)-1/32*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2
*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/256*c*erf(2*arcsinh(a*x)^(1/2))*Pi^(
1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/256*c*erfi(2*arcsinh(a*x)^(
1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*x*(a^2*c*x^2+c)
^(3/2)*arcsinh(a*x)^(1/2)+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

### 3.472.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.45

$$\int (c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx=\frac{c\sqrt{c+a^2cx^2}\left(-\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-4\text{arcsinh}(ax)\right)-8\sqrt{2}\sqrt{-\text{arcsinh}(ax)}\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(-(Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]])
 - 8*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]] + Sqrt[ArcSin
h[a*x]]*(32*ArcSinh[a*x]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[a*x]] - Ga
mma[3/2, 4*ArcSinh[a*x]])))/(128*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

### 3.472.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6201, 6200, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\sqrt{\text{arcsinh}(ax)}dx+\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(-\frac{a\sqrt{a^2cx^2+c}\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 6195

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(-\frac{\sqrt{a^2cx^2+c}\int \frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 5971

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}-\frac{\sqrt{a^2cx^2+c}\int \frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 27

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}-\frac{\sqrt{a^2cx^2+c}\int \frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3042

3.472. $\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx$

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}-\frac{\sqrt{a^2cx^2+c}\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2 -\right.$$
$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 26

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{i\sqrt{a^2cx^2+c}\int \frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c\right.$$
$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3789

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{8a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(a}}{\sqrt{a^2x^2+1}}}{2\sqrt{a^2x^2+1}}\right.$$
$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2611

$$-\frac{ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(\frac{i\sqrt{a^2cx^2+c}\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}}{2\sqrt{a^2x^2+1}}\right.$$
$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2633

3.472. $\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx$

$$\frac{3}{4}c\left(\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}\right.$$

$$\frac{ac\sqrt{a^2cx^2+c}\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{3}{4}c\left(\frac{\sqrt{a^2cx^2+c}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a\sqrt{a^2x^2+1}}\right.$$

$$\frac{ac\sqrt{a^2cx^2+c}\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{ac\sqrt{a^2cx^2+c}\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{8\sqrt{a^2x^2+1}}+$$

$$\frac{3}{4}c\left(\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a\sqrt{a^2x^2+1}}+\right.$$

$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6234}$$

$$-\frac{c\sqrt{a^2cx^2+c}\int\frac{ax(a^2x^2+1)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+$$

$$\frac{3}{4}c\left(\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a\sqrt{a^2x^2+1}}+\right.$$

$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{5971}$$

3.472. $\int\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{arcsinh}(ax)}\,dx$

$$-\frac{c\sqrt{a^2cx^2+c}\int\left(\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}+\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a\sqrt{a^2x^2+1}}+\right.$$
$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2009

$$-\frac{c\sqrt{a^2cx^2+c}\left(-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\right.}{8a\sqrt{a^2x^2+1}}$$
$$\frac{3}{4}c\left(\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a\sqrt{a^2x^2+1}}+\right.$$
$$\frac{1}{4}x\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{3/2}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

output
```
(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]])/4 - (c*Sqrt[c + a^2*c*x^2]*(-
1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]]) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[A
rcSinh[a*x]]])/8 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[Pi/2]*
Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8))/(8*a*Sqrt[1 + a^2*x^2]) + (3*c*((x*S
qrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x]^(3/2))/(3*a*Sqrt[1 + a^2*x^2]) + ((I/8)*Sqrt[c + a^2*c*x^2]*((-1/2*I)*S
qrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*
Sqrt[ArcSinh[a*x]]]))/(a*Sqrt[1 + a^2*x^2])))/4
```

### 3.472.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.472. $\int (c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx$

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.472.4 Maple [F]

$$\int \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(a x\right)}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

### 3.472.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.472.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{asinh}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x)), x)
```

### 3.472.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x)), x)
```

### 3.472.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.472.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int \sqrt{\operatorname{asinh}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

input
```
int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

# 3.473 $\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx$



## 3.473.1 Optimal result

Integrand size = 23, antiderivative size = 186

$$\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}+\frac{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}$$

output
```
1/3*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*erf(2^
(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+
1)^(1/2)-1/32*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2
+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/2*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

## 3.473.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.56

$$\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(16\text{arcsinh}(ax)^2-3\sqrt{2}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-2\text{arcsinh}(ax)\right)-3\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},2\text{arc}\right.\right.}{48a\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(16*ArcSinh[a*x]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*
Gamma[3/2, -2*ArcSinh[a*x]] - 3*Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, 2*Ar
cSinh[a*x]]))/(48*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

## 3.473.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6200, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{6200}$$

$$-\frac{a\sqrt{a^2cx^2+c}\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{6195}$$

$$-\frac{\sqrt{a^2cx^2+c}\int \frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}-\frac{\sqrt{a^2cx^2+c}\int \frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}-\frac{\sqrt{a^2cx^2+c}\int \frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}-\frac{\sqrt{a^2cx^2+c}\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{i\sqrt{a^2cx^2+c}\int \frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{3789}$$

$$\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{8a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{2611}$$

$$\frac{i\sqrt{a^2cx^2+c}\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{2633}$$

$$\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{2634}$$

---

3.473. $\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx$

$$\frac{\sqrt{a^2cx^2+c}\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+$$
$$\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a\sqrt{a^2x^2+1}}+$$
$$\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{6198}$$

$$\frac{i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a\sqrt{a^2x^2+1}}+$$
$$\frac{\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/2 + (Sqrt[c + a^2*c*x^2]*ArcSin
h[a*x]^(3/2))/(3*a*Sqrt[1 + a^2*x^2]) + ((I/8)*Sqrt[c + a^2*c*x^2]*((-1/2*
I)*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt
[2]*Sqrt[ArcSinh[a*x]]]))/(a*Sqrt[1 + a^2*x^2])
```

### 3.473.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

## 3.473. $\int \sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}\,dx$

### 3.473.4 Maple [F]

$$\int \sqrt{a^2c\,x^2+c}\,\sqrt{\operatorname{arcsinh}\left(a x\right)}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

### 3.473.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.473.6 Sympy [F]

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{asinh}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*asinh(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x)), x)
```

### 3.473.7 Maxima [F]

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{a^2cx^2+c}\sqrt{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x)), x)
```

### 3.473.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.473.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{asinh}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

# 3.474 $\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$



## 3.474.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}$$

output
```
2/3*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

## 3.474.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{3a\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c*(1 + a^2*x^2)])
```

## 3.474.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2cx^2+c}} \, dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

input
```
Int[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

### 3.474.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.474.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{2\,\text{arcsinh}(ax)^{\frac{3}{2}}\sqrt{a^2x^2+1}}{3a\sqrt{c(a^2x^2+1)}}$ | 36 |

input
```
int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

### 3.474.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.474.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(asinh(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

### 3.474.7 Maxima [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

### 3.474.8 Giac [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

### 3.474.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2), x)
```

# 3.475 $\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$



## 3.475.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{x\sqrt{\text{arcsinh}(ax)}}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)\sqrt{\text{arcsinh}(ax)}},x\right)}{2c\sqrt{c+a^2cx^2}}$$

output
```
x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*x^2+1)^(1/2)*Uninteg
rable(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+c)^(1/2)
```

## 3.475.2 Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2), x]
```

## 3.475.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6202, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x\sqrt{\text{arcsinh}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)\sqrt{\text{arcsinh}(ax)}}dx}{2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{x\sqrt{\text{arcsinh}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)\sqrt{\text{arcsinh}(ax)}}dx}{2c\sqrt{a^2cx^2+c}}$$

input
```
Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

### 3.475.3.1 Defintions of rubi rules used

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.475.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(ax\right)}}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

### 3.475.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.475.6 Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(ax\right)}}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

### 3.475.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\text{arsinh}\,(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.475.8 Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\text{arsinh}\,(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.475.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\text{asinh}\,(a\,x)}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2), x)
```

# 3.476 $\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$



## 3.476.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x\sqrt{\text{arcsinh}(ax)}}{3c(c+a^2cx^2)^{3/2}} + \frac{2x\sqrt{\text{arcsinh}(ax)}}{3c^2\sqrt{c+a^2cx^2}} - \frac{a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)^2\sqrt{\text{arcsinh}(ax)}},x\right)}{6c^2\sqrt{c+a^2cx^2}} - \frac{a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)\sqrt{\text{arcsinh}(ax)}},x\right)}{3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/3*x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arcsinh(a*x)^(1/2)/c^
2/(a^2*c*x^2+c)^(1/2)-1/6*a*(a^2*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2
/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)-1/3*a*(a^2*x^2+1)^(1/2)*Uni
ntegrable(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)
```

## 3.476.2 Mathematica [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2), x]
```

### 3.476.3 Rubi [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6203, 6202, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}dx}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\int \frac{\sqrt{\text{arcsinh}(ax)}}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\sqrt{\text{arcsinh}(ax)}}{3c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}dx}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\sqrt{\text{arcsinh}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)\sqrt{\text{arcsinh}(ax)}}dx}{2c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\sqrt{\text{arcsinh}(ax)}}{3c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}dx}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\sqrt{\text{arcsinh}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)\sqrt{\text{arcsinh}(ax)}}dx}{2c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\sqrt{\text{arcsinh}(ax)}}{3c\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

---

3.476. $\int \frac{\sqrt{\text{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}dx$

output
```
$Aborted
```

### 3.476.3.1 Defintions of rubi rules used

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.476.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

### 3.476.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.476.6 Sympy [N/A]

Not integrable

Time = 23.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

### 3.476.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

### 3.476.8 Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

### 3.476.9 Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2), x)
```

# 3.477 $\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2} dx$



## 3.477.1 Optimal result

Integrand size = 23, antiderivative size = 449

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2} dx = -\frac{27c\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{256a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{32\sqrt{1+a^2x^2}} - \frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{32a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2} + \frac{1}{4}x(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}{20a\sqrt{1+a^2x^2}} + \frac{3c\sqrt{\pi}\sqrt{c-}}{}$$

output
```
1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2)+3/8*c*x*arcsinh(a*x)^(3/2)*(a
^2*c*x^2+c)^(1/2)+3/20*c*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2
+1)^(1/2)+3/128*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*
x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*
2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*c*erf(2*ar
csinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*
c*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1
/2)-3/32*c*(a^2*x^2+1)^(3/2)*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a-27/2
56*c*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/32*a*c*x
^2*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

### 3.477.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.41

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{c\sqrt{c+a^2cx^2}\Big(384\operatorname{arcsinh}(ax)^3 - 480\operatorname{arcsinh}(ax)\cosh(2\operatorname{arcsinh}(ax)) + 60\sqrt{2}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(384*ArcSinh[a*x]^3 - 480*ArcSinh[a*x]*Cosh[2*ArcSi
nh[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]
] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 5*
Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]] - 5*Sqrt[ArcSinh[a*x]]*Gam
ma[5/2, 4*ArcSinh[a*x]] + 640*ArcSinh[a*x]^2*Sinh[2*ArcSinh[a*x]]))/(2560*
a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

### 3.477.3 Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.89, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {6201, 6200, 6192, 6198, 6213, 6206, 3042, 3793, 2009, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\sqrt{\operatorname{arcsinh}(ax)}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\operatorname{arcsinh}(ax)^{3/2}dx+\frac{1}{4}x\operatorname{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\sqrt{\text{arcsinh}(ax)}dx}{8\sqrt{a^2x^2+1}}+$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\int x\sqrt{\text{arcsinh}(ax)}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}\right)+$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\big\downarrow$ 6192

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\sqrt{\text{arcsinh}(ax)}dx}{8\sqrt{a^2x^2+1}}+$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\big\downarrow$ 6198

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsi}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\sqrt{\text{arcsinh}(ax)}dx}{8\sqrt{a^2x^2+1}}+\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\big\downarrow$ 6213

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsi}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\int\frac{\left(a^2x^2+1\right)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}dx}{8a}\right)}{8\sqrt{a^2x^2+1}}+\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\big\downarrow$ 6206

---

3.477. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}\,dx$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsi}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\int\frac{\left(a^2x^2+1\right)^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^2}\right)}{8\sqrt{a^2x^2+1}}+$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3042

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsi}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\int\frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^2}\right)}{8\sqrt{a^2x^2+1}}+$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3793

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsi}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\int\left(\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}+\frac{\cosh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}+\frac{3}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{8a^2}\right)}{8\sqrt{a^2x^2+1}}+$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2009

---

3.477. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}\,dx$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsi}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right.}{8a^2}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 6234

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{a^2x^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsin}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(\right.}{8a^2}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3042

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int-\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{ar}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(\right.}{8a^2}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 25

3.477. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}\,dx$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}+\frac{\int\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arc}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(\right.}{8a^2}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{\int\left(\frac{1}{2\sqrt{\text{arcsinh}(ax)}}-\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{4a^2}+\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a}}{5a\sqrt{a^2x^2+}}\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(\right.}{8a^2}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}{4a^2}-\frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsin}\right.}{8a^2}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\sqrt{\text{arcsinh}(ax)}}{4a^2}\right)}{4\sqrt{a^2x^2+1}}-\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}$$

---

3.477. $\int \left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{3/2}\,dx$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

output
```
(x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2))/4 - (3*a*c*Sqrt[c + a^2*c*x^2
]*(((1 + a^2*x^2)^2*Sqrt[ArcSinh[a*x]])/(4*a^2) - ((3*Sqrt[ArcSinh[a*x]])/
4 + (Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt
[ArcSinh[a*x]]])/4 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[Pi/2
]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4)/(8*a^2)))/(8*Sqrt[1 + a^2*x^2]) + (
3*c*((x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/2 + (Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(5/2))/(5*a*Sqrt[1 + a^2*x^2]) - (3*a*Sqrt[c + a^2*c*x^2]*((x^
2*Sqrt[ArcSinh[a*x]])/2 - (-Sqrt[ArcSinh[a*x]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4)/(
4*a^2)))/(4*Sqrt[1 + a^2*x^2])))/4
```

### 3.477.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.477. $\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx$

### 3.477.4 Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

### 3.477.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.477.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(3/2),x)
```

output
```
Timed out
```

### 3.477.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{3/2} \, dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arsinh}\left(ax\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2), x)
```

### 3.477.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.477.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{3/2} \, dx = \int \operatorname{asinh}(a\,x)^{3/2} \left(c\,a^2\,x^2+c\right)^{3/2} \, dx$$

input
```
int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

# 3.478 $\int \sqrt{c+a^2cx^2}\mathbf{arcsinh}(ax)^{3/2}\,dx$



## 3.478.1 Optimal result

Integrand size = 23, antiderivative size = 271

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = -\frac{3\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{16a\sqrt{1+a^2x^2}}-\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{8\sqrt{1+a^2x^2}}+\frac{1}{2}x\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}+\frac{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}{5a\sqrt{1+a^2x^2}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$

output
```
1/2*x*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)+1/5*arcsinh(a*x)^(5/2)*(a^2*c
*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^
(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erfi(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/
2)-3/16*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*a*x
^2*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

### 3.478.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.46

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(15\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+15\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\right.\right.}{}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 15
*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(-15*C
osh[2*ArcSinh[a*x]] + 4*ArcSinh[a*x]*(4*ArcSinh[a*x] + 5*Sinh[2*ArcSinh[a*
x]]))))/(640*a*Sqrt[1 + a^2*x^2])
```

### 3.478.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6200, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 6200$$

$$-\frac{3a\sqrt{a^2cx^2+c}\int x\sqrt{\text{arcsinh}(ax)}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \; 6192$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int \frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \; 6198$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int\frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{6234}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{a^2x^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int-\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}+\frac{\int\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{\int\left(\frac{1}{2\sqrt{\text{arcsinh}(ax)}}-\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{4a^2}+\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{2009}$$

3.478. $\int\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-\sqrt{\operatorname{arcsinh}(ax)}}{4a^2}\right)}{4\sqrt{a^2x^2+1}}+\frac{\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\operatorname{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSin
h[a*x]^(5/2))/(5*a*Sqrt[1 + a^2*x^2]) - (3*a*Sqrt[c + a^2*c*x^2]*((x^2*Sqr
t[ArcSinh[a*x]])/2 - (-Sqrt[ArcSinh[a*x]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[A
rcSinh[a*x]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4)/(4*a^2
)))/(4*Sqrt[1 + a^2*x^2])
```

### 3.478.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.478.4 Maple [F]

$$\int \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{a^2 c\, x^2 + c}dx$$

input
```
int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

## 3.478.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c + a^2cx^2}\text{arcsinh}(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.478.6 Sympy [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\,\text{asinh}^{\frac{3}{2}}\left(ax\right)dx$$

input
```
integrate(asinh(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2), x)
```

### 3.478.7 Maxima [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = \int \sqrt{a^2cx^2+c}\,\text{arsinh}\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2), x)
```

### 3.478.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.478.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}\,dx = \int \text{asinh}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

# 3.479 $\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$



## 3.479.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}$$

output
```
2/5*arcsinh(a*x)^(5/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

## 3.479.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c*(1 + a^2*x^2)])
```

## 3.479.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

### 3.479.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.479.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{2\operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{a^2x^2+1}}{5a\sqrt{c(a^2x^2+1)}}$ | 36 |

input
```
int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*arcsinh(a*x)^(5/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

### 3.479.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.479.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(asinh(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

### 3.479.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

### 3.479.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

### 3.479.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2), x)
```

# 3.480 $\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## 3.480.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\operatorname{arcsinh}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{3a\sqrt{1+a^2x^2}\operatorname{Int}\left(\frac{x\sqrt{\operatorname{arcsinh}(ax)}}{1+a^2x^2},x\right)}{2c\sqrt{c+a^2cx^2}}$$

output
```
x*arcsinh(a*x)^(3/2)/c/(a^2*c*x^2+c)^(1/2)-3/2*a*(a^2*x^2+1)^(1/2)*Uninteg
rable(x*arcsinh(a*x)^(1/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

## 3.480.2 Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2), x]
```

## 3.480.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6202, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x\text{arcsinh}(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}-\frac{3a\sqrt{a^2x^2+1}\int\frac{x\sqrt{\text{arcsinh}(ax)}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{6239}$$

$$\frac{x\text{arcsinh}(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}-\frac{3a\sqrt{a^2x^2+1}\int\frac{x\sqrt{\text{arcsinh}(ax)}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

### 3.480.3.1 Defintions of rubi rules used

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.480.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}\left(a x\right)^{\frac{3}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

### 3.480.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.480.6 Sympy [N/A]

Not integrable

Time = 9.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(asinh(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

### 3.480.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\text{arsinh}\,(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.480.8 Giac [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\text{arsinh}\,(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.480.9 Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\text{asinh}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2), x)
```

# 3.481 $\int (c + a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx$



## 3.481.1 Optimal result

Integrand size = 23, antiderivative size = 514

$$\int (c + a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{225}{512}cx\sqrt{c + a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}$$
$$+ \frac{15}{256}cx(1 + a^2x^2)\sqrt{c + a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{45c\sqrt{c + a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{256a\sqrt{1 + a^2x^2}}$$
$$- \frac{15acx^2\sqrt{c + a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32\sqrt{1 + a^2x^2}} - \frac{5c(1 + a^2x^2)^{3/2}\sqrt{c + a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32a}$$
$$+ \frac{3}{8}cx\sqrt{c + a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}x(c + a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c + a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{28a\sqrt{1 + a^2x^2}} + \frac{15c\sqrt{\pi}\sqrt{c}}{}$$

output
```
1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2)-5/32*c*(a^2*x^2+1)^(3/2)*arcs
inh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2
+c)^(1/2)-45/256*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1
/2)-15/32*a*c*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)
+3/28*c*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/512*
c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(
a^2*x^2+1)^(1/2)-15/512*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2
)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/16384*c*erf(2*arcsinh(a*x)^(1
/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/16384*c*erfi(2*ar
csinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+225/512
*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)+15/256*c*x*(a^2*x^2+1)*(a^2*c*
x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

## 3.481.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.39

$$\int (c +a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{c\sqrt{c+a^2cx^2}\Big(1536\operatorname{arcsinh}(ax)^4 - 4480\operatorname{arcsinh}(ax)^2\cosh(2\operatorname{arcsinh}(ax)) + 42}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(1536*ArcSinh[a*x]^4 - 4480*ArcSinh[a*x]^2*Cosh[2*A
rcSinh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[
a*x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]
] - 7*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 7*Sqrt[ArcSinh[a*x
]]*Gamma[7/2, 4*ArcSinh[a*x]] + 3360*ArcSinh[a*x]*Sinh[2*ArcSinh[a*x]] + 3
584*ArcSinh[a*x]^3*Sinh[2*ArcSinh[a*x]]))/(14336*a*Sqrt[1 + a^2*x^2]*Sqrt[
ArcSinh[a*x]])
```

## 3.481.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\operatorname{arcsinh}(ax)^{5/2}dx+\frac{1}{4}x\operatorname{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\int x\operatorname{arcsinh}(ax)^{3/2}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\operatorname{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6192}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsin}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a$$

$$\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6213}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\int\left(a^2x^2+1\right)^{3/2}\sqrt{\text{arcsinh}(ax)}dx}{8a}\right)}{8\sqrt{a^2x^2+1}}+$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6201}$$

3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\cdots\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\int\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\sqrt{\text{arcsin}\cdots}\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 6200$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{1}{4}a\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{1}{2}x\sqrt{\cdots}\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\cdots\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 6195$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(-\frac{\int\frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a}+\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(a\cdots}}{\sqrt{a^2x^2+1}}\cdots\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\cdots\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 5971$$

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3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}\right.\right.}{4a}}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}\right.\right.}{8a}}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 3042$$

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3.481. $\int \left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsi}}{8a}\right.\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 26$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}}{8a}\right.\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 3789$$

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3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsi}\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.}{8\sqrt{a^2x^2+1}\cdots}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\cdots\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 2611$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}\cdots}\right.}{\cdots}\right.\right.}{\cdots}\right.}{8\sqrt{a^2x^2+1}\cdots}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\cdots\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 2633$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{a}\cdots}\right.}{8a}\cdots\right.\right.}{\cdots}\right.}{8\sqrt{a^2x^2+1}\cdots}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\cdots\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

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3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$\downarrow$ 2634

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsin}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 6198

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(a\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 6227

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)\right)}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

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3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$\downarrow 6195$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcs}\cdots}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\downarrow 5971$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{a}\cdots}}{2a\cdots}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\downarrow 27$$

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3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{a}\cdots}}{2a\cdots}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\cdots}{\cdots}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 26$$

---

3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{ar}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 3789$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{8a^3}-\frac{\int}{}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 2611$$

---

3.481. $\int \left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\frac{\int\ldots}{}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\frac{\int\ldots}{}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\ldots}\right.\right.}{8a^3}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

---

3.481. $\int\left(c+a^2cx^2\right)^{3/2}\text{arcsinh}(ax)^{5/2}\,dx$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4} x \operatorname{arcsinh}(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} + \frac{3}{4}c\left(\frac{\operatorname{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}} + \frac{1}{2}x\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c} - \frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2} - \frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\right.}{\ldots}\right.\right.}{\ldots}\right. - \frac{5ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^{3/2}}{4a^2} - \frac{3\left(-\frac{1}{8}a\int\frac{x\left(a^2x^2+1\right)}{\sqrt{\operatorname{arcsinh}(ax)}}dx + \frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)} + \frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcs}\ldots\right.\right.}{\ldots}\right.\right.}{\ldots}\right.}{8\sqrt{a^2x^2+1}\ldots}$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{4} x \operatorname{arcsinh}(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} + \frac{3}{4}c\left(\frac{\operatorname{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}} + \frac{1}{2}x\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c} - \frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2} - \frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\right.}{\ldots}\right.\right.}{\ldots}\right. - \frac{5ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\operatorname{arcsinh}(ax)^{3/2}}{4a^2} - \frac{3\left(-\frac{\int\frac{ax\left(a^2x^2+1\right)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{8a} + \frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)} + \frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\ldots\right.\right.}{\ldots}\right.\right.}{\ldots}\right.}{8\sqrt{a^2x^2+1}\ldots}$$

$$\Big\downarrow \text{5971}$$

---

3.481. $\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\, dx$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{\int\left(\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}+\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{8a}+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{a\cdots}\right.\right.}{\cdots}\right.}{8\sqrt{a^2x\cdots}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}+$$

$$\frac{3}{4}c\left(\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

output
```
$Aborted
```

### 3.481.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.481. $\int (c + a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2} \, dx$

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.481. $\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx$

### 3.481.4 Maple [F]

$$\int \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x\right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

### 3.481.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.481.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

### 3.481.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2), x)
```

### 3.481.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.481.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx=\int \operatorname{asinh}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

# 3.482 $\int \sqrt{c+a^2cx^2}\,\textbf{arcsinh}(ax)^{5/2}\,dx$



## 3.482.1 Optimal result

Integrand size = 23, antiderivative size = 298

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} + \frac{15\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} - 15$$

output
```
1/2*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)-5/16*arcsinh(a*x)^(3/2)*(a^2*
c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-5/8*a*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2
+c)^(1/2)/(a^2*x^2+1)^(1/2)+1/7*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(
a^2*x^2+1)^(1/2)+15/512*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/512*erfi(2^(1/2)*arcsinh(a*x)^(1
/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/32*x*(a^2
*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

### 3.482.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.45

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \frac{\sqrt{c\left(1+a^2x^2\right)}\left(105\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-105\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsi}\right.\right.}{}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 1
05*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(64*
ArcSinh[a*x]^3 - 140*ArcSinh[a*x]*Cosh[2*ArcSinh[a*x]] + 7*(15 + 16*ArcSin
h[a*x]^2)*Sinh[2*ArcSinh[a*x]])))/(3584*a*Sqrt[1 + a^2*x^2])
```

### 3.482.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {6200, 6192, 6198, 6227, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 6200$$

$$-\frac{5a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^{3/2}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \; 6192$$

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int \frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \; 6198$$

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\Big\downarrow$ 6227

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\Big\downarrow$ 6195

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(a}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\Big\downarrow$ 5971

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsin}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\Big\downarrow$ 27

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsin}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\Big\downarrow$ 3042

3.482. $\int\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx$

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcs}\cdots}}{2a^2}\cdots\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 26

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\int \frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}\cdots}}{2a^2}\cdots\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 3789

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{8a^3}-\frac{\int \frac{\sqrt{\text{ar}\cdots}}{\sqrt{\cdots}}}{\cdots}\cdots\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 2611

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\frac{\int \frac{\sqrt{\text{arcs}\cdots}}{\sqrt{a^2\cdots}}}{2\cdots}\cdots\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 2633

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\frac{\int \frac{\sqrt{\text{arcs}\cdots}}{\sqrt{a^2\cdots}}}{2\cdots}\cdots\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 2634

---

3.482. $\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx$

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsi}}\right.\right.}{8a^3}\right.\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-$$
$$\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a^3}-\frac{\text{arcsinh}(ax)^3}{3a^3}\right.\right.}{4\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSin
h[a*x]^(7/2))/(7*a*Sqrt[1 + a^2*x^2]) - (5*a*Sqrt[c + a^2*c*x^2]*((x^2*Arc
Sinh[a*x]^(3/2))/2 - (3*a*((x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(2*a^2
) - ArcSinh[a*x]^(3/2)/(3*a^3) + ((I/8)*((-1/2*I)*Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]]))/a
^3))/4))/(4*Sqrt[1 + a^2*x^2])
```

### 3.482.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

---

## 3.482. $\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx$

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

### 3.482.4 Maple [F]

$$\int \operatorname{arcsinh}(ax)^{\frac{5}{2}} \sqrt{a^2c\,x^2+c}dx$$

input
```
int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

### 3.482.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.482.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(asinh(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

### 3.482.7 Maxima [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \int \sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2), x)
```

### 3.482.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.482.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx=\int \operatorname{asinh}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

# 3.483 $\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$



## 3.483.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}$$

output
```
2/7*arcsinh(a*x)^(7/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

## 3.483.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c*(1 + a^2*x^2)])
```

## 3.483.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

### 3.483.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.483.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{2\operatorname{arcsinh}(ax)^{\frac{7}{2}}\sqrt{a^2x^2+1}}{7a\sqrt{c(a^2x^2+1)}}$ | 36 |

input
```
int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*arcsinh(a*x)^(7/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

### 3.483.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.483.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

### 3.483.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

### 3.483.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

### 3.483.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\operatorname{asinh}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

# 3.484 $\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## 3.484.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} - \frac{5a\sqrt{1+a^2x^2}\text{Int}\left(\frac{x\text{arcsinh}(ax)^{3/2}}{1+a^2x^2},x\right)}{2c\sqrt{c+a^2cx^2}}$$

output
```
x*arcsinh(a*x)^(5/2)/c/(a^2*c*x^2+c)^(1/2)-5/2*a*(a^2*x^2+1)^(1/2)*Uninteg
rable(x*arcsinh(a*x)^(3/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

## 3.484.2 Mathematica [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2), x]
```

## 3.484.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6202, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x\operatorname{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\int \frac{x\operatorname{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{6239}$$

$$\frac{x\operatorname{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\int \frac{x\operatorname{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

### 3.484.3.1 Defintions of rubi rules used

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.484.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

### 3.484.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.484.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

### 3.484.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.484.8 Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

### 3.484.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{{\mathrm{asinh}\left(a\,x\right)}^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

# 3.485 $\int (a^2+x^2)^{3/2} \sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} \, dx$



## 3.485.1 Optimal result

Integrand size = 22, antiderivative size = 309

$$\int (a^2+x^2)^{3/2} \sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} \, dx = \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} + \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}} + \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\text{erfi}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

output
```
1/4*a^3*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a^3*erf(
2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(
1/2)-1/32*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^
(1/2)/(1+x^2/a^2)^(1/2)+1/256*a^3*erf(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2+
x^2)^(1/2)/(1+x^2/a^2)^(1/2)-1/256*a^3*erfi(2*arcsinh(x/a)^(1/2))*Pi^(1/2)
*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2
)+3/8*a^2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)
```

### 3.485.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.50

$$\int (a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{a^3\sqrt{a^2+x^2}\left(-\sqrt{-\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2},-4\text{arcsinh}\left(\frac{x}{a}\right)\right)-8\sqrt{2}\sqrt{-\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2},\right.\right.}{128}$$

input
```
Integrate[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

output
```
(a^3*Sqrt[a^2 + x^2]*(-(Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -4*ArcSinh[x/a]]) -
 8*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -2*ArcSinh[x/a]] + Sqrt[ArcSinh[
x/a]]*(32*ArcSinh[x/a]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[x/a]] - Gamm
a[3/2, 4*ArcSinh[x/a]])))/(128*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

### 3.485.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {6201, 27, 6200, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$$

$$\Big\downarrow \text{6201}$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx - \frac{a\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{a^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx - \frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 6200$$

$$\frac{3}{4}a^2\left(-\frac{\sqrt{a^2+x^2}\int \frac{x}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a\sqrt{\frac{x^2}{a^2}+1}} + \frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right) - \frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 6195$$

$$\frac{3}{4}a^2\left(-\frac{a\sqrt{a^2+x^2}\int \frac{x\sqrt{\frac{x^2}{a^2}+1}}{a\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}} + \frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right) - \frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 5971$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}} + \frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}} - \frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right) + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 27$$

---

3.485. $\int \left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}(\frac{x}{a})\right)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+$$
$$\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int -\frac{i\sin\left(2i\text{arcsinh}(\frac{x}{a})\right)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)$$
$$\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\int \frac{\sin\left(2i\text{arcsinh}(\frac{x}{a})\right)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+$$
$$\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{3789}$$

---

3.485. $\int (a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)-\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\downarrow$ 2611

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(i\int e^{2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\downarrow$ 2633

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\downarrow$ 2634

$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\downarrow$ 6198

---

3.485. $\int\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{6234}$$

$$-\frac{a^3\sqrt{a^2+x^2}\int \frac{x\left(\frac{x^2}{a^2}+1\right)^{3/2}}{a\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{a^3\sqrt{a^2+x^2}\int \left(\frac{\sinh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{4\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}+\frac{\sinh\left(4\text{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}\right)d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-$$
$$\frac{a^3\sqrt{a^2+x^2}\left(-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\right.}{8\sqrt{\frac{x^2}{a^2}+1}}$$

3.485. $\int \left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

input
```
Int[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

output
```
(x*(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]])/4 - (a^3*Sqrt[a^2 + x^2]*(-1/32*(
Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[x/a]]]) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh
[x/a]]])/8 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[x/a]]])/32 + (Sqrt[Pi/2]*Erfi[S
qrt[2]*Sqrt[ArcSinh[x/a]]])/8))/(8*Sqrt[1 + x^2/a^2]) + (3*a^2*((x*Sqrt[a^
2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/(3
*Sqrt[1 + x^2/a^2]) + ((I/8)*a*Sqrt[a^2 + x^2]*((-1/2*I)*Sqrt[Pi/2]*Erf[Sq
rt[2]*Sqrt[ArcSinh[x/a]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a
]]]))/Sqrt[1 + x^2/a^2]))/4
```

### 3.485.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

3.485. $\int (a^2+x^2)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\, dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

---

3.485. $\int (a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.485.4 Maple [F]

$$\int \left(a^{2}+x^{2}\right)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

output
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

### 3.485.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a^2+x^2\right)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.485.6 Sympy [F]

$$\int \left(a^2+x^2\right)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \left(a^{2}+x^{2}\right)^{\frac{3}{2}} \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a**2+x**2)**(3/2)*asinh(x/a)**(1/2),x)
```

output
```
Integral((a**2 + x**2)**(3/2)*sqrt(asinh(x/a)), x)
```

### 3.485.7 Maxima [F]

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \left(a^2+x^2\right)^{\frac{3}{2}}\sqrt{\text{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

### 3.485.8 Giac [F]

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \left(a^2+x^2\right)^{\frac{3}{2}}\sqrt{\text{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

### 3.485.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \sqrt{\text{asinh}\left(\frac{x}{a}\right)}\,\left(a^2+x^2\right)^{3/2}\,dx$$

input
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2), x)
```

# 3.486 $\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$



## 3.486.1 Optimal result

Integrand size = 22, antiderivative size = 176

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\,\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

output
```
1/3*a*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a*erf(2^(1
/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)
-1/32*a*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/
(1+x^2/a^2)^(1/2)+1/2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)
```

### 3.486.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$$
$$= \frac{a\sqrt{a^2+x^2}\left(16\text{arcsinh}\left(\frac{x}{a}\right)^2-3\sqrt{2}\sqrt{-\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2},-2\text{arcsinh}\left(\frac{x}{a}\right)\right)-3\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2},2\text{arcsinh}(\right.\right.}{48\sqrt{1+\frac{x^2}{a^2}}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}$$

input
```
Integrate[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

output
```
(a*Sqrt[a^2 + x^2]*(16*ArcSinh[x/a]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamm
a[3/2, -2*ArcSinh[x/a]] - 3*Sqrt[2]*Sqrt[ArcSinh[x/a]]*Gamma[3/2, 2*ArcSin
h[x/a]]))/(48*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

### 3.486.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6200, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$$
$$\Big\downarrow \text{ 6200}$$
$$-\frac{\sqrt{a^2+x^2}\int \frac{x}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$
$$\Big\downarrow \text{ 6195}$$

$$-\frac{a\sqrt{a^2+x^2}\int \frac{x\sqrt{\frac{x^2}{a^2}+1}}{a\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 5971$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int -\frac{i\sin\left(2i\text{arcsinh}\left(\frac{x}{a}\right)\right)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 26$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\int \frac{\sin\left(2i\text{arcsinh}\left(\frac{x}{a}\right)\right)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 3789$$

---

3.486. $\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

$$\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\big\downarrow$ 2611

$$\frac{ia\sqrt{a^2+x^2}\left(i\int e^{2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\big\downarrow$ 2633

$$\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\big\downarrow$ 2634

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$\big\downarrow$ 6198

---

3.486. $\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

$$\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

input
```
Int[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

output
```
(x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]
^(3/2))/(3*Sqrt[1 + x^2/a^2]) + ((I/8)*a*Sqrt[a^2 + x^2]*((-1/2*I)*Sqrt[Pi
/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[A
rcSinh[x/a]]]))/Sqrt[1 + x^2/a^2]
```

### 3.486.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

3.486. $\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

## 3.486.4 Maple [F]

$$\int \sqrt{a^2+x^2}\,\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

output
```
int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

---

3.486. $\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$

### 3.486.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.486.6 Sympy [F]

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a**2+x**2)**(1/2)*asinh(x/a)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + x**2)*sqrt(asinh(x/a)), x)
```

### 3.486.7 Maxima [F]

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

### 3.486.8 Giac [F]

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \sqrt{a^2+x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

### 3.486.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,\sqrt{a^2+x^2}\,dx$$

input
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2), x)
```

# 3.487 $\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$



## 3.487.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

output
```
2/3*a*arcsinh(x/a)^(3/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)
```

## 3.487.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

input
```
Integrate[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

## 3.487.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

input
```
Int[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

### 3.487.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.487.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| default | $\frac{2\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} a\sqrt{\frac{a^2+x^2}{a^2}}}{3\sqrt{a^2+x^2}}$ | 34 |

input
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arcsinh(x/a)^(3/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

### 3.487.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \frac{2}{3}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{3}{2}}$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*log((x + sqrt(a^2 + x^2))/a)^(3/2)
```

### 3.487.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(1/2),x)
```

output
```
Integral(sqrt(asinh(x/a))/sqrt(a**2 + x**2), x)
```

### 3.487.7 Maxima [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

### 3.487.8 Giac [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

### 3.487.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2), x)
```

# 3.488 $\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx$



## 3.488.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\,\text{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

output
```
x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(1/2)-1/2*(1+x^2/a^2)^(1/2)*Unintegrabl
e(x/(1+x^2/a^2)/arcsinh(x/a)^(1/2),x)/a^3/(a^2+x^2)^(1/2)
```

## 3.488.2 Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2), x]
```

### 3.488.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6202, 27, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^2x}{(a^2+x^2)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a^3\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2+x^2}}$$

input
```
Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

output
```
$Aborted
```

### 3.488.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.488.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

output
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

## 3.488.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.488.6 Sympy [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(3/2),x)
```

output
```
Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(3/2), x)
```

### 3.488.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

## 3.488.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\text{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

## 3.488.9 Mupad [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\text{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2), x)
```

# 3.489 $\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$



## 3.489.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx = \frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}} + \frac{2x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\operatorname{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\operatorname{Int}\left(\frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2+x^2}}$$

output
```
1/3*x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(3/2)+2/3*x*arcsinh(x/a)^(1/2)/a^4/
(a^2+x^2)^(1/2)-1/6*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1+x^2/a^2)^2/arcsinh
(x/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)-1/3*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1
+x^2/a^2)/arcsinh(x/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)
```

## 3.489.2 Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

output
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2), x]
```

## 3.489.3 Rubi [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6203, 27, 6202, 27, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{2\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}dx}{3a^2}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^4x}{\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{6a^5\sqrt{a^2+x^2}}+\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}dx}{3a^2}+\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^2x}{\left(a^2+x^2\right)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a^3\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)^2\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\sqrt{\text{arcsinh}(\frac{x}{a})}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)^2\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\sqrt{\text{arcsinh}(\frac{x}{a})}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

input
```
Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

output
```
$Aborted
```

### 3.489.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

3.489. $\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{(a^2+x^2)^{5/2}}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.489.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^{2}+x^{2}\right)^{\frac{5}{2}}}dx$$

input
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

output
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

### 3.489.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.489.6 Sympy [N/A]

Not integrable

Time = 23.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(5/2),x)
```

output
```
Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(5/2), x)
```

### 3.489.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

### 3.489.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

### 3.489.9 Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2),x)
```

output
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2), x)
```

# 3.490 $\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx$



## 3.490.1 Optimal result

Integrand size = 22, antiderivative size = 433

$$\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x(a^2+x^2)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a^3\sqrt{\pi}\sqrt{a^2+}}{20}$$

output
```
1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2)+3/8*a^2*x*arcsinh(x/a)^(3/2)*(a^2
+x^2)^(1/2)+3/20*a^3*arcsinh(x/a)^(5/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+
3/128*a^3*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)
/(1+x^2/a^2)^(1/2)+3/128*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(
1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erf(2*arcsinh(x/a)^(1/2)
)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erfi(2*arcsinh(x/a
)^(1/2))*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/32*(a^2+x^2)^(5/2)*a
rcsinh(x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)-27/256*a^3*(a^2+x^2)^(1/2)*arcsinh(x
/a)^(1/2)/(1+x^2/a^2)^(1/2)-9/32*a*x^2*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/
(1+x^2/a^2)^(1/2)
```

### 3.490.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.48

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \frac{a^3\sqrt{a^2+x^2}\left(384\operatorname{arcsinh}\left(\frac{x}{a}\right)^3 - 480\operatorname{arcsinh}\left(\frac{x}{a}\right)\cosh\left(2\operatorname{arcsinh}\left(\frac{x}{a}\right)\right) + 60\sqrt{2\pi}\sqrt{a}\right.}{}$$

input
```
Integrate[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

output
```
(a^3*Sqrt[a^2 + x^2]*(384*ArcSinh[x/a]^3 - 480*ArcSinh[x/a]*Cosh[2*ArcSinh
[x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]]
+ 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 5*Sq
rt[-ArcSinh[x/a]]*Gamma[5/2, -4*ArcSinh[x/a]] - 5*Sqrt[ArcSinh[x/a]]*Gamma
[5/2, 4*ArcSinh[x/a]] + 640*ArcSinh[x/a]^2*Sinh[2*ArcSinh[x/a]]))/(2560*Sq
rt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

### 3.490.3 Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6201, 27, 6200, 6192, 6198, 6213, 6206, 3042, 3793, 2009, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx$$

$$\Big\downarrow \; 6201$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx - \frac{3a\sqrt{a^2+x^2}\int \frac{x\left(a^2+x^2\right)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2}dx}{8\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}dx-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 6200

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\int x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\right)-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 6192

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{a}\right.$$

$$-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 6198

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcs}\right.$$

$$-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 6213

---

3.490. $\int \left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcs}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^3\int\frac{\left(\frac{x^2}{a^2}+1\right)^{3/2}}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx\right)}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6206}$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcs}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\int\frac{\left(\frac{x^2}{a^2}+1\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcs}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\int\frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)+\frac{\pi}{2}\right)^4}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{3793}$$

3.490. $\int\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcs}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\int\left(\frac{\cosh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}+\frac{\cosh\left(4\text{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}+\frac{3}{8\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}\right)d\text{arcsin}}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 2009

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcs}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\text{er}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$\downarrow$ 6234

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int\frac{x^2}{a^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\text{er}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$\downarrow$ 3042

---

3.490. $\int\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int -\frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\operatorname{er}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}+\frac{1}{4}a^2\int \frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\operatorname{er}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$$\Big\downarrow \; 3793$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}a^2\int \left(\frac{1}{2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}-\frac{\cosh\left(2\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\right)d\operatorname{arcsinh}\left(\frac{x}{a}\right)+\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}}{5\sqrt{}}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\operatorname{er}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$$\Big\downarrow \; 2009$$

3.490. $\int \left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)-\sqrt{\operatorname{arcs}\right.\right.\right.}{4a\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\operatorname{er}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

input
```
Int[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

output
```
(x*(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2))/4 - (3*Sqrt[a^2 + x^2]*(((a^2 + x
^2)^2*Sqrt[ArcSinh[x/a]])/4 - (a^4*((3*Sqrt[ArcSinh[x/a]])/4 + (Sqrt[Pi]*E
rf[2*Sqrt[ArcSinh[x/a]]])/32 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]]
)/4 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[x/a]]])/32 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*
Sqrt[ArcSinh[x/a]]])/4))/8))/(8*a*Sqrt[1 + x^2/a^2]) + (3*a^2*((x*Sqrt[a^2
 + x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(5*
Sqrt[1 + x^2/a^2]) - (3*Sqrt[a^2 + x^2]*((x^2*Sqrt[ArcSinh[x/a]])/2 - (a^2
*(-Sqrt[ArcSinh[x/a]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/4 + (
Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/4))/4))/(4*a*Sqrt[1 + x^2/a^2
])))/4
```

### 3.490.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.490. $\int \left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

3.490. $\int (a^2 + x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.490.4 Maple [F]

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

output
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

## 3.490.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.490.6 Sympy [F(-1)]

Timed out.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a**2+x**2)**(3/2)*asinh(x/a)**(3/2),x)
```

output
```
Timed out
```

### 3.490.7 Maxima [F]

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

### 3.490.8 Giac [F]

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

### 3.490.9 Mupad [F(-1)]

Timed out.

$$\int (a^2+x^2)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}\left(a^2+x^2\right)^{3/2}\,dx$$

input
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2), x)
```

# 3.491 $\int \sqrt{a^2+x^2}\,\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$



## 3.491.1 Optimal result

Integrand size = 22, antiderivative size = 259

$$\int \sqrt{a^2+x^2}\,\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx = -\frac{3a\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\,\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\,\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$

output
```
1/2*x*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)+1/5*a*arcsinh(x/a)^(5/2)*(a^2+x^2
)^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*
Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erfi(2^(1/2)*arcsinh(x/
a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/16*a*(a^2+x
^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-3/8*x^2*(a^2+x^2)^(1/2)*arc
sinh(x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)
```

### 3.491.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\frac{a\sqrt{a^2+x^2}\left(15\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right.}{}$$

input
```
Integrate[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

output
```
(a*Sqrt[a^2 + x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 15*Sqr
t[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 8*Sqrt[ArcSinh[x/a]]*(16*ArcSin
h[x/a]^2 - 15*Cosh[2*ArcSinh[x/a]] + 20*ArcSinh[x/a]*Sinh[2*ArcSinh[x/a]])
))/(640*Sqrt[1 + x^2/a^2])
```

### 3.491.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6200, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{6200}$$

$$-\frac{3\sqrt{a^2+x^2}\int x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6192}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6234}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int\frac{x^2}{a^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int-\frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}+\frac{1}{4}a^2\int\frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}a^2\int\left(\frac{1}{2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}-\frac{\cosh\left(2\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\right)d\operatorname{arcsinh}\left(\frac{x}{a}\right)+\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

---

3.491. $\int\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)-\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right.\right.}{4a\sqrt{\frac{x^2}{a^2}+1}}$$

$$\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

input
```
Int[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

output
```
(x*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]
^(5/2))/(5*Sqrt[1 + x^2/a^2]) - (3*Sqrt[a^2 + x^2]*((x^2*Sqrt[ArcSinh[x/a]
])/2 - (a^2*(-Sqrt[ArcSinh[x/a]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/
a]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/4))/4))/(4*a*Sqrt[
1 + x^2/a^2])
```

### 3.491.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

3.491. $\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.491.4 Maple [F]

$$\int \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} \sqrt{a^2+x^2}dx$$

input
```
int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

output
```
int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

### 3.491.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.491.6 Sympy [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2+x^2}\,\text{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)dx$$

input
```
integrate(asinh(x/a)**(3/2)*(a**2+x**2)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + x**2)*asinh(x/a)**(3/2), x)
```

### 3.491.7 Maxima [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2+x^2}\,\text{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

### 3.491.8 Giac [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2+x^2}\,\text{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

### 3.491.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \text{asinh}\left(\frac{x}{a}\right)^{3/2}\sqrt{a^2+x^2}\,dx$$

input
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2), x)
```

# 3.492 $\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$



## 3.492.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

output
```
2/5*a*arcsinh(x/a)^(5/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)
```

## 3.492.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

input
```
Integrate[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

## 3.492.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

input
```
Int[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

### 3.492.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.492.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| default | $\frac{2\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{5}{2}} a\sqrt{\frac{a^2+x^2}{a^2}}}{5\sqrt{a^2+x^2}}$ | 34 |

input
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*arcsinh(x/a)^(5/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

### 3.492.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\frac{2}{5}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{5}{2}}$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*log((x + sqrt(a^2 + x^2))/a)^(5/2)
```

### 3.492.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(1/2),x)
```

output
```
Integral(asinh(x/a)**(3/2)/sqrt(a**2 + x**2), x)
```

### 3.492.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx=\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

### 3.492.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

### 3.492.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$$

input
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2), x)
```

# 3.493 $\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx$



## 3.493.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx = \frac{x\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}} - \frac{3\sqrt{1+\frac{x^2}{a^2}}\,\text{Int}\left(\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{1+\frac{x^2}{a^2}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

output
```
x*arcsinh(x/a)^(3/2)/a^2/(a^2+x^2)^(1/2)-3/2*(1+x^2/a^2)^(1/2)*Unintegrabl
e(x*arcsinh(x/a)^(1/2)/(1+x^2/a^2),x)/a^3/(a^2+x^2)^(1/2)
```

## 3.493.2 Mathematica [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx = \int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx$$

input
```
Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

output
```
Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2), x]
```

## 3.493.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6202, 27, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^2x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a^3\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}$$

input
```
Int[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

output
```
$Aborted
```

### 3.493.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.493.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^{2}+x^{2}\right)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

output
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

### 3.493.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.493.6 Sympy [N/A]

Not integrable

Time = 9.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(3/2),x)
```

output
```
Integral(asinh(x/a)**(3/2)/(a**2 + x**2)**(3/2), x)
```

### 3.493.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

### 3.493.8 Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

### 3.493.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx$$

input
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

# 3.494 $\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}}\,dx$



## 3.494.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}}\,dx = -\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(x)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(x)}\right)$$

output
```
-1/2*erf(arcsinh(x)^(1/2))*Pi^(1/2)+1/2*erfi(arcsinh(x)^(1/2))*Pi^(1/2)
```

## 3.494.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\text{arcsinh}(x)}}\,dx = \frac{1}{2}\left(\frac{\sqrt{-\text{arcsinh}(x)}\Gamma\left(\frac{1}{2},-\text{arcsinh}(x)\right)}{\sqrt{\text{arcsinh}(x)}}+\Gamma\left(\frac{1}{2},\text{arcsinh}(x)\right)\right)$$

input
```
Integrate[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

output
```
((Sqrt[-ArcSinh[x]]*Gamma[1/2, -ArcSinh[x]])/Sqrt[ArcSinh[x]] + Gamma[1/2,
 ArcSinh[x]])/2
```

### 3.494.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6234, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{arcsinh}(x)}}dx$$
$$\Big\downarrow \text{6234}$$
$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(x)}}d\operatorname{arcsinh}(x)$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\sin(i\operatorname{arcsinh}(x))}{\sqrt{\operatorname{arcsinh}(x)}}d\operatorname{arcsinh}(x)$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\sin(i\operatorname{arcsinh}(x))}{\sqrt{\operatorname{arcsinh}(x)}}d\operatorname{arcsinh}(x)$$
$$\Big\downarrow \text{3789}$$
$$-i\left(\frac{1}{2}i\int \frac{e^{\operatorname{arcsinh}(x)}}{\sqrt{\operatorname{arcsinh}(x)}}d\operatorname{arcsinh}(x)-\frac{1}{2}i\int \frac{e^{-\operatorname{arcsinh}(x)}}{\sqrt{\operatorname{arcsinh}(x)}}d\operatorname{arcsinh}(x)\right)$$
$$\Big\downarrow \text{2611}$$
$$-i\left(i\int e^{\operatorname{arcsinh}(x)}d\sqrt{\operatorname{arcsinh}(x)}-i\int e^{-\operatorname{arcsinh}(x)}d\sqrt{\operatorname{arcsinh}(x)}\right)$$
$$\Big\downarrow \text{2633}$$
$$-i\left(\frac{1}{2}i\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(x)}\right)-i\int e^{-\operatorname{arcsinh}(x)}d\sqrt{\operatorname{arcsinh}(x)}\right)$$
$$\Big\downarrow \text{2634}$$
$$-i\left(\frac{1}{2}i\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(x)}\right)-\frac{1}{2}i\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(x)}\right)\right)$$

input
```
Int[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

output
```
(-I)*((-1/2*I)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[x]]] + (I/2)*Sqrt[Pi]*Erfi[Sqrt[A
rcSinh[x]]])
```

### 3.494.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.494. $\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx$

### 3.494.4 Maple [F]

$$\int \frac{x}{\sqrt{x^2+1}\,\sqrt{\operatorname{arcsinh}(x)}}dx$$

input
```
int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

output
```
int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

### 3.494.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.494.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{asinh}(x)}}\,dx$$

input
```
integrate(x/(x**2+1)**(1/2)/asinh(x)**(1/2),x)
```

output
```
Integral(x/(sqrt(x**2 + 1)*sqrt(asinh(x))), x)
```

### 3.494.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{arsinh}\left(x\right)}}\,dx$$

input
```
integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

### 3.494.8 Giac [F]

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{arsinh}\left(x\right)}}\,dx$$

input
```
integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

### 3.494.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\operatorname{arcsinh}(x)}}\,dx = \int \frac{x}{\sqrt{\operatorname{asinh}\left(x\right)}\,\sqrt{x^2+1}}\,dx$$

input
```
int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)),x)
```

output
```
int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)), x)
```

# 3.495 $\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx$



## 3.495.1 Optimal result

Integrand size = 23, antiderivative size = 396

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{8a\sqrt{1+a^2x^2}}$$
$$+ \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{6}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}} + \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$
$$+ \frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{6}\sqrt{\text{arcsinh}(ax)}\right)}{64a\sqrt{1+a^2x^2}}$$

output
```
1/384*c^2*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(
1/2)/a/(a^2*x^2+1)^(1/2)+1/384*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2
)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/128*c^2*erf(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/
2)+15/128*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2
+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*
(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erfi(2*arcsinh(a*x)^(1/2)
)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+5/8*c^2*(a^2*c*x^2+c)^(
1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

## 3.495.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.50

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\Big(240\operatorname{arcsinh}(ax)+\sqrt{6}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\big(\frac{1}{2},-6\operatorname{arcsinh}(ax)\big)+18\sqrt{-\mathrm{a}}}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(240*ArcSinh[a*x] + Sqrt[6]*Sqrt[-ArcSinh[a*x]]*G
amma[1/2, -6*ArcSinh[a*x]] + 18*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[
a*x]] + 45*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - 45*Sq
rt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] - 18*Sqrt[ArcSinh[a*x]
]*Gamma[1/2, 4*ArcSinh[a*x]] - Sqrt[6]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*Arc
Sinh[a*x]]))/(384*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

## 3.495.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{c^2\sqrt{a^2cx^2+c}\int \frac{(a^2x^2+1)^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{a^2cx^2+c}\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^6}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{c^2\sqrt{a^2cx^2+c}\int \left(\frac{15\cosh(2\text{arcsinh}(ax))}{32\sqrt{\text{arcsinh}(ax)}}+\frac{3\cosh(4\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{\cosh(6\text{arcsinh}(ax))}{32\sqrt{\text{arcsinh}(ax)}}+\frac{5}{16\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{a^2cx^2+c}\left(\frac{3}{64}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{15}{64}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\text{erf}\left(\sqrt{6}\sqrt{\text{arcsinh}(ax)}\right)+\frac{3}{64}\sqrt{\pi}\right.}{a\sqrt{a^2x^2 +}}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*((5*Sqrt[ArcSinh[a*x]])/8 + (3*Sqrt[Pi]*Erf[2*Sqr
t[ArcSinh[a*x]]])/64 + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/64
+ (Sqrt[Pi/6]*Erf[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/64 + (3*Sqrt[Pi]*Erfi[2*Sqr
t[ArcSinh[a*x]]])/64 + (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/64
 + (Sqrt[Pi/6]*Erfi[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/64))/(a*Sqrt[1 + a^2*x^2]
)
```

### 3.495.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

---

3.495. $\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{arcsinh}(ax)}}dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.495.4 Maple [F]

$$\int \frac{(a^2 c\, x^2 + c)^{\frac{5}{2}}}{\sqrt{\operatorname{arcsinh}(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

## 3.495.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2 c x^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.495.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
```

output
```
Timed out
```

### 3.495.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

### 3.495.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

### 3.495.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c + a^2 c x^2)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(c\,a^2\,x^2 + c)^{5/2}}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2), x)
```

# 3.496 $\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx$



## 3.496.1 Optimal result

Integrand size = 23, antiderivative size = 264

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{3c\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}$$

output
```
1/8*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)
/a/(a^2*x^2+1)^(1/2)+1/8*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erf(2*arcsinh(a*x)^(1/2)
)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erfi(2*arcsinh(a
*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/4*c*(a^2*c*x
^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

### 3.496.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.53

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-4\text{arcsinh}(ax)\right)+4\sqrt{2}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\text{ar}\right.}{32a\sqrt{1}}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] +
4*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[a
*x]]*(24*Sqrt[ArcSinh[a*x]] - 4*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] - Gamma
[1/2, 4*ArcSinh[a*x]])))/(32*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

### 3.496.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx$$

$$\Big\downarrow \; 6206$$

$$\frac{c\sqrt{a^2cx^2+c}\int \frac{(a^2x^2+1)^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{c\sqrt{a^2cx^2+c}\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \; 3793$$

$$\frac{c\sqrt{a^2cx^2+c}\int \left(\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}+\frac{\cosh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}+\frac{3}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{a^2cx^2+c}\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}}{a\sqrt{a^2x^2+1}}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*((3*Sqrt[ArcSinh[a*x]])/4 + (Sqrt[Pi]*Erf[2*Sqrt[Ar
cSinh[a*x]]])/32 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4 + (Sqrt[
Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh
[a*x]]])/4))/(a*Sqrt[1 + a^2*x^2])
```

### 3.496.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

3.496. $\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}\,dx$

### 3.496.4 Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arcsinh}(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

### 3.496.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.496.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(asinh(a*x)), x)
```

### 3.496.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

### 3.496.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

### 3.496.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2), x)
```

# 3.497 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$



## 3.497.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}}$$

output
```
1/8*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a
/(a^2*x^2+1)^(1/2)+1/8*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1
/2)/a/(a^2*x^2+1)^(1/2)
```

## 3.497.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(8\operatorname{arcsinh}(ax)+\sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)-\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},2\operatorname{arcsinh}\right.\right.}{8a\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(8*ArcSinh[a*x] + Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma
[1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a
*x]]))/(8*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

## 3.497.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\text{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{\sqrt{a^2cx^2+c}\int \frac{a^2x^2+1}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2cx^2+c}\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\sqrt{a^2cx^2+c}\int \left(\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}+\frac{1}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2cx^2+c}\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\sqrt{\text{arcsinh}(ax)}\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(Sqrt[ArcSinh[a*x]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[Ar
cSinh[a*x]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4))/(a*Sqr
t[1 + a^2*x^2])
```

### 3.497.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

## 3.497.4 Maple [F]

$$\int \frac{\sqrt{a^{2} c \,x^{2}+c}}{\sqrt{\operatorname{arcsinh}\left(a x \right)}}d x$$

input
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

### 3.497.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.497.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(asinh(a*x)), x)
```

### 3.497.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

### 3.497.8 Giac [F]

$$\int \frac{\sqrt{c + a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2 + c}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

### 3.497.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c + a^2cx^2}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2 + c}}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2), x)
```

# 3.498 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}\,dx$



## 3.498.1 Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}\,dx=\frac{2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{c+a^2cx^2}}$$

output
```
2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

## 3.498.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}\,dx=\frac{2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c*(1 + a^2*x^2)])
```

## 3.498.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

### 3.498.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.498.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{\text{arcsinh}(ax)}\,\sqrt{a^2x^2+1}}{a\sqrt{c(a^2x^2+1)}}$ | 36 |

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

### 3.498.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.498.6 Sympy [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x))), x)
```

### 3.498.7 Maxima [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

### 3.498.8 Giac [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

### 3.498.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

# 3.499 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx$



## 3.499.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \text{Int}\left(\frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}},x\right)$$

output
```
Unintegrable(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

## 3.499.2 Mathematica [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

## 3.499.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\text{arcsinh}(ax)}\,(a^2cx^2+c)^{3/2}}\,dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{\sqrt{\text{arcsinh}(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
```

output
```
$Aborted
```

### 3.499.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.499.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{3}{2}}\sqrt{\text{arcsinh}\,(ax)}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

### 3.499.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.499.6 Sympy [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{3}{2}}\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x))), x)
```

### 3.499.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

### 3.499.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

### 3.499.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

# 3.500 $\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}}\,dx$



## 3.500.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \text{Int}\left(\frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}},x\right)$$

output
```
Unintegrable(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

## 3.500.2 Mathematica [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}}\,dx = \int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{arcsinh}(ax)}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

## 3.500.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\text{arcsinh}(ax)}\,(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{\sqrt{\text{arcsinh}(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
```

output
```
$Aborted
```

### 3.500.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.500.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\text{arcsinh}\,(ax)}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

### 3.500.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.500.6 Sympy [N/A]

Not integrable

Time = 58.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{5}{2}}\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*sqrt(asinh(a*x))), x)
```

### 3.500.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

### 3.500.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

### 3.500.9 Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

# 3.501 $\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$



## 3.501.1 Optimal result

Integrand size = 23, antiderivative size = 391

$$\begin{aligned}\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = &-\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}}\\ &-\frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}}\\ &-\frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}\\ &-\frac{c^2\sqrt{\frac{3\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}\\ &+\frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a\sqrt{1+a^2x^2}}\\ &+\frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}\\ &+\frac{c^2\sqrt{\frac{3\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a\sqrt{1+a^2x^2}}\end{aligned}$$

output
```
-15/32*c^2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^
(1/2)/a/(a^2*x^2+1)^(1/2)+15/32*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/
2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*c^2*erf(2*arcsinh(
a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/8*c^2*erfi(
2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/3
2*c^2*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)
/a/(a^2*x^2+1)^(1/2)+1/32*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^
(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)^(5/2)*(a^2*x
^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

### 3.501.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{c^2e^{-6\operatorname{arcsinh}(ax)}\sqrt{c+a^2cx^2}\left(-1-6e^{2\operatorname{arcsinh}(ax)}+e^{4\operatorname{arcsinh}(ax)}-52e^{6\operatorname{arcsinh}(ax)}+e^{8\operatorname{arcsinh}\cdots}\right.}{\cdots}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(-1 - 6*E^(2*ArcSinh[a*x]) + E^(4*ArcSinh[a*x]) -
 52*E^(6*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]) - 6*E^(10*ArcSinh[a*x]) - E^(1
2*ArcSinh[a*x]) - 64*a^2*E^(6*ArcSinh[a*x])*x^2 - 16*E^(6*ArcSinh[a*x])*Sq
rt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 16*E^(6*ArcS
inh[a*x])*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] +
 Sqrt[6]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -6*ArcSinh[a*x]
] + 12*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]]
- Sqrt[2]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x
]] - Sqrt[2]*E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*
x]] + 12*E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]
+ Sqrt[6]*E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[a*x]]
))/(32*a*E^(6*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

### 3.501.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.59, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

$$\Big\downarrow \; 6205$$

$$\frac{12ac^2\sqrt{a^2cx^2+c}\int \frac{x\left(a^2x^2+1\right)^2}{\sqrt{\operatorname{arcsinh}(ax)}}dx}{\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \; 6234$$

$$\frac{12c^2\sqrt{a^2cx^2+c}\int \frac{ax\left(a^2x^2+1\right)^{5/2}}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \; 5971$$

$$\frac{12c^2\sqrt{a^2cx^2+c}\int \left(\frac{5\sinh(2\operatorname{arcsinh}(ax))}{32\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\sinh(4\operatorname{arcsinh}(ax))}{8\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\sinh(6\operatorname{arcsinh}(ax))}{32\sqrt{\operatorname{arcsinh}(ax)}}\right)d\operatorname{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{12c^2\sqrt{a^2cx^2+c}\left(-\frac{1}{32}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{5}{64}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{64}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\sqrt{6}\sqrt{\operatorname{arcsinh}(ax)}\right)+\right.}{a\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{5/2}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

3.501. $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

output
```
(-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(5/2))/(a*Sqrt[ArcSinh[a*x]]) + (12*
c^2*Sqrt[c + a^2*c*x^2]*(-1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]]) - (5*S
qrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/64 - (Sqrt[Pi/6]*Erf[Sqrt[6]*Sq
rt[ArcSinh[a*x]]])/64 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32 + (5*Sqrt
[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/64 + (Sqrt[Pi/6]*Erfi[Sqrt[6]*Sqr
t[ArcSinh[a*x]]])/64))/(a*Sqrt[1 + a^2*x^2])
```

### 3.501.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.501. $\int \frac{(c+a^2cx^2)^{5/2}}{\text{arcsinh}(ax)^{3/2}}\,dx$

### 3.501.4 Maple [F]

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

### 3.501.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.501.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
```

output
```
Timed out
```

### 3.501.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

### 3.501.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

### 3.501.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\text{asinh}\,(a\,x)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2), x)
```

# 3.502 $\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{3/2}}\,dx$



## 3.502.1 Optimal result

Integrand size = 23, antiderivative size = 256

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

output
```
-1/2*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2
)/a/(a^2*x^2+1)^(1/2)+1/2*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/4*c*erf(2*arcsinh(a*x)^(1/2)
)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*c*erfi(2*arcsinh(a*
x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)
^(3/2)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

### 3.502.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{ce^{-4\text{arcsinh}(ax)}\sqrt{c+a^2cx^2}\left(1+14e^{4\text{arcsinh}(ax)}+e^{8\text{arcsinh}(ax)}+16a^2e^{4\text{arcsinh}(ax)}x^2+4e^{4\text{arcsinh}(ax)}\sqrt{2\pi}\sqrt{\text{arcsinh}}\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

output
```
-1/8*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]
) + 16*a^2*E^(4*ArcSinh[a*x])*x^2 + 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi]*Sqrt[A
rcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 4*E^(4*ArcSinh[a*x])*Sqrt[2
*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 2*E^(4*ArcSinh[
a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - 2*E^(4*ArcSinh[a*x
])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]))/(a*E^(4*ArcSinh[a*x])*S
qrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

### 3.502.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\text{arcsinh}(ax)^{3/2}}\,dx$$

$$\Big\downarrow \ 6205$$

$$\frac{8ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx}{\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \ 6234$$

$$\frac{8c\sqrt{a^2cx^2+c}\int \frac{ax(a^2x^2+1)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{8c\sqrt{a^2cx^2+c}\int\left(\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}+\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8c\sqrt{a^2cx^2+c}\left(-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{e}\right.}{a\sqrt{a^2x^2+1}}$$
$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]) + (8*c
*Sqrt[c + a^2*c*x^2]*(-1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]]) - (Sqrt[P
i/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*
x]]])/32 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8))/(a*Sqrt[1 + a
^2*x^2])
```

### 3.502.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

3.502. $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\text{arcsinh}(ax)^{3/2}}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.502.4 Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

### 3.502.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.502.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(3/2), x)
```

### 3.502.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

### 3.502.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

### 3.502.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{(c\,a^2\,x^2+c)^{3/2}}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2), x)
```

# 3.503 $\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$



## 3.503.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a\sqrt{1+a^2x^2}}$$

output
```
-1/2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/
a/(a^2*x^2+1)^(1/2)+1/2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*
(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^
(1/2)/a/arcsinh(a*x)^(1/2)
```

## 3.503.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\left(4+4a^2x^2+\sqrt{2\pi}\sqrt{\operatorname{arcsinh}(ax)}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-\sqrt{2\pi}\sqrt{\operatorname{arcsinh}(ax)}\operatorname{erfi}\left(\sqrt{2}\sqrt{\text{arcsin}\dots}\right.\right.}{2a\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

output
```
-1/2*(Sqrt[c + a^2*c*x^2]*(4 + 4*a^2*x^2 + Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*E
rf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]]))/(a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

### 3.503.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6205, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arcsinh}(ax)^{3/2}}dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{4a\sqrt{a^2cx^2+c}\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}dx}{\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{6195}$$

$$\frac{4\sqrt{a^2cx^2+c}\int \frac{ax\sqrt{a^2x^2+1}}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{4\sqrt{a^2cx^2+c}\int \frac{\sinh(2\operatorname{arcsinh}(ax))}{2\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{a^2cx^2+c}\int \frac{\sinh(2\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\sqrt{a^2cx^2+c}\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\sqrt{a^2cx^2+c}\int \frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 3789}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 2611}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\sqrt{a^2cx^2+c}\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\sqrt{a^2cx^2+c}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

3.503. $\int \frac{\sqrt{c+a^2cx^2}}{\text{arcsinh}(ax)^{3/2}}\,dx$

output
```
(-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*Sqrt[ArcSinh[a*x]]) - ((2*I)
*Sqrt[c + a^2*c*x^2]*((-1/2*I)*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]]
+ (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]]))/(a*Sqrt[1 + a^2*x^2]
)
```

### 3.503.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

## 3.503. $\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

## 3.503.4 Maple [F]

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

## 3.503.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.503.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(3/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(3/2), x)
```

### 3.503.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

### 3.503.8 Giac [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

### 3.503.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{\operatorname{asinh}\left(a\,x\right)^{3/2}} \,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2), x)
```

# 3.504 $\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx$



## 3.504.1 Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)}}$$

output
```
-2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

## 3.504.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c\,(1+a^2x^2)}\sqrt{\text{arcsinh}(ax)}}$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c*(1 + a^2*x^2)]*Sqrt[ArcSinh[a*x]])
```

### 3.504.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{6198}$$
$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

#### 3.504.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

### 3.504.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}\,a\sqrt{c(a^2x^2+1)}}$ | 36 |

input
```
int(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

### 3.504.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{(a^3cx^2+ac)\sqrt{\log\left(ax+\sqrt{a^2x^2+1}\right)}}$$

input
```
integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*sqrt(log(a*x +
 sqrt(a^2*x^2 + 1))))
```

### 3.504.6 Sympy [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2)), x)
```

### 3.504.7 Maxima [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

### 3.504.8 Giac [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

### 3.504.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

# 3.505 $\int \frac{1}{(c+a^2cx^2)^{3/2}\text{arcsinh}(ax)^{3/2}}\,dx$



## 3.505.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a\,(c+a^2cx^2)^{3/2}\sqrt{\text{arcsinh}(ax)}} - \frac{4a\sqrt{1+a^2x^2}\text{Int}\left(\frac{x}{(1+a^2x^2)^2\sqrt{\text{arcsinh}(ax)}},x\right)}{c\sqrt{c+a^2cx^2}}$$

output
```
-2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2)-4*a*(a^2*x^2
+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+
c)^(1/2)
```

## 3.505.2 Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{3/2}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)), x]
```

### 3.505.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{3/2} (a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \text{6205}$$
$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}\,(a^2cx^2+c)^{3/2}}$$
$$\Big\downarrow \text{6239}$$
$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\sqrt{\text{arcsinh}(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

output
```
$Aborted
```

#### 3.505.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.505.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

### 3.505.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.505.6 Sympy [N/A]

Not integrable

Time = 25.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**(3/2)), x)
```

### 3.505.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} \operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

### 3.505.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} \operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

### 3.505.9 Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

# 3.506 $\int \frac{1}{(c+a^2cx^2)^{5/2}\text{arcsinh}(ax)^{3/2}}\,dx$



## 3.506.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a\,(c+a^2cx^2)^{5/2}\,\sqrt{\text{arcsinh}(ax)}} - \frac{8a\sqrt{1+a^2x^2}\text{Int}\left(\frac{x}{(1+a^2x^2)^3\sqrt{\text{arcsinh}(ax)}},x\right)}{c^2\sqrt{c+a^2cx^2}}$$

output
```
-2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2)-8*a*(a^2*x^2
+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^
2+c)^(1/2)
```

## 3.506.2 Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)), x]
```

### 3.506.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3\sqrt{\text{arcsinh}(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3\sqrt{\text{arcsinh}(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}\left(a^2cx^2+c\right)^{5/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

output
```
$Aborted
```

#### 3.506.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.506.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

### 3.506.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.506.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2} \operatorname{arcsinh}(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
```

output
```
Timed out
```

### 3.506.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

### 3.506.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

### 3.506.9 Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

# 3.507 $\int \frac{(c+a^2cx^2)^{3/2}}{\textbf{arcsinh}(ax)^{5/2}}\,dx$



## 3.507.1 Optimal result

Integrand size = 23, antiderivative size = 296

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}(c+a^2cx^2)^{3/2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{16cx(1+a^2x^2)\sqrt{c+a^2cx^2}}{3\sqrt{\operatorname{arcsinh}(ax)}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

output
```
-2/3*(a^2*c*x^2+c)^(3/2)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)+2/3*c*erf(
2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+2/3
*c*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(
1/2)+2/3*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^
(1/2)/a/(a^2*x^2+1)^(1/2)+2/3*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*P
i^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-16/3*c*x*(a^2*x^2+1)*(a^2*
c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

### 3.507.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.89

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{ce^{-4\operatorname{arcsinh}(ax)}\sqrt{c+a^2cx^2}\left(1+14e^{4\operatorname{arcsinh}(ax)}+e^{8\operatorname{arcsinh}(ax)}+16a^2e^{4\operatorname{arcsinh}(ax)}x^2-8\operatorname{arcsinh}(ax)+8e^{8\operatorname{arcsinh}(a\right.}}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

output
```
-1/24*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x
]) + 16*a^2*E^(4*ArcSinh[a*x])*x^2 - 8*ArcSinh[a*x] + 8*E^(8*ArcSinh[a*x])
*ArcSinh[a*x] + 64*a*E^(4*ArcSinh[a*x])*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] +
 16*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] +
 16*Sqrt[2]*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh
[a*x]] + 16*Sqrt[2]*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*Arc
Sinh[a*x]] + 16*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh
[a*x]]))/(a*E^(4*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))
```

### 3.507.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6205, 6229, 6206, 3042, 3793, 2009, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{8ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\operatorname{arcsinh}(ax)^{3/2}}dx}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{3a\operatorname{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6229}$$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{2\int \frac{\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx}{a}+8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$\big\downarrow$ 6206

$$\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\int \frac{a^2x^2+1}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}$$

$\big\downarrow$ 3793

$$\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\int \left(\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}+\frac{1}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^2}-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$\big\downarrow$ 2009

$$\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\right.}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}(a^2cx^2+c)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$\big\downarrow$ 6234

---

3.507. $\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{5/2}}dx$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{8\int \frac{a^2x^2\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\sqrt{\text{arcsinh}(\right.}{a^2}}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{8\int \left(\frac{\cosh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}-\frac{1}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^2}+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arc}\right.}{a^2}}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{8\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\text{arcsinh}(ax)}\right)}{a^2}+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(a\right.}{}}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(3*a*ArcSinh[a*x]^(3/2)) + (8
*a*c*Sqrt[c + a^2*c*x^2]*((-2*x*(1 + a^2*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]
) + (8*(-1/4*Sqrt[ArcSinh[a*x]] + (Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/32
+ (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32))/a^2 + (2*(Sqrt[ArcSinh[a*x]]
+ (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/4))/a^2))/(3*Sqrt[1 + a^2*x^2])
```

## 3.507. $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\text{arcsinh}(ax)^{5/2}}\,dx$

### 3.507.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

## 3.507. $\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{5/2}}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.507.4 Maple [F]

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

### 3.507.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.507.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(5/2), x)
```

### 3.507.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

### 3.507.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

### 3.507.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\operatorname{asinh}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2), x)
```

# 3.508 $\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx$



## 3.508.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\operatorname{arcsinh}(ax)}} + \frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

output
```
2/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a
/(a^2*x^2+1)^(1/2)+2/3*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2/3*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)
^(1/2)/a/arcsinh(a*x)^(3/2)-8/3*x*(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

## 3.508.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{c+a^2cx^2}\left(1+a^2x^2+4ax\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)+\sqrt{2}(-\operatorname{arcsinh}(ax))^{3/2}\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)+\sqrt{2}\operatorname{arc}\right.}{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

output
```
(-2*Sqrt[c + a^2*c*x^2]*(1 + a^2*x^2 + 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x
] + Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*Ar
cSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]))/(3*a*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^(3/2))
```

### 3.508.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6205, 6193, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\text{arcsinh}(ax)^{5/2}}dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{4a\sqrt{a^2cx^2+c}\int \frac{x}{\text{arcsinh}(ax)^{3/2}}dx}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6193}$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\int \frac{\cosh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{4a\sqrt{a^2cx^2+c}\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int \frac{\sin\left(2i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}\right)}{3\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{4a\sqrt{a^2cx^2+c}\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}\right)}{3\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \; 26$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \; 2611$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \; 2633$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \; 2634$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(3*a*ArcSinh[a*x]^(3/2)) + (4*a
*Sqrt[c + a^2*c*x^2]*((-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2
*((Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/2))/a^2))/(3*Sqrt[1 + a^2*x^2])
```

## 3.508. $\int \frac{\sqrt{c+a^2cx^2}}{\text{arcsinh}(ax)^{5/2}}\,dx$

### 3.508.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

3.508. $\int \frac{\sqrt{c+a^2cx^2}}{\text{arcsinh}(ax)^{5/2}}\,dx$

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

### 3.508.4 Maple [F]

$$\int \frac{\sqrt{a^2 c\,x^2+c}}{\operatorname{arcsinh}\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

### 3.508.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.508.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\operatorname{asinh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(5/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(5/2), x)
```

### 3.508.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

### 3.508.8 Giac [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

### 3.508.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\operatorname{asinh}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2), x)
```

# 3.509 $\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx$



## 3.509.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)
```

## 3.509.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c(1+a^2x^2)}\text{arcsinh}(ax)^{3/2}}$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c*(1 + a^2*x^2)]*ArcSinh[a*x]^(3/2))
```

## 3.509.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{6198}$$

$$-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))
```

### 3.509.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.509.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{a^2x^2+1}}{3\,\text{arcsinh}(ax)^{\frac{3}{2}}\,a\sqrt{c(a^2x^2+1)}}$ | 36 |

input
```
int(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

### 3.509.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{3\left(a^3cx^2+ac\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*log(a*x + sq
rt(a^2*x^2 + 1))^(3/2))
```

### 3.509.6 Sympy [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\,\text{asinh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

input
```
integrate(1/asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(5/2)), x)
```

### 3.509.7 Maxima [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

### 3.509.8 Giac [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

### 3.509.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

# 3.510 $\int \frac{1}{(c+a^2cx^2)^{3/2}\text{arcsinh}(ax)^{5/2}}\,dx$



## 3.510.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{3/2}} - \frac{4a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)^2\text{arcsinh}(ax)^{3/2}},x\right)}{3c\sqrt{c+a^2cx^2}}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2)-4/3*a*(a^2
*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(3/2),x)/c/(a^2*c*
x^2+c)^(1/2)
```

## 3.510.2 Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)), x]
```

### 3.510.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6205}$$

$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}dx}{3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}dx}{3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
$Aborted
```

#### 3.510.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.510.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x\right)^{\frac{5}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

### 3.510.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.510.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

### 3.510.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

### 3.510.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

### 3.510.9 Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

# 3.511 $\int \frac{1}{(c+a^2cx^2)^{5/2}\text{arcsinh}(ax)^{5/2}}\,dx$



## 3.511.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\,(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{3/2}} - \frac{8a\sqrt{1+a^2x^2}\,\text{Int}\left(\frac{x}{(1+a^2x^2)^3\text{arcsinh}(ax)^{3/2}},x\right)}{3c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2)-8/3*a*(a^2
*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(3/2),x)/c^2/(a^2*
c*x^2+c)^(1/2)
```

## 3.511.2 Mathematica [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)), x]
```

### 3.511.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6205}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3\text{arcsinh}(ax)^{3/2}}dx}{3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3\text{arcsinh}(ax)^{3/2}}dx}{3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
$Aborted
```

#### 3.511.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

### 3.511.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a^2 c x^2 + c)^{\frac{5}{2}} \operatorname{arcsinh}(ax)^{\frac{5}{2}}} dx$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

### 3.511.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c + a^2 cx^2)^{5/2} \operatorname{arcsinh}(ax)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.511.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c + a^2 cx^2)^{5/2} \operatorname{arcsinh}(ax)^{5/2}} dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

### 3.511.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

### 3.511.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

### 3.511.9 Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{\operatorname{asinh}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

# 3.512 $\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$



## 3.512.1 Optimal result

Integrand size = 28, antiderivative size = 235

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=-\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}}$$
$$+\frac{2^{-2(3+n)}e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$
$$-\frac{2^{-2(3+n)}e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}$$

output
```
-1/8*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+1)^
(1/2)+(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d
)^(1/2)/(2^(6+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)-exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(
c^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
(1/2)
```

### 3.512.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$$
$$=\frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{8(a+b\text{arcsinh}(cx))}{b(1+n)}+4^{-n}e^{-\frac{4a}{b}}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right.\right.}{64c^3\sqrt{d\left(1+c^2x^2\right)}}$$

input
```
Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b*
(1 + n)) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/
b] - E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcS
inh[c*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c
^3*Sqrt[d*(1 + c^2*x^2)])
```

### 3.512.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\,dx$$
$$\Big\downarrow \text{6234}$$
$$\frac{\sqrt{c^2dx^2+d}\int(a+b\text{arcsinh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^3\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{5971}$$
$$\frac{\sqrt{c^2dx^2+d}\int\left(\frac{1}{8}(a+b\text{arcsinh}(cx))^n\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}(a+b\text{arcsinh}(cx))^n\right)d(a+b\text{arcsinh}(cx))}{bc^3\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcsinh}(cx))^{n+1}}{8(n+1)}+b2^{-2(n+3)}e^{-\frac{4a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arcs}}{b}\right.\right.}{bc^3\sqrt{c^2x^2+1}}$$

input
```
Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-1/8*(a + b*ArcSinh[c*x])^(1 + n)/(1 + n) + (b*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n)
)*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (b*E^((4*a)/b)*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*((a +
b*ArcSinh[c*x])/b)^n)))/(b*c^3*Sqrt[1 + c^2*x^2])
```

### 3.512.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.512.4 Maple [F]

$$\int x^2(a+b\operatorname{arcsinh}(cx))^n\sqrt{c^2d\,x^2+d}dx$$

input
```
int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

### 3.512.5 Fricas [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

### 3.512.6 Sympy [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int x^2\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^n\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

### 3.512.7 Maxima [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

### 3.512.8 Giac [F]

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

### 3.512.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

# 3.513 $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$



## 3.513.1 Optimal result

Integrand size = 26, antiderivative size = 355

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{1+c^2x^2}} + \frac{e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{1+c^2x^2}} + \frac{e^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8c^2\sqrt{1+c^2x^2}} + \frac{3^{-1-n}e^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8c^2\sqrt{1+c^2x^2}}$$

output
```
1/8*3^(-1-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*
d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2
)+1/8*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*exp(a
/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/
2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*3^(-1-n)*exp(3*a/b
)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/
2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

### 3.513.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.65

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$$
$$= \frac{de^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(3e^{\frac{4a}{b}}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)^{-n}\Gamma\left(1+n,\frac{a}{b}+\text{arcsinh}(cx)\right)+\left(-\frac{a+b\text{arcsin}}{b}\right.\right.}{}$$

input
```
Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((3*E^((4*a)/b)*Gamma[1 + n, a
/b + ArcSinh[c*x]])/(a/b + ArcSinh[c*x])^n + (Gamma[1 + n, (-3*(a + b*ArcS
inh[c*x]))/b]/3^n + 3*E^((2*a)/b)*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)]
+ (E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*Ar
cSinh[c*x]))/b])/(3^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n))/(-((a + b*ArcSin
h[c*x])/b))^n))/(24*c^2*E^((3*a)/b)*Sqrt[d*(1 + c^2*x^2)])
```

### 3.513.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n\,dx$$
$$\Big\downarrow \text{6234}$$
$$\frac{\sqrt{c^2dx^2+d}\int -(a+b\text{arcsinh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^2\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{\sqrt{c^2dx^2+d}\int\left(\frac{1}{4}\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{1}{4}\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)(a+b\text{arcsinh}(\right.}{bc^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{1}{8}b3^{-n-1}e^{-\frac{3a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{8}be^{-\frac{a}{b}}(a+b\right.}{}$$

input
```
Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(Sqrt[d + c^2*d*x^2]*((3^(-1 - n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-
3*(a + b*ArcSinh[c*x]))/b])/(8*E^((3*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n)
+ (b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(8*E^
(a/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (b*E^(a/b)*(a + b*ArcSinh[c*x])^n*G
amma[1 + n, (a + b*ArcSinh[c*x])/b])/(8*((a + b*ArcSinh[c*x])/b)^n) + (3^(
-1 - n)*b*E^((3*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSin
h[c*x]))/b])/(8*((a + b*ArcSinh[c*x])/b)^n)))/(b*c^2*Sqrt[1 + c^2*x^2])
```

### 3.513.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.513. $\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.513.4 Maple [F]

$$\int x(a+b\,\text{arcsinh}\,(cx))^n\,\sqrt{c^2d\,x^2+d}dx$$

input
```
int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

### 3.513.5 Fricas [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^nx\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

### 3.513.6 Sympy [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int x\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^n\,\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

### 3.513.7 Maxima [F]

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^nx\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

### 3.513.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.513.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx=\int x\,(a+b\,\text{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

# 3.514 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx$



## 3.514.1 Optimal result

Integrand size = 25, antiderivative size = 235

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}} - \frac{2^{-3-n}e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

output
```
1/2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(1/
2)+2^(-3-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2
^(-3-n)*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)*
(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

### 3.514.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

$$\int \sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^n\,dx$$
$$= \frac{d\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{4a+4b\operatorname{arcsinh}(cx)}{b+bn}+2^{-n}e^{-\frac{2a}{b}}\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\operatorname{arcsinh}}{b}\right.\right.}{8c\sqrt{d\left(1+c^2x^2\right)}}$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((4*a + 4*b*ArcSinh[c*x])/(b +
 b*n) + Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a +
 b*ArcSinh[c*x])/b))^n) - (E^((2*a)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]
))/b])/(2^n*(a/b + ArcSinh[c*x])^n)))/(8*c*Sqrt[d*(1 + c^2*x^2)])
```

### 3.514.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^n\,dx$$
$$\Big\downarrow \text{ 6206}$$
$$\frac{\sqrt{c^2dx^2+d}\int (a+b\operatorname{arcsinh}(cx))^n\cosh^2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)d(a+b\operatorname{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{c^2dx^2+d}\int (a+b\operatorname{arcsinh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\operatorname{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^2d(a+b\operatorname{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{ 3793}$$

$$\frac{\sqrt{c^2dx^2+d}\int \left(\frac{1}{2}\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{1}{2}(a+b\text{arcsinh}(cx))^n\right)d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{(a+b\text{arcsinh}(cx))^{n+1}}{2(n+1)}+b2^{-n-3}e^{-\frac{2a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcsinh}(\right.\right.}{b}}{bc\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])^(1 + n)/(2*(1 + n)) + (2^(-3 -
n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^
((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*b*E^((2*a)/b)*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/((a + b*ArcSin
h[c*x])/b)^n))/(b*c*Sqrt[1 + c^2*x^2])
```

### 3.514.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

### 3.514.4 Maple [F]

$$\int (a+b\,\mathrm{arcsinh}\,(cx))^n\,\sqrt{c^2d\,x^2+d}dx$$

input
```
int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

### 3.514.5 Fricas [F]

$$\int \sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n\,dx=\int \sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

### 3.514.6 Sympy [F]

$$\int \sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))^n\,dx=\int \sqrt{d\,(c^2x^2+1)}(a+b\,\mathrm{asinh}\,(cx))^n\,dx$$

input
```
integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

### 3.514.7 Maxima [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

### 3.514.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.514.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\,dx = \int (a+b\,\text{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

# 3.515 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$



## 3.515.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \frac{de^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}} + \frac{de^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{d+c^2dx^2}} + d\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}},x\right)$$

output
```
1/2*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1
/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/2*d*exp(a/b
)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(
((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d*Unintegrable((a+b*arcsinh(
c*x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

### 3.515.2 Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

output
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x, x]
```

### 3.515.3 Rubi [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n}{x}\,dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{c^2dx(a+b\text{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{d(a+b\text{arcsinh}(cx))^n}{x\sqrt{c^2dx^2+d}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{c^2dx^2+d}}dx+\frac{de^{-\frac{a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{c^2dx^2+d}}+\frac{de^{a/b}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2\sqrt{c^2dx^2+d}}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

output
```
$Aborted
```

### 3.515.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

## 3.515.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n \sqrt{c^2 d\, x^2+d}}{x}dx$$

input
```
int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
```

output
```
int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
```

## 3.515.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^n}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

### 3.515.6 Sympy [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^n}{x}\,dx$$

input
```
integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x, x)
```

### 3.515.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

### 3.515.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.515.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,\sqrt{d\,c^2\,x^2+d}}{x} \,d x$$

input
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x, x)
```

# 3.516 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$



## 3.516.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\frac{cd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{b(1+n)\sqrt{d+c^2dx^2}}+d\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}},x\right)$$

output
```
c*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)^(1/2)
+d*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

## 3.516.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

output
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

### 3.516.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^n}{x^2}dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{c^2d(a+b\text{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{d(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{c^2dx^2+d}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{c^2dx^2+d}}dx+\frac{cd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{n+1}}{b(n+1)\sqrt{c^2dx^2+d}}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

#### 3.516.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

### 3.516.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^n\,\sqrt{c^2d\,x^2+d}}{x^2}dx$$

input
```
int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

output
```
int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

### 3.516.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

### 3.516.6 Sympy [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))^n}{x^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x**2, x)
```

### 3.516.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

### 3.516.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.516.9 Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

# 3.517 $\int x^2(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^n\,dx$



## 3.517.1 Optimal result

Integrand size = 28, antiderivative size = 616

$$\begin{aligned}
\int x^2\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx &= -\frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}} \\
&+\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+\frac{2^{-7-2n}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&-\frac{2^{-7-n}de^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+\frac{2^{-7-n}de^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&-\frac{2^{-7-2n}de^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&-\frac{2^{-7-n}3^{-1-n}de^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-1/16*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+
1)^(1/2)+2^(-7-n)*3^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsin
h(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/
(c^2*x^2+1)^(1/2)+2^(-7-2*n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n
)/(c^2*x^2+1)^(1/2)-2^(-7-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n
)/(c^2*x^2+1)^(1/2)+2^(-7-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2
*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/
(c^2*x^2+1)^(1/2)-2^(-7-2*n)*d*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4
*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/
(c^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d*exp(6*a/b)*(a+b*arcsinh(c*x))^n*GAMM
A(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))
/b)^n)/(c^2*x^2+1)^(1/2)
```

### 3.517.2 Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.70

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n \, dx =$$

$$-\frac{2^{-7-2n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-n}\left(-2^nb(1+n)\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right.}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
-((2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-
(2^n*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]
))/b]) - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 +
n, (-4*(a + b*ArcSinh[c*x]))/b] + 2^n*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(a/b
 + ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b] - 2^n*3^(1 +
n)*b*E^((8*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcSinh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*ArcSinh
[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^n*E^((6*a)/b)*(2
^(3 + n)*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^2))^n
+ b*E^((6*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a +
 b*ArcSinh[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-(
(a + b*ArcSinh[c*x])^2/b^2))^n))
```

### 3.517.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^n \, dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n \cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right) \sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right) d(a+b\text{arcsinh}(cx))}{bc^3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \left(\frac{1}{32}\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{1}{16}\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)(a+\text{barc}\right.}{bc^3\sqrt{c^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$d\sqrt{c^2dx^2+d}\left(-\frac{(a+b\text{arcsinh}(cx))^{n+1}}{16(n+1)}+b2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{6(a+}{}\right.\right.$$

input
```
Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(-1/16*(a + b*ArcSinh[c*x])^(1 + n)/(1 + n) + (2^(-
7 - n)*3^(-1 - n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh
[c*x]))/b])/(E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - 2*n)*b*
(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(E^((4*a
)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-7 - n)*b*(a + b*ArcSinh[c*x])^n
*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*(-((a + b*ArcSinh
[c*x])/b))^n) + (2^(-7 - n)*b*E^((2*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 +
 n, (2*(a + b*ArcSinh[c*x]))/b])/((a + b*ArcSinh[c*x])/b)^n - (2^(-7 - 2*n
)*b*E^((4*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]
))/b])/((a + b*ArcSinh[c*x])/b)^n - (2^(-7 - n)*3^(-1 - n)*b*E^((6*a)/b)*(
a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b])/((a + b*Ar
cSinh[c*x])/b)^n))/(b*c^3*Sqrt[1 + c^2*x^2])
```

### 3.517.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.517.4 Maple [F]

$$\int x^2 \left(c^2 d\,x^2+d\right)^{\frac{3}{2}} \left(a+b \operatorname{arcsinh}\left(c x\right)\right)^n dx$$

input
```
int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

output
```
int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

## 3.517.5 Fricas [F]

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))^n\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}} (b\operatorname{arsinh}\left(cx\right)+a)^n x^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="frica
s")
```

output
```
integral((c^2*d*x^4 + d*x^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x
)
```

### 3.517.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.517.7 Maxima [F]

$$\int x^2 \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n\,dx = \int \left(c^2 d x^2+d\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^n x^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

### 3.517.8 Giac [F]

$$\int x^2 \left(d+c^2 d x^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n\,dx = \int \left(c^2 d x^2+d\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^n x^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

### 3.517.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d + c^2 dx^2)^{3/2} (a + b\text{arcsinh}(cx))^n \, dx = \int x^2 (a + b\,\text{asinh}(c\,x))^n \left(d\,c^2\,x^2 + d\right)^{3/2} dx$$

input
```
int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

# 3.518 $\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx$



## 3.518.1 Optimal result

Integrand size = 26, antiderivative size = 542

$$\int x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arcsi}}{b}\right.}{32c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{-n}de^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$
$$+\frac{de^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{1+c^2x^2}}$$
$$+\frac{de^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{-n}de^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-1-n}de^{\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32c^2\sqrt{1+c^2x^2}}$$

output
```
1/32*5^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(c
^2*d*x^2+d)^(1/2)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(
1/2)+1/32*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/(3^n)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)+1/16*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1
/16*d*exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*d*exp(
3*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d
)^(1/2)/(3^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*5^(-1-
n)*d*exp(5*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^
2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

### 3.518.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.72

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx = \frac{15^{-1-n}d^2e^{-\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-2n}\left(2\;15^{1+n}e^{\frac{6a}{b}}\left(-\frac{a}{}\right.\right.}{}$$

input
```
Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(15^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2*15^(1 + n)*E^
((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*
Gamma[1 + n, a/b + ArcSinh[c*x]] + 3*(a/b + ArcSinh[c*x])^n*(3^n*(-((a + b
*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b] + 5^(1
+ n)*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b
*ArcSinh[c*x]))/b] + 2*3^n*5^(1 + n)*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2
/b^2))^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(
-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b]
+ 3^n*E^((10*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (5*(a +
b*ArcSinh[c*x]))/b])))/(32*c^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*A
rcSinh[c*x])^2/b^2))^(2*n))
```

### 3.518.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{d\sqrt{c^2dx^2+d}\int -(a+b\text{arcsinh}(cx))^n\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{d\sqrt{c^2dx^2+d}\int\left(\frac{1}{16}\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{3}{16}\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)(a+ba\ldots}{bc^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{c^2dx^2+d}\left(\frac{1}{32}b5^{-n-1}e^{-\frac{5a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{32}b3^{-n}e^{-\ldots}\right.}{}$$

input
```
Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((5^(-1 - n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
(-5*(a + b*ArcSinh[c*x]))/b])/(32*E^((5*a)/b)*(-((a + b*ArcSinh[c*x])/b))^
n) + (b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/
(32*3^n*E^((3*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (b*(a + b*ArcSinh[c*x
])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*E^(a/b)*(-((a + b*ArcSin
h[c*x])/b))^n) + (b*E^(a/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*Arc
Sinh[c*x])/b])/(16*((a + b*ArcSinh[c*x])/b)^n) + (b*E^((3*a)/b)*(a + b*Arc
Sinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*((a + b*Arc
Sinh[c*x])/b)^n) + (5^(-1 - n)*b*E^((5*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[
1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(32*((a + b*ArcSinh[c*x])/b)^n)))/(b*c
^2*Sqrt[1 + c^2*x^2])
```

### 3.518.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.518. $\int x(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))^n \, dx$

### 3.518.4 Maple [F]

$$\int x(c^2d\,x^2+d)^{\frac{3}{2}}(a+b\operatorname{arcsinh}(cx))^n\,dx$$

input
```
int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

output
```
int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

### 3.518.5 Fricas [F]

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\int(c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^nx\,dx$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas"
)
```

output
```
integral((c^2*d*x^3 + d*x)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

### 3.518.6 Sympy [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Timed out}$$

input
```
integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

output
```
Timed out
```

### 3.518.7 Maxima [F]

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\int (c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^n x\,dx$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

### 3.518.8 Giac [F(-2)]

Exception generated.

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.518.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\int x\,(a+b\operatorname{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

# 3.519 $\int (d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))^n \, dx$



## 3.519.1 Optimal result

Integrand size = 25, antiderivative size = 420

$$\int (d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))^n \, dx = \frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}$$
$$+ \frac{2^{-2(3+n)}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$+ \frac{2^{-3-n}de^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$- \frac{2^{-3-n}de^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$- \frac{2^{-2(3+n)}de^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

output
```
3/8*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(
1/2)+d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+
d)^(1/2)/(2^(6+2*n))/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
(1/2)+2^(-3-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(
c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1
/2)-2^(-3-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*
x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-
d*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/(2^(6+2*n))/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

### 3.519.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.69

$$\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n \, dx = \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n \left(-\frac{8(a+b\text{arcsinh}(cx))}{b(1+n)}+8\left(\frac{4a+4b\text{arcsinh}(cx)}{b+bn}+2^{-n}e^{-\frac{2a}{b}}\left(\right.\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(
b*(1 + n)) + 8*((4*a + 4*b*ArcSinh[c*x])/(b + b*n) + Gamma[1 + n, (-2*(a +
 b*ArcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (E^
((2*a)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(2^n*(a/b + ArcSinh[c*
x])^n)) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b
] - E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSi
nh[c*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c*
Sqrt[d + c^2*d*x^2])
```

### 3.519.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.72, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2} (a+b\text{arcsinh}(cx))^n \, dx$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n \cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n \sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^4d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$\downarrow$ 3793

$$\frac{d\sqrt{c^2dx^2+d}\int\left(\frac{1}{8}\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{1}{2}\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsi}}{bc\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$d\sqrt{c^2dx^2+d}\left(\frac{3(a+b\text{arcsinh}(cx))^{n+1}}{8(n+1)}+b2^{-2(n+3)}e^{-\frac{4a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arcs}}{b}\right.\right.$$

input
```
Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((3*(a + b*ArcSinh[c*x])^(1 + n))/(8*(1 + n)) + (b*
(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3
 + n))*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-3 - n)*b*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*(-(
(a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*b*E^((2*a)/b)*(a + b*ArcSinh[c*x
])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/((a + b*ArcSinh[c*x])/b)^n
- (b*E^((4*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x
]))/b])/(2^(2*(3 + n))*((a + b*ArcSinh[c*x])/b)^n)))/(b*c*Sqrt[1 + c^2*x^2
])
```

### 3.519.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.519. $\int\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx$

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

### 3.519.4 Maple [F]

$$\int \left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{n}d x$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

output
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

### 3.519.5 Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n\,dx = \int \left(c^{2} d x^{2} + d\right)^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n} \,d x$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

output
```
integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

### 3.519.6 Sympy [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))^n\,dx = \text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

output
```
Timed out
```

### 3.519.7 Maxima [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

### 3.519.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.519.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n\,dx=\int (a+b\,\text{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

# 3.520 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$



## 3.520.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \frac{3^{-1-n}d^2e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5d^2e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5d^2e^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{3^{-1-n}d^2e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+d^2\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}},x\right)$$

output
```
1/8*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(
c^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2
)+5/8*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1
)^(1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+5/8*d^2*e
xp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(
1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*3^(-1-n)*d^2*exp(3
*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(
1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^2*Unintegrable((a+b*
arcsinh(c*x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

### 3.520.2 Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x}\,dx=\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x}\,dx$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

output
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

### 3.520.3 Rubi [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x}\,dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{2c^2d^2x(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{d^2(a+b\operatorname{arcsinh}(cx))^n}{x\sqrt{c^2dx^2+d}}+\frac{c^4d^2x^3(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d^2\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{c^2dx^2+d}}dx+\frac{d^23^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{c^2dx^2+d}}+\frac{5d^2e^{-\frac{a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{c^2dx^2+d}}+\frac{5d^2e^{a/b}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{c^2dx^2+d}}+\frac{d^23^{-n-1}e^{\frac{3a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{c^2dx^2+d}}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

output
```
$Aborted
```

### 3.520.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

## 3.520.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(c^2d\,x^2+d)^{\frac{3}{2}}\,(a+b\text{ arcsinh}\,(cx))^n}{x}dx$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
```

output
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
```

### 3.520.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas"
)
```

output
```
integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

### 3.520.6 Sympy [N/A]

Not integrable

Time = 125.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^n}{x}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**n/x, x)
```

### 3.520.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

### 3.520.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.520.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{3/2}}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x, x)
```

# 3.521 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$



## 3.521.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \frac{3cd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{2b(1+n)\sqrt{d+c^2dx^2}}$$
$$+\frac{2^{-3-n}cd^2e^{-\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$-\frac{2^{-3-n}cd^2e^{\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$+d^2\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}},x\right)$$

output
```
3/2*c*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)
^(1/2)+2^(-3-n)*c*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))
/b)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)
^(1/2)-2^(-3-n)*c*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arc
sinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(
1/2)+d^2*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

### 3.521.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

output
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

### 3.521.3 Rubi [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}(a+b\operatorname{arcsinh}(cx))^n}{x^2}\,dx$$

$$\Big\downarrow \text{6235}$$

$$\int \left(\frac{2c^2d^2(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{d^2(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{c^2dx^2+d}}+\frac{c^4d^2x^2(a+b\operatorname{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d^2\int \frac{(a+b\operatorname{arcsinh}(cx))^n}{x^2\sqrt{c^2dx^2+d}}dx+\frac{3cd^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{n+1}}{2b(n+1)\sqrt{c^2dx^2+d}}+\frac{cd^22^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^n\left(-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{\sqrt{c^2dx^2+d}}-\frac{cd^22^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^n\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

### 3.521.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

## 3.521.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{n}}{x^{2}}d x$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

output
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

## 3.521.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{{\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n}}{x^{2}}\,{d x}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="frica
s")
```

output
```
integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

### 3.521.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x**2,x)
```

output
```
Timed out
```

### 3.521.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

### 3.521.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.521.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

# 3.522 $\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$



## 3.522.1 Optimal result

Integrand size = 28, antiderivative size = 816

$$
\begin{aligned}
\int x^2(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx &= -\frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}} \\
&+ \frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+ \frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+ \frac{2^{-2(4+n)}d^2e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&- \frac{2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&+ \frac{2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&- \frac{2^{-2(4+n)}d^2e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&- \frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}} \\
&- \frac{2^{-11-3n}d^2e^{\frac{8a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{8(a+b\text{arcsinh}(cx))}{b}\right)}{c^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

output
```
-5/128*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x
^2+1)^(1/2)+2^(-11-3*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-8*(a+b*arcsinh
(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(8*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(
c^2*x^2+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(
a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*
x))/b)^n)/(c^2*x^2+1)^(1/2)+d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arc
sinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsi
nh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1
+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arc
sinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*
x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arc
sinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAM
MA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/(((a+b*
arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+
b*arcsinh(c*x))^n*GAMMA(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^
3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-11-3*n)*d^2*exp(8*a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,8*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)
/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

## 3.522.2 Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 667, normalized size of antiderivative = 0.82

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx =$$

$$-\frac{2^{-11-3n}3^{-1-n}d^3e^{-\frac{8a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-n}\left(-3^{1+n}b(1+n)\left(\frac{a}{b}+\text{arcsinh}(c\right.\right.}{}$$

input
```
Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
-((2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(
-(3^(1 + n)*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSi
nh[c*x]))/b]) - 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gam
ma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)
*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b]
+ 3^(1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1
 + n, (-2*(a + b*ArcSinh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*3^(1 + n)*
a*(-((a + b*ArcSinh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*ArcSinh[c*
x]*(-((a + b*ArcSinh[c*x])^2/b^2))^n - 3^(1 + n)*4^(2 + n)*b*E^((2*a)/b)*(
1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))
/b] + 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b)
)^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6*a)/b)*(-(
(a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b] + 4^(2
 + n)*b*E^((6*a)/b)*n*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b
*ArcSinh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n
*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-((
a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])))/(b*c
^3*E^((8*a)/b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))
^n))
```

### 3.522.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c^2dx^2+d\right)^{5/2} (a+b\text{arcsinh}(cx))^n \, dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^3\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\frac{1}{128}\cosh\left(\frac{8a}{b}-\frac{8(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{1}{32}\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)(a+ba$$

$\downarrow$ 2009

$$d^2\sqrt{c^2dx^2+d}\left(-\frac{5(a+b\text{arcsinh}(cx))^{n+1}}{128(n+1)}+b2^{-3n-11}e^{-\frac{8a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{8(a+ba\right.$$

input
```
Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((-5*(a + b*ArcSinh[c*x])^(1 + n))/(128*(1 + n))
+ (2^(-11 - 3*n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh[
c*x]))/b])/(E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*3^(-1
 - n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/
(E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (b*(a + b*ArcSinh[c*x])^n*Ga
mma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(4 + n))*E^((4*a)/b)*(-((a
+ b*ArcSinh[c*x])/b))^n) - (2^(-7 - n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 +
n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^
n) + (2^(-7 - n)*b*E^((2*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a +
 b*ArcSinh[c*x]))/b])/((a + b*ArcSinh[c*x])/b)^n - (b*E^((4*a)/b)*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(4 + n))*((
a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*b*E^((6*a)/b)*(a + b*Ar
cSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b])/((a + b*ArcSinh[c*
x])/b)^n - (2^(-11 - 3*n)*b*E^((8*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n
, (8*(a + b*ArcSinh[c*x]))/b])/((a + b*ArcSinh[c*x])/b)^n))/(b*c^3*Sqrt[1
+ c^2*x^2])
```

### 3.522.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.522.4 Maple [F]

$$\int x^2 (c^2 d\, x^2 + d)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(cx))^n\, dx$$

input
```
int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

output
```
int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

### 3.522.5 Fricas [F]

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))^n\, dx = \int (c^2 dx^2 + d)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^n x^2\, dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="frica
s")
```

output
```
integral((c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c^2*d*x^2 + d)*(b*ar
csinh(c*x) + a)^n, x)
```

### 3.522.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))^n\, dx = \text{Timed out}$$

input
```
integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

output
```
Timed out
```

### 3.522.7 Maxima [F]

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^nx^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

### 3.522.8 Giac [F]

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^nx^2\,dx$$

input
```
integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

### 3.522.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

# 3.523 $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$



## 3.523.1 Optimal result

Integrand size = 26, antiderivative size = 745

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7(a+b\text{arcs}\ldots}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-n}d^2e^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{1-n}d^2e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5d^2e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5d^2e^{a/b}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{3^{1-n}d^2e^{\frac{3a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{5^{-n}d^2e^{\frac{5a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$
$$+\frac{7^{-1-n}d^2e^{\frac{7a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{128c^2\sqrt{1+c^2x^2}}$$

output
```
1/128*7^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-7*(a+b*arcsinh(c*x))/b)
*(c^2*d*x^2+d)^(1/2)/c^2/exp(7*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1
)^(1/2)+1/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*
(c^2*d*x^2+d)^(1/2)/(5^n)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*
x^2+1)^(1/2)+1/128*3^(1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n
)/(c^2*x^2+1)^(1/2)+5/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh
(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^
2*x^2+1)^(1/2)+5/128*d^2*exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1
)^(1/2)+1/128*3^(1-n)*d^2*exp(3*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b
*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*
x^2+1)^(1/2)+1/128*d^2*exp(5*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*ar
csinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(5^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)+1/128*7^(-1-n)*d^2*exp(7*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1
+n,7*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)
^n)/(c^2*x^2+1)^(1/2)
```

## 3.523.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 685, normalized size of antiderivative = 0.92

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{105^{-1-n}d^3e^{-\frac{7a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right.}{}$$

input
```
Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(105^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(5^(2 + n)*21^(
1 + n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x
])^2/b^2))^n*Gamma[1 + n, a/b + ArcSinh[c*x]] + 15^(1 + n)*(-((a + b*ArcSi
nh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-7*(a + b*ArcSinh[c*x]))/b] + E^((2*a
)/b)*(5*21^(1 + n)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*
(a + b*ArcSinh[c*x]))/b] + 9*35^(1 + n)*E^((2*a)/b)*(-((a + b*ArcSinh[c*x]
)^2/b^2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b] + 5^(2 + n)*21^(
1 + n)*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a
 + b*ArcSinh[c*x])/b)] + 35^(1 + n)*E^((8*a)/b)*(a/b + ArcSinh[c*x])^n*(-(
(a + b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] +
8*35^(1 + n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSi
nh[c*x])^2/b^2))^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] - 3^(2 + n)*7^
(1 + n)*E^((10*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[c*x])/b))^(3
*n)*Gamma[1 + n, (5*(a + b*ArcSinh[c*x]))/b] + 8*21^(1 + n)*E^((10*a)/b)*(
-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1
 + n, (5*(a + b*ArcSinh[c*x]))/b] + 15^(1 + n)*E^((12*a)/b)*(a/b + ArcSinh
[c*x])^n*(-((a + b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 + n, (7*(a + b*ArcSinh[
c*x]))/b])))/(128*c^2*E^((7*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*
x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n))
```

### 3.523.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int -(a+b\text{arcsinh}(cx))^n\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d^2\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{d^2\sqrt{c^2dx^2+d}\int\left(\frac{1}{64}\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{5}{64}\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)(a+ba$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\left(\frac{1}{128}b7^{-n-1}e^{-\frac{7a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{128}b5^{-n}e$$

input
```
Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((7^(-1 - n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n
, (-7*(a + b*ArcSinh[c*x]))/b])/(128*E^((7*a)/b)*(-((a + b*ArcSinh[c*x])/b
))^n) + (b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b
])/(128*5^n*E^((5*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (3^(1 - n)*b*(a +
 b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(128*E^((3*a
)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (5*b*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, -((a + b*ArcSinh[c*x])/b)])/(128*E^(a/b)*(-((a + b*ArcSinh[c*x])/b))^
n) + (5*b*E^(a/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])
/b])/(128*((a + b*ArcSinh[c*x])/b)^n) + (3^(1 - n)*b*E^((3*a)/b)*(a + b*Ar
cSinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(128*((a + b*ArcSi
nh[c*x])/b)^n) + (b*E^((5*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (5*(a
+ b*ArcSinh[c*x]))/b])/(128*5^n*((a + b*ArcSinh[c*x])/b)^n) + (7^(-1 - n)*
b*E^((7*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))
/b])/(128*((a + b*ArcSinh[c*x])/b)^n)))/(b*c^2*Sqrt[1 + c^2*x^2])
```

### 3.523.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.523. $\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.523.4 Maple [F]

$$\int x\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b\,\operatorname{arcsinh}\left(c x \right)\right)^{n}d x$$

input
```
int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

output
```
int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

### 3.523.5 Fricas [F]

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^{2} d x^{2} + d\right)^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n} x \,d x$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas"
)
```

output
```
integral((c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x)*sqrt(c^2*d*x^2 + d)*(b*arcs
inh(c*x) + a)^n, x)
```

### 3.523.6 Sympy [F(-1)]

Timed out.

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\text{Timed out}$$

input
```
integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

output
```
Timed out
```

### 3.523.7 Maxima [F]

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^n x\,dx$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

### 3.523.8 Giac [F(-2)]

Exception generated.

$$\int x\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.523.9 Mupad [F(-1)]

Timed out.

$$\int x(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int x\,(a+b\,\text{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

# 3.524 $\int (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^n \, dx$



## 3.524.1 Optimal result

Integrand size = 25, antiderivative size = 632

$$\int (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))^n \, dx = \frac{5d^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}$$
$$+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$+\frac{3\ 2^{-7-2n}d^2e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$+\frac{15\ 2^{-7-n}d^2e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$-\frac{15\ 2^{-7-n}d^2e^{\frac{2a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$-\frac{3\ 2^{-7-2n}d^2e^{\frac{4a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$
$$-\frac{2^{-7-n}3^{-1-n}d^2e^{\frac{6a}{b}}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{c\sqrt{1+c^2x^2}}$$

output
```
5/16*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1
)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsi
nh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(
c^2*x^2+1)^(1/2)+3*2^(-7-2*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*a
rcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^
n)/(c^2*x^2+1)^(1/2)+15*2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+
b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/
b)^n)/(c^2*x^2+1)^(1/2)-15*2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GA
MMA(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))
/b)^n)/(c^2*x^2+1)^(1/2)-3*2^(-7-2*n)*d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*
GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x
))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arcsinh(c
*x))^n*GAMMA(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcs
inh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

### 3.524.2 Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.84

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\frac{2^{-7-2n}3^{-1-n}d^3e^{-\frac{6a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}\right)^{-2n}\left(2^nb(1+n\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2^n
*b*(1 + n)*(a/b + ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[
1 + n, (-6*(a + b*ArcSinh[c*x]))/b] + 3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b
 + ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (-4*(a +
 b*ArcSinh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSin
h[c*x])^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-2*(a + b*ArcSin
h[c*x]))/b] - E^((6*a)/b)*(5*2^n*3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-((a + b
*ArcSinh[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a
+ b*ArcSinh[c*x]))/b] + 3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x
])^n*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]
))/b] + 2^n*(-5*2^(3 + n)*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh
[c*x])^2/b^2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((a
+ b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b]))))/(
b*c*E^((6*a)/b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2)
)^(2*n))
```

### 3.524.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (a+b\text{arcsinh}(cx))^n\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^6d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\frac{1}{32}\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)(a+b\text{arcsinh}(cx))^n+\frac{3}{16}\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)(a+\text{bar}$$

$bc\sqrt{c^2}$

$$\downarrow \text{2009}$$

$$d^2\sqrt{c^2dx^2+d}\left(\frac{5(a+b\text{arcsinh}(cx))^{n+1}}{16(n+1)}+b2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{6(a+\right.$$

input
```
Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((5*(a + b*ArcSinh[c*x])^(1 + n))/(16*(1 + n)) +
(2^(-7 - n)*3^(-1 - n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*Ar
cSinh[c*x]))/b])/(E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (3*2^(-7 -
2*n)*b*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(
E^((4*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) + (15*2^(-7 - n)*b*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*(-((a +
 b*ArcSinh[c*x])/b))^n) - (15*2^(-7 - n)*b*E^((2*a)/b)*(a + b*ArcSinh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/((a + b*ArcSinh[c*x])/b)^n -
 (3*2^(-7 - 2*n)*b*E^((4*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a +
 b*ArcSinh[c*x]))/b])/((a + b*ArcSinh[c*x])/b)^n - (2^(-7 - n)*3^(-1 - n)*
b*E^((6*a)/b)*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))
/b])/((a + b*ArcSinh[c*x])/b)^n))/(b*c*Sqrt[1 + c^2*x^2])
```

### 3.524.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

3.524. $\int (d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx$

### 3.524.4 Maple [F]

$$\int \left(c^2 d\,x^2+d\right)^{\frac{5}{2}} \left(a+b \operatorname{arcsinh}\left(c x\right)\right)^{n} dx$$

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

output
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

### 3.524.5 Fricas [F]

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^n \, dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}} (b\,\text{arsinh}\left(cx\right)+a)^n \, dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsin
h(c*x) + a)^n, x)
```

### 3.524.6 Sympy [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))^n \, dx = \text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

output
```
Timed out
```

### 3.524.7 Maxima [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^n\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n, x)
```

### 3.524.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.524.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n\,dx=\int (a+b\,\text{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

# 3.525 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$



## 3.525.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx = \frac{5^{-1-n}d^3e^{-\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}}{32\sqrt{d+c^2dx^2}}$$
$$-\frac{5\ 3^{-1-n}d^3e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}}$$
$$+\frac{3^{-n}d^3e^{-\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{11d^3e^{-\frac{a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{d+c^2dx^2}}$$
$$+\frac{11d^3e^{a/b}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{d+c^2dx^2}}$$
$$-\frac{5\ 3^{-1-n}d^3e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}}$$
$$+\frac{3^{-n}d^3e^{\frac{3a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{d+c^2dx^2}}$$
$$+\frac{5^{-1-n}d^3e^{\frac{5a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{d+c^2dx^2}}$$
$$+d^3\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{d+c^2dx^2}},x\right)$$

output
```
1/32*5^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*
(c^2*x^2+1)^(1/2)/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/
2)-5/32*3^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/
b)*(c^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^
(1/2)+1/8*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2
*x^2+1)^(1/2)/(3^n)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(
1/2)+11/16*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*
x^2+1)^(1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+11/1
6*d^3*exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x
^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)-5/32*3^(-1-n)*d
^3*exp(3*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*
x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*d^3*exp(3*
a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1
/2)/(3^n)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/32*5^(-1-n)*d^3
*exp(5*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^2*x^
2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^3*Unintegrable
((a+b*arcsinh(c*x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

## 3.525.2 Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

output
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

## 3.525.3 Rubi [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.525. $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}\,dx$

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}dx$$

$\big\downarrow$ 6235

$$\int \left(\frac{3c^2d^3x(a+b\text{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{d^3(a+b\text{arcsinh}(cx))^n}{x\sqrt{c^2dx^2+d}}+\frac{c^6d^3x^5(a+b\text{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{3c^4d^3x^3(a+b\text{arcsinh}(cx$$

$\big\downarrow$ 2009

$$d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x\sqrt{c^2dx^2+d}}dx+\frac{d^3 5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{c^2dx^2+d}}-\frac{5d^3 3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{c^2dx^2+d}}+\frac{d^3 3^{-n}e^{-\frac{3a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{c^2dx^2+d}}+\frac{11d^3e^{-\frac{a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{c^2dx^2+d}}+\frac{11d^3e^{a/b}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\text{arcsinh}(cx)}{b}\right)}{16\sqrt{c^2dx^2+d}}-\frac{5d^3 3^{-n-1}e^{\frac{3a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{c^2dx^2+d}}+\frac{d^3 3^{-n}e^{\frac{3a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{c^2dx^2+d}}+\frac{d^3 5^{-n-1}e^{\frac{5a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{c^2dx^2+d}}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

output
```
$Aborted
```

3.525. $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n}{x}dx$

#### 3.525.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] &&  !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

### 3.525.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{n}}{x}d x$$

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

output
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

### 3.525.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^n}{x}\,dx=\int \frac{{\left(c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n}}{x} \,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas"
)
```

output
```
integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsin
h(c*x) + a)^n/x, x)
```

### 3.525.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)^n}{x}\,dx=\text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x,x)
```

output
```
Timed out
```

### 3.525.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)^n}{x}\,dx=\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^n}{x}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

### 3.525.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)^n}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.525.9 Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^n}{x}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{5/2}}{x}\,dx$$

input
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x, x)
```

# 3.526 $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$



## 3.526.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\frac{15cd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{1+n}}{8b(1+n)\sqrt{d+c^2dx^2}}$$
$$+\frac{2^{-2(3+n)}cd^3e^{-\frac{4a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$+\frac{2^{-2-n}cd^3e^{-\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$-\frac{2^{-2-n}cd^3e^{\frac{2a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$-\frac{2^{-2(3+n)}cd^3e^{\frac{4a}{b}}\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{d+c^2dx^2}}$$
$$+d^3\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{d+c^2dx^2}},x\right)$$

output
```
15/8*c*d^3*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d
)^(1/2)+c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2
*x^2+1)^(1/2)/(2^(6+2*n))/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^
2+d)^(1/2)+2^(-2-n)*c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c
*x))/b)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^
2+d)^(1/2)-2^(-2-n)*c*d^3*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b
*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+
d)^(1/2)-c*d^3*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*
x))/b)*(c^2*x^2+1)^(1/2)/(2^(6+2*n))/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2
+d)^(1/2)+d^3*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

### 3.526.2 Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

output
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

### 3.526.3 Rubi [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6235, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx$$

$$\Big\downarrow \text{ 6235}$$

$$\int \left(\frac{3c^2d^3(a+b\text{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{d^3(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{c^2dx^2+d}}+\frac{c^6d^3x^4(a+b\text{arcsinh}(cx))^n}{\sqrt{c^2dx^2+d}}+\frac{3c^4d^3x^2(a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}\right.$$

$$\Big\downarrow \text{2009}$$

$$d^3\int \frac{(a+b\text{arcsinh}(cx))^n}{x^2\sqrt{c^2dx^2+d}}dx+\frac{15cd^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{n+1}}{8b(n+1)\sqrt{c^2dx^2+d}}+$$
$$\frac{cd^32^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{c^2dx^2+d}}+$$
$$\frac{cd^32^{-n-2}e^{-\frac{2a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{c^2dx^2+d}}-$$
$$\frac{cd^32^{-n-2}e^{\frac{2a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{c^2dx^2+d}}-$$
$$\frac{cd^32^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^n\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

### 3.526.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6235
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n/Sq
rt[d + e*x^2], (f*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d,
e, f, m, n}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && !IGtQ[(m + 1)/2, 0
] && (EqQ[m, -1] || EqQ[m, -2])
```

### 3.526.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(c^2d\,x^2+d)^{\frac{5}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))^n}{x^2}dx$$

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

output
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

### 3.526.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))^n}{x^2}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="frica
s")
```

output
```
integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsin
h(c*x) + a)^n/x^2, x)
```

### 3.526.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))^n}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x**2,x)
```

output
```
Timed out
```

### 3.526.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

### 3.526.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.526.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^n}{x^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^n\,(d\,c^2\,x^2+d)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

# 3.527 $\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} dx$



## 3.527.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} dx = \operatorname{Int}\left(\frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}, x\right)$$

output
```
Unintegrable(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

## 3.527.2 Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} dx = \int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} dx$$

input
```
Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

## 3.527.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}} dx$$
$$\Big\downarrow \text{ 6239}$$
$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}} dx$$

input
```
Int[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
$Aborted
```

### 3.527.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.527.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

input
```
int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

### 3.527.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.527.6 Sympy [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m\,\text{asinh}^n\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x**m*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

### 3.527.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.527.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \operatorname{arsinh}{(ax)}^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.527.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m\,{\mathrm{asinh}\left(a\,x\right)}^n}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.528 $\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$



## 3.528.1 Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{x^3\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax))}{8a^4} - \frac{3(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{8a^4} - \frac{3\Gamma(1+n,\text{arcsinh}(ax))}{8a^4} + \frac{3^{-1-n}\Gamma(1+n,3\text{arcsinh}(ax))}{8a^4}$$

output
```
1/8*3^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/a^4/((-arcsinh(a*x)
)^n)-3/8*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^4/((-arcsinh(a*x))^n)-3
/8*GAMMA(1+n,arcsinh(a*x))/a^4+1/8*3^(-1-n)*GAMMA(1+n,3*arcsinh(a*x))/a^4
```

## 3.528.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{x^3\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\left(\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax)) - 3^{2+n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax\right.}{8a^4}$$

input
```
Integrate[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
(3^(-1 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]] - 3^(2 + n)*ArcS
inh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(3^(2 + n)*Ga
mma[1 + n, ArcSinh[a*x]]) + Gamma[1 + n, 3*ArcSinh[a*x]])))/(8*a^4*(-ArcSi
nh[a*x])^n)
```

### 3.528.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6234, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{\int a^3x^3\text{arcsinh}(ax)^n d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int i\text{arcsinh}(ax)^n \sin(i\text{arcsinh}(ax))^3 d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int \text{arcsinh}(ax)^n \sin(i\text{arcsinh}(ax))^3 d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{i\int \left(\frac{3}{4}iax\text{arcsinh}(ax)^n - \frac{1}{4}i\text{arcsinh}(ax)^n \sinh(3\text{arcsinh}(ax))\right) d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\left(-\frac{1}{8}i3^{-n-1}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-3\text{arcsinh}(ax)) + \frac{3}{8}i\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-\right.}{a^4}$$

input
```
Int[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
(I*(((-1/8*I)*3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(-A
rcSinh[a*x])^n + (((3*I)/8)*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-
ArcSinh[a*x])^n + ((3*I)/8)*Gamma[1 + n, ArcSinh[a*x]] - (I/8)*3^(-1 - n)*
Gamma[1 + n, 3*ArcSinh[a*x]]))/a^4
```

### 3.528.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.528.4 Maple [F]

$$\int \frac{x^3 \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

input
```
int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

output
```
int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

### 3.528.5 Fricas [F]

$$\int \frac{x^3\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3 \,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.528.6 Sympy [F]

$$\int \frac{x^3\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3 \,\text{asinh}^n\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x**3*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

### 3.528.7 Maxima [F]

$$\int \frac{x^3\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^3 \,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.528.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.528.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}} \, dx = \int \frac{x^3 \operatorname{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}} \, dx$$

input
```
int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.529 $\int \frac{x^2 \textbf{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$



## 3.529.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{x^2\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{arcsinh}(ax)^{1+n}}{2a^3(1+n)} + \frac{2^{-3-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-2\operatorname{arcsinh}(ax))}{a^3} - \frac{2^{-3-n}\Gamma(1+n,2\operatorname{arcsinh}(ax))}{a^3}$$

output
```
-1/2*arcsinh(a*x)^(1+n)/a^3/(1+n)+2^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arc
sinh(a*x))/a^3/((-arcsinh(a*x))^n)-2^(-3-n)*GAMMA(1+n,2*arcsinh(a*x))/a^3
```

## 3.529.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{x^2\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{2^{-3-n}(-\operatorname{arcsinh}(ax))^{-n}\left((1+n)\operatorname{arcsinh}(ax)^n\Gamma(1+n,-2\operatorname{arcsinh}(ax)) - (-\operatorname{arcsinh}(ax))^n\left(2^{2+n}\operatorname{arcsinh}(a\right.\right.}{a^3(1+n)}$$

input
```
Integrate[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
(2^(-3 - n)*((1 + n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] - (-ArcS
inh[a*x])^n*(2^(2 + n)*ArcSinh[a*x]^(1 + n) + (1 + n)*Gamma[1 + n, 2*ArcSi
nh[a*x]])))/(a^3*(1 + n)*(-ArcSinh[a*x])^n)
```

### 3.529.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{\int a^2x^2\text{arcsinh}(ax)^nd\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\text{arcsinh}(ax)^n\sin(i\text{arcsinh}(ax))^2d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \text{arcsinh}(ax)^n\sin(i\text{arcsinh}(ax))^2d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{\int \left(\frac{1}{2}\text{arcsinh}(ax)^n-\frac{1}{2}\text{arcsinh}(ax)^n\cosh(2\text{arcsinh}(ax))\right)d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\text{arcsinh}(ax)^{n+1}}{2(n+1)}+2^{-n-3}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-2\text{arcsinh}(ax))-2^{-n-3}\Gamma(n+1,2\text{arcsinh}(ax))}{a^3}$$

input
```
Int[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
(-1/2*ArcSinh[a*x]^(1 + n)/(1 + n) + (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 +
n, -2*ArcSinh[a*x]])/(-ArcSinh[a*x])^n - 2^(-3 - n)*Gamma[1 + n, 2*ArcSinh
[a*x]])/a^3
```

### 3.529.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.529.4 Maple [F]

$$\int \frac{x^2 \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

input
```
int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

output
```
int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

### 3.529.5 Fricas [F]

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.529.6 Sympy [F]

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x**2*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

### 3.529.7 Maxima [F]

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.529.8 Giac [F]

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.529.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^2\,\text{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.530 $\int \frac{x\,\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$



## 3.530.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{x\,\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{2a^2} + \frac{\Gamma(1+n,\text{arcsinh}(ax))}{2a^2}$$

output
```
1/2*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^2/((-arcsinh(a*x))^n)+1/2*GA
MMA(1+n,arcsinh(a*x))/a^2
```

## 3.530.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x\,\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))+\Gamma(1+n,\text{arcsinh}(ax))}{2a^2}$$

input
```
Integrate[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[1
+ n, ArcSinh[a*x]])/(2*a^2)
```

### 3.530.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6234, 3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int ax\text{arcsinh}(ax)^nd\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\text{arcsinh}(ax)^n\sin(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \text{arcsinh}(ax)^n\sin(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{i\left(\frac{1}{2}i\int e^{\text{arcsinh}(ax)}\text{arcsinh}(ax)^nd\text{arcsinh}(ax)-\frac{1}{2}i\int e^{-\text{arcsinh}(ax)}\text{arcsinh}(ax)^nd\text{arcsinh}(ax)\right)}{a^2}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{i\left(\frac{1}{2}i\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-\text{arcsinh}(ax))+\frac{1}{2}i\Gamma(n+1,\text{arcsinh}(ax))\right)}{a^2}$$

input
```
Int[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

output
```
((-I)*(((I/2)*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^
n + (I/2)*Gamma[1 + n, ArcSinh[a*x]]))/a^2
```

### 3.530.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.530.4 Maple [F]

$$\int \frac{x \operatorname{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}dx$$

input
```
int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

output
```
int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

### 3.530.5 Fricas [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.530.6 Sympy [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{asinh}^n\left(ax\right)}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

### 3.530.7 Maxima [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x\,\text{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.530.8 Giac [F]

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

### 3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx=\int \frac{x\,\text{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

# 3.531 $\int \frac{\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$



## 3.531.1 Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^{1+n}}{a(1+n)}$$

output
```
arcsinh(a*x)^(1+n)/a/(1+n)
```

## 3.531.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{arcsinh}(ax)^{1+n}}{a(1+n)}$$

input
```
Integrate[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

## 3.531.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{\text{arcsinh}(ax)^{n+1}}{a(n+1)}$$

input
```
Int[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

output
```
ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

### 3.531.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

## 3.531.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{arcsinh}(ax)^{1+n}}{a(1+n)}$ | 18 |
| default | $\frac{\text{arcsinh}(ax)^{1+n}}{a(1+n)}$ | 18 |

input
```
int(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(a*x)^(1+n)/a/(1+n)
```

### 3.531.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.88

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\cosh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)\sinh\left(n\log\left(\log\left(ax+\right.\right.\right.}{an+a}$$

input
```
integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(cosh(n*log(log(a*x + sqrt(a^2*x^2 + 1))))*log(a*x + sqrt(a^2*x^2 + 1)) +
log(a*x + sqrt(a^2*x^2 + 1))*sinh(n*log(log(a*x + sqrt(a^2*x^2 + 1)))))/(a
*n + a)
```

### 3.531.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(12) = 24.

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge n=-1 \\ 0^n x & \text{for } a=0 \\ \frac{\log(\operatorname{asinh}(ax))}{a} & \text{for } n=-1 \\ \frac{\operatorname{asinh}(ax)\operatorname{asinh}^n(ax)}{an+a} & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(n, -1)), (0**n*x, Eq(a, 0)), (log(asinh(a*
x))/a, Eq(n, -1)), (asinh(a*x)*asinh(a*x)**n/(a*n + a), True))
```

### 3.531.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arsinh}\left(ax\right)^{n+1}}{a(n+1)}$$

input
```
integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(a*x)^(n + 1)/(a*(n + 1))
```

### 3.531.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^{n+1}}{a(n+1)}$$

input
```
integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
log(a*x + sqrt(a^2*x^2 + 1))^(n + 1)/(a*(n + 1))
```

### 3.531.9 Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{arcsinh}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \begin{cases} \frac{\ln(\operatorname{asinh}(a\,x))}{a} & \text{if } n=-1 \\ \frac{\operatorname{asinh}(a\,x)^{n+1}}{a\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

input
```
int(asinh(a*x)^n/(a^2*x^2 + 1)^(1/2),x)
```

output
```
piecewise(n == -1, log(asinh(a*x))/a, n ~= -1, asinh(a*x)^(n + 1)/(a*(n +
1)))
```

# 3.532 $\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$



## 3.532.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \text{Int}\left(\frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}},x\right)$$

output
```
Unintegrable(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

## 3.532.2 Mathematica [N/A]

Not integrable

Time = 5.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\text{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

input
```
Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

## 3.532.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \text{ 6239}$$

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{a^2x^2+1}}dx$$

input
```
Int[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
```

output
```
$Aborted
```

### 3.532.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.532.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(ax\right)^{n}}{x\sqrt{a^{2}x^{2}+1}}dx$$

input
```
int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

output
```
int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

### 3.532.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^3 + x), x)
```

### 3.532.6 Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^n(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**n/x/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**n/(x*sqrt(a**2*x**2 + 1)), x)
```

### 3.532.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.532.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}x}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

### 3.532.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^n}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.533 $\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$



## 3.533.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}},x\right)$$

output
```
Unintegrable(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

## 3.533.2 Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

input
```
Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

## 3.533.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^n}{x^2\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \text{6239}$$

$$\int \frac{\text{arcsinh}(ax)^n}{x^2\sqrt{a^2x^2+1}}dx$$

input
```
Int[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
```

output
```
$Aborted
```

### 3.533.3.1 Defintions of rubi rules used

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

## 3.533.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\text{arcsinh}\left(ax\right)^n}{x^2\sqrt{a^2x^2+1}}dx$$

input
```
int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

output
```
int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

### 3.533.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^4 + x^2), x)
```

### 3.533.6 Sympy [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{asinh}^n\left(ax\right)}{x^2\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(asinh(a*x)**n/x**2/(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(asinh(a*x)**n/(x**2*sqrt(a**2*x**2 + 1)), x)
```

### 3.533.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{\sqrt{a^2x^2+1}x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

### 3.533.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

### 3.533.9 Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx=\int \frac{\operatorname{asinh}(a\,x)^n}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

# 3.534 $\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\mathbf{arcsinh}(cx))\,dx$



## 3.534.1 Optimal result

Integrand size = 35, antiderivative size = 416

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} - \frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx)) + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

output
```
3/8*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*d
^2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*I*d^2*(c
^2*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-2/3*I*b
*d^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*d^2*
x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d^2*
x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*d^2*x
^4*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*d^2*(a+b*arc
sinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.534.2 Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.87

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{48ad^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}(16i+9cx+16ic^2x^2-6c^3x^3)+720ad^{5/2}\sqrt{f}\sqrt{1}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(48*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*(16*I + 9*
c*x + (16*I)*c^2*x^2 - 6*c^3*x^3) + 720*a*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2
]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 144
*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^2 - Cosh[2*ArcS
inh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]) - (64*I)*b*d^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + C
osh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]) + 9*b*d^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]
*Sinh[4*ArcSinh[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

### 3.534.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d^2(icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 6253

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int\Big(-c^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x^2+2ic\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x+\sqrt{c^2x^2+1}(a}{\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2009

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\Bigg(\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{2i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c}-\frac{1}{4}c^2x^3\sqrt{c^2x^2+1}(a+ba}{\sqrt{c^2x^2+1}}$$

input
```
Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(((-2*I)/3)*b*x - (3*b*c*x^2)/16
- ((2*I)/9)*b*c^2*x^3 + (b*c^3*x^4)/16 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/8 - (c^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 + (((2*
I)/3)*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (5*(a + b*ArcSinh[c*x]
)^2)/(16*b*c)))/Sqrt[1 + c^2*x^2]
```

### 3.534.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.534.4 Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx)) \sqrt{-icfx+f}dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

### 3.534.5 Fricas [F]

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-(b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x
 - a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.534.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

output
```
Timed out
```

### 3.534.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.534.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.534.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{5/2}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

# 3.535 $\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$



## 3.535.1 Optimal result

Integrand size = 35, antiderivative size = 304

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = -\frac{ibdx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}-\frac{ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))+\frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c}+\frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
1/2*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/3*I*d*(c^
2*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-1/3*I*b*
d*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*d*x^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*I*b*c^2*d*x^3*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*d*(a+b*arcsinh(c*x))
^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.535.2 Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{12ad\sqrt{d+icdx}\sqrt{f-icfx}(2i+3cx+2ic^2x^2)+36ad^{3/2}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+ic}\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(12*a*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*I + 3*c*x + (2*I)*c^2*x^2)
+ 36*a*d^(3/2)*Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]] + (9*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*
x]^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1
 + c^2*x^2] - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*Ar
cSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c
*x]]))/Sqrt[1 + c^2*x^2])/(72*c)
```

### 3.535.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d(icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int\left(icx\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))\right)dx}{\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 2009

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))+\frac{i(c^2x^2+1)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c}+\frac{(a+b\operatorname{arcsinh}(cx))^2}{4bc}-\frac{1}{9}ibc^2\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1/3*I)*b*x - (b*c*x^2)/4 - (I/9)
*b*c^2*x^3 + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + ((I/3)*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (a + b*ArcSinh[c*x])^2/(4*b*c)))/Sq
rt[1 + c^2*x^2]
```

### 3.535.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.535.4 Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx)) \sqrt{-icfx+f}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

### 3.535.5 Fricas [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((I*b*c*d*x + b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (I*a*c*d*x + a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
), x)
```

### 3.535.6 Sympy [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int (id(cx-i))^{\frac{3}{2}}\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))\,\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x
)
```

### 3.535.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.535.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

### 3.535.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a + b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

# 3.536 $\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$



## 3.536.1 Optimal result

Integrand size = 35, antiderivative size = 147

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
1/2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b*c*x^2*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x)
)^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

## 3.536.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.59

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{1}{2}ax\sqrt{id(-i+cx)}\sqrt{-if(i+cx)} + \frac{a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\right)}{2c} - \frac{b\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}(\cosh(2\text{arcsinh}(cx))-2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\text{si}}{8c\sqrt{-((-id+cdx)(if+cfx))}\sqrt{1+c^2x^2}}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(a*x*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)])/2 + (a*Sqrt[d]*Sqrt[f]*L
og[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])
/(2*c) - (b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(
1 + c^2*x^2))]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh
[2*ArcSinh[c*x]])))/(8*c*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 +
c^2*x^2])
```

### 3.536.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6200}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*c)))/Sqrt[1 + c^2*
x^2]
```

### 3.536.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

## 3.536.4 Maple [F]

$$\int (a+b\ \mathrm{arcsinh}\,(cx))\,\sqrt{icdx+d}\,\sqrt{-icfx+f}dx$$

input
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

### 3.536.5 Fricas [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \int \sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1
)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a, x)
```

### 3.536.6 Sympy [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \int \sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}(cx))\;dx$$

input
```
integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)
```

### 3.536.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.536.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

### 3.536.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$
$$= \int (a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

# 3.537 $\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$



## 3.537.1 Optimal result

Integrand size = 35, antiderivative size = 158

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
-I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+
I*b*f*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*f*(a+b*a
rcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

## 3.537.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$$
$$= \frac{2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right) - 2ib\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx) + b\sqrt{d+ic}}{2cd\sqrt{1+c^2x^2}}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
((2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) -
 (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
] + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqr
t[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]])/(2*c*d*Sqrt[1 + c^2*x^2])
```

### 3.537.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{c^2x^2+1}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f\sqrt{c^2x^2+1}\int \left(\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\sqrt{c^2x^2+1}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc}+ibx\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
(f*Sqrt[1 + c^2*x^2]*(I*b*x - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c
 + (a + b*ArcSinh[c*x])^2/(2*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

### 3.537.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.537.4 Maple [F]

$$\int \frac{(a + b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx + f}}{\sqrt{icdx + d}}dx$$

input
```
int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

### 3.537.5 Fricas [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))}{\sqrt{d + icdx}}\,dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)}{\sqrt{i\,cdx + d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2
 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*d*x - I*d), x)
```

### 3.537.6 Sympy [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))}{\sqrt{d + icdx}}\,dx = \int \frac{\sqrt{-if\,(cx + i)}(a + b\,\text{asinh}\,(cx))}{\sqrt{id\,(cx - i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)
```

### 3.537.7 Maxima [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))}{\sqrt{d + icdx}}\,dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)}{\sqrt{i\,cdx + d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
a*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + b*i
ntegrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x + d)
, x)
```

### 3.537.8 Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}\,(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

### 3.537.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\text{i}}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

# 3.538 $\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$



## 3.538.1 Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bf^2\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
2*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-1/2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*f^2*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)
```

## 3.538.2 Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx =$$

$$-\frac{-\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{-i+cx} + 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) + \frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(\operatorname{arcsinh}(cx)\left(-4i\right.\right.}{}}{}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
-1/2*((-4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) + 2*a*Sqrt[d]*
Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
 + (b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSi
nh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2
] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[
1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 +
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^2)
```

## 3.538.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-\frac{2i(cx+i)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(\frac{2i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{2b\log(-cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^2*(1 + c^2*x^2)^(3/2)*(((2*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt
[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^2/(2*b*c) - (2*b*Log[I - c*x])/c))/(
(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.538.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.538.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

## 3.538.5 Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*x^2 - 2*I*c*d^2*x
- d^2), x)
```

## 3.538.6 Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2), x
)
```

## 3.538.7 Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*d
^2*sqrt(f/d))) + b*integrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1
))/(I*c*d*x + d)^(3/2), x)
```

### 3.538.8 Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(3/2), x)
```

### 3.538.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\text{i}}}{(d+c\,d\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

# 3.539 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$



## 3.539.1 Optimal result

Integrand size = 35, antiderivative size = 187

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{2ibf^3(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if^3(1-icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^3(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
2/3*I*b*f^3*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)+1/3*I*f^3*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)+1/3*b*f^3*(c^2*x^2+1)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/
2)/(f-I*c*f*x)^(5/2)
```

## 3.539.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-\left((i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)\right)-b(i+}{3cd^3(-i+cx)^2\sqrt{1+}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-((I + c*x)*((-I)*b + b*c*x + a*Sqrt
[1 + c^2*x^2])) - b*(I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(-I + c*x
)^2*Log[d + I*c*d*x]))/(3*c*d^3*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

### 3.539.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6211, 27, 6252, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\int \frac{i(1-icx)^3}{3c(c^2x^2+1)^2}dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\int \frac{(1-icx)^3}{(c^2x^2+1)^2}dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{456}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\int \frac{1-icx}{(icx+1)^2}dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\int \left(\frac{i}{cx-i}-\frac{2}{(cx-i)^2}\right)dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\left(\frac{i\log(-cx+i)}{c}-\frac{2}{c(-cx+i)}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(5/2)*(((I/3)*(1 - I*c*x)^3*(a + b*ArcSinh[c*x]))/(c*(1
 + c^2*x^2)^(3/2)) - (I/3)*b*(-2/(c*(I - c*x)) + (I*Log[I - c*x])/c)))/((d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.539.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

## 3.539. $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$

### 3.539.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

### 3.539.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(142) = 284$.

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.93

$$\int \frac{\sqrt{f-icfx}(a+b\mathrm{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx =$$

$$-\frac{4\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,bcx+2\left(bc^2x^2+2i\,bcx-b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2\right.}{}$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(
b*c^2*x^2 + 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) - (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d^3)*s
qrt(b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sqrt(
c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*x^4 + 2*c^8
*d^3*x^3 + I*c^7*d^3*x^2 + 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(b*c^3*x^3
- I*b*c^2*x^2 + b*c*x - I*b)) + (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x -
 I*c*d^3)*sqrt(b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b
*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*d^3
*x^4 - 2*c^8*d^3*x^3 - I*c^7*d^3*x^2 - 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))
/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*(a*c^2*x^2 + 2*I*a*c*x - a)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d
^3*x - I*c*d^3)
```

### 3.539.6 Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}{(cx)})}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(5/2), x
)
```

### 3.539.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx &= \frac{1}{3}\,bc\left(\frac{6\sqrt{f}}{3i\,c^3d^{\frac{5}{2}}x+3\,c^2d^{\frac{5}{2}}}+\frac{\sqrt{f}\log{(cx-i)}}{c^2d^{\frac{5}{2}}}\right)\\ &\quad-\frac{1}{3}\,b\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\operatorname{arsinh}{(cx)}\\ &\quad-\frac{1}{3}\,a\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\end{aligned}$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algori
thm="maxima")
```

output
```
1/3*b*c*(6*sqrt(f)/(3*I*c^3*d^(5/2)*x + 3*c^2*d^(5/2)) + sqrt(f)*log(c*x -
 I)/(c^2*d^(5/2))) - 1/3*b*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I
*c^2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3
))*arcsinh(c*x) - 1/3*a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c^
2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))
```

### 3.539.8 Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}\left(b\,\text{arsinh}\,(cx)+a\right)}{\left(i\,cdx+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)
```

### 3.539.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)\,\sqrt{f-c\,f\,x\,1\text{i}}}{{\left(d+c\,d\,x\,1\text{i}\right)}^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

# 3.540 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.540.1 Optimal result

Integrand size = 35, antiderivative size = 459

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5(1+c^2x^2)^{3/2}} - \frac{5bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{2ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15(1+c^2x^2)^{3/2}} - \frac{bc^3dx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25(1+c^2x^2)^{3/2}} + \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)} + \frac{id(d+\cdots}{}$$

output
```
-1/5*I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*
c*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-2/15*I*b*c^2
*d*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*d*
x^4*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/25*I*b*c^4*d*x
^5*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*c*d*
x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)^(3/2)*(
f-I*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*d*(d+I*c*d*x)^(3/2)
*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

### 3.540.2 Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.49

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{-1200ibcd^2fx\sqrt{d+icdx}\sqrt{f-icfx}+1920iad^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((-1200*I)*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (1920*I)*a*d^
2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d^2*f
*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (3840*I)*a*c^2*
d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c
^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1920
*I)*a*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 1800*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b
*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(5/
2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]] - (200*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*Sinh[3*ArcSinh[c*x]] + 60*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Ar
cSinh[c*x]*((10*I)*Cosh[3*ArcSinh[c*x]] + (2*I)*Cosh[5*ArcSinh[c*x]] + 5*(
(4*I)*Sqrt[1 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]]))
- (24*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])
/(9600*c*Sqrt[1 + c^2*x^2])
```

### 3.540.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int d(icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int (icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(icx(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}+(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}\right)dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{i\left(c^2x^2+1\right)^{5/2}}{}\right.}{\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((-1/5*I)*b*x - (5*b*c*x^2)/16
- ((2*I)/15)*b*c^2*x^3 - (b*c^3*x^4)/16 - (I/25)*b*c^4*x^5 + (3*x*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/4 + ((I/5)*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/c + (3*(a + b*
ArcSinh[c*x])^2)/(16*b*c)))/(1 + c^2*x^2)^(3/2)
```

### 3.540.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## 3.540. $\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.540.4 Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))\,dx$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

### 3.540.5 Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((I*b*c^3*d^2*f*x^3 + b*c^2*d^2*f*x^2 + I*b*c*d^2*f*x + b*d^2*f)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^
3*d^2*f*x^3 + a*c^2*d^2*f*x^2 + I*a*c*d^2*f*x + a*d^2*f)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f), x)
```

### 3.540.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

### 3.540.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.540.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

### 3.540.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\,\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

# 3.541 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.541.1 Optimal result

Integrand size = 35, antiderivative size = 247

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} - \frac{bc^3x^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} + \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)} + \frac{3(d+ic}{}$$

output
```
-5/16*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b
*c^3*x^4*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*
c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*x*(d+I*c*d*x)^(3/2)*
(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+3/16*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

## 3.541.2 Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.43

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{80acdfx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+32ac^3dfx^3\sqrt{d+icdx}\sqrt{f-icf}}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(80*a*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32*a
*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 24*b*
d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 16*b*d*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - b*d*f*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 48*a*d^(3/2)*f^(3/2)*Sqrt[1 + c^
2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
+ 4*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(8*Sinh[2*ArcSi
nh[c*x]] + Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

### 3.541.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.61, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6211, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{6201}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+ba\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{244}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int \left(c^2x^3+x\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+bar\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{c^2x^4}{4}\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\big\downarrow$ 6200

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2x^2+\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\big\downarrow$ 15

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3}\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\big\downarrow$ 6198

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsi}}{4bc}\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(-1/4*(b*c*(x^2/2 + (c^2*x^4)/4))
 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (
x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*
c)))/4))/(1 + c^2*x^2)^(3/2)
```

### 3.541.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

### 3.541.4 Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))\, dx$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

### 3.541.5 Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((b*c^2*d*f*x^2 + b*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d*f*x^2 + a*d*f)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f), x)
```

### 3.541.6 Sympy [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (id(cx-i))^{\frac{3}{2}}\,(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))
, x)
```

### 3.541.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.541.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.541.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

# 3.542 $\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.542.1 Optimal result

Integrand size = 35, antiderivative size = 304

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{ibfx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
1/2*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/3*I*f*(c^
2*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/3*I*b*
f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*f*x^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/9*I*b*c^2*f*x^3*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))
^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.542.2 Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{12af\sqrt{d+icdx}\sqrt{f-icfx}(-2i+3cx-2ic^2x^2)+36a\sqrt{d}f^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+\right.}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(12*a*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*I + 3*c*x - (2*I)*c^2*x^2)
 + 36*a*Sqrt[d]*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]] + (9*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c
*x]^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[
1 + c^2*x^2] + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*A
rcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[
c*x]]))/Sqrt[1 + c^2*x^2])/(72*c)
```

### 3.542.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f(1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int (1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int\left(\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c}+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}+\frac{1}{9}ibc^2\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((I/3)*b*x - (b*c*x^2)/4 + (I/9)*b*
c^2*x^3 + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - ((I/3)*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (a + b*ArcSinh[c*x])^2/(4*b*c)))/Sqrt[
1 + c^2*x^2]
```

### 3.542.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.542.4 Maple [F]

$$\int (-icfx+f)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx)) \sqrt{icdx+d}dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

### 3.542.5 Fricas [F]

$$\int \sqrt{d+icdx}(f - icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((-I*b*c*f*x + b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a*c*f*x + a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f), x)
```

### 3.542.6 Sympy [F]

$$\int \sqrt{d+icdx}(f - icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{id(cx-i)}(-if(cx+i))^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x)), x
)
```

### 3.542.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.542.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

### 3.542.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

# 3.543 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$



## 3.543.1 Optimal result

Integrand size = 35, antiderivative size = 266

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
-2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)-1/2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)+2*I*b*f^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1
/4*b*c*f^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*f
^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

### 3.543.2 Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{16ibcfx\sqrt{d+icdx}\sqrt{f-icfx}-16iaf\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c}}{}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
((16*I)*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a*f*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*f*x*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*f*(4*I + c*x)*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*f*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Cosh[2*ArcSinh[c*x]] + 12*a*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f
*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*d*Sqrt[1 +
 c^2*x^2])
```

### 3.543.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$$

$$\Big\downarrow \ 6211$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^2(1-icx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 27$$

$$\frac{f^2\sqrt{c^2x^2+1}\int \frac{(1-icx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 6253$$

$$\frac{f^2\sqrt{c^2x^2+1}\int \left(-\frac{c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}-\frac{2ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\sqrt{c^2x^2+1}\left(-\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{2i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{3(a+b\text{arcsinh}(cx))^2}{4bc}+\frac{1}{4}bcx^2+2ibx\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
(f^2*Sqrt[1 + c^2*x^2]*((2*I)*b*x + (b*c*x^2)/4 - ((2*I)*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x]))/c - (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 +
(3*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

### 3.543.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

3.543. $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

### 3.543.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b\,\text{arcsinh}\,(cx))}{\sqrt{icdx+d}}dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

### 3.543.5 Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f))/(c*d*x - I*d), x)
```

### 3.543.6 Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{\sqrt{id\,(cx-i)}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x
)
```

### 3.543.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.543.8 Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

### 3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

# 3.544 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$



## 3.544.1 Optimal result

Integrand size = 35, antiderivative size = 284

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = -\frac{ibf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{3f^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^3(1+c^2x^2)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
-I*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*(
1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)-3/2*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2
)/(f-I*c*f*x)^(3/2)-4*b*f^3*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2
)/(f-I*c*f*x)^(3/2)
```

### 3.544.2 Mathematica [A] (verified)

Time = 7.01 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.81

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{\frac{2af(5+icx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^2(-i+cx)} - \frac{6af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}} - \frac{bf\sqrt{d}}{}}{}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x
]
```

output
```
((2*a*f*(5 + I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^2*(-I + c*x))
- (6*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]])/d^(3/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*
((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSi
nh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x
]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])) + (2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh
[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh
[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[
c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] -
(-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

### 3.544.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\int \left(\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{4i(cx+i)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{4i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{4b\log(-cx+i)}{c}-ibx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(3/2)*((-I)*b*x + ((4*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x
]))/(c*Sqrt[1 + c^2*x^2]) + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c -
 (3*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (4*b*Log[I - c*x])/c))/((d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.544.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.544. $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.544.4 Maple [F]

$$\int \frac{(-icfx + f)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))}{(icdx + d)^{\frac{3}{2}}} dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

## 3.544.5 Fricas [F]

$$\int \frac{(f - icfx)^{3/2}(a + b\operatorname{arcsinh}(cx))}{(d + icdx)^{3/2}} dx = \int \frac{(-i\,cfx + f)^{\frac{3}{2}}(b \operatorname{arsinh}(cx) + a)}{(i\,cdx + d)^{\frac{3}{2}}} dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((I*b*c*f*x - b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (I*a*c*f*x - a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

### 3.544.6 Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2)
, x)
```

### 3.544.7 Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*I
*sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d
^2*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3/2)*log(c*x + sqrt(c^2*x^2 +
 1))/(I*c*d*x + d)^(3/2), x)
```

### 3.544.8 Giac [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="giac")
```

output
```
Timed out
```

### 3.544.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

# 3.545 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$



## 3.545.1 Optimal result

Integrand size = 35, antiderivative size = 364

$$\begin{aligned}\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx &= \frac{4ibf^4(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{bf^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{2if^4(1-icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{f^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8bf^4(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

output
```
4/3*I*b*f^4*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)-1/2*b*f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+2/3*I*f^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x
)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x
))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x
)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*f^4*(c^2*
x^2+1)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.545.2 Mathematica [A] (verified)

Time = 9.81 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.94

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{-\frac{16af(-i+2cx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^3(-i+cx)^2}+\frac{12af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{5/2}}-}{}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x
]
```

output
```
((-16*a*f*(-I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^3*(-I + c*x
)^2) + (12*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/d^(5/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[Ar
cSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 +
(3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (7*I)
*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]]
 - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*Log[1 + c^2*x^2])) +
2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[
c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I +
3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]
))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Ar
cSinh[c*x]/2])^4) + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[A
rcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(A
rcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1 + c^2*x^2]) + C
osh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*
x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]
 + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*(4 + (2
+ Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c
*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

### 3.545.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$$

$\Big\downarrow$ 6211

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6252

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(-bc\int \left(\frac{2i(1-icx)^3}{3c(c^2x^2+1)^2}-\frac{2i(1-icx)}{c(c^2x^2+1)}+\frac{\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}\right)dx+\frac{2i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{2i(1-icx)(a+b\text{arcsin}}{c\sqrt{c^2x^2+1}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 2009

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(\frac{2i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{2i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}+\frac{\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c}-bc\left(\frac{\text{arcsinh}(cx}{2c^2}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(5/2)*((((2*I)/3)*(1 - I*c*x)^3*(a + b*ArcSinh[c*x]))/(
c*(1 + c^2*x^2)^(3/2)) - ((2*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt[
1 + c^2*x^2]) + (ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*(((-4*I)/3)/(c
^2*(I - c*x)) + ArcSinh[c*x]^2/(2*c^2) - (8*Log[I - c*x])/(3*c^2))))/((d +
 I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.545.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

### 3.545.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{(icdx+d)^{\frac{5}{2}}}dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

### 3.545.5 Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

### 3.545.6 Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/(I*d*(c*x - I))**(5/2)
, x)
```

### 3.545.7 Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*a*(-3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(-3*I*c^4*d^4*x^3 - 9*c^3*d^4*x^2 +
 9*I*c^2*d^4*x + 3*c*d^4) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*f/(c^3*d^3*x^2 - 2
*I*c^2*d^3*x - c*d^3) + 21*I*sqrt(c^2*d*f*x^2 + d*f)*f/(3*I*c^2*d^3*x + 3*
c*d^3) - 3*f^2*arcsinh(c*x)/(c*d^3*sqrt(f/d))) + b*integrate((-I*c*f*x + f
)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(5/2), x)
```

### 3.545.8 Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x
)
```

### 3.545.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

# 3.546 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.546.1 Optimal result

Integrand size = 35, antiderivative size = 344

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\,(1+c^2x^2)^{5/2}}$$
$$-\frac{5bc^3x^4(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\,(1+c^2x^2)^{5/2}}-\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}$$
$$+\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{16\,(1+c^2x^2)^2}+\frac{5x(d+i}{}$$

output
```
-25/96*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)-5/96*
b*c^3*x^4*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)+1/6*x*(d+I
*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))+5/16*x*(d+I*c*d*x)^(5/2
)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5
/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+5/32*(d+I*c*d*x)^(5/2
)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(5/2)-1/36*b*(d+I
*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(c^2*x^2+1)^(1/2)/c
```

### 3.546.2 Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.40

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{1584acd^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+1248ac^3d^2f^2x^3\sqrt{d+icdx}\sqrt{}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(1584*a*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 1248*a*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
x^2] + 384*a*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
c^2*x^2] + 360*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^
2 - 270*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]]
 - 27*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] -
 2*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSinh[c*x]] + 72
0*a*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]] + 12*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + S
inh[6*ArcSinh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2])
```

### 3.546.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.58, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {6211, 6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \ 6211$$

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\int \left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{5/2}}$$

$$\Big\downarrow \ 6201$$

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\int\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}bc\int x\left(c^2x^2+1\right)^2dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 241

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\int\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{b(c^2x^2}{36}\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 6201

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a-\right.\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 244

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int\left(c^2x^3+x\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+\right.\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 2009

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\right.\right.\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 6200

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(c^2x\right.\right.\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 15

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(c^2x^2+\right.\right.\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 6198

---

3.546. $\int(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\right.\right.\right.}{(c^2x^2+1)^{5/2}}$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(-1/36*(b*(1 + c^2*x^2)^3)/c + (x
*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*(-1/4*(b*c*(x^2/2 + (c^2
*x^4)/4)) + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*(-1/4*(b*c
*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x]
)^2/(4*b*c)))/4))/6))/(1 + c^2*x^2)^(5/2)
```

### 3.546.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

## 3.546.4 Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (-icfx+f)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))\,dx$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

## 3.546.5 Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((b*c^4*d^2*f^2*x^4 + 2*b*c^2*d^2*f^2*x^2 + b*d^2*f^2)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^4*d^2*f^2*x^
4 + 2*a*c^2*d^2*f^2*x^2 + a*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f),
 x)
```

### 3.546.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

### 3.546.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.546.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.546.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

# 3.547 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.547.1 Optimal result

Integrand size = 35, antiderivative size = 459

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5(1+c^2x^2)^{3/2}} - \frac{5bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} + \frac{2ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}}{15(1+c^2x^2)^{3/2}} - \frac{bc^3fx^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16(1+c^2x^2)^{3/2}} + \frac{ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}}{25(1+c^2x^2)^{3/2}} + \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{8(1+c^2x^2)} - \frac{if(d+\cdots}{\cdots}$$

output
```
1/5*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*c
*f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+2/15*I*b*c^2*
f*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*f*x
^4*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/25*I*b*c^4*f*x^
5*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I*c*d*x
)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*f*x*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*f*(d+I*c*d*x)^(3/2)*
(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

### 3.547.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.49

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{1200ibcdf^2x\sqrt{d+icdx}\sqrt{f-icfx}-1920iadf^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+}}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((1200*I)*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (1920*I)*a*d*f
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d*f^2*
x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (3840*I)*a*c^2*d
*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^
3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1920*
I)*a*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
 1800*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*
d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(3/2
)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]] + (200*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Sinh[3*ArcSinh[c*x]] + 60*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Arc
Sinh[c*x]*((-10*I)*Cosh[3*ArcSinh[c*x]] - (2*I)*Cosh[5*ArcSinh[c*x]] + 5*(
(-4*I)*Sqrt[1 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]]))
 + (24*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]]
)/(9600*c*Sqrt[1 + c^2*x^2])
```

### 3.547.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int f(1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int(1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int\left(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-icx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{i(c^2x^2+1)^{5/2}}{\right.}{\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((I/5)*b*x - (5*b*c*x^2)/16 + (
(2*I)/15)*b*c^2*x^3 - (b*c^3*x^4)/16 + (I/25)*b*c^4*x^5 + (3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
]))/4 - ((I/5)*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/c + (3*(a + b*Arc
Sinh[c*x])^2)/(16*b*c)))/(1 + c^2*x^2)^(3/2)
```

### 3.547.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.547. $\int(d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.547.4 Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx)) \, dx$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

### 3.547.5 Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx)) \, dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a) \, dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((-I*b*c^3*d*f^2*x^3 + b*c^2*d*f^2*x^2 - I*b*c*d*f^2*x + b*d*f^2)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*
c^3*d*f^2*x^3 + a*c^2*d*f^2*x^2 - I*a*c*d*f^2*x + a*d*f^2)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f), x)
```

### 3.547.6 Sympy [F(-1)]

Timed out.

$$\int (d + icdx)^{3/2}(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

### 3.547.7 Maxima [F(-2)]

Exception generated.

$$\int (d + icdx)^{3/2}(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.547.8 Giac [F(-2)]

Exception generated.

$$\int (d + icdx)^{3/2}(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

### 3.547.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\,\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

# 3.548 $\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$



## 3.548.1 Optimal result

Integrand size = 35, antiderivative size = 416

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{2ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c} + \frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

output
```
3/8*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*f
^2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3*I*f^2*(c
^2*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+2/3*I*b
*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*f^2*
x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f^2*
x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*f^2*x
^4*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*f^2*(a+b*arc
sinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.548.2 Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.36

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{576ibcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-768iaf^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+432acf^2x}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((576*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (768*I)*a*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 432*a*c*f^2*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (768*I)*a*c^2*f^2*x^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 288*a*c^3*f^2*x^3*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 360*b*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 144*b*f^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 9*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*Cosh[4*ArcSinh[c*x]] + 720*a*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[
c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (64*I)*b*
f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 12*b*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-48*I)*Sqrt[1 + c^2*x^2]
- (16*I)*Cosh[3*ArcSinh[c*x]] + 24*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh
[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

### 3.548.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f^2(1-icx)^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 27$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int (1-icx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 6253$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int \left(-c^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x^2-2ic\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x+\sqrt{c^2x^2+1}(a\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 2009$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{2i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c}-\frac{1}{4}c^2x^3\sqrt{c^2x^2+1}(a+ba\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(((2*I)/3)*b*x - (3*b*c*x^2)/16 +
 ((2*I)/9)*b*c^2*x^3 + (b*c^3*x^4)/16 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x]))/8 - (c^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 - (((2*I
)/3)*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (5*(a + b*ArcSinh[c*x])
^2)/(16*b*c)))/Sqrt[1 + c^2*x^2]
```

### 3.548.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.548. $\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.548.4 Maple [F]

$$\int (-icfx+f)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx)) \sqrt{icdx+d}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

### 3.548.5 Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-(b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x
 - a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.548.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

output
```
Timed out
```

### 3.548.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.548.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.548.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

# 3.549 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$



## 3.549.1 Optimal result

Integrand size = 35, antiderivative size = 381

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
-11/3*I*f^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)-3/2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+1/3*I*c*f^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)+11/3*I*b*f^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I
*c*f*x)^(1/2)+3/4*b*c*f^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)-1/9*I*b*c^2*f^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+5/4*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(
1/2)/(f-I*c*f*x)^(1/2)
```

### 3.549.2 Mathematica [A] (verified)

Time = 7.17 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.22

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{264ibcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-8ibc^3f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
((264*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (8*I)*b*c^3*f^2*x
^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (264*I)*a*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (24*I)*a*c^2*f^2*x^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^2 + 27*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[
2*ArcSinh[c*x]] - 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]
*(9*(5*I + 2*c*x)*Sqrt[1 + c^2*x^2] - I*Cosh[3*ArcSinh[c*x]]) + 180*a*Sqrt
[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]])/(72*c*d*Sqrt[1 + c^2*x^2])
```

### 3.549.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}dx$$

$$\Big\downarrow \ 6211$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 27$$

$$\frac{f^3\sqrt{c^2x^2+1}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \; 6253$$

$$\frac{f^3\sqrt{c^2x^2+1}\int \left(\frac{ic^3(a+b\text{arcsinh}(cx))x^3}{\sqrt{c^2x^2+1}}-\frac{3c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}-\frac{3ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \; 2009$$

$$\frac{f^3\sqrt{c^2x^2+1}\left(\frac{1}{3}icx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{3}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{11i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}+\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
(f^3*Sqrt[1 + c^2*x^2]*(((11*I)/3)*b*x + (3*b*c*x^2)/4 - (I/9)*b*c^2*x^3 -
 (((11*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c - (3*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/2 + (I/3)*c*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]) + (5*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

### 3.549.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.549. $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.549.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))}{\sqrt{icdx+d}}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

## 3.549.5 Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(((I*b*c^2*f^2*x^2 - 2*b*c*f^2*x - I*b*f^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*f^2*x^2 - 2*a*c*f^2
*x - I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

### 3.549.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

output
```
Timed out
```

### 3.549.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.549.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.549.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

# 3.550 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$



## 3.550.1 Optimal result

Integrand size = 35, antiderivative size = 518

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx &= -\frac{3ibf^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &+ \frac{bcf^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &+ \frac{15bf^4(1+c^2x^2)^{3/2}\text{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^4(1-icx)^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &+ \frac{15if^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5if^4(1-icx)(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\ &- \frac{15f^4(1+c^2x^2)^{3/2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bf^4(1+c^2x^2)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\end{aligned}$$

output
```
-3/2*I*b*f^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*f
^4*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*f^4*(1-
I*c*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*f^
4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2
*I*f^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)+15/2*I*f^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3
/2)/(f-I*c*f*x)^(3/2)+5/2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))
/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*f^4*(c^2*x^2+1)^(3/2)*arcsinh(
c*x)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^4*(c^2
*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

### 3.550.2 Mathematica [A] (verified)

Time = 9.86 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.50

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{\frac{4af^2\sqrt{d+icdx}\sqrt{f-icfx}(24+7icx+c^2x^2)}{d^2(-i+cx)} - \frac{60af^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}}}{}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x
]
```

output
```
((4*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 + (7*I)*c*x + c^2*x^2))/
(d^2*(-I + c*x)) - (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]])/d^(3/2) - (4*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2
]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((
4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2
] + I*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 + I*Sinh[ArcSinh[c*x]/2])) + (16*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*
x - 4*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSin
h[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2]
)*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/
(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) +
(b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSi
nh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - (Cosh[2*ArcSinh[c*x]] + 8*((2*I)*c*
x + (4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(
8 - 8*Sqrt[1 + c^2*x^2] + I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*(
(8*I)*(1 + Sqrt[1 + c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(d^2*Sqrt[1 + c^2
*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

### 3.550.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.550. $\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}dx$$
$$\Big\downarrow \text{ 6211}$$
$$\frac{(c^2x^2+1)^{3/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{f^4(c^2x^2+1)^{3/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$\Big\downarrow \text{ 6252}$$
$$\frac{f^4(c^2x^2+1)^{3/2}\left(-bc\int \left(\frac{x}{2}-\frac{15\text{arcsinh}(cx)}{2c\sqrt{c^2x^2+1}}+\frac{4i}{c}+\frac{8i(1-icx)}{c(c^2x^2+1)}\right)dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{4i\sqrt{c^2x^2+1}(a+\cdots}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{f^4(c^2x^2+1)^{3/2}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{8i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{15\text{arcsi}\cdots}{}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(3/2)*(((8*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt
[1 + c^2*x^2]) + ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (x*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (15*ArcSinh[c*x]*(a + b*ArcSinh[c
*x]))/(2*c) - b*c*(((4*I)*x)/c + x^2/4 - (15*ArcSinh[c*x]^2)/(4*c^2) + (8*
Log[I - c*x])/c^2)))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.550.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.550. $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}dx$

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

## 3.550.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))}{(icdx+d)^{\frac{3}{2}}}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

## 3.550.5 Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x
 - a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x
 - d^2), x)
```

### 3.550.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

output
```
Timed out
```

### 3.550.7 Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="maxima")
```

output
```
1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x
)/(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a + b*integrat
e((-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/2), x
)
```

### 3.550.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.550.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

# 3.551 $\int \frac{(f-icfx)^{5/2}(a+b\,\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$



## 3.551.1 Optimal result

Integrand size = 35, antiderivative size = 472

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\,\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx &= \frac{ibf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{8ibf^5(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{2if^5(1-icx)^4(1+c^2x^2)(a+b\,\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{10if^5(1-icx)^2(1+c^2x^2)^2(a+b\,\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{5if^5(1+c^2x^2)^3(a+b\,\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{5f^5(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\,\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28bf^5(1+c^2x^2)^{5/2}\log(i-cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

output
```
I*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*f^
5*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*f^
5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2
/3*I*f^5*(1-I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)-10/3*I*f^5*(1-I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/
(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5*I*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x
))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*f^5*(c^2*x^2+1)^(5/2)*arcsinh(c
*x)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*f^5*(c
^2*x^2+1)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.551.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1005 vs. 2(472) = 944.

Time = 12.47 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.13

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x
]
```

output
```
(((-4*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 - (34*I)*c*x + 3*c
^2*x^2))/(d^3*(-I + c*x)^2) + (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/d^(5/2) - (2*b*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(
3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Ta
nh[ArcSinh[c*x]/2]] + (7*I)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*A
rcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2
 + 21*Log[1 + c^2*x^2])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 +
(56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x
^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((2*I)*b*f^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I
)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
- (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] -
 (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sq
rt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcS
inh[c*x]/2]] + (I/2)*(4 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[
ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*
x]/2])^4) + (b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(I*Cosh[ArcSinh[...
```

### 3.551.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^5(1-icx)^5(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^5(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\left(-bc\int \left(\frac{2i(1-icx)^4}{3c(c^2x^2+1)^2}-\frac{20i(1-icx)}{3c(c^2x^2+1)}+\frac{5\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}-\frac{5i}{3c}\right)dx+\frac{2i(1-icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{20i(1-icx)(a\ldots}{3c\sqrt{\ldots}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\left(\frac{2i(1-icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{20i(1-icx)(a+b\text{arcsinh}(cx))}{3c\sqrt{c^2x^2+1}}-\frac{5i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}+\frac{5\text{arcsinh}(cx)(a\ldots}{c}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^5*(1 + c^2*x^2)^(5/2)*((((2*I)/3)*(1 - I*c*x)^4*(a + b*ArcSinh[c*x]))/(
c*(1 + c^2*x^2)^(3/2)) - (((20*I)/3)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*
Sqrt[1 + c^2*x^2]) - (((5*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c
+ (5*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*(((-I)*x)/c - ((8*I)/3)/(c
^2*(I - c*x)) + (5*ArcSinh[c*x]^2)/(2*c^2) - (28*Log[I - c*x])/(3*c^2))))/
((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.551.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

## 3.551.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}}\,(a+b\,\operatorname{arcsinh}\,(cx))}{(icdx+d)^{\frac{5}{2}}}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

### 3.551.5 Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((-I*b*c^2*f^2*x^2 + 2*b*c*f^2*x + I*b*f^2)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*f^2*x^2 + 2*a*c*f
^2*x + I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c
^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

### 3.551.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

output
```
Timed out
```

### 3.551.7 Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*(3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*d^5*x^4 - 4*I*c^4*d^5*x^3 - 6*c^3
*d^5*x^2 + 4*I*c^2*d^5*x + c*d^5) - 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*f/(-3*I
*c^4*d^4*x^3 - 9*c^3*d^4*x^2 + 9*I*c^2*d^4*x + 3*c*d^4) + 10*I*sqrt(c^2*d*
f*x^2 + d*f)*f^2/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 105*I*sqrt(c^2*d*
f*x^2 + d*f)*f^2/(3*I*c^2*d^3*x + 3*c*d^3) - 15*f^3*arcsinh(c*x)/(c*d^3*sq
rt(f/d)))*a + b*integrate((-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1)
)/(I*c*d*x + d)^(5/2), x)
```

## 3.551.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## 3.551.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

# 3.552 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$



## 3.552.1 Optimal result

Integrand size = 35, antiderivative size = 381

$$\begin{aligned}\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx &= -\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{11id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{icd^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
11/3*I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-3/2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)-1/3*I*c*d^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)-11/3*I*b*d^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*
c*f*x)^(1/2)+3/4*b*c*d^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+1/9*I*b*c^2*d^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+5/4*d^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1
/2)/(f-I*c*f*x)^(1/2)
```

### 3.552.2 Mathematica [A] (verified)

Time = 7.34 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.22

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{-264ibcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+8ibc^3d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
((-264*I)*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (8*I)*b*c^3*d^2*
x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (264*I)*a*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (24*I)*a*c^2*d^2*x^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*ArcSinh[c*x]^2 + 27*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[2*ArcSinh[c*x]] - 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]*(9*(-5*I + 2*c*x)*Sqrt[1 + c^2*x^2] + I*Cosh[3*ArcSinh[c*x]]) + 180*a*d^
(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]])/(72*c*f*Sqrt[1 + c^2*x^2])
```

### 3.552.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d^3(icx+1)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^3\sqrt{c^2x^2+1}\int \frac{(icx+1)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^3\sqrt{c^2x^2+1}\int \left(-\frac{ic^3(a+b\text{arcsinh}(cx))x^3}{\sqrt{c^2x^2+1}}-\frac{3c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}+\frac{3ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\sqrt{c^2x^2+1}\left(-\frac{1}{3}icx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{3}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{11i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
(d^3*Sqrt[1 + c^2*x^2]*(((-11*I)/3)*b*x + (3*b*c*x^2)/4 + (I/9)*b*c^2*x^3
+ (((11*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c - (3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/2 - (I/3)*c*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x]) + (5*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

### 3.552.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

## 3.552. $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.552.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))}{\sqrt{-icfx+f}}dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

## 3.552.5 Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(((-I*b*c^2*d^2*x^2 - 2*b*c*d^2*x + I*b*d^2)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*d^2*x^2 - 2*a*c*d
^2*x + I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

### 3.552.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

output
```
Timed out
```

### 3.552.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.552.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.552.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

# 3.553 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$



## 3.553.1 Optimal result

Integrand size = 35, antiderivative size = 266

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = -\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)-1/2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-2*I*b*d^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/
4*b*c*d^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*d^
2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)
```

### 3.553.2 Mathematica [A] (verified)

Time = 5.55 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{-16ibcdx\sqrt{d+icdx}\sqrt{f-icfx}+16iad\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
((-16*I)*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a*d*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*d*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*d*(-4*I + c*x)*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*d*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*Cosh[2*ArcSinh[c*x]] + 12*a*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*f*Sqrt[1
 + c^2*x^2])
```

### 3.553.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d^2(icx+1)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\sqrt{c^2x^2+1}\int \frac{(icx+1)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \; 6253$$

$$\frac{d^2\sqrt{c^2x^2+1}\int\left(-\frac{c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}+\frac{2ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^2\sqrt{c^2x^2+1}\left(-\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{2i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{3(a+b\text{arcsinh}(cx))^2}{4bc}+\frac{1}{4}bcx^2-2ibx\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
(d^2*Sqrt[1 + c^2*x^2]*((-2*I)*b*x + (b*c*x^2)/4 + ((2*I)*Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x]))/c - (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 +
 (3*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
)
```

### 3.553.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

3.553. $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

### 3.553.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}}(a+b\,\mathrm{arcsinh}(cx))}{\sqrt{-icfx+f}}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

### 3.553.5 Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f))/(c*f*x + I*f), x)
```

### 3.553.6 Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x
)
```

### 3.553.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.553.8 Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

### 3.553.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

# 3.554 $\int \frac{\sqrt{d+icdx}(a+b\,\mathbf{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$



## 3.554.1 Optimal result

Integrand size = 35, antiderivative size = 158

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = -\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I
*b*d*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*d*(a+b*ar
csinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

## 3.554.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{-2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right)+2ib\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b\sqrt{d+}}{2cf\sqrt{1+c^2x^{}}}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
((-2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2])
+ (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x] + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sq
rt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]])/(2*c*f*Sqrt[1 + c^2*x^2])
```

### 3.554.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\sqrt{c^2x^2+1}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\sqrt{c^2x^2+1}\int \left(\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{c^2x^2+1}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc}-ibx\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
(d*Sqrt[1 + c^2*x^2]*((-I)*b*x + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
)/c + (a + b*ArcSinh[c*x])^2/(2*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
])
```

### 3.554.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.554.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{icdx+d}}{\sqrt{-icfx+f}}dx$$

input
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

### 3.554.5 Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2
+ 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*f*x + I*f), x)
```

### 3.554.6 Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\operatorname{asinh}(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)
```

### 3.554.7 Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
a*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + b*i
ntegrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x + f)
, x)
```

### 3.554.8 Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

### 3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

# 3.555 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$



## 3.555.1 Optimal result

Integrand size = 35, antiderivative size = 59

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)
```

## 3.555.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.92

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{a\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (a*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]])/(c*Sqrt[d]*Sqrt[f])
```

### 3.555.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {6211, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

#### 3.555.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

### 3.555.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}dx$$

input
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

### 3.555.5 Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f*x^2 + d*f), x)
```

### 3.555.6 Sympy [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{a+b\,\mathrm{asinh}\,(cx)}{\sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))), x)
```

### 3.555.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{b\,\text{arsinh}\,(cx)^2}{2\,\sqrt{df}c}+\frac{a\,\text{arsinh}\,(cx)}{\sqrt{df}c}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a*arcsinh(c*x)/(sqrt(d*f)*c)
```

### 3.555.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)
```

### 3.555.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

# 3.556 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$



## 3.556.1 Optimal result

Integrand size = 35, antiderivative size = 111

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
f*(c*x+I)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)-b*f*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

## 3.556.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b(i-cx)\text{lo}\right.}{cd^2f(-i+cx)\sqrt{1+c^2x^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x] + b*(I - c*x)*Log[d + I*c*d*x]))/(c*d^2*f*(-I + c*x)*Sq
rt[1 + c^2*x^2])
```

### 3.556.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {6211, 27, 6252, 27, 451, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(c^2x^2+1)^{3/2}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-bc\int \frac{cx+i}{c(c^2x^2+1)}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-b\int \frac{cx+i}{c^2x^2+1}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{451}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(b\int \frac{1}{i-cx}dx+\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{b\log(-cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f*(1 + c^2*x^2)^(3/2)*(((I + c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt[1 + c^2*x
^2]) - (b*Log[I - c*x])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.556.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 451
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c^2/a   In
t[1/(c - d*x), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

## 3.556.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{(icdx + d)^{\frac{3}{2}} \sqrt{-icfx + f}} dx$$

input
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

### 3.556.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(89) = 178$.

Time = 0.31 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.99

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \frac{2\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}b\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(c^2d^2fx - i\,cd^2f\right)\sqrt{\frac{b^2}{c^2d^3}}}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
 + (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2
+ 2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f) - (I*c^9*d^2*f*x^4 + 2*c^8*d^2*f*x^3 + I*c^7*d^2*f*x^2 + 2*c^6*d^2*f
*x)*sqrt(b^2/(c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^2
*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c
^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) -
 (-I*c^9*d^2*f*x^4 - 2*c^8*d^2*f*x^3 - I*c^7*d^2*f*x^2 - 2*c^6*d^2*f*x)*sq
rt(b^2/(c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*f*x - I*c*d^2*f)
```

### 3.556.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{\left(id\,(cx - i)\right)^{\frac{3}{2}}\sqrt{-if\,(cx + i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))),
 x)
```

### 3.556.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{i\sqrt{c^2dfx^2+df}b\,\text{arsinh}\,(cx)}{i\,c^2d^2fx+cd^2f} + \frac{i\sqrt{c^2dfx^2+df}a}{i\,c^2d^2fx+cd^2f} - \frac{b\log\,(i\,cx+1)}{cd^{\frac{3}{2}}\sqrt{f}}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(I*c^2*d^2*f*x + c*d^2*f) + I*sqr
t(c^2*d*f*x^2 + d*f)*a/(I*c^2*d^2*f*x + c*d^2*f) - b*log(I*c*x + 1)/(c*d^(
3/2)*sqrt(f))
```

### 3.556.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)), x
)
```

### 3.556.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(d+c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

# 3.557 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$



## 3.557.1 Optimal result

Integrand size = 35, antiderivative size = 295

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
1/3*I*b*f^2*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)+2/3*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)-1/3*I*b*f^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*f^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d
+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.557.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.48

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-2i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)+b(-2i+cx\right.}{3cd^3f(-i+cx)^2\sqrt{1+c^2}}$$

input

```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

output

```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-2*I + c*x)*((-I)*b + b*c*x + a*Sqr
t[1 + c^2*x^2]) + b*(-2*I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*(-I +
c*x)^2*Log[d + I*c*d*x]))/(3*c*d^3*f*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

### 3.557.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\left(-bc\int \left(\frac{x}{3(c^2x^2+1)}+\frac{2i(1-icx)}{3c(c^2x^2+1)^2}\right)dx+\frac{x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{2i(1-icx)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2(c^2x^2+1)^{5/2}\left(\frac{x(a+b\operatorname{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{2i(1-icx)(a+b\operatorname{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(\frac{i\arctan(cx)}{3c^2}+\frac{i(cx+i)}{3c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{6c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f^2*(1 + c^2*x^2)^(5/2)*((((2*I)/3)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*
(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b*
c*(((I/3)*(I + c*x))/(c^2*(1 + c^2*x^2)) + ((I/3)*ArcTan[c*x])/c^2 + Log[1
 + c^2*x^2]/(6*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.557.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

3.557. $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

### 3.557.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}dx$$

input
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

### 3.557.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(228) = 456$.

Time = 0.34 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.95

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx =$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx-2\left(bc^2x^2-i\,bcx+2\,b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
-1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(
b*c^2*x^2 - I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (c^4*d^3*f*x^3 - I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*
d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^
4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*f*x
^4 + 2*c^8*d^3*f*x^3 + I*c^7*d^3*f*x^2 + 2*c^6*d^3*f*x)*sqrt(b^2/(c^2*d^5*
f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^4*d^3*f*x^3 - I*c^3*d^3
*f*x^2 + c^2*d^3*f*x - I*c*d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6
*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f) + (-I*c^9*d^3*f*x^4 - 2*c^8*d^3*f*x^3 - I*c^7*d^3*f*x^2 - 2*c^6
*d^3*f*x)*sqrt(b^2/(c^2*d^5*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b))
- 2*(a*c^2*x^2 - I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4
*d^3*f*x^3 - I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*d^3*f)
```

### 3.557.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{(id(cx-i))^{\frac{5}{2}}\sqrt{-if(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I))),
 x)
```

### 3.557.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.79

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{1}{3}\,bc\left(\frac{3}{3i\,c^3d^{\frac{5}{2}}\sqrt{f}x+3\,c^2d^{\frac{5}{2}}\sqrt{f}} - \frac{\log(cx-i)}{c^2d^{\frac{5}{2}}\sqrt{f}}\right) - \frac{1}{3}\,b\left(\frac{i\sqrt{c^2dfx^2+df}}{c^3d^3fx^2-2i\,c^2d^3fx-cd^3f} - \frac{3i\sqrt{c^2dfx^2+df}}{3i\,c^2d^3fx+3\,cd^3f}\right)\operatorname{arsinh}(cx) - \frac{1}{3}\,a\left(\frac{i\sqrt{c^2dfx^2+df}}{c^3d^3fx^2-2i\,c^2d^3fx-cd^3f} - \frac{3i\sqrt{c^2dfx^2+df}}{3i\,c^2d^3fx+3\,cd^3f}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
1/3*b*c*(3/(3*I*c^3*d^(5/2)*sqrt(f)*x + 3*c^2*d^(5/2)*sqrt(f)) - log(c*x -
 I)/(c^2*d^(5/2)*sqrt(f))) - 1/3*b*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*f*x
^2 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3
*f*x + 3*c*d^3*f))*arcsinh(c*x) - 1/3*a*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^
3*f*x^2 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^
2*d^3*f*x + 3*c*d^3*f))
```

### 3.557.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)), x
)
```

### 3.557.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{(d+c\,d\,x\,1\text{i})^{5/2}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

# 3.558 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$



## 3.558.1 Optimal result

Integrand size = 35, antiderivative size = 517

$$
\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx &= \frac{3ibd^4x(1+c^2x^2)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{bcd^4x^2(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\,(1+c^2x^2)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{15bd^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)^2}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{15id^4(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{5id^4(1+icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{15d^4(1+c^2x^2)^{3/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8bd^4(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

output
```
3/2*I*b*d^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*d^
4*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*d^4*(1+I
*c*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*d^4
*(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*
I*d^4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-15/2*I*d^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/
2)/(f-I*c*f*x)^(3/2)-5/2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/
c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*d^4*(c^2*x^2+1)^(3/2)*arcsinh(c
*x)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^4*(c^2*
x^2+1)^(3/2)*ln(c*x+I)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

## 3.558.2 Mathematica [A] (verified)

Time = 10.37 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.51

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{4ad^2\sqrt{d+icdx}\sqrt{f-icfx}(24-7icx+c^2x^2)}{f^2(i+cx)} - \frac{60ad^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}}}{}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x
]
```

output
```
((4*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 - (7*I)*c*x + c^2*x^2))/
(f^2*(I + c*x)) - (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]])/f^(3/2) + (4*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))
+ 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4
*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2
] + Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])) + (16*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x
- 4*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x
]/2] + Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*
Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f
^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (b
*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (16*c*x + 32*ArcTan[Tanh[ArcSinh[c*x]
/2]] + I*Cosh[2*ArcSinh[c*x]] + (8*I)*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*
x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 -
8*Sqrt[1 + c^2*x^2] - I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((-8*
I)*(1 + Sqrt[1 + c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(f^2*Sqrt[1 + c^2*x^
2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

### 3.558.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \; 6211$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{d^4(icx+1)^4(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^4(c^2x^2+1)^{3/2}\int \frac{(icx+1)^4(a+b\operatorname{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$\downarrow$ 6252

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\left(-bc\int\left(\frac{x}{2}-\frac{15\text{arcsinh}(cx)}{2c\sqrt{c^2x^2+1}}-\frac{4i}{c}-\frac{8i(icx+1)}{c(c^2x^2+1)}\right)dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{4i\sqrt{c^2x^2+1}(a+b}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$\downarrow$ 2009

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{8i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{15\text{arcsin}}{}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(3/2)*(((-8*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqr
t[1 + c^2*x^2]) - ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (x*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (15*ArcSinh[c*x]*(a + b*ArcSinh[
c*x]))/(2*c) - b*c*(((-4*I)*x)/c + x^2/4 - (15*ArcSinh[c*x]^2)/(4*c^2) + (
8*Log[I + c*x])/c^2)))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.558.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

3.558. $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

### 3.558.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))}{(-icfx+f)^{\frac{3}{2}}} dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

### 3.558.5 Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x
 - a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x
 - f^2), x)
```

### 3.558.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

### 3.558.7 Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x
)/(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a + b*integrat
e((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x
)
```

### 3.558.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

# 3.559 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$



## 3.559.1 Optimal result

Integrand size = 35, antiderivative size = 283

$$\begin{aligned}\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx &= \frac{ibd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\ &- \frac{4id^3(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\ &- \frac{3d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bd^3(1+c^2x^2)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\end{aligned}$$

output
```
I*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(1
+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)-3/2*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2)
/(f-I*c*f*x)^(3/2)-4*b*d^3*(c^2*x^2+1)^(3/2)*ln(c*x+I)/c/(d+I*c*d*x)^(3/2)
/(f-I*c*f*x)^(3/2)
```

### 3.559.2 Mathematica [A] (verified)

Time = 7.29 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.82

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{2ad(5-icx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(i+cx)} - \frac{6ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}} + \frac{bd\sqrt{d+}}{}}{}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x
]
```

output
```
((2*a*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x)) -
 (6*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]])/f^(3/2) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]
^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)
*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c
*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/
2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2
])) + (2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[A
rcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c
*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/
2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-
2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cos
h[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

### 3.559.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \; 6259$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{4i(i-cx)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{4i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{4b\log(cx+i)}{c}+ibx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(3/2)*(I*b*x - ((4*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))
/(c*Sqrt[1 + c^2*x^2]) - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c - (3
*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (4*b*Log[I + c*x])/c))/((d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2))
```

### 3.559.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

---

3.559. $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

### 3.559.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}}\,(a+b\operatorname{arcsinh}(cx))}{(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

### 3.559.5 Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((-I*b*c*d*x - b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (-I*a*c*d*x - a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
+ f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

### 3.559.6 Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2)
, x)
```

### 3.559.7 Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6*
I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c
*f^2*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2)*log(c*x + sqrt(c^2*x^2
+ 1))/(-I*c*f*x + f)^(3/2), x)
```

### 3.559.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to transpose Error: Bad Argu
ment Valuesym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error:
```

### 3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

# 3.560 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$



## 3.560.1 Optimal result

Integrand size = 35, antiderivative size = 180

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = -\frac{2id^2(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bd^2\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
-2*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-1/2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d^2*(c^2*x^2+1)^(3/2)*ln(c*x+I)/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)
```

## 3.560.2 Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{i+cx} - 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right.}{}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
((4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 2*a*Sqrt[d]*Sqrt[f]
*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (b*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] +
Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*
x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]
*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c*f^2)
```

## 3.560.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(c^2x^2+1)^{3/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^2(c^2x^2+1)^{3/2}\int \left(-\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-\frac{2i(i-cx)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2(c^2x^2+1)^{3/2}\left(-\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{2b\log(cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^2*(1 + c^2*x^2)^(3/2)*(((-2*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqr
t[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^2/(2*b*c) - (2*b*Log[I + c*x])/c))/
((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.560.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.560.4 Maple [F]

$$\int \frac{(a + b\,\mathrm{arcsinh}\,(cx))\,\sqrt{icdx + d}}{(-icfx + f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

## 3.560.5 Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*f^2*x^2 + 2*I*c*f^2*x
- f^2), x)
```

## 3.560.6 Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2), x
)
```

## 3.560.7 Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(c
*f^2*sqrt(d/f))) + b*integrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 +
1))/(-I*c*f*x + f)^(3/2), x)
```

## 3.560.8 Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(3/2), x)
```

## 3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

# 3.561 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$



## 3.561.1 Optimal result

Integrand size = 35, antiderivative size = 112

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
-d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)-b*d*(c^2*x^2+1)^(3/2)*ln(c*x+I)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)
```

## 3.561.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{f-icfx}\left(a+iacx+(b+ibcx)\text{arcsinh}(cx)-ib\sqrt{1+c^2x^2}\log(d(-1+ic\right.}{cf^2(i+cx)\sqrt{d+icdx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

output
```
(Sqrt[f - I*c*f*x]*(a + I*a*c*x + (b + I*b*c*x)*ArcSinh[c*x] - I*b*Sqrt[1
+ c^2*x^2]*Log[d*(-1 + I*c*x)]))/(c*f^2*(I + c*x)*Sqrt[d + I*c*d*x])
```

### 3.561.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6211, 27, 6252, 25, 27, 451, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(c^2x^2+1)^{3/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d(c^2x^2+1)^{3/2}\left(-bc\int -\frac{i-cx}{c(c^2x^2+1)}dx-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{d(c^2x^2+1)^{3/2}\left(bc\int \frac{i-cx}{c(c^2x^2+1)}dx-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(c^2x^2+1)^{3/2}\left(b\int \frac{i-cx}{c^2x^2+1}dx-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{451}$$

$$\frac{d(c^2x^2+1)^{3/2}\left(-b\int \frac{1}{cx+i}dx-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{d(c^2x^2+1)^{3/2}\left(-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{b\log(cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(3/2)*(-(((I - c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt[1 + c^2
*x^2])) - (b*Log[I + c*x])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.561.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 451
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c^2/a   In
t[1/(c - d*x), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

3.561. $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$

### 3.561.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{(-icfx + f)^{\frac{3}{2}} \sqrt{icdx + d}} dx$$

input
```
int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

### 3.561.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(88) = 176.

Time = 0.30 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.96

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{3/2}} dx = \frac{2\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}b\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \left(c^2df^2x + i\,cdf^2\right)\sqrt{\frac{b^2}{c^2df^3}}}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
 - (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2
 + 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f) - (I*c^9*d*f^2*x^4 - 2*c^8*d*f^2*x^3 + I*c^7*d*f^2*x^2 - 2*c^6*d*f^
2*x)*sqrt(b^2/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^
2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2 + 2*b
*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
 - (-I*c^9*d*f^2*x^4 + 2*c^8*d*f^2*x^3 - I*c^7*d*f^2*x^2 + 2*c^6*d*f^2*x)*
sqrt(b^2/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f^2*x + I*c*d*f^2)
```

### 3.561.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{id\,(cx-i)}\,(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)),
 x)
```

### 3.561.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=-\frac{i\sqrt{c^2dfx^2+df}\,b\,\text{arsinh}\,(cx)}{-i\,c^2df^2x+cdf^2}-\frac{i\sqrt{c^2dfx^2+df}\,a}{-i\,c^2df^2x+cdf^2}-\frac{b\log\,(i\,cx-1)}{c\sqrt{d}f^{\frac{3}{2}}}$$

input
```
integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
-I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(-I*c^2*d*f^2*x + c*d*f^2) - I*s
qrt(c^2*d*f*x^2 + d*f)*a/(-I*c^2*d*f^2*x + c*d*f^2) - b*log(I*c*x - 1)/(c*
sqrt(d)*f^(3/2))
```

### 3.561.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)), x
)
```

### 3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

# 3.562 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$



## 3.562.1 Optimal result

Integrand size = 35, antiderivative size = 103

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b(1+c^2x^2)^{3/2}\log(1+c^2x^2)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b
*(c^2*x^2+1)^(3/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

## 3.562.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\frac{i\sqrt{f-icfx}\left(2acx+2bcx\text{arcsinh}(cx)-b\sqrt{1+c^2x^2}\log(d(-1+icx))-b\right.}{2cdf^2(i+cx)\sqrt{d+icdx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x
]
```

output
```
((I/2)*Sqrt[f - I*c*f*x]*(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*
x^2]*Log[d*(-1 + I*c*x)] - b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(c*d*f^2
*(I + c*x)*Sqrt[d + I*c*d*x])
```

### 3.562.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {6211, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
((1 + c^2*x^2)^(3/2)*((x*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*Log[
1 + c^2*x^2])/(2*c)))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.562.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

## 3.562.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{3}{2}}\,(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

## 3.562.5 Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
1/4*(4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*x*log(c*x + sqrt(c^2*x^2 + 1
)) + 4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*x + (c^2*d^2*f^2*x^2 + d^2*f
^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^
4*x^4 + 2*b*c^2*x^2 + b)) - (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*
f^3))*log((b*c^2*x^4 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 +
b)) - 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3
 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2
/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sq
rt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)
) + 4*(c^2*d^2*f^2*x^2 + d^2*f^2)*integral(-sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f)*b*c*x/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*
f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f^2)
```

## 3.562.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{a+b\operatorname{asinh}(cx)}{(id(cx-i))^{\frac{3}{2}}(-if(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3/2
)), x)
```

## 3.562.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\frac{bx\operatorname{arsinh}(cx)}{\sqrt{c^2dfx^2+df}\,df}+\frac{ax}{\sqrt{c^2dfx^2+df}\,df}-\frac{b\sqrt{\frac{1}{df}}\log\left(x^2+\frac{1}{c^2}\right)}{2\,cdf}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a*x/(sqrt(c^2*d*f*x^2 + d
*f)*d*f) - 1/2*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*f)
```

### 3.562.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right)+a}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2)),
 x)
```

### 3.562.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{\left(d+c\,d\,x\,1\text{i}\right)^{3/2}\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

# 3.563 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$



## 3.563.1 Optimal result

Integrand size = 35, antiderivative size = 282

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{ibf(1+c^2x^2)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf(1+c^2x^2)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
1/6*I*b*f*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+
1/3*f*(c*x+I)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-1/6*I*b*f*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)-1/3*b*f*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2
)/(f-I*c*f*x)^(5/2)
```

### 3.563.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.71

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{f-icfx}\left(4ia+8acx+8iac^2x^2+2b\sqrt{1+c^2x^2}+4b(i+2cx+2ic^2x^2)\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x
]
```

output
```
(Sqrt[f - I*c*f*x]*((4*I)*a + 8*a*c*x + (8*I)*a*c^2*x^2 + 2*b*Sqrt[1 + c^2
*x^2] + 4*b*(I + 2*c*x + (2*I)*c^2*x^2)*ArcSinh[c*x] + 3*b*(-1 - I*c*x)*Sq
rt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] - 5*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*
x] - (5*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*d^2*f^2*Sqrt[d +
 I*c*d*x]*(c + c^3*x^2))
```

### 3.563.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f(c^2x^2+1)^{5/2}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6252}$$

$$\frac{f(c^2x^2+1)^{5/2}\left(-bc\int \left(\frac{2x}{3(c^2x^2+1)}+\frac{cx+i}{3c(c^2x^2+1)^2}\right)dx+\frac{2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{(cx+i)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f(c^2x^2+1)^{5/2}\left(\frac{2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{(cx+i)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(\frac{i\arctan(cx)}{6c^2}-\frac{1-icx}{6c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{3c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
(f*(1 + c^2*x^2)^(5/2)*(((I + c*x)*(a + b*ArcSinh[c*x]))/(3*c*(1 + c^2*x^2
)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b*c*(-1/6*(1
 - I*c*x)/(c^2*(1 + c^2*x^2)) + ((I/6)*ArcTan[c*x])/c^2 + Log[1 + c^2*x^2]
/(3*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.563.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

3.563. $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

### 3.563.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

### 3.563.5 Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
-1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*
(2*b*c^2*x^2 - 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) - 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f
^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c
^2*x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 3*(c^4*d^3*f^2*
x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f
^3))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*
f*x*sqrt(b^2/(c^2*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^
2 + b*c*x + I*b)) + 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x
 - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c^2*
x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 3*(c^4*d^3*f^2*x^3
 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3)
)*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*
x*sqrt(b^2/(c^2*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2
+ b*c*x + I*b)) + 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x -
 I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + b*c^2*x^3 + b*
x)/(b*c^2*x^2 + b)) - 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*...
```

### 3.563.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

### 3.563.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.84

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\frac{1}{12}\,bc\left(-\frac{2i\,\sqrt{d}\sqrt{f}}{c^3d^3f^2x-i\,c^2d^3f^2}-\frac{3\,\log\left(cx+i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}-\frac{5\,\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}\right)-\frac{1}{3}\,b\left(-\frac{3i}{3i\,\sqrt{c^2dfx^2+df}\,c^2d^2fx+3\,\sqrt{c^2dfx^2+df}\,cd^2f}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(-\frac{3i}{3i\,\sqrt{c^2dfx^2+df}\,c^2d^2fx+3\,\sqrt{c^2dfx^2+df}\,cd^2f}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
1/12*b*c*(-2*I*sqrt(d)*sqrt(f)/(c^3*d^3*f^2*x - I*c^2*d^3*f^2) - 3*log(c*x
 + I)/(c^2*d^(5/2)*f^(3/2)) - 5*log(c*x - I)/(c^2*d^(5/2)*f^(3/2))) - 1/3*
b*(-3*I/(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*
f)*c*d^2*f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))*arcsinh(c*x) - 1/3*a*(-
3*I/(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c
*d^2*f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))
```

### 3.563.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
,0,0]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,
0,0,0]ext
```

### 3.563.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{3/2}} \, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(d + c\,d\,x\,1\text{i})^{5/2}\,(f - c\,f\,x\,1\text{i})^{3/2}} \, dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

# 3.564 $\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$



## 3.564.1 Optimal result

Integrand size = 35, antiderivative size = 470

$$\begin{aligned}\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx &= -\frac{ibd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{8ibd^5(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &- \frac{2id^5(1+icx)^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{10id^5(1+icx)^2(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{5id^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\ &+ \frac{5d^5(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{28bd^5(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

output
```
-I*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*d
^5*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*d
^5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
2/3*I*d^5*(1+I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+10/3*I*d^5*(1+I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*
x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*d^5*(c^2*x^2+1)^(5/2)*arcsinh(
c*x)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*d^5*(
c^2*x^2+1)^(5/2)*ln(c*x+I)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.564.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1083 vs. $2(470) = 940$.

Time = 15.42 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.30

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x
]
```

output
```
(((4*I)*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 + (34*I)*c*x + 3*c^
2*x^2))/(f^3*(I + c*x)^2) + (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/f^(5/2) - ((2*I)*b*d^2*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Co
sh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I
/2)*Log[1 + c^2*x^2])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*Ar
cTan[Coth[ArcSinh[c*x]/2]] + ((3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*Ar
cSinh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt
[1 + c^2*x^2]*((2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + L
og[1 + c^2*x^2]))/2)*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (2*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSin
h[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] - 7*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSi
nh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 21*Log[
1 + c^2*x^2]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(Ar
cSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] +
 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSin
h[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - (I*b*d^2*Sqrt[d + I*c*d*x]*Sqr...
```

### 3.564.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d^5(icx+1)^5(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^5(c^2x^2+1)^{5/2}\int \frac{(icx+1)^5(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d^5(c^2x^2+1)^{5/2}\left(-bc\int \left(-\frac{2i(icx+1)^4}{3c(c^2x^2+1)^2}+\frac{20i(icx+1)}{3c(c^2x^2+1)}+\frac{5\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}+\frac{5i}{3c}\right)dx-\frac{2i(1+icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{20i(1+icx)}{3c}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^5(c^2x^2+1)^{5/2}\left(-\frac{2i(1+icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{20i(1+icx)(a+b\text{arcsinh}(cx))}{3c\sqrt{c^2x^2+1}}+\frac{5i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}+5\text{arcsinh}(cx)(\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^5*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*(1 + I*c*x)^4*(a + b*ArcSinh[c*x]))/
(c*(1 + c^2*x^2)^(3/2)) + (((20*I)/3)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c
*Sqrt[1 + c^2*x^2]) + (((5*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c
 + (5*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*((I*x)/c - ((8*I)/3)/(c^2
*(I + c*x)) + (5*ArcSinh[c*x]^2)/(2*c^2) - (28*Log[I + c*x])/(3*c^2))))/((
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.564.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

## 3.564.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))}{(-icfx+f)^{\frac{5}{2}}}dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

### 3.564.5 Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral((((I*b*c^2*d^2*x^2 + 2*b*c*d^2*x - I*b*d^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*d^2*x^2 + 2*a*c*d^2
*x - I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2
*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

### 3.564.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

### 3.564.7 Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*(-3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*f^5*x^4 + 4*I*c^4*f^5*x^3 - 6*c^
3*f^5*x^2 - 4*I*c^2*f^5*x + c*f^5) + 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*d/(3*I
*c^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9*I*c^2*f^4*x + 3*c*f^4) - 10*I*sqrt(c^2*d*
f*x^2 + d*f)*d^2/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 105*I*sqrt(c^2*d*
f*x^2 + d*f)*d^2/(-3*I*c^2*f^3*x + 3*c*f^3) - 15*d^3*arcsinh(c*x)/(c*f^3*s
qrt(d/f)))*a + b*integrate((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1)
)/(-I*c*f*x + f)^(5/2), x)
```

### 3.564.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.564.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

# 3.565 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$



## 3.565.1 Optimal result

Integrand size = 35, antiderivative size = 362

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{4ibd^4(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^4(1+icx)^3\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2id^4(1+icx)\,(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8bd^4(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
4/3*I*b*d^4*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)-1/2*b*d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)-2/3*I*d^4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x
)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x
))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x
)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*d^4*(c^2*
x^2+1)^(5/2)*ln(c*x+I)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.565.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.95

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{-\frac{16ad(i+2cx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^3(i+cx)^2}+\frac{12ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{5/2}}-\frac{2i}{}}{}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x
]
```

output
```
((-16*a*d*(I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^3*(I + c*x)^
2) + (12*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/f^(5/2) - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cos
h[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(A
rcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*Log[1 + c^2*x^2])) +
Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]
] + ((3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c*x] + (4*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2*x^2]*((2*I)*ArcS
inh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))/2)*Sinh
[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^4) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]
/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh
[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 7*Log[1 + c^2*
x^2]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 -
(84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) - (2*I)*(4 + (4
*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
+ 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[
c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[Ar
cSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*
x]/2])^4))/(12*c)
```

### 3.565.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$$

$\big\downarrow$ 6211

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d^4(icx+1)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{d^4(c^2x^2+1)^{5/2}\int \frac{(icx+1)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\big\downarrow$ 6252

$$\frac{d^4(c^2x^2+1)^{5/2}\left(-bc\int\left(-\frac{2i(icx+1)^3}{3c(c^2x^2+1)^2}+\frac{2i(icx+1)}{c(c^2x^2+1)}+\frac{\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}\right)dx-\frac{2i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{2i(1+icx)(a+b\text{arcs}}{c\sqrt{c^2x^2+1}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\big\downarrow$ 2009

$$\frac{d^4(c^2x^2+1)^{5/2}\left(-\frac{2i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}+\frac{\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c}-bc\left(\frac{\text{arcsinh}(}{2c^2}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*(1 + I*c*x)^3*(a + b*ArcSinh[c*x]))/
(c*(1 + c^2*x^2)^(3/2)) + ((2*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt
[1 + c^2*x^2]) + (ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*(((-4*I)/3)/(
c^2*(I + c*x)) + ArcSinh[c*x]^2/(2*c^2) - (8*Log[I + c*x])/(3*c^2))))/((d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.565.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.565. $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

### 3.565.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))}{(-icfx+f)^{\frac{5}{2}}}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

### 3.565.5 Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

### 3.565.6 Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(5/2)
, x)
```

### 3.565.7 Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*a*(3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(3*I*c^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9
*I*c^2*f^4*x + 3*c*f^4) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*d/(c^3*f^3*x^2 + 2*I
*c^2*f^3*x - c*f^3) - 21*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-3*I*c^2*f^3*x + 3*c
*f^3) - 3*d^2*arcsinh(c*x)/(c*f^3*sqrt(d/f))) + b*integrate((I*c*d*x + d)^
(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(5/2), x)
```

### 3.565.8 Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x
)
```

### 3.565.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

# 3.566 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$



## 3.566.1 Optimal result

Integrand size = 35, antiderivative size = 185

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\frac{2ibd^3(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{id^3(1+icx)^3\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^3(1+c^2x^2)^{5/2}\log(i+cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
2/3*I*b*d^3*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)-1/3*I*d^3*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)+1/3*b*d^3*(c^2*x^2+1)^(5/2)*ln(c*x+I)/c/(d+I*c*d*x)^(5/
2)/(f-I*c*f*x)^(5/2)
```

## 3.566.2 Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=-\frac{id\sqrt{f-icfx}\left((-i+cx)\left(-ia+acx+b\sqrt{1+c^2x^2}\right)+b(-i+cx)^2\text{arcsinh}(cx)-b(i+cx)\sqrt{1+c^2x^2}\log(d\right.}{3cf^3(i+cx)^2\sqrt{d+icdx}}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
((-1/3*I)*d*Sqrt[f - I*c*f*x]*((-I + c*x)*((-I)*a + a*c*x + b*Sqrt[1 + c^2
*x^2]) + b*(-I + c*x)^2*ArcSinh[c*x] - b*(I + c*x)*Sqrt[1 + c^2*x^2]*Log[d
*(-1 + I*c*x)]))/(c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

## 3.566.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6211, 27, 6252, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(c^2x^2+1)^{5/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d^3(c^2x^2+1)^{5/2}\left(-bc\int -\frac{i(icx+1)^3}{3c(c^2x^2+1)^2}dx-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(c^2x^2+1)^{5/2}\left(\frac{1}{3}ib\int \frac{(icx+1)^3}{(c^2x^2+1)^2}dx-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{456}$$

$$\frac{d^3(c^2x^2+1)^{5/2}\left(\frac{1}{3}ib\int \frac{icx+1}{(1-icx)^2}dx-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(\frac{1}{3}ib\int\left(-\frac{i}{cx+i}-\frac{2}{(cx+i)^2}\right)dx-\frac{i(1+icx)^3(a+b\operatorname{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(\frac{1}{3}ib\left(\frac{2}{c(cx+i)}-\frac{i\log(cx+i)}{c}\right)-\frac{i(1+icx)^3(a+b\operatorname{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(5/2)*(((-1/3*I)*(1 + I*c*x)^3*(a + b*ArcSinh[c*x]))/(c
*(1 + c^2*x^2)^(3/2)) + (I/3)*b*(2/(c*(I + c*x)) - (I*Log[I + c*x])/c)))/(
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.566.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

---

3.566. $\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

### 3.566.4 Maple [F]

$$\int \frac{(a + b\ \mathrm{arcsinh}\,(cx))\ \sqrt{icdx + d}}{(-icfx + f)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

### 3.566.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(141) = 282$.

Time = 0.33 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{d + icdx}(a + b\mathrm{arcsinh}(cx))}{(f - icfx)^{5/2}} dx =$$

$$-\frac{4\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,bcx + 2\left(bc^2x^2 - 2i\,bcx - b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx + \sqrt{c^2x^2 \cdots\right.}{\cdots}$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(
b*c^2*x^2 - 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c*f^3)*s
qrt(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*sqrt
(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*f^3*x^4 - 2*c^
8*f^3*x^3 + I*c^7*f^3*x^2 - 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(b*c^3*x^3
 + I*b*c^2*x^2 + b*c*x + I*b)) - (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x
+ I*c*f^3)*sqrt(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I
*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*f
^3*x^4 + 2*c^8*f^3*x^3 - I*c^7*f^3*x^2 + 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)
))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*(a*c^2*x^2 - 2*I*a*c*x - a
)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2
*f^3*x + I*c*f^3)
```

## 3.566.6 Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(5/2), x
)
```

## 3.566.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = &-\frac{1}{3}\,bc\left(\frac{6\,\sqrt{d}}{3i\,c^3f^{\frac{5}{2}}x-3\,c^2f^{\frac{5}{2}}}-\frac{\sqrt{d}\log\left(cx+i\right)}{c^2f^{\frac{5}{2}}}\right)\\ &-\frac{1}{3}\,b\left(-\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2f^3x+3\,cf^3}\right)\text{arsinh}\left(cx\right)\\ &-\frac{1}{3}\,a\left(-\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2f^3x+3\,cf^3}\right)\end{aligned}$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*b*c*(6*sqrt(d)/(3*I*c^3*f^(5/2)*x - 3*c^2*f^(5/2)) - sqrt(d)*log(c*x
+ I)/(c^2*f^(5/2))) - 1/3*b*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2
*I*c^2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*
f^3))*arcsinh(c*x) - 1/3*a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2*
I*c^2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f
^3))
```

## 3.566.8 Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)
```

## 3.566.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}}{(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

# 3.567 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$



## 3.567.1 Optimal result

Integrand size = 35, antiderivative size = 294

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx &= \frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}\\ &- \frac{2id^2(1+icx)\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\ &+ \frac{ibd^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^2(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

output
```
1/3*I*b*d^2*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)-2/3*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+1/3*I*b*d^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*d^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d
+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.567.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{f-icfx}\left((2i+cx)\left(a+iacx+ib\sqrt{1+c^2x^2}\right)+ib(2+icx+c^2x^2)\,\text{arcsin}\right.}{3cf^3(i+cx)^2\sqrt{d+icdx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

output
```
(Sqrt[f - I*c*f*x]*((2*I + c*x)*(a + I*a*c*x + I*b*Sqrt[1 + c^2*x^2]) + I*
b*(2 + I*c*x + c^2*x^2)*ArcSinh[c*x] + b*(1 - I*c*x)*Sqrt[1 + c^2*x^2]*Log
[d*(-1 + I*c*x)]))/(3*c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

### 3.567.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}\left(-bc\int \left(\frac{x}{3(c^2x^2+1)}-\frac{2i(icx+1)}{3c(c^2x^2+1)^2}\right)dx+\frac{x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2(c^2x^2+1)^{5/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(-\frac{i\arctan(cx)}{3c^2}+\frac{i(-cx+i)}{3c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{6c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

output
```
(d^2*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c
*(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b
*c*(((I/3)*(I - c*x))/(c^2*(1 + c^2*x^2)) - ((I/3)*ArcTan[c*x])/c^2 + Log[
1 + c^2*x^2]/(6*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.567.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

---

3.567. $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

### 3.567.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}dx$$

input
```
int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

### 3.567.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(228) = 456.

Time = 0.33 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.96

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx-2\left(bc^2x^2+i\,bcx+2\,b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2\right.}}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
-1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(
b*c^2*x^2 + I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) - (c^4*d*f^3*x^3 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*
d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c
^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d*f^3*
x^4 - 2*c^8*d*f^3*x^3 + I*c^7*d*f^3*x^2 - 2*c^6*d*f^3*x)*sqrt(b^2/(c^2*d*f
^5)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^4*d*f^3*x^3 + I*c^3*d*
f^3*x^2 + c^2*d*f^3*x + I*c*d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c
^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f) + (-I*c^9*d*f^3*x^4 + 2*c^8*d*f^3*x^3 - I*c^7*d*f^3*x^2 + 2*c
^6*d*f^3*x)*sqrt(b^2/(c^2*d*f^5)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)
) - 2*(a*c^2*x^2 + I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c
^4*d*f^3*x^3 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*d*f^3)
```

### 3.567.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{a+b\operatorname{asinh}(cx)}{\sqrt{id(cx-i)}\,(-if(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2)),
 x)
```

### 3.567.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.79

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=-\frac{1}{3}\,bc\left(\frac{3}{3i\,c^3\sqrt{d}f^{\frac{5}{2}}x-3\,c^2\sqrt{d}f^{\frac{5}{2}}}+\frac{\log(cx+i)}{c^2\sqrt{d}f^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(-\frac{i\sqrt{c^2dfx^2+df}}{c^3df^3x^2+2i\,c^2df^3x-cdf^3}+\frac{3i\sqrt{c^2dfx^2+df}}{-3i\,c^2df^3x+3\,cdf^3}\right)\operatorname{arsinh}(cx)-\frac{1}{3}\,a\left(-\frac{i\sqrt{c^2dfx^2+df}}{c^3df^3x^2+2i\,c^2df^3x-cdf^3}+\frac{3i\sqrt{c^2dfx^2+df}}{-3i\,c^2df^3x+3\,cdf^3}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
-1/3*b*c*(3/(3*I*c^3*sqrt(d)*f^(5/2)*x - 3*c^2*sqrt(d)*f^(5/2)) + log(c*x
+ I)/(c^2*sqrt(d)*f^(5/2))) - 1/3*b*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d*f^3
*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*
d*f^3*x + 3*c*d*f^3))*arcsinh(c*x) - 1/3*a*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^
3*d*f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3
*I*c^2*d*f^3*x + 3*c*d*f^3))
```

### 3.567.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{5/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{i\,cdx + d}(-i\,cfx + f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)), x
)
```

### 3.567.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{\sqrt{d + c\,d\,x\,1\text{i}}\,(f - c\,f\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

# 3.568 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$



## 3.568.1 Optimal result

Integrand size = 35, antiderivative size = 282

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx &= \frac{ibd(1+c^2x^2)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}}\\ &-\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2dx(1+c^2x^2)^2\left(a+b\text{arcsinh}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}\\ &+\frac{ibd(1+c^2x^2)^{5/2}\arctan(cx)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd(1+c^2x^2)^{5/2}\log\left(1+c^2x^2\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

output
```
1/6*I*b*d*(c^2*x^2+1)^(5/2)/c/(c*x+I)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
1/3*d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+1/6*I*b*d*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)-1/3*b*d*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2
)/(f-I*c*f*x)^(5/2)
```

### 3.568.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.72

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{f-icfx}\left(4ia-8acx+8iac^2x^2-2b\sqrt{1+c^2x^2}+4ib(1+2icx+2c^2x^2\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x
]
```

output
```
(Sqrt[f - I*c*f*x]*((4*I)*a - 8*a*c*x + (8*I)*a*c^2*x^2 - 2*b*Sqrt[1 + c^2
*x^2] + (4*I)*b*(1 + (2*I)*c*x + 2*c^2*x^2)*ArcSinh[c*x] + 5*b*(1 - I*c*x)
*Sqrt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] + 3*b*Sqrt[1 + c^2*x^2]*Log[d + I*c
*d*x] - (3*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*c*d*f^3*(I +
c*x)^2*Sqrt[d + I*c*d*x])
```

### 3.568.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6252}$$

$$\frac{d\left(c^2x^2+1\right)^{5/2}\left(-bc\int \left(\frac{2x}{3\left(c^2x^2+1\right)}-\frac{i-cx}{3c\left(c^2x^2+1\right)^2}\right)dx+\frac{2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{3c\left(c^2x^2+1\right)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(c^2x^2+1)^{5/2}\left(\frac{2x(a+b\operatorname{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{(-cx+i)(a+b\operatorname{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(-\frac{i\arctan(cx)}{6c^2}-\frac{1+icx}{6c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{3c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(5/2)*(-1/3*((I - c*x)*(a + b*ArcSinh[c*x]))/(c*(1 + c^2*
x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b*c*(-1/6
*(1 + I*c*x)/(c^2*(1 + c^2*x^2)) - ((I/6)*ArcTan[c*x])/c^2 + Log[1 + c^2*x
^2]/(3*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.568.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

3.568. $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

### 3.568.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{(icdx + d)^{\frac{3}{2}} (-icfx + f)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

### 3.568.5 Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{3/2}(f - icfx)^{5/2}} dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(i\,cdx + d)^{\frac{3}{2}} (-i\,cfx + f)^{\frac{5}{2}}} dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*
(2*b*c^2*x^2 + 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) - 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f
^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c
^2*x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 5*(c^4*d^2*f^3*
x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f
^5))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^
2*x*sqrt(b^2/(c^2*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^
2 + b*c*x + I*b)) + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x
 + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c^2*
x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 5*(c^4*d^2*f^3*x^3
 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5)
)*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*
x*sqrt(b^2/(c^2*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2
+ b*c*x + I*b)) + 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x +
 I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + b*c^2*x^3 + b*
x)/(b*c^2*x^2 + b)) - 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*...
```

### 3.568.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

### 3.568.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx &= \frac{1}{12}\,bc\left(\frac{2i\,\sqrt{d}\sqrt{f}}{c^3d^2f^3x+i\,c^2d^2f^3}-\frac{5\,\log\left(cx+i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}-\frac{3\,\log\left(cx-i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}\right)\\&-\frac{1}{3}\,b\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}\,c^2df^2x+3\,\sqrt{c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,df^2}\right)\operatorname{arsinh}\left(cx\right)\\&-\frac{1}{3}\,a\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}\,c^2df^2x+3\,\sqrt{c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,df^2}\right)\end{aligned}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
1/12*b*c*(2*I*sqrt(d)*sqrt(f)/(c^3*d^2*f^3*x + I*c^2*d^2*f^3) - 5*log(c*x
+ I)/(c^2*d^(3/2)*f^(5/2)) - 3*log(c*x - I)/(c^2*d^(3/2)*f^(5/2))) - 1/3*b
*(3*I/(-3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f
)*c*d*f^2) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))*arcsinh(c*x) - 1/3*a*(3*
I/(-3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*
d*f^2) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))
```

### 3.568.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
,0,0]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,
0,0,0]ext
```

### 3.568.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{{(d+c\,d\,x\,1\mathrm{i})}^{3/2}\,{(f-c\,f\,x\,1\mathrm{i})}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

# 3.569 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$



## 3.569.1 Optimal result

Integrand size = 35, antiderivative size = 203

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{b(1+c^2x^2)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
1/6*b*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*(c^2*x
^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x*(c^2*x^
2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*(c^2*x
^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

## 3.569.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{i\sqrt{f-icfx}\Big(6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\,\text{arcsinh}(cx)}{6cd}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x
]
```

output
```
((I/6)*Sqrt[f - I*c*f*x]*(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*
b*c*x*(3 + 2*c^2*x^2)*ArcSinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[d*(-1 + I
*c*x)] - 2*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x] - 2*b*c^2*x^2*Sqrt[1 + c^2
*x^2]*Log[d + I*c*d*x]))/(c*d^2*f^3*(-I + c*x)*(I + c*x)^2*Sqrt[d + I*c*d*
x])
```

## 3.569.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6211, 6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6203}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\int \frac{x}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
((1 + c^2*x^2)^(5/2)*(b/(6*c*(1 + c^2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*
(1 + c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b
*Log[1 + c^2*x^2])/(2*c)))/3))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.569.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.569. $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

## 3.569.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

## 3.569.5 Fricas [F]

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/6*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x^2 - 2*(
2*b*c^2*x^3 + 3*b*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c
^2*x^2 + 1)) - (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c
^2*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c
*d^2*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^4 + b*x^2)/(b*c^4*x^4 + 2*b
*c^2*x^2 + b)) + (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/
(c^2*d^5*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*c*d^2*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^4 - b*x^2)/(b*c^4*x^4 +
2*b*c^2*x^2 + b)) + 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt
(b^2/(c^2*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f)*c*d^2*f^2*x*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 +
b)) - 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d^5*
f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f
^2*x*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 2*(2*a*
c^2*x^3 + 3*a*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 6*(c^4*d^3*f^3*x^4
 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*integral(-2/3*sqrt(c^2*x^2 + 1)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f)*b*c*x/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^
3*f^3), x))/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

### 3.569.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

### 3.569.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}f^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}}df}+\frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f^2}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}}df}+\frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f^2}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
1/6*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^2
 + 1)/(c^2*d^(5/2)*f^(5/2))) + 1/3*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f) +
2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a*(x/((c^2*d*f*x
^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))
```

### 3.569.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
,0,0]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,
0,0,0]ext
```

### 3.569.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(d + c\,d\,x\,1\text{i})^{5/2}\,(f - c\,f\,x\,1\text{i})^{5/2}}\, dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

# 3.570 $\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.570.1 Optimal result

Integrand size = 37, antiderivative size = 680

$$\begin{aligned}
\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{8ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c} \\
&+\frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx}-\frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} \\
&+\frac{4ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} \\
&-\frac{15b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} \\
&-\frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&-\frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&-\frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+\frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&+\frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c} \\
&+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
8/9*I*b^2*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*d^2*x*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*d^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*
c*f*x)^(1/2)+4/27*I*b^2*d^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)/c+3/8*d^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4
*c^2*d^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*
I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
/c-15/64*b^2*d^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x
^2+1)^(1/2)-4/3*I*b*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)
^(1/2)/(c^2*x^2+1)^(1/2)-3/8*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1
/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-4/9*I*b*c^2*d^2*x^3*(a+b*arcsinh(c
*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*d^2*x
^4*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2
)+5/24*d^2*(a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c
^2*x^2+1)^(1/2)
```

### 3.570.2 Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.31

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-6912iabcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+4608ia^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+69}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((-6912*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (4608*I)*a^2*
d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (6912*I)*b^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*d^2*
x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (4608*I)*a^2*c^2
*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*
c^3*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b
^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*d^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (256*I)*b^2*d^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*d^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*S
qrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]] + 864*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2
*ArcSinh[c*x]] - (768*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[
3*ArcSinh[c*x]] - 27*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*Ar
cSinh[c*x]] + 12*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((
-576*I)*b*c*x + (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] +
 (192*I)*a*Cosh[3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*
ArcSinh[c*x]] - (64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]])
 + 72*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (48
*I)*b*Sqrt[1 + c^2*x^2] + (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*A...
```

### 3.570.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d^2(icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int\left(-c^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+2icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\sqrt{c^2x^2+1}\right.}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{8}bc^3x^4(a+b\text{arcsinh}(cx))-\frac{4}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)\right.$$

input
```
Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((8*I)/9)*b^2*Sqrt[1 + c^2*x^2]
)/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 - (b^2*c^2*x^3*Sqrt[1 + c^2*x^2])/32
 + (((4*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c - (15*b^2*ArcSinh[c*x])/(64*c) -
 ((4*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8 -
 ((4*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (b*c^3*x^4*(a + b*ArcSinh[c*x]
))/8 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*x^3*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x])^2)/4 + (((2*I)/3)*(1 + c^2*x^2)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(24*b*c)))/Sqrt[1 + c^2
*x^2]
```

### 3.570.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.570.4 Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))^2 \sqrt{-icfx+f}dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

## 3.570.5 Fricas [F]

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a + b\text{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*d^2*x^2 - 2
*I*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) - (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.570.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

output
```
Timed out
```

### 3.570.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.570.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.570.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

# 3.571 $\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.571.1 Optimal result

Integrand size = 37, antiderivative size = 508

$$\begin{aligned}
&\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\frac{4ib^2d\sqrt{d+icdx}\sqrt{f-icfx}}{9c}\\
&+\frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx}+\frac{2ib^2d\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c}\\
&-\frac{b^2d\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}\\
&-\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}}\\
&-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\
&-\frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}\\
&+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\
&+\frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c}\\
&+\frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
4/9*I*b^2*d*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*d*x*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)+2/27*I*b^2*d*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*
f*x)^(1/2)/c+1/2*d*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1
/2)+1/3*I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)
^(1/2)/c-1/4*b^2*d*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2
*x^2+1)^(1/2)-2/3*I*b*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)
^(1/2)/(c^2*x^2+1)^(1/2)-1/2*b*c*d*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2
)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d*x^3*(a+b*arcsinh(c*x))
*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*d*(a+b*arcsinh(
c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.571.2 Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.39

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-108iabcdx\sqrt{d+icdx}\sqrt{f-icfx}+72ia^2d\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+108ib^2d\cdots}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((-108*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (72*I)*a^2*d*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (108*I)*b^2*d*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*d*x*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*d*x^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*d*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*Cosh[3*ArcSinh[c*x]] + 108*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c
*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*d*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a + (3*I)*b*Sqrt[1 + c^2*x^2]
 + I*b*Cosh[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*d*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*d*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((-9*I
)*b*c*x + (9*I)*a*Sqrt[1 + c^2*x^2] + (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*S
inh[2*ArcSinh[c*x]] - I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2]
)
```

### 3.571.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d(icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int \left(icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$d\sqrt{d+icdx}\sqrt{f-icfx}\left(-\frac{2}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(\right.$$

input
```
Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((4*I)/9)*b^2*Sqrt[1 + c^2*x^2])/
c + (b^2*x*Sqrt[1 + c^2*x^2])/4 + (((2*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c -
 (b^2*ArcSinh[c*x])/(4*c) - ((2*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (b*c*x^2*
(a + b*ArcSinh[c*x]))/2 - ((2*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (x*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 + ((I/3)*(1 + c^2*x^2)^(3/2)*(a
+ b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(6*b*c)))/Sqrt[1 + c^2*x^2
]
```

#### 3.571.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.571.4 Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2 \sqrt{-icfx+f}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

### 3.571.5 Fricas [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((I*b^2*c*d*x + b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*d*x - a*b*d)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*d*x + a^2*d)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.571.6 Sympy [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (id(cx-i))^{\frac{3}{2}}\sqrt{-if(cx+i)}(a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2
, x)
```

### 3.571.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.571.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

### 3.571.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a + b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

# 3.572 $\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\text{arcsinh}(cx))^2\,dx$



## 3.572.1 Optimal result

Integrand size = 37, antiderivative size = 244

$$\begin{aligned}
&\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\text{arcsinh}(cx))^2\,dx \\
&= \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b^2\sqrt{d+icdx}\sqrt{f-icfx}\,\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}} \\
&\quad + \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\text{arcsinh}(cx))^2 \\
&\quad + \frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
1/4*b^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*
(f-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(a+b*arcsinh(c*x))*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsinh(c*x))^3*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.572.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.44

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{12a^2cx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+4b^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)^3-6ab\sqrt{d+icdx}\sqrt{f-icf}}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(12*a^2*c*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 6*a*b*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 12*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1
+ c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]] + 3*b^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - 6*b
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(b*Cosh[2*ArcSinh[c*x]]
- 2*a*Sinh[2*ArcSinh[c*x]]) + 6*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcS
inh[c*x]^2*(2*a + b*Sinh[2*ArcSinh[c*x]]))/(24*c*Sqrt[1 + c^2*x^2])
```

### 3.572.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {6211, 6200, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6200}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-bc\int x(a+b\text{arcsinh}(cx))dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2\right)}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 6191

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(\right.}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 262

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)+\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\right.}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 222

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}\right.\right.}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 6198

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right.\right.\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])^2)/2 + (a + b*ArcSinh[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*ArcSinh[c*x
]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)))/
Sqrt[1 + c^2*x^2]
```

### 3.572.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

### 3.572.4 Maple [F]

$$\int (a + b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{icdx + d}\,\sqrt{-icfx + f}dx$$

input
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

### 3.572.5 Fricas [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\int \sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2, x)
```

### 3.572.6 Sympy [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\int \sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))^2\;dx$$

input
```
integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2, x
)
```

### 3.572.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.572.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

### 3.572.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \int (a+b\operatorname{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

# 3.573 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$



## 3.573.1 Optimal result

Integrand size = 37, antiderivative size = 259

$$\begin{aligned}\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx &= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2ib^2f(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\&\quad+\frac{2ib^2fx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\\&\quad-\frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\&\quad+\frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
-2*I*b^2*f*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*f*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*a*b*f*x*
(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*b^2*f*x*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*f*(a+b*arcs
inh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

### 3.573.2 Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$$
$$= \frac{-3i\sqrt{d+icdx}\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)+6ib\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{\right.}}{}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
((-3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2
*x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) + (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*(a - I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 +
 c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]])/(3*c*d*Sqrt[1 + c^2*x^2])
```

### 3.573.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$$
$$\Big\downarrow \text{ 6211}$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{f(1-icx)(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{f\sqrt{c^2x^2+1}\int \frac{(1-icx)(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \text{ 6253}$$

$$\frac{f\sqrt{c^2x^2+1}\int\left(\frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{icx(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\sqrt{c^2x^2+1}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{c}+\frac{(a+b\operatorname{arcsinh}(cx))^3}{3bc}+2iabx+2ib^2x\operatorname{arcsinh}(cx)-\frac{2ib^2\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
(f*Sqrt[1 + c^2*x^2]*((2*I)*a*b*x - ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c + (2*I
)*b^2*x*ArcSinh[c*x] - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (a
 + b*ArcSinh[c*x])^3/(3*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

### 3.573.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

3.573. $\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

### 3.573.4 Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{\sqrt{icdx+d}}dx$$

input
```
int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

### 3.573.5 Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2 - 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c
^2*x^2 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*d*x - I*d),
x)
```

### 3.573.6 Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)), x
)
```

### 3.573.7 Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
a^2*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + i
ntegrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(I*c*d*
x + d) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*
x + d), x)
```

### 3.573.8 Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

### 3.573.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{f-c\,f\,x\,1\text{i}}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

# 3.574 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$



## 3.574.1 Optimal result

Integrand size = 37, antiderivative size = 544

$$\begin{aligned}\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx &= \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{f^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{8ibf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4bf^2(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4b^2f^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{4b^2f^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2b^2f^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\end{aligned}$$

output
```
2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)+2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)+2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(
f-I*c*f*x)^(3/2)-1/3*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c
*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x
))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b
*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/
c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(
2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*
f^2*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(
3/2)/(f-I*c*f*x)^(3/2)-2*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

### 3.574.2 Mathematica [A] (verified)

Time = 5.32 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \frac{\frac{6a^2\sqrt{d+icdx}\sqrt{f-icfx}}{-i+cx} - 3a^2\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-ic}\right.}{}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x
]
```

output
```
((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) - 3*a^2*Sqrt[d]*Sq
rt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] -
 (3*a*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[Arc
Sinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]
/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Lo
g[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1
+ c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*((-6 + 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]
/2] - Sinh[ArcSinh[c*x]/2]) - ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2]) + (12*I)*Pi*(Log[1 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^Ar
cSinh[c*x]] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*
x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 24*PolyLog[2, I
/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSi
nh[c*x]*(Pi - (4*I)*Log[1 - I/E^ArcSinh[c*x]])*((-I)*Cosh[ArcSinh[c*x]/2]
+ Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])))/(3*c*d^2)
```

### 3.574.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2i(cx+i)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(-\frac{8ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{2x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}(c}{3bc}\right.}{}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^2*(1 + c^2*x^2)^(3/2)*((2*(a + b*ArcSinh[c*x])^2)/c + ((2*I)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (2*x*(a + b*ArcSinh[c*x])^2)/Sqrt[1
+ c^2*x^2] - (a + b*ArcSinh[c*x])^3/(3*b*c) - ((8*I)*b*(a + b*ArcSinh[c*x]
)*ArcTan[E^ArcSinh[c*x]])/c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSi
nh[c*x])])/c - (4*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c + (4*b^2*PolyLog[
2, I*E^ArcSinh[c*x]])/c - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d
+ I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.574.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.574.4 Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

## 3.574.5 Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}\,(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x
^2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d^2*x^2 - 2*I*c*
d^2*x - d^2), x)
```

## 3.574.6 Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3/2)
, x)
```

## 3.574.7 Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c
*d^2*sqrt(f/d))) + integrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2/(I*c*d*x + d)^(3/2) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2
*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

## 3.574.8 Giac [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\,dx = \int \frac{\sqrt{-i\,cfx + f}\,(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(3/2), x
)
```

## 3.574.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2\,\sqrt{f - c\,f\,x\,1\text{i}}}{(d + c\,d\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

# 3.575 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$



## 3.575.1 Optimal result

Integrand size = 37, antiderivative size = 518

$$\begin{aligned}
\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= -\frac{f^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4ib^2f^3(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{if^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4bf^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4b^2f^3(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
-1/3*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-4/3*I*b^2*f^3*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*x))
/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*f^3*(c^2*x^2+1)^(5/2)*(a+b*ar
csinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+2/3*b*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*a
rcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*f^3*(c^2*x^2+1)
^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*
I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*b*f^3*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)+4/3*b^2*f^3*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(
c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.575.2 Mathematica [A] (warning: unable to verify)

Time = 9.85 (sec) , antiderivative size = 783, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(-\frac{2ia^2}{3d^3(-i+cx)^2}-\frac{a^2}{3d^3(-i+cx)}\right)}{c}$$

$$+\frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\,(1+c^2x^2)}\left(\cosh\left(\frac{1}{2}\text{arcsinh}(cx)\right)-i\sinh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\left(-i\cosh\right.}{}$$

$$+\frac{ib^2(i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\,(1+c^2x^2)}\left((-1+i)\text{arcsinh}(cx)^2-\frac{2\text{arcsinh}(cx)(-2i+\text{arcsi}}{-i+cx}\right.}{}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x
]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-2*I)/3)*a^2)/(d^3*(-I +
c*x)^2) - a^2/(3*d^3*(-I + c*x))))/c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]
*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] -
2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[
c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*L
og[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Cot
h[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*A
rcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]
/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSi
nh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*(I + c*x)*Sqrt[I*((-I
)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I
)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*
I)*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x
] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(P
i + (2*I)*ArcSinh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh
[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2
] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x)
)]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
```

### 3.575.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\int\left(\frac{i(a+b\text{arcsinh}(cx))^2}{(cx-i)\sqrt{c^2x^2+1}}-\frac{2(a+b\text{arcsinh}(cx))^2}{(cx-i)^2\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{3c}+\frac{4b\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{i\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))}{3c}\right.}{}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(5/2)*(-1/3*(a + b*ArcSinh[c*x])^2/c - (((4*I)/3)*b^2*C
ot[Pi/4 + (I/2)*ArcSinh[c*x]])/c - ((I/3)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4
+ (I/2)*ArcSinh[c*x]])/c + (2*b*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcS
inh[c*x]]^2)/(3*c) + ((I/3)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSin
h[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/c + (4*b*(a + b*ArcSinh[c*x])*Lo
g[1 + I*E^ArcSinh[c*x]])/(3*c) + (4*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(
3*c)))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.575.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.575. $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.575.4 Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2 \sqrt{-icfx + f}}{(icdx + d)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

## 3.575.5 Fricas [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}} dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{5}{2}}} dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algo
rithm="fricas")
```

output
```
-1/3*((b^2*c*x + I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 - 3*(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)*integral(1/3*(
3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*
c*d^3*x + I*d^3), x))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

### 3.575.6 Sympy [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}}\,dx = \int \frac{\sqrt{-if\,(cx+i)}(a+b\,\text{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(5/2)
, x)
```

### 3.575.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.575.8 Giac [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}}\,dx = \int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(5/2), x
)
```

### 3.575.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

# 3.576 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.576.1 Optimal result

Integrand size = 37, antiderivative size = 774

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{8ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c}$$
$$+\frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2}+\frac{16ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c(1+c^2x^2)}+\frac{15b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2}}{64(1+c^2x^2)}+$$

output
```
8/225*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*d*x*(d+I*c*d*
x)^(3/2)*(f-I*c*f*x)^(3/2)+16/75*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/
2)/c/(c^2*x^2+1)+15/64*b^2*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^
2+1)+2/125*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64*
b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)
-2/5*I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x
^2+1)^(3/2)-3/8*b*c*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh
(c*x))/(c^2*x^2+1)^(3/2)-4/15*I*b*c^2*d*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^
(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)-2/25*I*b*c^4*d*x^5*(d+I*c*d*x)^
(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*
c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*d*x*(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)
^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*d*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8
*b*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1
/2)/c
```

### 3.576.2 Mathematica [A] (verified)

Time = 4.44 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.40

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-72000iabcd^2fx\sqrt{d+icdx}\sqrt{f-icfx}+57600ia^2d^2f\sqrt{d+icdx}\sqrt{f-ic\ldots}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
((-72000*I)*a*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (57600*I)*
a^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72000*I
)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000
*a^2*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (11
5200*I)*a^2*c^2*d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + 72000*a^2*c^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1
 + c^2*x^2] + (57600*I)*a^2*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*Sqrt[1 + c^2*x^2] + 36000*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*ArcSinh[c*x]^3 - 72000*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[2*ArcSinh[c*x]] + (4000*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[3*ArcSinh[c*x]] - 4500*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[4*ArcSinh[c*x]] + (288*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*Cosh[5*ArcSinh[c*x]] + 108000*a^2*d^(5/2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b
^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - (12000
*I)*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1
125*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1
800*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (20
*I)*b*Sqrt[1 + c^2*x^2] + (10*I)*b*Cosh[3*ArcSinh[c*x]] + (2*I)*b*Cosh[5*A
rcSinh[c*x]] + 40*b*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) - ...
```

### 3.576.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int d(icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int (icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(icx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2\right)dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{2}{25}ibc^4x^5(a+b\text{arcsinh}(cx))-\frac{4}{15}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+\right.$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((((16*I)/75)*b^2*Sqrt[1 + c^2*
x^2])/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 + (((8*I)/225)*b^2*(1 + c^2*x^2)
^(3/2))/c + (b^2*x*(1 + c^2*x^2)^(3/2))/32 + (((2*I)/125)*b^2*(1 + c^2*x^2
)^(5/2))/c - (9*b^2*ArcSinh[c*x])/(64*c) - ((2*I)/5)*b*x*(a + b*ArcSinh[c*
x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8 - ((4*I)/15)*b*c^2*x^3*(a + b*Arc
Sinh[c*x]) - ((2*I)/25)*b*c^4*x^5*(a + b*ArcSinh[c*x]) - (b*(1 + c^2*x^2)^
2*(a + b*ArcSinh[c*x]))/(8*c) + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)^2)/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + ((I/5)*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(8*b*c)))/
(1 + c^2*x^2)^(3/2)
```

### 3.576.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.576.4 Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2\,dx$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

output
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

### 3.576.5 Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((I*b^2*c^3*d^2*f*x^3 + b^2*c^2*d^2*f*x^2 + I*b^2*c*d^2*f*x + b^2*
d^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2
 - 2*(-I*a*b*c^3*d^2*f*x^3 - a*b*c^2*d^2*f*x^2 - I*a*b*c*d^2*f*x - a*b*d^2
*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I
*a^2*c^3*d^2*f*x^3 + a^2*c^2*d^2*f*x^2 + I*a^2*c*d^2*f*x + a^2*d^2*f)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.576.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.576.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.576.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

### 3.576.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

# 3.577 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.577.1 Optimal result

Integrand size = 37, antiderivative size = 396

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}+\frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64(1+c^2x^2)}-\frac{9b^2(d}{}$$

output
```
1/32*b^2*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)+15/64*b^2*x*(d+I*c*d*x)^(3/
2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)-9/64*b^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3
/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)-3/8*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*
f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(
3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/8*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3
/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

## 3.577.2 Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.32

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{160a^2cdfx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+64a^2c^3dfx^3\sqrt{d+icdx}\sqrt{f-}}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
(160*a^2*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6
4*a^2*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 64*a*b*d*f
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 4*a*b*d*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 96*a^2*d^(3/2)*f^(3
/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]] + 32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*Arc
Sinh[c*x]] + b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*
x]] + 8*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(12*a + 8
*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) - 4*b*d*f*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(16*b*Cosh[2*ArcSinh[c*x]] + b*Cosh[4*A
rcSinh[c*x]] - 4*a*(8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(256*
c*Sqrt[1 + c^2*x^2])
```

### 3.577.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.69, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {6211, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6201}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{4}\right.}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6200}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-bc\int x(a+ba\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\downarrow$ 6191

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{}{\sqrt{}}\right.\right.\right.}{\left(c^2x^2-\right.}$$

$\downarrow$ 262

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x}{}\right.\right.\right.\right.$$

$\downarrow$ 222

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+}\right.\right.}{(}$$

$\downarrow$ 6198

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}\right.}{\left(c^2x\right.}$$

$\downarrow$ 6213

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int\left(c^2x^2+1\right)^{3/2}dx}{4c}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(\right.$$

$\downarrow$ 211

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\right)}{4c}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/\right.$$

$\downarrow$ 211

3.577. $\int(d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)+\right.$$

$\Big\downarrow$ 222

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}}{2c}\right.\right.}{}\right.\right.$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((x*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x])^2)/4 + (3*((x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (a
 + b*ArcSinh[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((
x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)))/4 - (b*c*(((1 +
 c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4
+ (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)))/2))/(1 +
c^2*x^2)^(3/2)
```

### 3.577.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

---

3.577. $\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.577.4 Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2\,dx$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

output
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

### 3.577.5 Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((b^2*c^2*d*f*x^2 + b^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d*f*x^2 + a*b*d*f)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^2*d*f*x^2
+ a^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.577.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.577.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.577.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.577.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

# 3.578 $\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.578.1 Optimal result

Integrand size = 37, antiderivative size = 508

$$\begin{aligned}\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = &-\frac{4ib^2f\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx}\\ &-\frac{2ib^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c}-\frac{b^2f\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}\\ &+\frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}}\\ &-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\ &+\frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}\\ &+\frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\ &-\frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c}\\ &+\frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-4/9*I*b^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*f*x*(d+I*c*d*x)
^(1/2)*(f-I*c*f*x)^(1/2)-2/27*I*b^2*f*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c
*f*x)^(1/2)/c+1/2*f*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(
1/2)-1/3*I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x
)^(1/2)/c-1/4*b^2*f*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^
2*x^2+1)^(1/2)+2/3*I*b*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x
)^(1/2)/(c^2*x^2+1)^(1/2)-1/2*b*c*f*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/
2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f*x^3*(a+b*arcsinh(c*x)
)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*f*(a+b*arcsinh
(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

### 3.578.2 Mathematica [A] (verified)

Time = 3.09 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.39

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{108iabcfx\sqrt{d+icdx}\sqrt{f-icfx}-72ia^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-108ib^2f\sqrt{}}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((108*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (72*I)*a^2*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (108*I)*b^2*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*f*x*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*f*x^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*f*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Cosh[3*ArcSinh[c*x]] + 108*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*
d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a - (3*I)*b*Sqrt[1 + c^2*x^2]
- I*b*Cosh[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*f*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((9*I)*
b*c*x - (9*I)*a*Sqrt[1 + c^2*x^2] - (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sin
h[2*ArcSinh[c*x]] + I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

### 3.578.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f(1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int (1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int \left(\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$f\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{2}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-\frac{i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((-4*I)/9)*b^2*Sqrt[1 + c^2*x^2])
/c + (b^2*x*Sqrt[1 + c^2*x^2])/4 - (((2*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c
- (b^2*ArcSinh[c*x])/(4*c) + ((2*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (b*c*x^2
*(a + b*ArcSinh[c*x]))/2 + ((2*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (x*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - ((I/3)*(1 + c^2*x^2)^(3/2)*(a
 + b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(6*b*c)))/Sqrt[1 + c^2*x^
2]
```

### 3.578.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.578.4 Maple [F]

$$\int (-icfx+f)^{\frac{3}{2}} (a+b\operatorname{arcsinh}(cx))^2 \sqrt{icdx+d}dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

### 3.578.5 Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((-I*b^2*c*f*x + b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*f*x - a*b*f)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*f*x + a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.578.6 Sympy [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int \sqrt{id\,(cx-i)}(-if\,(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2
, x)
```

### 3.578.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.578.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

### 3.578.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

# 3.579 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$



## 3.579.1 Optimal result

Integrand size = 37, antiderivative size = 436

$$\begin{aligned}\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx &= -\frac{4ib^2f^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}}\\&+\frac{b^2f^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{4ibf^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}}\\&+\frac{bcf^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\&-\frac{f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
-4*I*b^2*f^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*f^2
*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*f^2*(c^2*x^2+1)*(a+
b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*f^2*x*(c^2*x^2
+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*f^2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*I*b*
f^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)+1/2*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2
)/(f-I*c*f*x)^(1/2)+1/2*f^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+
I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

### 3.579.2 Mathematica [A] (verified)

Time = 8.52 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.22

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{32iabcfx\sqrt{d+icdx}\sqrt{f-icfx}-16ia^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1}}{}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x
]
```

output
```
((32*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (32*I)*b^2*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*f*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]^3 + 2*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]
*((4*I)*(4*b*c*x + a*(-4 + I*c*x)*Sqrt[1 + c^2*x^2]) + b*Cosh[2*ArcSinh[c*
x]]) + 12*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt
[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*ArcSinh[c*x]^2*(6*a - (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]
))/(8*c*d*Sqrt[1 + c^2*x^2])
```

### 3.579.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\sqrt{c^2x^2+1}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 6258}$$

$$\frac{f^2\sqrt{c^2x^2+1}\int \left(c-ic^2x\right)^2(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{f^2\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2(c-c\sin(i\text{arcsinh}(cx)))^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 3798}$$

$$\frac{f^2\sqrt{c^2x^2+1}\int \left(-x^2(a+b\text{arcsinh}(cx))^2c^4-2ix(a+b\text{arcsinh}(cx))^2c^3+(a+b\text{arcsinh}(cx))^2c^2\right)d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^2\sqrt{c^2x^2+1}\left(\frac{1}{2}bc^4x^2(a+b\text{arcsinh}(cx))+4ibc^3x(a+b\text{arcsinh}(cx))-2ic^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{c^2(a+b}{}\right.}{c^3\sqrt{d+icdx}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
(f^2*Sqrt[1 + c^2*x^2]*((-4*I)*b^2*c^2*Sqrt[1 + c^2*x^2] - (b^2*c^3*x*Sqrt
[1 + c^2*x^2])/4 + (b^2*c^2*ArcSinh[c*x])/4 + (4*I)*b*c^3*x*(a + b*ArcSinh
[c*x]) + (b*c^4*x^2*(a + b*ArcSinh[c*x]))/2 - (2*I)*c^2*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x])^2 - (c^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/
2 + (c^2*(a + b*ArcSinh[c*x])^3)/(2*b)))/(c^3*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

### 3.579.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.579. $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

### 3.579.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b\operatorname{arcsinh}(cx))^2}{\sqrt{icdx+d}}dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

### 3.579.5 Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

### 3.579.6 Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{\sqrt{id\,(cx-i)}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I))
, x)
```

### 3.579.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.579.8 Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x
)
```

### 3.579.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{3/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

# 3.580 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$



## 3.580.1 Optimal result

Integrand size = 37, antiderivative size = 752

$$\begin{aligned}
\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = & -\frac{2iabf^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2f^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& -\frac{2ib^2f^3x(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& +\frac{4f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4f^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& +\frac{if^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{f^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& -\frac{16ibf^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& -\frac{8bf^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& -\frac{8b^2f^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& +\frac{8b^2f^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
& -\frac{4b^2f^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

output
```
-2*I*a*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b
^2*f^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^2*f^3*x*(
c^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*f
^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+
4*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(
3/2)/(f-I*c*f*x)^(3/2)-16*I*b*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arc
tan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^3*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(
c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*b^2*f^3*(c
^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(
f-I*c*f*x)^(3/2)-4*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(
1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

### 3.580.2 Mathematica [A] (verified)

Time = 14.85 (sec) , antiderivative size = 1174, normalized size of antiderivative = 1.56

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\text{Too large to display}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2)
,x]
```

output
```
((I/3)*f*(-3*a^2*(-5*I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
 c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 9*a^2*Sqrt[d]*
Sqrt[f]*(-I + c*x)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*
x]/2]) + 6*a*b*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh
[c*x]/2]*(-(c*x) + (2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2
 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2]) + I*(-(c*x) + (-2
+ Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + (3*I)*a*b*(I - c
*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x
]*(-4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*Log[1 + c
^2*x^2]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSin
h[c*x]/2]] + 2*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + I*b^2*(I - c*x)*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((6 - 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSinh[c*x]*(I*Pi + 4*Log[1 - I/E^ArcSinh[c*x
]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 24*PolyLog[2, I/E^Ar
cSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 12*Pi*(Log[1
 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^ArcSinh[c*x]] - 2*Log[Cosh[ArcSinh[c*x]
/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*((-I)*Cosh[ArcSinh[c*x]/2...
```

### 3.580.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\int\left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4i(cx+i)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$f^3(c^2x^2+1)^{3/2}\left(-\frac{16ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{4x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{4i(a+b\right.$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(3/2)*((-2*I)*a*b*x + ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c -
 (2*I)*b^2*x*ArcSinh[c*x] + (4*(a + b*ArcSinh[c*x])^2)/c + ((4*I)*(a + b*A
rcSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (4*x*(a + b*ArcSinh[c*x])^2)/Sqrt[
1 + c^2*x^2] + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c - (a + b*Arc
Sinh[c*x])^3/(b*c) - ((16*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]]
)/c - (8*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c - (8*b^2*Po
lyLog[2, (-I)*E^ArcSinh[c*x]])/c + (8*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c
- (4*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2))
```

### 3.580.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.580. $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.580.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b\,\operatorname{arcsinh}(cx))^2}{(icdx+d)^{\frac{3}{2}}}dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

## 3.580.5 Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(((I*b^2*c*f*x - b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*f*x + a*b*f)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*f*x - a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

### 3.580.6 Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3
/2), x)
```

### 3.580.7 Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6
*I*sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c
*d^2*sqrt(f/d))) + integrate((-I*c*f*x + f)^(3/2)*b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2/(I*c*d*x + d)^(3/2) + 2*(-I*c*f*x + f)^(3/2)*a*b*log(c*x + sqrt
(c^2*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

### 3.580.8 Giac [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

### 3.580.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(f-c\,f\,x\,1\text{i})^{3/2}}{(d+c\,d\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

# 3.581 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$



## 3.581.1 Optimal result

Integrand size = 37, antiderivative size = 580

$$\begin{aligned}
\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= -\frac{8f^4(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{f^4(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{8ib^2f^4(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{8if^4(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4bf^4(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2if^4(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{32bf^4(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{32b^2f^4(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
-8/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+1/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)-8/3*I*b^2*f^4*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*
arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-8/3*I*f^4*(c^2*x^2+1)^
(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+4/3*b*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1
/4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*f^
4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*cs
c(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*
b*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))
/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*b^2*f^4*(c^2*x^2+1)^(5/2)*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.581.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1609 vs. 2(580) = 1160.

Time = 17.16 (sec) , antiderivative size = 1609, normalized size of antiderivative = 2.77

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-4*I)/3)*a^2*f)/(d^3*(-I
+ c*x)^2) - (8*a^2*f)/(3*d^3*(-I + c*x))))/c + (a^2*f^(3/2)*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(5/2))
+ ((I/3)*a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*
f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Co
sh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*
Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (
6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1
+ c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
 c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log
[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)
*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]
)^4) - (a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f
*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*
ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh
[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*
(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c
*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSin
h[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]]
 + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*Ar...
```

### 3.581.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^4(c^2x^2+1)^{5/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6259}$$

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\int\left(\frac{4i(a+b\text{arcsinh}(cx))^2}{(cx-i)\sqrt{c^2x^2+1}}-\frac{4(a+b\text{arcsinh}(cx))^2}{(cx-i)^2\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(\frac{(a+b\text{arcsinh}(cx))^3}{3bc}-\frac{8(a+b\text{arcsinh}(cx))^2}{3c}+\frac{32b\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{8i\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}\right.}{}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(5/2)*((-8*(a + b*ArcSinh[c*x])^2)/(3*c) + (a + b*ArcSi
nh[c*x])^3/(3*b*c) - (((8*I)/3)*b^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c - ((
(8*I)/3)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c + (4*b*(
a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((2*I)/3)*(
a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcS
inh[c*x]]^2)/c + (32*b*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*
c) + (32*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c)))/((d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2))
```

### 3.581.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.581. $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.581.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}}dx$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

output
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

## 3.581.5 Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

### 3.581.6 Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(5
/2), x)
```

### 3.581.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.581.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.581.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2\,{\left(f-c\,f\,x\,1\text{i}\right)}^{3/2}}{{\left(d+c\,d\,x\,1\text{i}\right)}^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

# 3.582 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.582.1 Optimal result

Integrand size = 37, antiderivative size = 548

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{245b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1152\left(1+c^2x^2\right)^2}+\frac{65b}{}$$

output
```
1/108*b^2*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)+245/1152*b^2*x*(d+I*c*d*x)
^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^2+65/1728*b^2*x*(d+I*c*d*x)^(5/2)*(f-
I*c*f*x)^(5/2)/(c^2*x^2+1)-115/1152*b^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2
)*arcsinh(c*x)/c/(c^2*x^2+1)^(5/2)-5/16*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f
*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(5/2)+1/6*x*(d+I*c*d*x)^(5/2)*(f-
I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2+5/16*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(
5/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*
x)^(5/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+5/48*(d+I*c*d*x)^(5/2)*(f-I*c*f*
x)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(5/2)-5/48*b*(d+I*c*d*x)^(5/
2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/c/(c^2*x^2+1)^(1/2)-1/18*b*(d+I*c*
d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.582.2 Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.34

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{9504a^2cd^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+7488a^2c^3d^2f^2x^3\sqrt{d+ic}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
(9504*a^2*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2
] + 7488*a^2*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
c^2*x^2] + 2304*a^2*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sq
rt[1 + c^2*x^2] + 1440*b^2*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Arc
Sinh[c*x]^3 - 3240*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*
ArcSinh[c*x]] - 324*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4
*ArcSinh[c*x]] - 24*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6
*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 1620*b^2*d^2*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 81*b^2*d^2*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d^2*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[6*ArcSinh[c*x]] - 12*b*d^2*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] +
27*b*Cosh[4*ArcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSin
h[c*x]] - 108*a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d
^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + 45*b*Sin
h[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(1
3824*c*Sqrt[1 + c^2*x^2])
```

### 3.582.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.78, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {6211, 6201, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$\downarrow$ 6211

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\int \left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2dx}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 6201

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 6201

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx)\right.\right.}{(c}$$

$\downarrow$ 6200

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx)\right.\right.}{}$$

$\downarrow$ 6191

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx)\right.\right.}{}$$

$\downarrow$ 262

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx\right.\right.}{}$$

$\downarrow$ 222

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx)\right.\right.}{}$$

$\downarrow$ 6198

3.582. $\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx)\right.\right.$$

$\downarrow$ 6213

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\int(c^2x^2+1)^{5/2}dx}{6c}\right)+\frac{5}{6}\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(\right.\right.\right.}{4c^2}$$

$\downarrow$ 211

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\int(c^2x^2+1)^{3/2}dx+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)+\frac{5}{6}\left(-\frac{1}{2}bc\left((c^2\right.\right.\right.$$

$\downarrow$ 211

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)\right.$$

$\downarrow$ 211

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\right.}{6c}\right.\right.$$

$\downarrow$ 222

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsi}}{}\right.\right.\right.}{}\right.\right.$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

3.582. $\int(d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

output
```
((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*((x*(1 + c^2*x^2)^(5/2)*(a + b*Ar
cSinh[c*x])^2)/6 - (b*c*(((1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2) -
(b*((x*(1 + c^2*x^2)^(5/2))/6 + (5*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqr
t[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/6))/(6*c)))/3 + (5*((x*(1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*((x*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])^2)/2 + (a + b*ArcSinh[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*Arc
Sinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)
))/2)))/4 - (b*c*(((1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*
(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c))
)/4))/(4*c)))/2))/6))/(1 + c^2*x^2)^(5/2)
```

### 3.582.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.582.4 Maple [F]

$$\int (icdx+d)^{\frac{5}{2}} (-icfx+f)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))^2\,dx$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

output
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

### 3.582.5 Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((b^2*c^4*d^2*f^2*x^4 + 2*b^2*c^2*d^2*f^2*x^2 + b^2*d^2*f^2)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^
4*d^2*f^2*x^4 + 2*a*b*c^2*d^2*f^2*x^2 + a*b*d^2*f^2)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^4*d^2*f^2*x^4 + 2*a^
2*c^2*d^2*f^2*x^2 + a^2*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.582.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.582.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.582.8 Giac [F(-2)]

Exception generated.

$$\int (d + icdx)^{5/2}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.582.9 Mupad [F(-1)]

Timed out.

$$\int (d + icdx)^{5/2}(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))^2\,dx = \int \left(a + b\,\text{asinh}(c\,x)\right)^2\left(d + c\,d\,x\,1\text{i}\right)^{5/2}\left(f - c\,f\,x\,1\text{i}\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

# 3.583 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.583.1 Optimal result

Integrand size = 37, antiderivative size = 774

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = -\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c}$$
$$+\frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}-\frac{16ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{75c(1+c^2x^2)}+\frac{15b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2}}{64(1+c^2x^2)}-$$

output
```
-8/225*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*f*x*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(3/2)-16/75*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3
/2)/c/(c^2*x^2+1)+15/64*b^2*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x
^2+1)-2/125*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64
*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2
)+2/5*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*
x^2+1)^(3/2)-3/8*b*c*f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsin
h(c*x))/(c^2*x^2+1)^(3/2)+4/15*I*b*c^2*f*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)
^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+2/25*I*b*c^4*f*x^5*(d+I*c*d*x)
^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I
*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*f*x*(d+I*c*d*x)^(
3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x
)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*f*(d+I*c*
d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/
8*b*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(
1/2)/c
```

### 3.583.2 Mathematica [A] (verified)

Time = 4.63 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.40

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{72000iabcdf^2x\sqrt{d+icdx}\sqrt{f-icfx}-57600ia^2df^2\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
((72000*I)*a*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (57600*I)*a
^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72000*I)
*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*
a^2*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (115
200*I)*a^2*c^2*d*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
x^2] + 72000*a^2*c^3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1
+ c^2*x^2] - (57600*I)*a^2*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Sqrt[1 + c^2*x^2] + 36000*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*ArcSinh[c*x]^3 - 72000*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[2*ArcSinh[c*x]] - (4000*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[3*ArcSinh[c*x]] - 4500*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[4*ArcSinh[c*x]] - (288*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Cosh[5*ArcSinh[c*x]] + 108000*a^2*d^(3/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log
[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^
2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + (12000*
I)*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 11
25*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 18
00*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (20*
I)*b*Sqrt[1 + c^2*x^2] - (10*I)*b*Cosh[3*ArcSinh[c*x]] - (2*I)*b*Cosh[5*Ar
cSinh[c*x]] + 40*b*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) + (...
```

### 3.583.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int f(1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int (1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2-icx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2\right)dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{2}{25}ibc^4x^5(a+b\text{arcsinh}(cx))+\frac{4}{15}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+ba\right.$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((((-16*I)/75)*b^2*Sqrt[1 + c^2
*x^2])/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 - (((8*I)/225)*b^2*(1 + c^2*x^2
)^(3/2))/c + (b^2*x*(1 + c^2*x^2)^(3/2))/32 - (((2*I)/125)*b^2*(1 + c^2*x^
2)^(5/2))/c - (9*b^2*ArcSinh[c*x])/(64*c) + ((2*I)/5)*b*x*(a + b*ArcSinh[c
*x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8 + ((4*I)/15)*b*c^2*x^3*(a + b*Ar
cSinh[c*x]) + ((2*I)/25)*b*c^4*x^5*(a + b*ArcSinh[c*x]) - (b*(1 + c^2*x^2)
^2*(a + b*ArcSinh[c*x]))/(8*c) + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 - ((I/5)*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(8*b*c)))
/(1 + c^2*x^2)^(3/2)
```

### 3.583.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.583. $\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

### 3.583.4 Maple [F]

$$\int (icdx+d)^{\frac{3}{2}} (-icfx+f)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))^2 \, dx$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

output
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

### 3.583.5 Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2 \, dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2 \, dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((-I*b^2*c^3*d*f^2*x^3 + b^2*c^2*d*f^2*x^2 - I*b^2*c*d*f^2*x + b^2
*d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^
2 - 2*(I*a*b*c^3*d*f^2*x^3 - a*b*c^2*d*f^2*x^2 + I*a*b*c*d*f^2*x - a*b*d*f
^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-
I*a^2*c^3*d*f^2*x^3 + a^2*c^2*d*f^2*x^2 - I*a^2*c*d*f^2*x + a^2*d*f^2)*sqr
t(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.583.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.583.7 Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.583.8 Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

### 3.583.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}\,(f-c\,f\,x\,1\text{i})^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

# 3.584 $\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.584.1 Optimal result

Integrand size = 37, antiderivative size = 680

$$\begin{aligned}
&\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \\
&-\frac{8ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} \\
&-\frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}-\frac{4ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} \\
&-\frac{15b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} \\
&+\frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} \\
&-\frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+\frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} \\
&+\frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+\frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2 \\
&-\frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c} \\
&+\frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-8/9*I*b^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*f^2*x*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I
*c*f*x)^(1/2)-4/27*I*b^2*f^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/
2)/c+3/8*f^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/
4*c^2*f^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3
*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)/c-15/64*b^2*f^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*
x^2+1)^(1/2)+4/3*I*b*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x
)^(1/2)/(c^2*x^2+1)^(1/2)-3/8*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+4/9*I*b*c^2*f^2*x^3*(a+b*arcsinh(
c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*f^2*
x^4*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/
2)+5/24*f^2*(a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(
c^2*x^2+1)^(1/2)
```

### 3.584.2 Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.31

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{6912iabcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-4608ia^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-6912}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((6912*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (4608*I)*a^2*f
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (6912*I)*b^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*f^2*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (4608*I)*a^2*c^2*
f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c
^3*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^
2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (256*I)*b^2*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*f^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*Sqrt[d]*f^
(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]] + 864*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*
ArcSinh[c*x]] + (768*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3
*ArcSinh[c*x]] - 27*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*Arc
Sinh[c*x]] + 12*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((5
76*I)*b*c*x - (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] - (
192*I)*a*Cosh[3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*Ar
cSinh[c*x]] + (64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) +
 72*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (48*I
)*b*Sqrt[1 + c^2*x^2] - (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*Arc...
```

### 3.584.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f^2(1-icx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int (1-icx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int\left(-c^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-2icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\sqrt{c^2x^2+1}\right.}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{8}bc^3x^4(a+b\text{arcsinh}(cx))+\frac{4}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)\right.}{}$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((-8*I)/9)*b^2*Sqrt[1 + c^2*x^2
])/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 - (b^2*c^2*x^3*Sqrt[1 + c^2*x^2])/3
2 - (((4*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c - (15*b^2*ArcSinh[c*x])/(64*c)
+ ((4*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8
+ ((4*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (b*c^3*x^4*(a + b*ArcSinh[c*x
]))/8 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*x^3*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/4 - (((2*I)/3)*(1 + c^2*x^2)^(3/2)*(a
+ b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(24*b*c)))/Sqrt[1 + c^
2*x^2]
```

### 3.584.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.584. $\int\sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.584.4 Maple [F]

$$\int (-icfx+f)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))^2 \sqrt{icdx+d}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

### 3.584.5 Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*f^2*x^2 + 2
*I*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) - (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

### 3.584.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

output
```
Timed out
```

### 3.584.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.584.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.584.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

# 3.585 $\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$



## 3.585.1 Optimal result

Integrand size = 37, antiderivative size = 615

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx &= -\frac{68ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3b^2f^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{22ibf^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3bcf^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{11if^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{icf^3x^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
-68/9*I*b^2*f^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*
f^3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2/27*I*b^2*f^3*(c^2*
x^2+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-11/3*I*f^3*(c^2*x^2+1)*(a+b
*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*f^3*x*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*I*c*f^3*x^
2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4
*b^2*f^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+22/3*I*b*f^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)+3/2*b*c*f^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+
I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2/9*I*b*c^2*f^3*x^3*(a+b*arcsinh(c*x))*(c
^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*f^3*(a+b*arcsinh(c
*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

### 3.585.2 Mathematica [A] (verified)

Time = 14.05 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.18

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{1620iabcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-792ia^2f^2\sqrt{d+icdx}\sqrt{f-icf}}{}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x
]
```

output
```
((1620*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (792*I)*a^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1620*I)*b^2*f^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*f^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*f^2*
x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(-4*b*c*x
*(-33 + c^2*x^2) + 27*a*(-5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] + 3*a*Cosh[3*Ar
cSinh[c*x]])) + 540*a^2*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*f^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(30*a - (45*I)*b*Sqrt[1 + c^2*x^2] + I*
b*Cosh[3*ArcSinh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*d*Sqrt[1 + c^2*
x^2])
```

### 3.585.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}dx$$

$$\Big\downarrow \ 6211$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 27$$

$$\frac{f^3\sqrt{c^2x^2+1}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 6258$$

$$\frac{f^3\sqrt{c^2x^2+1}\int \left(c-ic^2x\right)^3(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 3042$$

$$\frac{f^3\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2(c-c\sin(i\text{arcsinh}(cx)))^3d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 3798$$

$$\frac{f^3\sqrt{c^2x^2+1}\int \left(ix^3(a+b\text{arcsinh}(cx))^2c^6-3x^2(a+b\text{arcsinh}(cx))^2c^5-3ix(a+b\text{arcsinh}(cx))^2c^4+(a+b\text{arcsinh}(\right.}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \ 2009$$

$$\frac{f^3\sqrt{c^2x^2+1}\left(-\frac{2}{9}ibc^6x^3(a+b\text{arcsinh}(cx))+\frac{3}{2}bc^5x^2(a+b\text{arcsinh}(cx))+\frac{22}{3}ibc^4x(a+b\text{arcsinh}(cx))+\frac{5c^3(a+b\text{arcsi}}{6b}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
(f^3*Sqrt[1 + c^2*x^2]*(((-68*I)/9)*b^2*c^3*Sqrt[1 + c^2*x^2] - (3*b^2*c^4
*x*Sqrt[1 + c^2*x^2])/4 + ((2*I)/27)*b^2*c^3*(1 + c^2*x^2)^(3/2) + (3*b^2*
c^3*ArcSinh[c*x])/4 + ((22*I)/3)*b*c^4*x*(a + b*ArcSinh[c*x]) + (3*b*c^5*x
^2*(a + b*ArcSinh[c*x]))/2 - ((2*I)/9)*b*c^6*x^3*(a + b*ArcSinh[c*x]) - ((
11*I)/3)*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2 - (3*c^4*x*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (I/3)*c^5*x^2*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])^2 + (5*c^3*(a + b*ArcSinh[c*x])^3)/(6*b)))/(c^4*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x])
```

#### 3.585.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

3.585. $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

### 3.585.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(cx))^2}{\sqrt{icdx+d}}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

### 3.585.5 Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(((I*b^2*c^2*f^2*x^2 - 2*b^2*c*f^2*x - I*b^2*f^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*f^2*x^
2 + 2*a*b*c*f^2*x + I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*f^2*x^2 - 2*a^2*c*f^2*x - I*a^2*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

### 3.585.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

output
```
Timed out
```

### 3.585.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.585.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

# 3.586 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

## 3.586.1 Optimal result

Integrand size = 37, antiderivative size = 972

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = -\frac{8iabf^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8ib^2f^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{b^2f^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{b^2f^4(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2f^4x(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{bcf^4x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8if^4(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8f^4x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{8f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^4(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{f^4x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{5f^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{32ibf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{16bf^4(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{16b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$+\frac{16b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$-\frac{8b^2f^4(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
4*I*f^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)-8*I*a*b*f^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+1/4*b^2*f^4*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*f
^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*
I*b^2*f^4*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*c*f^4*
x^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-8*I*b^2*f^4*x*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f
*x)^(3/2)+8*f^4*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)+8*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3
/2)/(f-I*c*f*x)^(3/2)+8*I*f^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*f^4*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d
+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/2*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c
*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-32*I*b*f^4*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-16*b*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-16*b^2*f^4*(c^2*x
^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)+16*b^2*f^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1
/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*f^4*(c^2*x^2+1)^(3/2)*po
lylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/...
```

### 3.586.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2492 vs. $2(972) = 1944$.

Time = 23.48 (sec) , antiderivative size = 2492, normalized size of antiderivative = 2.56

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((4*I)*a^2*f^2)/d^2 + (a^2*c
*f^2*x)/(2*d^2) + (8*a^2*f^2)/(d^2*(-I + c*x))))/c - (15*a^2*f^(5/2)*Log[c
*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*
c*d^(3/2)) + ((4*I)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*
x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c
*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[Ar
cSinh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]]) + I*(-(c*x) - 2*ArcSinh[c*x
] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcS
inh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2
*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c
*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(Arc
Sinh[c*x]*(-4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*L
og[Sqrt[1 + c^2*x^2]]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcT
an[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2])
)/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[A
rcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (b^2*f^2*Sqrt[I*((-I)*d + c*d*
x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]
/2]*((6*I)*Pi*ArcSinh[c*x] + (6 - 6*I)*ArcSinh[c*x]^2 + ArcSinh[c*x]^3 + 1
2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - (24*I)*Pi*Log[...
```

## 3.586.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^4(c^2x^2+1)^{3/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$\Big\downarrow$ 6259

$$\frac{f^4\left(c^2x^2+1\right)^{3/2}\int\left(\frac{c^2x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{4icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{7(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{8i(cx+i)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{f^4\left(c^2x^2+1\right)^{3/2}\left(-\frac{32ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsin}}{c}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(3/2)*((-8*I)*a*b*x + ((8*I)*b^2*Sqrt[1 + c^2*x^2])/c +
 (b^2*x*Sqrt[1 + c^2*x^2])/4 - (b^2*ArcSinh[c*x])/(4*c) - (8*I)*b^2*x*ArcS
inh[c*x] - (b*c*x^2*(a + b*ArcSinh[c*x]))/2 + (8*(a + b*ArcSinh[c*x])^2)/c
 + ((8*I)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (8*x*(a + b*ArcS
inh[c*x])^2)/Sqrt[1 + c^2*x^2] + ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - (5*(a + b*Arc
Sinh[c*x])^3)/(2*b*c) - ((32*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*
x]])/c - (16*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c - (16*b
^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c + (16*b^2*PolyLog[2, I*E^ArcSinh[c*x
]])/c - (8*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))
```

### 3.586.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.586. $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.586.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

## 3.586.5 Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(((b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*f^2*x^2 + 2
*I*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) + (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

### 3.586.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

output
```
Timed out
```

### 3.586.7 Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x
)/(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a^2 + integrat
e((-I*c*f*x + f)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(I*c*d*x + d)^(3
/2) + 2*(-I*c*f*x + f)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d
)^(3/2), x)
```

### 3.586.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.586.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(f-c\,f\,x\,1\text{i})^{5/2}}{(d+c\,d\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

# 3.587 $\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$



## 3.587.1 Optimal result

Integrand size = 37, antiderivative size = 790

$$\begin{aligned}
\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= \frac{2iabf^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ib^2f^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{28f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5(1+c^2x^2)^3(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{5f^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{16ib^2f^5(1+c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{28if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4if^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112bf^5(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{112b^2f^5(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
2*I*a*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^
2*f^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b^2*f^5*x*(c
^2*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*f^5*
(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)-I*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x
)^(5/2)+5/3*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/
2)/(f-I*c*f*x)^(5/2)-16/3*I*b^2*f^5*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arc
sinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*I*f^5*(c^2*x^2+1)^(5
/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2
)/(f-I*c*f*x)^(5/2)+8/3*b*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4
*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*f^5*
(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(
1/4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+112/3*b
*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/
c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+112/3*b^2*f^5*(c^2*x^2+1)^(5/2)*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.587.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2622 vs. 2(790) = 1580.

Time = 24.70 (sec) , antiderivative size = 2622, normalized size of antiderivative = 3.32

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*f^2)/d^3 - (((8*I)
/3)*a^2*f^2)/(d^3*(-I + c*x)^2) - (28*a^2*f^2)/(3*d^3*(-I + c*x))))/c + (5
*a^2*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*
f*(I + c*x)]])/(c*d^(5/2)) + ((I/3)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[
(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*S
inh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcT
an[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2
]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqr
t[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcS
inh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[
Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/
(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x
]/2] + I*Sinh[ArcSinh[c*x]/2])^4) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*
Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x]
)*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2
*x^2]]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (
6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4
- (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/
2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*...
```

### 3.587.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f^5(1-icx)^5(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f^5(c^2x^2+1)^{5/2}\int \frac{(1-icx)^5(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\int\left(-\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{12i(a+b\text{arcsinh}(cx))^2}{(cx-i)\sqrt{c^2x^2+1}}-\frac{8(a+b\text{arcsinh}(cx))^2}{(cx-i)^2\sqrt{c^2x^2+1}}+\frac{5(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{5(a+b\text{arcsinh}(cx))^3}{3bc}-\frac{28(a+b\text{arcsinh}(cx))^2}{3c}+\frac{112b\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+}{3c}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^5*(1 + c^2*x^2)^(5/2)*((2*I)*a*b*x - ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c +
(2*I)*b^2*x*ArcSinh[c*x] - (28*(a + b*ArcSinh[c*x])^2)/(3*c) - (I*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(3*b*c) -
 (((16*I)/3)*b^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c - (((28*I)/3)*(a + b*Ar
cSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c + (8*b*(a + b*ArcSinh[c*x]
)*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((4*I)/3)*(a + b*ArcSinh[c*x]
)^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/c + (
112*b*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c) + (112*b^2*Pol
yLog[2, (-I)*E^ArcSinh[c*x]])/(3*c)))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2))
```

### 3.587.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.587. $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

### 3.587.4 Maple [F]

$$\int \frac{(-icfx+f)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}}dx$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

output
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

### 3.587.5 Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(((-I*b^2*c^2*f^2*x^2 + 2*b^2*c*f^2*x + I*b^2*f^2)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*f^2*x^
2 - 2*a*b*c*f^2*x - I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*f^2*x^2 + 2*a^2*c*f^2*x + I*a^2*f^2)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c
*d^3*x + I*d^3), x)
```

### 3.587.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

output
```
Timed out
```

### 3.587.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.587.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.587.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(f-c\,f\,x\,1\text{i})^{5/2}}{(d+c\,d\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

# 3.588 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$



## 3.588.1 Optimal result

Integrand size = 37, antiderivative size = 615

$$
\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx &= \frac{68ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{3b^2d^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{22ibd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{3bcd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{11id^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{icd^3x^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

output
```
68/9*I*b^2*d^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*d
^3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2/27*I*b^2*d^3*(c^2*x
^2+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+11/3*I*d^3*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*d^3*x*(c^2*x^2+1
)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*c*d^3*x^2
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*
b^2*d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)-22/3*I*b*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(
f-I*c*f*x)^(1/2)+3/2*b*c*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2/9*I*b*c^2*d^3*x^3*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*d^3*(a+b*arcsinh(c*
x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

### 3.588.2 Mathematica [A] (verified)

Time = 13.88 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.18

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{-1620iabcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+792ia^2d^2\sqrt{d+icdx}\sqrt{f-ic}}{}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x
]
```

output
```
((-1620*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (792*I)*a^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1620*I)*b^2*d^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*d^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*d^2
*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*d^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*d^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*d^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(4*b*c*x
*(-33 + c^2*x^2) + 27*a*(5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] - 3*a*Cosh[3*Arc
Sinh[c*x]])) + 540*a^2*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqr
t[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*d^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(30*a + (45*I)*b*Sqrt[1 + c^2*x^2] - I*b
*Cosh[3*ArcSinh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*d^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*f*Sqrt[1 + c^2*x
^2])
```

### 3.588.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}dx$$
$$\Big\downarrow \text{6211}$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \text{27}$$
$$\frac{d^3\sqrt{c^2x^2+1}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \text{6258}$$
$$\frac{d^3\sqrt{c^2x^2+1}\int \left(ixc^2+c\right)^3(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{d^3\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2(\sin(i\text{arcsinh}(cx))c+c)^3d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \text{3798}$$
$$\frac{d^3\sqrt{c^2x^2+1}\int \left(-ix^3(a+b\text{arcsinh}(cx))^2c^6-3x^2(a+b\text{arcsinh}(cx))^2c^5+3ix(a+b\text{arcsinh}(cx))^2c^4+(a+b\text{arcsin}}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \text{2009}$$
$$d^3\sqrt{c^2x^2+1}\Big(\tfrac{2}{9}ibc^6x^3(a+b\text{arcsinh}(cx))+\tfrac{3}{2}bc^5x^2(a+b\text{arcsinh}(cx))-\tfrac{22}{3}ibc^4x(a+b\text{arcsinh}(cx))+\frac{5c^3(a+b\text{arcsinh}}{6b}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
(d^3*Sqrt[1 + c^2*x^2]*(((68*I)/9)*b^2*c^3*Sqrt[1 + c^2*x^2] - (3*b^2*c^4*
x*Sqrt[1 + c^2*x^2])/4 - ((2*I)/27)*b^2*c^3*(1 + c^2*x^2)^(3/2) + (3*b^2*c
^3*ArcSinh[c*x])/4 - ((22*I)/3)*b*c^4*x*(a + b*ArcSinh[c*x]) + (3*b*c^5*x^
2*(a + b*ArcSinh[c*x]))/2 + ((2*I)/9)*b*c^6*x^3*(a + b*ArcSinh[c*x]) + ((1
1*I)/3)*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2 - (3*c^4*x*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - (I/3)*c^5*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])^2 + (5*c^3*(a + b*ArcSinh[c*x])^3)/(6*b)))/(c^4*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x])
```

### 3.588.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

3.588. $\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

### 3.588.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{\sqrt{-icfx+f}}dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

### 3.588.5 Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(((-I*b^2*c^2*d^2*x^2 - 2*b^2*c*d^2*x + I*b^2*d^2)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*d^2*x^
2 + 2*a*b*c*d^2*x - I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*d^2*x^2 - 2*a^2*c*d^2*x + I*a^2*d^2)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

### 3.588.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

output
```
Timed out
```

### 3.588.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.588.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.588.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

# 3.589 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$



## 3.589.1 Optimal result

Integrand size = 37, antiderivative size = 436

$$\begin{aligned}\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx &= \frac{4ib^2d^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\ &- \frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2d^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}\\ &- \frac{4ibd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}}\\ &+ \frac{bcd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}\\ &- \frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
4*I*b^2*d^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*d^2*
x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*d^2*(c^2*x^2+1)*(a+b
*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*d^2*x*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*d^2*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4*I*b*d
^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+1/2*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)+1/2*d^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

### 3.589.2 Mathematica [A] (verified)

Time = 8.55 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{-32iabcdx\sqrt{d+icdx}\sqrt{f-icfx}+16ia^2d\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1}}{}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x
]
```

output
```
((-32*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a^2*d*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (32*I)*b^2*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*d*x*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*ArcSinh[c*x]^3 + 2*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Co
sh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]*((-16*I)*b*c*x - 4*a*(-4*I + c*x)*Sqrt[1 + c^2*x^2] + b*Cosh[2*ArcSinh[c
*x]]) + 12*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqr
t[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*ArcSinh[c*x]^2*(6*a + (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]
]))/(8*c*f*Sqrt[1 + c^2*x^2])
```

### 3.589.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{c^2x^2+1}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 6258

$$\frac{d^2\sqrt{c^2x^2+1}\int \left(ixc^2+c\right)^2(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3042

$$\frac{d^2\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2(\sin(i\text{arcsinh}(cx))c+c)^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3798

$$\frac{d^2\sqrt{c^2x^2+1}\int \left(-x^2(a+b\text{arcsinh}(cx))^2c^4+2ix(a+b\text{arcsinh}(cx))^2c^3+(a+b\text{arcsinh}(cx))^2c^2\right)d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 2009

$$\frac{d^2\sqrt{c^2x^2+1}\left(\frac{1}{2}bc^4x^2(a+b\text{arcsinh}(cx))-4ibc^3x(a+b\text{arcsinh}(cx))+2ic^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{c^2(a+b\text{a}}{}\right.}{c^3\sqrt{d+icdx}\sqrt{}}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
(d^2*Sqrt[1 + c^2*x^2]*((4*I)*b^2*c^2*Sqrt[1 + c^2*x^2] - (b^2*c^3*x*Sqrt[
1 + c^2*x^2])/4 + (b^2*c^2*ArcSinh[c*x])/4 - (4*I)*b*c^3*x*(a + b*ArcSinh[
c*x]) + (b*c^4*x^2*(a + b*ArcSinh[c*x]))/2 + (2*I)*c^2*Sqrt[1 + c^2*x^2]*(
a + b*ArcSinh[c*x])^2 - (c^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2
 + (c^2*(a + b*ArcSinh[c*x])^3)/(2*b)))/(c^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x])
```

### 3.589.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

---

3.589. $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

### 3.589.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2}{\sqrt{-icfx+f}}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

### 3.589.5 Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

### 3.589.6 Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I))
, x)
```

### 3.589.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

### 3.589.8 Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x
)
```

### 3.589.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

# 3.590 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$



## 3.590.1 Optimal result

Integrand size = 37, antiderivative size = 259

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2d(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
2*I*b^2*d*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I*d*(c^2*x^2+1
)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*a*b*d*x*(
c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*b^2*d*x*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*d*(a+b*arcsi
nh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

### 3.590.2 Mathematica [A] (verified)

Time = 2.64 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$$
$$= \frac{3i\sqrt{d+icdx}\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)-6ib\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1\right.}}{}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
((3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*
x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) - (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*(a + I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 +
c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
])/(3*c*f*Sqrt[1 + c^2*x^2])
```

### 3.590.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$$
$$\Big\downarrow \ 6211$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \ 27$$
$$\frac{d\sqrt{c^2x^2+1}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\sqrt{c^2x^2+1}\int \left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{c^2x^2+1}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{(a+b\text{arcsinh}(cx))^3}{3bc}-2iabx-2ib^2x\text{arcsinh}(cx)+\frac{2ib^2\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
(d*Sqrt[1 + c^2*x^2]*((-2*I)*a*b*x + ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c - (2*
I)*b^2*x*ArcSinh[c*x] + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (
a + b*ArcSinh[c*x])^3/(3*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

### 3.590.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

---

## 3.590. $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

### 3.590.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2\,\sqrt{icdx+d}}{\sqrt{-icfx+f}}dx$$

input
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

### 3.590.5 Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\mathrm{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^
2*x^2 + 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*f*x + I*f), x
)
```

### 3.590.6 Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)), x
)
```

### 3.590.7 Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
a^2*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + i
ntegrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(-I*c*f*
x + f) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*
x + f), x)
```

### 3.590.8 Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

### 3.590.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

# 3.591 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$



## 3.591.1 Optimal result

Integrand size = 37, antiderivative size = 59

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)
```

## 3.591.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 168 vs. 2(59) = 118.

Time = 2.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.85

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{ab\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^3}{3c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{a^2\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^3)/(3*c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) + (a^2*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/(c*Sqrt[d]*Sqrt[f])
```

## 3.591.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {6211, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

### 3.591.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

## 3.591.4 Maple [F]

$$\int \frac{(a + b\ \mathrm{arcsinh}\,(cx))^2}{\sqrt{icdx + d}\,\sqrt{-icfx + f}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

## 3.591.5 Fricas [F]

$$\int \frac{(a + b\mathrm{arcsinh}(cx))^2}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx) + a)^2}{\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^
2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d*f*x^2 + d*f), x
)
```

### 3.591.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))),
 x)
```

### 3.591.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \frac{b^2\,\text{arsinh}\,(cx)^3}{3\,\sqrt{df}c} + \frac{ab\,\text{arsinh}\,(cx)^2}{\sqrt{df}c} + \frac{a^2\,\text{arsinh}\,(cx)}{\sqrt{df}c}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
1/3*b^2*arcsinh(c*x)^3/(sqrt(d*f)*c) + a*b*arcsinh(c*x)^2/(sqrt(d*f)*c) +
a^2*arcsinh(c*x)/(sqrt(d*f)*c)
```

### 3.591.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x
)
```

### 3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,\sqrt{f-c\,f\,x\,1{}\mathrm{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

# 3.592 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$



## 3.592.1 Optimal result

Integrand size = 37, antiderivative size = 464

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx &= \frac{if(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{fx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4ibf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2bf(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{b^2f(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\end{aligned}$$

output
```
I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+
f*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)-4*I*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(
1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*f*(c^2*x^2+1)^(3/2)*(a+b*a
rcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)-2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c
/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,I
*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*f*(c^2
*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)
```

### 3.592.2 Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-1+i)b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2\left(\cosh\left(\frac{1}{2}\text{arcsinh}(c\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 + I)*b^2*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + (I*a^2 + a^2*c*x
 - (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Pi
*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x
^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - (4*
I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi*Sqrt[
1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2]
 + I*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh
[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*(I*Cosh[ArcSinh[c*x]/2]*(2*a - b*Pi + (4*I)*b*Log[1 - I/E^
ArcSinh[c*x]]) + (2*a + b*Pi - (4*I)*b*Log[1 - I/E^ArcSinh[c*x]])*Sinh[Arc
Sinh[c*x]/2])))/(c*d^2*f*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Si
nh[ArcSinh[c*x]/2]))
```

### 3.592.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(c^2x^2+1)^{3/2}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f(c^2x^2+1)^{3/2}\int \left(\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}-\frac{icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(-\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{c}\cdots\right.}{(d+icdx\cdots}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f*(1 + c^2*x^2)^(3/2)*((a + b*ArcSinh[c*x])^2/c + (I*(a + b*ArcSinh[c*x])
^2)/(c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] -
 ((4*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/c - (2*b*(a + b*Arc
Sinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c - (2*b^2*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/c + (2*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c - (b^2*PolyLog[2, -E^
(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.592.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.592.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

### 3.592.5 Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}\sqrt{-i\,cfx + f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
 (c^2*d^2*f*x - I*c*d^2*f)*integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + I
*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(
c^3*d^2*f*x^3 - I*c^2*d^2*f*x^2 + c*d^2*f*x - I*d^2*f), x))/(c^2*d^2*f*x -
 I*c*d^2*f)
```

### 3.592.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{(a + b\,\text{asinh}\,(cx))^2}{(id\,(cx - i))^{\frac{3}{2}}\,\sqrt{-if\,(cx + i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I)
)), x)
```

### 3.592.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}\sqrt{-i\,cfx + f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*sqrt(-I*
c*f*x + f)), x) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(I*c^2*d^2*
f*x + c*d^2*f) + I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(I*c^2*d^2*f*x + c*d^2*f) -
 2*a*b*log(I*c*x + 1)/(c*d^(3/2)*sqrt(f))
```

### 3.592.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)),
 x)
```

### 3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

# 3.593 $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

### 3.593.1 Optimal result

Integrand size = 37, antiderivative size = 942

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+c^2x^2)^{5/2}\text{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcf^2x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ibf^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2bf^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2b^2f^2(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2f^2(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
2/3*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-2/3*b^2*f^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3
*b^2*f^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+1/3*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)-2/3*I*b^2*f^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-1/3*b*c*f^2*x^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)-4/3*I*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arc
tan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*f^2*x
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*
c^2*f^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I
*c*f*x)^(5/2)+1/3*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*b*f^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x)
)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-2/3*b^2*f^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))
/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*f^2*(c^2*x^2+1)^(5/2)*polyl
og(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*
b^2*f^2*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.593.2 Mathematica [A] (warning: unable to verify)

Time = 6.91 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.56

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{a^2(-2i+cx)}{(-i+cx)^2} - \frac{ab\left(-i\cosh\left(\frac{3}{2}\text{arcsinh}(cx)\right)\left(\text{arcsinh}(cx)-2\,\text{arct}\right.\right.}{}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(-2*I + c*x))/(-I + c*x)^2 - (a
*b*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*
x]/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 - (3*I)*ArcSin
h[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*
((-1 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[
Coth[ArcSinh[c*x]/2]] + (I/2)*(-2 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^
2]))*Sinh[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])^3) - (b^2*((1 - I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(-2
*I + ArcSinh[c*x]))/(-I + c*x) + 2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^
ArcSinh[c*x]] + I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cos
h[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 4*PolyLog[
2, I/E^ArcSinh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSi
nh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[Arc
Sinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c
^2*x^2]))/(3*c*d^3*f)
```

### 3.593.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^2(1-icx)^2(a+b\operatorname{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^2(a+b\operatorname{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\int \left(-\frac{c^2x^2(a+b\operatorname{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}-\frac{2icx(a+b\operatorname{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}+\frac{(a+b\operatorname{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 2009

$$f^2(c^2x^2+1)^{5/2}\left(-\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{bcx^2(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\sqrt{c^2x^2+1}}+\frac{x(a+b\text{arcsinh}}{3(c^2x^2+1)^{\cdot}}\right.$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f^2*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (2*b^2*
x)/(3*Sqrt[1 + c^2*x^2]) + (b^2*ArcSinh[c*x])/(3*c) + (b*(a + b*ArcSinh[c*
x]))/(3*c*(1 + c^2*x^2)) - (((2*I)/3)*b*x*(a + b*ArcSinh[c*x]))/(1 + c^2*x
^2) - (b*c*x^2*(a + b*ArcSinh[c*x]))/(3*(1 + c^2*x^2)) + (a + b*ArcSinh[c*
x])^2/(3*c) + (((2*I)/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) +
 (x*(a + b*ArcSinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) - (c^2*x^3*(a + b*ArcS
inh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sqr
t[1 + c^2*x^2]) - (((4*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]]
)/c - (2*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) - (2*b^
2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) + (2*b^2*PolyLog[2, I*E^ArcSinh[c
*x]])/(3*c) - (b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)
^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.593.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.593.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

### 3.593.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*((b^2*c*x - 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1))^2 + 3*(c^3*d^3*f*x^2 - 2*I*c^2*d^3*f*x - c*d^3*f)*integra
l(-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*a*b + (b^2*c*x - 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^3*f*x^4
- 2*I*c^3*d^3*f*x^3 - 2*I*c*d^3*f*x - d^3*f), x))/(c^3*d^3*f*x^2 - 2*I*c^2
*d^3*f*x - c*d^3*f)
```

### 3.593.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{(id(cx-i))^{\frac{5}{2}}\sqrt{-if(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I)
)), x)
```

### 3.593.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.593.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)),
 x)
```

### 3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,\sqrt{f-c\,f\,x\,1{}\mathrm{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

# 3.594 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

### 3.594.1 Optimal result

Integrand size = 37, antiderivative size = 972

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{8iabd^4x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x(1+c^2x^2)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2d^4(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2d^4x(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8id^4(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8d^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^4(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d^4x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{5d^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{32ibd^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16bd^4(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{16b^2d^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{16b^2d^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8b^2d^4(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
-8*I*b^2*d^4*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^4*(
c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/
4*b^2*d^4*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*d^4*
(c^2*x^2+1)^(3/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+32*I*
b*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c
/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*c*d^4*x^2*(c^2*x^2+1)^(3/2)*(a+
b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b^2*d^4*x*(c^2*x^2
+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*x*(c^2*x^
2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*(c^2*x
^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I
*a*b*d^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*d^4*x
*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/
2*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*
f*x)^(3/2)-8*I*d^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)-16*b*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+16*b^2*d^4*(c^2*
x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-16*b^2*d^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(
1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^4*(c^2*x^2+1)^(3/2)*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3...
```

### 3.594.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2143 vs. $2(972) = 1944$.

Time = 25.04 (sec) , antiderivative size = 2143, normalized size of antiderivative = 2.20

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-4*I)*a^2*d^2)/f^2 + (a^2*
c*d^2*x)/(2*f^2) + (8*a^2*d^2)/(f^2*(I + c*x))))/c - (15*a^2*d^(5/2)*Log[c
*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*
c*f^(3/2)) - ((4*I)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*
x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c
*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[Ar
cSinh[c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - ((-I)*c*x - (2*I)*ArcSinh
[c*x] + I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 + (4*I)*ArcTan[C
oth[ArcSinh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c
*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) - (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*
Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*
(8*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + 4
*Log[Sqrt[1 + c^2*x^2]])) + (ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) - (8*I)*Ar
cTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2
]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(I*Co
sh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d^2*(-I + c*x)*Sqrt[I*(
(-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-18*
Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (
2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[...
```

### 3.594.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{d^4(icx+1)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^4(c^2x^2+1)^{3/2}\int \frac{(icx+1)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^4(c^2x^2+1)^{3/2}\int\left(\frac{c^2x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{7(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{8i(i-cx)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^4(c^2x^2+1)^{3/2}\left(\frac{32ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(\right.}{c}}{}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(3/2)*((8*I)*a*b*x - ((8*I)*b^2*Sqrt[1 + c^2*x^2])/c +
(b^2*x*Sqrt[1 + c^2*x^2])/4 - (b^2*ArcSinh[c*x])/(4*c) + (8*I)*b^2*x*ArcSi
nh[c*x] - (b*c*x^2*(a + b*ArcSinh[c*x]))/2 + (8*(a + b*ArcSinh[c*x])^2)/c
- ((8*I)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (8*x*(a + b*ArcSi
nh[c*x])^2)/Sqrt[1 + c^2*x^2] - ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - (5*(a + b*ArcS
inh[c*x])^3)/(2*b*c) + ((32*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x
]])/c - (16*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c + (16*b^
2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c - (16*b^2*PolyLog[2, I*E^ArcSinh[c*x]
])/c - (8*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2))
```

### 3.594.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.594. $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.594.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

## 3.594.5 Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(((b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d^2*x^2 - 2
*I*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) + (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

### 3.594.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

### 3.594.7 Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x
)/(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a^2 + integrat
e((I*c*d*x + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(-I*c*f*x + f)^(3
/2) + 2*(I*c*d*x + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f
)^(3/2), x)
```

### 3.594.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}}{{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

# 3.595 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$



## 3.595.1 Optimal result

Integrand size = 37, antiderivative size = 752

$$\begin{aligned}\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx &= \frac{2iabd^3x(1+c^2x^2)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{2ib^2d^3(1+c^2x^2)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2ib^2d^3x(1+c^2x^2)^{3/2}\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{4id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{4d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{id^3(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{16ibd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{8bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{8b^2d^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{8b^2d^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{4b^2d^3(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\end{aligned}$$

output
```
2*I*a*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^
2*d^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b^2*d^3*x*(c
^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(
c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*d^
3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4
*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3
/2)/(f-I*c*f*x)^(3/2)+16*I*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^3*(c
^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^3*(c^
2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)-4*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

### 3.595.2 Mathematica [A] (warning: unable to verify)

Time = 17.84 (sec) , antiderivative size = 1346, normalized size of antiderivative = 1.79

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*d)/f^2 + (4*a^2*d)
/(f^2*(I + c*x))))/c - (3*a^2*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I
*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(3/2)) - ((2*I)*a*b*d*Sqrt[I*
((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cos
h[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x
] - I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] - (2*I)*Log[Sqrt[1 +
 c^2*x^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] + I*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 2*Log[Sqrt[1
 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f +
 c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]
)) - (a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(
1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*(
ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 + c^2*x^2]])) + (ArcSinh[
c*x]*(-4*I + ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sq
rt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I
*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x
]/2])) - (b^2*d*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x
)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-18*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x
]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[
c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(Pi + (2*I)*ArcS...
```

## 3.595.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(c^2x^2+1)^{3/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \; 6259$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\int\left(-\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4i(i-cx)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\left(\frac{16ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{4x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4i(a+b\text{ar}}{c\sqrt{c}}\right.}{}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(3/2)*((2*I)*a*b*x - ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c +
(2*I)*b^2*x*ArcSinh[c*x] + (4*(a + b*ArcSinh[c*x])^2)/c - ((4*I)*(a + b*Ar
cSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (4*x*(a + b*ArcSinh[c*x])^2)/Sqrt[1
 + c^2*x^2] - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c - (a + b*ArcS
inh[c*x])^3/(b*c) + ((16*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])
/c - (8*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c + (8*b^2*Pol
yLog[2, (-I)*E^ArcSinh[c*x]])/c - (8*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c -
 (4*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*c
*f*x)^(3/2))
```

### 3.595.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

---

3.595. $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

### 3.595.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2}{(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

### 3.595.5 Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(((-I*b^2*c*d*x - b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*d*x + a*b*d)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*d*x - a^2*d)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

### 3.595.6 Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3
/2), x)
```

### 3.595.7 Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) -
6*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/
(c*f^2*sqrt(d/f))) + integrate((I*c*d*x + d)^(3/2)*b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2/(-I*c*f*x + f)^(3/2) + 2*(I*c*d*x + d)^(3/2)*a*b*log(c*x + sqr
t(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

### 3.595.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to transpose Error: Bad Argu
ment ValueUnable to transpose Error: Bad Argument Valuesym2poly/r2sym(cons
t gen & e
```

### 3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

# 3.596 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}} dx$



## 3.596.1 Optimal result

Integrand size = 37, antiderivative size = 544

$$\begin{aligned}
&\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}} dx = \\
&-\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{2d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{8ibd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{4bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{4b^2d^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{4b^2d^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{2b^2d^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

output
```
-2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)+2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/
(f-I*c*f*x)^(3/2)-1/3*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*
x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*
b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2
*d^2*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)-2*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

### 3.596.2 Mathematica [A] (warning: unable to verify)

Time = 6.69 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{\frac{6a^2\sqrt{d+icdx}\sqrt{f-icfx}}{i+cx} - 3a^2\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-ic}\right.}{}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x
]
```

output
```
((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 3*a^2*Sqrt[d]*Sqr
t[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] -
(b^2*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-18*Pi*ArcSinh[c*x] -
 (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x]
)*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-C
os[(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I
)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*
x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2
]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[A
rcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSin
h[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Co
sh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcS
inh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2)
```

### 3.596.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2i(i-cx)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\left(\frac{8ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{2x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}(cx)}{3bc}\right.}{}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^2*(1 + c^2*x^2)^(3/2)*((2*(a + b*ArcSinh[c*x])^2)/c - ((2*I)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (2*x*(a + b*ArcSinh[c*x])^2)/Sqrt[1
+ c^2*x^2] - (a + b*ArcSinh[c*x])^3/(3*b*c) + ((8*I)*b*(a + b*ArcSinh[c*x]
)*ArcTan[E^ArcSinh[c*x]])/c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSi
nh[c*x])])/c + (4*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c - (4*b^2*PolyLog[
2, I*E^ArcSinh[c*x]])/c - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d
+ I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.596.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.596.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2\,\sqrt{icdx+d}}{(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

## 3.596.5 Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x
^2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*f^2*x^2 + 2*I*c*
f^2*x - f^2), x)
```

### 3.596.6 Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3/2)
, x)
```

### 3.596.7 Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/
(c*f^2*sqrt(d/f))) + integrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2/(-I*c*f*x + f)^(3/2) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^
2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

### 3.596.8 Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(3/2), x
)
```

### 3.596.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}}{(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

# 3.597 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$



## 3.597.1 Optimal result

Integrand size = 37, antiderivative size = 464

$$
\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx &= -\frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{dx(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{4ibd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{2bd(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+\frac{2b^2d(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{2b^2d(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&-\frac{b^2d(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

output
```
-I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)+d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+4*I*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^
(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f
*x)^(3/2)+2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/
c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,
I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*d*(c^
2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(
f-I*c*f*x)^(3/2)
```

## 3.597.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-1-i)b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2\left(\cosh\left(\frac{1}{2}\text{arcsinh}(c\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 - I)*b^2*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ((-I)*a^2 + a^2*
c*x + (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - (2*I)*b^2
*Pi*Sqrt[1 + c^2*x^2]*Log[1 + I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^
2*x^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] +
(2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4
*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]])*(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^
ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 +
 c^2*x^2]*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2]*(2*a + 3*b*Pi - (4*I)*b*
Log[1 + I/E^ArcSinh[c*x]]) + (2*a - 3*b*Pi + (4*I)*b*Log[1 + I/E^ArcSinh[c
*x]])*Sinh[ArcSinh[c*x]/2])))/(c*d*f^2*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[
c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))
```

### 3.597.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\left(c^2x^2+1\right)^{3/2}\int \left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(c^2x^2+1\right)^{3/2}\left(\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{c}-\right.}{(d+icdx)}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(3/2)*((a + b*ArcSinh[c*x])^2/c - (I*(a + b*ArcSinh[c*x])
^2)/(c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] +
 ((4*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/c - (2*b*(a + b*Arc
Sinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c + (2*b^2*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/c - (2*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c - (b^2*PolyLog[2, -E^
(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.597.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

## 3.597.4 Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

### 3.597.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
 (c^2*d*f^2*x + I*c*d*f^2)*integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - I*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c
^3*d*f^2*x^3 + I*c^2*d*f^2*x^2 + c*d*f^2*x + I*d*f^2), x))/(c^2*d*f^2*x +
I*c*d*f^2)
```

### 3.597.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}\,(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2
)), x)
```

### 3.597.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(I*c*d*x + d)*(-I*c*f*x
+ f)^(3/2)), x) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(-I*c^2*d*f
^2*x + c*d*f^2) - I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(-I*c^2*d*f^2*x + c*d*f^2)
 - 2*a*b*log(I*c*x - 1)/(c*sqrt(d)*f^(3/2))
```

### 3.597.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)),
 x)
```

### 3.597.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

# 3.598 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$



## 3.598.1 Optimal result

Integrand size = 37, antiderivative size = 224

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b(1+c^2x^2)^{3/2}\,(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b^2(1+c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+(c^
2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-
2*b*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c
/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

### 3.598.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 488 vs. 2(224) = 448.

Time = 3.05 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.18

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{a^2cx+2abcx\text{arcsinh}(cx)-2ib^2\pi\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b^2cx\text{arcsinh}(c}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
,x]
```

output
```
(a^2*c*x + 2*a*b*c*x*ArcSinh[c*x] - (2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + b^2*c*x*ArcSinh[c*x]^2 - b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + I*
b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log
[1 + I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^
ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 + E^ArcSinh[c*x]] - a
*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Co
s[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[
ArcSinh[c*x]/2]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[
c*x])/4]] + 2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + 2*b^
2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c*d*f*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x])
```

### 3.598.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.62, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {6211, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 6202$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-2bc\int\frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 6212$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int\frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 26$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\int(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 4201$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\int\frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 2620$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\right.}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 2715$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\right.\right.}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 2838$$

---

3.598. $\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}dx$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcs}}{2}}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
((1 + c^2*x^2)^(3/2)*((x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + ((2*I
)*b*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Lo
g[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/c)
)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

### 3.598.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

### 3.598.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}\,(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

### 3.598.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2
 + (c^2*d^2*f^2*x^2 + d^2*f^2)*integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*c*
x - sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1))
)/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 +
d^2*f^2)
```

### 3.598.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}\,(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(
3/2)), x)
```

### 3.598.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*(-I*c*f*
x + f)^(3/2)), x) + 2*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a
^2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f) - a*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c
*d*f)
```

## 3.598.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2)
), x)
```

## 3.598.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+c\,d\,x\,1\text{i})^{3/2}\,(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

# 3.599 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$



## 3.599.1 Optimal result

Integrand size = 37, antiderivative size = 743

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx &= -\frac{ib^2f(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2fx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{ibfx(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2fx(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2f(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ibf(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4bf(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2f(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{b^2f(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2f(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
-1/3*I*b^2*f*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f
*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*f*(c^2*x^2+1)^(
3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*b*f*x*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1
/3*I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+1/3*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+2/3*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/
2)/(f-I*c*f*x)^(5/2)-2/3*I*b*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan
(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*f*(c^2
*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x
+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*f*(c^2*
x^2+1)^(5/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I
*c*f*x)^(5/2)-2/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

## 3.599.2 Mathematica [A] (warning: unable to verify)

Time = 9.91 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(-\frac{ia^2}{6d^3f^2(-i+cx)^2}+\frac{5a^2}{12d^3f^2(-i+cx)}+\frac{a^2}{4d^3f^2(i+cx)}\right.}{c}$$
$$+\frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\left(4cx\operatorname{arcsinh}(cx)+2i\operatorname{arcsinh}(cx)\cosh(2\operatorname{arcsinh}(cx))+\sqrt{1+c^2x^2}\left(1-2\right.\right.}{3cd^2f(-i+cx)\sqrt{-((-id+}$$
$$+\frac{ib^2\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{1+c^2x^2}\left(7\pi\operatorname{arcsinh}(cx)+\frac{(2+i\operatorname{arcsinh}(cx))\operatorname{arcsinh}(cx)}{-i+cx}-(1+4i)\operatorname{arcsin}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2))
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-1/6*I)*a^2)/(d^3*f^2*(-I
+ c*x)^2) + (5*a^2)/(12*d^3*f^2*(-I + c*x)) + a^2/(4*d^3*f^2*(I + c*x))))/
c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*Ar
cSinh[c*x] + (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(
1 - (2*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2
]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d^2*f*
(-I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2
))]) + ((I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1
 + c^2*x^2]*(7*Pi*ArcSinh[c*x] + ((2 + I*ArcSinh[c*x])*ArcSinh[c*x])/(-I +
 c*x) - (1 + 4*I)*ArcSinh[c*x]^2 - 5*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E
^ArcSinh[c*x]] + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 1
6*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]]
 + 16*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 5*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x]
)/4]] + (6*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (10*I)*PolyLog[2, I/E^ArcS
inh[c*x]] + ((3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2]) + ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]
)/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + ((-4 + 5*ArcSinh[c*x
]^2)*Sinh[ArcSinh[c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/
2])))/(c*d^2*f*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2
*x^2))])
```

### 3.599.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \ 6211$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{f\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f(c^2x^2+1)^{5/2}\int\left(\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}-\frac{icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$f(c^2x^2+1)^{5/2}\left(-\frac{2ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{ibx(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{b(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(cx}{3\sqrt{c^2x^2+1}}\right.$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
(f*(1 + c^2*x^2)^(5/2)*(((-1/3*I)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (b^2*x)/(3*
Sqrt[1 + c^2*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*(1 + c^2*x^2)) - ((I/3)
*b*x*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2) + (2*(a + b*ArcSinh[c*x])^2)/(3*c
) + ((I/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) + (x*(a + b*Arc
Sinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sq
rt[1 + c^2*x^2]) - (((2*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]
])/c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) - (b^2
*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) + (b^2*PolyLog[2, I*E^ArcSinh[c*x]
])/(3*c) - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.599.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.599. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.599.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

### 3.599.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^
2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*integral(1/3*(-3*I*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*a^2 - 2*(3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*
b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^3*f^2*x^5 - I*c^4*d^3*
f^2*x^4 + 2*c^3*d^3*f^2*x^3 - 2*I*c^2*d^3*f^2*x^2 + c*d^3*f^2*x - I*d^3*f^
2), x))/(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2
)
```

### 3.599.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

### 3.599.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

### 3.599.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.599.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

# 3.600 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$



## 3.600.1 Optimal result

Integrand size = 37, antiderivative size = 794

$$\begin{aligned}
&\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \\
&-\frac{2iabd^5x(1+c^2x^2)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5(1+c^2x^2)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&-\frac{2ib^2d^5x(1+c^2x^2)^{5/2}\,\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{28d^5(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{id^5(1+c^2x^2)^3\,(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{112bd^5(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&-\frac{112b^2d^5(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{8bd^5(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{16ib^2d^5(1+c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+\frac{28id^5(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&-\frac{4id^5(1+c^2x^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
-2*I*a*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b
^2*d^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^2*d^5*x*(
c^2*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*d^5
*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+5/3*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+112/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(
1+I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-112/3*b
^2*d^5*(c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*
x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*
sec(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+16/
3*I*b^2*d^5*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)+28/3*I*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2
*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*
I*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x)
)^2*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.600.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2552 vs. 2(794) = 1588.

Time = 24.35 (sec) , antiderivative size = 2552, normalized size of antiderivative = 3.21

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a^2*d^2)/f^3 + (((8*I)/3)
*a^2*d^2)/(f^3*(I + c*x)^2) - (28*a^2*d^2)/(3*f^3*(I + c*x))))/c + (5*a^2*
d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I
+ c*x)]])/(c*f^(5/2)) - ((I/3)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*
(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[A
rcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[
ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I
+ 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2
*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh
[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(
c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqr
t[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*
ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x
^2]]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 -
(84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(
4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x
]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14...
```

### 3.600.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^5(icx+1)^5(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^5(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\int\left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{12i(a+b\text{arcsinh}(cx))^2}{(cx+i)\sqrt{c^2x^2+1}}-\frac{8(a+b\text{arcsinh}(cx))^2}{(cx+i)^2\sqrt{c^2x^2+1}}+\frac{5(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{5(a+b\text{arcsinh}(cx))^3}{3bc}+\frac{28(a+b\text{arcsinh}(cx))^2}{3c}+\frac{112b\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b}{3c}\right.}{}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^5*(1 + c^2*x^2)^(5/2)*((-2*I)*a*b*x + ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c -
 (2*I)*b^2*x*ArcSinh[c*x] + (28*(a + b*ArcSinh[c*x])^2)/(3*c) + (I*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(3*b*c)
+ (112*b*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c) - (112*b^2*
PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c) + (8*b*(a + b*ArcSinh[c*x])*Sec[Pi/
4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((16*I)/3)*b^2*Tan[Pi/4 + (I/2)*ArcSin
h[c*x]])/c + (((28*I)/3)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c
*x]])/c - (((4*I)/3)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]
^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^
(5/2))
```

### 3.600.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.600. $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

### 3.600.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{5}{2}}}dx$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

output
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

### 3.600.5 Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(((I*b^2*c^2*d^2*x^2 + 2*b^2*c*d^2*x - I*b^2*d^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*d^2*x^
2 - 2*a*b*c*d^2*x + I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x - I*a^2*d^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*
f^3*x - I*f^3), x)
```

### 3.600.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

### 3.600.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.600.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.600.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}}{(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

# 3.601 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$



## 3.601.1 Optimal result

Integrand size = 37, antiderivative size = 584

$$\begin{aligned}
\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx &= \frac{8d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{d^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{32bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{32b^2d^4(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4bd^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8ib^2d^4(1+c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{8id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2id^4(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
8/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+1/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+32/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln
(1+I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-32/3*b
^2*d^4*(c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*
x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*
sec(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3
*I*b^2*d^4*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*t
an(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*
d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^
2*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.601.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1617 vs. 2(584) = 1168.

Time = 17.78 (sec) , antiderivative size = 1617, normalized size of antiderivative = 2.77

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((4*I)/3)*a^2*d)/(f^3*(I +
c*x)^2) - (8*a^2*d)/(3*f^3*(I + c*x))))/c + (a^2*d^(3/2)*Log[c*d*f*x + Sqr
t[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(5/2)) - (
(I/3)*a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(
1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*
ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sq
rt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan
[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x
^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^
2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Lo
g[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((
-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]
/2])^4) + (a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(
d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[
(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[
Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*
(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*Ar
cSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x
^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)...
```

## 3.601.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^4(icx+1)^4(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^4(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^4(c^2x^2+1)^{5/2}\int\left(-\frac{4i(a+b\text{arcsinh}(cx))^2}{(cx+i)\sqrt{c^2x^2+1}}-\frac{4(a+b\text{arcsinh}(cx))^2}{(cx+i)^2\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^4(c^2x^2+1)^{5/2}\left(\frac{(a+b\text{arcsinh}(cx))^3}{3bc}+\frac{8(a+b\text{arcsinh}(cx))^2}{3c}+\frac{32b\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}+\frac{8i\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsi}\right.}{}\right.}{}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(5/2)*((8*(a + b*ArcSinh[c*x])^2)/(3*c) + (a + b*ArcSin
h[c*x])^3/(3*b*c) + (32*b*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/
(3*c) - (32*b^2*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c) + (4*b*(a + b*ArcSi
nh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((8*I)/3)*b^2*Tan[Pi/4
 + (I/2)*ArcSinh[c*x]])/c + (((8*I)/3)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (
I/2)*ArcSinh[c*x]])/c - (((2*I)/3)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)
*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c))/((d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2))
```

### 3.601.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.601. $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

## 3.601.4 Maple [F]

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\operatorname{arcsinh}(cx))^2}{(-icfx+f)^{\frac{5}{2}}}dx$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

output
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

## 3.601.5 Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x
 - I*f^3), x)
```

### 3.601.6 Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(5
/2), x)
```

### 3.601.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.601.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.601.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

# 3.602 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$



## 3.602.1 Optimal result

Integrand size = 37, antiderivative size = 522

$$\begin{aligned}
\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx &= \frac{d^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4b^2d^3(1+c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4ib^2d^3(1+c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{id^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
1/3*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+4/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*x+(c^2
*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b^2*d^3*(c^2*x^2
+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+2/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi+1/2*
I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*b^2*d^3*(c^2
*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+1/3*I*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I
*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*d^3*(c^2*x^2+1)
^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+1/
2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.602.2 Mathematica [A] (warning: unable to verify)

Time = 10.07 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(\frac{2ia^2}{3f^3(i+cx)^2}-\frac{a^2}{3f^3(i+cx)}\right)}{c}$$

$$-\frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left(\cosh\left(\frac{1}{2}\text{arcsinh}(cx)\right)+i\sinh\left(\frac{1}{2}\text{arcsinh}(cx)\right)\right)\left(-\cosh\right.}{}$$

$$-\frac{ib^2(-i+cx)\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df\left(1+c^2x^2\right)}\left((-1-i)\text{arcsinh}(cx)^2-\frac{2\text{arcsinh}(cx)(2i+\text{arc}}{i+cx}\right.}{}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x
]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((2*I)/3)*a^2)/(f^3*(I + c*
x)^2) - a^2/(3*f^3*(I + c*x))))/c - ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcT
an[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/
2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt
[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth
[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I
)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x
]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[Ar
cSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*(-I + c*x)*Sqrt[I*
((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1
 - I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (
2*I)*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh
[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4
]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4
*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSi
nh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSin
h[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f
+ c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x...
```

### 3.602.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^3(c^2x^2+1)^{5/2}\int\left(-\frac{i(a+b\text{arcsinh}(cx))^2}{(cx+i)\sqrt{c^2x^2+1}}-\frac{2(a+b\text{arcsinh}(cx))^2}{(cx+i)^2\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(c^2x^2+1)^{5/2}\left(\frac{(a+b\text{arcsinh}(cx))^2}{3c}+\frac{4b\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}+\frac{i\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx))^2}{3c}\right.}{}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(5/2)*((a + b*ArcSinh[c*x])^2/(3*c) + (4*b*(a + b*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c) - (4*b^2*PolyLog[2, (-I)/E^ArcSi
nh[c*x]])/(3*c) + (2*b*(a + b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]
^2)/(3*c) + (((4*I)/3)*b^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c + ((I/3)*(a +
 b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c - ((I/3)*(a + b*ArcSi
nh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]]
)/c))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.602.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

### 3.602.4 Maple [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2\sqrt{icdx+d}}{(-icfx+f)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

### 3.602.5 Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
-1/3*((b^2*c*x - I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 - 3*(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)*integral(1/3*(
-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
+ f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3
*c*f^3*x - I*f^3), x))/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)
```

### 3.602.6 Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(5/2)
, x)
```

### 3.602.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.602.8 Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(5/2), x
)
```

### 3.602.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}}{{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

# 3.603 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

## 3.603.1 Optimal result

Integrand size = 37, antiderivative size = 942

$$\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx &= \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2d^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{4ibd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}$$

output
```
2/3*I*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)-2/3*b^2*d^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+1/3*b^2*d^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+1/3*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/
2)/(f-I*c*f*x)^(5/2)-2/3*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*d^2*x^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh
(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*b*d^2*(c^2*x^2+1)^(5/2)*(
a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+1/3*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)-1/3*c^2*d^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(
d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(
c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*b^2*d^2*(c^2*x^2+1)^2/
c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*b^2*d^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))
/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*d^2*(c^2*x^2+1)^(5/2)*polyl
og(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*
b^2*d^2*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

## 3.603.2 Mathematica [A] (warning: unable to verify)

Time = 6.96 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.56

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{a^2(2i+cx)}{(i+cx)^2} - \frac{ab\left(i\cosh\left(\frac{3}{2}\text{arcsinh}(cx)\right)\left(\text{arcsinh}(cx)-2\arctan(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(2*I + c*x))/(I + c*x)^2 - (a*b
*(I*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]
] + (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 + (3*I)*ArcSinh[c*x
] + (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*(I +
(-1 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[C
oth[ArcSinh[c*x]/2]] - (I/2)*(2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2])*Sin
h[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcS
inh[c*x]/2])^3) - (b^2*((1 + I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(2*I + ArcS
inh[c*x]))/(I + c*x) + 2*(I*Pi + 2*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]]
 + I*Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*
I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) - 4*PolyLog[2, (-I)/E^
ArcSinh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*
x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]
))/(3*c*d*f^3)
```

### 3.603.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(c^2x^2+1)^{5/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2(c^2x^2+1)^{5/2}\int \left(-\frac{c^2x^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}+\frac{2icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$d^2\left(c^2x^2+1\right)^{5/2}\left(\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{bcx^2(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\sqrt{c^2x^2+1}}+\frac{x(a+b\text{arcsinh}(c}{3(c^2x^2+1)^{3/2}}\right.$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

output
```
(d^2*(1 + c^2*x^2)^(5/2)*((((2*I)/3)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (2*b^2*x
)/(3*Sqrt[1 + c^2*x^2]) + (b^2*ArcSinh[c*x])/(3*c) + (b*(a + b*ArcSinh[c*x
]))/(3*c*(1 + c^2*x^2)) + (((2*I)/3)*b*x*(a + b*ArcSinh[c*x]))/(1 + c^2*x^
2) - (b*c*x^2*(a + b*ArcSinh[c*x]))/(3*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x
])^2/(3*c) - (((2*I)/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) +
(x*(a + b*ArcSinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) - (c^2*x^3*(a + b*ArcSi
nh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sqrt
[1 + c^2*x^2]) + (((4*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])
/c - (2*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) + (2*b^2
*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) - (2*b^2*PolyLog[2, I*E^ArcSinh[c*
x]])/(3*c) - (b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2))
```

### 3.603.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

3.603. $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.603.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

### 3.603.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*((b^2*c*x + 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1))^2 + 3*(c^3*d*f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3)*integra
l(-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*a*b + (b^2*c*x + 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d*f^3*x^4
+ 2*I*c^3*d*f^3*x^3 + 2*I*c*d*f^3*x - d*f^3), x))/(c^3*d*f^3*x^2 + 2*I*c^2
*d*f^3*x - c*d*f^3)
```

### 3.603.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}\,(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2
)), x)
```

### 3.603.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

### 3.603.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)),
 x)
```

### 3.603.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

# 3.604 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$



## 3.604.1 Optimal result

Integrand size = 37, antiderivative size = 743

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx &= \frac{ib^2d(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2dx(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{ibdx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{id(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2dx(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2ibd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4bd(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2d(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}\end{aligned}$$

output
```
1/3*I*b^2*d*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*
x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*d*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*b*d*x*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/
3*I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^
(5/2)+2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+2/3*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2
)/(f-I*c*f*x)^(5/2)+2/3*I*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(
c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*d*(c^2*
x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+
(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*(c^2*x
^2+1)^(5/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)-2/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.604.2 Mathematica [A] (warning: unable to verify)

Time = 10.01 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\left(\frac{a^2}{4d^2f^3(-i+cx)}+\frac{ia^2}{6d^2f^3(i+cx)^2}+\frac{5a^2}{12d^2f^3(i+cx)}\right)}{c}$$
$$-\frac{iab\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\left(4cx\text{arcsinh}(cx)-2i\text{arcsinh}(cx)\cosh(2\text{arcsinh}(cx))+\sqrt{1+c^2x^2}(1+\right.}{3cdf^2(i+cx)\sqrt{-((-id+}$$
$$-\frac{ib^2\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{1+c^2x^2}\left(-9\pi\text{arcsinh}(cx)+\frac{(2-i\text{arcsinh}(cx))\text{arcsinh}(cx)}{i+cx}-(1-4i)\text{arcs}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2))
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(a^2/(4*d^2*f^3*(-I + c*x)) +
 ((I/6)*a^2)/(d^2*f^3*(I + c*x)^2) + (5*a^2)/(12*d^2*f^3*(I + c*x))))/c -
((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSin
h[c*x] - (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 +
(2*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] +
 (2*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d*f^2*(I +
 c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
- ((I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^
2*x^2]*(-9*Pi*ArcSinh[c*x] + ((2 - I*ArcSinh[c*x])*ArcSinh[c*x])/(I + c*x)
 - (1 - 4*I)*ArcSinh[c*x]^2 + 3*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcS
inh[c*x]] - 5*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 16*Pi*
Log[1 + E^ArcSinh[c*x]] + 5*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 16
*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]
 - (10*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[2, I/E^ArcSinh[c
*x]] - ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])^3 + (I*(4 - 5*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2
])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) - ((3*I)*ArcSinh[c*x]^2
*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(
c*d*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]
)
```

### 3.604.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d(c^2x^2+1)^{5/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\left(c^2x^2+1\right)^{5/2}\int\left(\frac{icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$d\left(c^2x^2+1\right)^{5/2}\left(\frac{2ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}+\frac{ibx(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{b(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\sqrt{c^2x^2+1}}\right.$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(5/2)*(((I/3)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (b^2*x)/(3*Sqr
t[1 + c^2*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*(1 + c^2*x^2)) + ((I/3)*b*
x*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2) + (2*(a + b*ArcSinh[c*x])^2)/(3*c) -
 ((I/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSin
h[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sqrt[
1 + c^2*x^2]) + (((2*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/
c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) + (b^2*Po
lyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) - (b^2*PolyLog[2, I*E^ArcSinh[c*x]])/
(3*c) - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)^(5/
2)*(f - I*c*f*x)^(5/2))
```

### 3.604.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

### 3.604.4 Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{(icdx + d)^{\frac{3}{2}} (-icfx + f)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

### 3.604.5 Fricas [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{5/2}} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}} (-i\,cfx + f)^{\frac{5}{2}}} dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^
2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*integral(1/3*(3*I*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*a^2 - 2*(-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*
b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^2*f^3*x^5 + I*c^4*d^2*
f^3*x^4 + 2*c^3*d^2*f^3*x^3 + 2*I*c^2*d^2*f^3*x^2 + c*d^2*f^3*x + I*d^2*f^
3), x))/(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3
)
```

### 3.604.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

### 3.604.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

### 3.604.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.604.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

# 3.605 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$



## 3.605.1 Optimal result

Integrand size = 37, antiderivative size = 386

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = -\frac{b^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4b(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2(1+c^2x^2)^{5/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
-1/3*b^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*(c^2*x^
2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x
*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/
3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)-4/3*b*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2
))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*(c^2*x^2+1)^(5/2)*poly
log(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

### 3.605.2 Mathematica [A] (verified)

Time = 8.66 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{4a^2cx(3+2c^2x^2)-b^2\Big(cx-6cx\text{arcsinh}(cx)^2+4i\pi\text{arcsinh}(cx)\cosh(3\text{ar}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
,x]
```

output
```
(4*a^2*c*x*(3 + 2*c^2*x^2) - b^2*(c*x - 6*c*x*ArcSinh[c*x]^2 + (4*I)*Pi*Ar
cSinh[c*x]*Cosh[3*ArcSinh[c*x]] + 2*ArcSinh[c*x]^2*Cosh[3*ArcSinh[c*x]] -
(2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*C
osh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + (2*I)*Pi*Cosh[3*ArcSinh[c*
x]]*Log[1 + I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1
+ I/E^ArcSinh[c*x]] - (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + E^ArcSinh[c*x]
] - (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] +
 (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[Cosh[ArcSinh[c*x]/2]] + (2*I)*Pi*Cosh[3
*ArcSinh[c*x]]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + 2*Sqrt[1 + c^2*x^2]
*(((-3*I)*Pi + 6*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + I*((2*I)*ArcSin
h[c*x] + 6*Pi*ArcSinh[c*x] - (3*I)*ArcSinh[c*x]^2 + 3*(Pi - (2*I)*ArcSinh[
c*x])*Log[1 + I/E^ArcSinh[c*x]] - 12*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log
[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 12*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 3*
Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])) - 16*(1 + c^2*x^2)^(3/2)*PolyLo
g[2, (-I)/E^ArcSinh[c*x]] - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[
c*x]] + Sinh[3*ArcSinh[c*x]] - 2*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]]) + 2*
a*b*(Sqrt[1 + c^2*x^2]*(2 - 3*Log[1 + c^2*x^2]) - Cosh[3*ArcSinh[c*x]]*Log
[1 + c^2*x^2] + 2*ArcSinh[c*x]*(3*c*x + Sinh[3*ArcSinh[c*x]])))/(12*d^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(c + c^3*x^2))
```

### 3.605.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.59, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {6211, 6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}dx$$

$\Big\downarrow$ 6211

$$\frac{(c^2x^2+1)^{5/2}\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6203

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{3(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6202

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx\right)+\frac{x(a+b\text{arcsinh}(cx)}{3(c^2x^2+1)^{3/2}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6212

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c}\right)+\frac{x(a+b\text{ar}}{3(c^2}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}}{c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 26

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(c}{c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 4201

$$\frac{(c^2x^2+1)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c}\right)-\frac{2}{3}bc\int\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 2620

---

3.605. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}dx$

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\right.\right.}{}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 2715

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\right.\right.}{}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 2838

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\right.\right.}{c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 6213

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\left(\frac{b\int\frac{1}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\right.\right.}{}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 208

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b}{}\right.}{c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
((1 + c^2*x^2)^(5/2)*((x*(a + b*ArcSinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) -
 (2*b*c*((b*x)/(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*(1 +
c^2*x^2))))/3 + (2*((x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + ((2*I)*
b*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/c))/
3))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

3.605. $\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

### 3.605.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

---

3.605. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.605.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{5}{2}}\,(-icfx+f)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

### 3.605.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*b^2*c^2*x^3 + 3*b^2*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^
3)*integral(1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f)*a*b - (2*b^2*c^3*x^3 + 3*b^2*c*x)*sqrt(c^2*x
^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1))
)/(c^6*d^3*f^3*x^6 + 3*c^4*d^3*f^3*x^4 + 3*c^2*d^3*f^3*x^2 + d^3*f^3), x))
/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

### 3.605.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

### 3.605.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
1/3*a*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x
^2 + 1)/(c^2*d^(5/2)*f^(5/2))) + 2/3*a*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f
) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a^2*(x/((c^2
*d*f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2)) + b^2*
integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(5/2)*(-I*c*f*x +
f)^(5/2)), x)
```

### 3.605.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

### 3.605.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+c\,d\,x\,1\text{i})^{5/2}\,(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

# 3.606 $\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx$



## 3.606.1 Optimal result

Integrand size = 18, antiderivative size = 312

$$\begin{aligned}
&\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx\\
&=-\frac{b(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4)\sqrt{1+c^2x^2}}{315c^9}\\
&\quad-\frac{4be(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)(1+c^2x^2)^{3/2}}{945c^9}\\
&\quad-\frac{2be^2(63c^4d^2-90c^2de+35e^2)(1+c^2x^2)^{5/2}}{525c^9}\\
&\quad-\frac{4b(9c^2d-7e)e^3(1+c^2x^2)^{7/2}}{441c^9}-\frac{be^4(1+c^2x^2)^{9/2}}{81c^9}\\
&\quad+d^4x(a+b\text{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx))+\frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx))
\end{aligned}$$

output
```
-4/945*b*e*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*(c^2*x^2+1)^(3
/2)/c^9-2/525*b*e^2*(63*c^4*d^2-90*c^2*d*e+35*e^2)*(c^2*x^2+1)^(5/2)/c^9-4
/441*b*(9*c^2*d-7*e)*e^3*(c^2*x^2+1)^(7/2)/c^9-1/81*b*e^4*(c^2*x^2+1)^(9/2
)/c^9+d^4*x*(a+b*arcsinh(c*x))+4/3*d^3*e*x^3*(a+b*arcsinh(c*x))+6/5*d^2*e^
2*x^5*(a+b*arcsinh(c*x))+4/7*d*e^3*x^7*(a+b*arcsinh(c*x))+1/9*e^4*x^9*(a+b
*arcsinh(c*x))-1/315*b*(315*c^8*d^4-420*c^6*d^3*e+378*c^4*d^2*e^2-180*c^2*
d*e^3+35*e^4)*(c^2*x^2+1)^(1/2)/c^9
```

### 3.606.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)^4 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{315ax(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)-\frac{b\sqrt{1+c^2x^2}\left(4480e^4-320c^2e^3\left(81d+7ex^2\right)+48c^4e^2\left(1323d^2\right.\right.}{}}{}$$

input
```
Integrate[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

output
```
(315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e
^4*x^8) - (b*Sqrt[1 + c^2*x^2]*(4480*e^4 - 320*c^2*e^3*(81*d + 7*e*x^2) +
48*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) - 8*c^6*e*(11025*d^3 + 39
69*d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 44100*d^3*
e*x^2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 315*b*x
*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*x^8)*
ArcSinh[c*x])/99225
```

### 3.606.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6207, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^4 (a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6207}$$

$$-bc\int \frac{x\left(35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4\right)}{315\sqrt{c^2x^2+1}}dx+d^4x(a+b\operatorname{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\operatorname{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\operatorname{arcsinh}(cx))+\frac{4}{7}de^3x^7(a+b\operatorname{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\operatorname{arcsinh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{315}bc\int \frac{x(35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4)}{\sqrt{c^2x^2+1}}dx+d^4x(a+b\text{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx))+\frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{630}bc\int \frac{35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4}{\sqrt{c^2x^2+1}}dx^2+d^4x(a+b\text{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx))+\frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2389}$$

$$-\frac{1}{630}bc\int \left(\frac{35(c^2x^2+1)^{7/2}e^4}{c^8}+\frac{20(9c^2d-7e)(c^2x^2+1)^{5/2}e^3}{c^8}+\frac{6(63d^2c^4-90dec^2+35e^2)(c^2x^2+1)^{3/2}e^2}{c^8}+\right.$$
$$d^4x(a+b\text{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx))+\frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^4x(a+b\text{arcsinh}(cx))+\frac{4}{3}d^3ex^3(a+b\text{arcsinh}(cx))+\frac{6}{5}d^2e^2x^5(a+b\text{arcsinh}(cx))+\frac{4}{7}de^3x^7(a+b\text{arcsinh}(cx))+\frac{1}{9}e^4x^9(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{630}bc\left(\frac{40e^3(c^2x^2+1)^{7/2}(9c^2d-7e)}{7c^{10}}+\frac{70e^4(c^2x^2+1)^{9/2}}{9c^{10}}+\frac{12e^2(c^2x^2+1)^{5/2}(63c^4d^2-90c^2de+35e^2)}{5c^{10}}+\frac{8e(\right.$$

input
```
Int[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/630*(b*c*((2*(315*c^8*d^4 - 420*c^6*d^3*e + 378*c^4*d^2*e^2 - 180*c^2*d
*e^3 + 35*e^4)*Sqrt[1 + c^2*x^2])/c^10 + (8*e*(105*c^6*d^3 - 189*c^4*d^2*e
 + 135*c^2*d*e^2 - 35*e^3)*(1 + c^2*x^2)^(3/2))/(3*c^10) + (12*e^2*(63*c^4
*d^2 - 90*c^2*d*e + 35*e^2)*(1 + c^2*x^2)^(5/2))/(5*c^10) + (40*(9*c^2*d -
 7*e)*e^3*(1 + c^2*x^2)^(7/2))/(7*c^10) + (70*e^4*(1 + c^2*x^2)^(9/2))/(9*
c^10))) + d^4*x*(a + b*ArcSinh[c*x]) + (4*d^3*e*x^3*(a + b*ArcSinh[c*x]))/
3 + (6*d^2*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcSinh[c
*x]))/7 + (e^4*x^9*(a + b*ArcSinh[c*x]))/9
```

## 3.606. $\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx$

### 3.606.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

## 3.606.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.36

| method | result |
|---|---|
| parts | $a\left(\frac{1}{9}e^4x^9+\frac{4}{7}d\,e^3x^7+\frac{6}{5}d^2e^2x^5+\frac{4}{3}d^3e\,x^3+d^4x\right)+$ $b\Bigg(\frac{c\ \mathrm{arcsinh}(cx)e^4x^9}{9}+\frac{4c\ \mathrm{arcsinh}(cx)d\,e^3x^7}{7}+$ 6c arcsin… |
| derivativedivides | $\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+$ $b\Bigg(\mathrm{arcsinh}(cx)d^4c^9x+\frac{4\ \mathrm{arcsinh}(cx)d^3c^9e\,x^3}{3}+\frac{6\ \mathrm{arcsinh}(cx)d^2c^9e^2x^5}{5}$ … |
| default | $\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+$ $b\Bigg(\mathrm{arcsinh}(cx)d^4c^9x+\frac{4\ \mathrm{arcsinh}(cx)d^3c^9e\,x^3}{3}+\frac{6\ \mathrm{arcsinh}(cx)d^2c^9e^2x^5}{5}$ … |

input
```
int((e*x^2+d)^4*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^4*x^9+4/7*d*e^3*x^7+6/5*d^2*e^2*x^5+4/3*d^3*e*x^3+d^4*x)+b/c*(1/9
*c*arcsinh(c*x)*e^4*x^9+4/7*c*arcsinh(c*x)*d*e^3*x^7+6/5*c*arcsinh(c*x)*d^
2*e^2*x^5+4/3*c*arcsinh(c*x)*d^3*e*x^3+arcsinh(c*x)*d^4*c*x-1/315/c^8*(35*
e^4*(1/9*c^8*x^8*(c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(c^2*x^2+1)^(1/2)+16/105*c
^4*x^4*(c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(c^2*x^2+1)^(1/2)+128/315*(c^2*x^2
+1)^(1/2))+315*d^4*c^8*(c^2*x^2+1)^(1/2)+180*d*c^2*e^3*(1/7*c^6*x^6*(c^2*x
^2+1)^(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-
16/35*(c^2*x^2+1)^(1/2))+378*d^2*c^4*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/
15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+420*d^3*c^6*e*(1/3*c^
2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

### 3.606.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.07

$$\int (d+ex^2)^4 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{11025\,ac^9e^4x^9+56700\,ac^9de^3x^7+119070\,ac^9d^2e^2x^5+132300\,ac^9d^3ex^3+99225\,ac^9d^4x+315\,(35\,bc^9e^4x^9}{}$$

input
```
integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^
2*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9
 + 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315
*b*c^9*d^4*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*e^4*x^8 + 99225*b
*c^8*d^4 - 88200*b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 - 25920*b*c^2*d*e^3 + 1
00*(81*b*c^8*d*e^3 - 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^
2 - 1620*b*c^6*d*e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e - 7938*b*
c^6*d^2*e^2 + 3240*b*c^4*d*e^3 - 560*b*c^2*e^4)*x^2)*sqrt(c^2*x^2 + 1))/c^
9
```

### 3.606.6 Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.90

$$\int \left(d+ex^2\right)^4 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\begin{cases} ad^4x+\frac{4ad^3ex^3}{3}+\frac{6ad^2e^2x^5}{5}+\frac{4ade^3x^7}{7}+\frac{ae^4x^9}{9}+bd^4x\operatorname{asinh}(cx)+\frac{4bd^3ex^3\operatorname{asinh}(cx)}{3}+\frac{6bd^2e^2x^5\operatorname{asinh}(cx)}{5}+\frac{4bde^3x^7}{} \\ a\left(d^4x+\frac{4d^3ex^3}{3}+\frac{6d^2e^2x^5}{5}+\frac{4de^3x^7}{7}+\frac{e^4x^9}{9}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**4*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**
3*x**7/7 + a*e**4*x**9/9 + b*d**4*x*asinh(c*x) + 4*b*d**3*e*x**3*asinh(c*x
)/3 + 6*b*d**2*e**2*x**5*asinh(c*x)/5 + 4*b*d*e**3*x**7*asinh(c*x)/7 + b*e
**4*x**9*asinh(c*x)/9 - b*d**4*sqrt(c**2*x**2 + 1)/c - 4*b*d**3*e*x**2*sqr
t(c**2*x**2 + 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 4
*b*d*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 + 1
)/(81*c) + 8*b*d**3*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 8*b*d**2*e**2*x**2*sq
rt(c**2*x**2 + 1)/(25*c**3) + 24*b*d*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c*
*3) + 8*b*e**4*x**6*sqrt(c**2*x**2 + 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c
**2*x**2 + 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5)
- 16*b*e**4*x**4*sqrt(c**2*x**2 + 1)/(945*c**5) + 64*b*d*e**3*sqrt(c**2*x*
*2 + 1)/(245*c**7) + 64*b*e**4*x**2*sqrt(c**2*x**2 + 1)/(2835*c**7) - 128*
b*e**4*sqrt(c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), (a*(d**4*x + 4*d**3*e*x
**3/3 + 6*d**2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

### 3.606.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.33

$$\int (d+ex^2)^4 (a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{9}\,ae^4x^9 + \frac{4}{7}\,ade^3x^7 + \frac{6}{5}\,ad^2e^2x^5 + \frac{4}{3}\,ad^3ex^3 + \frac{4}{9}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^3e + \frac{2}{25}\left(15\,x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bd^2e^2 + \frac{4}{245}\left(35\,x^7\operatorname{arsinh}(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bde^3 + \frac{1}{2835}\left(315\,x^9\operatorname{arsinh}(cx) - \left(\frac{35\sqrt{c^2x^2+1}x^8}{c^2} - \frac{40\sqrt{c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{c^2x^2+1}x^4}{c^6} - \frac{64\sqrt{c^2x^2+1}x^2}{c^8} \right.\right. + ad^4x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^4}{c}$$

input
```
integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/
9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)
/c^4))*b*d^3*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2
- 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^2*e^2 + 4/
245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 +
 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b
*d*e^3 + 1/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40
*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^
2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*e^4 + a*d^4*x + (c*x*arc
sinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^4/c
```

### 3.606.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^4 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.606.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^4 (a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(e\,x^2+d\right)^4\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2)^4,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2)^4, x)
```

# 3.607 $\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx$



## 3.607.1 Optimal result

Integrand size = 18, antiderivative size = 221

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx = -\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\sqrt{1+c^2x^2}}{35c^7} - \frac{be(35c^4d^2-42c^2de+15e^2)(1+c^2x^2)^{3/2}}{105c^7} - \frac{3b(7c^2d-5e)e^2(1+c^2x^2)^{5/2}}{175c^7} - \frac{be^3(1+c^2x^2)^{7/2}}{49c^7} + d^3x(a+b\text{arcsinh}(cx)) + d^2ex^3(a+b\text{arcsinh}(cx)) + \frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx)) + \frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

output
```
-1/105*b*e*(35*c^4*d^2-42*c^2*d*e+15*e^2)*(c^2*x^2+1)^(3/2)/c^7-3/175*b*(7
*c^2*d-5*e)*e^2*(c^2*x^2+1)^(5/2)/c^7-1/49*b*e^3*(c^2*x^2+1)^(7/2)/c^7+d^3
*x*(a+b*arcsinh(c*x))+d^2*e*x^3*(a+b*arcsinh(c*x))+3/5*d*e^2*x^5*(a+b*arcs
inh(c*x))+1/7*e^3*x^7*(a+b*arcsinh(c*x))-1/35*b*(35*c^6*d^3-35*c^4*d^2*e+2
1*c^2*d*e^2-5*e^3)*(c^2*x^2+1)^(1/2)/c^7
```

### 3.607.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right) - \frac{b\sqrt{1+c^2x^2}\left(-240e^3+24c^2e^2(49d+5ex^2)-2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex\right.}{3675c^7} + b\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)\text{arcsinh}(cx)$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 + c^2*x^
2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x
^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3
*x^6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)
*ArcSinh[c*x]
```

### 3.607.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6207, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6207$$

$$-bc\int \frac{x\left(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3\right)}{35\sqrt{c^2x^2+1}}dx+d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{35}bc\int \frac{x\left(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3\right)}{\sqrt{c^2x^2+1}}dx+d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{70}bc\int \frac{5e^3x^6+21de^2x^4+35d^2ex^2+35d^3}{\sqrt{c^2x^2+1}}dx^2+d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2389}$$

$$-\frac{1}{70}bc\int \left(\frac{5(c^2x^2+1)^{5/2}e^3}{c^6}+\frac{3(7c^2d-5e)(c^2x^2+1)^{3/2}e^2}{c^6}+\frac{(35d^2c^4-42dec^2+15e^2)\sqrt{c^2x^2+1}e}{c^6}+\frac{35d^3c^6}{}\right.$$
$$d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{70}bc\left(\frac{6e^2(c^2x^2+1)^{5/2}(7c^2d-5e)}{5c^8}+\frac{10e^3(c^2x^2+1)^{7/2}}{7c^8}+\frac{2e(c^2x^2+1)^{3/2}(35c^4d^2-42c^2de+15e^2)}{3c^8}+\frac{2\sqrt{c^2x^2}}{}\right.$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/70*(b*c*((2*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Sqrt[1 +
 c^2*x^2])/c^8 + (2*e*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*(1 + c^2*x^2)^(3/
2))/(3*c^8) + (6*(7*c^2*d - 5*e)*e^2*(1 + c^2*x^2)^(5/2))/(5*c^8) + (10*e^
3*(1 + c^2*x^2)^(7/2))/(7*c^8))) + d^3*x*(a + b*ArcSinh[c*x]) + d^2*e*x^3*
(a + b*ArcSinh[c*x]) + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (e^3*x^7*(a
+ b*ArcSinh[c*x]))/7
```

### 3.607.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

## 3.607.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| parts | $a\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b\left(\frac{c\,\operatorname{arcsinh}(cx)e^3x^7}{7}+\frac{3c\,\operatorname{arcsinh}(cx)d\,e^2x^5}{5}+c\,\operatorname{arcsinh}(cx)d^2e\,x^3+\operatorname{arc}\right.}{}$ |
| derivativedivides | $\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arcsinh}(cx)d^3c^7x+\operatorname{arcsinh}(cx)d^2c^7e\,x^3+\frac{3\operatorname{arcsinh}(cx)d\,c^7e^2x^5}{5}+\frac{\operatorname{arcsinh}(cx)e^3c^7x}{7}\right.}{}$ |
| default | $\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arcsinh}(cx)d^3c^7x+\operatorname{arcsinh}(cx)d^2c^7e\,x^3+\frac{3\operatorname{arcsinh}(cx)d\,c^7e^2x^5}{5}+\frac{\operatorname{arcsinh}(cx)e^3c^7x}{7}\right.}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^3*x^7+3/5*d*e^2*x^5+d^2*e*x^3+d^3*x)+b/c*(1/7*c*arcsinh(c*x)*e^3*
x^7+3/5*c*arcsinh(c*x)*d*e^2*x^5+c*arcsinh(c*x)*d^2*e*x^3+arcsinh(c*x)*c*x
*d^3-1/35/c^6*(5*e^3*(1/7*c^6*x^6*(c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(c^2*x^2+
1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(1/2))+35*d^3*c^
6*(c^2*x^2+1)^(1/2)+21*d*c^2*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x
^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+35*d^2*c^4*e*(1/3*c^2*x^2*(c^
2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

### 3.607.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.09

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx = \frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\,(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex^3}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 +
3675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^
2*e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*e^3*x^6
 + 3675*b*c^6*d^3 - 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^
2 - 10*b*c^4*e^3)*x^4 - 240*b*e^3 + (1225*b*c^6*d^2*e - 588*b*c^4*d*e^2 +
120*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))/c^7
```

### 3.607.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.76

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx = \begin{cases} ad^3x+ad^2ex^3+\frac{3ade^2x^5}{5}+\frac{ae^3x^7}{7}+bd^3x\,\text{asinh}\,(cx)+bd^2ex^3\,\text{asinh}\,(cx)+\frac{3bde^2x^5\,\text{asinh}\,(cx)}{5}+\frac{be^3x^7\,\text{asinh}\,(cx)}{7}- \\ a\left(d^3x+d^2ex^3+\frac{3de^2x^5}{5}+\frac{e^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 +
b*d**3*x*asinh(c*x) + b*d**2*e*x**3*asinh(c*x) + 3*b*d*e**2*x**5*asinh(c*x
)/5 + b*e**3*x**7*asinh(c*x)/7 - b*d**3*sqrt(c**2*x**2 + 1)/c - b*d**2*e*x
**2*sqrt(c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c)
 - b*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) + 2*b*d**2*e*sqrt(c**2*x**2 + 1)
/(3*c**3) + 4*b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(25*c**3) + 6*b*e**3*x**4*
sqrt(c**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5)
- 8*b*e**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*sqrt(c**2*x**2
+ 1)/(245*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e
**3*x**7/7), True))
```

### 3.607.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.30

$$\begin{aligned}\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx &= \frac{1}{7}\,ae^3x^7 + \frac{3}{5}\,ade^2x^5 + ad^2ex^3 \\ &+ \frac{1}{3}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2e \\ &+ \frac{1}{25}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bde^2 \\ &+ \frac{1}{245}\left(35\,x^7\,\text{arsinh}\,(cx) - \left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\,\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)be^3 \\ &+ ad^3x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^3}{c}\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arcsinh(c*x) -
c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(1
5*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^
2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arcsinh(c*x) -
 (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x
^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arcs
inh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

### 3.607.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.607.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2)^3, x)
```

# 3.608 $\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx$



## 3.608.1 Optimal result

Integrand size = 18, antiderivative size = 147

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx = -\frac{b(15c^4d^2-10c^2de+3e^2)\sqrt{1+c^2x^2}}{15c^5} - \frac{2b(5c^2d-3e)\,e(1+c^2x^2)^{3/2}}{45c^5} - \frac{be^2(1+c^2x^2)^{5/2}}{25c^5} + d^2x(a+b\text{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

output
```
-2/45*b*(5*c^2*d-3*e)*e*(c^2*x^2+1)^(3/2)/c^5-1/25*b*e^2*(c^2*x^2+1)^(5/2)
/c^5+d^2*x*(a+b*arcsinh(c*x))+2/3*d*e*x^3*(a+b*arcsinh(c*x))+1/5*e^2*x^5*(
a+b*arcsinh(c*x))-1/15*b*(15*c^4*d^2-10*c^2*d*e+3*e^2)*(c^2*x^2+1)^(1/2)/c
^5
```

### 3.608.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{225}\left(15ax\left(15d^2+10dex^2+3e^2x^4\right)-\frac{b\sqrt{1+c^2x^2}(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5}+15bx\left(15d^2+10dex^2+3e^2x^4\right)\text{arcsinh}(cx)\right)$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 + c^2*x^2]*(24*e^2 -
 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 +
 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSinh[c*x])/225
```

### 3.608.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6207, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$\Big\downarrow \text{6207}$$
$$-bc\int \frac{x\left(3e^2x^4+10dex^2+15d^2\right)}{15\sqrt{c^2x^2+1}}dx+d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$
$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bc\int \frac{x(3e^2x^4+10dex^2+15d^2)}{\sqrt{c^2x^2+1}}dx+d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}bc\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{c^2x^2+1}}dx^2+d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}bc\int \left(\frac{3(c^2x^2+1)^{3/2}e^2}{c^4}+\frac{2(5c^2d-3e)\sqrt{c^2x^2+1}e}{c^4}+\frac{15d^2c^4-10dec^2+3e^2}{c^4\sqrt{c^2x^2+1}}\right)dx^2+d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{4e(c^2x^2+1)^{3/2}(5c^2d-3e)}{3c^6}+\frac{6e^2(c^2x^2+1)^{5/2}}{5c^6}+\frac{2\sqrt{c^2x^2+1}(15c^4d^2-10c^2de+3e^2)}{c^6}\right)$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/30*(b*c*((2*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Sqrt[1 + c^2*x^2])/c^6 +
(4*(5*c^2*d - 3*e)*e*(1 + c^2*x^2)^(3/2))/(3*c^6) + (6*e^2*(1 + c^2*x^2)^(
5/2))/(5*c^6))) + d^2*x*(a + b*ArcSinh[c*x]) + (2*d*e*x^3*(a + b*ArcSinh[c
*x]))/3 + (e^2*x^5*(a + b*ArcSinh[c*x]))/5
```

### 3.608.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

## 3.608.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.28

| method | result |
|---|---|
| parts | $a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\left(\frac{c\,\mathrm{arcsinh}(cx)e^2x^5}{5}+\frac{2c\,\mathrm{arcsinh}(cx)de\,x^3}{3}+\mathrm{arcsinh}(cx)cx\,d^2-\frac{3e^2\left(\frac{c^4x^4\sqrt{c^2x^2+1}}{5}-\right.}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\mathrm{arcsinh}(cx)d^2c^5x+\frac{2\,\mathrm{arcsinh}(cx)d\,c^5e\,x^3}{3}+\frac{\mathrm{arcsinh}(cx)e^2c^5x^5}{5}-\frac{e^2\left(\frac{c^4x^4\sqrt{c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{}}{15}\right.}{5}\right.}{c^4}}{c}$ |
| default | $\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\mathrm{arcsinh}(cx)d^2c^5x+\frac{2\,\mathrm{arcsinh}(cx)d\,c^5e\,x^3}{3}+\frac{\mathrm{arcsinh}(cx)e^2c^5x^5}{5}-\frac{e^2\left(\frac{c^4x^4\sqrt{c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{}}{15}\right.}{5}\right.}{c^4}}{c}$ |

input
```
int((e*x^2+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*c*arcsinh(c*x)*e^2*x^5+2/3*c*ar
csinh(c*x)*d*e*x^3+arcsinh(c*x)*c*x*d^2-1/15/c^4*(3*e^2*(1/5*c^4*x^4*(c^2*
x^2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+15*d^2
*c^4*(c^2*x^2+1)^(1/2)+10*d*c^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*
x^2+1)^(1/2))))
```

### 3.608.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-}{225\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^
5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)
) - (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 - 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*
c^4*d*e - 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 + 1))/c^5
```

### 3.608.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.63

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\operatorname{asinh}(cx)+\frac{2bdex^3\operatorname{asinh}(cx)}{3}+\frac{be^2x^5\operatorname{asinh}(cx)}{5}-\frac{bd^2\sqrt{c^2x^2+1}}{c}-\frac{2bdex^2\sqrt{c^2x^2+1}}{9c}-\frac{be^{}}{} \\ a\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asinh(c*x)
 + 2*b*d*e*x**3*asinh(c*x)/3 + b*e**2*x**5*asinh(c*x)/5 - b*d**2*sqrt(c**2
*x**2 + 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(c
**2*x**2 + 1)/(25*c) + 4*b*d*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 4*b*e**2*x**
2*sqrt(c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 + 1)/(75*c**5),
Ne(c, 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

### 3.608.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.22

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{5}ae^2x^5 + \frac{2}{3}adex^3 + \frac{2}{9}\left(3x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bde + \frac{1}{75}\left(15x^5\,\text{arsinh}\,(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)be^2 + ad^2x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arcsinh(c*x)
 - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))
*b*d^2/c
```

### 3.608.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.608.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx=\int \left(a+b\,\mathrm{asinh}(c\,x)\right)\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2)^2, x)
```

# 3.609 $\int (d+ex^2) (a+b\text{arcsinh}(cx)) \, dx$



## 3.609.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int (d+ex^2) (a+b\text{arcsinh}(cx)) \, dx = -\frac{b(3c^2d-e)\sqrt{1+c^2x^2}}{3c^3} - \frac{be(1+c^2x^2)^{3/2}}{9c^3} + dx(a+b\text{arcsinh}(cx)) + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

output
```
-1/9*b*e*(c^2*x^2+1)^(3/2)/c^3+d*x*(a+b*arcsinh(c*x))+1/3*e*x^3*(a+b*arcsi
nh(c*x))-1/3*b*(3*c^2*d-e)*(c^2*x^2+1)^(1/2)/c^3
```

## 3.609.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int (d+ex^2) (a+b\text{arcsinh}(cx)) \, dx = \frac{1}{9}\left(3ax(3d+ex^2) - \frac{b\sqrt{1+c^2x^2}(-2e+c^2(9d+ex^2))}{c^3} + 3bx(3d+ex^2) \text{arcsinh}(cx)\right)$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(3*a*x*(3*d + e*x^2) - (b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))/c^
3 + 3*b*x*(3*d + e*x^2)*ArcSinh[c*x])/9
```

### 3.609.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6207, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\,(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6207}$$

$$-bc\int \frac{x(ex^2+3d)}{3\sqrt{c^2x^2+1}}dx+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}bc\int \frac{x(ex^2+3d)}{\sqrt{c^2x^2+1}}dx+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 353}$$

$$-\frac{1}{6}bc\int \frac{ex^2+3d}{\sqrt{c^2x^2+1}}dx^2+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{1}{6}bc\int \left(\frac{3c^2d-e}{c^2\sqrt{c^2x^2+1}}+\frac{e\sqrt{c^2x^2+1}}{c^2}\right)dx^2+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\sqrt{c^2x^2+1}(3c^2d-e)}{c^4}+\frac{2e(c^2x^2+1)^{3/2}}{3c^4}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/6*(b*c*((2*(3*c^2*d - e)*Sqrt[1 + c^2*x^2])/c^4 + (2*e*(1 + c^2*x^2)^(3
/2))/(3*c^4))) + d*x*(a + b*ArcSinh[c*x]) + (e*x^3*(a + b*ArcSinh[c*x]))/3
```

### 3.609.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

## 3.609.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

| method | result | size |
|---|---|---|
| parts | $a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\,\operatorname{arcsinh}(cx)x^3e}{3}+\operatorname{arcsinh}(cx)dcx-\frac{e\left(\frac{c^2x^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)+3d\,c^2\sqrt{c^2x^2+1}}{3c^2}\right)}{c}$ | 97 |
| derivativedivides | $\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^3x+\frac{\operatorname{arcsinh}(cx)e\,c^3x^3}{3}-\frac{e\left(\frac{c^2x^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{c^2x^2+1}\right)}{c^2}}{c}$ | 109 |
| default | $\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^3x+\frac{\operatorname{arcsinh}(cx)e\,c^3x^3}{3}-\frac{e\left(\frac{c^2x^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{c^2x^2+1}\right)}{c^2}}{c}$ | 109 |

input
```
int((e*x^2+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arcsinh(c*x)*x^3*e+arcsinh(c*x)*d*c*x-1/3/c^2
*(e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d*c^2*(c^2*x^2
+1)^(1/2)))
```

### 3.609.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx = \frac{3\,ac^3ex^3+9\,ac^3dx+3\,(bc^3ex^3+3\,bc^3dx)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^2ex^2+9\,bc^2d-2\,be)\sqrt{c^2x^2+1}}{9\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
 sqrt(c^2*x^2 + 1)) - (b*c^2*e*x^2 + 9*b*c^2*d - 2*b*e)*sqrt(c^2*x^2 + 1))
/c^3
```

### 3.609.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx = \begin{cases} adx+\frac{aex^3}{3}+bdx\,\text{asinh}\,(cx)+\frac{bex^3\,\text{asinh}\,(cx)}{3}-\frac{bd\sqrt{c^2x^2+1}}{c}-\frac{bex^2\sqrt{c^2x^2+1}}{9c}+\frac{2be\sqrt{c^2x^2+1}}{9c^3} & \text{for } c\neq 0 \\ a\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**3/3 + b*d*x*asinh(c*x) + b*e*x**3*asinh(c*x)/3 -
 b*d*sqrt(c**2*x**2 + 1)/c - b*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) + 2*b*e*sq
rt(c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

### 3.609.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d+ex^2)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{1}{3}aex^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)be$$
$$+adx+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*s
qrt(c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1)
)*b*d/c
```

### 3.609.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.609.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\,\operatorname{asinh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2), x)
```

# 3.610 $\int (a+b\text{arcsinh}(cx))\,dx$



## 3.610.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a+b\text{arcsinh}(cx))\,dx = ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\text{arcsinh}(cx)$$

output
```
a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c
```

## 3.610.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a+b\text{arcsinh}(cx))\,dx = ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\text{arcsinh}(cx)$$

input
```
Integrate[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

## 3.610.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}$$

input
```
Int[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

### 3.610.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## 3.610.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| default | $ax+\frac{b\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 31 |
| parts | $ax+\frac{b\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 31 |
| derivativedivides | $\frac{cxa+b\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 33 |

input
```
int(a+b*arcsinh(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

### 3.610.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int (a+b\text{arcsinh}(cx))\,dx=\frac{bcx\log\left(cx+\sqrt{c^2x^2+1}\right)+acx-\sqrt{c^2x^2+1}b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

output
```
(b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

### 3.610.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (a+b\text{arcsinh}(cx))\,dx=ax+b\left(\begin{cases} x\,\text{asinh}\,(cx)-\frac{\sqrt{c^2x^2+1}}{c} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*asinh(c*x),x)
```

output
```
a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tr
ue))
```

### 3.610.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a+b\text{arcsinh}(cx))\,dx=ax+\frac{\left(cx\,\text{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

### 3.610.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (a + b\text{arcsinh}(cx))\,dx = \left(x\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \frac{\sqrt{c^2x^2 + 1}}{c}\right)b + ax$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

output
```
(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

### 3.610.9 Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (a + b\text{arcsinh}(cx))\,dx = a\,x - \frac{b\,\sqrt{c^2\,x^2 + 1}}{c} + b\,x\,\text{asinh}(c\,x)$$

input
```
int(a + b*asinh(c*x),x)
```

output
```
a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

# 3.611 $\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx$



## 3.611.1 Optimal result

Integrand size = 18, antiderivative size = 485

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx &= \frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad-\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad+\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad-\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-
(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^
2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2
)+1/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2
)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(
c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1
/2)-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d
+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(
1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2
)/e^(1/2)+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-
c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

### 3.611.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.89

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx$$
$$= \frac{2a\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-b\sqrt{d}\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)+b\sqrt{d}\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx}}{-c\sqrt{-d}+\sqrt{-c^2d}}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

output
```
(2*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 +
 (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*A
rcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d
) + e])] + b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^Ar
cSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*PolyLog[2, (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*PolyLo
g[2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - b*Sq
rt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e]))] + b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

### 3.611.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex^2}\,dx$$

$$\Big\downarrow \; 6208$$

$$\int \left(\frac{\sqrt{-d}(a+b\operatorname{arcsinh}(cx))}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\operatorname{arcsinh}(cx))}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(a+b\operatorname{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\operatorname{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\operatorname{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\operatorname{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) + ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1
+ (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]
*Sqrt[e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[
2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt
[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
```

#### 3.611.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.611.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.91 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.46

| method | result |
|---|---|
| parts | $\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{bc\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)} \frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-cx \ldots}{\ldots}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)} \ldots\right.}{2}$ |
| derivativedivides | $\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)} \frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-c \ldots}{\ldots}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)} \ldots\right.}{2} \ldots\right.}{\ldots}$ |
| default | $\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)} \frac{\mathrm{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-c \ldots}{\ldots}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)} \ldots\right.}{2} \ldots\right.}{\ldots}$ |

input
```
int((a+b*arcsinh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c*sum(1/_R1/(_R1^2*e+2*c^2*d-e
)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-(c^2*x^
2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/2*b*c*sum(_R1
/(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dil
og((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+
e))
```

### 3.611.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

### 3.611.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\text{asinh}\left(cx\right)}{d+ex^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2), x)
```

### 3.611.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.611.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

### 3.611.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2), x)
```

# 3.612 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx$



## 3.612.1 Optimal result

Integrand size = 18, antiderivative size = 707

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = &-\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\\
&-\frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}\\
&-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

output
```
-1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+(c
^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/
2)-1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/
2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+
(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(
1/2)+1/4*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*
d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^
(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/
2)/e^(1/2)-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*c*arctan((-c^2*x*(-d)^(1/2)+e^(
1/2))/(c^2*d-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)-1/4*b*c
*arctan((c^2*x*(-d)^(1/2)+e^(1/2))/(c^2*d-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c
^2*d-e)^(1/2)/e^(1/2)+1/4*(-a-b*arcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)-x*e^(1
/2))+1/4*(a+b*arcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)+x*e^(1/2))
```

### 3.612.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 622, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = \frac{1}{2}\left(\frac{ax}{d^2+dex^2}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}\right.$$
$$+\frac{b\left(-2\sqrt{d}\left(-\frac{\text{arcsinh}(cx)}{i\sqrt{d}+\sqrt{e}x}+\frac{c\arctan\left(\frac{\sqrt{e}-ic^2\sqrt{d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right)+2i\sqrt{d}\left(\frac{\text{arcsinh}(cx)}{\sqrt{d}+i\sqrt{e}x}+\frac{c\text{arctanh}\left(\frac{i\sqrt{e}-c^2\sqrt{d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right)+\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

output
```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*(-2*Sqrt[d]*(-(ArcSinh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*ArcTan[(Sq
rt[e] - I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d
- e]) + (2*I)*Sqrt[d]*(ArcSinh[c*x]/(Sqrt[d] + I*Sqrt[e]*x) + (c*ArcTanh[(
I*Sqrt[e] - c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*
d - e]) + I*(ArcSinh[c*x]*(-ArcSinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c
*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) + e])] + Log[1 + (Sqrt[e]*E^ArcSinh[c*x]
)/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[
c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])] + 2*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]))]) - I*(ArcSinh[c*x]*(-Arc
Sinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c
^2*d) + e])] + Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) + e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sq
rt[-(c^2*d) + e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] +
 Sqrt[-(c^2*d) + e])])))/(4*d^(3/2)*Sqrt[e]))/2
```

### 3.612.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(-\frac{e(a+b\text{arcsinh}(cx))}{2d\left(-de-e^2x^2\right)}-\frac{e(a+b\text{arcsinh}(cx))}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\text{arcsinh}(cx))}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2x^2+1}\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\sqrt{c^2d-e}}-\frac{bc\arctan\left(\frac{c^2\sqrt{-d}x+\sqrt{e}}{\sqrt{c^2x^2+1}\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\sqrt{c^2d-e}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

output
```
-1/4*(a + b*ArcSinh[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcS
inh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTan[(Sqrt[e] - c^
2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*S
qrt[e]) - (b*c*ArcTan[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 +
 c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2
)*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[c*x
])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4
*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLo
g[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(4*(-
d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) -
 (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])]
)/(4*(-d)^(3/2)*Sqrt[e])
```

3.612. $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx$

### 3.612.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.612.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.96 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| parts | $\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\operatorname{arcsinh}(cx)x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{c^2\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(cx)\ln\left(\_R\ldots\right.}{\ldots}\ldots\right.}{4d}\ldots\right.}{\ldots}$ |
| derivativedivides | $\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arcsinh}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(cx)\ln\left(\_R\ldots\right.}{\ldots}}{4c^2d}\ldots\right.}{\ldots}$ |
| default | $\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arcsinh}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(cx)\ln\left(\_R\ldots\right.}{\ldots}}{4c^2d}\ldots\right.}{\ldots}$ |

input
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arcsinh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4/d*c^2*sum(1/_R1/(_R1^2*e+2*c^2*d
-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-(c^2*
x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/4/d*c^2*sum
(_R1/(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)
+dilog((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_
Z^2+e))+1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(2*(c^2*d*(c^
2*d-e))^(1/2)*c^2*d+2*c^4*d^2-2*c^2*d*e-(c^2*d*(c^2*d-e))^(1/2)*e)*c^2*arc
tanh(e*(c*x+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^
(1/2))/d/(c^2*d-e)/e^3-1/2*(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2
)*(2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*arctanh(e*(c*x+(c^2*x^2+1)^(1/2))/
((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/2))*c^2/d/e^3+1/2*((2*c^2*d+
2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(-2*(c^2*d*(c^2*d-e))^(1/2)*c^2*d+2*
c^4*d^2-2*c^2*d*e+(c^2*d*(c^2*d-e))^(1/2)*e)*c^2*arctan(e*(c*x+(c^2*x^2+1)
^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/(c^2*d-e)/e^3-1
/2*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*
d-e))^(1/2)-e)*arctan(e*(c*x+(c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-
e))^(1/2)-e)*e)^(1/2))*c^2/d/e^3)
```

### 3.612.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

### 3.612.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

### 3.612.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.612.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(ex^2 + d)^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

### 3.612.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} \, dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{(e\,x^2 + d)^2} \, dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^2, x)
```

# 3.613 $\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$

### 3.613.1 Optimal result

Integrand size = 20, antiderivative size = 559

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))^2\,dx = 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} - \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{6}{125}b^2de^2x^5 - \frac{12b^2e^3x^5}{1225c^2} + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{4bd^2e\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} - \frac{16bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^5} + \frac{32be^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^7} - \frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} + \frac{8bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^3} - \frac{16be^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^5} - \frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} + \frac{12be^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^3} - \frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{49c} + d^3x(a+b\operatorname{arcsinh}(cx))^2 + d^2ex^3(a+b\operatorname{arcsinh}(cx))^2 + \frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))^2$$

output
```
2*b^2*d^3*x-4/3*b^2*d^2*e*x/c^2+16/25*b^2*d*e^2*x/c^4-32/245*b^2*e^3*x/c^6
+2/9*b^2*d^2*e*x^3-8/75*b^2*d*e^2*x^3/c^2+16/735*b^2*e^3*x^3/c^4+6/125*b^2
*d*e^2*x^5-12/1225*b^2*e^3*x^5/c^2+2/343*b^2*e^3*x^7+d^3*x*(a+b*arcsinh(c*
x))^2+d^2*e*x^3*(a+b*arcsinh(c*x))^2+3/5*d*e^2*x^5*(a+b*arcsinh(c*x))^2+1/
7*e^3*x^7*(a+b*arcsinh(c*x))^2-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2
)/c+4/3*b*d^2*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/25*b*d*e^2*(a+
b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+32/245*b*e^3*(a+b*arcsinh(c*x))*(c^2
*x^2+1)^(1/2)/c^7-2/3*b*d^2*e*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+8
/25*b*d*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/245*b*e^3*x^2*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5-6/25*b*d*e^2*x^4*(a+b*arcsinh(c*x
))*(c^2*x^2+1)^(1/2)/c+12/245*b*e^3*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/
2)/c^3-2/49*b*e^3*x^6*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.613.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.79

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{11025a^2c^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-210ab\sqrt{1+c^2x^2}(-240e^3+24c^2e^2(49d+5ex^2)-2c^4e}{}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*
a*b*Sqrt[1 + c^2*x^2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1
225*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441
*d*e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(-25200*e^3 + 840*c^2*e^2*(147*d + 5
*e*x^2) - 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3
+ 42875*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*
(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 + c^2*x^2]*(
-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2 +
 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6
)))*ArcSinh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 +
 5*e^3*x^6)*ArcSinh[c*x]^2)/(385875*c^7)
```

### 3.613.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(d^3(a+b\text{arcsinh}(cx))^2+3d^2ex^2(a+b\text{arcsinh}(cx))^2+3de^2x^4(a+b\text{arcsinh}(cx))^2+e^3x^6(a+b\text{arcsinh}(cx))^2\right) d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2bd^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{2bd^2ex^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}-\frac{6bde^2x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c}-\frac{2be^3x^6\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{49c}+\frac{32be^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^7}-\frac{16bde^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c^5}-\frac{16be^3x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^5}+\frac{4bd^2e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^3}+\frac{8bde^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c^3}+\frac{12be^3x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^3}+d^3x(a+b\text{arcsinh}(cx))^2+d^2ex^3(a+b\text{arcsinh}(cx))^2+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))^2+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))^2-\frac{32b^2e^3x}{245c^6}+\frac{16b^2de^2x}{25c^4}+\frac{16b^2e^3x^3}{735c^4}-\frac{4b^2d^2ex}{3c^2}-\frac{8b^2de^2x^3}{75c^2}-\frac{12b^2e^3x^5}{1225c^2}+2b^2d^3x+\frac{2}{9}b^2d^2ex^3+\frac{6}{125}b^2de^2x^5+\frac{2}{343}b^2e^3x^7$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) - (32*b^
2*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) + (1
6*b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225*
c^2) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/c + (4*b*d^2*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^3) - (16*b
*d*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c^5) + (32*b*e^3*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x]))/(3*c) + (8*b*d*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/(245*c^5) - (6*b*d*e^2*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
(25*c) + (12*b*e^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^3) -
 (2*b*e^3*x^6*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(49*c) + d^3*x*(a +
b*ArcSinh[c*x])^2 + d^2*e*x^3*(a + b*ArcSinh[c*x])^2 + (3*d*e^2*x^5*(a + b
*ArcSinh[c*x])^2)/5 + (e^3*x^7*(a + b*ArcSinh[c*x])^2)/7
```

### 3.613.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.613.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{c^4d^2e\left(9\operatorname{arcsinh}(cx)^2x^3c^3 \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{c^4d^2e\left(9\operatorname{arcsinh}(cx)^2x^3c^3 \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$ |
| parts | $a^2\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b^2\left(55125\operatorname{arcsinh}(cx)^2c^7x^7e^3+231525\operatorname{arcsinh}(cx)^2c^7x^5d\,e^2+3858 \cdots\right.}{\cdots}$ |

input
```
int((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+b
^2/c^6*(c^6*d^3*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+1/9*c^4*d^2*e*(9*arcsinh(c*x)^2*x^3*c^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
*x^2*c^2+2*c^3*x^3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-12*c*x)+1/375*c^2*d*e
^2*(225*arcsinh(c*x)^2*c^5*x^5-90*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4+1
8*c^5*x^5+120*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-40*c^3*x^3-240*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)+240*c*x)+1/25725*e^3*(3675*arcsinh(c*x)^2*c^7*x^7
-1050*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6*x^6+150*c^7*x^7+1260*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)*x^4*c^4-252*c^5*x^5-1680*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
*x^2*c^2+560*c^3*x^3+3360*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-3360*c*x))+2*a*b/
c^6*(arcsinh(c*x)*d^3*c^7*x+arcsinh(c*x)*d^2*c^7*e*x^3+3/5*arcsinh(c*x)*d*
c^7*e^2*x^5+1/7*arcsinh(c*x)*e^3*c^7*x^7-1/7*e^3*(1/7*c^6*x^6*(c^2*x^2+1)^
(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*
(c^2*x^2+1)^(1/2))-d^3*c^6*(c^2*x^2+1)^(1/2)-d^2*c^4*e*(1/3*c^2*x^2*(c^2*x
^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-3/5*d*c^2*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^
(1/2)-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))))
```

### 3.613.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.05

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1125\,(49\,a^2+2\,b^2)c^7e^3x^7+189\,(49\,(25\,a^2+2\,b^2)c^7de^2-20\,b^2c^5e^3)x^5+35\,(1225\,(9\,a^2+2\,b^2)c^7d^2e-1176}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/385875*(1125*(49*a^2 + 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 + 2*b^2)*c^7
*d*e^2 - 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 + 2*b^2)*c^7*d^2*e - 1176*b
^2*c^5*d*e^2 + 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^
7*d*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*log(c*x + sqrt(c^2*
x^2 + 1))^2 + 105*(3675*(a^2 + 2*b^2)*c^7*d^3 - 4900*b^2*c^5*d^2*e + 2352*
b^2*c^3*d*e^2 - 480*b^2*c*e^3)*x + 210*(525*a*b*c^7*e^3*x^7 + 2205*a*b*c^7
*d*e^2*x^5 + 3675*a*b*c^7*d^2*e*x^3 + 3675*a*b*c^7*d^3*x - (75*b^2*c^6*e^3
*x^6 + 3675*b^2*c^6*d^3 - 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 - 240*b^
2*e^3 + 9*(49*b^2*c^6*d*e^2 - 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e -
588*b^2*c^4*d*e^2 + 120*b^2*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqr
t(c^2*x^2 + 1)) - 210*(75*a*b*c^6*e^3*x^6 + 3675*a*b*c^6*d^3 - 2450*a*b*c^
4*d^2*e + 1176*a*b*c^2*d*e^2 - 240*a*b*e^3 + 9*(49*a*b*c^6*d*e^2 - 10*a*b*
c^4*e^3)*x^4 + (1225*a*b*c^6*d^2*e - 588*a*b*c^4*d*e^2 + 120*a*b*c^2*e^3)*
x^2)*sqrt(c^2*x^2 + 1))/c^7
```

### 3.613.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.77

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} a^2d^3x + a^2d^2ex^3 + \frac{3a^2de^2x^5}{5} + \frac{a^2e^3x^7}{7} + 2abd^3x\operatorname{asinh}(cx) + 2abd^2ex^3\operatorname{asinh}(cx) + \frac{6abde^2x^5\operatorname{asinh}(cx)}{5} + \frac{2abe^3 \cdots}{} \\ a^2\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e*
*3*x**7/7 + 2*a*b*d**3*x*asinh(c*x) + 2*a*b*d**2*e*x**3*asinh(c*x) + 6*a*b
*d*e**2*x**5*asinh(c*x)/5 + 2*a*b*e**3*x**7*asinh(c*x)/7 - 2*a*b*d**3*sqrt
(c**2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 + 1)/(3*c) - 6*a*b*d*
e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 + 1)
/(49*c) + 4*a*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c**3) + 8*a*b*d*e**2*x**2*sq
rt(c**2*x**2 + 1)/(25*c**3) + 12*a*b*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c*
*3) - 16*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(
c**2*x**2 + 1)/(245*c**5) + 32*a*b*e**3*sqrt(c**2*x**2 + 1)/(245*c**7) + b
**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*asinh(c*x)**2
+ 2*b**2*d**2*e*x**3/9 + 3*b**2*d*e**2*x**5*asinh(c*x)**2/5 + 6*b**2*d*e**
2*x**5/125 + b**2*e**3*x**7*asinh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**
2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**
2 + 1)*asinh(c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(25*c) - 2*b**2*e**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/(49*c) - 4*b**2
*d**2*e*x/(3*c**2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(122
5*c**2) + 4*b**2*d**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 8*b**2*d
*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c**3) + 12*b**2*e**3*x**4*sq
rt(c**2*x**2 + 1)*asinh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*
b**2*e**3*x**3/(735*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*...
```

3.613. $\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$

### 3.613.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.22

$$\int \left(d+ex^2\right)^3 (a+b\mathrm{arcsinh}(cx))^2\,dx = \frac{1}{7}b^2e^3x^7\,\mathrm{arsinh}\,(cx)^2 + \frac{1}{7}a^2e^3x^7 + \frac{3}{5}b^2de^2x^5\,\mathrm{arsinh}\,(cx)^2 + \frac{3}{5}a^2de^2x^5 + b^2d^2ex^3\,\mathrm{arsinh}\,(cx)^2 + a^2d^2ex^3 + b^2d^3x\,\mathrm{arsinh}\,(cx)^2 + \frac{2}{3}\left(3\,x^3\,\mathrm{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abd^2e - \frac{2}{9}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\mathrm{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2d^2e + \frac{2}{25}\left(15\,x^5\,\mathrm{arsinh}\,(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abde^2 - \frac{2}{375}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\,\mathrm{arsinh}\,(cx) - \frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4}\right)b + \frac{2}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)abe^3 - \frac{2}{25725}\left(105\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\,\mathrm{arsinh}\,(cx) - \frac{75\,c^6}{} \right. + 2\,b^2d^3\left(x - \frac{\sqrt{c^2x^2+1}\,\mathrm{arsinh}\,(cx)}{c}\right) + a^2d^3x + \frac{2\left(cx\,\mathrm{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd^3}{c}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*e^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arcsi
nh(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arcsinh(c*x)^2 + a^2*d^2*e*x
^3 + b^2*d^3*x*arcsinh(c*x)^2 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^
2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c
^2)*b^2*d^2*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 -
 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/3
75*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt
(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*
b^2*d*e^2 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*
sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2
+ 1)/c^8)*c)*a*b*e^3 - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(
c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/
c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^
6)*b^2*e^3 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x
+ 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

### 3.613.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.613.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^3, x)
```

# 3.614 $\int (d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^2 \, dx$



## 3.614.1 Optimal result

Integrand size = 20, antiderivative size = 329

$$\begin{aligned}\int (d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^2 \, dx &= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} \\ &\quad + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} \\ &\quad + \frac{8bde\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c^3} \\ &\quad - \frac{16be^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{75c^5} \\ &\quad - \frac{4bdex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c} \\ &\quad + \frac{8be^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{75c^3} \\ &\quad - \frac{2be^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} \\ &\quad + d^2x(a+b\operatorname{arcsinh}(cx))^2 + \frac{2}{3}dex^3(a+b\operatorname{arcsinh}(cx))^2 \\ &\quad + \frac{1}{5}e^2x^5(a+b\operatorname{arcsinh}(cx))^2\end{aligned}$$

output
```
2*b^2*d^2*x-8/9*b^2*d*e*x/c^2+16/75*b^2*e^2*x/c^4+4/27*b^2*d*e*x^3-8/225*b
^2*e^2*x^3/c^2+2/125*b^2*e^2*x^5+d^2*x*(a+b*arcsinh(c*x))^2+2/3*d*e*x^3*(a
+b*arcsinh(c*x))^2+1/5*e^2*x^5*(a+b*arcsinh(c*x))^2-2*b*d^2*(a+b*arcsinh(c
*x))*(c^2*x^2+1)^(1/2)/c+8/9*b*d*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^
3-16/75*b*e^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5-4/9*b*d*e*x^2*(a+b*
arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+8/75*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x
^2+1)^(1/2)/c^3-2/25*b*e^2*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

### 3.614.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \frac{225a^2c^5x(15d^2+10dex^2+3e^2x^4)-30ab\sqrt{1+c^2x^2}(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 + c^2*x^2
]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x
^4)) + 2*b^2*c*x*(360*e^2 - 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*
d*e*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^
4) + b*Sqrt[1 + c^2*x^2]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2
 + 50*d*e*x^2 + 9*e^2*x^4)))*ArcSinh[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e
*x^2 + 3*e^2*x^4)*ArcSinh[c*x]^2)/(3375*c^5)
```

### 3.614.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(d^2(a+b\text{arcsinh}(cx))^2+2dex^2(a+b\text{arcsinh}(cx))^2+e^2x^4(a+b\text{arcsinh}(cx))^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2bd^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{4bdex^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c}-\frac{2be^2x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c}-\frac{16be^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{75c^5}+\frac{8bde\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c^3}+\frac{8be^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{75c^3}+d^2x(a+b\text{arcsinh}(cx))^2+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2+\frac{16b^2e^2x}{75c^4}-\frac{8b^2dex}{9c^2}-\frac{8b^2e^2x^3}{225c^2}+2b^2d^2x+\frac{4}{27}b^2dex^3+\frac{2}{125}b^2e^2x^5$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e
*x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (8*b*d*e*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(75*c^5) - (4*b*d*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + (
8*b*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75*c^3) - (2*b*e^2*x^
4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + d^2*x*(a + b*ArcSinh[c*
x])^2 + (2*d*e*x^3*(a + b*ArcSinh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcSinh[c*x
])^2)/5
```

### 3.614.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

### 3.614.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^2\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{2c^2de\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\operatorname{arcsinh}(\right.}{}\right.}{}}{}$ |
| default | $\frac{\frac{a^2\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{2c^2de\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\operatorname{arcsinh}(\right.}{}\right.}{}}{}$ |
| parts | $a^2\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b^2\left(675\operatorname{arcsinh}(cx)^2c^5x^5e^2+2250\operatorname{arcsinh}(cx)^2c^5x^3de+3375\operatorname{arcsinh}(cx)^2c^5x\,d^2\right.}{}$ |

input
```
int((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b^2/c^4*(c^4*d^2*
(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+2/27*c^2*d*e*(
9*arcsinh(c*x)^2*x^3*c^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^
3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-12*c*x)+1/1125*e^2*(225*arcsinh(c*x)^2
*c^5*x^5-90*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4+18*c^5*x^5+120*arcsinh(
c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2-40*c^3*x^3-240*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)+240*c*x))+2*a*b/c^4*(arcsinh(c*x)*d^2*c^5*x+2/3*arcsinh(c*x)*d*c^5*e*x^
3+1/5*arcsinh(c*x)*e^2*c^5*x^5-1/5*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15
*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-d^2*c^4*(c^2*x^2+1)^(1/
2)-2/3*d*c^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

### 3.614.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.16

$$\int \left(d+ex^2\right)^2 (a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{27\,(25\,a^2+2\,b^2)c^5e^2x^5+10\,(25\,(9\,a^2+2\,b^2)c^5de-12\,b^2c^3e^2)x^3+225\,(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c^5d}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 + 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 + 2*b^2)*c^5*d*e -
 12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^
2*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 15*(225*(a^2 + 2*b^2)*c^5*d^
2 - 200*b^2*c^3*d*e + 48*b^2*c*e^2)*x + 30*(45*a*b*c^5*e^2*x^5 + 150*a*b*c
^5*d*e*x^3 + 225*a*b*c^5*d^2*x - (9*b^2*c^4*e^2*x^4 + 225*b^2*c^4*d^2 - 10
0*b^2*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c^4*d*e - 6*b^2*c^2*e^2)*x^2)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 30*(9*a*b*c^4*e^2*x^4 + 225*a
*b*c^4*d^2 - 100*a*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e - 6*a*b*c^2*
e^2)*x^2)*sqrt(c^2*x^2 + 1))/c^5
```

### 3.614.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.81

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))^2\,dx = \begin{cases} a^2d^2x + \frac{2a^2dex^3}{3} + \frac{a^2e^2x^5}{5} + 2abd^2x\,\text{asinh}\,(cx) + \frac{4abdex^3\,\text{asinh}\,(cx)}{3} + \frac{2abe^2x^5\,\text{asinh}\,(cx)}{5} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{4abdex^2\sqrt{c}}{9c} \\ a^2\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2
*x*asinh(c*x) + 4*a*b*d*e*x**3*asinh(c*x)/3 + 2*a*b*e**2*x**5*asinh(c*x)/5
 - 2*a*b*d**2*sqrt(c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(
9*c) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) + 8*a*b*d*e*sqrt(c**2*x*
*2 + 1)/(9*c**3) + 8*a*b*e**2*x**2*sqrt(c**2*x**2 + 1)/(75*c**3) - 16*a*b*
e**2*sqrt(c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d*
*2*x + 2*b**2*d*e*x**3*asinh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x*
*5*asinh(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 + 1
)*asinh(c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c) - 2*
b**2*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**
2) - 8*b**2*e**2*x**3/(225*c**2) + 8*b**2*d*e*sqrt(c**2*x**2 + 1)*asinh(c*
x)/(9*c**3) + 8*b**2*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**3) +
16*b**2*e**2*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75
*c**5), Ne(c, 0)), (a**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

### 3.614.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.30

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))^2 dx$$
$$= \frac{1}{5}b^2e^2x^5\operatorname{arsinh}(cx)^2 + \frac{1}{5}a^2e^2x^5 + \frac{2}{3}b^2dex^3\operatorname{arsinh}(cx)^2 + \frac{2}{3}a^2dex^3 + b^2d^2x\operatorname{arsinh}(cx)^2 + \frac{4}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abde - \frac{4}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3-6x}{c^2}\right)b^2de + \frac{2}{75}\left(15x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abe^2 - \frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\operatorname{arsinh}(cx) - \frac{9c^4x^5-20c^2x^3+120x}{c^4}\right) + 2b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2d^2x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*e^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arcsinh
(c*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arcsinh(c*x)^2 + 4/9*(3*x^3*arcsinh(
c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e -
4/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*
x) - (c^2*x^3 - 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^
2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6
)*c)*a*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 +
1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2
*x^3 + 120*x)/c^4)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)
/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

### 3.614.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.614.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^2, x)
```

# 3.615 $\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx$



## 3.615.1 Optimal result

Integrand size = 18, antiderivative size = 153

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx = 2b^2dx-\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+\frac{4be\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3}-\frac{2bex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c}+dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2$$

output
```
2*b^2*d*x-4/9*b^2*e*x/c^2+2/27*b^2*e*x^3+d*x*(a+b*arcsinh(c*x))^2+1/3*e*x^
3*(a+b*arcsinh(c*x))^2-2*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*
e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-2/9*b*e*x^2*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)/c
```

### 3.615.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

$$\int (d + ex^2) (a + b\text{arcsinh}(cx))^2 \, dx = \frac{9a^2c^3x(3d + ex^2) - 6ab\sqrt{1 + c^2x^2}(-2e + c^2(9d + ex^2)) + 2b^2cx(-6e + c^2(27d + ex^2)) - 6b(-3ac^3x(3d +}{27c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*
x^2)) + 2*b^2*c*x*(-6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x
^2) + b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))*ArcSinh[c*x] + 9*b^2
*c^3*x*(3*d + e*x^2)*ArcSinh[c*x]^2)/(27*c^3)
```

### 3.615.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2) (a + b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(d(a + b\text{arcsinh}(cx))^2 + ex^2(a + b\text{arcsinh}(cx))^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2bd\sqrt{c^2x^2+1}(a + b\text{arcsinh}(cx))}{c} - \frac{2bex^2\sqrt{c^2x^2+1}(a + b\text{arcsinh}(cx))}{9c} + \frac{4be\sqrt{c^2x^2+1}(a + b\text{arcsinh}(cx))}{9c^3} + dx(a + b\text{arcsinh}(cx))^2 + \frac{1}{3}ex^3(a + b\text{arcsinh}(cx))^2 - \frac{4b^2ex}{9c^2} + 2b^2dx + \frac{2}{27}b^2ex^3$$

input
```
Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
2*b^2*d*x - (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x]))/c + (4*b*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
))/(9*c^3) - (2*b*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + d*
x*(a + b*ArcSinh[c*x])^2 + (e*x^3*(a + b*ArcSinh[c*x])^2)/3
```

### 3.615.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.615.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{3}x^3e+dx\right)+\frac{b^2\left(\frac{e\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2c^3x^3+12\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}-12cx\right)}{27c^2}+d\left(\operatorname{arc}\right.\right.}{c}$ |
| derivativedivides | $\frac{a^2\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{e\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2\right.}{27}\right.}{c^2}$ |
| default | $\frac{a^2\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{e\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2\right.}{27}\right.}{c^2}$ |

input
```
int((e*x^2+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*x^3*e+d*x)+b^2/c*(1/27*e*(9*arcsinh(c*x)^2*x^3*c^3-6*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-12*
c*x)/c^2+d*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*
a*b/c*(1/3*c*arcsinh(c*x)*x^3*e+arcsinh(c*x)*d*c*x-1/3/c^2*(e*(1/3*c^2*x^2
*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d*c^2*(c^2*x^2+1)^(1/2)))
```

### 3.615.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.37

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{(9a^2+2b^2)c^3ex^3+9(b^2c^3ex^3+3b^2c^3dx)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+3(9(a^2+2b^2)c^3d-4b^2ce)x+6(3ab$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 + 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 3*(9*(a^2 + 2*b^2)*c^3*d - 4*b^2*c*e)*x + 6*(3*
a*b*c^3*e*x^3 + 9*a*b*c^3*d*x - (b^2*c^2*e*x^2 + 9*b^2*c^2*d - 2*b^2*e)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(a*b*c^2*e*x^2 + 9*a*b*c
^2*d - 2*a*b*e)*sqrt(c^2*x^2 + 1))/c^3
```

### 3.615.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.82

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} a^2dx+\frac{a^2ex^3}{3}+2abdx\operatorname{asinh}(cx)+\frac{2abex^3\operatorname{asinh}(cx)}{3}-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{2abex^2\sqrt{c^2x^2+1}}{9c}+\frac{4abe\sqrt{c^2x^2+1}}{9c^3}+b^2dx\operatorname{asinh} \\ a^2\left(dx+\frac{ex^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*asinh(c*x) + 2*a*b*e*x**3*
asinh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2
 + 1)/(9*c) + 4*a*b*e*sqrt(c**2*x**2 + 1)/(9*c**3) + b**2*d*x*asinh(c*x)**
2 + 2*b**2*d*x + b**2*e*x**3*asinh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d
*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 + 1)*asin
h(c*x)/(9*c) - 4*b**2*e*x/(9*c**2) + 4*b**2*e*sqrt(c**2*x**2 + 1)*asinh(c*
x)/(9*c**3), Ne(c, 0)), (a**2*(d*x + e*x**3/3), True))
```

### 3.615.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.42

$$\int \left(d+ex^2\right) (a+b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{1}{3}\, b^2 e x^3 \operatorname{arsinh}\left(cx\right)^2 + \frac{1}{3}\, a^2 e x^3 + b^2 d x \operatorname{arsinh}\left(cx\right)^2$$
$$+ \frac{2}{9}\left(3\, x^3 \operatorname{arsinh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right) abe$$
$$- \frac{2}{27}\left(3\, c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right) \operatorname{arsinh}\left(cx\right) - \frac{c^2x^3-6\,x}{c^2}\right) b^2 e$$
$$+ 2\, b^2 d\left(x - \frac{\sqrt{c^2x^2+1} \operatorname{arsinh}\left(cx\right)}{c}\right) + a^2 d x + \frac{2\left(cx \operatorname{arsinh}\left(cx\right) - \sqrt{c^2x^2+1}\right) abd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*e*x^3*arcsinh(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arcsinh(c*x)^2 + 2/
9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)
/c^4))*a*b*e - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/
c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2
 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))
*a*b*d/c
```

### 3.615.8 Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right) (a+b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.615.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\,(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2), x)
```

# 3.616 $\int (a + b\text{arcsinh}(cx))^2 \, dx$



## 3.616.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = 2b^2x - \frac{2b\sqrt{1+c^2x^2}(a + b\text{arcsinh}(cx))}{c} + x(a + b\text{arcsinh}(cx))^2$$

output
```
2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

## 3.616.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = \left(a^2 + 2b^2\right) x - \frac{2ab\sqrt{1+c^2x^2}}{c} + \frac{2b\left(acx - b\sqrt{1+c^2x^2}\right) \text{arcsinh}(cx)}{c} + b^2x\text{arcsinh}(cx)^2$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2,x]
```

output
```
(a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c
^2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

### 3.616.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\operatorname{arcsinh}(cx))^2-2bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x(a+b\operatorname{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)$$

$$\Big\downarrow \text{24}$$

$$x(a+b\operatorname{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)$$

input
```
Int[(a + b*ArcSinh[c*x])^2,x]
```

output
```
x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/c^2)
```

#### 3.616.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

## 3.616.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{cx\,a^2+b^2\left(\operatorname{arcsinh}(cx)^2 xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 72 |
| default | $\frac{cx\,a^2+b^2\left(\operatorname{arcsinh}(cx)^2 xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 72 |
| parts | $a^2x+\frac{b^2\left(\operatorname{arcsinh}(cx)^2 xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)}{c}+\frac{2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 73 |

input
```
int((a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^2+b^2*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*
x)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

## 3.616.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int (a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+(a^2+2\,b^2)cx-2\,\sqrt{c^2x^2+1}ab+2\left(abcx-\sqrt{c^2x^2+1}b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
(b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x
^2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 +
 1)))/c
```

### 3.616.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int (a+b\text{arcsinh}(cx))^2\,dx = \begin{cases} a^2x+2abx\,\text{asinh}\,(cx)-\frac{2ab\sqrt{c^2x^2+1}}{c}+b^2x\,\text{asinh}^2\,(cx)+2b^2x-\frac{2b^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{c} & \text{for } c\neq 0\\ a^2x & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**
2*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne
(c, 0)), (a**2*x, True))
```

### 3.616.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int (a+b\text{arcsinh}(cx))^2\,dx = b^2x\,\text{arsinh}\,(cx)^2+2\,b^2\left(x-\frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2x+\frac{2\,(cx\,\text{arsinh}\,(cx)-\sqrt{c^2x^2+1})ab}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*
x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

### 3.616.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int (a+b\text{arcsinh}(cx))^2\,dx$$
$$=2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2+1}\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2+a^2x$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x
+ sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))/c^2))*b^2 + a^2*x
```

### 3.616.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,dx$$

input
```
int((a + b*asinh(c*x))^2,x)
```

output
```
int((a + b*asinh(c*x))^2, x)
```

# 3.617 $\int \frac{(a+b\mathbf{arcsinh}(cx))^2}{d+ex^2}\,dx$

### 3.617.1 Optimal result

Integrand size = 20, antiderivative size = 739

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx = \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2 \log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2
)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+(c*x
+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^
(1/2)+1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d
)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(
1+(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2
)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))
*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2))
)/(-d)^(1/2)/e^(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcs
inh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+
e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/
2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+
(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(
1/2)+b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+
e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2
)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

### 3.617.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 985, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex^2}\,dx$$
$$= \frac{2a^2\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - 2ab\sqrt{d}\operatorname{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right) - b^2\sqrt{d}\operatorname{arcsinh}(cx)^2\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcs}}}{c\sqrt{-d}-\sqrt{}}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

output
```
(2*a^2*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*L
og[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b^2*S
qrt[d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) + e])] + 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*
x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log
[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + 2*a*
b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 - (Sqrt[e]*E^ArcSinh[c
*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1
 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - b^2*Sqrt[
d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^
2*d) + e])] + 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSi
nh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - 2*b*Sqrt[d]*(a + b*ArcSinh[c
*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) +
e])] - 2*a*b*Sqrt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + S
qrt[-(c^2*d) + e]))] - 2*b^2*Sqrt[d]*ArcSinh[c*x]*PolyLog[2, -((Sqrt[e]*E^
ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + 2*a*b*Sqrt[d]*PolyLog[
2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + 2*b^2*Sqr
t[d]*ArcSinh[c*x]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e])] - 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*S...
```

### 3.617.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

output
```
((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*S
qrt[e]) + ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^
2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*
Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcSin
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a +
b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt
[e]) + (b*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e])
 - (b^2*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) +
e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]
)
```

3.617. $\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx$

### 3.617.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.617.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{e\,x^2+d}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
```

## 3.617.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x^2 + d), x)
```

### 3.617.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex^2} \, dx = \int \frac{(a + b \operatorname{asinh}(cx))^2}{d + ex^2} \, dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x**2), x)
```

### 3.617.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.617.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex^2} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{ex^2 + d} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d), x)
```

### 3.617.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2), x)
```

# 3.618 $\int \frac{(d+ex^2)^3}{a+b\mathbf{arcsinh}(cx)}\,dx$

## 3.618.1 Optimal result

Integrand size = 20, antiderivative size = 670

$$\begin{aligned}
\int \frac{(d+ex^2)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = {} & \frac{d^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} \\
& - \frac{3d^2e\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} \\
& + \frac{3de^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} \\
& - \frac{5e^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc^7} \\
& + \frac{3d^2e\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} \\
& - \frac{9de^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
& + \frac{9e^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& + \frac{3de^2\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
& - \frac{5e^3\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& + \frac{e^3\cosh\left(\frac{7a}{b}\right)\operatorname{Chi}\left(\frac{7(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& - \frac{d^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} \\
& + \frac{3d^2e\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3} \\
& - \frac{3de^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} \\
& + \frac{5e^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc^7} \\
& - \frac{3d^2e\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3} \\
& + \frac{9de^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} \\
& - \frac{9e^3\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc^7} \\
& 3de^2\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{\phantom{b}}\right.
\end{aligned}$$

output
```
d^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-3/4*d^2*e*Chi((a+b*arcsinh(c*x
))/b)*cosh(a/b)/b/c^3+3/8*d*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5-
5/64*e^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^7+3/4*d^2*e*Chi(3*(a+b*ar
csinh(c*x))/b)*cosh(3*a/b)/b/c^3-9/16*d*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*co
sh(3*a/b)/b/c^5+9/64*e^3*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^7+3/1
6*d*e^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b/c^5-5/64*e^3*Chi(5*(a+b*
arcsinh(c*x))/b)*cosh(5*a/b)/b/c^7+1/64*e^3*Chi(7*(a+b*arcsinh(c*x))/b)*co
sh(7*a/b)/b/c^7-d^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+3/4*d^2*e*Shi(
(a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^3-3/8*d*e^2*Shi((a+b*arcsinh(c*x))/b)*
sinh(a/b)/b/c^5+5/64*e^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^7-3/4*d^2
*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+9/16*d*e^2*Shi(3*(a+b*arc
sinh(c*x))/b)*sinh(3*a/b)/b/c^5-9/64*e^3*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(
3*a/b)/b/c^7-3/16*d*e^2*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^5+5/64
*e^3*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^7-1/64*e^3*Shi(7*(a+b*arc
sinh(c*x))/b)*sinh(7*a/b)/b/c^7
```

### 3.618.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{(64c^6d^3-48c^4d^2e+24c^2de^2-5e^3)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+3e(16c^4d^2-12c^2de+3e^2)\cosh\left(\frac{3a}{b}\right)}{}$$

input
```
Integrate[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

output
```
((64*c^6*d^3 - 48*c^4*d^2*e + 24*c^2*d*e^2 - 5*e^3)*Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]] + 3*e*(16*c^4*d^2 - 12*c^2*d*e + 3*e^2)*Cosh[(3*a)/b]
*CoshIntegral[3*(a/b + ArcSinh[c*x])] + 12*c^2*d*e^2*Cosh[(5*a)/b]*CoshInt
egral[5*(a/b + ArcSinh[c*x])] - 5*e^3*Cosh[(5*a)/b]*CoshIntegral[5*(a/b +
ArcSinh[c*x])] + e^3*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] -
64*c^6*d^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 48*c^4*d^2*e*Sinh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e^2*Sinh[a/b]*SinhIntegra
l[a/b + ArcSinh[c*x]] + 5*e^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] -
 48*c^4*d^2*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 36*c^2*
d*e^2*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 9*e^3*Sinh[(3*a
)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 12*c^2*d*e^2*Sinh[(5*a)/b]*Sin
hIntegral[5*(a/b + ArcSinh[c*x])] + 5*e^3*Sinh[(5*a)/b]*SinhIntegral[5*(a/
b + ArcSinh[c*x])] - e^3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])
])/(64*b*c^7)
```

### 3.618.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d^3}{a+b\text{arcsinh}(cx)}+\frac{3d^2ex^2}{a+b\text{arcsinh}(cx)}+\frac{3de^2x^4}{a+b\text{arcsinh}(cx)}+\frac{e^3x^6}{a+b\text{arcsinh}(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5e^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^7}+\frac{9e^3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^7}-\frac{5e^3\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^7}+\frac{e^3\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^7}+\frac{5e^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64bc^7}-\frac{9e^3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^7}+\frac{5e^3\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^7}-\frac{e^3\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64bc^7}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5}-\frac{9de^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}+\frac{3de^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{3de^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5}+\frac{9de^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{3de^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{3d^2e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3}+\frac{3d^2e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}+\frac{3d^2e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3}-\frac{3d^2e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}+\frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

3.618. $\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx$

output
```
(d^3*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (3*d^2*e*Cosh
[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (3*d*e^2*Cosh[a/b]
*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) - (5*e^3*Cosh[a/b]*CoshIn
tegral[(a + b*ArcSinh[c*x])/b])/(64*b*c^7) + (3*d^2*e*Cosh[(3*a)/b]*CoshIn
tegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (9*d*e^2*Cosh[(3*a)/b]*Cos
hIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (9*e^3*Cosh[(3*a)/b]*C
oshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (3*d*e^2*Cosh[(5*a)/
b]*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (5*e^3*Cosh[(5*a
)/b]*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (e^3*Cosh[(7*a
)/b]*CoshIntegral[(7*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) - (d^3*Sinh[a/b]
*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) + (3*d^2*e*Sinh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) - (3*d*e^2*Sinh[a/b]*SinhIntegral[
(a + b*ArcSinh[c*x])/b])/(8*b*c^5) + (5*e^3*Sinh[a/b]*SinhIntegral[(a + b*
ArcSinh[c*x])/b])/(64*b*c^7) - (3*d^2*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(4*b*c^3) + (9*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(
a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (9*e^3*Sinh[(3*a)/b]*SinhIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) - (3*d*e^2*Sinh[(5*a)/b]*SinhIntegra
l[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (5*e^3*Sinh[(5*a)/b]*SinhInteg
ral[(5*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) - (e^3*Sinh[(7*a)/b]*SinhInteg
ral[(7*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7)
```

### 3.618.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.618. $\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx$

### 3.618.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 654, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $-\frac{e^3 e^{\frac{7a}{b}} \operatorname{Ei}_1\left(7\operatorname{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^6 b}-\frac{e^3 e^{-\frac{7a}{b}} \operatorname{Ei}_1\left(-7\operatorname{arcsinh}(cx)-\frac{7a}{b}\right)}{128c^6 b}-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d^3}{2b}+\frac{3\,e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d^2 e}{8c^2 b}$ |
| default | $-\frac{e^3 e^{\frac{7a}{b}} \operatorname{Ei}_1\left(7\operatorname{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^6 b}-\frac{e^3 e^{-\frac{7a}{b}} \operatorname{Ei}_1\left(-7\operatorname{arcsinh}(cx)-\frac{7a}{b}\right)}{128c^6 b}-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d^3}{2b}+\frac{3\,e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d^2 e}{8c^2 b}$ |

input
```
int((e*x^2+d)^3/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/128/c^6*e^3/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)-1/128/c^6*e^3/
b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)
+a/b)*d^3+3/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^2*e-3/16/c^4/b*exp(a
/b)*Ei(1,arcsinh(c*x)+a/b)*d*e^2+5/128/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/
b)*e^3-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2/b*exp(-a/b)*Ei(
1,-arcsinh(c*x)-a/b)*d^2*e-3/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*
e^2+5/128/c^6/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^3-3/8/c^2*e/b*exp(3*a/
b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d^2+9/32/c^4*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh
(c*x)+3*a/b)*d-9/128/c^6*e^3/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c
^2*e/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d^2+9/32/c^4*e^2/b*exp(-3*a
/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d-9/128/c^6*e^3/b*exp(-3*a/b)*Ei(1,-3*arcs
inh(c*x)-3*a/b)-3/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)*d+5/1
28/c^6*e^3/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32/c^4*e^2/b*exp(-5*a
/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*d+5/128/c^6*e^3/b*exp(-5*a/b)*Ei(1,-5*arcs
inh(c*x)-5*a/b))
```

### 3.618.5 Fricas [F]

$$\int \frac{(d+ex^2)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/(b*arcsinh(c*x) + a),
 x)
```

### 3.618.6 Sympy [F]

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+ex^2)^3}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)**3/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x**2)**3/(a + b*asinh(c*x)), x)
```

### 3.618.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

### 3.618.8 Giac [F]

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

### 3.618.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{{\left(e\,x^2+d\right)}^3}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x^2)^3/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x^2)^3/(a + b*asinh(c*x)), x)
```

# 3.619 $\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx$

### 3.619.1 Optimal result

Integrand size = 20, antiderivative size = 388

$$\int \frac{(d+ex^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} - \frac{de\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3} + \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} + \frac{de\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3} - \frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} + \frac{e^2\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{d^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} + \frac{de\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3} - \frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} - \frac{de\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3} + \frac{3e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{e^2\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}$$

output
```
d^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/2*d*e*Chi((a+b*arcsinh(c*x))
/b)*cosh(a/b)/b/c^3+1/8*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5+1/2*
d*e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-3/16*e^2*Chi(3*(a+b*arcs
inh(c*x))/b)*cosh(3*a/b)/b/c^5+1/16*e^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5
*a/b)/b/c^5-d^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/2*d*e*Shi((a+b*a
rcsinh(c*x))/b)*sinh(a/b)/b/c^3-1/8*e^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b
)/b/c^5-1/2*d*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+3/16*e^2*Shi
(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^5-1/16*e^2*Shi(5*(a+b*arcsinh(c*x
))/b)*sinh(5*a/b)/b/c^5
```

### 3.619.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{2(8c^4d^2-4c^2de+e^2)\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+(8c^2d-3e)\,e\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
(2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]
] + (8*c^2*d - 3*e)*e*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] +
 e^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 16*c^4*d^2*Sinh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*c^2*d*e*Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]] - 2*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 8*
c^2*d*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 3*e^2*Sinh[(3
*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - e^2*Sinh[(5*a)/b]*SinhIntegr
al[5*(a/b + ArcSinh[c*x])])/(16*b*c^5)
```

### 3.619.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d^2}{a+b\operatorname{arcsinh}(cx)}+\frac{2dex^2}{a+b\operatorname{arcsinh}(cx)}+\frac{e^2x^4}{a+b\operatorname{arcsinh}(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}+\frac{e^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5}+\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2bc^3}+\frac{de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3}+\frac{de\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2bc^3}-\frac{de\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (d*e*Cosh[a/b
]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(2*b*c^3) + (e^2*Cosh[a/b]*CoshInt
egral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) + (d*e*Cosh[(3*a)/b]*CoshIntegral
[(3*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - (3*e^2*Cosh[(3*a)/b]*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (e^2*Cosh[(5*a)/b]*CoshIntegra
l[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (d^2*Sinh[a/b]*SinhIntegral[(a
 + b*ArcSinh[c*x])/b])/(b*c) + (d*e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[
c*x])/b])/(2*b*c^3) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])
/(8*b*c^5) - (d*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(2*b*c^3) + (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])
/(16*b*c^5) - (e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])
/(16*b*c^5)
```

### 3.619.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

3.619. $\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx$

## 3.619.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $-\frac{e^2 e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4 b}-\frac{e^2 e^{-\frac{5a}{b}} \operatorname{Ei}_1\left(-5 \operatorname{arcsinh}(cx)-\frac{5a}{b}\right)}{32c^4 b}-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d^2}{2b}+\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) de}{4c^2 b}-$ |
| default | $-\frac{e^2 e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4 b}-\frac{e^2 e^{-\frac{5a}{b}} \operatorname{Ei}_1\left(-5 \operatorname{arcsinh}(cx)-\frac{5a}{b}\right)}{32c^4 b}-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d^2}{2b}+\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) de}{4c^2 b}-$ |

input
```
int((e*x^2+d)^2/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/c^4*e^2/b*
exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a
/b)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d*e-1/16/c^4/b*exp(a/b)*
Ei(1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^2+1/4
/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-ar
csinh(c*x)-a/b)*e^2-1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d+3/
32/c^4*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)
*Ei(1,-3*arcsinh(c*x)-3*a/b)*d+3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b))
```

## 3.619.5 Fricas [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex^2+d)^2}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arcsinh(c*x) + a), x)
```

### 3.619.6 Sympy [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+ex^2)^2}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x**2)**2/(a + b*asinh(c*x)), x)
```

### 3.619.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

### 3.619.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

### 3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x^2)^2/(a + b*asinh(c*x)), x)
```

# 3.620 $\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx$



## 3.620.1 Optimal result

Integrand size = 18, antiderivative size = 180

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} + \frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} - \frac{e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}$$

output
```
d*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/4*e*Chi((a+b*arcsinh(c*x))/b)*
cosh(a/b)/b/c^3+1/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-d*Shi(
(a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/4*e*Shi((a+b*arcsinh(c*x))/b)*sinh(a
/b)/b/c^3-1/4*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3
```

### 3.620.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{(4c^2d-e)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-4c^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\right.}{4bc^3}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

output
```
((4*c^2*d - e)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e*Cosh[(3*a)/b
]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c^2*d*Sinh[a/b]*SinhIntegral[a/
b + ArcSinh[c*x]] + e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - e*Sinh[
(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

### 3.620.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx$$
$$\Big\downarrow \text{6208}$$
$$\int \left(\frac{d}{a+b\text{arcsinh}(cx)}+\frac{ex^2}{a+b\text{arcsinh}(cx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}+\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (e*Cosh[a/b]*Co
shIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (e*Cosh[(3*a)/b]*CoshInteg
ral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (d*Sinh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(b*c) + (e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x
])/b])/(4*b*c^3) - (e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/
b])/(4*b*c^3)
```

### 3.620.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.620.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\,\mathrm{Ei}_1\left(-3\,\mathrm{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)e}{8c^2b}-}{c}$ |
| default | $\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\,\mathrm{Ei}_1\left(-3\,\mathrm{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)e}{8c^2b}-}{c}$ |

input
```
int((e*x^2+d)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/8*e/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8*e/c^2/b*exp(
3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d+
1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arcsinh(
c*x)-a/b)*d+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e)
```

### 3.620.5 Fricas [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

### 3.620.6 Sympy [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{d+ex^2}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x**2)/(a + b*asinh(c*x)), x)
```

### 3.620.7 Maxima [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

### 3.620.8 Giac [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

### 3.620.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{e\,x^2+d}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x)), x)
```

# 3.621 $\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx$



## 3.621.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

output
```
Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b
)/b/c
```

## 3.621.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(b*c)
```

### 3.621.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6189, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b] - Sinh[a/b]*SinhIntegral[(
a + b*ArcSinh[c*x])/b])/(b*c)
```

### 3.621.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

### 3.621.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$ | 56 |
| default | $\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$ | 56 |

input
```
int(1/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(
c*x)-a/b))
```

### 3.621.5 Fricas [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsinh(c*x) + a), x)
```

### 3.621.6 Sympy [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{1}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(a + b*asinh(c*x)), x)
```

### 3.621.7 Maxima [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

### 3.621.8 Giac [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

### 3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

input
```
int(1/(a + b*asinh(c*x)),x)
```

output
```
int(1/(a + b*asinh(c*x)), x)
```

# 3.622 $\int \frac{1}{(d+ex^2)(a+b\,\text{arcsinh}(cx))}\,dx$



## 3.622.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

## 3.622.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

### 3.622.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))}\,dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

#### 3.622.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

### 3.622.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\operatorname{arcsinh}\,(cx))}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

### 3.622.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

### 3.622.6 Sympy [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)), x)
```

### 3.622.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

### 3.622.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

### 3.622.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,(e\,x^2+d)} \, dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)), x)
```

# 3.623 $\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx$



## 3.623.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

## 3.623.2 Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

### 3.623.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

#### 3.623.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

### 3.623.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(e\,x^2+d\right)^2 \left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

### 3.623.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(ex^2+d)^2(b \operatorname{arsinh}(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b
*d^2)*arcsinh(c*x)), x)
```

### 3.623.6 Sympy [N/A]

Not integrable

Time = 65.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(a+b \operatorname{asinh}(cx)) (d+ex^2)^2} \, dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

### 3.623.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(ex^2+d)^2(b \operatorname{arsinh}(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

### 3.623.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

### 3.623.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^2), x)
```

# 3.624 $\int \frac{(d+ex^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$

## 3.624.1 Optimal result

Integrand size = 20, antiderivative size = 495

$$\int \frac{(d+ex^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{d^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{de\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{2b^2c^3}-\frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^5}-\frac{3de\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{2b^2c^3}+\frac{9e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c^5}-\frac{5e^2\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c^5}+\frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{de\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2b^2c^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8b^2c^5}+\frac{3de\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2b^2c^3}-\frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^5}+\frac{5e^2\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^5}$$

output
```
d^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/2*d*e*cosh(a/b)*Shi((a+b*a
rcsinh(c*x))/b)/b^2/c^3+1/8*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^
5+3/2*d*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-9/16*e^2*cosh(3*
a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^5+5/16*e^2*cosh(5*a/b)*Shi(5*(a+b*a
rcsinh(c*x))/b)/b^2/c^5-d^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/2*
d*e*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^3-1/8*e^2*Chi((a+b*arcsinh(c
*x))/b)*sinh(a/b)/b^2/c^5-3/2*d*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/
b^2/c^3+9/16*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^5-5/16*e^2*
Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^5-d^2*(c^2*x^2+1)^(1/2)/b/c/
(a+b*arcsinh(c*x))-2*d*e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*
x^4*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

## 3.624.2 Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx =$$
$$-\frac{\frac{16bc^4d^2\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}+\frac{32bc^4dex^2\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}+\frac{16bc^4e^2x^4\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}+2(8c^4d^2-4c^2de+e^2)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*((16*b*c^4*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (32*b*c^4*d
*e*x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (16*b*c^4*e^2*x^4*Sqrt[1
+ c^2*x^2])/(a + b*ArcSinh[c*x]) + 2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*CoshInt
egral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*(8*c^2*d - 3*e)*e*CoshIntegral[3*(
a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] + 5*e^2*CoshIntegral[5*(a/b + ArcSinh[c
*x])]*Sinh[(5*a)/b] - 16*c^4*d^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]
] + 8*c^2*d*e*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 2*e^2*Cosh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e*Cosh[(3*a)/b]*SinhIntegral[
3*(a/b + ArcSinh[c*x])] + 9*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSin
h[c*x])] - 5*e^2*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])])/(b^2*
c^5)
```

### 3.624.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(\frac{d^2}{(a+b\text{arcsinh}(cx))^2}+\frac{2dex^2}{(a+b\text{arcsinh}(cx))^2}+\frac{e^2x^4}{(a+b\text{arcsinh}(cx))^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}+\frac{5e^2\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}+\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2b^2c^3}-\frac{3de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}-\frac{de\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2b^2c^3}+\frac{3de\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{2dex^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{e^2x^4\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x^2*Sqrt[1
+ c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^4*Sqrt[1 + c^2*x^2])/(b*c*
(a + b*ArcSinh[c*x])) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b
])/(b^2*c) + (d*e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(2*b^2*c
^3) - (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b^2*c^5) - (
3*d*e*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(2*b^2*c^3)
+ (9*e^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c
^5) - (5*e^2*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b
^2*c^5) + (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (
d*e*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(2*b^2*c^3) + (e^2*Cos
h[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^5) + (3*d*e*Cosh[(3*
a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Cosh[
(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5) + (5*e^2*C
osh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5)
```

### 3.624.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.624.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(469) = 938$.

Time = 0.97 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.09

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1036 |
| default | Expression too large to display | 1036 |

input
```
int((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(
c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+b*arcsinh(c*x))+5/3
2*e^2/c^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/b*e^2/c^4*(16*c^5
*x^5+20*c^3*x^3+16*c^4*x^4*(c^2*x^2+1)^(1/2)+5*c*x+12*c^2*x^2*(c^2*x^2+1)^
(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-5/32/b^2*e^2/c^4*exp(-5*a/b)*E
i(1,-5*arcsinh(c*x)-5*a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d^2/b/(a+b*arcsinh
(c*x))+1/2*d^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/4*(-(c^2*x^2+1)^(1/2)
+c*x)*d*e/c^2/b/(a+b*arcsinh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arcsinh(c
*x)+a/b)+1/16*(-(c^2*x^2+1)^(1/2)+c*x)*e^2/c^4/b/(a+b*arcsinh(c*x))+1/16/c
^4*e^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c*x+(c^2*x^2+1)^(1/2
))/(a+b*arcsinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4/c^
2/b*d*e*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/4/c^2/b^2*d*e*exp(-a/
b)*Ei(1,-arcsinh(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arc
sinh(c*x))-1/16/c^4/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(4*c^3*x
^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*d*e/c^2/b/(a+b*arc
sinh(c*x))-3/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(
1/2))*e^2/c^4/b/(a+b*arcsinh(c*x))+3/4*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh
(c*x)+3*a/b)*d-9/32*e^2/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/
c^2*e/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a
+b*arcsinh(c*x))*d+3/32/c^4*e^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1...
```

### 3.624.5 Fricas [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c
*x) + a^2), x)
```

### 3.624.6 Sympy [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex^2)^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x**2)**2/(a + b*asinh(c*x))**2, x)
```

### 3.624.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e^2*x^7 + (2*c^3*d*e + c*e^2)*x^5 + c*d^2*x + (c^3*d^2 + 2*c*d*e)*x^
3 + (c^2*e^2*x^6 + (2*c^2*d*e + e^2)*x^4 + (c^2*d^2 + 2*d*e)*x^2 + d^2)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2
*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1
))) + integrate((5*c^5*e^2*x^8 + 2*(3*c^5*d*e + 5*c^3*e^2)*x^6 + (c^5*d^2
+ 12*c^3*d*e + 5*c*e^2)*x^4 + c*d^2 + 2*(c^3*d^2 + 3*c*d*e)*x^2 + (5*c^3*e
^2*x^6 + 3*(2*c^3*d*e + c*e^2)*x^4 - c*d^2 + (c^3*d^2 + 2*c*d*e)*x^2)*(c^2
*x^2 + 1) + (10*c^4*e^2*x^7 + (12*c^4*d*e + 13*c^2*e^2)*x^5 + 2*(c^4*d^2 +
 7*c^2*d*e + 2*e^2)*x^3 + (c^2*d^2 + 4*d*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5
*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 +
(c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c
^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a
*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.624.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(ex^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

### 3.624.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(e\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^2, x)
```

# 3.625 $\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.625.1 Optimal result

Integrand size = 18, antiderivative size = 247

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{d\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{e\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3}-\frac{3e\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}$$

output
```
d*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/4*e*cosh(a/b)*Shi((a+b*arcsi
nh(c*x))/b)/b^2/c^3+3/4*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-
d*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e*Chi((a+b*arcsinh(c*x))/b
)*sinh(a/b)/b^2/c^3-3/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-
d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+
b*arcsinh(c*x))
```

### 3.625.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.77

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\frac{4bc^2d\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{4bc^2ex^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+(4c^2d-e)\,\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+3e\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/4*((4*b*c^2*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e*x^2*
Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*c^2*d - e)*CoshIntegral[a/b +
 ArcSinh[c*x]]*Sinh[a/b] + 3*e*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(
3*a)/b] - 4*c^2*d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e*Cosh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - 3*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b
+ ArcSinh[c*x])])/(b^2*c^3)
```

### 3.625.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d}{(a+b\text{arcsinh}(cx))^2}+\frac{ex^2}{(a+b\text{arcsinh}(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}-\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x^2*Sqrt[1 + c^2*
x^2])/(b*c*(a + b*ArcSinh[c*x])) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]
*Sinh[a/b])/(b^2*c) + (e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(
4*b^2*c^3) - (3*e*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/
(4*b^2*c^3) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) -
 (e*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) + (3*e*Cos
h[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

### 3.625.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

### 3.625.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.77

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e}{8c^2b(a+b\ \mathrm{arcsinh}(cx))}+\frac{3e\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\ \mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8c^2b(a+b\ \mathrm{arcsinh}(cx))}-3e\,\mathrm{e}}{}$ |
| default | $\frac{\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e}{8c^2b(a+b\ \mathrm{arcsinh}(cx))}+\frac{3e\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\ \mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8c^2b(a+b\ \mathrm{arcsinh}(cx))}-3e\,\mathrm{e}}{}$ |

input
```
int((e*x^2+d)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e
/c^2/b/(a+b*arcsinh(c*x))+3/8*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a
/b)-1/8*e/c^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(
1/2))/(a+b*arcsinh(c*x))-3/8*e/c^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d/b/(a+b*arcsinh(c*x))-1/8*(-(c^2*x^2+1)
^(1/2)+c*x)*e/c^2/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+
a/b)*d-1/8/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e-1/2/b*(c*x+(c^2*x^2+1
)^(1/2))/(a+b*arcsinh(c*x))*d+1/8/c^2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsi
nh(c*x))*e-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d+1/8/c^2/b^2*exp(-a/
b)*Ei(1,-arcsinh(c*x)-a/b)*e)
```

### 3.625.5 Fricas [F]

$$\int \frac{d+ex^2}{(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{ex^2+d}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

### 3.625.6 Sympy [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{d+ex^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

### 3.625.7 Maxima [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex^2+d}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e*x^5 + (c^3*d + c*e)*x^3 + c*d*x + (c^2*e*x^4 + (c^2*d + e)*x^2 + d
)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
 + 1))) + integrate((3*c^5*e*x^6 + (c^5*d + 6*c^3*e)*x^4 + (2*c^3*d + 3*c*
e)*x^2 + (3*c^3*e*x^4 + (c^3*d + c*e)*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (6*
c^4*e*x^5 + (2*c^4*d + 7*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c^2*x^2 + 1))/
(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^
5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3
 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4
*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

### 3.625.8 Giac [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex^2+d}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

### 3.625.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{e\,x^2+d}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x))^2, x)
```

# 3.626 $\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.626.1 Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}$$

output
```
cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a
/b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

## 3.626.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - \text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b^2c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
(-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSin
h[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

### 3.626.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6188, 6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{c\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx}{b}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int-\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 3042

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 3779

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

$\downarrow$ 3782

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

---

3.626. $\int\frac{1}{(a+b\text{arcsinh}(cx))^2}dx$

input
```
Int[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
-(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) + (I*(I*CoshIntegral[(a +
b*ArcSinh[c*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x
])/b]))/(b^2*c)
```

#### 3.626.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

3.626. $\int \frac{1}{(a+b\text{arcsinh}(cx))^2} dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.626.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\operatorname{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\operatorname{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$ | 118 |
| default | $\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\operatorname{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\operatorname{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$ | 118 |

input
```
int(1/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei
(1,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/
b^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

### 3.626.5 Fricas [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

### 3.626.6 Sympy [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(a + b \operatorname{asinh}{(cx)})^2} \, dx$$

input
```
integrate(1/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(-2), x)
```

### 3.626.7 Maxima [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2} \, dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(
c*x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)
*(c^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (
c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 +
 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c
^2*x^2 + 1)), x)
```

### 3.626.8 Giac [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2} \, dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-2), x)
```

### 3.626.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

input
```
int(1/(a + b*asinh(c*x))^2,x)
```

output
```
int(1/(a + b*asinh(c*x))^2, x)
```

# 3.627 $\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.627.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

## 3.627.2 Mathematica [N/A]

Not integrable

Time = 12.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.627.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \; 6209$$

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.627.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.627.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\mathrm{arcsinh}\,(cx))^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

### 3.627.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex^2)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c*x)^2 + 2*(a*
b*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

### 3.627.6 Sympy [N/A]

Not integrable

Time = 33.78 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^2(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)), x)
```

### 3.627.7 Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 766, normalized size of antiderivative = 38.30

$$\int \frac{1}{(d+ex^2)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^4 + (c^3*d + c*e)*a*b*
x^2 + a*b*c*d + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*c^
2*e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (
a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^6 - (
c^5*d - 2*c^3*e)*x^4 - (2*c^3*d - c*e)*x^2 + (c^3*e*x^4 - (c^3*d - 3*c*e)*
x^2 + c*d)*(c^2*x^2 + 1) - c*d + (2*c^4*e*x^5 - (2*c^4*d - 5*c^2*e)*x^3 -
(c^2*d - 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^
2)*a*b*x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + 2*(c^3*d^
2 + c*d*e)*a*b*x^2 + (a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^
2)*(c^2*x^2 + 1) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6 + (c^5
*d^2 + 4*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*b^2*x^
2 + (b^2*c^3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1)
+ 2*(b^2*c^4*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^
4*d^2 + 2*c^2*d*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + 2*(a*b*c^4*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (
c^4*d^2 + 2*c^2*d*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

## 3.627.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

## 3.627.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\text{asinh}\,(c\,x)\right)^2\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)), x)
```

3.627. $\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}\,dx$

# 3.628 $\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.628.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

## 3.628.2 Mathematica [N/A]

Not integrable

Time = 25.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

## 3.628.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.628.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.628.4 Maple [N/A] (verified)

Not integrable

Time = 0.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2\,(a+b\,\operatorname{arcsinh}\,(cx))^2}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

### 3.628.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.90

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^2} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d
*e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^
2)*arcsinh(c*x)), x)
```

### 3.628.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

### 3.628.7 Maxima [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 1027, normalized size of antiderivative = 51.35

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^2} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^6 + (2*c^3*d*e + c*e
^2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2 + (b^2*c^3*e^2*x^6 +
 (2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e)*b^2*x^2 + (
b^2*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^
2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*e*x^6 - (c^5*d - 6*c^3*e)*x
^4 - (2*c^3*d - 3*c*e)*x^2 + (3*c^3*e*x^4 - (c^3*d - 5*c*e)*x^2 + c*d)*(c^
2*x^2 + 1) - c*d + (6*c^4*e*x^5 - (2*c^4*d - 11*c^2*e)*x^3 - (c^2*d - 4*e)
*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x
^8 + (3*c^5*d^2*e + 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e
+ 3*c*d*e^2)*a*b*x^4 + a*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2 + (a*b*
c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)
*(c^2*x^2 + 1) + (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (
3*c^5*d^2*e + 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*
d*e^2)*b^2*x^4 + b^2*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*b^2*x^2 + (b^2*c^3*e^
3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c^2*
x^2 + 1) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e^3)*b^2*x^7 + b^2*c^2*
d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*b^2*x^
3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^3*x^9 +
(3*c^4*d*e^2 + c^2*e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*...
```

### 3.628.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

### 3.628.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2), x)
```

# 3.629 $\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx$

### 3.629.1 Optimal result

Integrand size = 22, antiderivative size = 672

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = d^2x\sqrt{a+b\text{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)}+\frac{\sqrt{b}d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}dee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3}+\frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5}+\frac{\sqrt{b}dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3}-\frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5}+\frac{\sqrt{b}e^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}-\frac{\sqrt{b}d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}+\frac{\sqrt{b}dee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5}-\frac{\sqrt{b}dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5}-\frac{\sqrt{b}e^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}$$

output
```
1/1600*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2
)*5^(1/2)*Pi^(1/2)/c^5-1/1600*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^
(1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)+1/72*d*e*exp(3*a/b)*erf(3^(
1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^3-1/192*
e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/
2)*Pi^(1/2)/c^5-1/72*d*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^
(1/2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/192*e^2*erfi(3^(1/2)*(a+b*arcsinh(
c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)+1/4*d^2*exp(a
/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/8*d*e*exp(a
/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3+1/32*e^2*ex
p(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5-1/4*d^2*
erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/8*d*e
*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)-1/32
*e^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5/exp(a/b)+
d^2*x*(a+b*arcsinh(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arcsinh(c*x))^(1/2)+1/5*e^
2*x^5*(a+b*arcsinh(c*x))^(1/2)
```

### 3.629.2 Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^2\sqrt{a+b\text{arcsinh}(cx)}\,dx =$$

$$-\frac{be^{-\frac{5a}{b}}\left(450e^{\frac{6a}{b}}\left(8ac^4d^2\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+8bc^4d^2\text{arcsinh}(cx)\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+b\left(4c^2d-e\right)e\sqrt{-\frac{a+b}{}}\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
-1/7200*(b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcSinh[c*x]] + 8*b*c^
4*d^2*ArcSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*(4*c^2*d - e)*e*Sqrt[-((a
+ b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, a/b
+ ArcSinh[c*x]] + 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*A
rcSinh[c*x])^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcSinh[c*x]))/b] + 25*Sqrt[3]*
b*(8*c^2*d - 3*e)*e*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*Arc
Sinh[c*x])^2/b^2)]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] + 450*E^((4*a)/
b)*(8*a*c^4*d^2*Sqrt[-((a + b*ArcSinh[c*x])/b)] + 8*b*c^4*d^2*ArcSinh[c*x]
*Sqrt[-((a + b*ArcSinh[c*x])/b)] + b*e*(-4*c^2*d + e)*Sqrt[a/b + ArcSinh[c
*x]]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcSinh[c*x]
)/b)] - b*e*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcS
inh[c*x])^2/b^2)]*(25*Sqrt[3]*(8*c^2*d - 3*e)*Gamma[3/2, (3*(a + b*ArcSinh
[c*x]))/b] + 9*Sqrt[5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b
])))/(c^5*E^((5*a)/b)*(a + b*ArcSinh[c*x])^(3/2))
```

### 3.629.3 Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(d^2\sqrt{a+b\text{arcsinh}(cx)}+2dex^2\sqrt{a+b\text{arcsinh}(cx)}+e^2x^4\sqrt{a+b\text{arcsinh}(cx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}-\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^2e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}-\frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{\pi}\sqrt{b}d^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}d^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}+d^2x\sqrt{a+b\text{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)}$$

input
```
Int[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
d^2*x*Sqrt[a + b*ArcSinh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 +
(e^2*x^5*Sqrt[a + b*ArcSinh[c*x]])/5 + (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Er
f[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3) + (Sqrt[b]*e^2*E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5) + (Sqrt[b]*d*e*E^((3*a)
/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3) -
 (Sqrt[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(64*c^5) + (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sq
rt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) + (Sqrt[b]*d*e*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*
e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3*E^(
(3*a)/b)) + (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(64*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*S
qrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

## 3.629. $\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx$

### 3.629.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.629.4 Maple [F]

$$\int \left(e\,x^{2}+d\right)^{2} \sqrt{a+b\,\operatorname{arcsinh}\left(c x\right)}dx$$

input
```
int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

## 3.629.5 Fricas [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.629.6 Sympy [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\operatorname{asinh}\left(cx\right)}\left(d+ex^2\right)^2\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2, x)
```

### 3.629.7 Maxima [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(ex^2+d\right)^2\sqrt{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

### 3.629.8 Giac [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(ex^2+d\right)^2\sqrt{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

### 3.629.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\, dx = \int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(e\,x^2+d\right)^2\, dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2, x)
```

# 3.630 $\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)}\,dx$



## 3.630.1 Optimal result

Integrand size = 20, antiderivative size = 322

$$\begin{aligned}\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)}\,dx &= dx\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\text{arcsinh}(cx)}\\ &+\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}\\ &-\frac{\sqrt{b}ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}\\ &+\frac{\sqrt{b}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\\ &-\frac{\sqrt{b}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}\\ &+\frac{\sqrt{b}ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}\\ &-\frac{\sqrt{b}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\end{aligned}$$

output
```
1/144*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3
^(1/2)*Pi^(1/2)/c^3-1/144*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
*b^(1/2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/4*d*exp(a/b)*erf((a+b*arcsinh(c
*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/16*e*exp(a/b)*erf((a+b*arcsinh(c*
x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/4*d*erfi((a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/16*e*erfi((a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+d*x*(a+b*arcsinh(c*x))^(1/2)+1/3*
e*x^3*(a+b*arcsinh(c*x))^(1/2)
```

### 3.630.2 Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.99

$$\int (d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{de^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c} + \frac{ee^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{3}{2},-\right.\right.}{72c^3}$$

input
```
Integrate[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]]
)/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcSinh[c*x]]*(9
*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]
] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/
b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*
x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (
3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x
])^2/b^2)])
```

### 3.630.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(d\sqrt{a+b\operatorname{arcsinh}(cx)}+ex^2\sqrt{a+b\operatorname{arcsinh}(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}\sqrt{b}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}+dx\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\operatorname{arcsinh}(cx)}$$

input
```
Int[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
d*x*Sqrt[a + b*ArcSinh[c*x]] + (e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (Sqrt[
b]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt
[b]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (
Sqrt[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[
b]])/(4*c*E^(a/b)) + (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqr
t[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

### 3.630.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.630.4 Maple [F]

$$\int \left(e\,x^{2}+d\right)\sqrt{a+b\,\operatorname{arcsinh}\left(cx\right)}dx$$

input
```
int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

## 3.630.5 Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2)\,\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.630.6 Sympy [F]

$$\int (d+ex^2)\,\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\,\text{asinh}\,(cx)}\,(d+ex^2)\;dx$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2), x)
```

### 3.630.7 Maxima [F]

$$\int (d+ex^2)\,\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int (ex^2+d)\,\sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

### 3.630.8 Giac [F]

$$\int (d+ex^2)\,\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int (ex^2+d)\,\sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

### 3.630.9 Mupad [F(-1)]

Timed out.

$$\int (d + ex^2) \sqrt{a + b\text{arcsinh}(cx)}\, dx = \int \sqrt{a + b\,\text{asinh}\,(c\,x)}\, (e\,x^2 + d)\, dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2), x)
```

# 3.631 $\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx$



## 3.631.1 Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = x\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

output
```
1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*
erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*
arcsinh(c*x))^(1/2)
```

## 3.631.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}}\right)}{2c}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/
Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

### 3.631.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6187}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}bc\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6234}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}+x\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{26}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \; 3789$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{2c}$$

$$\Big\downarrow \; 2611$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{2c}$$

$$\Big\downarrow \; 2633$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

$$\Big\downarrow \; 2634$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
x*Sqrt[a + b*ArcSinh[c*x]] - ((I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/c
```

## 3.631. $\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$

### 3.631.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

## 3.631. $\int \sqrt{a + b\text{arcsinh}(cx)}\,dx$

### 3.631.4 Maple [F]

$$\int \sqrt{a+b\,\mathrm{arcsinh}\,(cx)}dx$$

input
```
int((a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^(1/2),x)
```

### 3.631.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\mathrm{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.631.6 Sympy [F]

$$\int \sqrt{a+b\mathrm{arcsinh}(cx)}\,dx = \int \sqrt{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x)), x)
```

### 3.631.7 Maxima [F]

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

### 3.631.8 Giac [F]

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

### 3.631.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2), x)
```

# 3.632 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx$



## 3.632.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

## 3.632.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
```

output
```
$Aborted
```

## 3.632.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2} \, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}dx$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
```

output
```
$Aborted
```

### 3.632.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.632.4 Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(cx)}}{e\,x^2+d}dx$$

input
```
int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

output
```
int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

### 3.632.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{d + ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.632.6 Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{d + ex^2}\,dx = \int \frac{\sqrt{a + b\,\text{asinh}\,(cx)}}{d + ex^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2), x)
```

### 3.632.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{d + ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.632.8 Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

### 3.632.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\,\text{asinh}\,(c\,x)}}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2), x)
```

# 3.633 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx$



## 3.633.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

## 3.633.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

## 3.633.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}dx$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

### 3.633.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.633.4 Maple [N/A] (verified)

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\,\operatorname{arcsinh}(cx)}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

### 3.633.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.633.6 Sympy [N/A]

Not integrable

Time = 14.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a+b\,\text{asinh}\,(cx)}}{(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d)**2,x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

### 3.633.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

### 3.633.8 Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

### 3.633.9 Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}\,dx=\int \frac{\sqrt{a+b\,\text{asinh}\,(c\,x)}}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2, x)
```

# 3.634 $\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx$



## 3.634.1 Optimal result

Integrand size = 20, antiderivative size = 427

$$\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx = -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} + dx(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{3b^{3/2}ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} + \frac{b^{3/2}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3} + \frac{3b^{3/2}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{3b^{3/2}ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} + \frac{b^{3/2}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}$$

output
```
d*x*(a+b*arcsinh(c*x))^(3/2)+1/3*e*x^3*(a+b*arcsinh(c*x))^(3/2)+1/288*b^(3
/2)*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/c^3+1/288*b^(3/2)*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+3/8*b^(3/2)*d*exp(a/b)*erf((a+b*arcsinh(c*x
))^(1/2)/b^(1/2))*Pi^(1/2)/c-3/32*b^(3/2)*e*exp(a/b)*erf((a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*Pi^(1/2)/c^3+3/8*b^(3/2)*d*erfi((a+b*arcsinh(c*x))^(1/2)/
b^(1/2))*Pi^(1/2)/c/exp(a/b)-3/32*b^(3/2)*e*erfi((a+b*arcsinh(c*x))^(1/2)/
b^(1/2))*Pi^(1/2)/c^3/exp(a/b)-3/2*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x)
)^(1/2)/c+1/3*b*e*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*e*x
^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

## 3.634.2 Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.80

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}\,dx = \frac{ade^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}$$
$$+\frac{aee^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{3}{2},-\frac{3(\ldots}{}\right.\right.}{72c^3\sqrt{-\ldots}}$$
$$+\frac{\sqrt{b}d\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)(\cos\ldots\right.}{8c}$$
$$+\frac{\sqrt{b}e\left(-9\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(a*d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x
]])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt
[-((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcSinh[c*x]
]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[
c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x
]))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSin
h[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/
2, (3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh
[c*x])^2/b^2)]) + (Sqrt[b]*d*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[
1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*c) + (Sqrt
[b]*e*(-9*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*
x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[
b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh
[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (
-2*a + b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh
[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-Cosh[3*
ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[3*ArcSinh[c*x]])))/(288*c^3)
```

### 3.634.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \ 6208$$

$$\int \left(d(a+b\text{arcsinh}(cx))^{3/2}+ex^2(a+b\text{arcsinh}(cx))^{3/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3bd\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{2c}-\frac{bex^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{6c}+\frac{be\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3}+dx(a+b\text{arcsinh}(cx))^{3/2}+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2}$$

input
```
Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-3*b*d*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + (b*e*Sqrt[1 +
c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 + c^2*x^2]*Sq
rt[a + b*ArcSinh[c*x]])/(6*c) + d*x*(a + b*ArcSinh[c*x])^(3/2) + (e*x^3*(a
 + b*ArcSinh[c*x])^(3/2))/3 + (3*b^(3/2)*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3) + (b^(3/2)*e*E^((3*a)/b)*Sqrt[Pi/3]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*d*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c*E^(a/b)) - (3*b^(3/2)*
e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(
3/2)*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^
3*E^((3*a)/b))
```

### 3.634.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

### 3.634.4 Maple [F]

$$\int \left(e\,x^{2}+d\right) \left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}d x$$

input
```
int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.634.5 Fricas [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right) (a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.634.6 Sympy [F]

$$\int \left(d+ex^2\right) (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int \left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}} \left(d+ex^{2}\right)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)*(d + e*x**2), x)
```

### 3.634.7 Maxima [F]

$$\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx = \int (ex^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}} dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.634.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

### 3.634.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) (a+b\text{arcsinh}(cx))^{3/2} dx = \int (a+b\,\text{asinh}(c\,x))^{3/2} (e\,x^2+d)\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)*(d + e*x^2), x)
```

# 3.635 $\int (a+b\text{arcsinh}(cx))^{3/2}\,dx$



## 3.635.1 Optimal result

Integrand size = 12, antiderivative size = 135

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}$$

output
```
x*(a+b*arcsinh(c*x))^(3/2)+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*
Pi^(1/2)/c/exp(a/b)-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

## 3.635.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.86

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)(\cosh}{8c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]]
)/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b
*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (
-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Si
nh[a/b])))/(8*c)
```

## 3.635.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{b\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}\right)$$

$$\Big\downarrow \text{6189}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{x(a+b\text{arcsinh}(cx))^{3/2}-\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$\downarrow$ 3788

$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{x(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}i\int-\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int\frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+bar$$

$\downarrow$ 26

$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{x(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int\frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsi}$$

$\downarrow$ 2611

$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{x(a+b\text{arcsinh}(cx))^{3/2}-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}}{2c^2}$$

$\downarrow$ 2633

$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{x(a+b\text{arcsinh}(cx))^{3/2}-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2c^2}$$

$\downarrow$ 2634

$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{x(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2c^2}\right)$$

---

3.635. $\int(a+b\text{arcsinh}(cx))^{3/2}\,dx$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
x*(a + b*ArcSinh[c*x])^(3/2) - (3*b*c*((Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSi
nh[c*x]])/c^2 - ((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E
^(a/b)))/(2*c^2)))/2
```

### 3.635.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

### 3.635.4 Maple [F]

$$\int (a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}} \, dx$$

input
```
int((a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^(3/2),x)
```

### 3.635.5 Fricas [F(-2)]

Exception generated.

$$\int (a + b\text{arcsinh}(cx))^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.635.6 Sympy [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2), x)
```

### 3.635.7 Maxima [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.635.8 Giac [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.635.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\operatorname{arcsinh}(cx))^{3/2} \, dx = \int (a + b \operatorname{asinh}(c\, x))^{3/2} \, dx$$

input
```
int((a + b*asinh(c*x))^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2), x)
```

# 3.636 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} dx$



## 3.636.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}, x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

## 3.636.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

output
```
$Aborted
```

## 3.636.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} \, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

output
```
$Aborted
```

### 3.636.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.636.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b \operatorname{arcsinh}(cx))^{\frac{3}{2}}}{e\,x^2+d}dx$$

input
```
int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

output
```
int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

### 3.636.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.636.6 Sympy [N/A]

Not integrable

Time = 9.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}{d+ex^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2), x)
```

### 3.636.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.636.8 Giac [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

### 3.636.9 Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2), x)
```

# 3.637 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx$



## 3.637.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

## 3.637.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx=\text{\$Aborted}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

## 3.637.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} \, dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

### 3.637.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.637.4 Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

### 3.637.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + ex^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.637.6 Sympy [N/A]

Not integrable

Time = 95.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + ex^2)^2} \, dx = \int \frac{(a + b \operatorname{asinh}(cx))^{\frac{3}{2}}}{(d + ex^2)^2} \, dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2)**2, x)
```

### 3.637.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + ex^2)^2} \, dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}{(ex^2 + d)^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

### 3.637.8 Giac [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

### 3.637.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2, x)
```

# 3.638 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$

## 3.638.1 Optimal result

Integrand size = 22, antiderivative size = 608

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \frac{d^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} - \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}$$

output
```
1/160*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)
*Pi^(1/2)/c^5/b^(1/2)+1/160*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)/b^(1/2)+1/12*d*e*exp(3*a/b)*erf(3^(1/
2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/b^(1/2)+1/12*d*e
*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3
*a/b)/b^(1/2)+1/2*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1
/2)/c/b^(1/2)-1/4*d*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1
/2)/c^3/b^(1/2)+1/16*e^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c^5/b^(1/2)+1/2*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)
/c/exp(a/b)/b^(1/2)-1/4*d*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2
)/c^3/exp(a/b)/b^(1/2)+1/16*e^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^
(1/2)/c^5/exp(a/b)/b^(1/2)-1/32*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*
x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^5/b^(1/2)-1/32*e^2*erfi(3^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)/b^(1/2)
```

### 3.638.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{5a}{b}}\left(-30(8c^4d^2-4c^2de+e^2)\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+3\sqrt{5}e^2\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\left(\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-30*(8*c^4*d^2 - 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Ga
mma[1/2, a/b + ArcSinh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcSinh[c*x])/b
)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b
)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b]
- 15*Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-
3*(a + b*ArcSinh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh
[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - 120*c^2*d*e*E^((4*a)/b)
*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 3
0*e^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcS
inh[c*x])/b)] - 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Ga
mma[1/2, (3*(a + b*ArcSinh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*e^2*E^
((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]
)/(480*c^5*E^((5*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.638.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(\frac{d^2}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2dex^2}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\text{arcsinh}(cx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{5}}e^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}-\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{5}}e^2e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}-\frac{\sqrt{\pi}dee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\pi}d^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

input
```
Int[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
 - (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]
*c^3) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*S
qrt[b]*c^5) + (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3
]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (e^2*E^((5*a)/b)*
Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^
5) + (d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^
(a/b)) - (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*
c^3*E^(a/b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*S
qrt[b]*c^5*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(4*Sqrt[b]*c^3*E^((3*a)/b)) - (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sq
rt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5
*E^((5*a)/b))
```

## 3.638. $\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$

### 3.638.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

## 3.638.4 Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{2}}{\sqrt{a+b\,\operatorname{arcsinh}\left(c x\right)}}dx$$

input
```
int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

## 3.638.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.638.6 Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{(d+ex^2)^2}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral((d + e*x**2)**2/sqrt(a + b*asinh(c*x)), x)
```

### 3.638.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{(ex^2+d)^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

### 3.638.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{(ex^2+d)^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

### 3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2), x)
```

# 3.639 $\int \frac{d+ex^2}{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$



## 3.639.1 Optimal result

Integrand size = 20, antiderivative size = 287

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx = {} & \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} \\ & + \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} \\ & + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} \\ & - \frac{ee^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} \\ & + \frac{ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}\end{aligned}$$

output
```
1/24*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi
^(1/2)/c^3/b^(1/2)+1/24*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)+1/2*d*exp(a/b)*erf((a+b*arcsinh(c*x
))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)-1/8*e*exp(a/b)*erf((a+b*arcsinh(c*x))
^(1/2)/b^(1/2))*Pi^(1/2)/c^3/b^(1/2)+1/2*d*erfi((a+b*arcsinh(c*x))^(1/2)/b
^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)-1/8*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^
(1/2))*Pi^(1/2)/c^3/exp(a/b)/b^(1/2)
```

## 3.639.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{3a}{b}}\left(-3(4c^2d-e)\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}e\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\text{arcsi}}{b}\right.\right.}{24c^3}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-3*(4*c^2*d - e)*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + Ar
cSinh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcSinh[c*x]))/b] + 3*(4*c^2*d - e)*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[
c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*e*E^((6*a)/b)*Sq
rt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^(
(3*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

## 3.639.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\pi}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

input
```
Int[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
 (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3
) + (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/(2*Sqrt[b]*c*E^(a/b)) - (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]
])/(8*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSin
h[c*x]])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

### 3.639.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

### 3.639.4 Maple [F]

$$\int \frac{e\,x^{2}+d}{\sqrt{a+b\,\operatorname{arcsinh}\left(c x\right)}}d x$$

input
```
int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

### 3.639.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.639.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{d+ex^2}{\sqrt{a+b\,\operatorname{asinh}\left(cx\right)}}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt(a + b*asinh(c*x)), x)
```

### 3.639.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{ex^2+d}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

### 3.639.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{ex^2+d}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

### 3.639.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{e\,x^2+d}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x))^(1/2), x)
```

# 3.640 $\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$



## 3.640.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

output
```
1/2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*
erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

## 3.640.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx =\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{2c\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) +
Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*
c*E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

## 3.640.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}}{bc}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

input
```
Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqr
t[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E^(a/b)))/(b*c)
```

### 3.640.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.640. $\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

### 3.640.4 Maple [F]

$$\int \frac{1}{\sqrt{a + b \operatorname{arcsinh}(cx)}} dx$$

input
```
int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

### 3.640.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\operatorname{arcsinh}(cx)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.640.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\operatorname{asinh}(cx)}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*asinh(c*x)), x)
```

### 3.640.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}(cx)+a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

### 3.640.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}(cx)+a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

### 3.640.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}} \,dx$$

input
```
int(1/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(1/2), x)
```

# 3.641 $\int \frac{1}{(d+ex^2)\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$



## 3.641.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx=\mathrm{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\,\mathrm{arcsinh}(cx)}},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

## 3.641.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx=\int \frac{1}{(d+ex^2)\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

## 3.641.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$
$$\Big\downarrow \; 6209$$
$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

### 3.641.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.641.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(e\,x^{2}+d\right)\sqrt{a+b\,\text{arcsinh}\left(c x\right)}}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

### 3.641.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.641.6 Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(cx)}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)), x)
```

### 3.641.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

### 3.641.8 Giac [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

### 3.641.9 Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)), x)
```

# 3.642 $\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx$



## 3.642.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

## 3.642.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

## 3.642.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

### 3.642.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.642.4 Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^2 \sqrt{a+b\,\operatorname{arcsinh}\,(cx)}}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

### 3.642.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.642.6 Sympy [N/A]

Not integrable

Time = 41.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\operatorname{asinh}(cx)}\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

### 3.642.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)^2\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

### 3.642.8 Giac [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)^2 \sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

### 3.642.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2), x)
```

# 3.643 $\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.643.1 Optimal result

Integrand size = 20, antiderivative size = 349

$$\begin{aligned}\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}\\ &-\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{ee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}\\ &-\frac{ee^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}\\ &-\frac{ee^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{ee^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}\end{aligned}$$

output
```
-d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+1/4*e
*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+d*erf
i((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*e*erfi
((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1/4*e*exp
(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(
3/2)/c^3+1/4*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c^3/exp(3*a/b)-2*d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(
1/2)-2*e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

### 3.643.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.87

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(\left(4c^2d-e\right)e^{\frac{4a}{b}+3\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
((4*c^2*d - e)*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*e*E^(3*ArcSinh[c*x])*Sqrt[-((a + b*Arc
Sinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + (4*c^2*d - e)*E^(
(2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a
+ b*ArcSinh[c*x])/b)] + E^((3*a)/b)*(-((1 + E^(2*ArcSinh[c*x]))*(4*c^2*d*E
^(2*ArcSinh[c*x]) + e*(-1 + E^(2*ArcSinh[c*x]))^2)) + Sqrt[3]*e*E^(3*(a/b
+ ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x
]))/b]))/(4*b*c^3*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.643.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d}{(a+b\text{arcsinh}(cx))^{3/2}}+\frac{ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}ee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}ee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}de^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}de^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2d\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2ex^2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (2*e*x^2*Sqrt[1
+ c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
 + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (e*E^((3*a)/b)*Sqrt[3*Pi]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) + (d*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b)) - (e*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b)) + (e*S
qrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3
*E^((3*a)/b))
```

### 3.643.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

3.643. $\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

### 3.643.4 Maple [F]

$$\int \frac{e\,x^2+d}{(a+b\,\operatorname{arcsinh}(cx))^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.643.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.643.6 Sympy [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{d+ex^2}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((d + e*x**2)/(a + b*asinh(c*x))**(3/2), x)
```

### 3.643.7 Maxima [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.643.8 Giac [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

### 3.643.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{e\,x^2+d}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

# 3.644 $\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.644.1 Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=-\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

output
```
-exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c^2*x^2+1)^
(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

## 3.644.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\frac{e^{-\frac{a+b\text{arcsinh}(cx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(cx)}\right)+e^{\frac{2a}{b}+\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\right.\right.}{bc\sqrt{a+b\text{arcs}}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output
```
(-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a
 + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a +
 b*ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

### 3.644.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6188}$$

$$\frac{2c\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int\frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + ((2*I)*((I/2)*Sqrt
[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b
]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/(b^2*c)
```

3.644. $\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

### 3.644.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

## 3.644. $\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

### 3.644.4 Maple [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.644.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.644.6 Sympy [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(-3/2), x)
```

### 3.644.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

### 3.644.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

### 3.644.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(c\,x))^{3/2}}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(3/2), x)
```

# 3.645 $\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.645.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

## 3.645.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

## 3.645.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

### 3.645.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.645.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.645.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.645.6 Sympy [N/A]

Not integrable

Time = 7.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)), x)
```

### 3.645.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

### 3.645.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

### 3.645.9 Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)), x)
```

# 3.646 $\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx$



## 3.646.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

## 3.646.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

## 3.646.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}} \, dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

### 3.646.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.646.4 Maple [N/A] (verified)

Not integrable

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^2 \left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

### 3.646.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.646.6 Sympy [N/A]

Not integrable

Time = 139.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^{3/2}} dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}} (d+ex^2)^2} dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)**2), x)
```

### 3.646.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\operatorname{arcsinh}(cx))^{3/2}} dx = \int \frac{1}{(ex^2+d)^2 (b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}} dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

### 3.646.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

### 3.646.9 Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2), x)
```

# 3.647 $\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$



## 3.647.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\text{arcsinh}(cx)),x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

## 3.647.2 Mathematica [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

## 3.647.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

### 3.647.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.647.4 Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\,\text{arcsinh}\,(cx))\,\sqrt{e\,x^2+d}dx$$

input
```
int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

### 3.647.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

### 3.647.6 Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*sqrt(d + e*x**2), x)
```

### 3.647.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.647.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

### 3.647.9 Mupad [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2)^(1/2), x)
```

# 3.648 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$



## 3.648.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

## 3.648.2 Mathematica [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

## 3.648.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{6209}$$

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

### 3.648.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.648.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\text{arcsinh}\left(cx\right)}{\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

### 3.648.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

### 3.648.6 Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\operatorname{asinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/sqrt(d + e*x**2), x)
```

### 3.648.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.648.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

### 3.648.9 Mupad [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(1/2), x)
```

# 3.649 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx$



## 3.649.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

output
```
-b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d/e^(1/2)+x*(a+b*a
rcsinh(c*x))/d/(e*x^2+d)^(1/2)
```

## 3.649.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x\left(-bcx\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{ex^2}{d}\right)+2(a+b\text{arcsinh}(cx))\right)}{2d\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(-(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*
x^2)/d)]) + 2*(a + b*ArcSinh[c*x])))/(2*d*Sqrt[d + e*x^2])
```

### 3.649.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6207, 27, 353, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{6207}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-bc\int \frac{x}{d\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx^2}{2d}$$

$$\Big\downarrow \text{66}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2+1}}{\sqrt{ex^2+d}}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTanh[(Sqrt[e]*Sqrt[1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

### 3.649.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

## 3.649.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\left(cx\right)}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

### 3.649.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.66

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{3/2}}\,dx = \left[\frac{4\sqrt{ex^2 + d}bex\log\left(cx + \sqrt{c^2x^2 + 1}\right) + 4\sqrt{ex^2 + d}aex + (bex^2 + bd)\sqrt{e}\log\left(8c^4\right.}{4}\right.$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(e*x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 4*sqrt(e*x^2
+ d)*a*e*x + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d
*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^
2 + 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2))/(d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(e*
x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(e*x^2 + d)*a*e*x + (b
*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 +
 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)
))/(d*e^2*x^2 + d^2*e)]
```

### 3.649.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2)**(3/2), x)
```

### 3.649.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

### 3.649.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

### 3.649.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(3/2), x)
```

# 3.650 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx$



## 3.650.1 Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=-\frac{bc\sqrt{1+c^2x^2}}{3d(c^2d-e)\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{2b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}$$

output
```
1/3*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(3/2)-2/3*b*arctanh(e^(1/2)*(c^2*x^2+
1)^(1/2)/c/(e*x^2+d)^(1/2))/d^2/e^(1/2)+2/3*x*(a+b*arcsinh(c*x))/d^2/(e*x^
2+d)^(1/2)-1/3*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(1/2)
```

## 3.650.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\frac{-\frac{bcd\sqrt{1+c^2x^2}(d+ex^2)}{c^2d-e}+ax(3d+2ex^2)-bcx^2(d+ex^2)\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2\right.}{3d^2(d+ex^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(-((b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^2))/(c^2*d - e)) + a*x*(3*d + 2*e*x^2
) - b*c*x^2*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2
*x^2), -((e*x^2)/d)] + b*x*(3*d + 2*e*x^2)*ArcSinh[c*x])/(3*d^2*(d + e*x^2
)^(3/2))
```

## 3.650.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6207, 27, 435, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{6207}$$

$$-bc\int \frac{x(2ex^2+3d)}{3d^2\sqrt{c^2x^2+1}(ex^2+d)^{3/2}}dx+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x(2ex^2+3d)}{\sqrt{c^2x^2+1}(ex^2+d)^{3/2}}dx}{3d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{bc\int \frac{2ex^2+3d}{\sqrt{c^2x^2+1}(ex^2+d)^{3/2}}dx^2}{6d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{bc\left(2\int \frac{1}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx^2+\frac{2d\sqrt{c^2x^2+1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$-\frac{bc\left(4\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2+1}}{\sqrt{ex^2+d}}+\frac{2d\sqrt{c^2x^2+1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2x(a+b\operatorname{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\left(\frac{4\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}+\frac{2d\sqrt{c^2x^2+1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x
]))/(3*d^2*Sqrt[d + e*x^2]) - (b*c*((2*d*Sqrt[1 + c^2*x^2])/((c^2*d - e)*S
qrt[d + e*x^2]) + (4*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2
])])/(c*Sqrt[e])))/(6*d^2)
```

### 3.650.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

3.650. $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx$

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

### 3.650.4 Maple [F]

$$\int \frac{a + b\ \mathrm{arcsinh}\left(cx\right)}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

### 3.650.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(122) = 244$.

Time = 0.32 (sec) , antiderivative size = 738, normalized size of antiderivative = 5.05

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(bc^2d^3 + (bc^2de^2 - be^3)x^4 - bd^2e + 2\,(bc^2d^2e - bde^2)x^2)\sqrt{e}\log\left(8\,c^4e^2x^4 + c^4d^2 + \right.}{}\right.$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e -
b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e
 + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 2*(2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3*(b*c^2*d^2*e -
 b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(2*(a*c^2*d*
e^2 - a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2 + b*c*d^2*
e)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3
 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2), 1/3*((b*c^2*d^3 + (b*c^2
*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-e)*ar
ctan(1/2*(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(
-e)/(c^3*e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)) + (2*(b*c^2*d*e^2 - b*e
^3)*x^3 + 3*(b*c^2*d^2*e - b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*
x^2 + 1)) + (2*(a*c^2*d*e^2 - a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (
b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e
- d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2)]
```

### 3.650.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2)**(5/2), x)
```

### 3.650.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

### 3.650.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

### 3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(5/2), x)
```

# 3.651 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx$



## 3.651.1 Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{15d(c^2d-e)(d+ex^2)^{3/2}} - \frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2(c^2d-e)^2\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{8b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

output
```
1/5*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arcsinh(c*x))/d^2/(
e*x^2+d)^(3/2)-8/15*b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))
/d^3/e^(1/2)-1/15*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(3/2)+8/15*x
*(a+b*arcsinh(c*x))/d^3/(e*x^2+d)^(1/2)-2/15*b*c*(3*c^2*d-2*e)*(c^2*x^2+1)
^(1/2)/d^2/(c^2*d-e)^2/(e*x^2+d)^(1/2)
```

### 3.651.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{ax(15d^2+20dex^2+8e^2x^4)-\frac{bcd\sqrt{1+c^2x^2}(d+ex^2)(-e(5d+4ex^2)+c^2d(7d+6ex^2))}{(-c^2d+e)^2}-4bcx^2(d+\cdots}{15\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x
^2)*(-(e*(5*d + 4*e*x^2)) + c^2*d*(7*d + 6*e*x^2)))/(-(c^2*d) + e)^2 - 4*b
*c*x^2*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^
2), -((e*x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcSinh[c*x])/(1
5*d^3*(d + e*x^2)^(5/2))
```

### 3.651.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6207, 27, 7266, 1193, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 6207}$$

$$-bc\int \frac{x(8e^2x^4+20dex^2+15d^2)}{15d^3\sqrt{c^2x^2+1}\,(ex^2+d)^{5/2}}dx+\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bc\int \frac{x(8e^2x^4+20dex^2+15d^2)}{\sqrt{c^2x^2+1}(ex^2+d)^{5/2}}dx}{15d^3}+\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{ 7266}$$

$$-\frac{bc\int \frac{8e^2x^4+20dex^2+15d^2}{\sqrt{c^2x^2+1}(ex^2+d)^{5/2}}dx^2}{30d^3}+\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{1193}$$

$$-\frac{bc\left(\frac{2\int \frac{3\left(4\left(c^2d-e\right)ex^2+d\left(7c^2d-6e\right)\right)}{\sqrt{c^2x^2+1}\left(ex^2+d\right)^{3/2}}dx^2}{3(c^2d-e)}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\left(\frac{2\int \frac{4\left(c^2d-e\right)ex^2+d\left(7c^2d-6e\right)}{\sqrt{c^2x^2+1}\left(ex^2+d\right)^{3/2}}dx^2}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{bc\left(\frac{2\left(4(c^2d-e)\int \frac{1}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx^2+\frac{2d\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}}\right)}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{66}$$

$$-\frac{bc\left(\frac{2\left(8(c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2+1}}{\sqrt{ex^2+d}}+\frac{2d\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}}\right)}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{221}$$

3.651. $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx$

$$\frac{8x(a+b\operatorname{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\operatorname{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{5d(d+ex^2)^{5/2}}-\frac{bc\left(\frac{2\left(\frac{8(c^2d-e)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}+\frac{2d\sqrt{c^2x^2+1}(3c^2d-2e)}{(c^2d-e)\sqrt{d+ex^2}}\right)}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcSinh[c*x
]))/(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcSinh[c*x]))/(15*d^3*Sqrt[d
 + e*x^2]) - (b*c*((2*d^2*Sqrt[1 + c^2*x^2])/((c^2*d - e)*(d + e*x^2)^(3/2
)) + (2*((2*d*(3*c^2*d - 2*e)*Sqrt[1 + c^2*x^2])/((c^2*d - e)*Sqrt[d + e*x
^2]) + (8*(c^2*d - e)*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^
2])])/(c*Sqrt[e])))/(c^2*d - e)))/(30*d^3)
```

### 3.651.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.651. $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx$

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

## 3.651.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\left(c x\right)}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}}d x$$

input
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

output
```
int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

## 3.651.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(193) = 386$.

Time = 0.39 (sec) , antiderivative size = 1354, normalized size of antiderivative = 5.96

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
[1/15*(2*(b*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e
^5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 +
3*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*log(8*c^4*e^2*x
^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^
3*d + c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + (8*(b*c^4*d^
2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 +
 b*d*e^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x
^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 +
 a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c
^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2
 + 2*(3*b*c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e
^3)*x^2)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^4*d^8*e - 2*c^2*d^7*e^2 +
d^6*e^3 + (c^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2
*c^2*d^5*e^4 + d^4*e^5)*x^4 + 3*(c^4*d^7*e^2 - 2*c^2*d^6*e^3 + d^5*e^4)*x^
2), 1/15*(4*(b*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 +
b*e^5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4
 + 3*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*arctan(1/2*
(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*
e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)) + (8*(b*c^4*d^2*e^3 - 2*b*c^2*d*
e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^3 +...
```

### 3.651.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d + ex^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(7/2),x)
```

output
```
Timed out
```

### 3.651.7 Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{7/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(ex^2 + d)^{\frac{7}{2}}} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*
x^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^
(7/2), x)
```

### 3.651.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{7/2}} \, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(ex^2 + d)^{\frac{7}{2}}} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

### 3.651.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{7/2}} \, dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{(e\,x^2 + d)^{7/2}} \, dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(7/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(7/2), x)
```

# 3.652 $\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$



## 3.652.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2,x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

## 3.652.2 Mathematica [N/A]

Not integrable

Time = 13.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

## 3.652.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2 \, dx$$
$$\Big\downarrow \text{6209}$$
$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

### 3.652.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.652.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(a+b\,\text{arcsinh}\left(cx\right)\right)^{2} \sqrt{e\,x^{2}+d}dx$$

input
```
int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

### 3.652.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d),
x)
```

### 3.652.6 Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}\,(cx))^2\,\sqrt{d+ex^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2*sqrt(d + e*x**2), x)
```

### 3.652.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.652.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2, x)
```

### 3.652.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2), x)
```

# 3.653 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx$



## 3.653.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

## 3.653.2 Mathematica [N/A]

Not integrable

Time = 8.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

## 3.653.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

### 3.653.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.653.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}{\sqrt{e \,x^{2}+d}}d x$$

input
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

### 3.653.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(e*x^2 + d),
x)
```

### 3.653.6 Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \int \frac{(a + b\,\text{asinh}\,(cx))^2}{\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/sqrt(d + e*x**2), x)
```

### 3.653.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

### 3.653.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

### 3.653.9 Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2), x)
```

# 3.654 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$



## 3.654.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

## 3.654.2 Mathematica [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

## 3.654.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

### 3.654.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.654.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

### 3.654.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(
e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

### 3.654.6 Sympy [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(3/2), x)
```

### 3.654.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

### 3.654.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

### 3.654.9 Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2), x)
```

# 3.655 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx$



## 3.655.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

## 3.655.2 Mathematica [N/A]

Not integrable

Time = 5.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

## 3.655.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{6209}$$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

### 3.655.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.655.4 Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

### 3.655.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(
e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

### 3.655.6 Sympy [N/A]

Not integrable

Time = 59.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(5/2), x)
```

### 3.655.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.50

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(
b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x^2 + d)^(5/2) + 2*a*b*log(c*x + sqr
t(c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

### 3.655.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

### 3.655.9 Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2), x)
```

# 3.656 $\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$



## 3.656.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)},x\right)$$

output
```
Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

## 3.656.2 Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

## 3.656.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

### 3.656.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.656.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\,\text{arcsinh}\,(cx)}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

### 3.656.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

### 3.656.6 Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x)), x)
```

### 3.656.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

### 3.656.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

### 3.656.9 Mupad [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{e\,x^2+d}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)), x)
```

# 3.657 $\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.657.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

## 3.657.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

## 3.657.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.657.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.657.4 Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(cx))\,\sqrt{e\,x^2+d}}dx$$

input
```
int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

### 3.657.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x
)
```

### 3.657.6 Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*sqrt(d + e*x**2)), x)
```

### 3.657.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

## 3.657.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

## 3.657.9 Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)), x)
```

# 3.658 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.658.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

## 3.658.2 Mathematica [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

## 3.658.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

### 3.658.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.658.4 Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

### 3.658.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2
*b*d*e*x^2 + b*d^2)*arcsinh(c*x)), x)
```

### 3.658.6 Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(3/2)), x)
```

### 3.658.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.658.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.658.9 Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)), x)
```

# 3.659 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx$



## 3.659.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

## 3.659.2 Mathematica [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

### 3.659.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

#### 3.659.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

### 3.659.4 Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

### 3.659.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.95

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^
3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsinh(c*x)), x)
```

### 3.659.6 Sympy [N/A]

Not integrable

Time = 7.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(5/2)), x)
```

### 3.659.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.659.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

### 3.659.9 Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)), x)
```

# 3.660 $\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.660.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.660.2 Mathematica [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

## 3.660.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

### 3.660.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.660.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}}{\left(a +b \,\operatorname{arcsinh}\left(c x \right)\right)^{2}}d x$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.660.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{ex^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2),
x)
```

### 3.660.6 Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{d+ex^2}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

### 3.660.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 575, normalized size of antiderivative = 26.14

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{ex^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 + 1)^(3/2)*sqrt(e*x^2 + d) + (c^3*x^3 + c*x)*sqrt(e*x^2 + d))/(
a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^
2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((
2*c^3*e*x^4 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (4*c^4*e*x^
5 + 2*(c^4*d + 2*c^2*e)*x^3 + (c^2*d + e)*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2
+ d) + (2*c^5*e*x^6 + (c^5*d + 4*c^3*e)*x^4 + 2*(c^3*d + c*e)*x^2 + c*d)*s
qrt(e*x^2 + d))/(a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*
e)*a*b*x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^
2*c^5*e*x^6 + (c^5*d + 2*c^3*e)*b^2*x^4 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*
d + (b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^5 + b^2
*c^2*d*x + (c^4*d + c^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) + 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt
(c^2*x^2 + 1)), x)
```

## 3.660.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{ex^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

## 3.660.9 Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{e\,x^2+d}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2, x)
```

# 3.661 $\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2} dx$



## 3.661.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2} dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}, x\right)$$

output
```
Unintegrable(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

## 3.661.2 Mathematica [N/A]

Not integrable

Time = 6.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2} dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

## 3.661.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.661.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.661.4 Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\,\text{arcsinh}\left(cx\right)\right)^{2}\sqrt{e\,x^{2}+d}}dx$$

input
```
int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

### 3.661.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}\left(b\operatorname{arsinh}(cx)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(
c*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

### 3.661.6 Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\operatorname{asinh}(cx)\right)^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*sqrt(d + e*x**2)), x)
```

### 3.661.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 591, normalized size of antiderivative = 26.86

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}\left(b\operatorname{arsinh}(cx)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*
a*b*c^2*x + (sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*b^2*c^2*x + (b^2*c^3*x^2 +
b^2*c)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*
c)*sqrt(e*x^2 + d)) + integrate((c^5*d*x^4 + 2*c^3*d*x^2 + (c^2*x^2 + 1)*(
(c^3*d - 2*c*e)*x^2 - c*d) + c*d + sqrt(c^2*x^2 + 1)*(2*(c^4*d - c^2*e)*x^
3 + (c^2*d - e)*x))/((a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt(e*
x^2 + d) + 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(
c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 +
 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x + (c^4*d + c^2*e)*b^2
*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e*x^6 + (c^5*d + 2*c^3*
e)*b^2*x^4 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sqrt(e*x^2 + d))*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*
d + c*e)*a*b*x^2 + a*b*c*d)*sqrt(e*x^2 + d)), x)
```

## 3.661.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

## 3.661.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)), x)
```

# 3.662 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$



## 3.662.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.662.2 Mathematica [N/A]

Not integrable

Time = 13.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.662.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.662.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.662.4 Maple [N/A] (verified)

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arcsinh}(cx))^2}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.662.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2
*x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*
e*x^2 + a*b*d^2)*arcsinh(c*x)), x)
```

### 3.662.6 Sympy [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2 (d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

### 3.662.7 Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 864, normalized size of antiderivative = 39.27

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt
(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^
2 + 1)*sqrt(e*x^2 + d) + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d)
*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e*x^4 + (c^3*d +
 c*e)*a*b*x^2 + a*b*c*d)*sqrt(e*x^2 + d)) - integrate((2*c^5*e*x^6 - (c^5*
d - 4*c^3*e)*x^4 - 2*(c^3*d - c*e)*x^2 + (2*c^3*e*x^4 - (c^3*d - 4*c*e)*x^
2 + c*d)*(c^2*x^2 + 1) - c*d + (4*c^4*e*x^5 - 2*(c^4*d - 4*c^2*e)*x^3 - (c
^2*d - 3*e)*x)*sqrt(c^2*x^2 + 1))/((a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 +
a*b*c^3*d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^2*x^7 + (2*c
^4*d*e + c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*a*b*x^3)
*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e^2*x^6 + 2*b^2*c^3*d*e*x^4
 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^2*x^7 + (
2*c^4*d*e + c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*b^2*x
^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^
3*e^2)*b^2*x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^
3*d^2 + c*d*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a
*b*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*a*b*x^6 + (c^5*d^2 + 4*c^3*d*e + c*
e^2)*a*b*x^4 + a*b*c*d^2 + 2*(c^3*d^2 + c*d*e)*a*b*x^2)*sqrt(e*x^2 + d)),
x)
```

### 3.662.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.662.9 Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)), x)
```

# 3.663 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\mathbf{arcsinh}(cx))^2}\,dx$



## 3.663.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

## 3.663.2 Mathematica [N/A]

Not integrable

Time = 22.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

## 3.663.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2} (a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^{5/2} (a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

### 3.663.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

## 3.663.4 Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b\,\operatorname{arcsinh}\left(c x\right)\right)^{2}}d x$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

### 3.663.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 6.77

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2
+ a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*ar
csinh(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^
3)*arcsinh(c*x)), x)
```

### 3.663.6 Sympy [N/A]

Not integrable

Time = 27.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(5/2)), x)
```

### 3.663.7 Maxima [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 1123, normalized size of antiderivative = 51.05

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x
^3 + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e^2*x^5
+ 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (
b^2*c^3*e^2*x^6 + (2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c
*d*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e^
2*x^6 + (2*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*
x^2)*sqrt(e*x^2 + d)) - integrate((4*c^5*e*x^6 - (c^5*d - 8*c^3*e)*x^4 - 2
*(c^3*d - 2*c*e)*x^2 + (4*c^3*e*x^4 - (c^3*d - 6*c*e)*x^2 + c*d)*(c^2*x^2
+ 1) - c*d + (8*c^4*e*x^5 - 2*(c^4*d - 7*c^2*e)*x^3 - (c^2*d - 5*e)*x)*sqr
t(c^2*x^2 + 1))/((a*b*c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*
x^4 + a*b*c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^3*x^9
+ (3*c^4*d*e^2 + c^2*e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e
^2)*a*b*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^
2 + d) + ((b^2*c^3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b
^2*c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^3*x^9 + (3*c^
4*d*e^2 + c^2*e^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*b^2
*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)
+ (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e + 6
*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4
+ b^2*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x...
```

### 3.663.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

### 3.663.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)), x)
```

CHAPTER 4

# Appendix



## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul):  #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func):  #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func):  #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args)))  #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

### 4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```